SCHOOL OF PHYSICS

FINAL YEAR PROJECT INTERIM REPORT

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1 Introduction and Literature Review

The study of complex systems, particularly those exhibiting collective behaviour arising from interactions among their constituent elements, has been a focal point across various scientific disciplines. The Ising model, a cornerstone in statistical mechanics, has been pivotal in exploring these phenomena. Initially introduced to model ferromagnetic materials, where spins on a lattice interact to result in a phase transition at a critical temperature [1], the model has since transcended its origins to understand diverse systems, ranging from neural networks to social dynamics [2, 3].

In the realm of economics, the Ising model provides a framework for modelling the binary decisions of agents, akin to the binary spin states in physical systems. This analogy extends naturally to financial markets, where the myriad of transactions and the interplay between various financial instruments can be thought of as a complex system with agents' decisions influencing market movements [4].

The intricate web of correlations present in financial data poses a challenge for traditional analysis techniques, which often fall short in capturing the underlying dynamics governing market behaviour. The Ising model, with its simplicity in modelling interactions and its power to explain emergent properties, offers an alternative approach. By framing market dynamics as a series of pairwise interactions within a network, the Ising model enables the identification of critical points akin to phase transitions, providing insight into the conditions leading to market stability or volatility [5].

The project at hand leverages these theoretical foundations and computational techniques to dissect the forex market, a quintessential example of a complex, adaptive system. By applying the Ising model to the forex market, the study seeks to unveil the subtle intricacies of currency interactions, paving the way for novel interpretations and strategies in financial analysis [6].

Moreover, the versatility of the Ising model, especially in the realm of inverse problems, extends its applicability far beyond traditional physics. This expansion into data science highlights the model's potential for extracting meaningful patterns from complex datasets, a principle that is central to our study of financial markets [7]. Similarly, the adaptability of the model in drawing parallels between seemingly disparate systems like neural networks and financial markets provides a unique perspective on understanding the stochastic nature of financial interactions, further justifying the application of the Ising model in our study [8]. The insights from these works not only underscore the methodological robustness of our approach but also situate our research within a broader, interdisciplinary scientific discourse.

2 Background Theory

2.1 Inverse Ising Inference

Inverse Ising inference is a technique employed mainly in statistical physics, computational biology, and machine learning to infer the parameters of an Ising model based on observed data [7]. The Ising model is commonly used to describe systems of interacting spins (binary variables) on a lattice, where each spin can take on one of two values, typically +1 or -1. The model's elegance lies in its simplicity, encapsulating the essence of phase transitions through local interactions between adjacent spins and an external magnetic field [9].

Mathematically, the energy of a configuration $\mathrm{s} = (s_1, s_2, \ldots, s_N)$ in an Ising model is given by the Hamiltonian function:

$$H = -\sum_{i,j} J_{ij} s_i s_j - \sum_i h_i s_i \tag{1}$$

where J_{ij} are the coupling constants describing the interaction between spins s_i and s_j, and h_i are external magnetic fields acting on each spin s_i. The goal of inverse Ising inference is to determine the values of parameters J_{ij} and h_i that best represent the observed data.

The problem is computationally challenging due to the large number of possible configurations (2^N for N spins), making it difficult to directly optimise the likelihood function. As a result, approximate methods, such as pseudo-likelihood maximisation, mean-field approximation, and Monte Carlo methods, have been developed to infer these parameters [7].

The successful application of these methods opens the door to not only a deeper understanding of financial markets but also to the potential for predicting critical transitions, thus contributing to a more robust financial system [10]. It is also employed in computational neuroscience to model the collective behaviour of genes or neurons [2], and in social network analysis to unravel the patterns of interaction within communities [8].

2.2 From Spins to Currencies

In translating the Ising model to foreign exchange markets, each 'spin' can be conceptually adapted to represent the directional movements in the value of a currency pair. The forex market, resembling a lattice, consists of interconnected currencies whose correlated movements can be modeled by the interaction strength J_{ij} in the equation (1). The external field h_i in financial adaptation corresponds to the impact of broader economic trends or policies on an individual currency pair [5].

The critical insight provided by the Ising model is the emergence of collective behaviour from simple rules of interaction. In financial markets, this translates to the phenomenon where global market trends emerge from the aggregate of individual currency fluctuations [11]. Identifying phases akin to ferromagnetic or paramagnetic states can signal periods of market stability or volatility, offering a predictive edge in financial analysis.

2.3 Challenges and Computational Techniques

In the pursuit of addressing the inverse Ising inference, our primary objective is to efficiently and accurately determine the interaction strengths $\hat{J}$ and external fields $\hat{h}$. One of the fundamental challenges in using the Ising model for financial data analysis is the computation of the partition function, a key component in the maximum likelihood estimation. This function, which requires summing over all possible states of the system, becomes computationally intractable with the increase in the number of variables, such as the 26 currencies in our study.

Moreover, the maximum likelihood estimation in the Ising model typically involves extensive Monte Carlo sampling to approximate the partition function. This sampling, which entails generating a large number of states and calculating the corresponding probabilities, is computationally expensive and time-consuming, particularly for large systems. The computational intensity and complexity inherent to both the partition function and the Monte Carlo sampling, as discussed, necessitate the consideration of an alternative computational strategy.

The pseudo-likelihood maximisation approach, as detailed by Aurell and Ekeberg [12], addresses these challenges by circumventing the direct computation of the partition function. Instead, it focuses on conditional probabilities, thus simplifying the computational process. The approach is encapsulated in the following objective function, where we aim to maximise this expression to estimate the model parameters:

$$\hat{J}, \hat{h} = \arg\max_{J,h} \sum_{m=1}^{M} \sum_{i=1}^{N} \left[s_i^m \left(h_i + \sum_{j \neq i} J_{ij} s_j^m \right) - \log \left(2\cosh \left(h_i + \sum_{j \neq i} J_{ij} s_j^m \right) \right) \right] \tag{2}$$

In this equation, $\hat{J}$ and $\hat{h}$ represent the estimated interaction matrix and external field vector, respectively. The summation over m runs through the different observational time points, and the summation over i runs through the different currency pairs. s_i^m denotes the state (e.g., increase or decrease in value) of currency pair i at time m.

In conclusion, the utilisation of pseudo-likelihood maximisation in our study strategically enhances our capability to decode the elaborate network of interactions in financial markets. This method efficiently handles the intricacies of correlation and interaction within financial data while providing a balance between computational feasibility and accuracy, contributing a novel perspective to the understanding of financial market dynamics.

3 Methodology

The methodology for analysing financial data using the Ising model was structured into three main stages: data preprocessing, parameter optimisation, and visualisation. This approach was designed to maintain data integrity, foster contextual understanding, and ensure compatibility with the Ising model's requirements.

3.1 Data Preprocessing

This phase was designed to convert historical forex data across multiple currency pairs into a binary format apt for analysis via the Ising model. Initially, we addressed potential inaccuracies due to trading gaps or errors by interpolating consecutive identical values. Following this, we normalised the data by calculating log returns, a process visually depicted by the transition from (Figure 1) to (Figure 2) and ensured consistency across starting dates, thereby standardising the dataset for uniform analysis.

To provide deeper insights and contextual understanding, we annotated significant financial events directly on the interactive plots (Figure 3). These annotations encompassed economic crises, policy shifts, and other pivotal macroeconomic variables that had historically exerted substantial influence on currency fluctuations. It was in the context of these significant events that outlier detection and cleansing were performed using the Median Absolute Deviation (MAD) method, thus ensuring that the cleansed data reflected true market anomalies and not just fluctuations due to major economic events.

The systematic filtering process not only cleansed the dataset but also embedded it with valuable historical context. The culmination of the preprocessing stage was the transformation of the cleansed dataset into a binary data matrix, where each element represents the daily directional movement of a currency pair, coded as '1' for an increase and '-1' for a decrease, thus aligning it with the Ising model's structural needs.

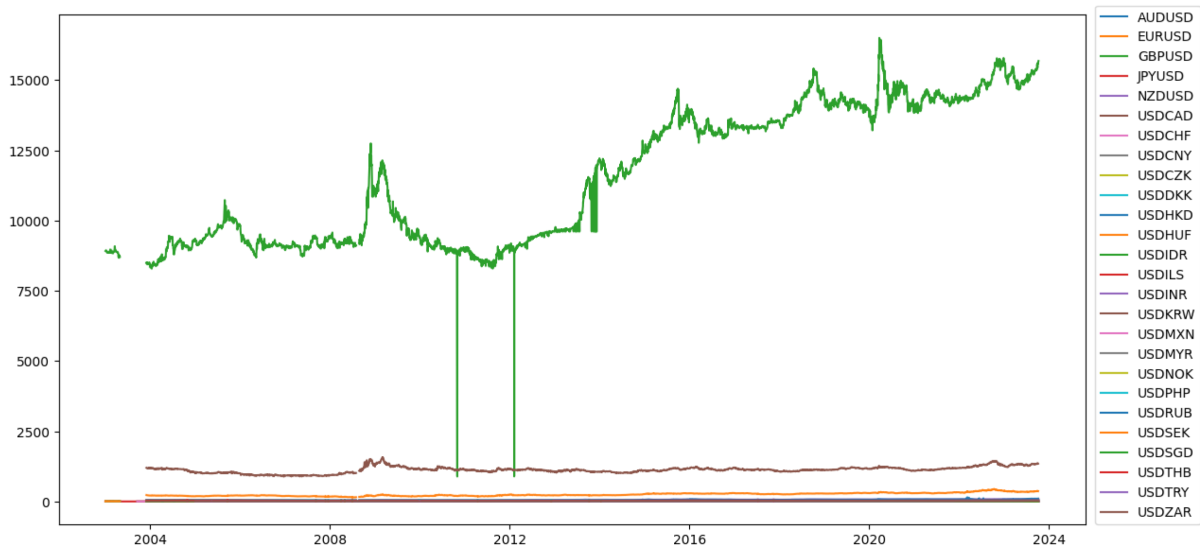


Figure 1: Initial raw data for 26 currency pairs, displaying the scale and scope of the financial data collected.

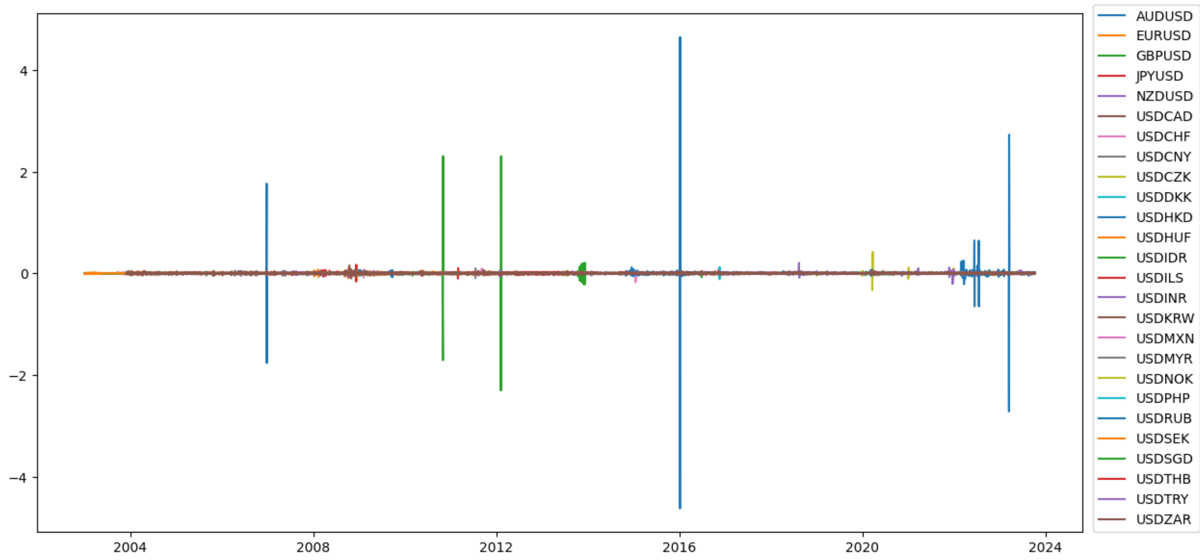


Figure 2: Normalised data using log returns, providing a standardised basis for comparative analysis.

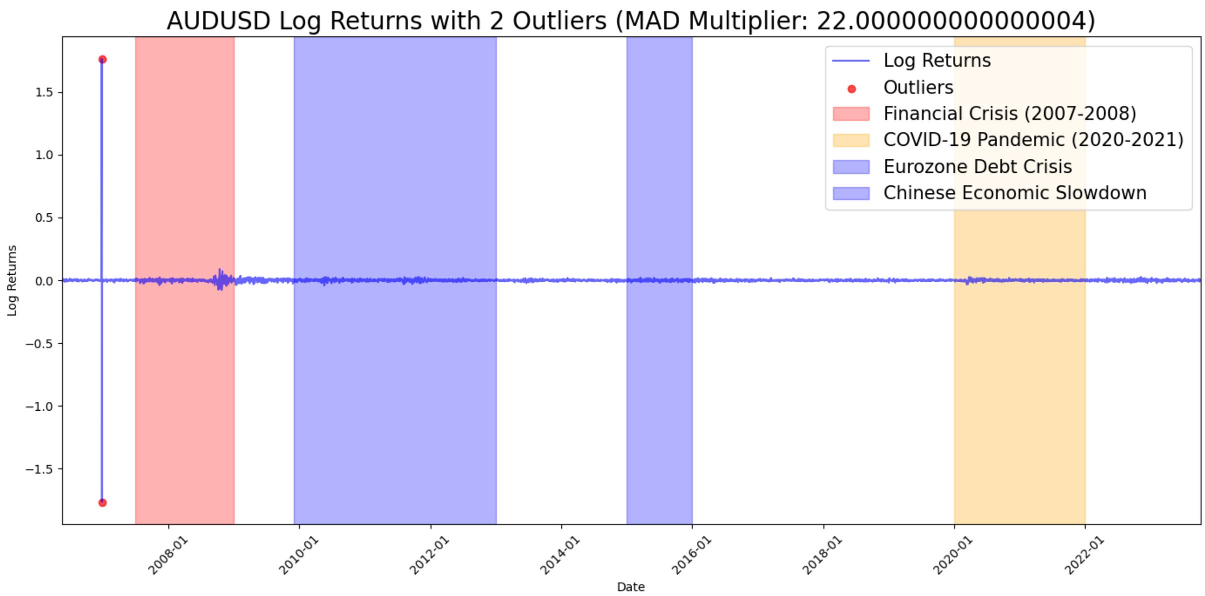


Figure 3: An example interactive plot showcasing outliers for the AUDUSD currency pair, representative of the 26 individual analyses conducted to detect outliers across all studied currency pairs. This plot includes annotations of significant financial events, derived from various government websites, to enhance comprehension of market dynamics.

3.2 Parameter Optimisation

In this critical stage of the methodology, we aimed to determine the optimal interaction matrix (J matrix) and external field vector (h vector), which are fundamental for capturing the complex dynamics within financial markets. To reflect the absence of self-interactions among currency pairs, the diagonal elements of the 26x26 J matrix were constrained to zero.

The optimisation challenge extended to 351 dimensions comprising 325 unique off-diagonal elements in J and 26 elements in h. Initially, the pursuit of optimising the full likelihood estimation met with unanticipated computational challenges. Despite the convexity of the pseudo-

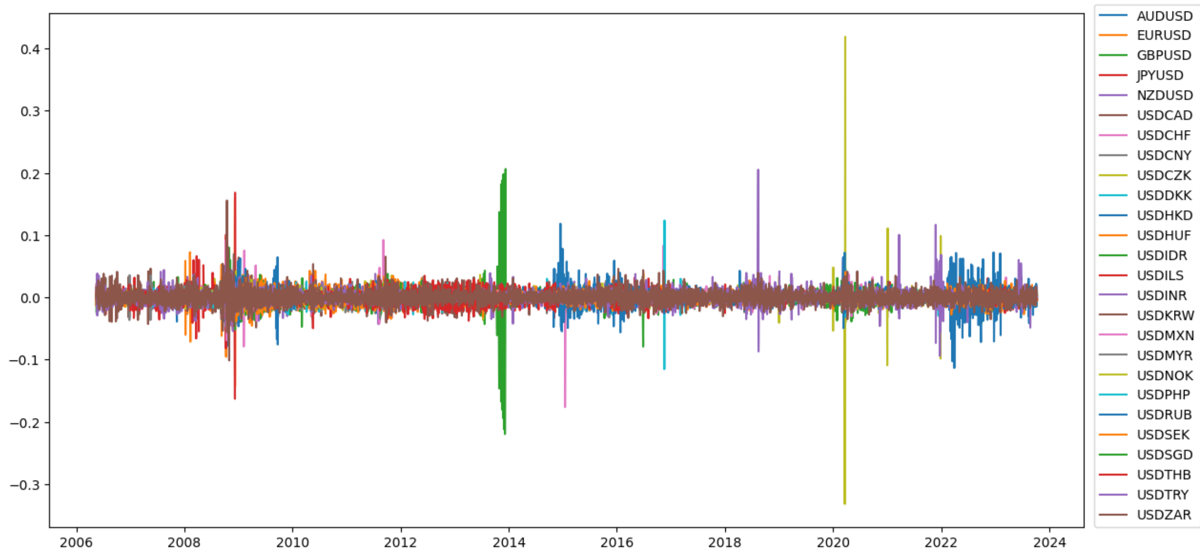


Figure 4: Data after the application of outlier removal techniques, resulting in a cleaned dataset ready for robust Ising model analysis.

likelihood function, convergence eluded us due to the high dimensionality, and the Hessian matrix suggested the absence of a global minimum, presenting a puzzling scenario.

In response to these challenges, we adjusted our strategy. We divided the binary data matrix into smaller subsets, specifically into groups of 7/7/6/6, to promote convergence. This step-by-step optimisation process allowed us to carefully merge the subsets and ensure the successful convergence of the pseudo-likelihood function.

The L-BFGS-B algorithm, chosen for its efficiency in large-scale problems and memory conservation, proved invaluable in navigating the complexity of our high-dimensional space, respecting bound constraints, and achieving a balance between computational speed and accuracy. This methodological pivot not only resolved the initial computational conundrums but also enabled us to reliably estimate the parameters of the Ising model across the complete dataset, as depicted in Figure 5a.

3.3 Visualisation

The visualisation process utilised the optimised J matrix and h vector to create insightful plots, thereby enabling a deeper understanding of the intricate network dynamics represented in the currency data. These included a heatmap of pairwise interaction strengths, a bar chart for external fields, and a network graph showing the interaction structure among currencies (Figures 5 to 7). These visualisations provided a comprehensive perspective on the complex dependencies within currency pairs, modelled by the Ising framework.

4 Results and Discussion

4.1 Analysis of Pairwise Interactions and Correlation Matrix

Investigative analysis revealed a significant correlation between the USDEUR and USDDKK currency pairs, as demonstrated by the pairwise interactions heatmap (Figure 5a) and the time series plots (Figures 5c and 5d). This correlation was partially attributable to the European Exchange Rate Mechanism (ERM II), which ties the Danish krone's value closely to the euro, enforcing a narrow band against the euro and establishing a direct linkage between their respective economic policies and market dynamics [13]. The visual similarity in the fluctuations of USDEUR and USDDKK, as depicted in the time series plots, corroborated the strong correlation coefficients observed in our analysis. The correlation matrix further substantiated this, highlighting the interconnected movements of these currencies and suggesting a common response to economic stimuli (Figure 5b).

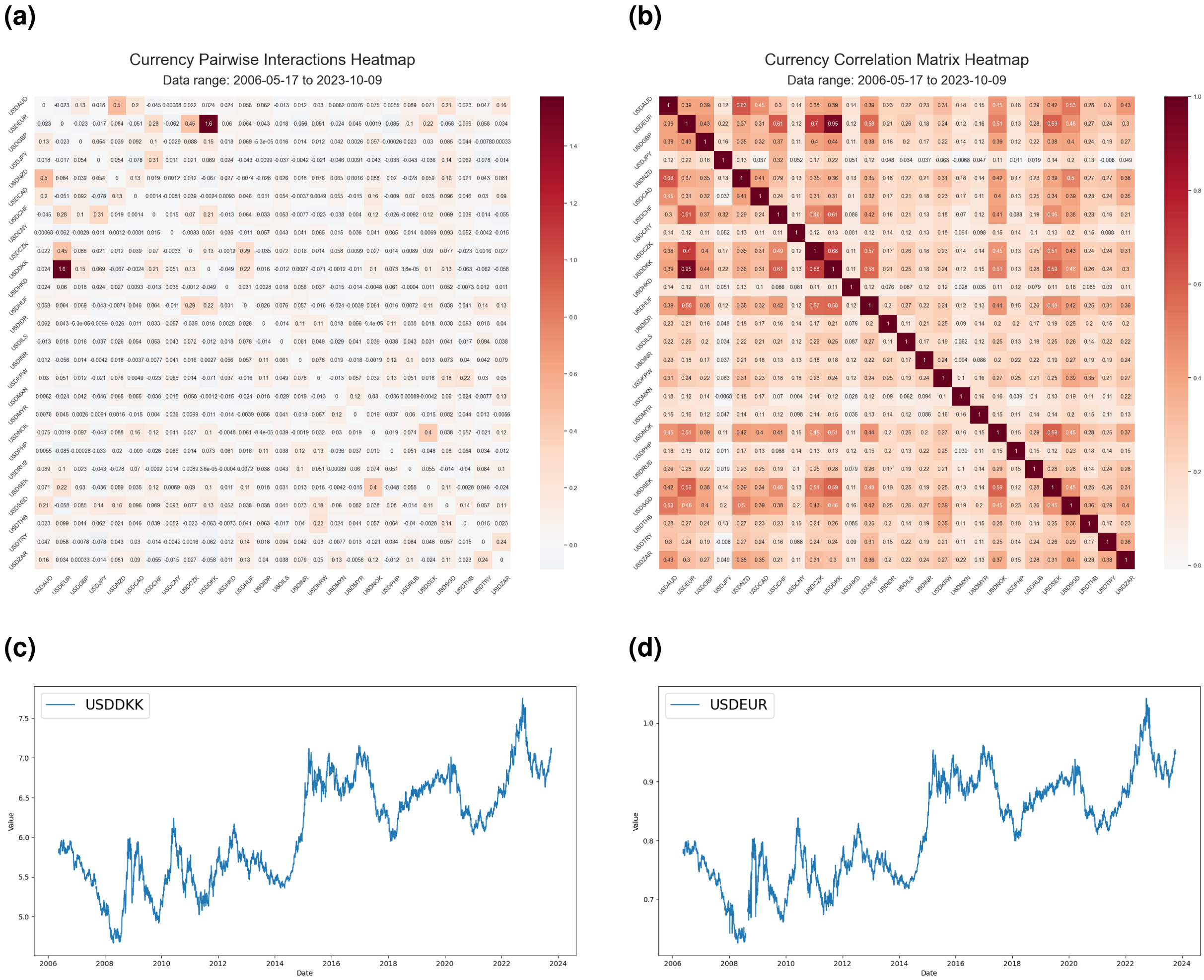


Figure 5: Composite figure of currency interaction and time series analysis: (a) Heatmap of the interaction matrix J and **(b)** Heatmap of the currency correlation matrix, both illustrating the strength of correlations between currency pairs. **(c)** USDDKK and **(d)** USDEUR time series plots showcasing their close correlation over the specified data range

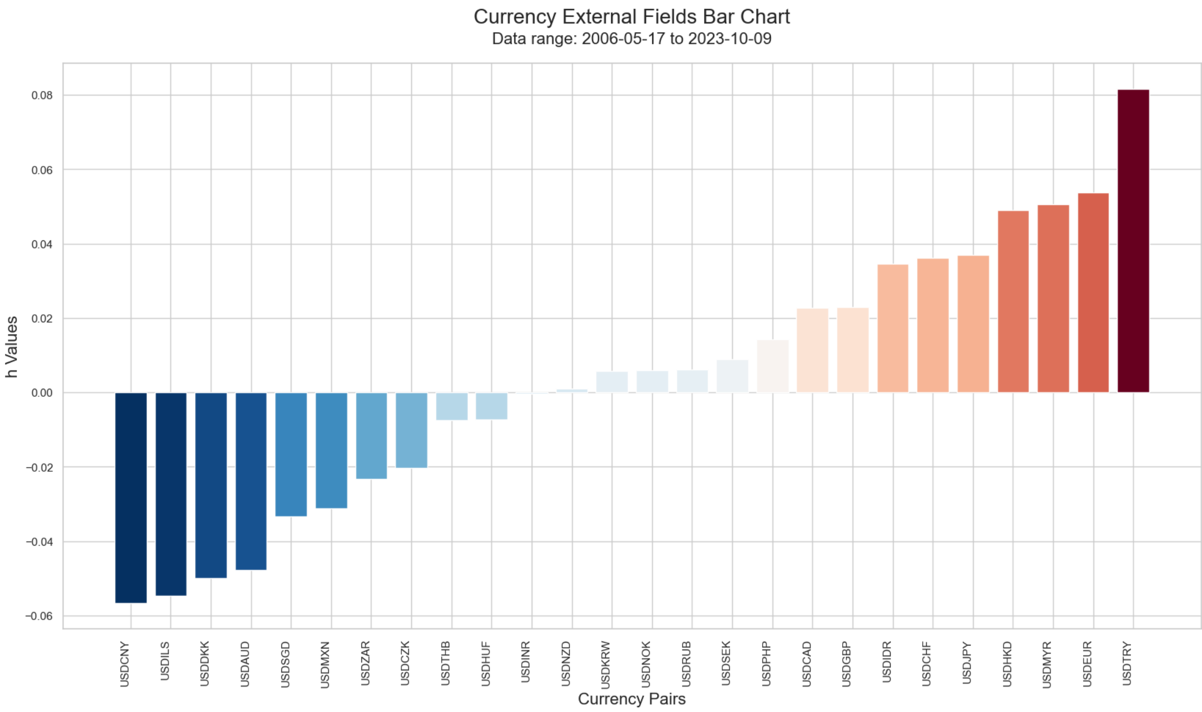
4.2 Analysis of External Fields

The bar chart of the external field vector h in Figure 6a provided a nuanced understanding of the external economic factors influencing currency pair movements. For instance, the USDTRY pair exhibited a strong positive external field, reflecting the impacts of Turkey's inflationary pressures and political uncertainties [14]. This influence was corroborated by the stark upward trend in the USDTRY time series (Figure 6d), signaling the Lira's significant depreciation.

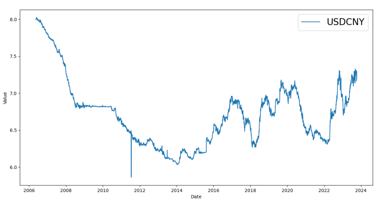
In contrast, the USDCNY pair, with its strong negative external field, mirrored China's policy-driven efforts to maintain the Yuan's stability [15], as seen in the USDCNY time series (Figure 6b). This cyclical pattern underscored the managed float regime of the People's Bank of China, contrasting with the steady, gradual depreciation of the USDINR pair shown in the USDINR time series (Figure 6c).

The neutral external field for USDINR suggested a more balanced response to external forces, likely due to India's consistent economic expansion and the interplay of various economic factors within the observed period.

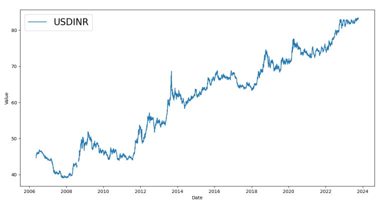
(a)



(b)



(c)



(d)

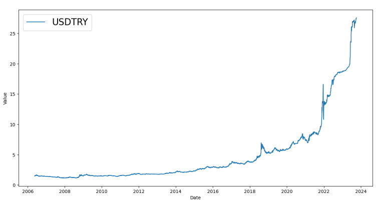


Figure 6: Composite figure of external field vector and time series analysis: (a) Bar chart of the external field vector h indicating the influence of external factors on each currency pair. **(b)** USDCNY, **(c)** USDINR, and **(d)** USDTRY time series plots demonstrating their distinct currency valuation trends over the specified data range.

4.3 Network Analysis of Currency Interactions

The constructed network graph demonstrated the intricate web of currency interactions (Figure 7). Notably, while the USDEUR and USDDKK exhibited significant strength, it was the USDSGD that was posited as the central hub. This was substantiated by its close trailing position behind the top two in terms of strength, and a multitude of economic factors reinforcing its centrality.

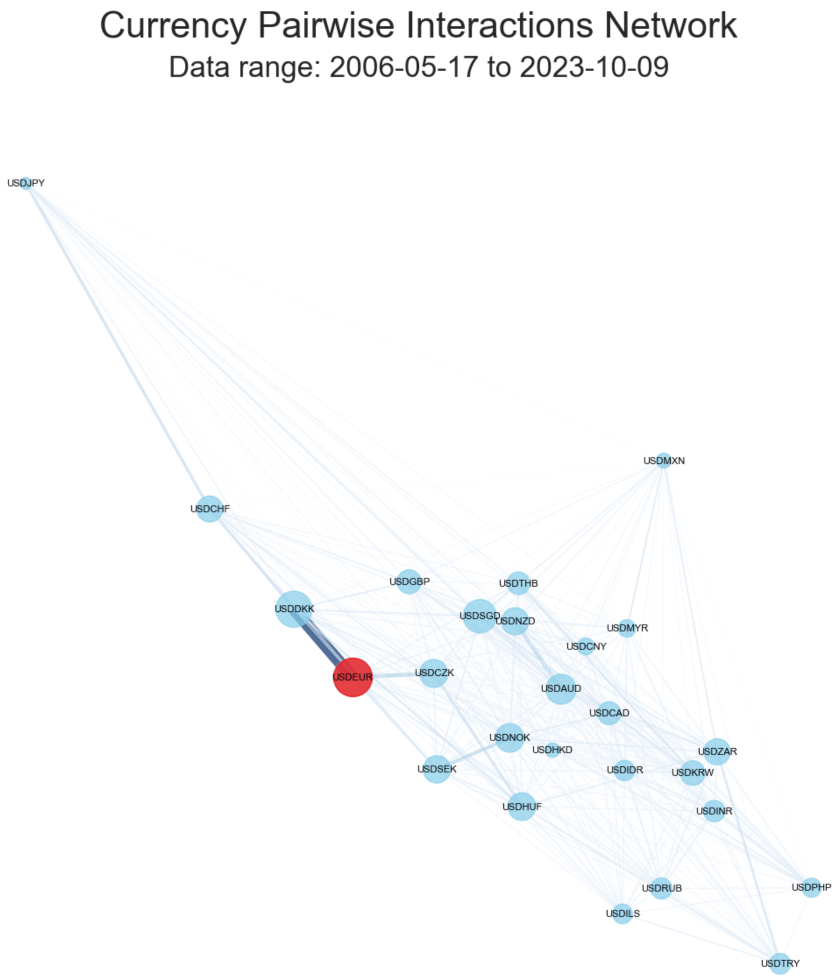


Figure 7: Network graph illustrating the cumulative strength of interactions between major currency pairs. Node sizes are proportional to the strength of each currency, with the red node (USDEUR) indicating the highest strength value of 2.78. The strengths of the top currency pairs are as follows: USDEUR: 2.78, USDDKK: 2.45, USDSGD: 2.08, USDAUD: 1.69, USDNOK: 1.53. The highlighted red node signifies the currency pair with the greatest cumulative interaction strength, which, upon further analysis, suggests the central hub is more likely the USDSGD given its robust trade and financial significance.

The USDSGD occupied a pivotal position due to Singapore's global trade prominence and bustling port activities that directly impacted the exchange rate. As a principal financial center in Asia-Pacific, it anchored a multitude of international banking and finance corporations, reinforcing its vital role in forex trading. Historically, Singapore's strategic location has served as a vital trade hub, a legacy that has transitioned into its current financial clout. Geographically at the heart of key shipping routes, its significance in financial markets echoed across Southeast

Asian economic patterns. Moreover, the high liquidity and trading volume of the USDSGD pair highlighted its extensive economic engagements [16].

Thus, the network graph not only illustrated the interconnectedness of currency pairs but also highlighted the pivotal role of USDSGD in the global currency exchange landscape.

4.4 Concluding Remarks

These findings articulated the multifaceted dynamics that govern currency valuations, influenced by domestic fiscal policies, international trade relationships, and broader economic conditions. While the observed correlations and external field values offered predictive insights for financial strategising, they also underscored the complex interdependencies within the global currency market.

5 Future Research Plans

5.1 Deciphering the Hidden Variable Governing USD

A paramount question arises: what governs the USD in the global financial landscape? Identifying this hidden variable is critical. Employing Boltzmann machines offers a promising avenue to unravel this enigma, allowing us to model complex systems where USD acts as a central node influenced by potentially discoverable factors [17].

5.2 Transitioning from Ising to Potts Model

The Ising model has provided initial insights; however, the Potts model could offer a more nuanced representation of economic behaviour [6]. By capturing interactions across multiple states, the Potts model may yield a better understanding of the multifaceted nature of currency markets.

5.3 Period-Specific Time Series Analysis

Time series analysis could be segmented into distinct economic epochs: pre-financial crisis, during the financial crisis, post-financial crisis (leading up to the COVID-19 pandemic), during the pandemic, and the post-pandemic era. This granular approach will enable us to examine the interaction matrix J across these periods, potentially revealing how specific temporal contexts induce phase transitions in currency market behaviours [2, 12]. Identifying such transitions could be pivotal for economic forecasting and policy-making.

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