# Scheduling Jobs across Geo-Distributed Datacenters with Max-Min Fairness

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***Abstract*—It has become routine for large volumes of data to be generated, stored, and processed across geographically distributed datacenters. To run a single data analytic job on such geo-distributed data, recent research proposed to distribute its tasks across datacenters, considering both data locality and network bandwidth across datacenters. Yet, it remains an open problem in the more general case, where *multiple* analytic jobs need to *fairly* share the resources at these geo-distributed datacenters. In this paper, we focus on the problem of assigning tasks belonging to multiple jobs across datacenters, with the specific objective of achieving *max-min fairness* across jobs sharing these datacenters, in terms of their job completion times. We formulate this problem as a lexicographical minimization problem, which is challenging to solve in practice due to its inherent multi-objective and discrete nature. To address these challenges, we iteratively solve its single-objective subproblems, which can be transformed to equivalent linear programming (LP) problems to be efficiently solved, thanks to their favorable properties. As a highlight of this paper, we have designed and implemented our proposed solution as a fair job scheduler based on Apache Spark, a modern data processing framework. With extensive evaluations of our real-world implementation on Amazon EC2, we have shown convincing evidence that max-min fairness has been achieved using our new job scheduler.**

## I. Introduction

It is increasingly common for large volumes of data to be generated and processed in a geographically distributed fashion, across multiple datacenters around the world. Popular data analytic frameworks, such as MapReduce [1] and Spark [2], are extensively employed to process such large volumes of data efficiently. A data analytic *job* typically proceeds in consecutive computation *stages*, each of which consisting of a number of computation *tasks* that are executed in parallel. To start a new computation stage, intermediate data from the preceding stage needs to be fetched, which may initiate multiple network flows.

When input data is located across multiple datacenters, a naive approach is to gather all the data to be processed locally within a single datacenter. Naturally, transferring huge amounts of data across datacenters may be slow and inefficient, since bandwidth on inter-datacenter network links is limited [3]. Existing research (*e.g.*, [4], [5]) has shown that better performance can be achieved if tasks in an analytic job can be distributed across datacenters, and located closer to the data to be processed. In this case, designing the best possible task assignment strategy to assign tasks to datacenters is important, since different strategies lead to different flow patterns across datacenters, and ultimately, different job completion times.

When designing optimal task assignment strategies, however, existing works in the literature [4], [5] only considered a single data analytic job. The problem of assigning tasks belonging to multiple jobs across datacenters remains open. Given the limited amount of resources at each datacenter, multiple jobs are inherently competing for resources with each other. It is, therefore, important to maintain *fairness* when allocating such a shared pool of resources, which cannot be achieved if tasks from one job are assigned without considering the other jobs.

In this paper, we propose a new task assignment strategy that is designed to achieve *max-min fairness* across multiple jobs with respect to their performance, as they compete for the limited pool of shared resources across multiple geo-distributed datacenters. To be more specific, we wish to minimize the job completion times across all concurrent jobs, while maintaining max-min fairness. Such a problem can be formally formulated as a *lexicographical minimization* problem, which has unique challenges that make it difficult to solve this problem with multiple objectives. The task assignment problem is essentially an integer optimization problem, which in general is NP-hard [6].

To address these challenges, we first consider the subproblem of minimizing the worst (longest) job completion time among all the concurrent jobs, which turns out to have a *totally unimodular* coefficient matrix for linear constraints, based on an in-depth investigation of the problem structure. Such a nice property guarantees that the extreme points in a feasible solution polyhedron are integers. Moreover, with several steps of non-trivial transformations, we show that the optimal solution to the original problem can be obtained by solving an equivalent problem with a *separable convex objective*. With these structures identified, we can then apply the $\lambda$-technique and linear relaxation to obtain a linear programming (LP) problem, which is guaranteed to have the same solution to the original problem. As a result, any LP solver can be used for minimizing the completion time of each job, and to efficiently compute the overall assignment decisions that achieve the optimal completion times with max-min fairness.

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To demonstrate the practicality of our proposed solution, we have designed and implemented a new job scheduler to assign tasks from multiple jobs to geo-distributed datacenters, in the context of Apache Spark. Our experimental results on multiple Amazon EC2 datacenters have shown that our new scheduler is effective in optimizing job completion times and achieving max-min fairness.

Highlights of our original contributions are as follows. *First*, as motivated by our example in Sec. II, we focus on jointly assigning tasks from multiple data parallel jobs across multiple datacenters, which considers the interplay between these jobs when sharing a limited pool of datacenter resources. *Second*, our problem, formulated as a lexicographical minimization problem in Sec. III, has both the discrete and multi-objective nature that make it challenging to solve. Fortunately, with a careful investigation of its structure, we are able to identify the favorable properties of totally unimodular constraints and a separable convex objective, thus transforming it into an equivalent LP problem to be efficiently and elegantly solved (Sec. IV and Sec. V). *Finally*, to show its practicality, we have completed a real-world implementation of our proposed solution within the Spark job scheduling framework, and conducted extensive evaluations across multiple datacenters in Amazon EC2 (Sec. VI).

## II. Background and Motivation

It is typical for a data analytic job to contain tens or hundreds of tasks, supported by a data parallel framework, such as MapReduce and Spark. These tasks are parallel to or dependent upon each other, and network flows are generated between dependent tasks, since tasks in the subsequent stage need to fetch intermediate data from tasks in the current stage. For example, in a MapReduce job, a set of map tasks are first launched to read input data partitions and generate intermediate results; then reduce tasks would fetch such intermediate data from map tasks for further processing, which involves transferring data over the network.

In the case of running tasks in a data analytic job across multiple datacenters, data may be transferred over inter-datacenter links, which may become bottlenecks due to their limited bandwidth availability. Our design objective in this paper is to compute the best way to assign tasks belonging to *multiple* jobs to geo-distributed datacenters, so that all jobs can achieve their best possible performance with respect to their completion times, without harming the performance of others. This implies that *max-min fairness* needs to be achieved across jobs sharing the datacenters, in terms of their job completion times.

For a better intuition of our problem, we use Fig. 1 to show an example with two data analytic jobs sharing three geo-distributed datacenters. For job A, both of its tasks, tA1 and tA2, require 100 MB of data from input dataset A1 stored in DC1, and 200 MB of data from A2 located at DC3. For job B, the amounts of data to be read by task tB1 from dataset B1 in DC2 and B2 in DC3 are both 200 MB; while task tB2 needs to read 200 MB of data from B1 and 300 MB from B2. These tasks are to be assigned to available computing slots in the three datacenters, each with two slots, two slots, and one slot, respectively. The amounts of available bandwidth of inter-datacenter links are illustrated in the figure, with the unit of MB/s.

For each job, a different assignment of its tasks will lead to flows of different sizes traversing different links, thus resulting in different job completion times. Moreover, both jobs must share and compete for the same pool of computing resources across these datacenters. For example, since DC3 only has one available computing slot, if we assign a task from one job, tasks from the other job cannot be assigned. In order to achieve the best possible performance for both jobs, we need to consider the placement of their tasks jointly, rather than independently.

We have illustrated two ways of assigning tasks to datacenters in Fig. 2 and Fig. 3. Intuitively, DC3 is a favorable location for tasks from both jobs, since they all have part of their input data stored in this datacenter, and the links of DC1-DC3 and DC2-DC3 both have high bandwidth. If the scheduler tries to optimize task assignment of these jobs independently, the result is shown in Fig.2. To optimize the assignment of job A, task tA2 would be assigned to the only available computing slot in DC3, and tA1 would be placed in DC2, which result in a job completion time of $\max\{100/80, 200/160, 100/150\} = 1.25$ seconds. Then, if the scheduler continues to optimize the assignment of job B, DC1 and DC2 would be selected to distribute task tB1 and tB2, respectively, resulting in the completion time of $\max\{200/80, 200/100, 300/160\} = 2.5$ seconds for job B.

However, this placement is not optimal when considering the performance of these jobs jointly. Instead, we show the optimal assignment for both jobs satisfying max-min fairness in Fig. 3. With this assignment, task tB2 of job B would occupy the computing slot in DC3, which avoids the transfer of 300 MB data from dataset B2. Task tB1 is assigned to DC2 rather than DC1, which takes advantage of the high bandwidth of the DC3-DC2 link. As a result, the flow patterns are illustrated in the figure. In this assignment, the completion times of job A and B are $\max\{200/100, 100/80, 200/160\} = 2$ seconds, and $\max\{200/160, 200/120\} = 5/3$ seconds, respectively. Compared with the independent assignment in Fig. 2 where the worst performance is 2.5 seconds, this assignment results in the worst completion time of 2 seconds (job A), which is optimal if we wish to minimize the worst completion time, and is fair in terms of the performance achieved by both jobs.

We are now ready to formally construct a mathematical model to study the problem of optimizing task assignment with max-min fairness across multiple jobs to be achieved.

## III. Model and Formulation

We consider a set of data parallel jobs $\mathcal{K} = \{1, 2, \cdots, K\}$ submitted to the scheduler for task assignment. The input data of these jobs are distributed across a set of geo-distributed datacenters, represented by $\mathcal{D} = \{1, 2, \cdots, J\}$. Each job $k \in \mathcal{K}$ has a set of parallel tasks $\mathcal{T}_k = \{1, 2, \cdots, n_k\}$ to

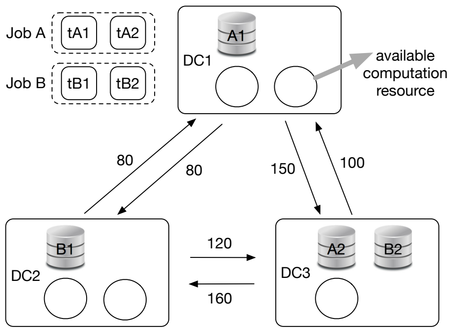

Fig. 1: An example of scheduling multiple jobs fairly across geo-distributed datacenters.

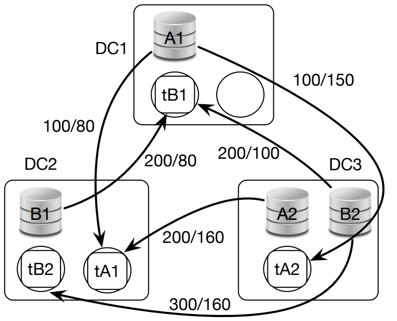

Fig. 2: A task assignment that favors the performance of job A at the cost of job B.

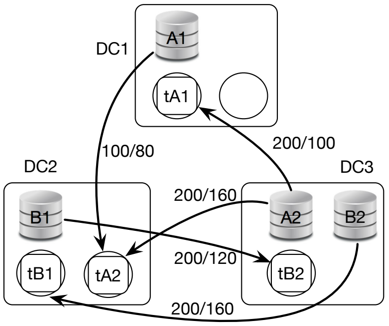

Fig. 3: The optimal assignment for both job A and job B.

be launched on available computing slots in these datacenters. We use $a_j$ to denote the capacity of available computing slots in datacenter $j \in \mathcal{D}$.

For each task $i \in \mathcal{T}_k$ of job $k$, the time it takes to complete consists of both the network transfer time, denoted by $c^k_{i,j}$, to fetch the input data if the task is assigned to datacenter $j$, and the execution time represented by $e^k_{i,j}$. The network transfer time is determined by both the amount of data to be read, and the bandwidth on the link the data traverses. Let $S^k_i$ denote the set of datacenters where the input data of task $i$ from job $k$ are stored, called the source datacenters of this task for convenience. The task needs to read the input data from each of its source datacenters $s \in S^k_i$, the amount of which is represented by $d^{k,s}_i$. Let $b_{s,j}$ [1] represent the bandwidth of the link from datacenter $s$ to datacenter $j$ ($s \neq j$). Hence, the network transfer time of task $i \in \mathcal{T}_k$, if assigned to datacenter $j$, is expressed as follows:

$$c^k_{i,j} = \begin{cases} 0, & \text{when } S^k_i = \{j\}; \\ \max\limits_{s \in S^k_i, s \neq j} d^{k,s}_i / b_{sj}, & \text{otherwise.} \end{cases} \tag{1}$$

which indicates that the network transfer completes when input data from all the source datacenters have been fetched.

The assignment of a task is represented with a binary variable $x^k_{i,j}$, indicating whether the $i$-th task of job $k$ is assigned to datacenter $j$. A job $k$ completes when its slowest task finishes, thus the job completion time of $k$, represented by $\tau_k$, is determined by the maximum completion time among all of its tasks, expressed as follows:

$$\tau_k = \max_{i \in \mathcal{T}_k, j \in \mathcal{D}} x^k_{i,j}(c^k_{i,j} + e^k_{i,j}) \tag{2}$$

As the computing slots in all the datacenters are shared by tasks from multiple jobs, we would like to obtain an optimal task assignment without exceeding the resource capacities. To be more specific, our scheduler would decide the assignment of all the tasks, aiming to optimize the worst performance achieved among all the jobs with respect to their job completion times, and then optimize the next worst performance without impacting the previous one, and so on. This is executed repeatedly until the completion times have been optimized for all the jobs. Such an objective can be rigorously formulated as a *lexicographical minimization* problem, with the following definitions as its basis.

***Definition 1:*** Let $\langle \boldsymbol{v} \rangle_k$ denote the $k$-th ($1 \leq k \leq K$) largest element of $\boldsymbol{v} \in \mathbb{Z}^K$, implying $\langle \boldsymbol{v} \rangle_1 \geq \langle \boldsymbol{v} \rangle_2 \geq \cdots \geq \langle \boldsymbol{v} \rangle_K$. Intuitively, $\langle \boldsymbol{v} \rangle = (\langle \boldsymbol{v} \rangle_1, \langle \boldsymbol{v} \rangle_2, \cdots, \langle \boldsymbol{v} \rangle_K)$ represents the non-increasingly sorted version of $\boldsymbol{v}$.

***Definition 2:*** For any $\boldsymbol{\alpha} \in \mathbb{Z}^K$ and $\boldsymbol{\beta} \in \mathbb{Z}^K$, if $\langle \boldsymbol{\alpha} \rangle_1 < \langle \boldsymbol{\beta} \rangle_1$ or $\exists k \in \{2, 3, \cdots, K\}$ such that $\langle \boldsymbol{\alpha} \rangle_k < \langle \boldsymbol{\beta} \rangle_k$ and $\langle \boldsymbol{\alpha} \rangle_i = \langle \boldsymbol{\beta} \rangle_i, \forall i \in \{1, \cdots, k-1\}$, then $\boldsymbol{\alpha}$ is *lexicographically smaller* than $\boldsymbol{\beta}$, represented as $\boldsymbol{\alpha} \prec \boldsymbol{\beta}$. Similarly, if $\langle \boldsymbol{\alpha} \rangle_k = \langle \boldsymbol{\beta} \rangle_k, \forall k \in \{1, 2, \cdots, K\}$ or $\boldsymbol{\alpha} \prec \boldsymbol{\beta}$, then $\boldsymbol{\alpha}$ is *lexicographically no greater* than $\boldsymbol{\beta}$, represented as $\boldsymbol{\alpha} \preceq \boldsymbol{\beta}$.

***Definition 3:*** $\operatorname*{lexmin}\limits_{\boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x})$ represents the *lexicographical minimization* of the vector $\boldsymbol{f} \in \mathbb{R}^N$, which consists of $N$ objective functions of $\boldsymbol{x}$. To be particular, the optimal solution $\boldsymbol{x}^* \in \mathbb{R}^K$ achieves the optimal $\boldsymbol{f}^*$, in the sense that $\boldsymbol{f}^* = \boldsymbol{f}(\boldsymbol{x}^*) \preceq \boldsymbol{f}(\boldsymbol{x}), \forall \boldsymbol{x} \in \mathbb{R}^K$.

With these definitions, we are now ready to formulate our optimal task assignment problem among sharing jobs as follows:

$$\operatorname*{lexmin}_{\boldsymbol{x}} \quad \boldsymbol{f} = (\tau_1, \tau_2, \cdots, \tau_K) \tag{3}$$

$$\text{s.t.} \quad \tau_k = \max_{i \in \mathcal{T}_k, j \in \mathcal{D}} x^k_{i,j}(c^k_{i,j} + e^k_{i,j}), \forall k \in \mathcal{K} \tag{4}$$

$$\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{T}_k} x^k_{i,j} \leq a_j, \quad \forall j \in \mathcal{D} \tag{5}$$

$$\sum_{j \in \mathcal{D}} x^k_{i,j} = 1, \quad \forall i \in \mathcal{T}_k, \ \forall k \in \mathcal{K} \tag{6}$$

$$x^k_{i,j} \in \{0, 1\}, \quad \forall i \in \mathcal{T}_k, \ \forall j \in \mathcal{D}, \ \forall k \in \mathcal{K} \tag{7}$$

where constraint (5) indicates that the total number of tasks to be assigned to datacenter $j$ does not exceed its capacity $a_j$, which is the total number of available computing slots. Constraint (6) implies that each task should be assigned to a single datacenter.

The objective is a vector $\boldsymbol{f} \in \mathbb{R}^K$ with $K$ elements, each standing for the completion time of a particular job

[1] On popular cloud platforms (e.g., Amazon EC2 and Google Could), inter-datacenter wide-area networks are provided as a shared service, where user-generated flows will compete with millions of other flows. As a result, each inter-datacenter TCP flow will get a fair share of the link capacity. Our measurement with iperf3 on EC2 verifies this assumption.

$k \in \mathcal{K}$. According to the previous definitions, the optimal $\boldsymbol{f}^*$ is lexicographically no greater than any $\boldsymbol{f}$ obtained with a feasible assignment, which means that when sorting them in a non-increasing order, if their $k$-th largest element satisfies $\langle \boldsymbol{f}^* \rangle_{k'} = \langle \boldsymbol{f} \rangle_{k'}, \forall k' < k$ and $\langle \boldsymbol{f}^* \rangle_k \neq \langle \boldsymbol{f} \rangle_k$, then we have $\langle \boldsymbol{f}^* \rangle_k < \langle \boldsymbol{f} \rangle_k$. This implies that the first largest element of $\boldsymbol{f}^*$, *i.e.*, the slowest completion time, is the minimum among all $\boldsymbol{f}$. Then among all $\boldsymbol{f}$ with the same worst completion time, the second worst completion time in $\boldsymbol{f}^*$ is the minimum, and so on. In this way, solving this problem would result in an optimal assignment vector $\boldsymbol{x}^*$, with which all the job completion times are minimized.

## IV. Optimizing the Worst Completion Time among Concurrent Jobs

Problem (3) is a vector optimization with multiple objectives. In this section, we consider the single-objective subproblem of optimizing the worst job performance as follows:

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \max_{k \in \mathcal{K}} \,(\tau_k) \qquad\qquad (8)\\ \text{s.t.} \quad & \text{Constraints } (4), (5), (6) \text{ and } (7). \end{aligned}$$

which is a primary step towards solving the original problem, to be elaborated in the next section.

Substituting the completion time $\tau_k$ in the objective with the expression in constraint (4), we have the following problem with the non-linear constraint (4) eliminated:

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \max_{k \in \mathcal{K}} \,\big( \max_{i \in \mathcal{T}_k, j \in \mathcal{D}} x^k_{i,j}(c^k_{i,j} + e^k_{i,j})\big) \qquad (9)\\ \text{s.t.} \quad & \text{Constraints } (5), (6) \text{ and } (7). \end{aligned}$$

Though this problem is an integer programming problem, we will show that it can be transformed into an equivalent linear programming (LP) problem after an in-depth investigation of its structure. As a result of such a transformation, it can be solved efficiently to obtain the optimal schedule vector $\boldsymbol{x}$. Our transformation takes advantage of its features of *separable convex objective* and *totally unimodular linear constraints*, and it involves three major steps to be elaborated in the following subsections.

### A. Separable Convex Objective

In the first step, we will show that the optimal solution for Problem (9) can be obtained by solving a problem with a separable convex objective function, which is represented as a summation of convex functions with respect to each single variable $x^k_{i,j}$.

We first show that the optimal solution of Problem (9) can be obtained by solving the following problem:

$$\begin{aligned} \operatorname*{lexmin}_{\boldsymbol{x}} \quad & \boldsymbol{g} = (\phi(x^1_{1,1}), \cdots, \phi(x^k_{i,j}), \cdots, \phi(x^K_{n_K,J}))\\ \text{s.t.} \quad & \text{Constraints } (5), (6) \text{ and } (7). \end{aligned}$$

where $\phi(x^k_{i,j}) = x^k_{i,j}(c^k_{i,j} + e^k_{i,j}), \forall i \in \mathcal{T}_k,\ \forall j \in \mathcal{D},\ \forall k \in \mathcal{K}$, and $\boldsymbol{g}$ is a vector with the dimension of $M = |\boldsymbol{g}| = J\sum_{k=1}^K n_k$. For this problem, the objective includes minimizing the maximum element in $\boldsymbol{g}$, which is the worst completion time across all the jobs. Therefore, the optimal assignment variables $\boldsymbol{x}^*$ that gives $\boldsymbol{g}^*$ is also the optimal solution for Problem (9).

Let $\varphi(\boldsymbol{g})$ define a function of $\boldsymbol{g}$:

$$\varphi(\boldsymbol{g}) = \sum_{m=1}^{|\boldsymbol{g}|} |\boldsymbol{g}|^{g_m} = \sum_{m=1}^{M} M^{g_m}$$

where $g_m$ is the $m$-th element of $\boldsymbol{g}$.

***Lemma 1:*** $\varphi(\cdot)$ preserves the order of *lexicographically no greater* ($\preceq$), *i.e.*, $\boldsymbol{g}(\boldsymbol{x}^*) \preceq \boldsymbol{g}(\boldsymbol{x}) \iff \varphi(\boldsymbol{g}(\boldsymbol{x}^*)) \leq \varphi(\boldsymbol{g}(\boldsymbol{x}))$.

*Proof:* We first consider $\boldsymbol{\alpha}, \boldsymbol{\beta} \in \mathbb{Z}^K$ that satisfies $\boldsymbol{\alpha} \prec \boldsymbol{\beta}$. If we use the integer $\tilde{k}(1 \leq \tilde{k} \leq K)$ to represent the first non-zero element of $\langle \boldsymbol{\alpha} \rangle - \langle \boldsymbol{\beta} \rangle$, we have $\langle \boldsymbol{\alpha} \rangle_k = \langle \boldsymbol{\beta} \rangle_k, \forall \tilde{k} \leq k \leq K$ and $\langle \boldsymbol{\alpha} \rangle_{\tilde{k}} < \langle \boldsymbol{\beta} \rangle_{\tilde{k}}$. Assume $\langle \boldsymbol{\alpha} \rangle_{\tilde{k}} = m$, then $\langle \boldsymbol{\beta} \rangle_{\tilde{k}} \geq m + 1$.

$$\begin{aligned} \varphi(\boldsymbol{\alpha}) &= \sum_{k=1}^{K} K^{\langle \boldsymbol{\alpha} \rangle_k} = \sum_{k=1}^{\tilde{k}-1} K^{\langle \boldsymbol{\alpha} \rangle_k} + K^{\langle \boldsymbol{\alpha} \rangle_{\tilde{k}}} + \sum_{k=\tilde{k}+1}^{K} K^{\langle \boldsymbol{\alpha} \rangle_k}\\ &\leq \textstyle\sum_{k=1}^{\tilde{k}-1} K^{\langle \boldsymbol{\alpha} \rangle_k} + K^{\langle \boldsymbol{\alpha} \rangle_{\tilde{k}}} + (K - \tilde{k}) K^{\langle \boldsymbol{\alpha} \rangle_{\tilde{k}}}\\ &= \textstyle\sum_{k=1}^{\tilde{k}-1} K^{\langle \boldsymbol{\alpha} \rangle_k} + (K + 1 - \tilde{k}) \cdot K^{\langle \boldsymbol{\alpha} \rangle_{\tilde{k}}}\\ &< \textstyle\sum_{k=1}^{\tilde{k}-1} K^{\langle \boldsymbol{\alpha} \rangle_k} + K \cdot K^m, \end{aligned}$$

where the first inequality holds as $\langle \boldsymbol{\alpha} \rangle_{\tilde{k}} \geq \langle \boldsymbol{\alpha} \rangle_k, \forall \tilde{k} + 1 \leq k \leq K$.

$$\begin{aligned} \varphi(\boldsymbol{\beta}) &= \sum_{k=1}^{K} K^{\langle \boldsymbol{\beta} \rangle_k} = \sum_{k=1}^{\tilde{k}-1} K^{\langle \boldsymbol{\beta} \rangle_k} + K^{\langle \boldsymbol{\beta} \rangle_{\tilde{k}}} + \sum_{k=\tilde{k}+1}^{K} K^{\langle \boldsymbol{\beta} \rangle_k}\\ &> \textstyle\sum_{k=1}^{\tilde{k}-1} K^{\langle \boldsymbol{\beta} \rangle_k} + K^{\langle \boldsymbol{\beta} \rangle_{\tilde{k}}} + (K - \tilde{k}) \cdot 0\\ &\geq \textstyle\sum_{k=1}^{\tilde{k}-1} K^{\langle \boldsymbol{\beta} \rangle_k} + K \cdot K^m. \end{aligned}$$

Given that $\sum_{k=1}^{\tilde{k}-1} K^{\langle \boldsymbol{\alpha} \rangle_k} = \sum_{k=1}^{\tilde{k}-1} K^{\langle \boldsymbol{\beta} \rangle_k}$, we have proved that $\varphi(\boldsymbol{\alpha}) < \varphi(\boldsymbol{\beta})$.

If $\boldsymbol{\alpha} = \boldsymbol{\beta}$, which means that $\langle \boldsymbol{\alpha} \rangle_k = \langle \boldsymbol{\beta} \rangle_k, \forall 1 \leq k \leq K$, it is trivially true that $\varphi(\boldsymbol{\alpha}) = \sum_{k=1}^{K} K^{\langle \boldsymbol{\alpha} \rangle_k} = \sum_{k=1}^{K} K^{\langle \boldsymbol{\beta} \rangle_k} = \varphi(\boldsymbol{\beta})$. Thus, we have proved $\boldsymbol{\alpha} \preceq \boldsymbol{\beta} \implies \varphi(\boldsymbol{\alpha}) \leq \varphi(\boldsymbol{\beta})$.

We further prove $\varphi(\boldsymbol{\alpha}) \leq \varphi(\boldsymbol{\beta}) \implies \boldsymbol{\alpha} \preceq \boldsymbol{\beta}$ by proving its contrapositive: $\neg(\boldsymbol{\alpha} \preceq \boldsymbol{\beta}) \implies \varphi(\boldsymbol{\alpha}) > \varphi(\boldsymbol{\beta})$. $\neg(\boldsymbol{\alpha} \preceq \boldsymbol{\beta})$ implies $\boldsymbol{\alpha} \neq \boldsymbol{\beta}$ and the first non-zero element of $\langle \boldsymbol{\alpha} \rangle - \langle \boldsymbol{\beta} \rangle$ is positive, which further indicates the first non-zero element of $\langle \boldsymbol{\beta} \rangle - \langle \boldsymbol{\alpha} \rangle$ is negative, *i.e.*, $\boldsymbol{\beta} \prec \boldsymbol{\alpha}$. Thus, the contrapositive is equivalent to $\boldsymbol{\beta} \prec \boldsymbol{\alpha} \implies \varphi(\boldsymbol{\beta}) < \varphi(\boldsymbol{\alpha})$, which has already been proved previously using the exchanged notations of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

With $\boldsymbol{\alpha} \preceq \boldsymbol{\beta} \iff \varphi(\boldsymbol{\alpha}) \leq \varphi(\boldsymbol{\beta})$ holding for any $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ of the same dimension, we complete the proof by letting $\boldsymbol{\alpha} = \boldsymbol{g}(\boldsymbol{x}^*)$ and $\boldsymbol{\beta} = \boldsymbol{g}(\boldsymbol{x})$. ■

Based on Lemma 1, we have

$$\operatorname*{lexmin}_{\boldsymbol{x}} \ \boldsymbol{g} \iff \min_{\boldsymbol{x}} \ \ \varphi(\boldsymbol{g}) = \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{T}_k} \sum_{j \in \mathcal{D}} M^{\phi(x^k_{i,j})}$$

where the objective function $\varphi(\boldsymbol{g})$ is a summation of the term $M^{\phi(x^k_{i,j})}$, which is a convex function of the single variable $x^k_{i,j}$.

Therefore, solving Problem (9) is equivalent to solving the following problem with a separable convex objective:

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{T}_k}\sum_{j\in\mathcal{D}} M^{\phi(x^k_{i,j})} \qquad (10) \\ \text{s.t.} \quad & \text{Constraints } (5), (6) \text{ and } (7). \end{aligned}$$

### B. Totally Unimodular Linear Constraints

In the second step, we investigate the coefficient matrix of linear constraints (5) and (6). An $m$-by-$n$ matrix is *totally unimodular* [6], if it satisfies two conditions: 1) any of its elements belongs to $\{-1, 0, 1\}$; 2) any row subset $R \subset \{1, 2, \cdots, m\}$ can be divided into two disjoint sets, $R_1$ and $R_2$, such that $|\sum_{i\in R_1} a_{ij} - \sum_{i\in R_2} a_{ij}| \le 1, \forall j \in \{1, 2, \cdots, n\}$.

***Lemma** 2:* The coefficients of constraints (5) and (6) form a totally unimodular matrix.

*Proof:* Let $\mathbf{A}_{m\times n}$ denote the coefficient matrix of all the linear constraints (5) and (6), where $m = J + \sum_{k=1}^{K} n_k$, representing the total number of the constraints, and $n = J\sum_{k=1}^{K} n_k$, denoting the dimension of the variable $\boldsymbol{x}$.

It is obvious that any element of $\mathbf{A}_{m\times n}$ is either 0 or 1, satisfying the first condition. For any row subset $R \subset \{1, 2, \cdots, m\}$, we can select all the elements that belong to $\{1, 2, \cdots, J\}$ to form the set $R_1$. As such, $R$ is divided into two disjoint sets, $R_1$ and $R_2 = R - R_1$. It is easy to check that for coefficient matrix of constraint (5), the summation of all its rows, represented by rows $\{1, 2, \cdots, J\}$, is a $1\times n$ vector with all the elements equal to 1. Similarly, for coefficient matrix of constraint (6), the summation of all its rows, represented by rows $\{J+1, J+2, \cdots, J+\sum_{k=1}^{K} n_k\}$, is also a $1 \times n$ vector whose elements are 1. Hence, we can easily derive that $\sum_{i\in R_1} a_{ij} \le 1, \sum_{i\in R_2} a_{ij} \le 1, \forall j \in \{1, 2, \cdots, n\}$. Eventually, we have $|\sum_{i\in R_1} a_{ij} - \sum_{i\in R_2} a_{ij}| \le 1, \forall j \in \{1, 2, \cdots, n\}$, and the second condition is satisfied.

In summary, we have shown that both conditions for total unimodularity are satisfied, thus the coefficient matrix $\mathbf{A}_{m\times n}$ is totally unimodular. ■

### C. Structure-Inspired Equivalent LP Transformation

In the final step, exploiting the problem structure of totally unimodular constraints and separable convex objective, we can use the $\lambda$-representation technique [7] to transform Problem (10) to a linear programming problem that has the same optimal solution.

For a single integer variable $y \in \mathcal{Y} = \{0, 1, \cdots, Y\}$ , the convex function $h : \mathcal{Y} \to \mathbb{R}$ can be linearized with the $\lambda$-representation as follows:

$$h(y) = \sum_{s\in\mathcal{Y}} h(s)\lambda_s, \quad y = \sum_{s\in\mathcal{Y}} s\lambda_s$$
$$\sum_{s\in\mathcal{Y}} \lambda_s = 1, \quad \lambda_s \in \mathbb{R}^+, \ \forall s \in \mathcal{Y}.$$

In our problem, we apply the $\lambda$-representation technique to each convex function $h^k_{i,j}(x^k_{i,j}) = M^{\phi(x^k_{i,j})} : \{0, 1\} \to \mathbb{R}$ as follows:

$$h^k_{i,j}(x^k_{i,j}) = \sum_{s\in\{0,1\}} M^{s(c^k_{i,j}+e^k_{i,j})}\lambda^{k,s}_{i,j} = \lambda^{k,0}_{i,j} + M^{c^k_{i,j}+e^k_{i,j}}\lambda^{k,1}_{i,j}$$

which removes the variable $x^k_{i,j}$ by sampling at each of its possible value $s \in \{0, 1\}$, weighted by the newly introduced variables $\lambda^{k,s}_{i,j} \in \mathbb{R}^+, \forall s \in \{0, 1\}$ that satisfy

$$x^k_{i,j} = \sum_{s\in\{0,1\}} s\lambda^{k,s}_{i,j} = \lambda^{k,1}_{i,j}$$
$$\sum_{s\in\{0,1\}} \lambda^{k,s}_{i,j} = \lambda^{k,0}_{i,j} + \lambda^{k,1}_{i,j} = 1$$

Further, with linear relaxation on the integer constraints (7), we obtain the following linear programming problem:

$$\begin{aligned} \min_{\boldsymbol{x},\boldsymbol{\lambda}} \quad & \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{T}_k}\sum_{j\in\mathcal{D}} (\lambda^{k,0}_{i,j} + M^{c^k_{i,j}+e^k_{i,j}}\lambda^{k,1}_{i,j}) \qquad (11) \\ \text{s.t.} \quad & x^k_{i,j} = \lambda^{k,1}_{i,j}, \ \ \forall k \in \mathcal{K}, i \in \mathcal{T}_k, j \in \mathcal{D} \\ & \lambda^{k,0}_{i,j} + \lambda^{k,1}_{i,j} = 1, \ \ \forall k \in \mathcal{K}, i \in \mathcal{T}_k, j \in \mathcal{D} \\ & \lambda^{k,0}_{i,j}, \lambda^{k,1}_{i,j}, x^k_{i,j} \in \mathbb{R}^+, \ \forall k \in \mathcal{K}, i \in \mathcal{T}_k, j \in \mathcal{D} \\ & \text{Constraints } (5) \text{ and } (6). \end{aligned}$$

***Theorem** 1:* An optimal solution to Problem (11) is an optimal solution to Problem (8).

*Proof:* The property of total unimodularity ensures that an optimal solution to the relaxed LP problem (11) has integer values of $x^k_{i,j}$, which is an optimal solution to Problem (10), and thus an optimal solution to Problem (9) as demonstrated in Sec. IV-A. Moreover, Problem (8) and (9) are equivalent forms, completing the proof. ■

Therefore, the optimal assignment that minimizes the worst completion time among all the jobs can be obtained by solving Problem (11) with efficient LP solvers, such as MOSEK [8].

## V. Iteratively Optimizing Worst Completion Times to Achieve Max-Min Fairness

With the subproblem of minimizing the worst completion time efficiently solved as an LP problem (11), we continue to solve our original multi-objective problem (3) by minimizing the next worst completion time repeatedly.

After solving the subproblem, it is known that the optimal worst completion time is achieved by job $k^*$, whose slowest task $i^*$ is assigned to datacenter $j^*$. We then fix the computed assignment of the slowest task of job $k^*$, which means that the corresponding schedule variable $x^{k^*}_{i^*,j^*}$ is removed from the variable set $\boldsymbol{x}$ for the next round. Also, since task $i^*$ is to be assigned to datacenter $j^*$, it is intuitive that all the assignment variables associated with it should be fixed as zero and removed from $\boldsymbol{x}$: $x^{k^*}_{i^*,j} = 0, \forall j \neq j^*, j \in \mathcal{D}$.

As we have fixed a part of the assignment, the resource capacities should be updated in our problem constraints in the next round. For example, if $x^{k^*}_{i^*,j^*} = 1$, which means that the $i^*$th task of job $k^*$ would be assigned to datacenter $j^*$, then for the problem in the next round, $x^{k^*}_{i^*,j^*}$ is no longer a variable. The resource capacity constraints should be updated as $\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{T}_k,(k,i)\neq(k^*,i^*)} x^k_{i,j^*} \le a_{j^*} - 1$.

Moreover, the completion time of job $k^*$ is obtained as $\phi(x^{k^*}_{i^*,j^*})$, the completion time of its slowest task $i^*$ assigned to datacenter $j^*$, yet the assignment of other tasks has not been

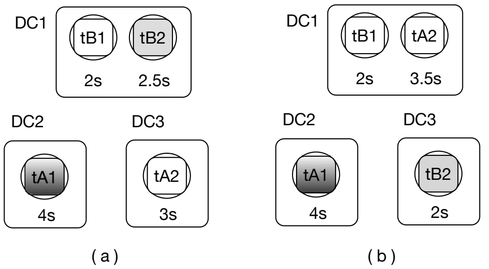

Fig. 4: Two possible assignments of tasks from two jobs.

fixed, which would be the variables $(x_{i,j}^{k^*}, \forall(i,j) \neq (i^*,j^*))$ of the problem in the next round. We set the associated completion times of these variables as $x_{i,j}^{k^*}(c_{i^*,j^*}^{k^*}+e_{i^*,j^*}^{k^*})$. The rationale is that no matter how fast other tasks of $k^*$ complete, the completion time is determined by the slowest task $i^*$. This ensures that the completion time optimized in the next round is achieved by another job, rather than a task of job $k^*$ (other than its slowest). This is better illustrated with the example in Fig. 4.

In this example, after the calculation in the first round, task tA1 assigned in DC2 achieves the worst completion time of 4s, among the two sharing jobs. In the second round, if we do not change the associated completion time for another task tA2, assignment (a) would be calculated as the optimal solution, since it achieves the completion times of $(4, 3, 2.5, 2)$ for the four tasks, which is lexicographically smaller than $(4, 3.5, 2, 2)$ achieved by assignment (b). However, assignment (a) minimizes the next worst completion time for another task of the same job A, which does not match our original objective to optimize the worst completion time for job B. Instead, if we set the associated completion times of all job A's tasks as 4, the objective function achieved by assignment (b) would be $(4, 4, 2, 2)$, better than $(4, 4, 2.5, 2)$ given by assignment (a). In this way, the optimization can correctly choose assignment (b) to achieve the optimal completion time of job B. Note that although the completion time of task tA2 in assignment (b) is longer than in assignment (a), the completion time of job A remains the same, which is 4s.

As a result, the subproblem in the next round is solved over a decreased set of variables with updated constraints and objectives, so that the next worst job completion time would be optimized, without impacting the worst job performance in this round. Such a procedure is repeatedly executed until the last worst completion time of jobs has been optimized, and the max-min fairness has been achieved, as summarized in Algorithm 1.

**Algorithm 1:** Performance-Optimal Task Assignment among Jobs with Max-Min Fairness.

```
Input:
    Input data sizes $d_i^{k,s}$ and link bandwidth $b_{sj}$ to obtain network transfer time $c_{i,j}^k$ (by Eq. 1); execution time $e_{i,j}^k$; datacenter resource capacity $a_j$;
Output:
    Task assignment $x_{i,j}^k, \forall k \in \mathcal{K}, \forall i \in \mathcal{T}_k, \forall j \in \mathcal{D}$;
Initialize $\mathcal{K}' = \mathcal{K}$;
while $\mathcal{K}' \neq \varnothing$ do
    Solve the LP Problem (11) to obtain the solution $\boldsymbol{x}$;
    Obtain $x_{k^*,i^*}^{j^*} = \text{argmax}_{x_{i,j}^k \in \boldsymbol{x}} \, \phi(x_{i,j}^k)$;
    Fix $x_{i^*,j}^{k^*}, \forall j \in \mathcal{D}$; remove them from variable set $\boldsymbol{x}$;
    Update the corresponding resource capacities in Constraints (5);
    Set $\phi(x_{i,j}^{k^*}) = x_{i,j}^{k^*}(c_{i^*,j^*}^{k^*} + e_{i^*,j^*}^{k^*}), \ \forall i \in \mathcal{T}_{k^*}, \forall j \in \mathcal{D}$;
    Remove $k^*$ from $\mathcal{K}'$;
end while
```

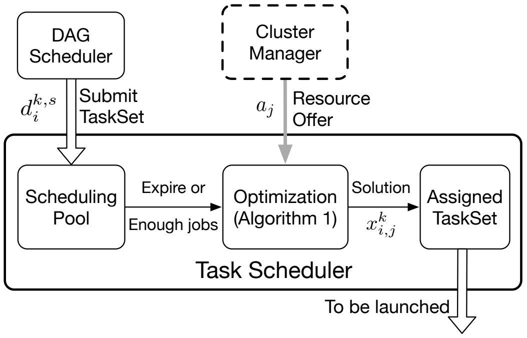

Fig. 5: The implementation of our task assignment algorithm in Spark.

## VI. Real-World Implementation and Performance Evaluation

Having proved the theoretical optimality and efficiency of our scheduling solution, we proceed to implement it in Apache Spark, and demonstrate its effectiveness in optimizing job completion times in real-world experiments.

### *A. Design and Implementation*

In Apache Spark [2], a job can be represented by a Directed Acyclic Graph (DAG), where each node represents a task and each directed edge indicates a precedence constraint. In general, the problem of assigning all the tasks in a DAG to a number of worker nodes, with the objective of minimizing the job completion time, is known as NP-Complete [9]. As a practical and efficient design, Spark schedules tasks stage by stage, which is handled by the *DAG scheduler*. When a job is submitted, it is transformed into a DAG of tasks, categorized into a set of stages. The DAG scheduler will then submit the tasks within each stage, called a TaskSet, to the task scheduler whenever the stage is ready to be scheduled, implying that all its parent stages have completed.

Fortunately, the task scheduler in Spark has access to most of the information that our algorithm needs as its input. As a result, we have implemented our new task assignment algorithm as an extension to Spark's TaskScheduler module. The design of our implementation is illustrated in Fig. 5. In our implementation, as soon as a TaskSet in a job has been submitted by the DAG scheduler to the task scheduler, they will be immediately queued in the scheduling pool. With a number of concurrent jobs waiting to be scheduled, our algorithm will be triggered when a preset timer expires or when the number of pending jobs in the pool exceeds a certain threshold.

To optimize the assignment of tasks, our algorithm requires knowledge about the size and location of the output data

TABLE I: Available bandwidth across geo-distributed datacenters (Mbps).

| | Virginia | Oregon | Ireland | Sing | Sydney | SP |
|---|---|---|---|---|---|---|
| Virginia | 1000 | 169 | 154 | 52 | 53 | 104 |
| Oregon | - | 1000 | 71 | 69 | 77 | 68 |
| Ireland | - | - | 1000 | 49 | 40 | 65 |
| Singapore | - | - | - | 1000 | 58 | 35 |
| Sydney | - | - | - | - | 1000 | 38 |
| San Paulo | - | - | - | - | - | 1000 |

“Sing" is short for “Singapore", and “SP" is short for “San Paulo".

from each map task, represented by $d_i^{k,s}$ in our formulation. Such knowledge can be obtained from the MapOutputTracker, which are further saved in the TaskSet of reduce tasks. The available bandwidth ($b_{sj}$) between each pair of datacenters ($s$ to $j$) can be measured with the iperf2 utility. In addition, information about the amount of available resources, corresponding to $a_j$ in our formulation, can be obtained from the cluster manager. Now that all the input required by our algorithm is ready, an optimal assignment will be computed for all the tasks in the scheduling pool, through iteratively formulating and solving updated versions of linear programming problems, solved by the LP solver in the Breeze optimization library [10].

After the assignment has been computed, it will be recorded in the corresponding TaskSet, overriding the original task assignment preferences. When the tasks are finally submitted for execution, these assignment preferences will be satisfied in a greedy manner. Since the scheduling decisions will satisfy resource constraints by considering *Resource Offers*, each task in the TaskSet can be launched in any available computing slot in the assigned datacenter.

### B. Experimental Setup

We are now ready to evaluate our real-world implementation with an extensive set of experiments deployed across 6 datacenters in Amazon EC2, located in a geographically distributed fashion across different continents. The available bandwidth between each pair of datacenters, measured with the iperf2 utility, is shown in Table I. Compared to the intra-datacenter network where the available bandwidth is around 1 Gbps, bandwidth on inter-datacenter links are much more limited: almost all inter-continental links have less than 100 Mbps of available bandwidth. This confirms the observation that transferring large volumes of data across datacenters is likely to be time-consuming.

In our experiments, we have used a total of 12 on-demand Virtual Machine (VM) instances as Spark workers in our Spark cluster, located across 6 datacenters. Two special VM instances in Virginia (us-east-1) have been used as the Spark master node and the Hadoop File System (HDFS) [11] Name Node, respectively. All instances are of type m3.large, each with 2 vCPUs, 7.5 GB of memory, and a 32GB Solid-State Drive. In each instance, we run Ubuntu Server 14.04 LTS 64-bit (HVM), with Java 1.8 and Scala 2.11.8 installed. Hadoop 2.6.4 is installed to provide HDFS support for Spark. Our own implementation of the task assignment algorithm is based on Spark 1.6.1, a recent release as of July 2016. Our Spark cluster runs in the standalone mode, with all configurations left as default. No external resource manager (*e.g.,* YARN) or database system is activated.

In order to illustrate the efficiency of our task assignment algorithm, we use the legacy Sort application as the benchmark workload. We choose this workload because it is simple but primitive. As one of the simplest MapReduce applications, Sort has only one map and one reduce stage. However, its sortByKey() operation is a basic building block for many complex data analytics applications, especially in Spark SQL. It triggers an all-to-all shuffle, which introduces heavier cross-node traffic than other reduce operations such as reduceByKey().

In our experiments, we have implemented a Sort application with multiple jobs, and submitted it to Spark for execution. The jobs are submitted to Spark in parallel threads, triggering concurrent jobs to share the resources in the cluster. Therefore, the task assignment decisions for these concurrent jobs will be made and enforced by our implementation in the TaskScheduler. To evaluate the performance under different workloads, we run the workload with 3, 4 and 5 concurrent jobs as separate experiments.

For each Sort job in our benchmark application, the default parallelism is set to 3. In other words, the job will trigger 3 reduce tasks to sort the input dataset, which has 3 partitions distributed on 3 randomly chosen worker nodes. The input dataset is prepared as a step in the map task. Each partition of the generated dataset is 100 MB in size, containing 10,000 key-value pairs. Then, as the start of the reduce task, these key-values will be shuffled over the network. Since each datacenter has only two workers, a fraction of the shuffled traffic will be sent over inter-datacenter network links, which are likely to be the performance bottleneck. Our task assignment algorithm is specifically designed to mitigate the negative effects of such bottlenecks.

### C. Experimental Results

We conducted three groups of experiments, with 3, 4, and 5 concurrent jobs, respectively. In the first two groups, each job has three tasks; while in the third group, the total number of tasks of the five jobs is set as the total number of available computing slots, which is 12 in our Spark cluster. In each experiment, the job completion times achieved with our optimal task assignment algorithm is compared with that achieved with the default scheduling in Spark, used as the baseline in our comparison study.

The results of our experiments are presented in Fig. 6 and Fig. 7, showing the worst completion times and the second worst completion times among concurrent jobs, respectively. With respect to the worst completion time, it is easy to see that our algorithm always performs better than the baseline in Spark, with a performance improvement of up to 66%, as shown in Fig. 6. With respect to the second worst completion time, our algorithm does not theoretically guarantee that it is smaller than that achieved with any unfair placement. Even without guarantees, our algorithm always shows better performance than the baseline in Spark. These experiments have

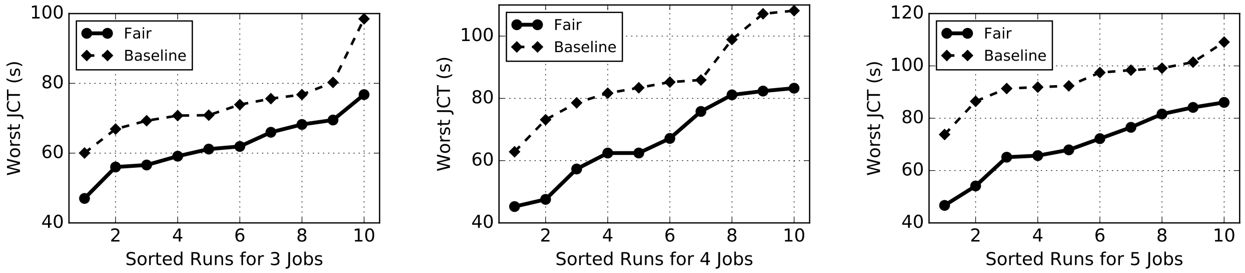

Fig. 6: The worst job completion time in a set of concurrent jobs.

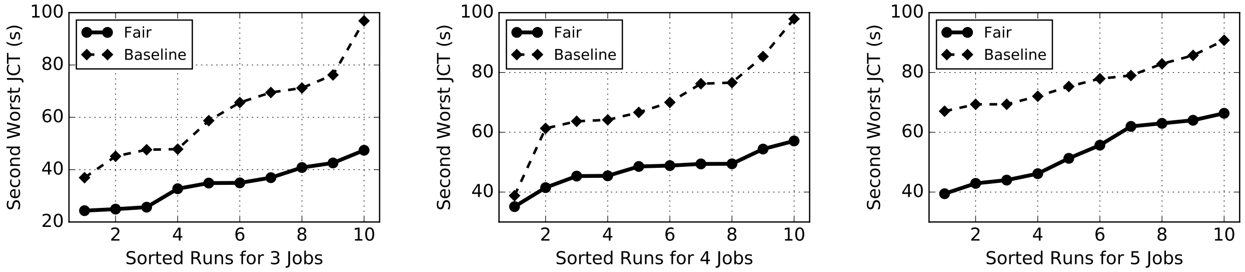

Fig. 7: The second-worst job completion time in a set of concurrent jobs.

shown convincing evidence that our algorithm — optimizing for max-min fairness — is effective in maximizing the worst completion time and achieving best possible performance for all concurrent jobs.

To offer a more in-depth examination and show why such a performance improvement can be achieved, we consider the 7-th run in the second group of our experiment, with the sharing relationship and bandwidth shown in Fig. 8. The six circles represent the six datacenters used in our experiment. A1 is located in the Virginia datacenter, representing the data required by tasks from job A. For each task in job A to be scheduled, a fraction of data needs to be read from all the three datasets (A1, A2 and A3). As each datacenter has two available computing slots, the assignment of 12 tasks from four jobs becomes a one-to-one mapping. In such a limited resource scenario, the assignment of a task is tightly coupled with each other. It becomes more difficult for the default strategy in Spark to find a good assignment, without optimizing across all the tasks. Moreover, the wide range of available bandwidth between datacenters is not taken into consideration by the default scheduling in Spark, which also explains the performance improvement of our strategy over the baseline.

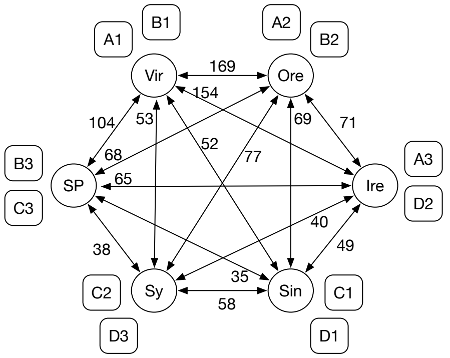

Fig. 8: The location of input data for four jobs across six datacenters.

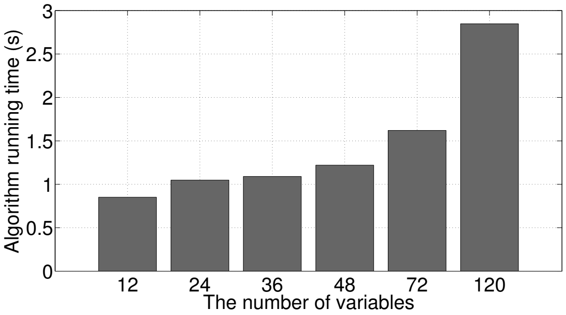

Fig. 9: The computation times of our algorithm at different scales.

Furthermore, to evaluate the practicality of our algorithm, we have recorded the time it takes to calculate optimal solutions. Fig. 9 illustrates the computation times, each averaged over 10 runs, with the number of variables varying from 12 to 120. The linear program in our algorithm is efficient, as it takes about 1 second to obtain the solution for 48 variables. The computation time is less than 3 seconds for 120 variables, which is acceptable compared with the transfer times across datacenters that could be tens or hundreds of seconds. In our experiment, the algorithm is running in the VM with 2 vCPUs. We envision that with more powerful servers for the scheduler in a production environment, the running time could be even smaller.

## VII. Related Work

With increasingly large volumes of data generated globally and stored in geo-distributed datacenters, it has received an increasing amount of research attention to deploying data analytic jobs across multiple datacenters. Based on their objectives, existing efforts can be roughly divided into two categories: reducing the amount of inter-datacenter network traffic to save operation costs, and reducing the job completion time to improve application performance.

Vulimiri *et al.* [3], [12] took the initiative to reduce the amount of data to be moved across datacenters when running geo-distributed data analytic jobs. To reduce the bandwidth cost, they formulated an integer programming problem to optimize the query execution plan and the data replication strategy. They also took advantage of the abundant storage resources to aggressively cache results of queries, to be leveraged by subsequent queries to reduce data transfers. Pixida [13] proposed to divide the DAG of a job into several parts, each to be executed in a datacenter, with the objective of minimizing the total amount of traffic among these divided parts. Despite reducing traffic across datacenters, these solutions do not necessarily shorten job completion times, as bandwidth availability varies across different links and over time.

As a representative work in the second category, Iridium [4] proposed an online heuristic to place both data and tasks across datacenters. Unfortunately, it assumes that the wide-area network that interconnects datacenters is free of congestion, which is far from realistic. Flutter [5] removed this unrealistic assumption, formulated a lexicographical minimization problem of task assignment for a single stage of one job, and obtained its optimal solution. However, all existing works focused on assigning tasks in a single job, without considering the inherent competition for resources among concurrent jobs. Despite using a similar theoretical foundation as [5], our problem considers multiple jobs, and is therefore remarkably different and more challenging.

Accounting for the scenario of multiple jobs sharing geo-distributed datacenters, Hung *et al.* [14] proposed a greedy scheduling heuristic to make job scheduling decisions across geo-distributed datacenters, with an objective of reducing the average job completion time. However, it assumes that the task assignment is predetermined, and the scheduling decision is the execution order of all the assigned tasks in each datacenter. Therefore, despite sharing a similar context of considering multiple jobs sharing the same pool of computing resources in geo-distributed datacenters, this work is orthogonal to our work, which aims to determine the best possible placement for tasks of all the sharing jobs with the consideration of fairness.

There are plenty of existing efforts [15]–[17] related to task assignment and job scheduling in big data analytic frameworks. To reduce job completion times, they proposed to improve data locality and fairness [15], [16], and to mitigate the negative impact of tasks that progress slowly, called stragglers ( [17]). However, they are all designed for frameworks deployed in a single datacenter, and do not work effectively across multiple datacenters.

## VIII. Concluding Remarks

In this paper, we have conducted a theoretical study of the task assignment problem among competing data analytic jobs, whose input data are distributed across geo-distributed datacenters. With tasks from multiple jobs competing for the computing slots in each datacenter, we have designed and implemented a new optimal scheduler to assign tasks across these datacenters, in order to better satisfy job requirements with max-min fairness achieved across their job completion times. To achieve this objective, we first formulated a lexicographical minimization problem to optimize all the job completion times, which is challenging due to the inherent complexity of both multi-objective and discrete optimizations. To address these challenges, we started from the single-objective subproblem and transformed it into an equivalent linear programming (LP) problem to be efficiently solved in practice, based on an in-depth investigation of the problem structure. An algorithm is further designed to repeatedly solve an updated version of the LP subproblems, which would eventually optimize all the job performance with max-min fairness achieved. Last but not the least, we have implemented our performance-optimal scheduler in the popular Spark framework, and demonstrated convincing evidence on the effectiveness of our new algorithm using real-world experiments.

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