

## Digital Logic Design

### Assignment # ①

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(Q1.)

State of a 16-bit register :

1000 0101 0011 1001

The two interpretations are :

- 1- BCD (Binary-Coded Decimal) : A code where each group of 4 bits represents a single decimal digit (0-9).
- 2- Gray code : A code where the sequence represents hexadecimal digits, but encoded in a special way where consecutive values differ by only one bit.

a)	1000	0101	0011	1001
	$8^4 2^3 1$	$8^3 4^2 1$	$8^2 1$	$8^1 2^0 1$
	( $8 \times 0$ ) + 8	(4+1)	(2+1)	(8+1)
	8	5	3	9

 $\therefore$  The BCD content is the four decimal digits :

8, 5, 3, 9.

(b)

e.g.	$\xrightarrow{15}$	BINARY	BINARY-CODED DECIMAL
		$\xrightarrow{15}$ 1111	0001 0101

b)	1000	0101	0011	1001
	1	1	1	1

Four hexadecimal digits,  
encoded in gray code -

Binary  $\rightarrow$  Gray code

Q9

$$\begin{array}{r} \oplus \\ \oplus \\ \oplus \\ \oplus \\ \text{Gray} \\ \text{Code} \end{array} \xrightarrow{\quad} \begin{array}{r} 10110 \\ \hline \end{array}$$

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$\oplus$  XOR

① In order to convert :

Gray code  $\rightarrow$  binary

$\Rightarrow$  e.g. 1 0 110

$$\downarrow \begin{array}{c} (1) \\ (0) \\ (1) \\ (0) \\ (0) \end{array}$$

$$\begin{array}{r} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{array}$$

$$\therefore \begin{array}{r} 1000 \\ 0101 \\ 0011 \\ 1001 \\ 1110 \end{array}$$

$$\begin{array}{r} 1000 \\ 0101 \\ 0011 \\ 1001 \\ 1110 \end{array}$$

$\Rightarrow$  Gray code  $\rightarrow$  binary

$\Rightarrow$  1111 0110 0010 1110

$\Rightarrow (8+4+2+1) \quad (4+2) \quad (2) \quad (8+4+2)$

$\Rightarrow 15 \quad 6 \quad 2 \quad 14$

$\Rightarrow F \quad 6 \quad 2 \quad E$

$\therefore$  As Gray code for hexadecimal : F62E (four hex digits)

②

In signed 2's complement representation for an 8-bit number :

$\Rightarrow$  The MSB is the sign bit.

$\hookrightarrow$  If MSB = 1  $\rightarrow$  number is negative (-ve)

$\hookrightarrow$  If MSB = 0  $\rightarrow$  number is positive (+ve).

e.g.: 11100010

$\Rightarrow$  00011101

+ 1

$\underline{\underline{00011110}}$

$$128 + 64 + 32 + 16 + 8 + 4 + 2 = 30$$

$\therefore$  Since the original number's MSB was 1 (negative)  
the decimal value is : -30.

(Apply XOR operation between successive bits and the ans is the gray code value)

$$(b) (5C7B.6)_{16} = (?)_2 = (?)_{10}$$

5      C      7      B      6

$$\sigma 1 \sigma 1 \quad 1100 \quad 0111 \quad 1011 \quad 0110$$

$$\therefore (0101110001111011.0110)_2$$

$$15 \text{ M } 13 \text{ M } 12 \text{ N } 10 \text{ Q } 9 \text{ Q } 7 \text{ C } 5 \text{ G } 4 \text{ H } -2 \text{ E } 2 \text{ D } 0$$

$$0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 1$$

$$\Rightarrow (2^{14}) + (2^{12}) + 2^{10} + 2^6 + 2^5 + 2^4 + 2^3 + 2^1 + 2^0$$

$$\Rightarrow 23679$$

For the fractional part . 0110, positions are negative powers of 2 (from left to right:  $2^{-1}, 2^{-2}, 2^{-3}, \dots$ )

$$\Rightarrow . 0 \ 1 \ 1 \ 0$$

$$2^{-1} \ 2^{-2} \ 2^{-3} \ 2^{-4}$$

$$\Rightarrow (2^{-2} \times 1) + (2^{-3} \times 1) = 0.25 + 0.125$$

$$\Rightarrow 0.25 + 0.125 = 0.375$$

$$\therefore (23679.375)_{10}$$

A	B	C	D	$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{D}$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{C}\bar{D}$	$\bar{B}\bar{C}\bar{D}$	$\bar{B}\bar{D}$	$\bar{C}\bar{D}$	$\bar{A}\bar{B}$	$F_D$	
0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	1
0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0	0	1	0	0	0	1	0	0
0	1	0	0	1	0	1	0	0	0	0	0	0	0	1	-1
0	1	0	1	0	1	0	1	0	0	0	0	0	0	1	-1
0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	1	1	0	0	0	1	-1	1
1	0	0	1	0	1	0	1	0	0	0	0	0	0	1	0
1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

∴ Simplified SOP form of the Boolean equation?

$$\begin{aligned}
 & \bar{A} \bar{B} \bar{C} \bar{D} + \bar{A} \bar{B} \bar{C} D + A \bar{B} \bar{C} \bar{D} + \bar{B} \bar{C} \bar{D} + \bar{A} B \\
 & \Rightarrow \bar{A} \bar{B} \bar{C} \bar{D} + A \bar{B} \bar{C} \bar{D} + \bar{A} \bar{B} \bar{C} D + \bar{B} \bar{C} \bar{D} + \bar{A} B \\
 & \Rightarrow \bar{B} \bar{C} \bar{D} (1) - \cancel{\bar{A} \bar{B} \bar{C} \bar{D}} + \bar{A} \bar{B} \bar{C} D + \bar{B} \bar{C} \bar{D} + \bar{A} B \\
 & \Rightarrow \bar{B} \bar{C} \bar{D} (1) - \cancel{\bar{A} \bar{B} \bar{C} \bar{D}} + \bar{B} \bar{C} \bar{D} + \bar{A} B \\
 & \Rightarrow F = \bar{A} \bar{C} \bar{D} + \bar{A} \bar{B} + B \bar{C} \bar{D} + \bar{B} \bar{C} \bar{D}
 \end{aligned}$$

∴ Yes, both methods yield the same result.

The most simplified SOP form derived through Boolean algebra and the equation obtained from the k-map method are functionally identical. In the k-map method, grouping adjacent 1's (and don't care terms) corresponds to the Boolean algebra law of distributivity and adjacency ( $X\bar{Y} + X\bar{Y} = X$ ). Any discrepancy in the number of literals per term confirms, rather than contradicts, their equivalence, as it demonstrates the successful application of minimization principles.

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Implementation of POS using NOR gates

$$f(A, B, C, D) = \sum m(0, 1, 6, 7, 14, 15) + d(2)$$

$$\Rightarrow \overline{m}(3, 4, 5, 8, 9, 10, 11, 12, 13) + d(2)$$

AB	CD	00	01	11	10
00	0	0	1	0	X
01	0 <sup>4</sup>	0 <sup>5</sup>	1	0 <sup>3</sup>	1 <sup>2</sup>
11	0 <sup>12</sup>	0 <sup>13</sup>	1	0 <sup>15</sup>	1 <sup>14</sup>
10	0 <sup>8</sup>	0 <sup>9</sup>	1	0 <sup>11</sup>	0 <sup>10</sup>

$$\therefore F = (\bar{B} + \bar{C})(\bar{B} + C)(\bar{A} + B)$$

$$\Rightarrow \bar{F} = (\bar{B} + \bar{C})(\bar{B} + C)(\bar{A} + B)$$

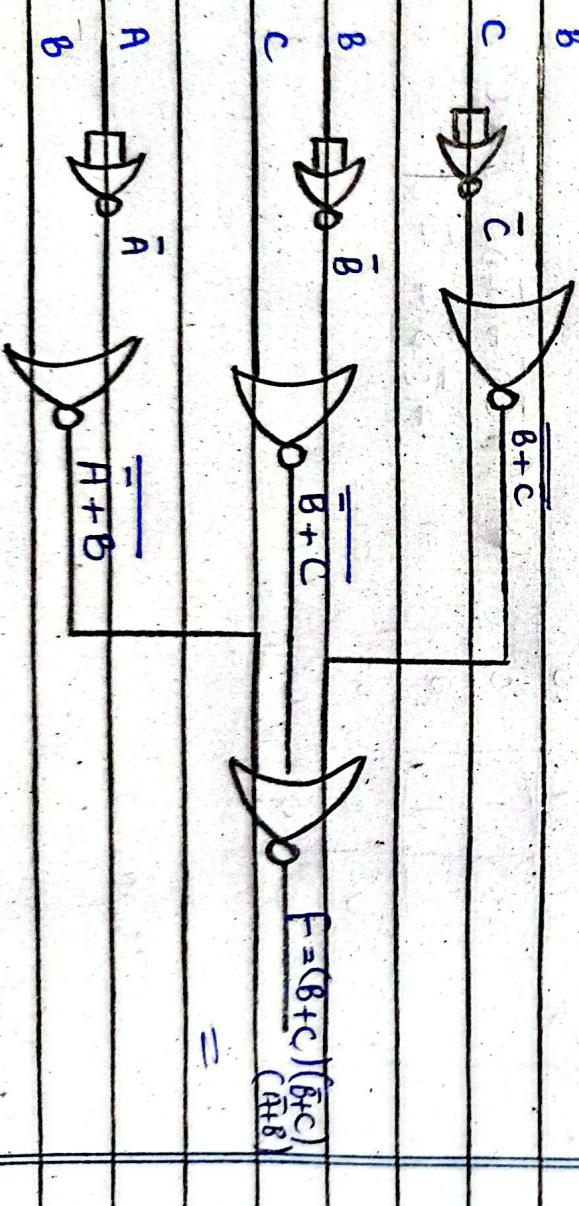
Demorgan's Theorem

$$F = (\bar{B} + \bar{C}) + (\bar{B} + C) + (\bar{A} + B)$$

$$F = \underset{\substack{\downarrow \\ \text{NOR}}}{\text{NOR}} + \underset{\substack{\downarrow \\ \text{NOR}}}{\text{NOR}} + \underset{\substack{\downarrow \\ \text{NOR}}}{\text{NOR}}$$

$$\text{NOR} = \bar{A} + B + C = \bar{A} \cdot \bar{B} \cdot \bar{C}$$

↙  
Two level implementation of NOR.



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(Q.S.)

Month	31	30	29	28			
A	B	C	D	m <sub>0</sub>	m <sub>1</sub>	m <sub>2</sub>	m <sub>3</sub>
0	0	0	1	1			
0	0	1	0			1	1
0	0	1	1	1			
0	1	0	0		1		
0	1	0	1	1			
0	1	1	0		1		
0	1	1	1	1			
1	0	0	0	1			
1	0	0	1		1		
1	0	1	0	1			
1	0	1	1		1		
1	1	0	0	1			

$$m_0 = (A\bar{C}\bar{D} + \bar{A}D)\bar{E} + (A\bar{C}\bar{D} + \bar{A}D)L$$

AB \ CD	00	01	11	10	
00	0	1	1	0	$\Rightarrow (A\bar{C}\bar{D} + \bar{A}D)(\bar{L} + C)$
01	0	1	1	0	$\Rightarrow A\bar{C}\bar{D} + \bar{A}D$
11	1	0	0	0	
10	1	0	0	0	

for m<sub>1</sub>, m<sub>2</sub>

AB \ CD	00	01	11	10	
00	0	0	0	0	$\Rightarrow \bar{L}(\bar{A}B\bar{D} + A\bar{B}D) +$
01	1	0	0	1	$\Rightarrow L(\bar{A}B\bar{D} + A\bar{B}D)$
11	0	0	0	0	$\Rightarrow \bar{A}B\bar{D} + \bar{A}\bar{B}D$
10	0	1	1	0	

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For  $m_2$ :

AB	CD	00	01	11	10	
00	0	0	0	1		$\Rightarrow L \bar{A} \bar{B} C \bar{D}$
01	0	0	0	0	0	
11	0	0	0	0	0	
10	0	0	0	0	0	

for  $m_1 \rightarrow \bar{L} \bar{A} \bar{B} C \bar{D}$

Schematic(s):

