Circuit Analysis

Date:

- A node is the junction of two or more branches.
- A branch is any portion of a circuit with two terminals connected to it.

⇒ In analysis, all wires that are directly connected (with no components in between) belong to the same node because they are at the same electrical potential (voltage).

In essence, a branch represents a single element such as a voltage source or a resistor.

- Two or more elements are in series if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in parallel if they ~~have~~ are connected to the same two nodes and consequently have the same voltage across them.

Q. The charge entering the positive terminal of an element is $q = 5\sin(4\pi t)$ mC while the voltage across the element (plus to minus) is $v = 3\cos(4\pi t)$ V.

(a) Find the power supplied to the element at $t = 0.3$ s.

(b) Calculate the energy delivered to the element between 0 and 0.6 s.

(a) $i(t) = \frac{dq}{dt}$; $i(t) = 4\pi(5)\cos(4\pi t) = 20\pi\cos(4\pi t)$ A

$\therefore$ Power $= VI = (3\cos(4\pi t))(20\pi\cos(4\pi t))$

$= 60\pi\cos^2(4\pi t)$ mW

$\Rightarrow$ Power @ $t = 0.3$s $= 60\pi\cos^2(4\pi(0.3)) = 123.372$ mW

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(b) Power tells us the energy transferred per second.

$\Rightarrow$ Power is the rate at which work is done.

$\therefore \; P = \frac{dW}{dt} \; ; \; dW = P\,dt$

$\Rightarrow \int dW = \int P\,dt \; ; \; W = \int_0^{0.6} P\,dt$

$\Rightarrow W = \int_0^{0.6} 60\pi \cos^2(4\pi t)\,dt = 60\pi \int_0^{0.6} \cos^2(4\pi t)\,dt$

energy delivered

Let $u = 4\pi x$; $du = 4\pi dx$

$\Rightarrow \frac{1}{4\pi} \int \cos^2(u)\,du$

$\Rightarrow$ Applying reduction formula ;

$\Rightarrow \int \cos^n(u)\,du = \frac{\cos^{n-1}(u)\sin(u)}{n} + \frac{n-1}{n} \int \cos^{n-2}(u)\,du$

$\Rightarrow \frac{\cos(u)\sin(u)}{2} + \frac{1}{2}\int 1\,du$

$\Rightarrow \left[\frac{\cos(u)\sin(u)}{2} + \frac{u}{2}\right] \times \frac{1}{4\pi}$

$\Rightarrow \frac{\cos(u)\sin(u)}{8\pi} + \frac{u}{8\pi}$

$\Rightarrow \frac{\cos(4\pi x)\sin(4\pi x)}{8\pi} + \frac{x}{2} + c$

$\left[\frac{\cos(4\pi x)\sin(4\pi x)}{8\pi} + \frac{x}{2}\right]_0^{0.6}$

$\Rightarrow 0.31169$

$\therefore \; W = 60\pi\,(0.31169)$

$W = 58.75 \text{ mJ}$

Pay attention to the units involved.

Real current source $\rightarrow$ Small internal resistance connected in series with the source

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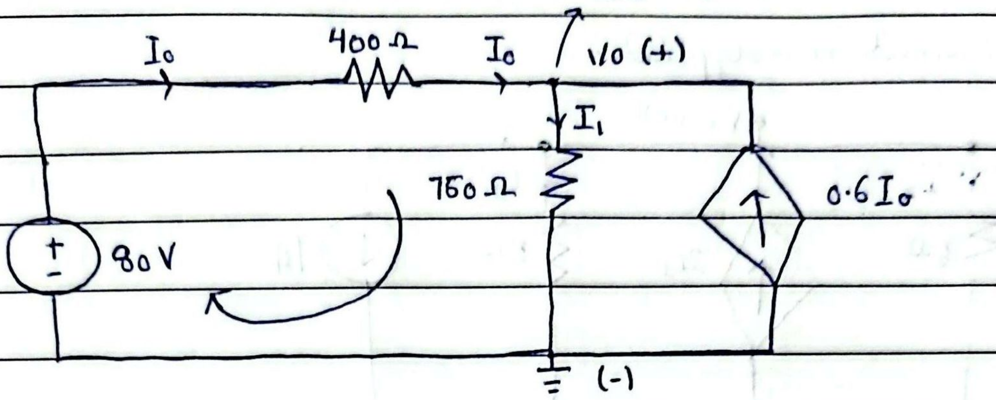
Q. Find the current I_o :-



Applying KCL at the node - 0 ;

$\Rightarrow \quad \sum I_{ent} = \sum I_{leaving}$

$\Rightarrow \quad I_o + 0.6 I_o = I_1$

$\Rightarrow \quad 1.6 I_o = I_1$

Applying KVL in the loop to express I_o in terms of V_o ;

$\Rightarrow \quad \sum V_{rise} = \sum V_{drop}$

$\Rightarrow \quad 80 = I_o(400) + V_o$

$$\frac{80 - V_o}{400} = I_o$$

$$\therefore \quad 1.6\left(\frac{80 - V_o}{400}\right) = \frac{V_o}{750} \quad ; \quad \frac{128 - 1.6V_o}{400} = \frac{V_o}{750}$$

$(128 - 1.6V_o)750 = V_o(400)$

$(128 - 1.6V_o)(1.875) = V_o$

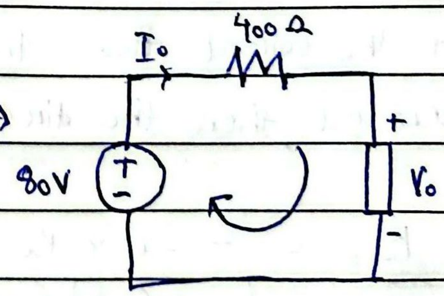
$240 - 3V_o = V_o$

$\Rightarrow \quad 240 = V_o + 3V_o$

$\Rightarrow \quad 240 = 4V_o$

$\Rightarrow \quad V_o = 60\text{ V}$

The circuit can be expressed in a simpler way ;



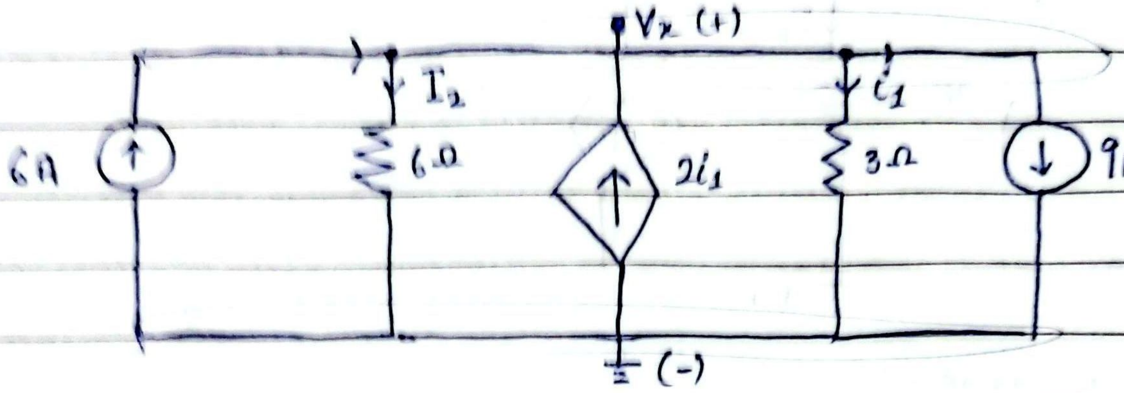
$\therefore$ Applying KVL ;

$\Rightarrow \quad \sum V_{rise} = \sum V_{drop}$

$\Rightarrow \quad 80 = I_o(400) + 60 \quad ; \quad I_o = 50\text{ mA}$

Q. Find i_1 :-

Calculate power for controlled current source and indicate whether the power is absorbed or supplied.



Applying KCL at V_x ;

$\Rightarrow \sum I_{ent} = \sum I_{leaving}$

$\Rightarrow 6 + 2i_1 = I_2 + i_1 + 9$

$\therefore 6 + 2\left[V_x/3\right] = \left[V_x/6\right] + \left[V_x/3\right] + 9$

$\Rightarrow V_x = 18V$

$\Rightarrow V = IR$

$\Rightarrow V_x = i_1 (3\Omega)$

$18 = i_1 (3)$

$i_1 = 6A$

- If the current flows from the negative terminal to the positive terminal, then the direction of current is opposite.

$\Rightarrow P_{2i_1} = -2i_1 \times V_x$

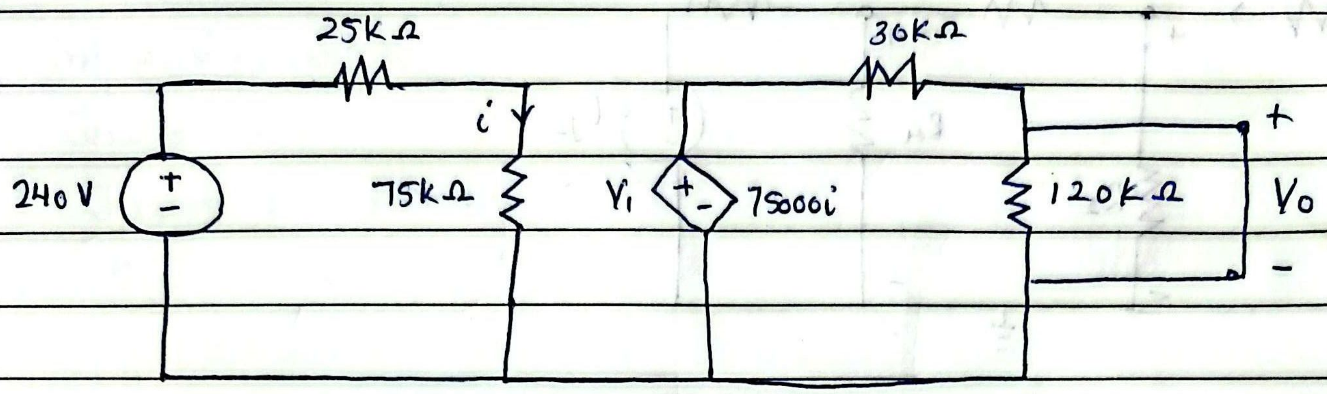
$= \left[-2(6)\right](18)$

$= (-12)(18) = -216 \text{ W}$

$2i_1$ is flowing from -ve to +ve terminal through current source.

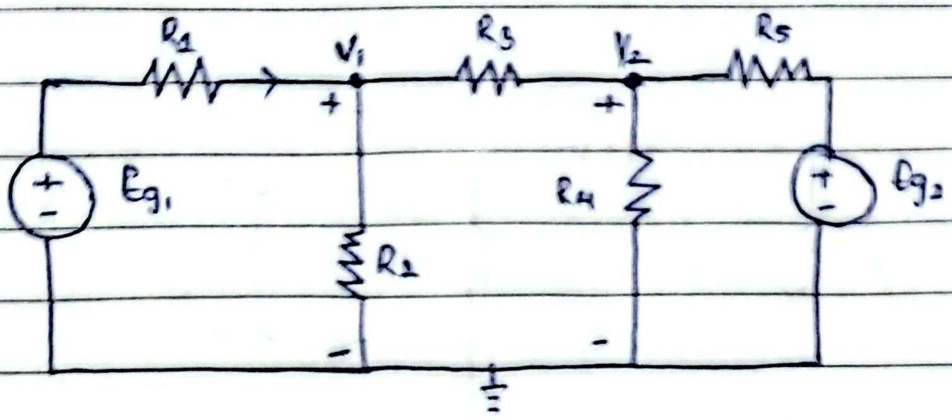
Date:

Q. Find V_o :-



NODAL ANALYSIS

Date:



$$\begin{bmatrix} I_{g_1} \\ I_{g_2} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_{n\times1} = G_{n\times n} V_{n\times1}$$

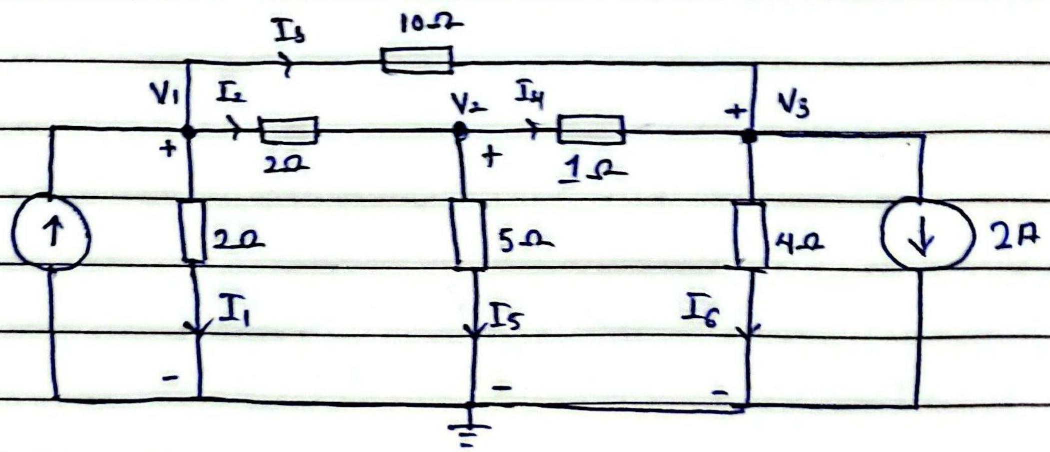
- $I_{g_1} = +E_{g_1}/R_1$, $I_{g_2} = E_{g_2}/R_5$

$G_{11} = G_1 + G_2 + G_3$

$G_{12} = G_{21} = -G_3$

$G_{22} = G_3 + G_4 + G_5$

e.g.



Nodal eqs :-

$$\begin{bmatrix} I_{g_1} \\ I_{g_2} \\ I_{g_3} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

; $I_{g_1} = 28A$;

$I_{g_2} = 0A$

$I_{g_3} = -2A$

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$$G_{11} = \tfrac{1}{2} + \tfrac{1}{2} + 1 = 1.125 \ \mho$$

$$G_{12} = G_{21} = -\tfrac{1}{2} = -0.525 \ \mho$$

$$G_{13} = G_{31} = -\tfrac{1}{10} = -0.1 \ \mho$$

$$G_{22} = 1 + \tfrac{1}{5} + \tfrac{1}{2} = 1.7 \ \mho$$

$$G_{23} = G_{32} = -1 \ \mho$$

$$G_{33} = \tfrac{1}{10} + 1 + \tfrac{1}{4} = 1.35 \ \mho$$

$$\therefore \begin{bmatrix} 28 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1.1 & -0.5 & -0.1 \\ -0.5 & 1.7 & -1 \\ -0.1 & -1 & 1.35 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} 28 &= 1.1V_1 - 0.5V_2 - 0.1V_3 \\ 0 &= -0.5V_1 + 1.7V_2 - V_3 \\ -2 &= -0.1V_1 - V_2 + 1.35V_3 \end{aligned}\right\}$$

Cramer's rule can be applied to solve this system of simultaneous equations ;

(■) If the voltage of a node is KNOWN, then we will not apply KCL at that node.

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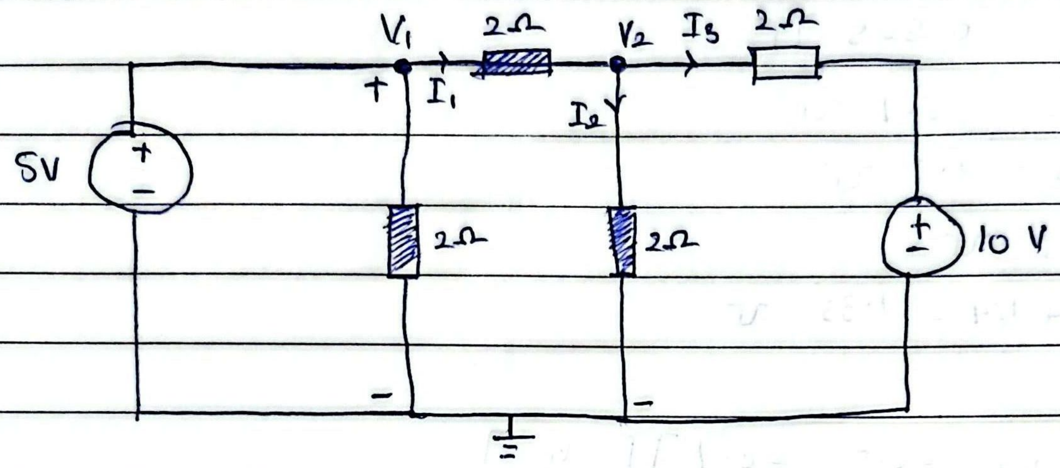


$V_1 = 5V$

⇒ Applying KCL at node - 2 :-

$\Sigma I_{ent} = \Sigma I_{Leaving}$

$I_1 = I_2 + I_3$

$\frac{5 - V_2}{\cancel{2}} = \frac{V_2}{\cancel{2}} + \frac{V_2 - 10}{\cancel{2}}$

$(5 - V_2) = V_2 + (V_2 - 10)$

$5 - V_2 = 2V_2 - 10$

$-V_2 - 2V_2 = -10 - 5$

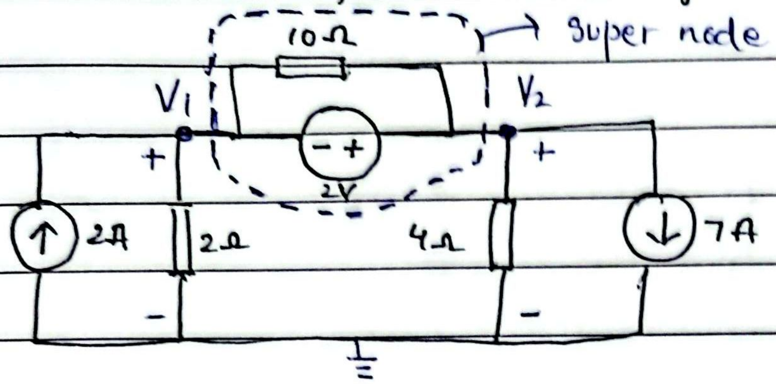
$\cancel{-}3V_2 = \cancel{-}15$

$V_2 = 5V$

■ Current flows from a higher (↑) (+) voltage towards a lower (↓) (−) voltage.

Date: ______

Q. For the circuit, find nodal voltages :-



Applying KCL on the super node ;

$\Rightarrow \sum I_{entering} = \sum I_{leaving}$

$\Rightarrow \quad 2 = i_{2\Omega} + i_{4\Omega} + 7$

$\Rightarrow \quad 2 - 7 = \dfrac{V_1}{2} + \dfrac{V_2}{4} \quad - (1)$

⇒ Applying KVL on the super node ;

$\Rightarrow \quad \sum V_r = \sum V_d$

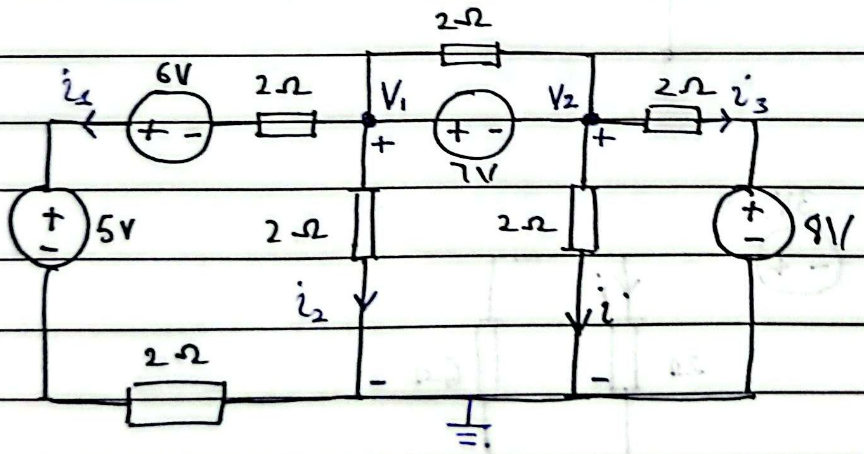
$\Rightarrow \quad V_1 + 2 = V_2 \quad - (2)$

Solving the two eq. simultaneously ;

$\therefore \quad V_1 = -7.33\ V$

$V_2 = -5.33\ V$

Q. For the circuit, find i :-



Applying KCL on the super node ;

$\Rightarrow \quad \sum I_{ent} = \sum I_{leav}$

$\Rightarrow \quad 0 = i_1 + i_2 + i + i_3$

$$\Rightarrow \quad 0 = \left(\frac{V_1 - 5 + 6}{4}\right) + \left(\frac{V_1}{2}\right) + \left(\frac{V_2}{2}\right) + \left(\frac{V_2 - 8}{2}\right)$$

$\Rightarrow \quad 0 = V_1 - 5 + 6 + 2V_2 + 2V_2 + 2V_2 - 16$

$0 = V_1 + 1 + 2V_1 + 2V_2 + 2V_2 - 16$

$0 = 3V_1 + 4V_2 - 15$

$-3V_1 = 4V_2 - 15 \quad ; \quad V_1 = \dfrac{\cancel{-}(15 - 4V_2)}{\cancel{-}3} \quad ; \quad V_1 = \dfrac{15 - 4V_2}{3}$

→ Applying KVL on super node :

⇒ $\sum V_r = \sum V_d$

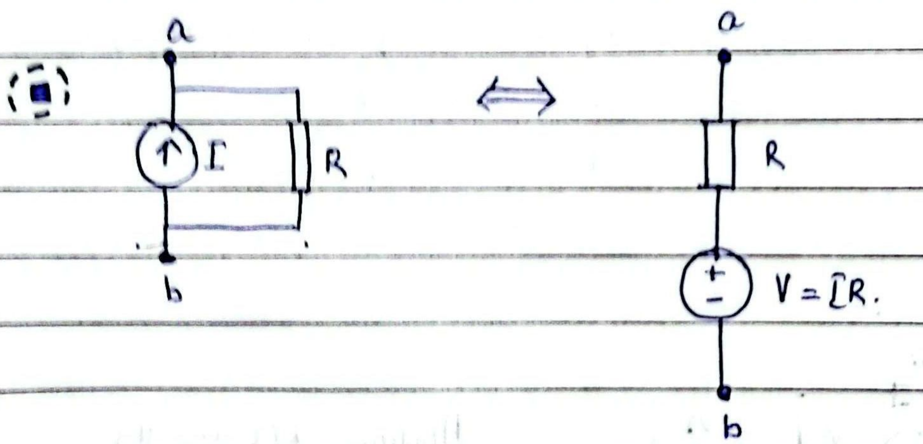
⇒ $V_2 + 7 = V_1$

Date:

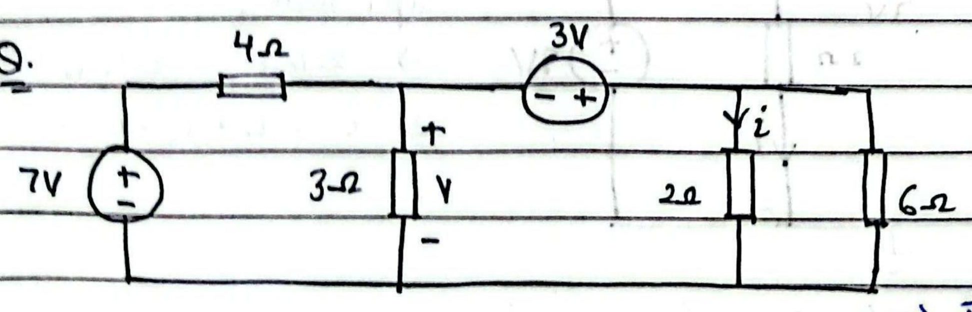
$V_1 = V_2 + 7$ — (2)

$V_1 = \frac{15 - V_1 / V_2}{3}$ — (1)

⇒ Solving the two eq. simultaneously ;



Q.



Total resistance of the different loops

Apply mesh analysis :-

$$\begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 & -3 & 0 \\ -3 & 5 & -2 \\ 0 & -2 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

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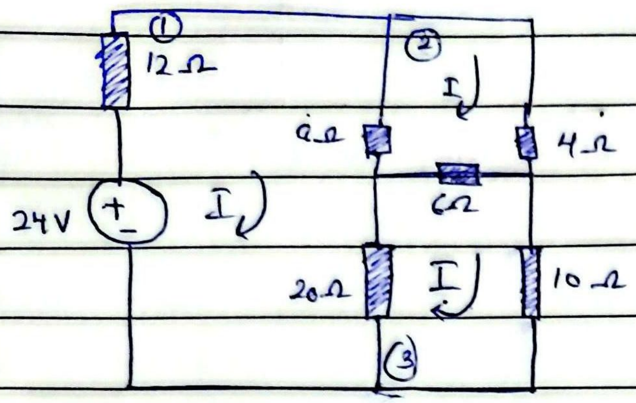
Q. Use mesh analysis to find the voltage across the 6Ω resistor :-



KVL to loop-1 :-

$$\Sigma V_R = \Sigma V_D$$

$$24 = 12I_1 + 8(I_1 - I_2) + 20(I_1 - I_3)$$

$$24 = 40I_1 - 8I_2 - 20I_3 \quad - (1)$$

KVL to loop - 2 :-

$$\Sigma_i V_R = \Sigma_i V_D$$

$$0 = 8(I_2 - I_1) + 6(I_2 - I_3) + 4I_2$$

$$0 = -8I_1 + 18I_2 - 6I_3 \quad - (2)$$

KVL to loop - 3 :-

$$\Sigma V_R = \Sigma V_D$$

$$0 = 20(I_3 - I_1) + 6(I_3 - I_2) + 10(I_3)$$

$$0 = -20I_1 + 20I_3 + 6I_3 - 6I_2 + 10I_3$$

$$0 = -20I_1 + 20I_3 + 6I_3 + 10I_3 - 6I_2$$

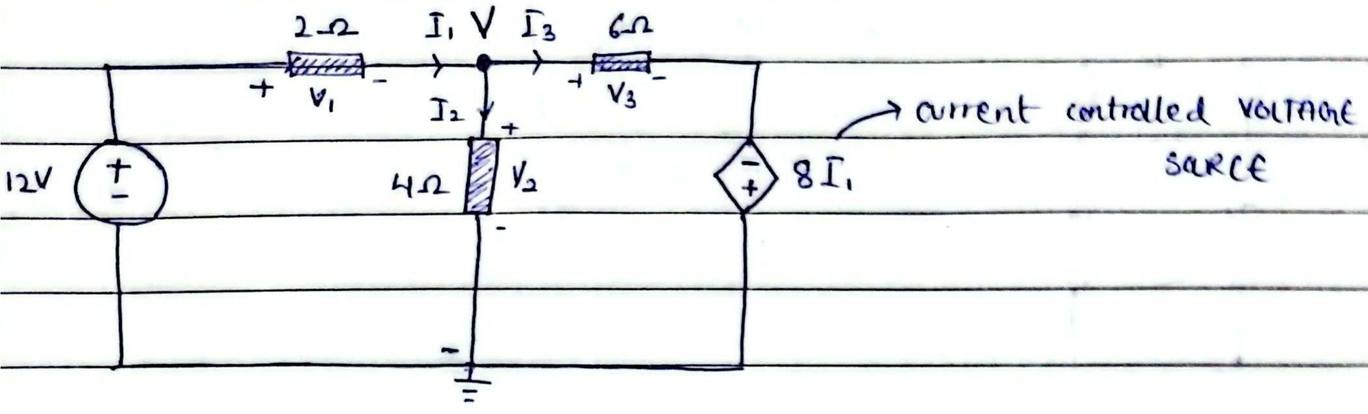
$$0 = -20I_1 + 36I_3 - 6I_2 \quad - (3)$$

Solving all the three equations simultaneously ;

$$\Rightarrow \quad I_1 = 1.125\ A$$

$$I_2 = I_3 = 0.75\ A$$

Q2. Calculate the unknown currents and voltages in the following circuit :-



Applying nodal analysis :-

$$\Sigma I_{ent} = \Sigma I_{leav}$$

$$I_1 = I_2 + I_3$$

$$\frac{12-V}{2} = \frac{V}{4} + \left(\frac{V + 8I_1}{6}\right)$$

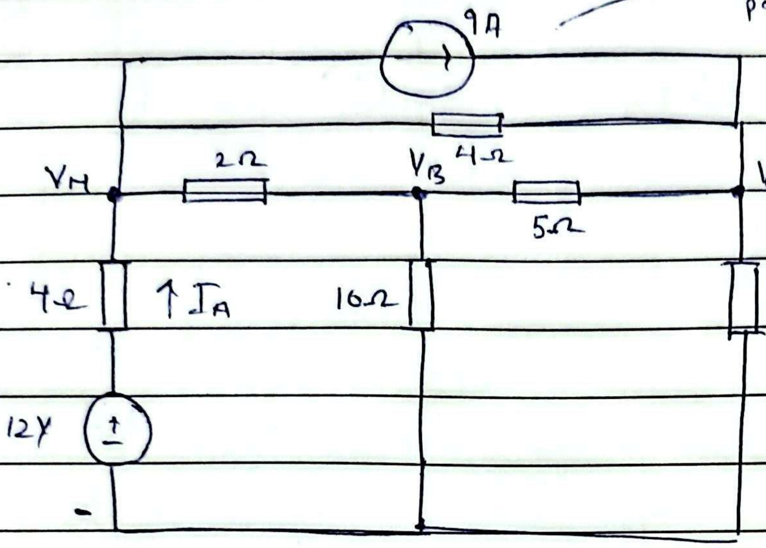
$$\Rightarrow \frac{12-V}{2} = \frac{V}{4} + \left(\frac{V + \left(\frac{12-V}{2}\right)}{6}\right)$$

Date:

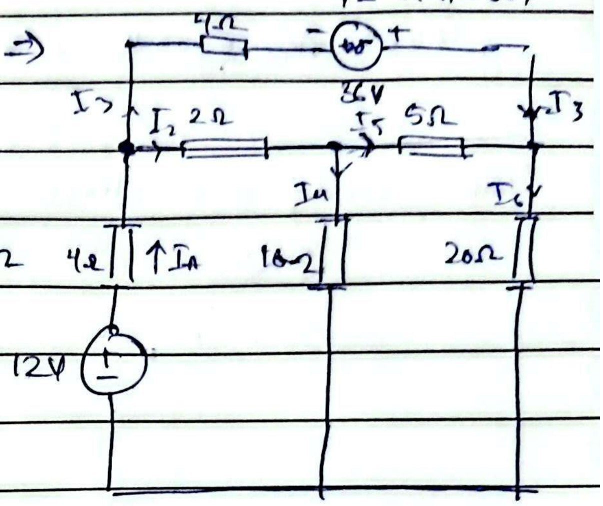
Exp. Find the voltage across 5Ω resistor and the current I_A through the 12V source.

current source connected in parallel with resistor

$V = 9(4) = 36V$



$\Rightarrow$



KCL at node - 0 :-

$\Sigma I_{ent} = \Sigma I_{leav}$

$\Rightarrow I_A = I_2 + I_3$

$\Rightarrow \frac{12 - V_A}{4} = \left(\frac{V_A - V_B}{2 \times 2}\right) + \left(\frac{V_A + 36 \mp V_C}{4}\right)$; $12 - V_A = 2V_A - 2V_B + V_A + 36 \mp V_C$

KCL at node - 1 :-

$\Sigma I_{ent} = \Sigma I_{leav}$

$\Rightarrow I_2 = I_4 + I_5$

$\Rightarrow \frac{V_A - V_B}{2 \times 5} = \frac{V_B}{10 \times 1} + \left(\frac{V_B - V_C}{5 \times 2}\right)$; $5V_A - 5V_B = V_B + 2V_B - 2V_C$

$\Rightarrow 5V_A - 5V_B = V_B + 2V_B - 2V_C$

KCL at node - 2 :-

$\Rightarrow \Sigma I_{ent} = \Sigma I_{leav}$

$\Rightarrow \cancel{I_8} = I_5 + I_3 = I_C$

$\Rightarrow \left(\frac{V_B - V_C}{5 \times 4}\right) + \left(\frac{V_A + 36 \mp V_C}{4 \times 5}\right) = \frac{V_C}{20}$; $4V_B - 4V_C + 5V_A + 180 - 5V_C = V_C$

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$$12 - V_A = 2V_A - 2V_B + V_A + 36 - V_C \quad - (1)$$

$$5V_A - 5V_B = V_B + 2V_B - 2V_C \quad - (2)$$

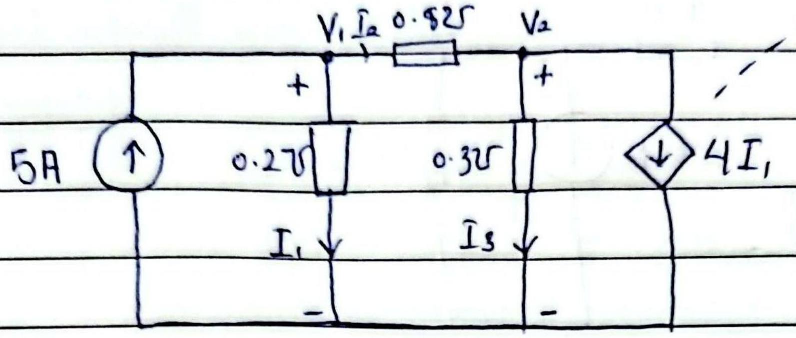
$$4V_B - 4V_C + 5V_A + 180 - 5V_C = V_C \quad - (3)$$

Simultaneously solve these eq. :-

If a current-controlled voltage source was connected, instead of the current source, then, $V_2 = 4I_1$; since there is no resistor connected.

Date:

Q. Find the voltage across $0.8\mho$ conductance :-



KCL at j-1 :-

$\Rightarrow \Sigma I_{ent} = \Sigma I_{ex}$

$\Rightarrow 5 = I_1 + I_2$

KCL at j-2 :-

$\Rightarrow \Sigma I_{ent} = \Sigma I_{ex}$

$\Rightarrow I_2 = I_3 + 4I_1$

- Since conductances are given, in calculations involving $V = IR$, the inverse will be taken.

$\Rightarrow$ KCL at j-1 :-

$\Rightarrow 5 = 0.2V_1 + 0.8(V_1 - V_2)$

$\Rightarrow 5 = V_1 - 0.8V_2$

$\Rightarrow$ KCL at j-2 :-

$\Rightarrow I_2 = I_3 + 4I_1$

$\Rightarrow 0.8(V_1 - V_2) = 0.3V_2 + 4(0.2V_1)$

$\Rightarrow \cancel{0.8V_1} - 0.8V_2 = 0.3V_2 + \cancel{0.8V_1}$

$\Rightarrow -0.8V_2 - 0.3V_2 = 0$

$\Rightarrow \therefore V_2 = 0$

$\therefore 5 = V_1 - 0.8(0)$

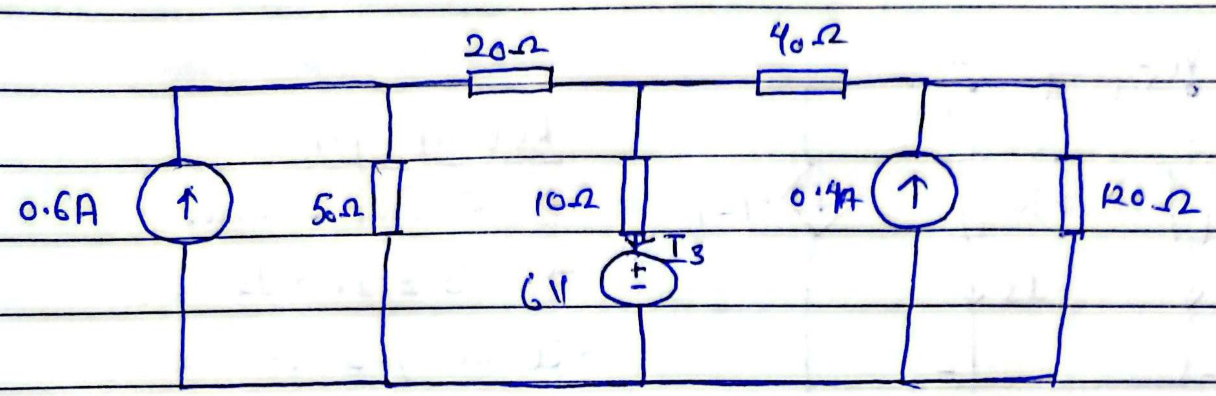
$\Rightarrow V_1 = 5$

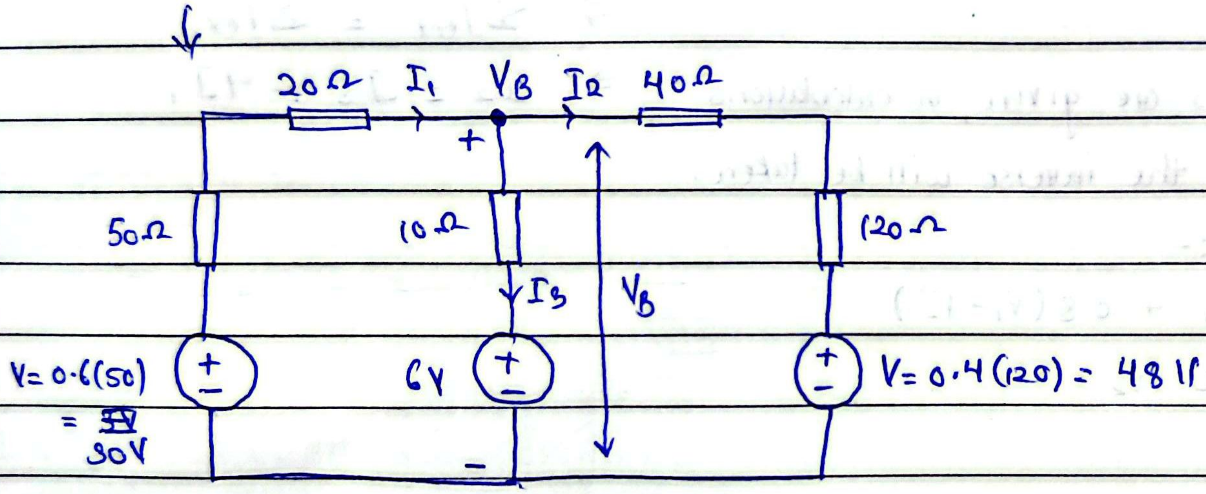
$\therefore$ Voltage ACROSS $0.8\mho$ conductance is

$\Rightarrow V_{0.8\mho} = 5 - 0 = 5V.$

Date:

Q. Calculate V_B and I_3 :-

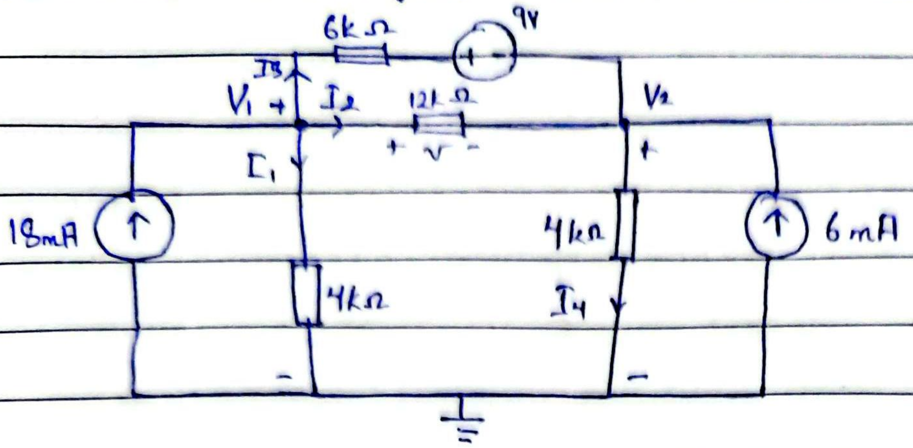




⇒

Date:

Q. Using nodal analysis, find V :-



KCL at j-1 :-

$\Rightarrow 18m = I_1 + I_2 + I_3$

$$18m = \frac{V_1}{4k} + \frac{V_1 - V_2}{12k} + \frac{V_1 - 9 - V_2}{6k}$$

$$18m + \frac{9}{6k} = \left[\frac{1}{4k} + \frac{1}{12k} + \frac{1}{6k}\right] V_1 + \left[\frac{-1}{6k} - \frac{1}{12k}\right] V_2 \quad - (1)$$

KCL at j-2 :-

$$I_3 + I_2 + 6m = I_4$$

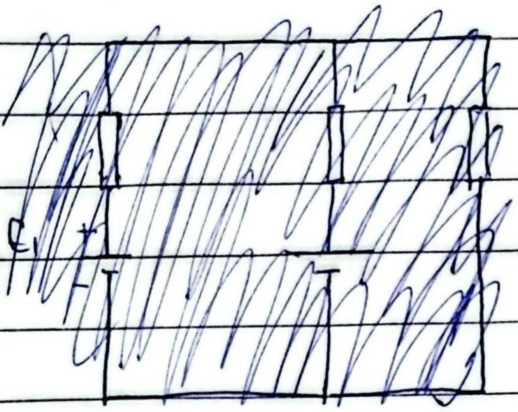
$$\left(\frac{V_1 - 9 - V_2}{6k}\right) + \left(\frac{V_1 - V_2}{12k}\right) + 6m = \frac{V_2}{4k}$$

$$6m - \frac{9}{6k} = \left[\frac{-1}{6k} - \frac{1}{12k}\right] V_1 + \left[\frac{1}{6k} + \frac{1}{12k} + \frac{1}{4k}\right] V_2 \quad - (2)$$

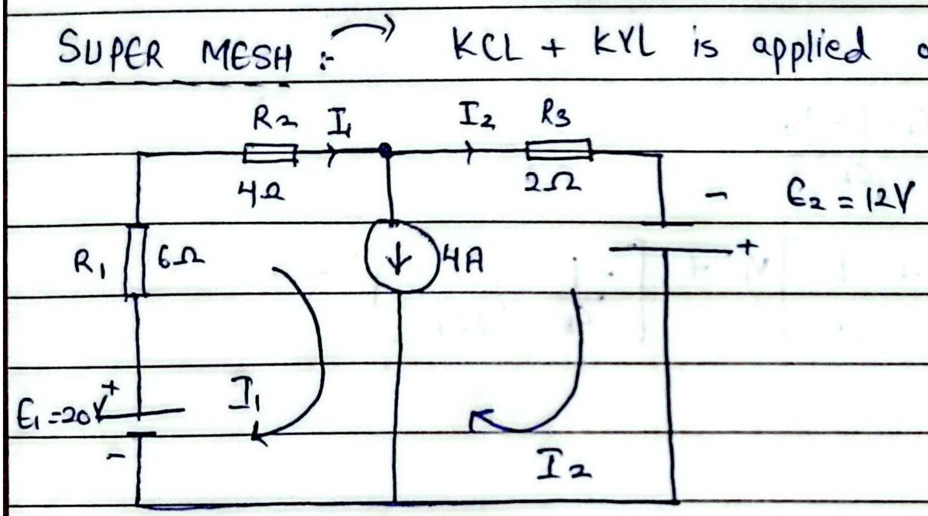
$\Rightarrow \quad I = G\,V$

$$\begin{bmatrix} 18m + 9/6k \\ 6m - 9/6k \end{bmatrix} = \begin{bmatrix} 1/6k + 1/12k + 1/4k & -1/12k - 1/6k \\ -1/12k - 1/6k & 1/4k + 1/12k + 1/6k \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

~~Q. Use mesh analysis for loop currents :-~~



SUPER MESH :→ KCL + KVL is applied on a super mesh (only once)



■ Identify the loops, assign loop currents in CW direction, remove the current source and apply KVL to resulting loops.

↳ Apply KVL collectively on all the loops.

KVL

$$\Sigma V_r = \Sigma V_d$$

$$\Rightarrow 20 + 12 = 6I_1 + 4I_1 + 2I_2$$

$$32 = 10I_1 + 2I_2 \quad - (1)$$

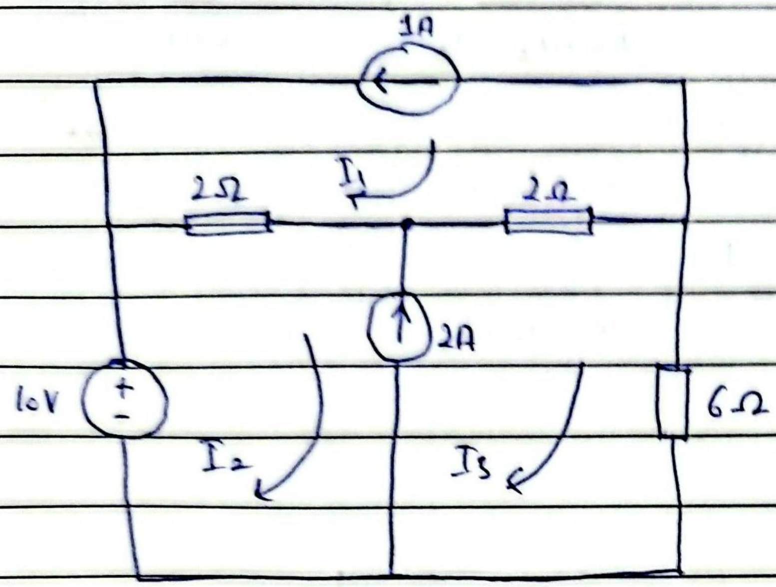
KCL

$$\Sigma I_{ent} = \Sigma I_{leav}$$

$$\Rightarrow I_1 = 4 + I_2 \quad - (2)$$

Date:

(Q.) Find the loop currents :-



Applying KVL on the supermesh :-

$\Rightarrow \quad \sum V_r = \sum V_d$

$\Rightarrow \quad 10 = 2(I_2 - I_1) + 2(I_3 - I_1) + 6I_3$

$10 = 2I_2 - 2I_1 + 2I_3 - 2I_1 + 6I_3$

$10 = -4I_1 + 2I_2 + 8I_3$

$10 = 2(-2I_1 + I_2 + 4I_3)$

$5 = -2I_1 + I_2 + 4I_3$

$\Rightarrow \quad 5 = -2I_1 + I_2 + 4I_3 \quad - (1)$

$\Rightarrow \quad I_1 = -1 \quad - (2)$

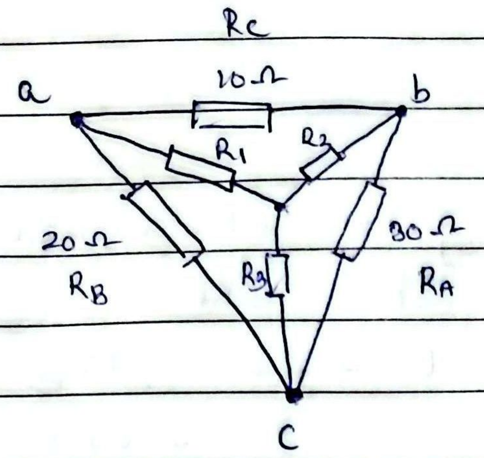
Applying KCL on the supermesh :-

$\Rightarrow \quad \sum I_{ent} = \sum I_{leav}$

$\Rightarrow \quad I_2 + 2 = I_3 \quad - (3)$

TRANSFORMATIONS

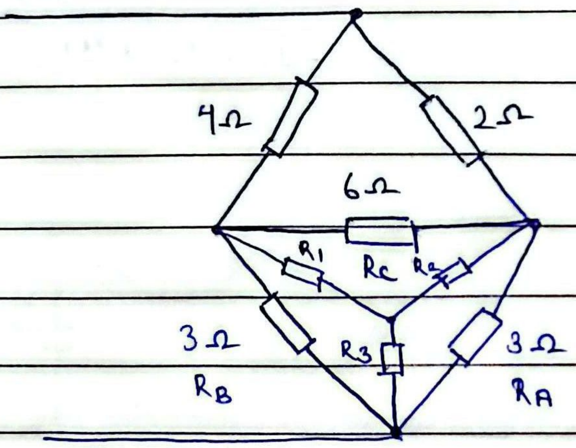
Date:



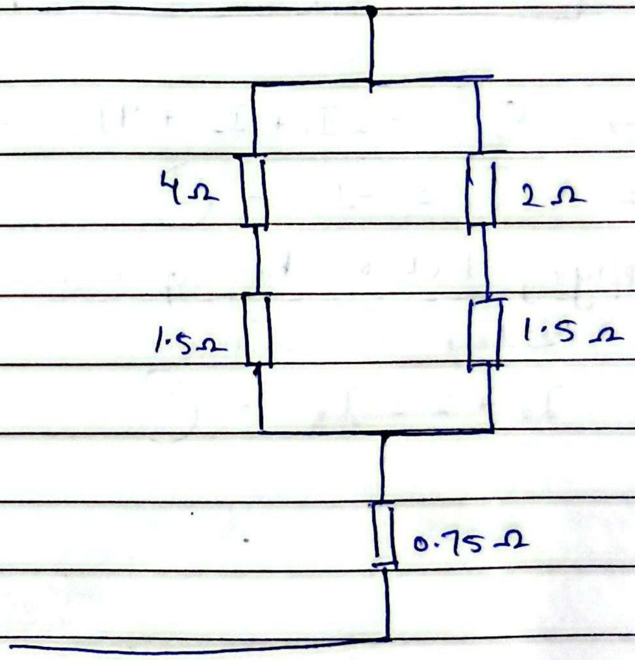
$$\Rightarrow \quad R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20)(10)}{30+20+10} = 3.33\,\Omega$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30)(10)}{30+20+10} = 5\,\Omega$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(30)(20)}{30+20+10} = 10\,\Omega$$



$\Rightarrow$



$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3)(6)}{3+3+6} = 1.5\,\Omega$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3)(6)}{3+3+6} = 1.5\,\Omega$$

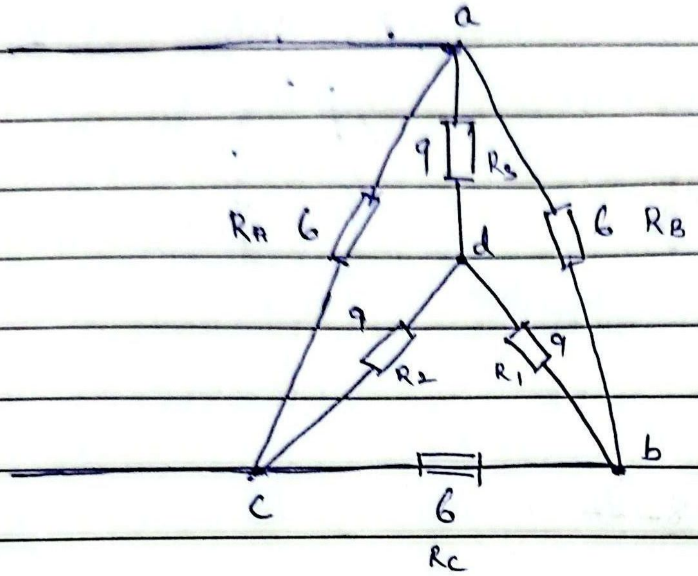
$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3)(3)}{3+3+6} = 0.75\,\Omega$$

$$R_T = [5.5 \parallel 3.5] + 0.75$$

■ If all the resistances in a Δ - form are the same, then the equivalent resistance in the Y - form are all the same; $\Rightarrow R_1 = R_2 = R_3 = \frac{R^2}{3R} = \frac{R}{3}$; R = resistance

Date:

Find the equivalent resistance, R_T = ?



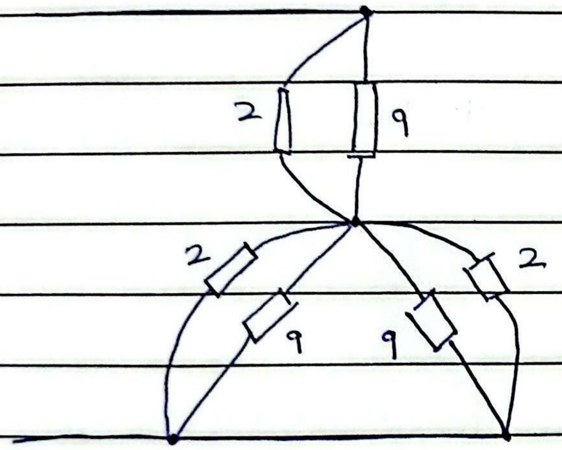
Transforming the outer;

$\Delta \rightarrow Y$:

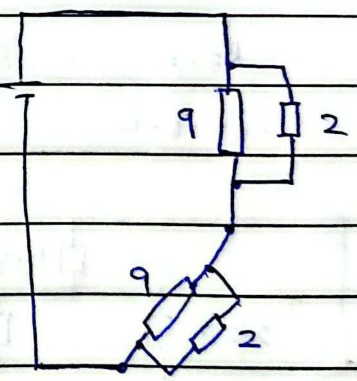
$$\Rightarrow R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{R}{3} = \frac{6}{3} = 2$$

$$\therefore R_1 = R_2 = R_3 = 2\,\Omega$$

$\Rightarrow$



The following circuit acts in the following way :

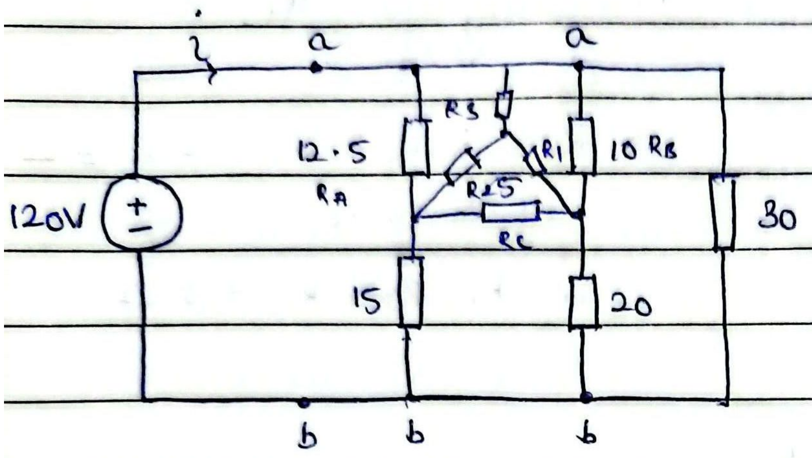


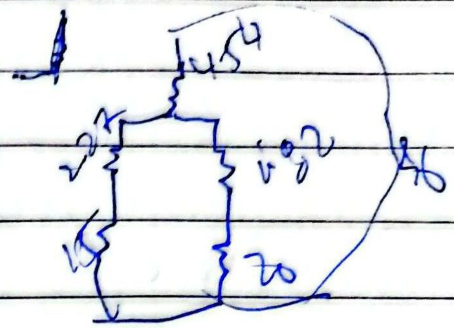
$$\Rightarrow R_T = \frac{(9)(2)}{11} + \frac{(9)(2)}{11} = 1.63 + 1.63 = \underline{3.27\,\Omega}$$

Date:

Q. Find $R_{ab} = ?$

$i = ?$

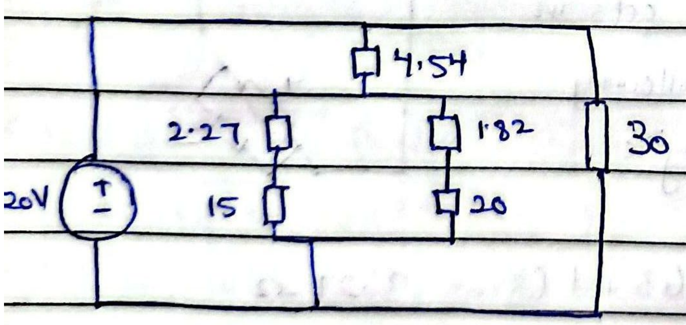




$$R_1 = \frac{R_B R_C}{R_A+R_B+R_C} = \frac{(10)(5)}{12.5+10+5} = \frac{50}{27.5} = 1.82\,\Omega$$

$$R_2 = \frac{R_A R_C}{R_A+R_B+R_C} = \frac{(12.5)(5)}{12.5+10+5} = \frac{62.5}{27.5} = 2.27\,\Omega$$

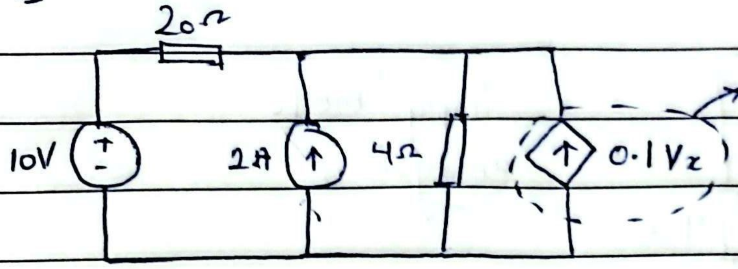
$$R_3 = \frac{R_B R_A}{R_A+R_B+R_C} = \frac{\cancel{(10)(5)}\,(12.5)(10)}{27.5} = \frac{125}{27.5} = 4.54\,\Omega$$



- The dependent source SHOULD NOT be charged, when applying the superposition principle (SPT).

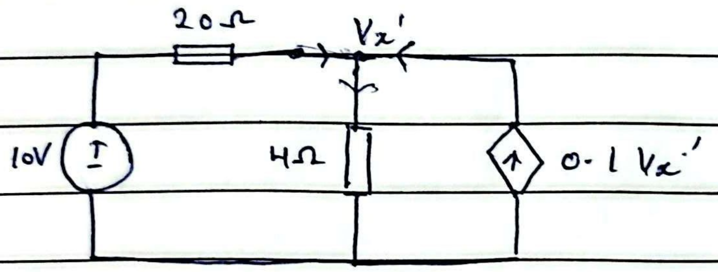
Date:

Q. Use SPT to find V_x :-



would remain uncharged

⇒ ① Let us consider 10V source alone :



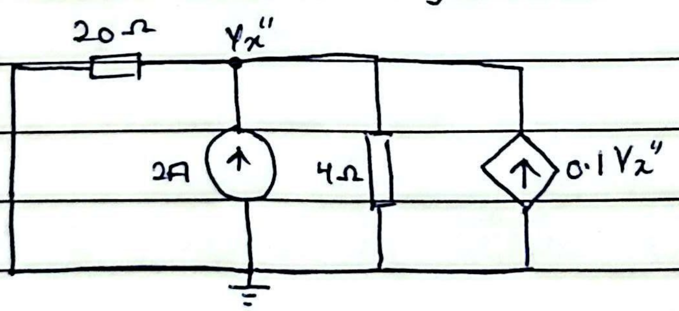
⇒ KCL

$$\Rightarrow 20\left(\frac{10 - V_x' + 0.1 V_x'}{20}\right) = \left(\frac{V_x'}{4}\right)20$$

$$\Rightarrow 10 - V_x' + 2V_x' = 5V_x'$$

$$V_x' = \frac{10}{4} = 2.5V$$

⇒ ② Let us consider only 2A current source in the circuit :



⇒ KCL

$$\Rightarrow 20\left(\frac{-V_x'' + 2 + 0.1 V_x''}{20}\right) = \left(\frac{V_x''}{4}\right)20$$

$$\Rightarrow V_x'' + 40 + 2V_x'' = 5V_x''$$

$$4V_x'' = 40$$

$$V_x'' = 10V$$

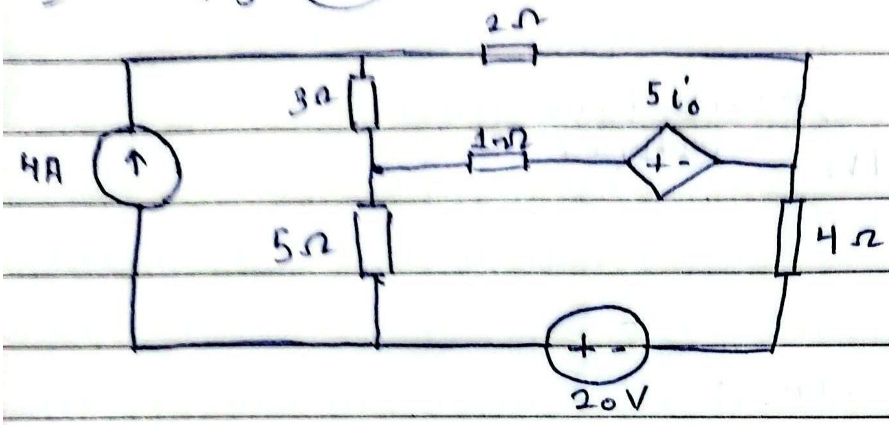
⇒ ③ Applying SPT :

$$\Rightarrow V_x = V_x' + V_x'' = \underline{\underline{12.5\,V}}$$

- The Superposition principle (SPT) is applied to find the DEPENDENT voltage or current source.

Date:

Q. Apply SPT to find $i_o = ?$

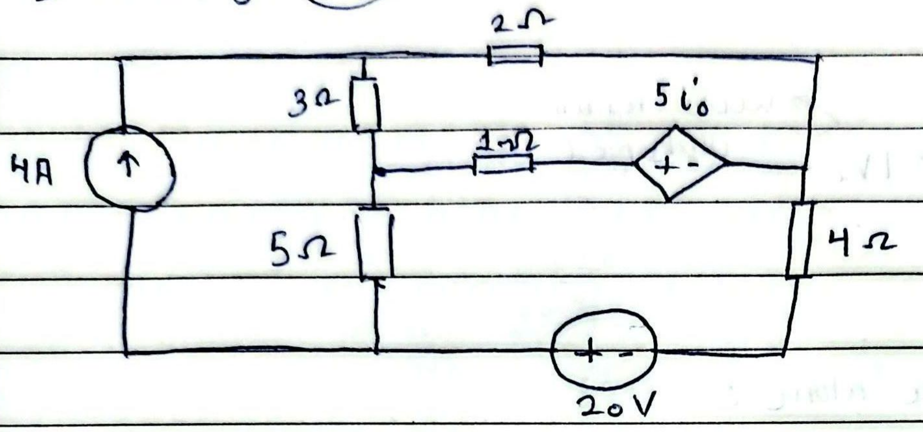


Apply mesh analysis;

- The Superposition principle (SPT) is applied to find the DEPENDENT voltage or current source.

Date:

Q. Apply SPT to find $i_o = ?$

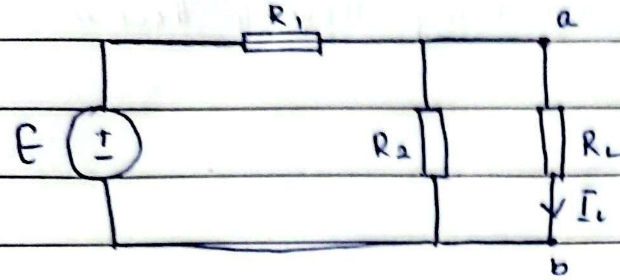


Apply mesh analysis ;

can be used to calculate the voltage and the current across a load resistor.

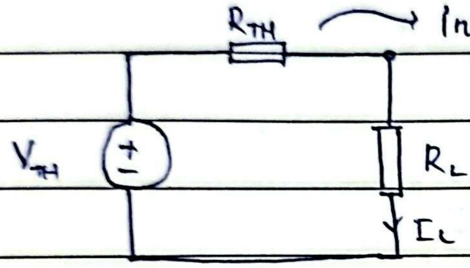
Thevenin's Theorem

Date:



Step-1 :- Draw the Thevenin equivalent circuit :-

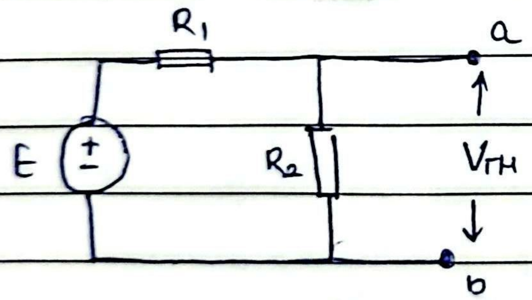
R_{TH} → Internal resistance of V_{TH}



$$\therefore I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

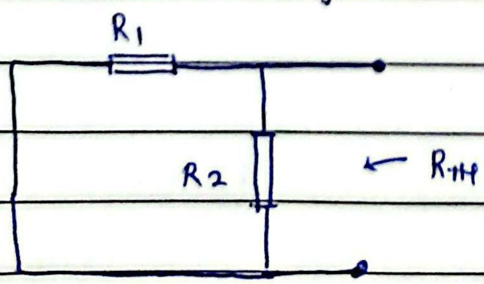
⇒ The Thevenin equivalent circuit is a series circuit with three resistors.

Step - 2 :- Open the circuit ; the terminals ⓐ and ⓑ in the original circuit are opened. V_{TH} is the voltage that would appear across ⓐ and ⓑ.



$$V_{TH} = \left(\frac{E}{R_1 + R_2}\right)(R_2)$$

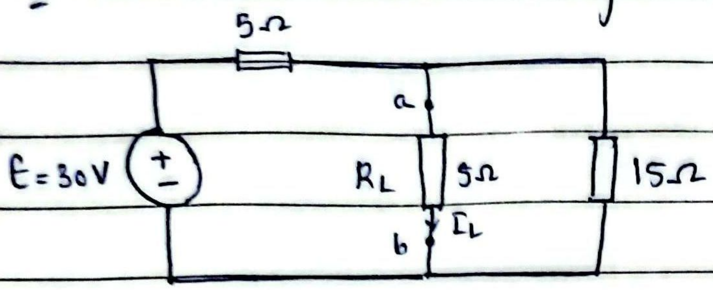
Step-3 :- Replace any independent source in the original circuits by its internal resistance. R_{TH} is the equivalent R while looking back through ⓐ and ⓑ.



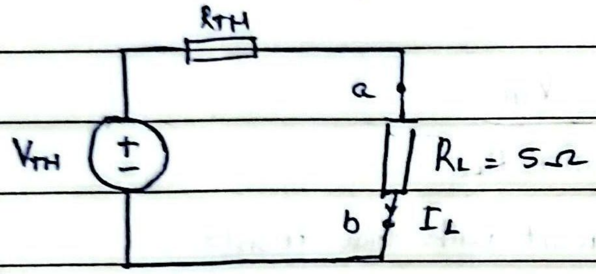
$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2}$$

Date:

Q. Find the current through 5Ω resistor using Thevenin theorem :-

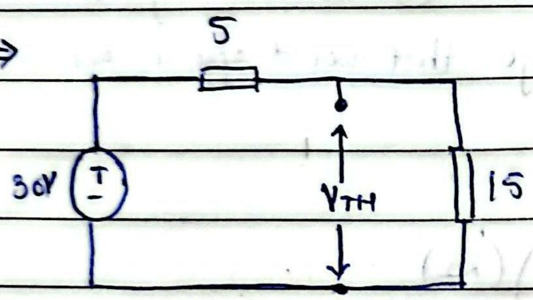


⇒ The Thevenin equivalent circuit :-



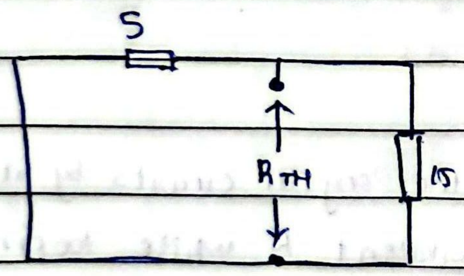
$$I_L = \frac{V_{TH}}{R_{TH} + 5}$$

⇒



$$V_{TH} = \left(\frac{30}{5+15}\right)(15) = 22.5\,V$$

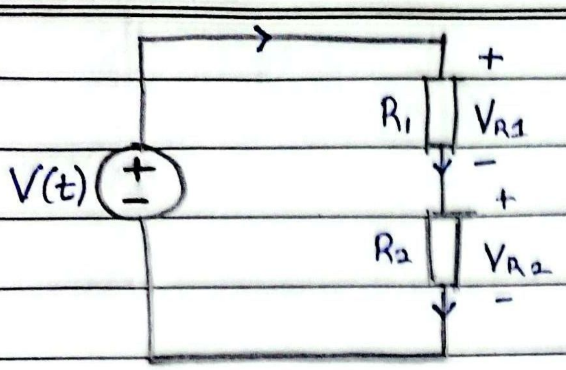
⇒



$$R_{TH} = \frac{15 \times 5}{20} = \frac{75}{20} = 3.75Ω$$

⇒ $\therefore \; I_L = \dfrac{22.5}{3.75 + 5} = 2.57\,A$

Date:



Applying Kirchoff's voltage law ;

$$\Rightarrow V_{R1} + V_{R2} - V(t) = 0$$

$$V(t) = V_{R1} + V_{R2}$$

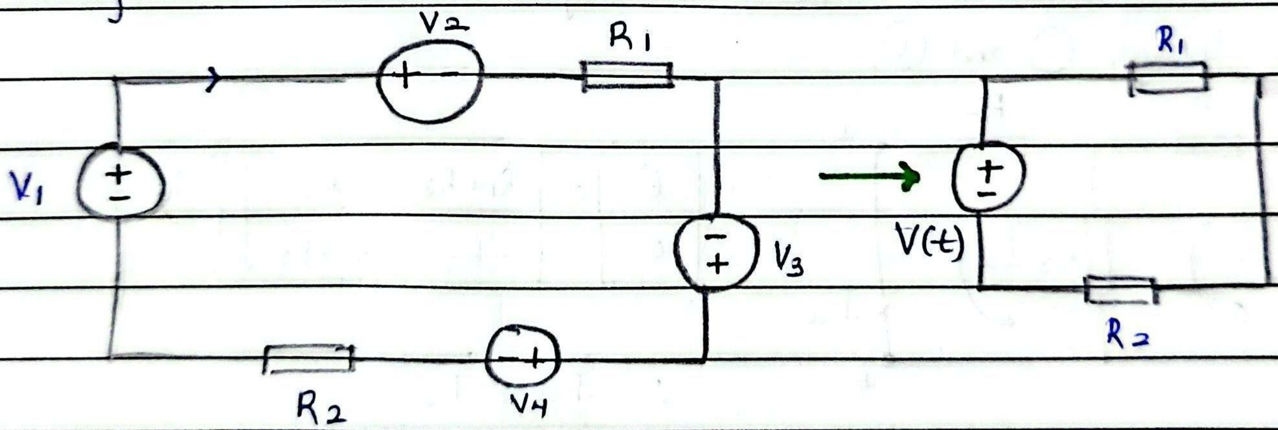
$$V(t) = i(t)R_1 + i(t)R_2$$

$$\Rightarrow i(t) = \frac{V(t)}{R_1 + R_2}$$

$$\therefore \boxed{\begin{aligned} V_{R1} &= \left(\frac{V(t)}{R_1 + R_2}\right)R_1 \\ V_{R2} &= \left(\frac{V(t)}{R_1 + R_2}\right)R_2 \end{aligned}}$$

Voltage dividers

- Voltage sources can also be combined, to form a singular voltage source, using KVL :

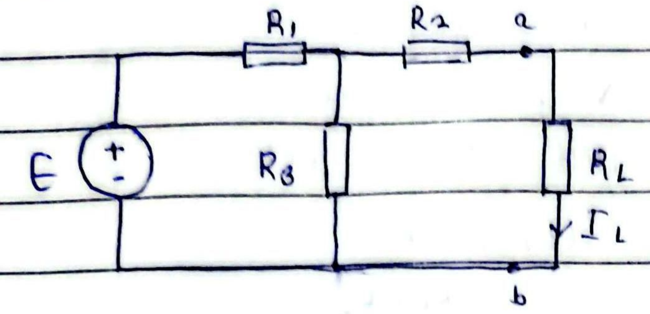


$$\Rightarrow V(t) = V_2 - V_3 + V_4 - V_1$$

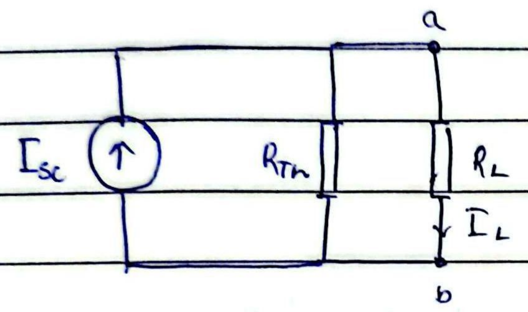
Norton's Theorem

Date:

↳ applies to linear bilateral circuits only

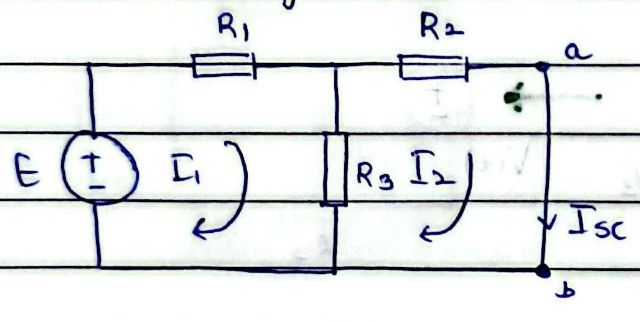


Step-1 :- Draw the Norton equivalent circuit.



$$I_L = \frac{I_{sc}\, R_{Th}}{R_{Th} + R_L}$$

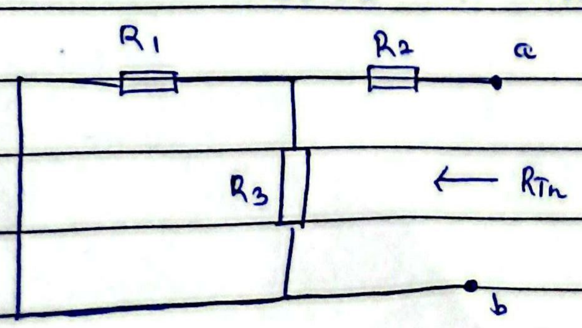
Step-2 :- Short circuit (a) and (b) in the original circuit. I_{sc} is the current through (a) and (b).



$$\begin{bmatrix} E \\ 0 \end{bmatrix} = \begin{bmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\therefore \quad I_{sc} = I_2$$

Step-3 :- Open circuit (a) and (b) in the original circuit, and replace all independent sources by the internal resistances. R_{ab} is the equivalent R when we look through (a) and (b).

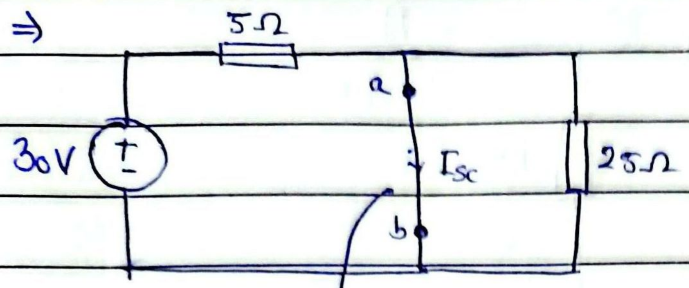


$$R_{Th} = \left[R_1 \parallel R_3 \right] + R_2$$

(■) In a circuit where a path of a current is short circuited, then the parallel circuit paths are ignored.

Date:

⇒



$\therefore I_{sc} = \frac{30}{5} = \underline{6A}$.

This path short circuits, rendering the 25Ω resistor useless.

(4.12)

Q.

MAXIMUM POWER TRANSFER THEOREM :-

Example : find the condition for max power $(R_L = R_{Th})$ transferred to the load resistor. Also Calculate the max power transferred.

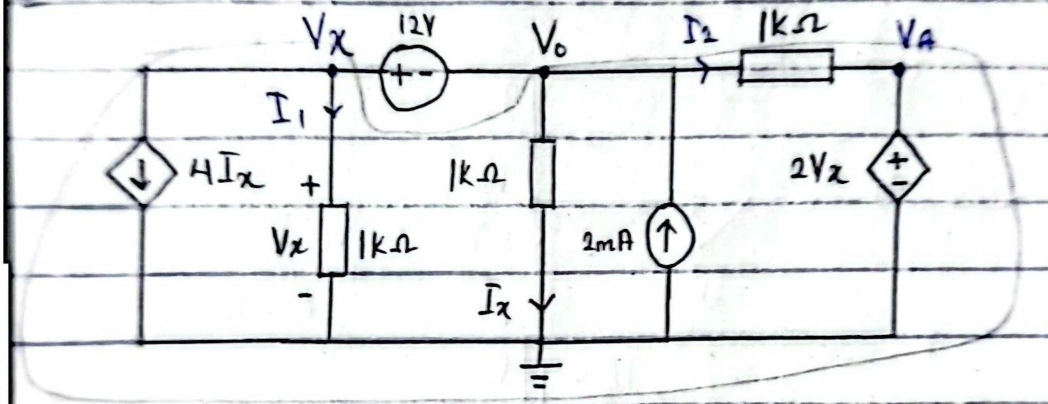
Circuit Analysis

- Current flows from high potential to low potential.

⇒ The ground node is at a lower potential, and therefore, the direction of current should be taken accordingly.

$V = IR \quad ; \quad \frac{V}{R} = I$

Q. Find the value of V_o :-



Since there is a voltage source connected between two essential nodes, node-x and node-0 ;

$\Rightarrow \quad V_x - V_o = 12 \quad - (1)$

Applying KCL at node-A :-

→ CANNOT apply, since the nodal voltage is known.

$\Rightarrow \quad V_A - 0 = 2V_x$

$V_A = 2V_x \quad - (2)$

Applying KCL at the supernode :-

$\Rightarrow \quad \Sigma I_{ent} = \Sigma I_{leav}$

$\Rightarrow \quad 2m = 4I_x + I_1 + I_x + I_2$

$$2m = 4\left(\frac{V_o}{1k}\right) + \left(\frac{V_x}{1k}\right) + \left(\frac{V_o}{1k}\right) + \left(\frac{V_o - V_A}{1k}\right)$$

$$2 = 4V_o + V_x + V_o + V_o - V_A$$

$$2 = 6V_o + V_x - (2V_x)$$

$\Rightarrow \quad 2 = 6V_o + V_x - 2V_x$

$2 = 6V_o - V_x$

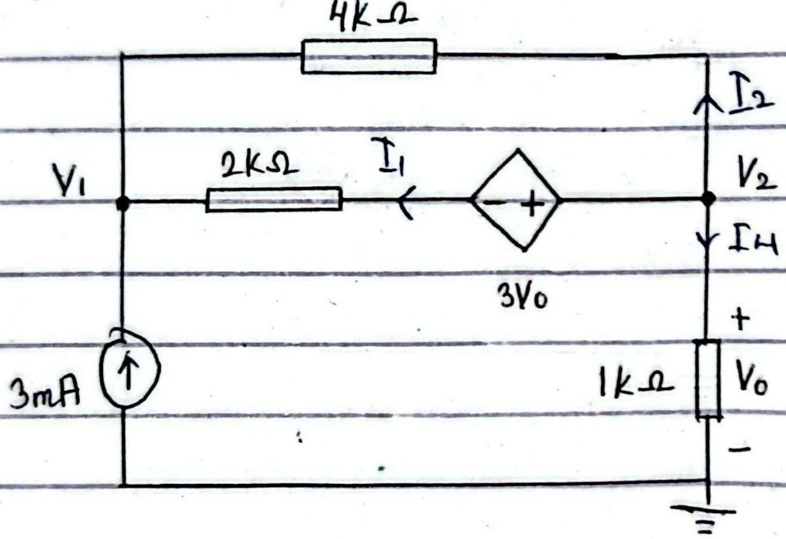
$\Rightarrow \quad 6V_o - V_x = 2$

$$\begin{array}{rrrl} \Rightarrow & -V_x & +\,6V_o & = 2 \\ + & V_x & -\,V_o & = 12 \\ \hline & & 5V_o & = 14 \end{array}$$

$V_o = 2.8V$

$\Rightarrow \quad \underline{\underline{V_o = 2.8V}}$

Q. For the circuit below, find V_1 and V_2 using nodal analysis.



$V_2 - 0 = V_o$

$V_2 = V_o$

- If there is a voltage source and a resistor connected in series between two nodes, then it is NOT a supernode.

Applying KCL at node-1 :-

$\Rightarrow \quad \Sigma I_{ent} = \Sigma I_{leav}$

$\Rightarrow \quad 3m + I_2 + I_1 = 0$

$$3m + \left(\frac{V_2 - V_1}{4K}\right) + \left(\frac{V_2 - 3V_o - V_1}{2K}\right) = 0$$

$\Rightarrow \quad 4(3m) + V_2 - V_1 + 2V_2 - 6V_o - 2V_1 = 0$

$4(3m) + V_2 + 2V_2 - V_1 - 2V_1 - 6V_o = 0$

$\Rightarrow \quad 12m + V_2 + 2V_2 - V_1 - 2V_1 - 6V_0 = 0$

$\Rightarrow \quad 12 + 3V_2 - 3V_1 - 6V_0 = 0$

$12 + 3V_2 - 3V_1 - 6V_2 = 0$

$12 + 3V_2 - 6V_2 - 3V_1 = 0$

$12 - 3V_2 - 3V_1 = 0$

$-3V_1 - 3V_2 + 12 = 0$

$-3(V_1 + V_2 - 4) = 0$

$V_1 + V_2 - 4 = 0 \quad - \quad (1)$

Applying KCL at node-2 :-

$\Rightarrow \quad \Sigma I_{ent} = \Sigma I_{leav}$

$0 = I_4 + I_2 + I_1$

$$0 = \left(\frac{V_2}{1k}\right) + \left(\frac{V_2 - V_1}{4k}\right) + \left(\frac{V_2 - 3V_0 - V_1}{2k}\right)$$

$0 = 4V_2 + V_2 - V_1 + 2V_2 - 6V_0 - 2V_1$

$0 = 4V_2 + V_2 + 2V_2 - 6V_2 - 2V_1 - V_1$

$0 = V_2 - 3V_1$

$\Rightarrow \quad -3V_1 + V_2 = 0$

$(V_1 + V_2 = 4) \times -1$

$$\begin{array}{r} -3V_1 + V_2 = 0 \\ + \; -V_1 - V_2 = -4 \\ \hline -4V_1 = -4 \\ V_1 = 1\,V \end{array}$$

$\Rightarrow \quad -3(1) + V_2 = 0$

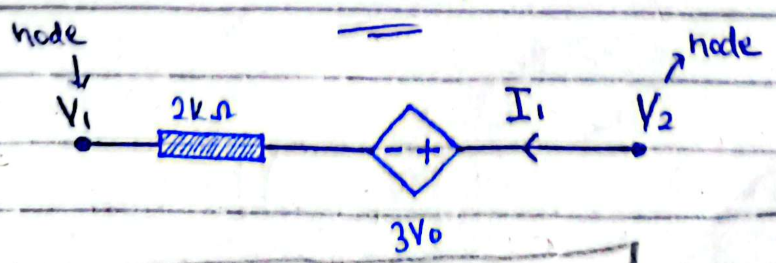
$-3 + V_2 = 0$

$V_2 = 3\,V$

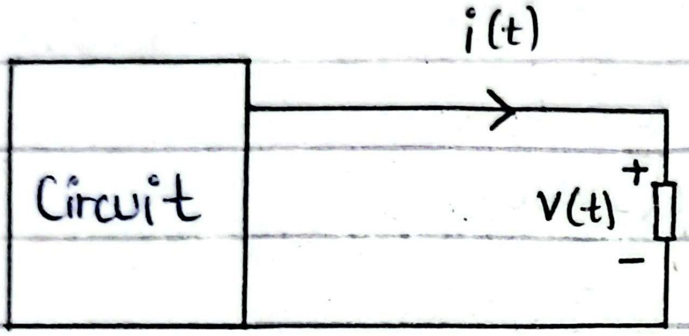
$\therefore \quad V_1 = 1\,V$

$V_2 = 3\,V$

$\Rightarrow$



$$\Rightarrow \quad \boxed{\frac{V_2 - 3V_0 - V_1}{2k} = I_1}$$



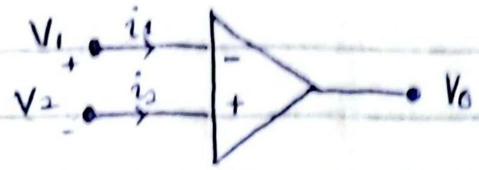
The current, by the passive sign convention enters the positive terminal of the component;

- When the power is calculated;
  - If the value is +ve, then that element is absorbing power.
  - If the value is -ve, then that element is supplying power.

- If an element is supplying power, then the current LEAVES the positive (+ve) terminal of the element.

...ge controlled voltage source.

Ideal Op Amp

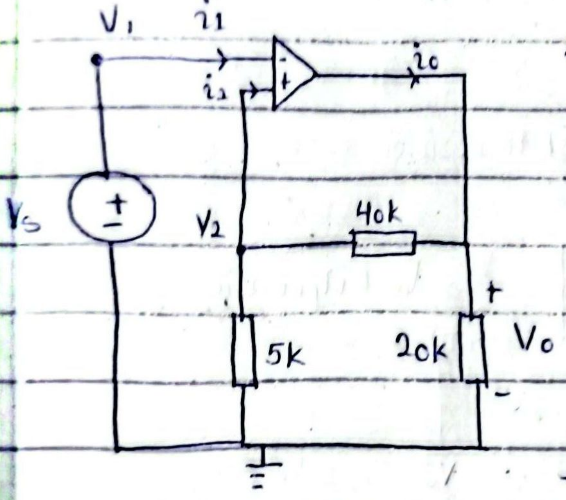


Assumptions of an ideal op-amp:

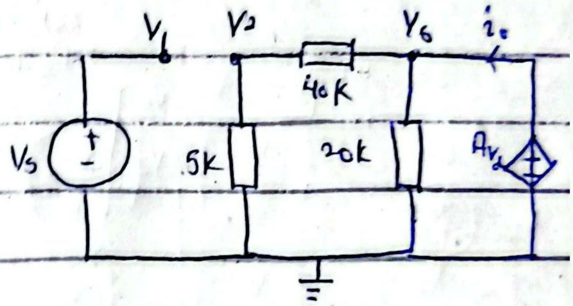
⇒ i_1 and $i_2 \approx 0$

⇒ $V_1 \approx V_2$ (i.e. $V_d \approx 0$) } voltage difference is zero.

e.g. Rework using ideal op-amp model:



Drawing the equivalent circuit;



In an ideal op-amp configuration;

⇒ $V_s = V_1$

and since $V_1 \approx V_2$;

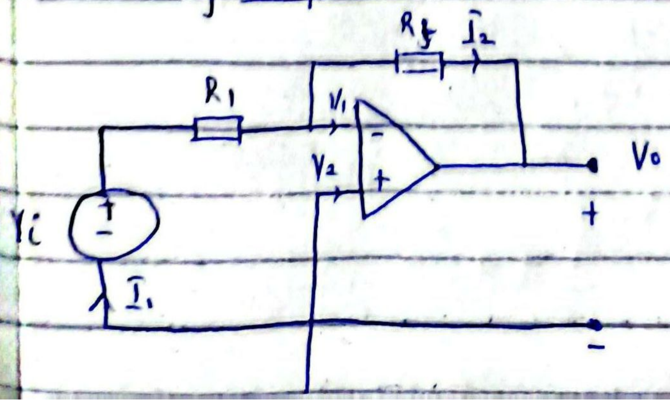
$\therefore\ V_2 = V_s$

Applying the voltage divider rule;

⇒ $V_2 = V_o\left(\frac{5k}{45k}\right) = \frac{1}{9}V_o$

⇒ $\frac{V_o}{V_s} = 9$

Inverting Amplifier



$V_1 = V_2 = 0$

KCL : $I_1 = I_2$

$$\frac{V_i - 0}{R_1} = \frac{0 - V_o}{R_f}$$

$$\frac{V_o}{R_f} = \frac{-V_i}{R_i}$$

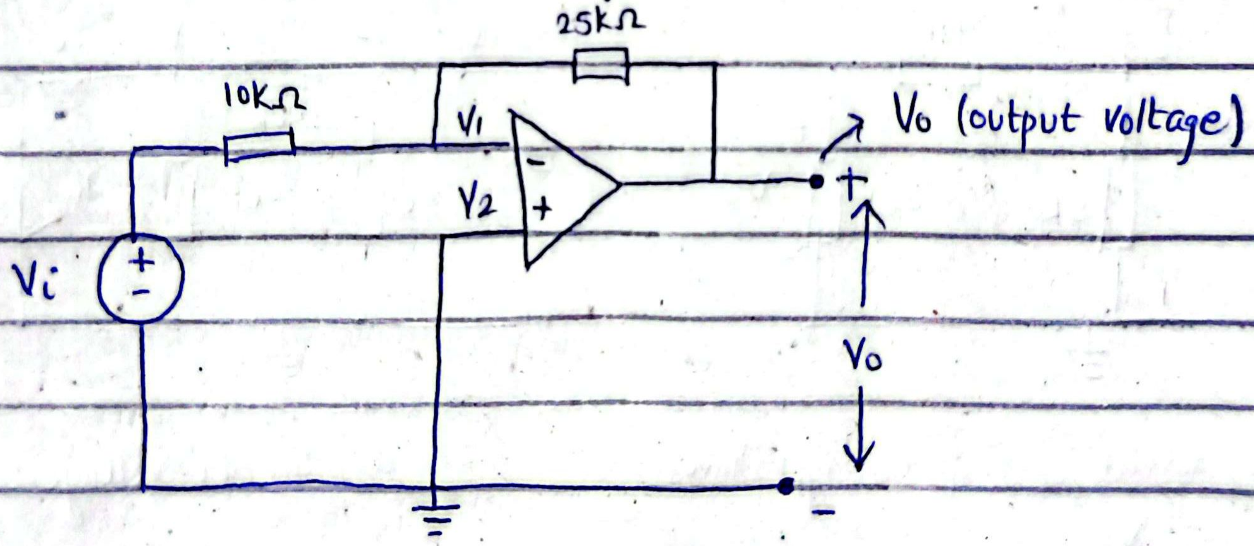
$$V_o = \left(-\frac{R_f}{R_i}\right) V_i$$

5.3

e.g. $V_i = 0.5V$

(a) Calculate output voltage V_o

(b) Calculate current through $10k\Omega$ resistor



$\Rightarrow V_o = -\frac{R_f}{R_1}(V_i) = -\left(\frac{25}{10}\right)(0.5) = -2.5V$

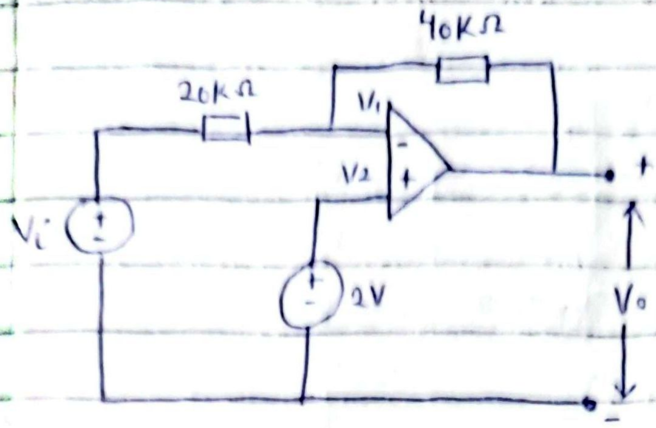
$\Rightarrow$ Since the circuit is an IDEAL op-amp;

$V_1 \approx V_2 = 0$

$$\frac{V_i - V_1}{10k} = i = \frac{V_i - 0}{10k} = \frac{0.5}{10k} = 50\mu A = i$$

5.4

e.g. Determine V_o



$\Rightarrow V_i = 6V$ (given)

In an ideal op-amp circuit; $V_1 \approx V_2$, and since V_1 is connected with the output voltage, V_o. $\therefore V_1 = V_o$.

$$\Rightarrow \frac{6-2}{20k} = \frac{2-V_o}{40k}$$

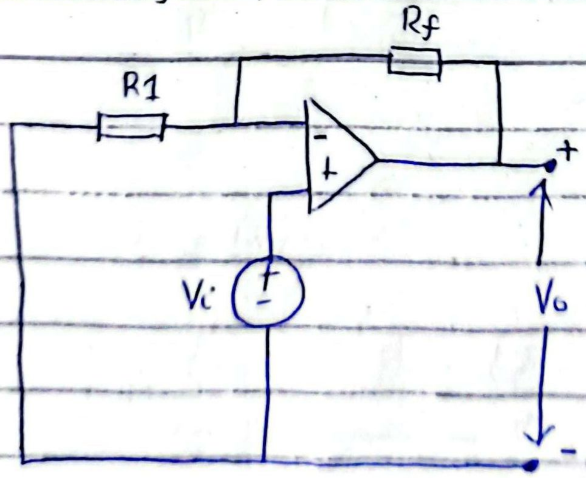
$$4(40k) = (20k)(2-V_o)$$

$$160 = 40 - 20V_o$$

$$120 = -20V_o$$

$$V_o = -6V \text{ (non-inverting)}$$

Non-inverting amplifier



The voltage signal is applied to the non-inverting end of the op-amp.

$\therefore$ In an ideal circuit;

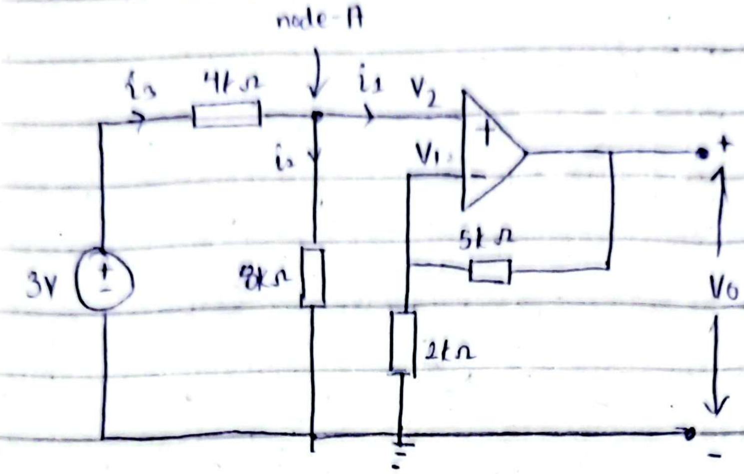
$\Rightarrow V_1 = V_2 = V_i$

$$\frac{0-V_i}{R_1} = \frac{V_i-V_o}{R_f}$$

$$\Rightarrow V_o = \left[1+\frac{R_f}{R_i}\right](V_i)$$

Any signal assigned at the non-inverting (+) end will positively amplify the signal.

PRACTICE PROBLEM 5.5



In an ideal circuit, $V_1 \approx V_2$.

Performing nodal-analysis on node-A;

In an ideal case, the input current = 0 A.

$\sum I_{ent} = \sum I_{leav}$

$\Rightarrow \quad i_3 = i_2 + i_1$ 0

$\cancel{V_1 \approx V_2}$

$$\frac{3-V_A}{4k} = \left(\frac{V_A}{8k}\right) + \cancel{\left(\frac{V_1}{2k}\right)} \quad 0$$

$$\left(\frac{3-V_2}{4k}\right) = \left(\frac{V_2}{8k}\right) + \cancel{\left(\frac{V_2}{2k\times 4}\right)}$$

$$\cancel{\frac{3-V_2}{4k} = \frac{V_2 + 4V_1}{8k}}$$

$$\frac{3-V_2}{4k} = \frac{V_2}{8k}$$

$$24k - 8kV_2 = 4kV_2$$

$$24k = 4kV_2 + 8kV_2$$

$$24k = 12kV_2$$

$$\frac{24000}{12000} = V_2 \; ; \; V_2 = 2V$$

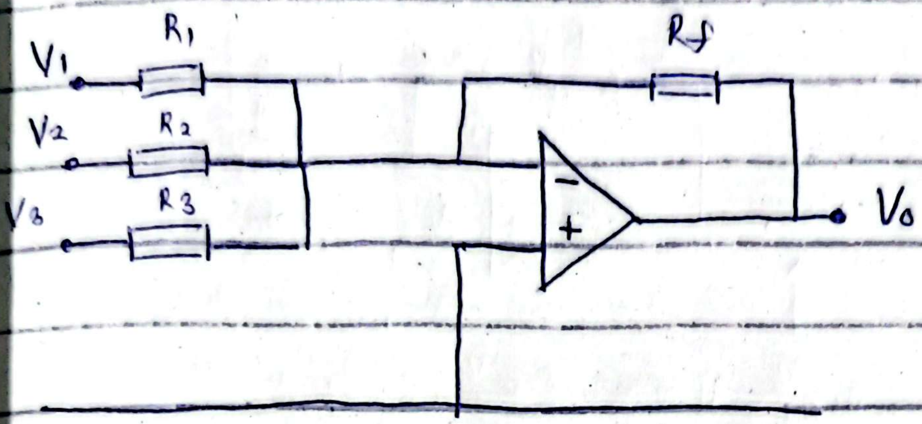
$\rightarrow \quad V_1 = V_2 = V_o\left(\frac{2}{2+5}\right) = V_o\left(\frac{2}{7}\right) = 2$

$\rightarrow \quad V_o(2) = 14$

$V_o = 7V$

- 5.6, 5.6
- Differential amplifiers

Summing amplifier



$$\Rightarrow \quad V_o = \left(-\frac{R_f}{R_1}\right)V_1 - \frac{R_f}{R_2}V_2 - \frac{R_f}{R_3}V_3$$

~~IDEAL~~ : $i_1 = 0$, $i_2 = 0$, $V_1 = V_2$

NONINVERTING : $V_o = (1 + R_f/R_1)(V_i)$

INVERTING : $V_o = (-R_f/R_1)(V_i)$

SUMMING : $V_o = -\left(\frac{R_f}{R_1}(V_1) + \frac{R_f}{R_2}(V_2) + \frac{R_f}{R_3}(V_3)\right)$

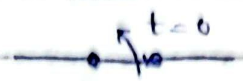
$$\frac{V_o + 5V_o}{10} = \frac{6V_o}{10} = \frac{6(-8)}{10V} = -4.8$$

A capacitor is a voltage sensitive device → resists instantaneous change of voltage.

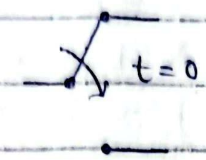
Transient Circuits

Switches : Electrical component that brings about a change in the state of the circuit.

Normally open switch



Normally closed switch



Change over switch

Transient state $\{t=0\}$

That state when the state of the circuit would change due to change in the position of the state.

Steady state $\{t=\infty\}$

When all changes due to change in the switch state die out.

⇒ At a transient state $(t=0)$, if the capacitor is uncharged, it will act as a short circuit $(V_C=0)$. [capacitor is voltage sensitive] → in the case of DC circuits.

assuming capacitor is initially uncharged

- **Capacitive Reactance :** $X_C = \dfrac{1}{i2\pi fC}$

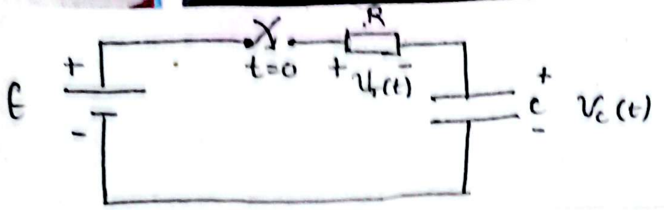
  For DC $f=0$;

  $X_C = \infty$ {In the steady state, capacitor behaves as open circuit}

- **Inductive Reactance :** $X_L = i2\pi fL$

  For DC $f=0$;

  $X_L = 0$ {In the steady state, inductor behaves as [illegible] circuit}



ASSUMPTION : The capacitor is initially uncharged.

At $\underline{t < 0}$

$i_C(0^-) = 0$, $V_C(0^-) = 0$ } since switch was open

At $\underline{t = 0}$

$i_C(0) = \frac{E}{R}$, $V_C(0) = 0$ } transient state

At $\underline{t > 0}$

$E = V_R(t) + V_C(t)$

$$E = R\,i(t) + \frac{1}{C}\int i(t)\,dt$$

Taking derivative wrt time on both sides ;

$$\cancel{\frac{dE}{dt}}^{\,0} = R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

Since there is no change in the direct voltage in a DC circuit ;

$$\frac{di(t)}{dt} + \frac{1}{RC}i(t) = 0 \quad - (1)$$

⇒ General form of sol. for (1)

$$i(t) = \underbrace{Ae^{mt}}_{\text{complementary sol.}} + \underbrace{K}_{\text{particular sol.}}$$

Since (1) is homogenous so particular sol. = 0 ;

So $i(t) = Ae^{mt} \quad - (2)$

Let $m = \frac{d}{dt}$ in

$$m\,i(t) + \frac{1}{RC}i(t) = 0$$

$$m = -1/RC \quad - (3)$$

Substituting (3) in (2) ;

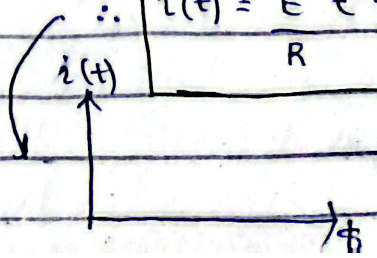
$i(t) = Ae^{(-1/RC)t}$

To find A, we use initial conditions $(t=0)$;

$i(0) = Ae^{0} = A = E/R$

$$\therefore \boxed{i(t) = \frac{E}{R}e^{-t/RC}}$$

$$\boxed{V_R(t) = R\,i(t) = Ee^{-t/RC}}$$



one energy storage device → first order circuit

$$V_c(t) = \frac{1}{C}\int i(t)\,dt + \cancel{R}\, k$$

$$V_c(t) = \frac{1}{C}\int \frac{E}{R} e^{-t/RC} dt + k\cancel{R} = \frac{E}{RC} \times (RC) . e^{-t/RC} + k$$

$$V_c(t) = R - Ee^{-t/RC}$$

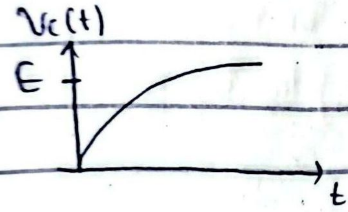
@ initial conditions ; $t = 0$;

$$V_c(0)^0 = R - Ee^0$$

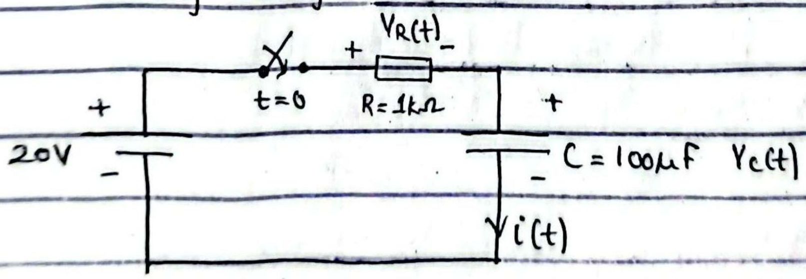
$$0 = R - E$$

$$R = E$$

$$\therefore \boxed{V_c(t) = E\left[1 - e^{-t/RC}\right]}$$



Q. Find $i(t)$, $V_R(t)$, $V_c(t)$ for the given circuit when switches closes at $t = 0$. Assume the capacitor is initially uncharged :-



At $t < 0$

$$i(\bar{0}) = 0, \quad V_c(\bar{0}) = 0$$

At $t = 0$

$$i(0) = \frac{E}{R} = \frac{20}{1\times10^3} = 20mA, \quad V_c(0) = 0$$

At $t > 0$

$$E = Ri(t) + \frac{1}{c}\int i(t)dt$$

time dependent relationship between voltage and current

$$V = L\frac{di}{dt}$$ } voltage across an inductor

where V = instantaneous voltage across the inductor

L = inductance in Henrys

$\frac{di}{dt}$ = Instantaneous rate of current change

$$V(t_2) - V(t_1) = \frac{1}{C}\int_{t_1}^{t_2} i(t)\,dt$$ } voltage across a capacitor

⇒ displaying a time dependent relationship between voltage and current.

STEADY-STATE BEHAVIOR IN DC CIRCUITS

For circuits consisting of DC sources, resistors, capacitors, inductors and switches, etc;

⇒ If all the switches all remain in a given state "for a long time...":

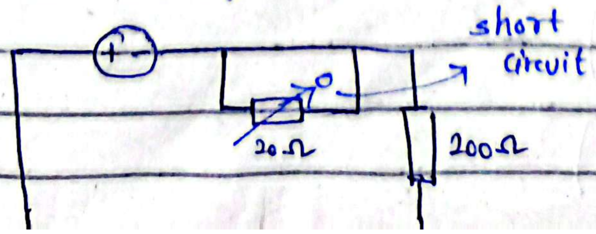
⇒ Inductors will behave like short circuits.

→ Capacitors will behave like open circuits.

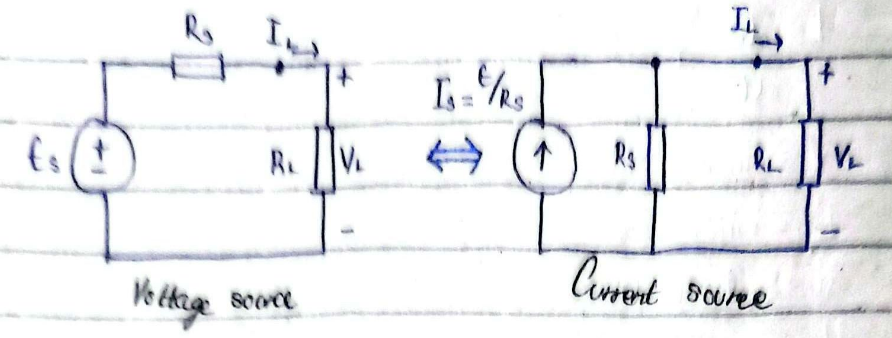
→ Steady-state behavior (in most cases $t(0^-)$) can be determined by applying the above two types of substitutions and analyzing the circuit.

If two ends of a resistor are connected to the same node, and essentially, there is zero resistance path, and therefore no current will flow through the resistor.

e.g: →



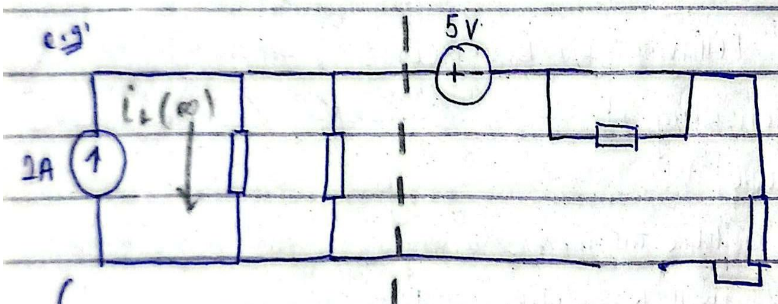
Amongst the many techniques that can be applied, source transformation(s) is one of them.



- $E_s = I_s R_s$ } when converting a current source in // with resistor
- $I_s = E_s / R_s$ } when converting a voltage source in series with resist

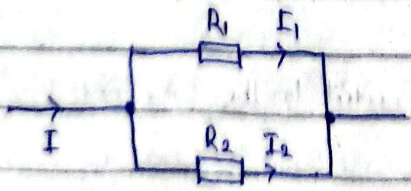
- If a voltage source is connected in series with an open circuit (a gap with infinite resistance, $R = \infty$), the current = 0 through every series component, including the voltage source itself.

e.g:



As time goes infinite, no current will flow in the right half of the circuit.

Another technique that can be used is current division.



$I = I_1 + I_2$

⇒ Since it is a parallel connection

⇒ $I_1 R_1 = I_2 R_2$

$$\Rightarrow I_1 = I\left(\frac{R_2}{R_1+R_2}\right)$$

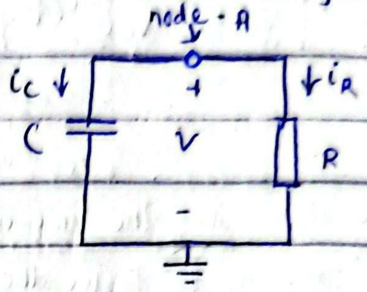
$$\Rightarrow I_2 = I\left(\frac{R_1}{R_1+R_2}\right)$$

Since :
- $V_L(t) = L \frac{di}{dt}$
- $i_C(t) = C \frac{dV}{dt}$

Current cannot change instantly through an inductor and voltage cannot change instantly in a capacitor.

A source-free RC circuit occurs when its DC source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors.

⇒ Consider a series combination of a resistor and an initially charged capacitor ;



Since the capacitor is initially charged, we can assume that at time $t = 0$, the initial voltage is :

$$V(0) = V_0$$

with the corresponding value of the energy stored as :

$$\boxed{W(0) = \tfrac{1}{2} C V_0^2}$$

Applying KCL at the top node (node-A) yields :

$$i_C + i_R = 0$$

By definition, $i_C = C\, dV/dt$ and $i_R = V/R$. Thus,

$$C\, dV/dt + V/R = 0$$

$$\Rightarrow \quad dV/dt + V/RC = 0$$

This is a first-order differential equation since only the first derivative of V is involved.

To solve it, we rearrange the terms as :

$$dV/V = -1/RC \, dt$$

Integrating both sides, we get :

$$\ln V = -t/RC + \ln A$$ } $\ln A$ is the integration constant

Taking powers of e produces :

$$V(t) = A e^{-t/RC}$$

⇒ From the initial conditions, $V(0) = A = V_0$.

$$\therefore \quad V(t) = V_0 e^{-t/RC}$$

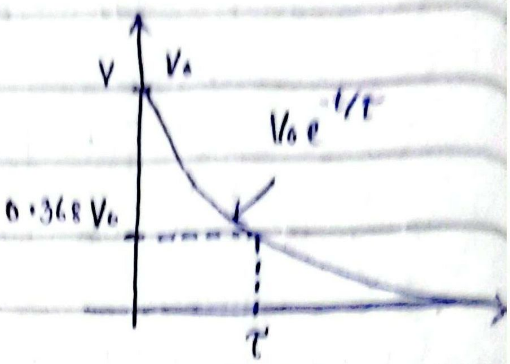
⇒ The above scenario assumes that there is no voltage source.

→ [CONTINUED FROM LAST PAGE]

This shows that the voltage response of the RC circuit is an exponential decay of the initial voltage.

In terms of the time constant;

$$\boxed{V(t) = V_0 e^{-t/\tau}}$$



- The smaller the time constant, the more rapidly the voltage decreases, that is, the faster the response.
  A circuit with a small time constant gives a fast response in that it reaches the steady state (or final state) quickly due to quick dissipation of energy stored.

⇒ With the voltage, $v(t)$ the current $i_R(t)$ can be found:

$$\boxed{i_R(t) = \frac{V(t)}{R} = \frac{V_0 e^{-t/\tau}}{R}}$$ current through the resistor

⇒ The power dissipated through the resistor up to time t is:

$$p(t) = V(t)\, i_R(t) = \left(V_0 e^{-t/\tau}\right)\left(\frac{V_0 e^{-t/\tau}}{R}\right) = \frac{V_0^2 e^{-2t/\tau}}{R}$$

$$\therefore \boxed{p(t) = \frac{V_0^2 e^{-2t/\tau}}{R}}$$

Q1. Refer to the circuit in fig. 7.7. Let $V_c(0) = 60V$. Determine V_c, V_x, and i_o for $t \geq 0$.

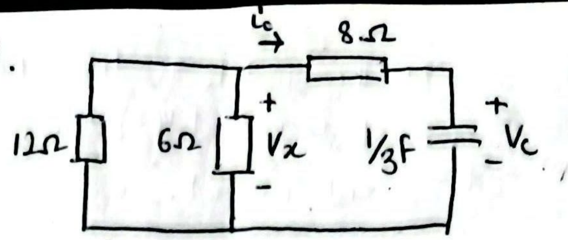
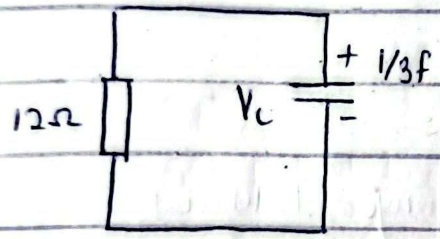


fig 7.7

At time $t = 0$; $V_c(0) = 60V$.

⇒ Finding the equivalent resistance, or, the Thevenin resistance at the capacitor terminals;

$$\frac{12(6)}{12+6} = \frac{72}{18} = 4 \ ; \ 4 + 8 = 12\,\Omega$$



⇒ Finding the time constant, τ:

$$\Rightarrow \tau = RC = 12(1/3) = 4\,s$$

⇒ Thus, the voltage stored across the capacitor for $t \geq 0$ is;

$$V(t) = V_c(0)e^{-t/\tau} = 60e^{-0.25t}\ V$$

⇒ Since voltages are same in parallel circuits, and only currents are different; (voltage of V_x is the same one across the 12Ω resisto

$$V_x = \left(60e^{-0.25t}\right)\left(\frac{\cancel{6}4}{4+8}\right) = \left(60e^{-0.25t}\right)(1/3) = 20e^{-0.25t}$$

$$\therefore\ V_x = 20e^{-0.25t}\ V \quad \text{\} through voltage division}$$

⇒ Applying voltage division to find the voltage across the 8Ω resistor:

$$V_{8\Omega} = \left(60e^{-0.25t}\right)\left(\frac{8}{4+8}\right) = 40e^{-0.25t}\ V$$

$$V = IR \ ; \ I = V/R = 40e^{-0.25t}/8 = 5e^{-0.25t}\ A$$

Q2. If the switch in fig. 7.10 opens at $t = 0$, find $v(t)$ for $t \geq 0$, and $w_C(0)$.

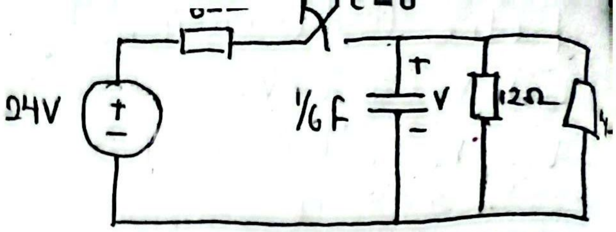


fig 7.10.

For $t < 0$, the switch was closed.

∴ The capacitor acts as an open circuit to DC steady state.

$$\Rightarrow \frac{12(4)}{12+4} = \frac{48}{16} = 3\,\Omega$$

Using voltage division ;

$$\Rightarrow V_{11} = 24\left(\frac{3}{3+6}\right) = 8V \; ; \; t < 0^-$$

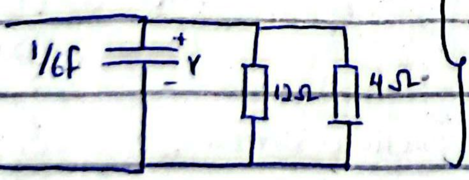
$$\Rightarrow V_C(t) = v(t) = 8\,V$$

∴ Since the capacitor voltage cannot change instantaneously, the voltage calculated at $t = 0^-$ is equal to the voltage at $t = 0$, or ;

$$\Rightarrow V_C(0) = V_0 = 8\,V$$

For $t \geq 0$, the switch is opened ;

The left half of the circuit is essentially discarded, since no current flows in that :



Finding the time constant, τ ;

$$\Rightarrow \tau = RC = (12 \| 4)(1/6)$$

$$= (3)(1/6) = 0.5\,s$$

Thus, the voltage across the capacitor for $t \geq 0$ is :

$$v(t) = V_C(0)\,e^{-t/\tau} = 8\,e^{-t/0.5} = 8e^{-2t}\ V$$

The initial energy stored in the capacitor is :

$$\Rightarrow W_C(0) = \frac{1}{2} C\,V_C(0)^2 = \frac{1}{2}(1/6)(8)^2 = 5.333\,J$$

SOURCE-FREE RL CIRCUIT

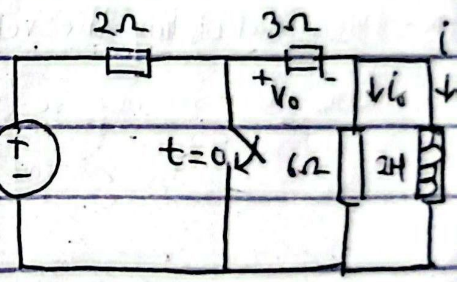
↳ circuit response = current $i(t)$ through the inductor

⇒ When a circuit has a single inductor and several resistors and dependent sources, the THEVENIN equivalent can be found at the terminals of the inductor to form a simple RL circuit.

⇒ Current enters the positive (+ve) side.
Current leaves the negative (-ve) side.

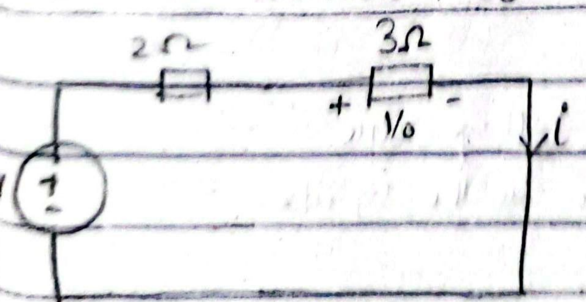
$$i(t) = i(0)\, e^{-t/\tau} \quad ; \quad \tau = \frac{L}{R}$$

Q. In the circuit shown in Fig. 7.19 find i_o, v_o and i for all time, assuming that the switch was open for a long time :-



fig

For $t<0$, the switch was open ;
The inductor acts like a short circuit in the DC steady state ;
∴ The 6Ω is also short circuited, since the current takes the path of least resistance ; $i_o = 0$; $t<0$



{ i is the current across the inductor ; and since this is a series circuit :

$\therefore\ 10 = i(2+3) = i(5)$; $i = 2A$; $t<0$

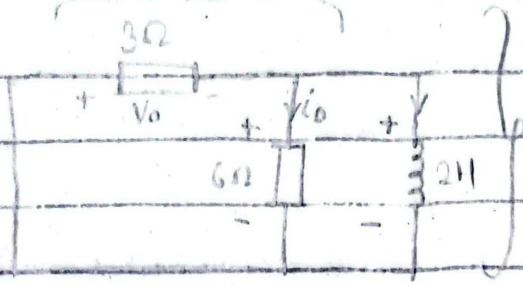
$V_o = 10\left(\frac{3}{3+2}\right) = \frac{30}{5} = 6V$; $t<0$ { using voltage division

For $t > 0$, the switch is closed;

⇒ The wire, where the switch is, short circuits the voltage source, in series with the resistor;

$3\,\Omega$ and $6\,\Omega$ are in parallel

⇒



Combining resistors;

$\Rightarrow (3 \| 6) = \frac{3(6)}{3+6} = 2\,\Omega$

$\Rightarrow \tau = \frac{L}{R} = \frac{2}{2} = 1\,s$

$\therefore\ i = i(0)e^{-t/\tau} = 2e^{-t}\ A\ ;\ t > 0$

⇒ Since the inductor is in parallel with the $6\,\Omega$ and the $3\,\Omega$ resistor;

$\therefore\ V_{2H} = -V_{3\Omega} = -V_o = -L\frac{di}{dt} = -2(-2e^{-t}) = 4e^{-t}$

$V_o = 4e^{-t}\ V\ ;\ t > 0$

$\Rightarrow V = IR\ ;\ V/R = I\ ;\ i_o = \frac{4e^{-t}}{6} = -\frac{2}{3}e^{-t}\ A\ ;\ t > 0$

When calculating the voltage of the inductor; $V(t) = L\frac{di}{dt}$; the $\frac{di}{dt}$ is negative (-ve), since the current after $t > 0$, is an exponential decay (decreasing).

Assuming the capacitor is initially charged: with a initial voltage, V_o

$$\Rightarrow v(t) = \begin{cases} V_o & t < 0 \\ V_s + (V_o - V_s)e^{-t/\tau} & t > 0 \end{cases}$$

where V_s = constant dc voltage source

V_o = initial voltage on the capacitor

If we assume that the capacitor is initially uncharged, then, $V_o = 0$;

$$\Rightarrow v(t) = \begin{cases} 0 & t < 0 \\ V_s(1 - e^{-t/\tau}) & t > 0 \end{cases}$$

Complete response = TRANSIENT RESPONSE (temporary part) + STEADY-STATE RESPONSE (permanent part)

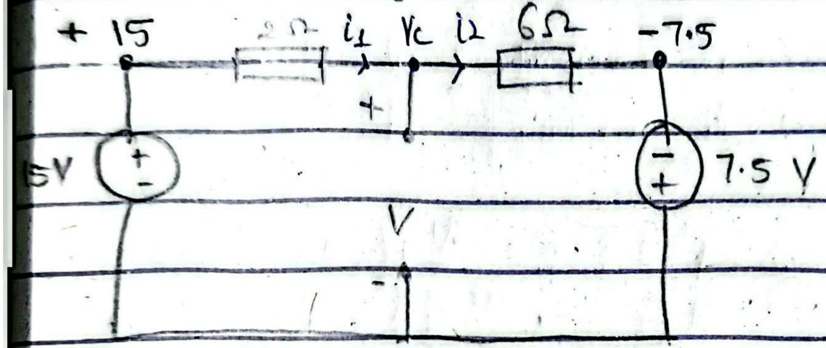
$$\Rightarrow \boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}}$$

where $v(0)$ = initial capacitor voltage

$v(\infty)$ = final capacitor voltage

τ = time constant

When $t = \infty$;



Applying KCL at node, V_c ;

$\Rightarrow i_1 = i_2$

$$\frac{15 - V_c \cancel{V_c - 15}}{2} = \frac{V_c - (-7.5)}{6} = \frac{V_c + 7.5}{6}$$

$$\Rightarrow \frac{15 - V_c \cancel{V_c - 15}}{2} = \frac{V_c + 7.5}{6}$$

$$90 - 6V_c \cancel{6V_c - 90} = 2V_c + 15$$

$$\cancel{-6V_c - 2V_c = 15 + 90}$$

$$90 - 15 = 2V_c + 6V_c$$

$$75 = 8V_c$$

$$\therefore V_c = 9.375 \text{ V}$$

For the step response of an RL circuit ;

$$\Rightarrow \boxed{i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}}$$