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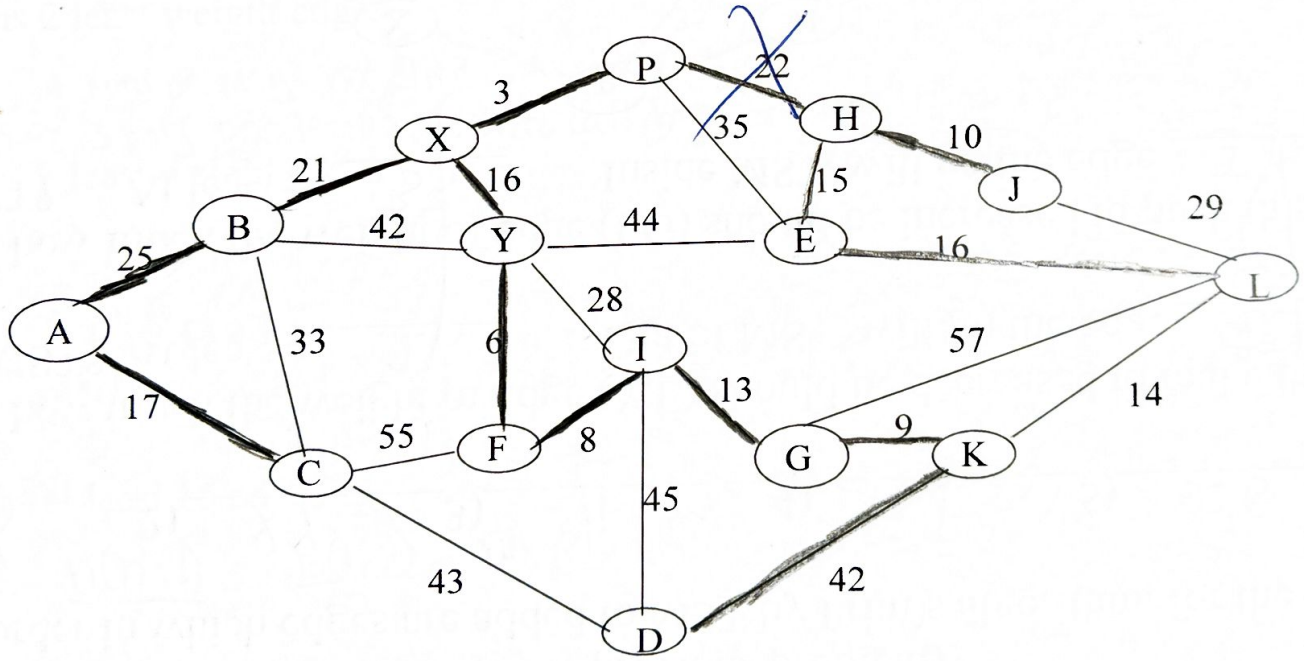
2. Given a graph below:
Draw (color edges or make them thick lines) minimum spanning tree (MST).
Give the order in which edges are added to MST by Kruskal's algorithm, for the first 5 edges:

10 1) PX 2) YF 3) FI 4) GK 5) HJ

5 What are the neighbors in the MST of the node G= K, I and the node I= F, G



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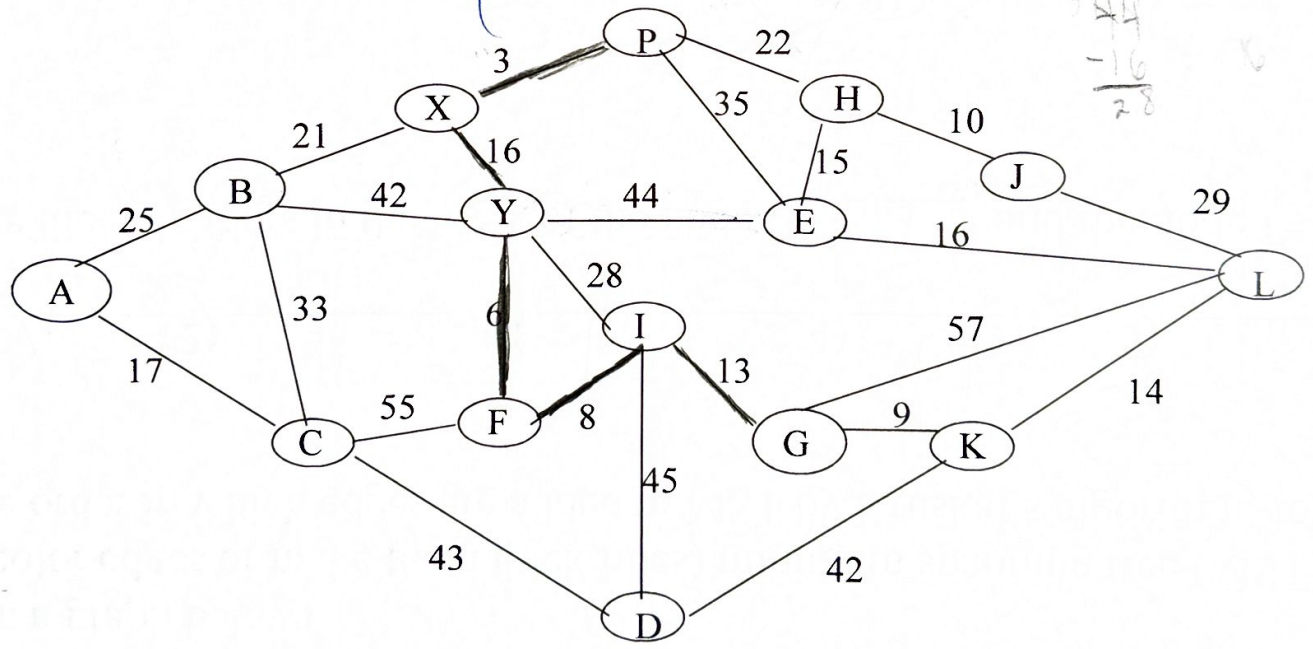
3. Given a graph below (same as for the previous problem)
Give the order in which edges are added to MST by Prim's algorithm, for the first 5 edges:

1) PX 2) XY 3) YF 4) FI 5) IG

• [EC] By how much the weight of edge (Y,E) should be decreased to make this edge added to MST? At least by 28 Out of MST will go the edge YI

• [EC] By how much the weight of edge (I,G) should be increased to push this edges out of MST? At least by 33 Inside MST will go the edge ID



4. Circle whichever applies to given claims, and give explanations

to your answers (why "yes", "no" or maybe why it is impossible to decide either way).
Answers with no explanations = no full credit.

The MST of a connected weighted graph (with no loops and no multiple edges) with 100 edges **always** 101 vertices

a. contains the least weight edge — (Yes) No

because... it has to contain at least one least edge because this is where you start the span tree (at the lowest value)

b. contains 2 least weight edges — (Yes) No ~~Maybe~~

because... it may or may not, like in example #2, in #2 there is only one least edge, there could have been more, but it is impossible to tell unless you know what

c. contains 3 least weight edges — Yes (No)

because... 3 least weight edges will cause a cycle.

d [EC]. has at least 14 edges — (Yes) No

because... $\frac{n(n-1)}{2} = \frac{14(14-1)}{2} = \frac{14 \times 13}{2} = \frac{182}{2} = 91 < 100$ ✓

e [EC]. has at least 15 edges — Yes (No)

because... $\frac{n(n-1)}{2} = \frac{15(15-1)}{2} = 105 > 100$ ✗

$15 \times 14 = 210$

$\frac{210}{2} = 105$

5. Given the following stable marriage instance:

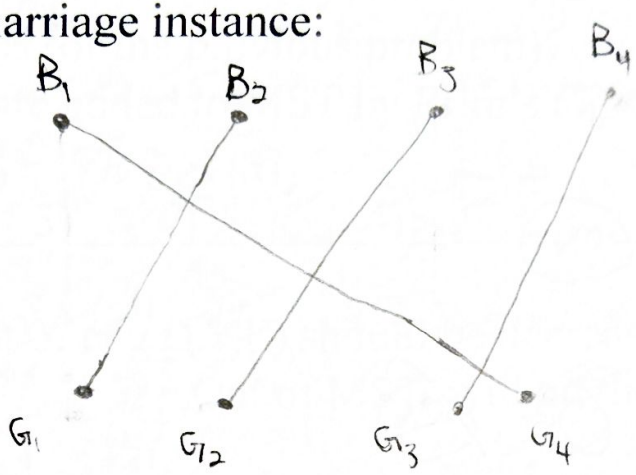
Boys preferences:

B1: G1, G2, G3, G4

B2: G1, G4, G3, G2

B3: G3, G1, G2, G4

B4: G3, G2, G4, G1



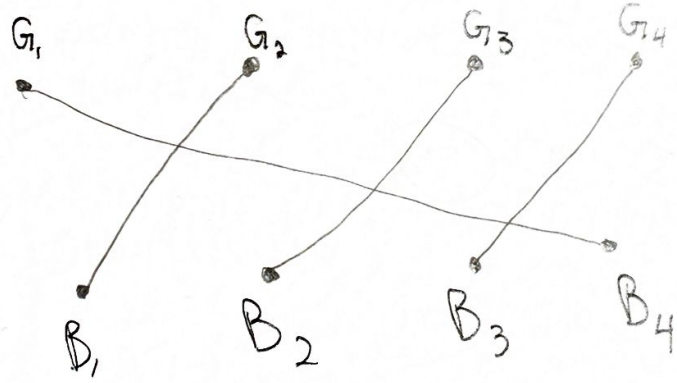
Girls preferences:

G1: B4, B3, B2, B1

G2: B4, B1, B2, B3

G3: B3, B4, B1, B2

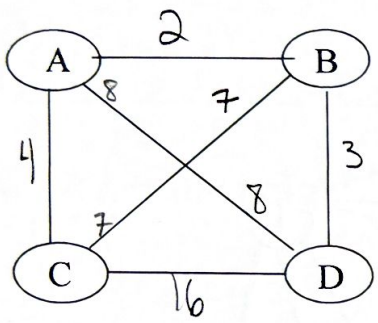
G4: B4, B3, B1, B2



Boys' best stable marriage is: B1 to G 4 ; B2 to G 1 ; B3 to G 2 ; B4 to G 3 .

Girls' best stable marriage is G1 to B 4 ; G2 to B 1 ; G3 to B 2 ; G4 to B 3 .

6. Run Floyd-Warshall algorithm, give all matrices on the way.
In the end, give reasoning at which "city" (=vertex) it would make most sense to build a hospital and why.



M_0

	A	B	C	D
A	0	2	4	8
B	2	0	7	3
C	4	7	0	16
D	8	3	16	0

M_1

	A	B	C	D
A	0	2	4	8
B	2	0	7	3
C	4	7	0	16
D	8	3	16	0

M_2

	A	B	C	D
A	0	2	4	8
B	2	0	6	3
C	4	6	0	16
D	8	3	16	0

M_3

	A	B	C	D
A	0	2	4	8
B	2	0	6	3
C	4	6	0	6
D	8	3	6	0

M_4

	A	B	C	D
A	0	2	4	5
B	2	0	6	3
C	4	6	0	6
D	5	3	6	0

which city?

20p

EC. In the following graph find

Maximum Independent Set See picture