INDIAN INSTITUTE OF TECHNOLOGY PALAKKAD

COMPUTER SCIENCE AND ENGINEERING
Indian Institute of Technology Palakkad
CS5016: Computational Methods and Applications

07 Mar, 2023

Coding Assignment 5

Least-Square Function Approximations

1. Write a function to compute the polynomial of degree up to n that is the best fit for a given set of points. It should take as its arguments an array of tuples and an integer n, and return an object of type `Polynomial` (developed in the last coding assignment). The function is also expected to produce a plot of the input points along with the best-fit polynomial together.

2. Write a function to compute the polynomial of degree up to n that best approximates the function $\sin(x) + \cos(x)$ in the interval $[0, \pi]$. It should take as its argument an integer n, and return an object of type `Polynomial` (developed in the last coding assignment). The function is also expected to plot the actual and approximate function together.

3. Write a function that uses the enhanced `Polynomial` class (developed in the last coding assignment) to compute the n^{th} *Legendre polynomial*. It should take as its argument an integer n, and return an object of type `Polynomial` that represents the n^{th} *Legendre polynomial*.

4. Write a function to compute the least-square approximation of e^x in the interval $[-1, 1]$ using the first n *Legendre polynomials*. It should take as its argument an integer n, and return an object of type `Polynomial`. Also, plot the actual and approximate function together.

5. Write a function that uses the enhanced `Polynomial` class (developed in the last coding assignment) to compute the n^{th} *Chebyshev polynomial*. It should take as its argument an integer n, and return an object of type `Polynomial` that represents the n^{th} *Chebyshev polynomial*.

6. Write a function to compute the first 5 *Chebyshev polynomials* and numerically demonstrate that they are *orthogonal* with respect the weight function $w(x) = 1/\sqrt{1 - x^2}$ in the interval $[-1, 1]$.

7. Write a function to compute coefficients of the best-fit *Fourier approximation* $S_n(x)$ of function e^x in the interval $[-\pi, \pi]$. It should take as its arguments an integer n, and print the coefficients of $S_n(x)$. The function is also expected to generate a plot with the actual and approximate function together.

8. Using Python's `scipy.fft` package, write a function, with time complexity of $O(n \log n)$, to multiply two large n-digit integers.

End of document