[Digital Signal Processing]

Homework 1

1.

For each of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, and (4) time invariant.

(a) $T(x[n]) = (\cos \pi n)x[n]$
(b) $T(x[n]) = x[n^2]$
(c) $T(x[n]) = x[n] \sum_{k=0}^{\infty} \delta[n-k]$
(d) $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$.

2.

An LTI system has impulse response defined by

$$h[n] = \begin{cases} 0 & n < 0 \\ 1 & n = 0, 1, 2, 3 \\ -2 & n = 4, 5 \\ 0 & n > 5 \end{cases}$$

Determine and plot the output $y[n]$ when the input $x[n]$ is:

(a) $u[n]$
(b) $u[n-4]$
(c) $u[n] - u[n-4]$.

3.

A sequence has the DTFT

$$X(e^{j\omega}) = \frac{1-a^2}{(1-ae^{-j\omega})(1-ae^{j\omega})}, \qquad |a| < 1.$$

(a) Find the sequence $x[n]$.
(b) Calculate $1/2\pi \int_{-\pi}^{\pi} X(e^{j\omega}) \cos(\omega) d\omega$.

□ (a) $T\{x[n]\} = (\cos \pi n)\, x[n]$

■ Stable : O

- 이유: $x[n]$이 Bounded 되어있을 때, $|\cos \pi n| = 1$ 이므로 $T\{x[n]\}$도 Bounded 되어 있다.

■ Causal : O

- 이유: 출력이 오로지 현재값 $x[n]$에만 의존, 미래값에 의존하지 않음.

■ Linear : O

- 이유: $y_1[n] = T\{x_1[n]\} = (\cos \pi n)\, x_1[n]$, $y_2[n] = T\{x_2[n]\} = (\cos \pi n)\, x_2[n]$

$\rightarrow y[n] = T\{a x_1[n] + b x_2[n]\} = (\cos \pi n)(a x_1[n] + b x_2[n])$

$= a(\cos \pi n)\, x_1[n] + b(\cos \pi n)\, x_2[n] = a y_1[n] + b y_2[n]$

■ Time-invariant : X

- 이유: ① $x[n-n_0] \longrightarrow T\{x[n-n_0]\} = (\cos \pi n)\, x[n-n_0]$

② $y[n-n_0] = \{\cos \pi(n-n_0)\}\, x[n-n_0]$ $\therefore$ ① $\neq$ ②

(b) $T\{x[n]\} = x[n^2]$

■ Stable : O

- 이유: $x[n]$이 Bounded 되어 있다면, $x[n^2]$도 Bounded 되어 있다.

■ Causal : X

- 이유 : n이 integer라면, $n < n^2$ → 즉, 출력이 미래값에 dependent 하다.

■ Linear : O

- 이유 : $y_1[n] = T\{x_1[n]\} = x_1[n^2]$, $y_2[n] = T\{x_2[n]\} = x_2[n^2]$

$\rightarrow y[n] = T\{a x_1[n] + b x_2[n]\} = a x_1[n^2] + b x_2[n^2] = a y_1[n] + b y_2[n]$

■ Time-invariant : X

- 이유 : ① $x[n-n_0] \rightarrow T\{x[n-n_0]\} = x[n^2-n_0]$

② $y[n] = x[n^2] \rightarrow y[n-n_0] = x[(n-n_0)^2]$

(c) $T\{x[n]\} = x[n]\sum_{k=0}^{\infty}\delta[n-k]$ $= x[n]u[n]$

■ Stable : O

- 이유 : $|x[n]| \le B < \infty \rightarrow |x[n]u[n]| \le B < \infty$

■ Causal : O

- 이유 : 출력이 미래값의 영향을 받지 않는다.

■ Linear : O

- 이유 : $y_1[n] = T\{x_1[n]\} = x_1[n]u[n]$, $y_2[n] = T\{x_2[n]\} = x_2[n]u[n]$

$\rightarrow y[n] = T\{ax_1[n] + bx_2[n]\} = (ax_1[n] + bx_2[n])u[n] = ax_1[n]u[n] + bx_2[n]u[n]$

$= ay_1[n] + by_2[n]$

■ Time-invariant : X

- 이유 : ① $x[n-n_0] \rightarrow T\{x[n-n_0]\} = x[n-n_0]u[n]$

② $y[n-n_0] = x[n-n_0]u[n-n_0]$

$\therefore$ ① $\neq$ ②

(d) $T\{x[n]\} = \sum_{k=n-1}^{\infty} x[k]$

■ Stable : X

- 이유 : $|x[k]| \le B < \infty \rightarrow \left|\sum_{k=n-1}^{\infty} x[k]\right|$ 은 ∞로 발산할 수 있다.

■ Causal : X

- 이유 : $\sum_{k=n-1}^{\infty} x[k] = x[n-1] + x[n] + x[n+1] + x[n+2] + \cdots$: 미래값에 의존

■ Linear : O

- 이유 : $y_1[n] = \sum_{k=n-1}^{\infty} x_1[n]$, $y_2[n] = \sum_{k=n-1}^{\infty} x_2[n]$

$\rightarrow y[n] = T\{ax_1[n] + bx_2[n]\} = \sum_{k=n-1}^{\infty}\{ax_1[n] + bx_2[n]\} = a\sum_{k=n-1}^{\infty} x_1[n] + b\sum_{k=n-1}^{\infty} x_2[n]$

■ Time-invariant : O

- 이유 : ① $x[n-n_0] \rightarrow T\{x[n-n_0]\} = x[n-n_0-1] + x[n-n_0] + x[n-n_0+1] + \cdots = \sum_{k=n-n_0-1}^{\infty} x[k]$

② $y[n] = \sum_{k=n-1}^{\infty} x[k] \rightarrow y[n-n_0] = \sum_{k=n-n_0-1}^{\infty} x[k]$

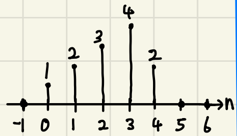
$\boxed{2}$ (a) $u[n]$

sol) $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$

$= \sum_{k=0}^{5} h[k]x[n-k]$

n	0	1	2	3	4	5	6	7 ..
$h[0]x[n]$	1	1	1	1	1	1	1	1
$h[1]x[n-1]$	0	1	1	1	1	1	1	1
$h[2]x[n-2]$	0	0	1	1	1	1	1	1
$h[3]x[n-3]$	0	0	0	1	1	1	1	1
$h[4]x[n-4]$	0	0	0	0	-2	-2	-2	-2
$h[5]x[n-5]$	0	0	0	0	0	-2	-2	-2
$y[n]$	1	2	3	4	2	0	0	0

$$\therefore y[n] = \begin{cases} n+1 & (0 \le n \le 3) \\ 2 & (n=4) \\ 0 & (\text{otherwise}) \end{cases}$$



(b) $u[n-4]$

sol) $y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$

$= \sum_{k=0}^{5} h[k]x[n-k]$

n	4	5	6	7	8	9	10	⋯
$h[0]x[n]$	1	1	1	1	1	1	1	⋯
$h[1]x[n-1]$	0	1	1	1	1	1	1	⋯
$h[2]x[n-2]$	0	0	1	1	1	1	1	⋯
$h[3]x[n-3]$	0	0	0	1	1	1	1	⋯
$h[4]x[n-4]$	0	0	0	0	-2	-2	-2	⋯
$h[5]x[n-5]$	0	0	0	0	0	-2	-2	⋯
$y[n]$	1	2	3	4	2	0	0	⋯

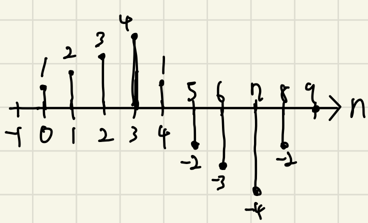
$$\therefore y[n] = \begin{cases} n-3 & (4 \le n \le 7) \\ 2 & (n=8) \\ 0 & (\text{otherwise}) \end{cases}$$

(c) $u[n] - u[n-4]$

sol) $y[n] = \sum_{k=0}^{5} h[k]x[n-k]$

n	0	1	2	3	4	5	6	7	8	9 ⋯
$h[0]x[n]$	1	1	1	1	0	0	0	0	0	0
$h[1]x[n-1]$	0	1	1	1	1	0	0	0	0	0
$h[2]x[n-2]$	0	0	1	1	1	1	0	0	0	0
$h[3]x[n-3]$	0	0	0	1	1	1	1	0	0	0
$h[4]x[n-4]$	0	0	0	0	-2	-2	-2	-2	0	0
$h[5]x[n-5]$	0	0	0	0	0	-2	-2	-2	-2	0
$y[n]$	1	2	3	4	1	-2	-3	-4	-2	0

$$\therefore y[n] = \begin{cases} n+1 & (0 \le n \le 3) \\ -3n+13 & (3 \le n \le 5) \\ -n+3 & (5 \le n \le 7) \\ 2n-18 & (7 \le n \le 9) \\ 0 & (\text{otherwise}) \end{cases}$$



3 (a) $x[n]$

sol) $X(e^{j\omega}) = \frac{1-a^2}{(1-ae^{-j\omega})(1-ae^{j\omega})}$

$= \frac{1}{1-ae^{-j\omega}} + \frac{ae^{j\omega}}{1-ae^{j\omega}}$

$\Rightarrow x[n] = a^n u[n] + a^{-n} u[-n-1]$

$= a^{|n|}$

틀림.

(b) $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) \cos\omega \, d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) \cdot \frac{e^{j\omega}+e^{-j\omega}}{2} d\omega$

$= \frac{1}{2} \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega} d\omega + \frac{1}{2} \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{-j\omega} d\omega$

■ 정의: $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$

따라서 $x[1] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega} d\omega$

$x[-1] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{-j\omega} d\omega$

따라서 $\frac{1}{2} \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega} d\omega + \frac{1}{2} \cdot \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{-j\omega} d\omega$

$= \frac{1}{2} x[1] + \frac{1}{2} x[-1]$

$= \frac{1}{2} \cdot 2a = a$

4.

An LTI discrete-time system has frequency response given by

$$H(e^{j\omega}) = \frac{(1 - je^{-j\omega})(1 + je^{-j\omega})}{1 - 0.8e^{-j\omega}} = \frac{1 + e^{-j2\omega}}{1 - 0.8e^{-j\omega}} = \frac{1}{1 - 0.8e^{-j\omega}} + \frac{e^{-j2\omega}}{1 - 0.8e^{-j\omega}}.$$

(a) Use one of the above forms of the frequency response to obtain an equation for the impulse response $h[n]$ of the system.

(b) From the frequency response, determine the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$ of the system.

(c) If the input to this system is

$$x[n] = 4 + 2\cos(\omega_0 n) \quad \text{for} - \infty < n < \infty,$$

for what value of ω_0 will the output be of the form

$$y[n] = A = \text{constant}$$

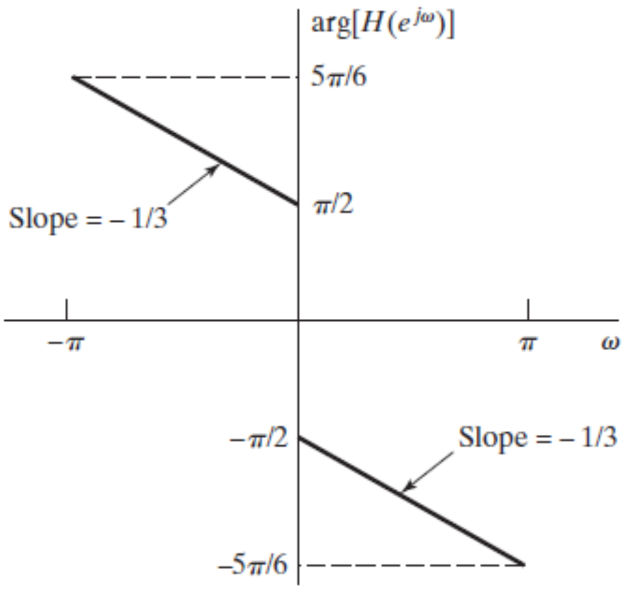
for $-\infty < n < \infty$? What is the constant A?

5.

Consider an LTI system with $|H(e^{j\omega})| = 1$, and let $\arg[H(e^{j\omega})]$ be as shown in Figure
If the input is

$$x[n] = \cos\left(\frac{3\pi}{2}n + \frac{\pi}{4}\right),$$

determine the output $y[n]$.



4

(a) $H(e^{j\omega}) = \dfrac{1}{1-0.8e^{-j\omega}} + \dfrac{e^{-j2\omega}}{1-0.8e^{-j\omega}}$

$$\Updownarrow$$

$$\boxed{h[n] = \left(\tfrac{4}{5}\right)^n u[n] + \left(\tfrac{4}{5}\right)^{n-2} u[n-2]}$$

(b) $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1+e^{-j2\omega}}{1-0.8e^{-j\omega}}$; $Y(e^{j\omega})(1-0.8e^{-j\omega}) = X(e^{j\omega})(1+e^{-j2\omega})$

$$\Updownarrow$$

$$\boxed{y[n] - 0.8y[n-1] = x[n] + x[n-2]}$$

(c) $y[n] = A$, $x[n] = 4 + e^{j\omega_0 n} + e^{-j\omega_0 n}$, $x[n-2] = 4 + e^{j\omega_0(n-2)} + e^{-j\omega_0(n-2)}$

$\Rightarrow$ (using LCCDE) $\underbrace{A - 0.8A}_{=0.2A} = 8 + e^{j\omega_0 n}\underbrace{(1+e^{-j2\omega_0})}_{=0} + e^{-j\omega_0 n}\underbrace{(1+e^{j2\omega_0})}_{=0}$

① $e^{-j2\omega_0} = -1$; $2\omega_0 = \pi \pm 2m\pi$ $(m=0,1,2,\cdots)$

② $e^{j2\omega_0} = -1$; $2\omega_0 = \pi \pm 2k\pi$ $(k=0,1,2,\cdots)$

한편: $0.2A = 8$; $A = 40$

답: $\omega_0 = \dfrac{\pi}{2} \pm m\pi$ $(m=0,1,2,\cdots)$

$A = 40$

$\boxed{5}$

sol) $H(e^{j\omega})$ 는 크기 1, 위상 변화만 일으키는 All-pass filter 이다.

따라서 $\omega = \frac{3\pi}{2}$ 일 때 $\arg[H(e^{j\omega})]$ 값을 구하자.

$$\therefore \arg[H(e^{j\omega})] = \begin{cases} -\frac{1}{3}\omega + \frac{\pi}{2} & (-\pi < \omega < 0) \\ -\frac{1}{3}\omega - \frac{\pi}{2} & (0 < \omega < \pi) \end{cases}$$

$$\therefore \arg[H(e^{j\frac{3}{2}\pi})] = \arg[H(e^{-j\frac{\pi}{2}})]$$

$$= -\frac{1}{3}\left(-\frac{\pi}{2}\right) + \frac{\pi}{2} = \frac{2}{3}\pi$$

따라서 입력에 시스템 위상이 추가되므로

- $x[n] = \mathrm{Re}\{e^{j(\frac{3\pi}{2}n + \frac{\pi}{4})}\}$

$$\boxed{\begin{aligned} \cdot\ y[n] &= \mathrm{Re}\{e^{j(\frac{3\pi}{2}n + \frac{\pi}{4})} \cdot e^{j\frac{2\pi}{3}}\} \\ &= \cos\left(\frac{3\pi}{2}n + \frac{11}{12}\pi\right) \end{aligned}}$$

6.

Consider an LTI system that is stable and for which $H(z)$, the z-transform of the impulse response, is given by

$$H(z) = \frac{3}{1 + \frac{1}{3}z^{-1}}.$$

Suppose $x[n]$, the input to the system, is a unit step sequence.

(a) Determine the output $y[n]$ by evaluating the discrete convolution of $x[n]$ and $h[n]$.

(b) Determine the output $y[n]$ by computing the inverse z-transform of $Y(z)$.

7.

For each of the following pairs of input and output z-transforms $X(z)$ and $Y(z)$, determine the ROC for the system function $H(z)$:

(a)

$$X(z) = \frac{1}{1 - \frac{3}{4}z^{-1}}, \qquad |z| > \frac{3}{4}$$

$$Y(z) = \frac{1}{1 + \frac{2}{3}z^{-1}}, \qquad |z| > \frac{2}{3}$$

(b)

$$X(z) = \frac{1}{1 + \frac{1}{3}z^{-1}}, \qquad |z| < \frac{1}{3}$$

$$Y(z) = \frac{1}{\left(1 - \frac{1}{6}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)}, \qquad \frac{1}{6} < |z| < \frac{1}{3}$$

8.

A causal LTI system has the following system function:

$$H(z) = \frac{4 + 0.25z^{-1} - 0.5z^{-2}}{(1 - 0.25z^{-1})(1 + 0.5z^{-1})}$$

(a) What is the ROC for $H(z)$?

6 (a) Discrete convolution을 이용한 $y[n]$ 구하기.

sol) ① Stable → ROC ⊇ unit circle

② ROC : $|z| > \frac{1}{3}$ ⇒ $h[n]$: right-sided signal.

$\therefore h[n] = 3 \cdot \left(-\frac{1}{3}\right)^n u[n]$

→ $y[n] = x[n] * h[n]$

$$= \sum_{k=-\infty}^{\infty} \left[3 \cdot \left(-\frac{1}{3}\right)^k u[k]\right] u[n-k]$$

$$= \sum_{k=0}^{n} 3\left(-\frac{1}{3}\right)^k = \frac{3\left\{1-\left(-\frac{1}{3}\right)^{n+1}\right\}}{1-\left(-\frac{1}{3}\right)} = \begin{cases} \frac{9}{4}\left\{1-\left(-\frac{1}{3}\right)^{n+1}\right\} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

(b) Z-역변환을 이용하여 $y[n]$ 구하기.

sol) $x[n] = u[n] \leftrightarrow X(z) = \frac{1}{1-z^{-1}}$ (ROC : $|z| > 1$)

$\therefore Y(z) = X(z)H(z)$

$$= \frac{3}{(1-z^{-1})(1+\frac{1}{3}z^{-1})}$$

$$= \frac{\frac{9}{4}}{1-z^{-1}} + \frac{\frac{3}{4}}{1+\frac{1}{3}z^{-1}}$$

$$= \frac{9}{4}\left(\frac{1}{1-z^{-1}} + \frac{1/3}{1+\frac{1}{3}z^{-1}}\right)$$

(ROC : $|z| > 1$)

⇔

$$y[n] = \frac{9}{4}\left\{u[n] + \frac{1}{3}\left(-\frac{1}{3}\right)^n u[n]\right\}$$

$$= \frac{9}{4}\left\{1-\left(-\frac{1}{3}\right)^{n+1}\right\} u[n]$$

$\boxed{7}$ (a) $X(z) = \dfrac{1}{1-\frac{3}{4}z^{-1}} \quad (|z|>\frac{3}{4})$, $Y(z) = \dfrac{1}{1+\frac{2}{3}z^{-1}} \quad (|z|>\frac{2}{3})$

sol) $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1-\frac{3}{4}z^{-1}}{1+\frac{2}{3}z^{-1}} \quad \Rightarrow \text{ROC} : |z| > \frac{2}{3}$

(b) $X(z) = \dfrac{1}{1+\frac{1}{3}z^{-1}} \quad (|z|<\frac{1}{3})$, $Y(z) = \dfrac{1}{(1-\frac{1}{6}z^{-1})(1+\frac{1}{3}z^{-1})} \quad (\frac{1}{6}<|z|<\frac{1}{3})$

sol) $H(z) = \dfrac{Y(z)}{X(z)}$

$$= \frac{\cancel{(1+\frac{1}{3}z^{-1})}}{(1-\frac{1}{6}z^{-1})\cancel{(1+\frac{1}{3}z^{-1})}}$$

$$= \frac{1}{1-\frac{1}{6}z^{-1}} \quad \Rightarrow \text{ROC} : |z| > \frac{1}{6}$$

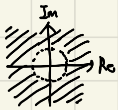
8 causal LTI, $H(z) = \dfrac{4+\left(\frac{1}{4}\right)z^{-1}-\left(\frac{1}{2}\right)z^{-2}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}$

sol) 문제 분석 : ① causal → $h[n]=0$ $(n<0)$ → $h[n]$은 right-sided signal

② LTI → $y[n] = x[n] * h[n]$

③ $H(z)$ → Table & 부분 분수

(a) ROC

sol) $h[n]$은 right sided signal →  꼴의 ROC를 갖는다.

$\therefore H(z) = \dfrac{4z^2+\left(\frac{1}{4}\right)z-\left(\frac{1}{2}\right)}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{2}\right)}$ ■ pole : $\frac{1}{4}, -\frac{1}{2}$

이 중 더 큰 값은 $\left|-\frac{1}{2}\right|$ 이므로

$\boxed{\text{ROC} : |z| > \frac{1}{2}}$

(b) Stable : O

• 이유 : unit circle ∈ ROC → DTFT 존재. → $\sum |h[n]| < \infty$

→ Stable.

(c) $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{4+0.25z^{-1}-0.5z^{-2}}{1+0.25z^{-1}-0.125z^{-2}}$

; $Y(z)(1+0.25z^{-1}-0.125z^{-2}) = X(z)(4+0.25z^{-1}-0.5z^{-2})$

$\Updownarrow$

$\boxed{y[n]+0.25y[n-1]-0.125y[n-2] = 4x[n]+0.25x[n-1]-0.5x[n-2]}$

(d) $1+0.25z^{-1}-0.125z^{-2}\ \overline{)\ 4+0.25z^{-1}-0.5z^{-2}}$ quotient 4

$$\begin{array}{r} 4+\ 1z^{-1}-0.5z^{-2} \\ \hline -0.75z^{-1} \end{array}$$

$$\therefore H(z)=\frac{4+0.25z^{-1}-0.5z^{-2}}{1+0.25z^{-1}-0.125z^{-2}}=4-\frac{0.75z^{-1}}{(1-0.25z^{-1})(1+0.5z^{-1})}$$

$$=4+\frac{-1}{1-0.25z^{-1}}+\frac{1}{1+0.5z^{-1}}$$

$$\Rightarrow \boxed{h[n]=4\delta[n]-\left(\frac{1}{4}\right)^n u[n]+\left(-\frac{1}{2}\right)^n u[n]}$$

(e) $x[n]=u[-n-1] \longleftrightarrow X(z)=\sum_{n=-\infty}^{\infty}x[n]z^{-n}=\sum_{n=-\infty}^{-1}z^{-n}$

$$=\sum_{n=1}^{\infty}z^{n}=\frac{z}{1-z}=\frac{-1}{1-z^{-1}}\quad(|z|<1)$$

$$\boxed{\therefore Y(z)=X(z)H(z)=\frac{-4-0.25z^{-1}+0.5z^{-2}}{(1-0.25z^{-1})(1+0.5z^{-1})(1-z^{-1})}\quad \left(\text{ROC}:\tfrac{1}{2}<|z|<1\right)}$$

(f) $$Y(z)=\frac{-4-0.25z^{-1}+0.5z^{-2}}{(1-\frac{1}{4}z^{-1})(1+\frac{1}{2}z^{-1})(1-z^{-1})}=\frac{-\frac{1}{3}}{1-\frac{1}{4}z^{-1}}+\frac{-\frac{1}{3}}{1+\frac{1}{2}z^{-1}}+\frac{-\frac{10}{3}}{1-z^{-1}}$$

$$\Updownarrow$$

$$y[n]=-\frac{1}{3}\left(\frac{1}{4}\right)^n u[n]-\frac{1}{3}\left(-\frac{1}{2}\right)^n u[n]+\frac{10}{3}[-n-1]$$

$$\boxed{\begin{array}{l} \because \\ \sum_{n=0}^{\infty}(1)^n z^{-n}=\frac{1}{1-z^{-1}}=\frac{z}{z-1} \\ \sum_{n=-\infty}^{-1}(1)^n z^{-n}=\sum_{n=1}^{\infty}z^{n}=\frac{z}{1-z}=-\frac{1}{1-z^{-1}} \end{array}}$$

(b) Determine if the system is stable or not.

(c) Determine the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$.

(d) Use a partial fraction expansion to determine the impulse response $h[n]$.

(e) Find $Y(z)$, the z-transform of the output, when the input is $x[n] = u[-n-1]$. Be sure to specify the ROC for $Y(z)$.

(f) Find the output sequence $y[n]$ when the input is $x[n] = u[-n-1]$.

9.

Consider an LTI system with impulse response

$$h[n] = \begin{cases} a^n, & n \geq 0, \\ 0, & n < 0, \end{cases}$$

and input

$$x[n] = \begin{cases} 1, & 0 \leq n \leq (N-1), \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the output $y[n]$ by explicitly evaluating the discrete convolution of $x[n]$ and $h[n]$.

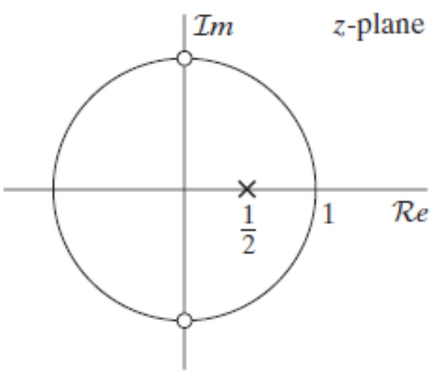
(b) Determine the output $y[n]$ by computing the inverse z-transform of the product of the z-transforms of $x[n]$ and $h[n]$.

10.

Let $x[n]$ be the sequence with the pole–zero plot shown in Figure Sketch the pole–zero plot for:

(a) $y[n] = \left(\frac{1}{2}\right)^n x[n]$

(b) $w[n] = \cos\left(\frac{\pi n}{2}\right)x[n]$



$\boxed{9}$ $h[n] = a^n u[n]$, $x[n] = u[n] - u[n-N]$

(a) $y[n] = x[n] * h[n]$

$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$= \begin{cases} 0 & (n<0) \\ \dfrac{1-a^{n+1}}{1-a} & (0 \le n < N-1) \\ a^{n-N+1}\dfrac{1-a^N}{1-a} & (n \ge N-1) \end{cases}$$

(b) $H(z) = \dfrac{1}{1-az^{-1}}$ $(|z|>a)$, $X(z) = \dfrac{1-z^{-N}}{1-z^{-1}}$ $(|z|>0)$

$\therefore Y(z) = X(z)H(z)$

$$= \frac{1-z^{-N}}{(1-az^{-1})(1-z^{-1})}$$

$$= \frac{1}{(1-az^{-1})(1-z^{-1})} - \frac{z^{-N}}{(1-az^{-1})(1-z^{-1})}$$

① $\dfrac{1}{(1-az^{-1})(1-z^{-1})} = \dfrac{\left(\frac{a}{a-1}\right)}{1-az^{-1}} + \dfrac{\left(\frac{1}{1-a}\right)}{1-z^{-1}}$

$$= \left(\frac{1}{1-a}\right)\left(\frac{1}{1-z^{-1}} - \frac{a}{1-az^{-1}}\right)$$

$$\Updownarrow$$

$$y_1[n] = \left(\frac{1}{1-a}\right)\left(u[n] - a(a)^n u[n]\right)$$

② • z 변환 성질

• $x[n-n_0] \leftrightarrow z^{-n_0}X(z)$

$\rightarrow \dfrac{z^{-N}}{(1-az^{-1})(1-z^{-1})} = z^{-N}\left(\dfrac{1}{1-a}\right)\left(\dfrac{1}{1-z^{-1}} - \dfrac{a}{1-az^{-1}}\right)$

$$\Downarrow$$

$$y_1[n-N] = \left(\frac{1}{1-a}\right)\left(u[n-N] - a(a)^{n-N}u[n-N]\right)$$

$\Rightarrow y[n] = y_1[n] - y_1[n-N]$

$$= \left(\frac{1}{1-a}\right)\left\{(u[n]-u[n-N]) - (a^{n+1}u[n] - a^{n-N+1}u[n-N])\right\}$$

$\boxed{10}$ sol) ■ pole : $z = \frac{1}{2}$

■ zero : $z = \pm j$

$$\Rightarrow X(z) = \frac{(z-j)(z+j)}{z-\frac{1}{2}} = \frac{z^2+1}{z-\frac{1}{2}}$$

(a) $y[n] = \left(\frac{1}{2}\right)^n x[n]$

sol) ■ z 변환 성질

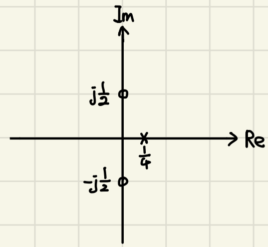
• $z_0^n x[n] \leftrightarrow X\left(\frac{z}{z_0}\right)$ (ROC : $|z_0| R_x$)

$\therefore z_0 = \frac{1}{2}$ 이므로

$$Y(z) = X(2z) = \frac{4z^2+1}{2z-\frac{1}{2}}$$

■ pole : $z = \frac{1}{4}$

■ zero : $z = \pm j\frac{1}{2}$



(b) $w[n] = \cos\left(\frac{\pi n}{2}\right) x[n]$

sol) $\cos\left(\frac{\pi n}{2}\right) = \frac{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}{2}$

■ z 변환 성질

• $z_0^n x[n] \leftrightarrow X(z/z_0)$

$$\therefore W(z) = \frac{1}{2}\left\{X(e^{-j\frac{\pi}{2}}z) + X(e^{j\frac{\pi}{2}}z)\right\}$$

• $x(j\frac{\pi}{2})$: $+90°$ 회전.

• $x(-j\frac{\pi}{2})$: $-90°$ 회전.

$\Rightarrow$ ■ pole : $\pm j\frac{1}{2}$

■ zero : ± 1