算法设计与分析——第五章作业

● 运行结果

一、 N 皇后问题

图 1 400000 皇后运行结果

n	2000	5000	10000	20000	50000	100000	200000	400000
m	1	1	1	1	1	1	1	1
t(n)	0.015	0.062	0.265	1.078	8.622	37.048	150.570	1125.060
lg t(n)	-1.824	-1.208	-0.577	0.032	0.935	1.569	2.178	3.051
2 <i>lg</i> n	6.602	7.398	8	8.602	9.398	10	10.602	11.204
3 <i>lg</i> n	9.903	11.097	12	12.903	14.097	15	15.903	16.806

表 1 N 皇后问题的运行结果

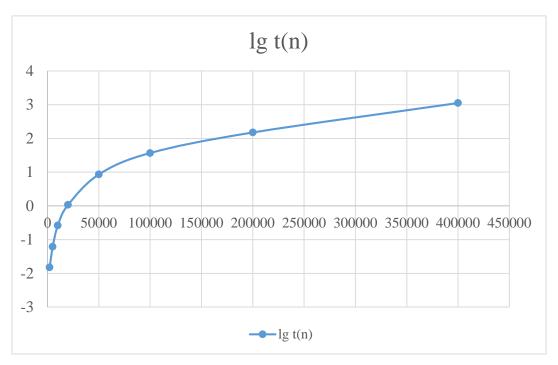


图 2 N 皇后问题中时间 Log t(n)和数量 n 之间的关系

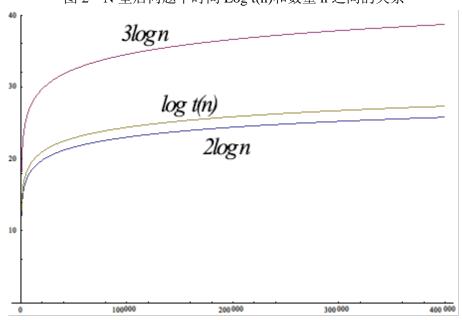


图 3 N 皇后问题中时间 log t(n)和数量 2log n、3log n 之间的关系 经过 Mathematica 拟合以后得到 log t(n)的曲线为:

20.70080791550514 + 2.1190267389563893 Log[x]

由图 3 可以看出 2 < k < 3,经过拟合后得到结果 $k \approx 2.12$,即算法的时间复杂度为 $O(n^{2.12})$ 规模。

二、 图的 M 着色问题

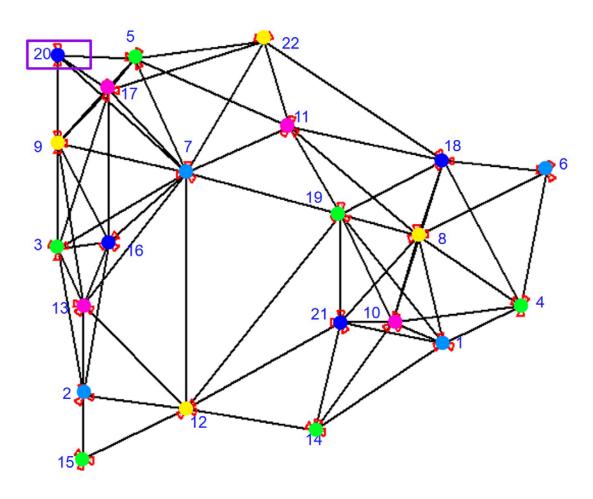
针对 22 个顶点构成的图:

最少需要5个颜色。

着色方案总数为1426560种。

运行总时间为11029毫秒。

遍历的节点总数 47028611 个。



程序输出的结果为:

1:No Ans.

The time is 0

2:No Ans.

The time is 0

3:No Ans.

The time is 0

4:No Ans.

The time is 31

The minimum number of color is 5

The sum is 1426560. And the time is 10998

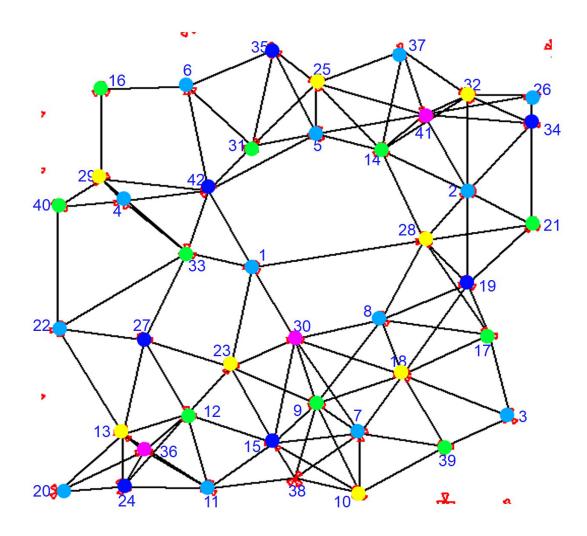
The nodes travelled:47028611

The color of node 12 is 3
The color of node 13 is 4
The color of node 14 is 2
The color of node 15 is 2
The color of node 16 is 5
The color of node 17 is 4
The color of node 18 is 5
The color of node 19 is 2
The color of node 20 is 5
The color of node 21 is 5
The color of node 22 is 3

针对 42 个顶点构成的图:

最少需要5个颜色。

运行总时间为 1.94992x106毫秒。



程序输出的结果:

1:No Ans.

The time is 0

2:No Ans.

The time is 0

3:No Ans.

The time is 37

4:No Ans.

The time is 1.94992e+006

The color of node 1 is 1
The color of node 2 is 1
The color of node 3 is 1
The color of node 4 is 1
The color of node 5 is 1
The color of node 6 is 1
The color of node 7 is 1

The color of node 8 is 1 The color of node 9 is 2

The color of node 10is 3

The color of node 12is 2
The color of node 13is 3
The color of node 14is 2
The color of node 15is 4
The color of node 16is 2
The color of node 17is 2
The color of node 18is 3
The color of node 19is 4

The color of node 20is 1

The color of node 11 is 1

The color of node 21 is 2 The color of node 32is 3 The color of node 22is 1 The color of node 33is 2 The color of node 23is 3 The color of node 34is 4 The color of node 24is 4 The color of node 35is 4 The color of node 25is 3 The color of node 36is 5 The color of node 26is 1 The color of node 37is 1 The color of node 27 is 4 The color of node 38is 5 The color of node 28is 3 The color of node 39is 2 The color of node 29is 3 The color of node 40is 2 The color of node 30is 5 The color of node 41 is 5 The color of node 31 is 2 The color of node 42is 4

三、旅行商问题

针对 22 个顶点构成的图:

最短路径为:

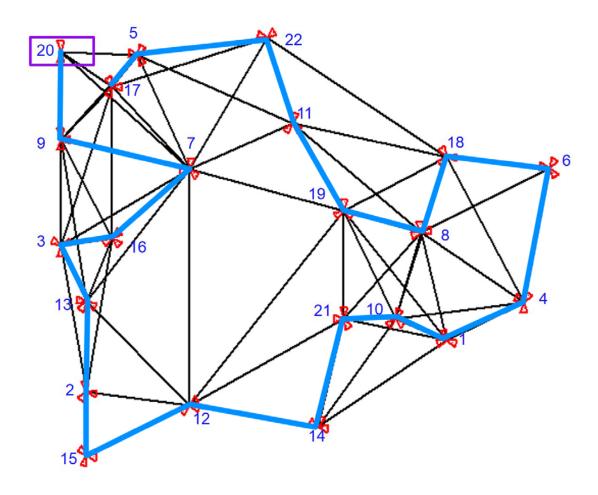
20->9->7->16->3->13->2->15->12->14->21->10->1->4->6->18->8->19-

>11->22->5->17

其总长度为: 7690.8

程序运行总时间为: 26932 毫秒

总遍历节点数: 405818247



程序输出的结果为:

The shortest length is 7690.8

The time cost is 26932

The node travelled: 405818247

The road is:

20->9->7->16->3->13->2->15->12->14->21->10->1->4->6->18->8->19->11->22->5->17

● 源程序代码

二、 图的 M 着色问题

```
#include <iostream>
1
    #include <cstdbool>
3
  #include <cstdio>
  #include <cmath>
4
   #include <ctime>
5
    #include "libxl.h" //用于读取excel文件
6
7
    using namespace libxl; //用于读取excel文件
8
    using namespace std;
9
10
    #define NUM1 22 //第一个文件是22个基站的邻接矩阵
11
    #define NUM2 42
12
13
    int colorNum; //颜色数量
14
   int arc[50][50]; //节点间是否连接
15
    int ans[50]; //可行解
16
    long int sum; //可行解总数量
17
    long long int nodeNum; //遍历节点数
18
    bool isFirst; //是否是第一个可行解
19
20
21
    void backtrack(int t, int num);
   bool ok(int k);
22
23
   //数据和文件处理
24
    int readData(Book* book, int arc[][50], wchar_t
    loadFileName[], int sheetNum, int n);
26
27
28
    int main(void){
       //从excel文件中读取数据
29
       Book* book = xlCreateBook();
30
31
       if (!book){
          cout << "Error when init book." << endl;</pre>
32
33
          return -1;
34
       }
35
    //首先是22个顶点构成的图
36
       //读取数据
37
       wchar_t loadFileName1[] = L"附件1-1.基站图的邻接矩阵-v1.xls";
38
       if (readData(book, arc, loadFileName1, 1, NUM1) <= 0)</pre>
39
40
          return -2;
```

```
41
42
        clock t start, finish;
43
        double duration;
44
        for (colorNum = 1; colorNum <= NUM1; colorNum++){</pre>
45
46
            //初始化
47
            isFirst = true;
            sum = 0;
48
           nodeNum = 0;
49
            start = clock();
50
            //开始遍历
51
52
           backtrack(1, NUM1);
53
            finish = clock();
           duration = (double)(finish - start);
54
            //输出结果
55
            if (sum != 0){
56
               cout << "The minimum number of color is " << colorNum</pre>
57
58
     << endl;
59
               cout << "The sum is " << sum << ".And the time is "</pre>
60
     << duration << endl;
               cout << "The nodes travelled:" << nodeNum << endl <<</pre>
61
62
     endl;
63
               break;
            }
64
65
            else{
               cout << colorNum << ':' << "No Ans." << endl;</pre>
66
67
               cout << "The time is " << duration << endl;</pre>
68
            }
        }
69
70
     //尝试遍历42个顶点
71
72
        Book* book2 = xlCreateBook();
73
        if (!book2){
74
            cout << "Error when init book." << endl;</pre>
75
           return -1;
76
77
        if (readData(book2, arc, loadFileName1, 0, NUM2) <= 0)</pre>
           return -2;
78
79
80
        for (colorNum = 1; colorNum <= NUM2; colorNum++){</pre>
            sum = 0;
81
           nodeNum = 0;
82
83
            start = clock();
84
           backtrack(1, NUM2);
```

```
finish = clock();
85
86
            duration = (double)(finish - start);
87
            if (sum != 0){
               cout << "The minimum number of color is " << colorNum</pre>
88
89
     << endl;
90
               cout << "The sum is " << sum << ".And the time is "
91
     << duration << endl;
               cout << "The nodes travelled:" << nodeNum << endl;</pre>
92
93
               break;
94
            else{
95
96
               cout << colorNum << ':' << "No Ans." << endl;</pre>
97
               cout << "The time is " << duration << endl;</pre>
98
           }
99
        }
100
101
        system("PAUSE");
102
        return 0;
103
     }
104
     void backtrack(int t, int num){ //对所有节点进行回溯染色
105
106
        nodeNum++;
        if (t > num){ //如果t>num, 说明所有的点都已经正确染色, 此时输出结果
107
108
            sum++;
109
            if (isFirst){
               for (int j = 1; j <= NUM1; j++){</pre>
110
                  cout << "The color of node " << j << "is " <<
111
112
     ans[j] << endl;</pre>
113
               }
               isFirst = false;
114
115
            }
116
        }
        else{ //此时还有点未被正确着色
117
            for (int i = 1; i <= colorNum; i++){</pre>
118
               ans[t] = i; //将第t个点染成第i色
119
               if (ok(t)) //如果不与前t个点冲突,说明可行,继续染下一个点
120
121
                  backtrack(t + 1, num);
               ans[t] = 0; //将这个点的颜色置空以回溯
122
            }
123
124
        }
     }
125
126
     bool ok(int k){ //判断第k个节点的颜色是否与前k-1个节点冲突
127
128
        for (int j = 1; j < k; j++)
```

```
if (arc[k][j] && (ans[j] == ans[k]))
129
130
               return false;
131
        return true;
     }
132
133
134
     //数据与文件处理
135
     int readData(Book* book, int arc[][50], wchar_t
     loadFileName[], int sheetNum, int n){ //将数据从excel文件中读出
136
        if (book->load(loadFileName)){ //读取book
137
           cout << "已读取文件。" << endl;
138
        }
139
        else{
140
141
           cout << "读取文件时错误。" << endl;
142
           return -2;
143
        }
        Sheet* sheet = book->getSheet(sheetNum);//读取excel文件中的
144
145
146
        if (sheet) { //将sheet中的数据复制到结构数组中
147
           for (int i = 0; i < n; i++)</pre>
               for (int j = 0; j < n; j++){</pre>
148
149
                  arc[i + 1][j + 1] = (int)sheet -> readNum(i + 2, j +
150
     2);
151
                  if (arc[i + 1][j + 1] == -1)
152
                     arc[i + 1][j + 1] = 0;
153
                  else
                     arc[i + 1][j + 1] = 1;
154
               }
155
156
        }
157
        book->release();
158
        return 1;
159
     }
```

三、旅行商问题

```
#include <iostream>
1
  #include <cstdbool>
2
    #include <cstdio>
3
    #include <cmath>
  #include <ctime>
    #include "libxl.h" //用于读取excel文件
6
7
    using namespace libxl; //用于读取excel文件
8
    using namespace std;
9
10
    #define NUM1 22 //第一个文件是22个基站的邻接矩阵
11
12
13
    double arc[50][50];
  int ans[50];
14
  int bestAns[50];
15
16 double value;
17
   double bestValue;
18
    int nodeNum;
19
20
    void tsp(int num);
    void backtrack(int i, int num);
21
22
23
    //数据和文件处理
24
25
    int readData(Book* book, double arc[][50], wchar_t
    loadFileName[], int sheetNum, int n);
26
27
    int main(void){
28
       //从excel文件中读取数据
29
30
       Book* book = xlCreateBook();
31
       if (!book){
          cout << "Error when init book." << endl;</pre>
32
33
          return -1;
34
       }
35
       //读取数据
36
       wchar_t loadFileName1[] = L"附件1-1.基站图的邻接矩阵-v1.xls";
37
       if (readData(book, arc, loadFileName1, 1, NUM1) <= 0)</pre>
38
          return -2;
39
40
       //计时
41
42
       clock_t start, finish;
```

```
43
        double duration;
44
45
        start = clock();
46
        tsp(NUM1);
        finish = clock();
47
48
        duration = (double)(finish - start);
49
        //输出结果
50
        cout << "The shortest length is " << bestValue << endl;</pre>
51
        cout << "The time cost is " << duration << endl;</pre>
52
53
        cout << "The node travelled: " << nodeNum << endl;</pre>
54
        cout << "The road is: ";</pre>
        for (int i = 1; i <= NUM1; i++){</pre>
55
           cout << bestAns[i] << "->";
56
57
        }
58
        cout << endl;</pre>
59
60
        system("PAUSE");
61
        return 0;
62
    }
63
    void tsp(int num){ //初始化变量并开始回溯过程
64
        for (int i = 1; i <= num; i++)</pre>
65
66
           ans[i] = i;
        bestValue = 10000;
67
        value = 0;
68
69
        nodeNum = 0;
70
        backtrack(2, num);
71
    }
72
    void backtrack(int i, int num){ //回溯遍历各个节点
73
74
        nodeNum++;
75
        int temp;
        if (i == num){ //如果满足条件,则将这个结果记为最优结果
76
77
           if (arc[ans[num - 1]][ans[num]] < 10000 &&</pre>
78
               arc[ans[num]][1] < 10000 &&
79
               (bestValue == 10000 || value + arc[ans[num -
    1]][ans[num]] + arc[ans[num]][1] < bestValue)){</pre>
80
81
               for (int j = 1; j <= num; j++)</pre>
82
                  bestAns[j] = ans[j];
83
               bestValue = value + arc[ans[num - 1]][ans[num]] +
    arc[ans[num]][1];
84
85
86
        }
```

```
else{ //如果不满足条件,交换i、j向下一个节点回溯
87
            for (int j = i; j <= num; j++)</pre>
88
89
               if (arc[ans[i - 1]][ans[j]] < 10000 && (bestValue ==</pre>
     10000 || value + arc[ans[i - 1]][ans[j]] < bestValue)){
90
91
                  temp = ans[i];
92
                  ans[i] = ans[j];
93
                  ans[j] = temp;
94
95
                  value += arc[ans[i - 1]][ans[i]];
96
                  backtrack(i + 1, num);
97
                  value -= arc[ans[i - 1]][ans[i]];
98
99
                  temp = ans[i];
100
                  ans[i] = ans[j];
101
                  ans[j] = temp;
               }
102
103
        }
104
     }
105
     //数据与文件处理
106
     int readData(Book* book, double arc[][50], wchar_t
107
     loadFileName[], int sheetNum, int n){ //将数据从excel文件中读出
108
        if (book->load(loadFileName)){ //读取book
109
110
            cout << "已读取文件。" << endl;
111
112
        else{
113
            cout << "读取文件时错误。" << endl;
114
           return -2;
115
        }
        Sheet* sheet = book->getSheet(sheetNum);//读取excel文件中的
116
117
     sheet2
        if (sheet) { //将sheet中的数据复制到结构数组中
118
119
            for (int i = 0; i < n; i++)</pre>
               for (int j = 0; j < n; j++){
120
121
                  arc[i + 1][j + 1] = sheet->readNum(i + 2, j + 2);
122
                  if (arc[i + 1][j + 1] == -1)
123
                     arc[i + 1][j + 1] = 10000;
               }
124
125
126
        book->release();
        return 1;
127
128
     }
129
```