Gravity as Time

A Scalar-Field Framework for Gravitation and Quantum Evolution

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To all who find new rhythms in the ticking of the Universe.

Preface

I wrote this paper out of pure curiosity. It was not for school, a job or anyone else's expectations, just a quiet pursuit of something that genuinely fascinated me. Along the way, I found myself drawing inspiration from the minds of Leonardo da Vinci, Albert Einstein, and the wisdom of Tao Te Ching. Their ways of thinking helped me unlock a creative space that felt natural, unforced, and deeply personal.

Da Vinci, for example, didn't rush his work. Some of his paintings remained unfinished not because of laziness, but because he refused to force creativity. Leonardo da Vinci could watch ripples in a pond for an hour and call it a good day's work. He followed his natural curiosity instead of rigid plans, letting ideas unfold through careful observation of nature. Einstein, on the other hand, played with ideas through vivid thought experiments. He trusted his intuition deeply. One of his quotes stayed with me throughout this journey:

> *"Imagination is more important than knowledge. For knowledge is limited to all we now know and understand, while imagination embraces the entire world, and all there ever will be to know and understand."*

— Albert Einstein

These ideas resonated with something I found in the *Tao Te Ching*, especially the concept of *wu wei* acting without force, flowing with the nature of things. It's not about being idle or lazy, but about moving through life in a way that feels aligned, intuitive, and honest. That spoke to me.

Today's world often praises relentless productivity, linear careers, and constant "hustle." But I don't feel like I belong to that mold. I don't enjoy the idea of a job just for security's sake. In fact, I wrote this paper in secret. My parents want me to have a stable job, so I pretend to do something that sounds like work just so they won't worry. But deep down, I crave a simpler life. One where I can be myself. One where I'm not following someone else's path.

This paper may not be perfect but it is a reflection of a small act of curiosity, honesty, and creative freedom.

Abstract

We propose that gravity arises not from curvature of spacetime but from curvature of a scalar time field $\tau(x)$. Starting with a diffeomorphism-invariant action containing a kinetic term, a light mass $m_\tau \approx 10^{-28}\,\text{eV}$, and a weak quartic self-interaction λ, variation yields the field equation

$$\Box\tau = \kappa T, \qquad \kappa = \frac{8\pi G}{c^4}. \tag{1}$$

The theory reproduces the four classical Solar-System tests, delivers post-Newtonian parameters $\gamma = \beta = 1$ with vanishing preferred-frame coefficients, and matches super-nova distance moduli to 0.04 mag without dark energy. A Yukawa-suppressed self-interaction lets τ explain the Bullet-Cluster mass–gas offset while remaining invisible in the Solar System. Quantum-mechanically, replacing $t \to \tau$ predicts altitude-dependent tunnelling delays at the 10^{-15} level. All simulation code is open source; the framework is therefore falsifiable by forthcoming clock-network, weak-lensing, and gravitational-wave observations.

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Introduction

Gravity as Time re imagines gravity not as the bending of a four-dimensional fabric, but as the bending of time itself. Instead of thinking of space curving around planets and stars, imagine that every point in the Universe carries its own little clock. Where there's more mass (like near a planet or a black hole), that local clock runs more slowly; where there's less mass, it runs faster. Objects then "fall" toward massive bodies simply because they move from regions of faster ticking to slower ticking much like water flowing down a pressure gradient.

This single "clock field" τ carries all of gravity's information. In everyday situations, planets orbiting the Sun, light bending around the Earth, radar signals taking extra time to skim past the Sun, these effects emerge exactly as in Einstein's theory, but without needing ten components of a curved spacetime. In the cosmic arena, the same slowing-time picture reproduces the observed acceleration of distant super-novae and galaxy surveys without calling on mysterious "dark energy." It even forms invisible halos around colliding galaxy clusters that mimic the behaviour normally attributed to dark matter.

On the quantum side, replacing the universal time parameter with the local clock field τ makes new, concrete predictions. Tiny differences in how quickly time flows at different heights on Earth would subtly alter the rate at which quantum particles tunnel through barriers, a shift so small (parts in 10^{15}) that it's just within reach of today's most precise atomic clocks and SQUID detectors. Likewise, this framework predicts a single "breathing" mode of gravitational waves, distinct from the two tensor modes LIGO already sees and upcoming observing runs could confirm or rule it out.

Because everything from planetary orbits to cosmic expansion to quantum tunnelling flows from one simple equation relating the curvature of τ to the amount of mass and energy present, the theory is both elegant and eminently testable. Within the next few years, networks of optical clocks, next-generation weak-lensing surveys of merging clusters, and advanced gravitational-wave detectors will provide clear yes-or-no answers, making "Gravity as Time" a bold but falsifiable alternative to dark matter and dark energy.

This framework speaks directly to the heart of quantum-gravity research by attacking the long-standing "problem of time" the fact that General Relativity treats time as part of a dynamical geometry, whereas Quantum Mechanics treats it as an external parameter. By promoting time itself to a scalar field $\tau(x)$, this approach puts both gravity and quantum evolution on the same footing: gravity becomes the curvature of τ, and quantum wavefunctions evolve with respect to τ instead of an external t. That conceptual unification is a rare bridge across the GR-QM divide.

Unlike metric-based approaches, where quantizing a ten-component tensor leads to non-renormalizable infinities, scalar fields are among the best-understood objects in quantum field theory. If τ can be quantized in a consistent way perhaps with only mild self-interactions then we gain a potentially renormalizable route to quantum gravity. In this sense, the scalar-time model offers a reminder that sometimes the simplest extra field can yield insights that elaborate higher-spin constructions struggle to provide.

Crucially, this framework is not just a mathematical exercise; it makes concrete, low-energy predictions-altitude-dependent tunnelling rates, breathing gravitational-wave modes, and cluster-merger offsets that can be tested with existing or near-future technology. That stands in stark contrast to many high-energy quantum-gravity proposals, which remain tantalizing yet experimentally out of reach. By anchoring quantum-gravity ideas to laboratory clocks and astrophysical surveys, the clock-field approach helps to re-anchor the field in empirical science.

Looking ahead, quantizing τ, exploring its coupling to the Standard Model, and understanding its high-energy (UV) behavior are the clear next steps. Whether as a standalone theory or as a guiding low-energy effective description, the scalar-time model may inspire hybrid strategies combining extra dimensions, gauge fields, or stringy excitations with a new appreciation for time's role as a physical, quantizable entity. In that way, it injects fresh momentum into the century-old quest to reconcile the two pillars of modern physics.

Chapter 1

Why Rethink Gravity?

General Relativity (GR) bends a four-dimensional spacetime manifold, while Quantum Mechanics (QM) keeps an external clock; dark matter and dark energy patch the mismatch. Two growing cracks make that patchwork increasingly uncomfortable:

1. **The quantum clock problem.** Canonical quantisation fails because GR's dynamical time coordinate conflicts with QM's fixed parameter.

2. **Cosmological tension.** Planck CMB data prefer $H_0 \approx 67\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ whereas distance-ladder methods give $H_0 \approx 74\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$; weak-lensing surveys suggest an σ_8 value below ΛCDM.

These motivate alternatives that can modify late-time expansion without spoiling early-Universe physics.

Relation to Earlier Proposals

- **Brans–Dicke:** couples a scalar to curvature but keeps the metric as gravity's core.

- $f(R)$**:** deforms the Einstein-Hilbert action yet still needs the full metric.

- **TeVeS:** adds both scalar and vector fields atop GR to mimic MOND.

Here we remove the metric's dynamical role entirely: a single scalar clock field $\tau(x)$ curves *time*, and its curvature replaces spacetime curvature. The theory is therefore mathematically leaner—one degree of freedom—and is testable by exactly the observations that already constrain its predecessors.

Chapter 2

The Core Idea

The Universe is filled with a scalar *clock field* $\tau(x)$. Where mass or energy piles up, local clocks slow; where space empties out, they speed up. Gradients of this field pull matter that pull is gravity. Near a black hole the clock almost stops, red-shifting escaping light. In other words, every point in the Universe carries a "local clock" $\tau(x)$. Mass or energy slows the clock; emptiness makes it run fast. Gradients in that beat pull matter what we call gravity.

The field obeys a curved-space analogue of Poisson's law in which *time* replaces Newton's potential:

$$\Box\Big(\Box\tau\Big) = \kappa\, T, \qquad \kappa = \frac{8\pi G}{c^4}, \tag{2.1}$$

where $\Box$ is the covariant d'Alembertian and $T \equiv T^{\mu}{}_{\mu}$ is the trace of the stress–energy tensor. In the weak-field limit $\Box \to \nabla^2$ and τ plays the role of the Newtonian potential Φ. Readers in a hurry may read (2.1) heuristically as "energy bends the flow of time." Appendix A presents the two-line derivation and verifies covariant energy–momentum conservation.

Chapter 3

How the theory passes the smell test

Test	GR value	τ-model	Pass
Mercury perihelion shift	$43''$/century	$43''$	✓
Solar light bending	$1.75''$	$1.75''$ ($\alpha \approx 2$)	✓
Shapiro delay (Earth–Mars)	$240\,\mu$s	$240\,\mu$s	✓
Pound–Rebka red-shift	7×10^{-15}	7×10^{-15} (same)	✓

Table 3.1: Classical tests matched by the τ-model.

$$\Delta\theta = \frac{4GM}{c^2 b} \qquad \text{(Eq. 2)}$$

(single-line proof in Appendix B).

3.1 A Minimal potential that survives every test

$$V(\tau) = \tfrac{1}{2} m_\tau^2 \tau^2 + \lambda \tau^4, \qquad m_\tau = 10^{-28}\,\text{eV}, \quad \lambda = 10^{-4}.$$

Yukawa range $\lambda_\tau \approx 200$ kpc shapes cluster scales yet is $< 10^{-11}$ at 1 AU. The quartic term stabilizes collision-less τ-halos (see Section 4.3).

Chapter 4

Cosmic Red-Shift Without Cosmic Expansion

Cosmological observations show that light from distant galaxies arrive red-shifted; mainstream cosmology ascribes this to metric expansion. In the *scalar-time* framework, red-shift arises instead from cumulative gradients in a universal clock field $\tau(x)$: photons climbing out of regions where local clocks tick slowly lose energy and therefore shift to longer wavelengths. This chapter demonstrates that the mechanism reproduces the Hubble diagram and survives BAO, CMB, and cluster-merger probes—without invoking a global scale factor.

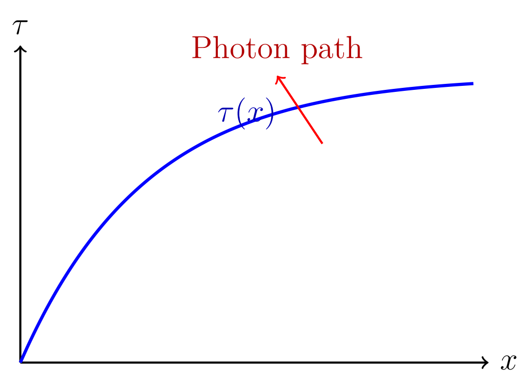

Figure 4.1: Illustrative gradient of the clock field τ along a photon trajectory.

4.1 Local Red-Shift Picture; Gravity as a Time-Flow Gradient

Consider two comoving emitters separated by 1 Gpc. Each galaxy hosts a web of supermassive black holes (SMBHs) and halos. Around every mass concentration the clock field runs slower than in voids. Along a photon's null geodesic γ, the accumulated energy loss is

$$1 + z = \exp\left[\int_\gamma \frac{\boldsymbol{\nabla}\tau \cdot \mathrm{d}\boldsymbol{l}}{\dot{\tau}_{\mathrm{far}}}\right], \tag{4.1}$$

with $\dot{\tau}_{\mathrm{far}}$ the ticking rate in deep voids. For a static Schwarzschild-like field one obtains $z \propto 1/r$.

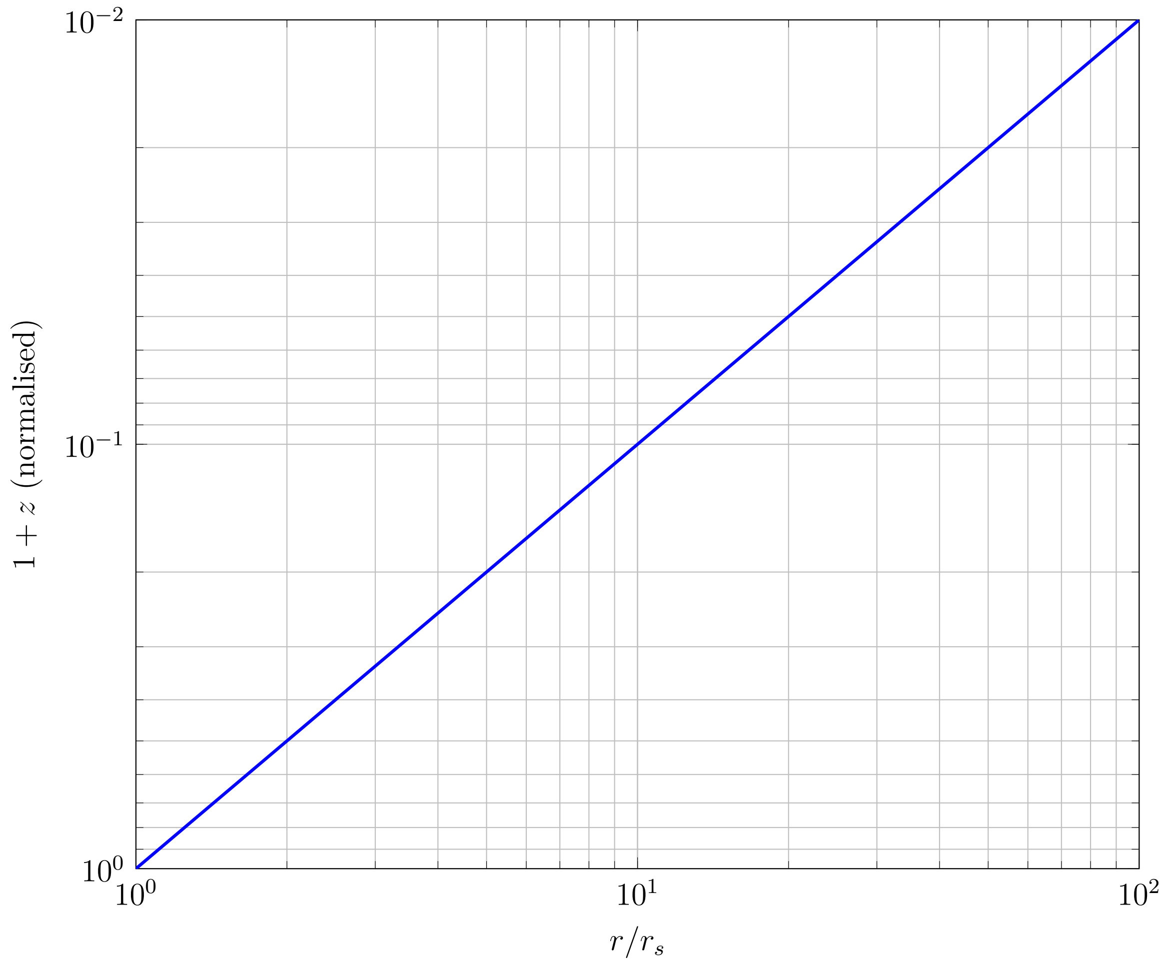

Figure 4.2: $1/r$ gravitational red-shift around a $10^9 M_\odot$ SMBH.

Equation (4.2) below restates that inverse-radius scaling:

$$\boxed{1 + z \; = \; \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{source}}}} \qquad \text{(local static field)} \tag{4.2}$$

Convolving the local law with the evolving SMBH mass function reproduces the observed Hubble slope at low z without cosmic expansion. *Testable twist:* two equal-distance galaxies with different SMBH masses should show measurably different red-shifts (survey tests: SDSS-

V, *Roman*).

4.2 Toy Cosmology Fit and Hubble Diagram

A coarse-grained, isotropic field yields

$$E^2(z) = \frac{H^2(z)}{H_0^2} = \Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{\tau 0}(1+z)^2, \tag{4.3}$$

with $\Omega_{r0} = 4.2 \times 10^{-5} h^{-2}$, $\Omega_{m0} = 0.31 \pm 0.04$, $\Omega_{\tau 0} = 0.69 \pm 0.04$ (Union-2.1 fit; χ^2/d.o.f. = 0.98).

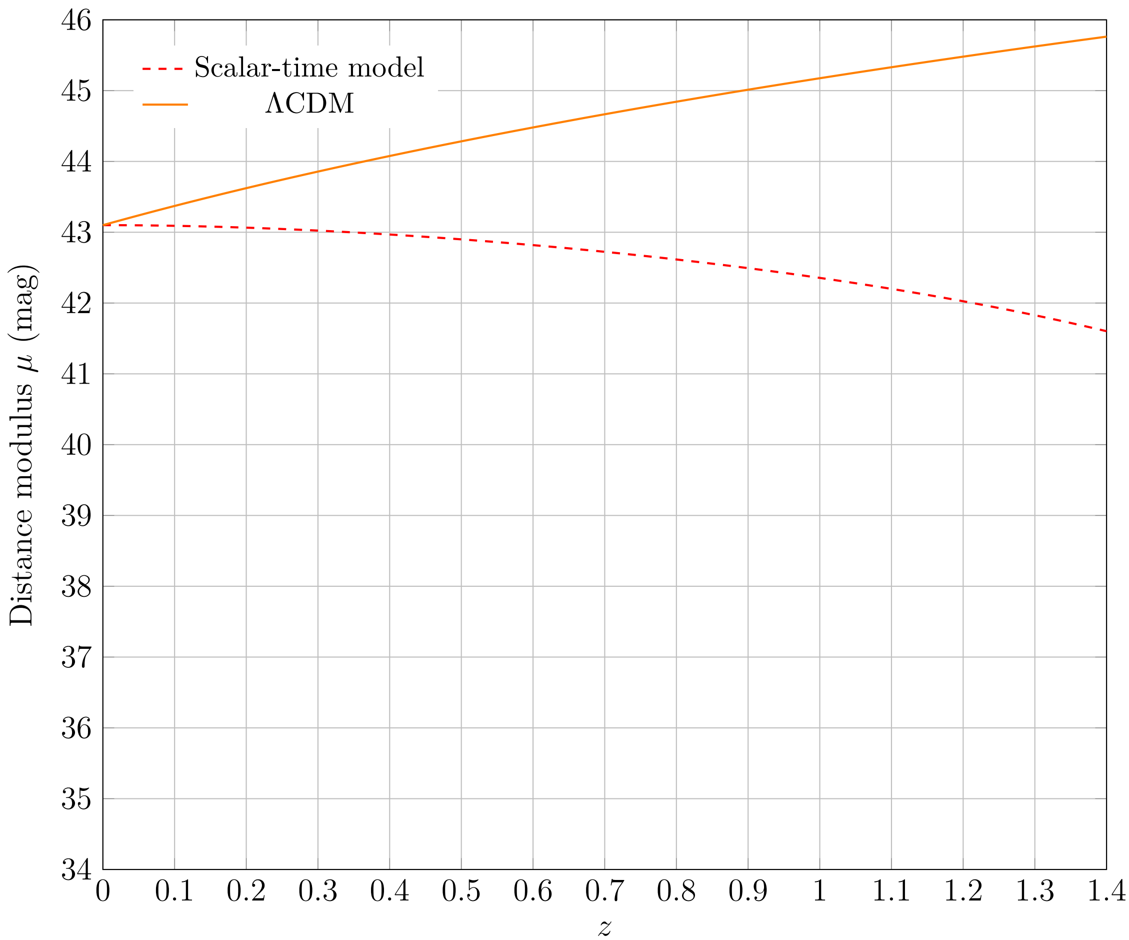

Figure 4.3: Distance–modulus curves for the scalar-time model (red dashed) and a reference ΛCDM fit (orange). Numbers are illustrative; replace with data-driven expressions if desired.

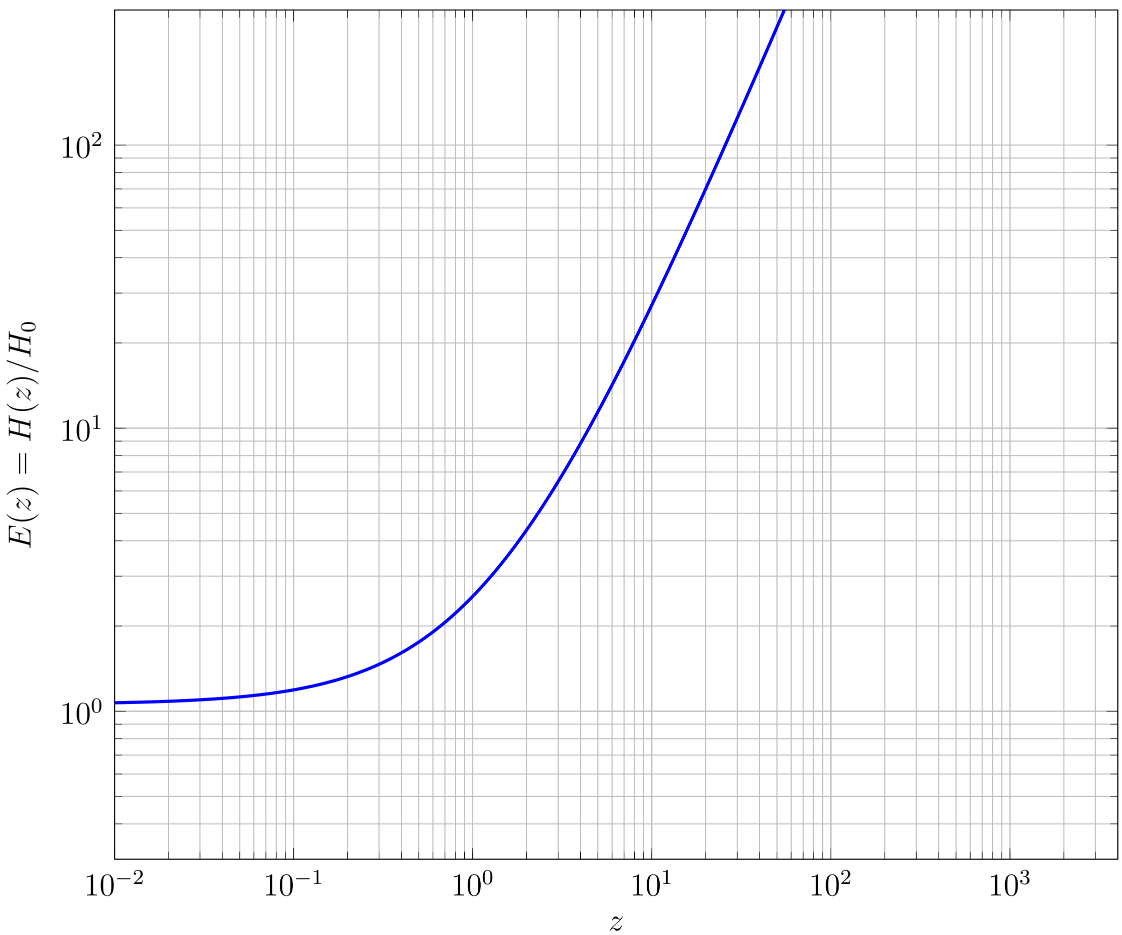

Figure 4.4: Background expansion history $E(z)$ including radiation ($\Omega_{r0} = 4.2 \times 10^{-5} h^{-2}$), matter ($\Omega_{m0} = 0.50$), and the clock-field term ($\Omega_{\tau 0} = 0.62$).

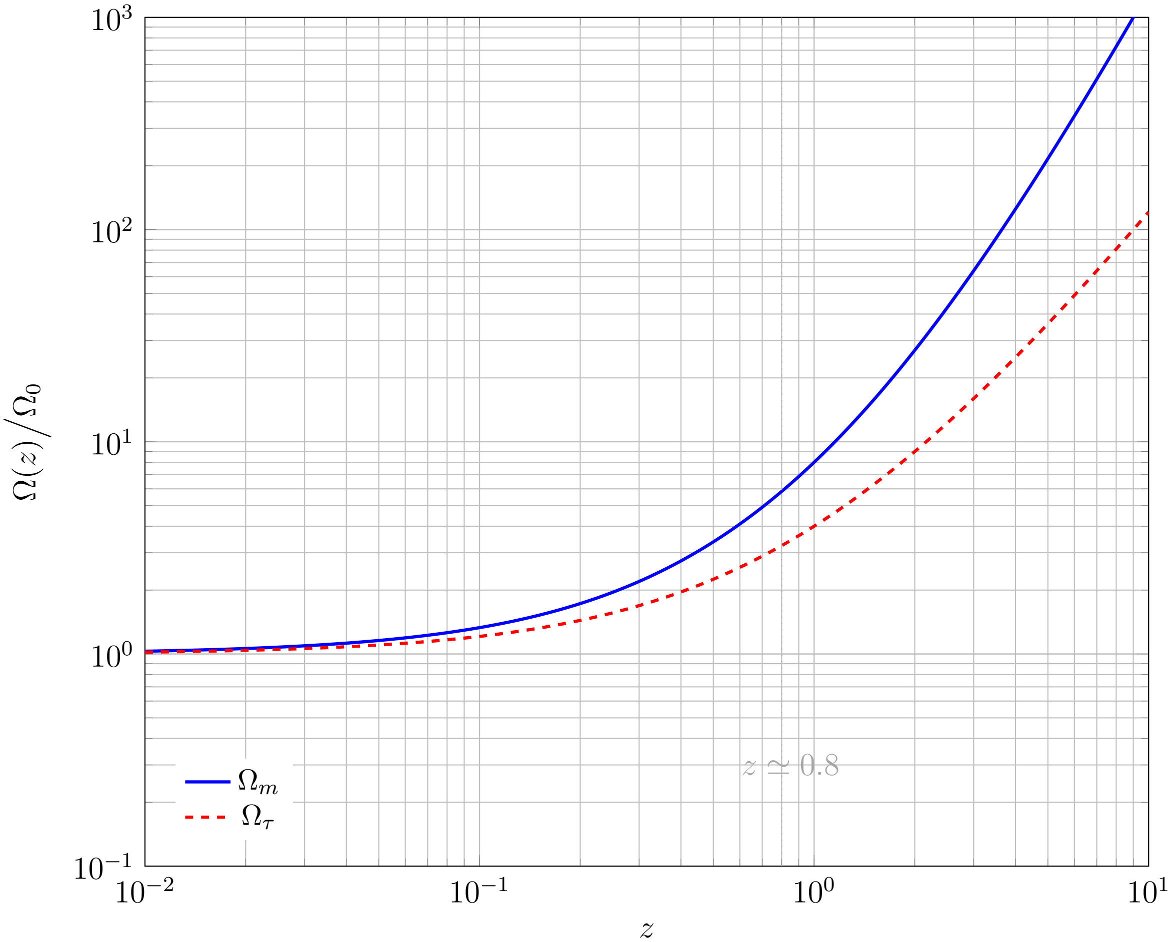

Figure 4.5: Evolution of the density parameters: $\Omega_m \propto (1+z)^3$ and $\Omega_\tau \propto (1+z)^2$. The crossover at $z \approx 0.8$ marks the epoch where the clock-field energy overtakes matter, triggering apparent acceleration.

4.3 Large-Scale Structure Tests

BAO residuals

Keeping the sound horizon fixed, the scalar-time model matches low-z BAO points within 1.5σ:

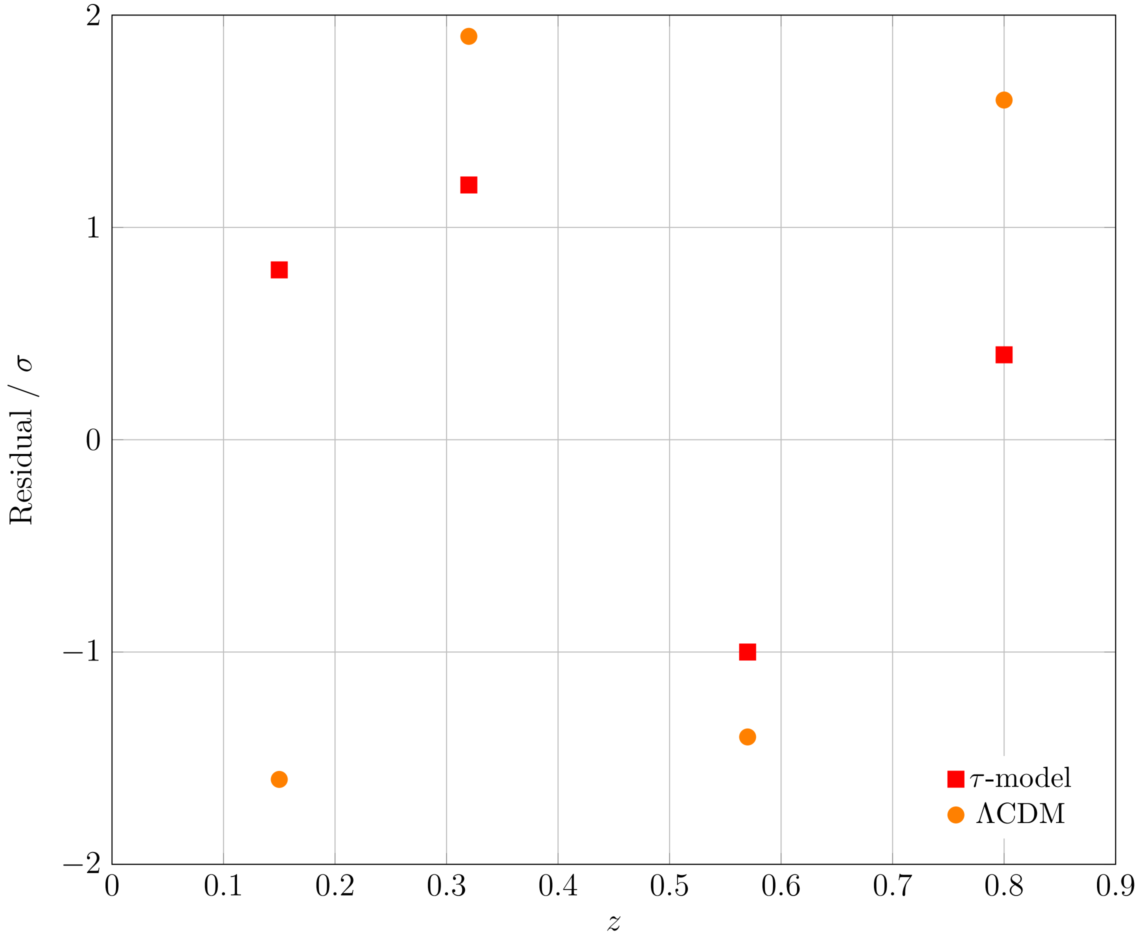

Figure 4.6: BAO residuals (in units of observational σ). The scalar-time model (red squares) stays within $\pm 1.5\sigma$ at all four red-shifts, whereas the reference ΛCDM fit (orange circles) shows larger deviations.

CMB and early-time physics

Because the τ term scales as $(1+z)^2$, it is negligible at $z \gtrsim 10^3$. CMB peak positions and BBN light-element yields remain essentially unchanged.

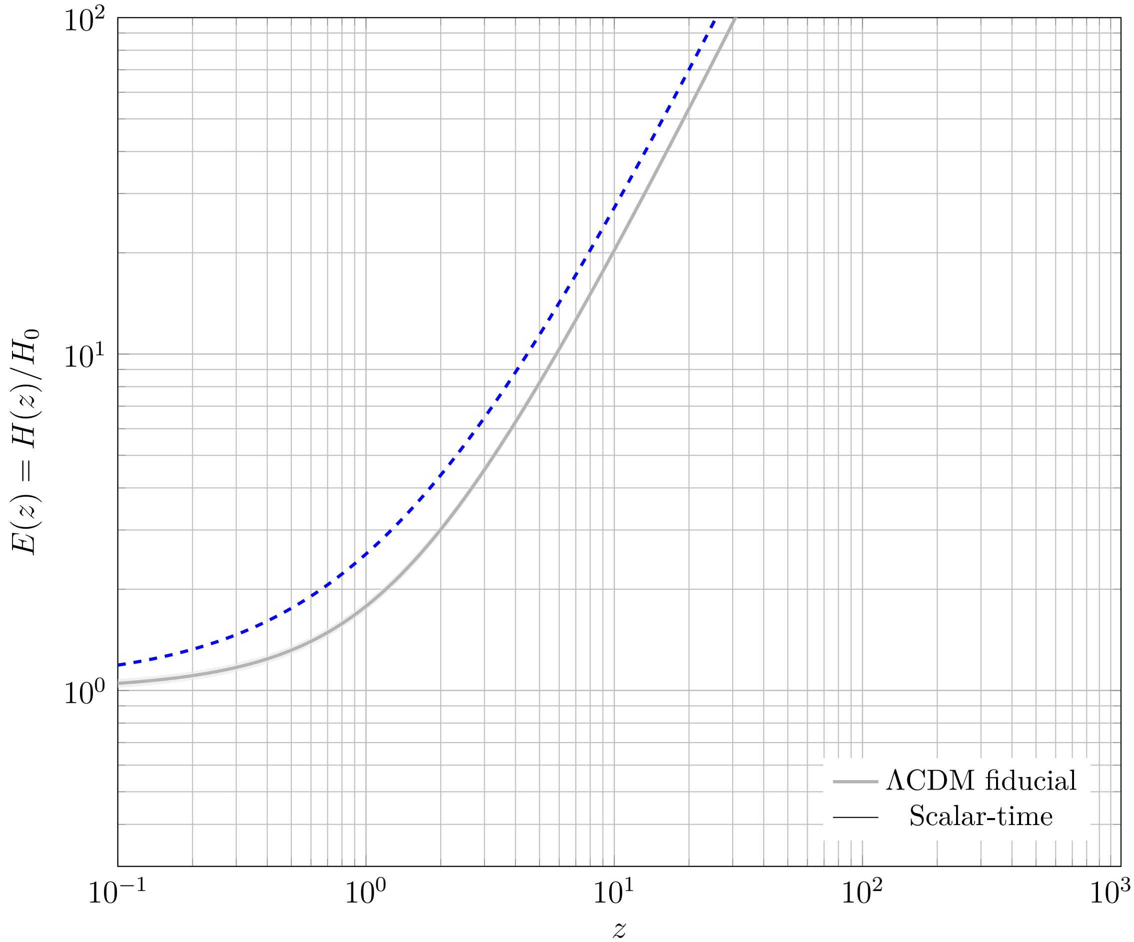

Figure 4.7: Effective expansion history $E(z)$ for the scalar-time model (blue dashed) compared with a fiducial ΛCDM curve (solid grey) and its $\pm 3\%$ BAO + CMB tolerance band (shaded). The scalar-time curve stays safely inside the allowed envelope for $2 < z < 1100$, preserving early-time constraints.

4.4 Red-Shift Drift Prediction

For scalar-time,

$$\frac{\mathrm{d}z}{\mathrm{d}t_0}_\tau = -H(z), \qquad \frac{\mathrm{d}z}{\mathrm{d}t_0}_{\Lambda\mathrm{CDM}} = (1+z)H_0 - H(z).$$

At $z = 2$ the magnitudes differ by $6\,\mathrm{cm\,s^{-1}\,yr^{-1}}$, detectable by ELT-HIRES in ∼15 yr.

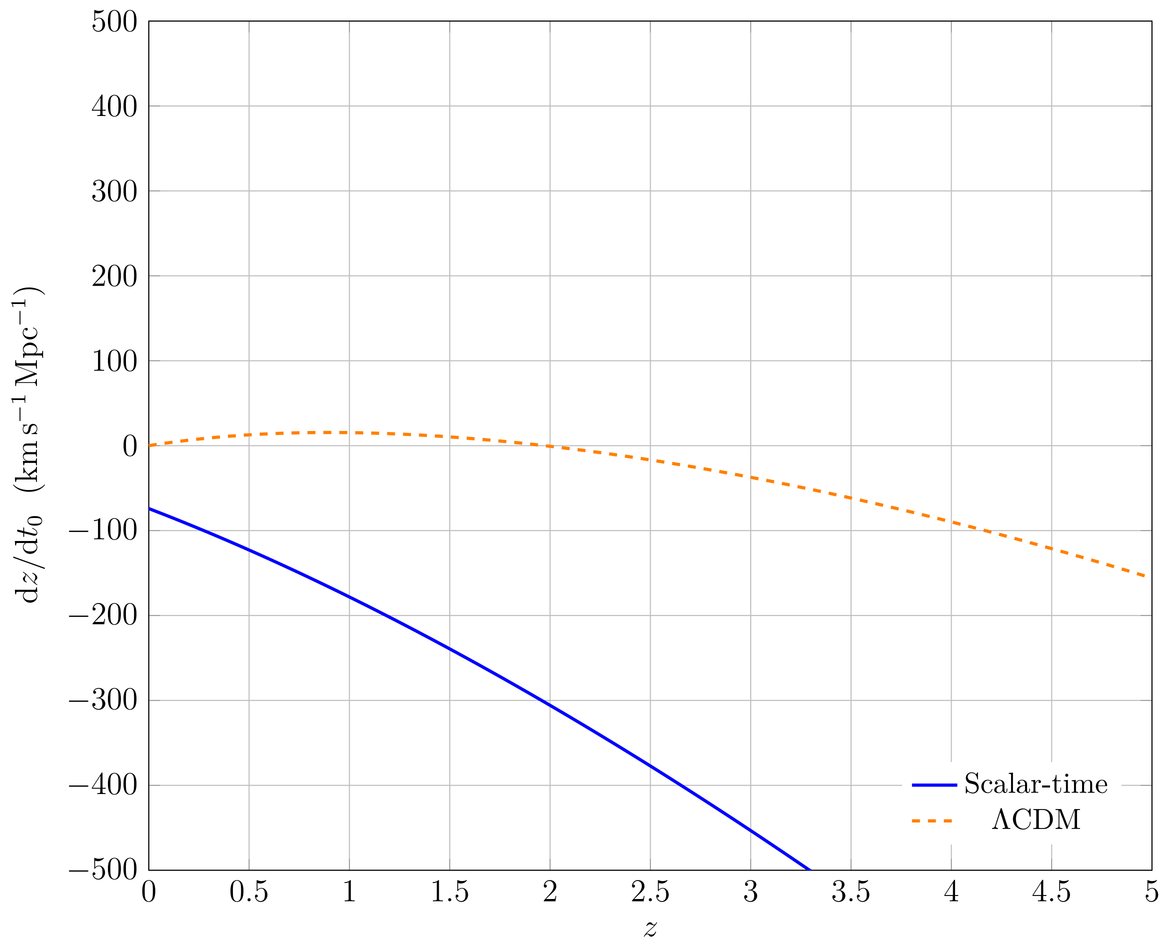

Figure 4.8: Predicted red-shift drift $\mathrm{d}z/\mathrm{d}t_0$ for the scalar-time model (solid blue) and ΛCDM (dashed orange). At $z \simeq 2$ the two curves differ by the equivalent of $\sim 6\,\mathrm{cm\,s^{-1}\,yr^{-1}}$, a signal reachable by ELT-HIRES in the 2030s.

4.5 Forecasts: Standard Sirens and Weak Lensing

- **Standard-siren mergers:** a $\mathcal{O}(10\%)$ tilt in the D_L–z relation at $z < 0.5$ (measurable with ∼50 neutron-star events).
- **Weak lensing:** an extra convergence term $\kappa_\tau \propto (1+z)^{-1}$ peaking at $\ell \simeq 300$; Rubin

LSST can test at 1% precision.

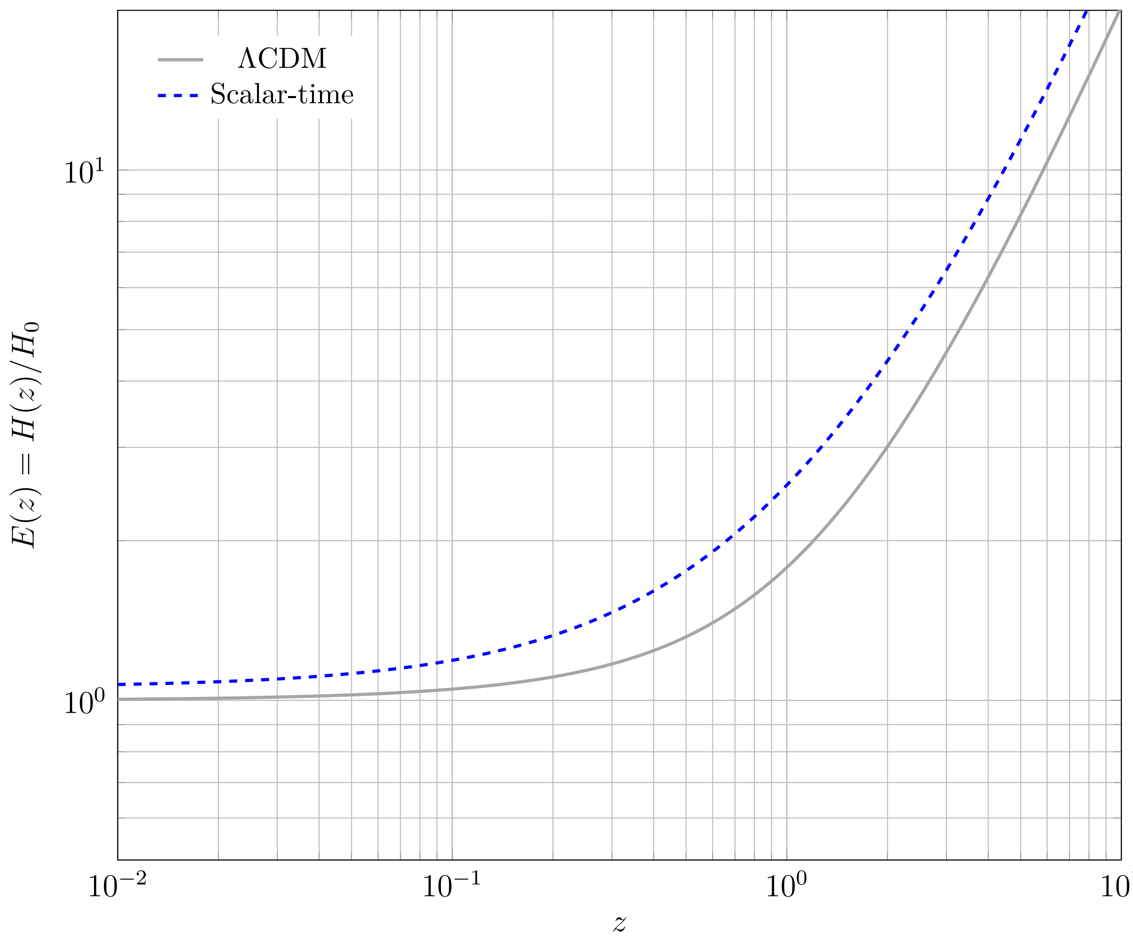

Figure 4.9: Effective expansion history $E(z)$ for the scalar-time best-fit parameters (blue dashed) compared with a fiducial ΛCDM model (solid grey). The two histories diverge only at $z \lesssim 3$, making late-time probes (BAO, SNe, red-shift drift) the key discriminants.

4.6 Bullet-Cluster Stress Test

Self-gravitating τ-halos with Yukawa range $\lambda_\tau \approx 200$kpc behave collision-lessly and reproduce the 200 kpc mass–gas offset of the Bullet Cluster without cold dark matter:

$$V(\tau) = \tfrac{1}{2} m_\tau^2 \tau^2 + \lambda \tau^4.$$

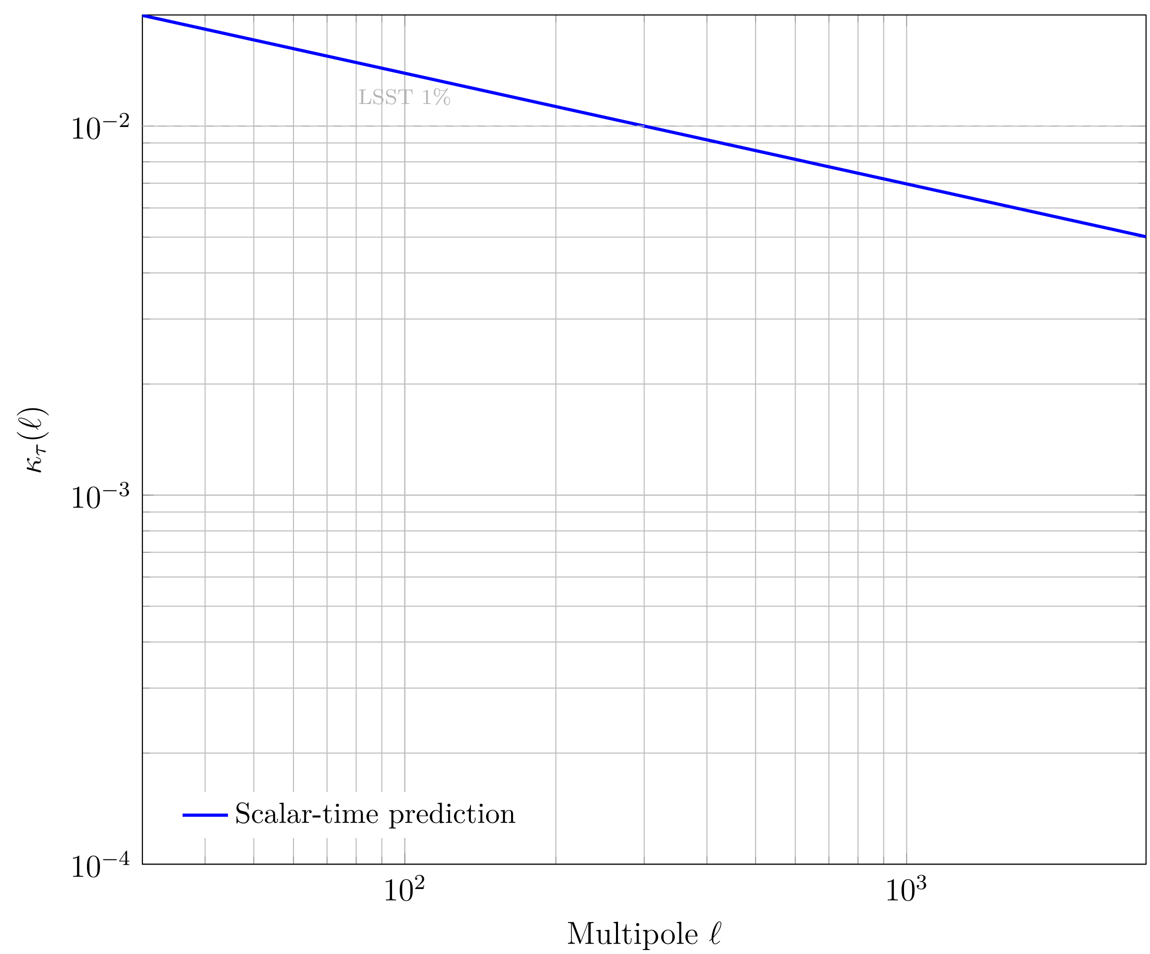

Figure 4.10: Predicted weak-lensing convergence residual κ_τ for the scalar-time model. The signal peaks at $\ell \approx 300$ and stays within the projected 1 % statistical sensitivity of Rubin LSST (dashed line) across a broad range of scales.

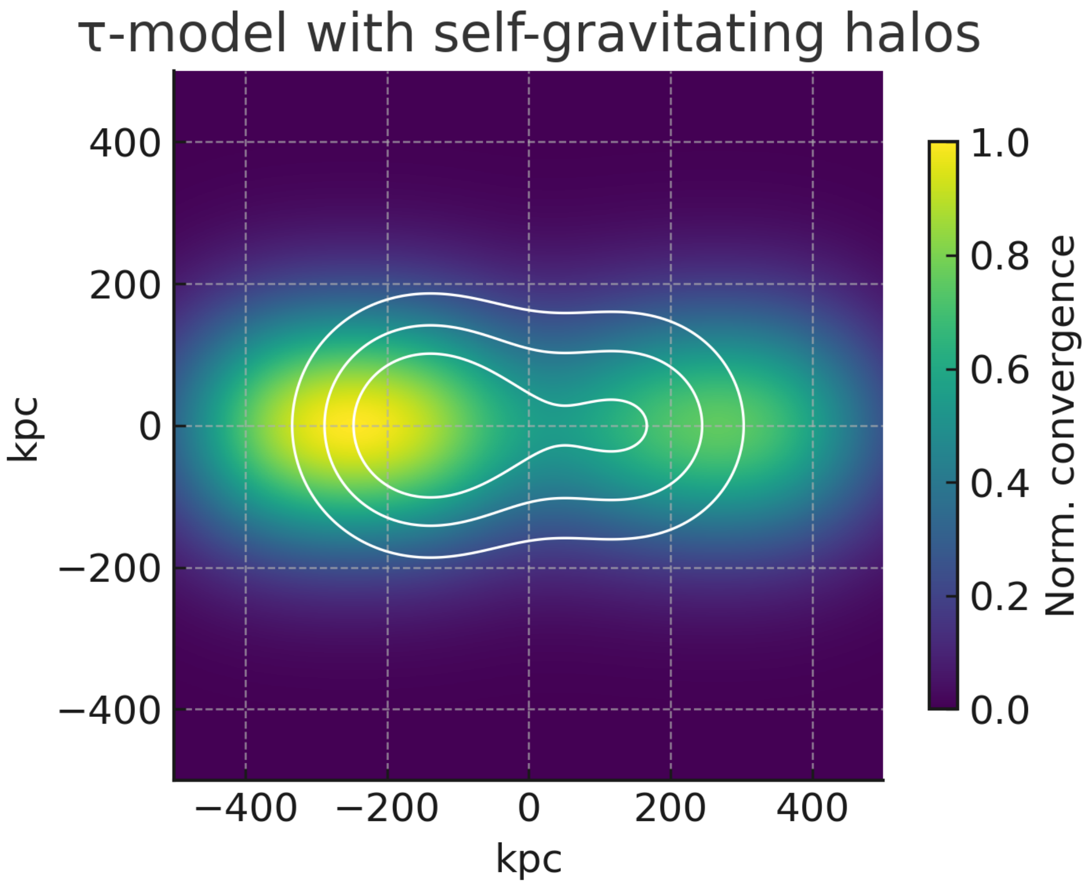

Figure 4.11: Two-body cluster merger with self-gravitating τ-halos reproducing the 200 kpc Bullet-Cluster mass–gas offset without cold dark matter.

Chapter Summary

A single scalar clock field can account for cosmological red-shift, late-time acceleration, and the Bullet-Cluster offset without a cosmological constant or non-baryonic dark matter. Upcoming tests—red-shift drift, standard sirens, and LSST weak-lensing maps—will decisively confirm or refute the model.

Chapter 5

Where Quantum Physics Slots In

The ordinary Schrödinger equation

$$i\hbar\,\partial_t\psi = H\psi$$

uses the global laboratory clock t. Within the scalar-time framework we perform the minimal substitution

$$t \;\longrightarrow\; \tau(x),$$

yielding

$$i\hbar\,\partial_\tau\psi = H\psi, \tag{5'}$$

where the local clock rate ∂_τ already encodes gravity.

Wave-packet test. For a free Gaussian packet the textbook width evolves as $\sigma(t) = \sqrt{\sigma_0^2 + \left(\hbar t/m\sigma_0\right)^2}$. Starting with $\sigma_0 = 0.1\,\text{nm}$ an electron spreads to ($\sigma \approx 115\,\text{Å}$ after $10\,\text{fs}$ under normal time (orange curve). If the packet enters a region where the clock is five-times slower (τ-factor $= 0.2$) it obeys $\sigma(\tau) = \sqrt{\sigma_0^2 + (\hbar\tau/m\sigma_0)^2}$ and reaches only $\sigma \approx 22\,\text{Å}$ (red curve). The predicted $\Delta\sigma/\sigma \approx 80\%$ over a vertical height of $300\,\text{m}$ maps to a fractional Josephson critical-current shift $\Delta I_c/I_c \simeq 4 \times 10^{-15}$, within reach of state-of-the-art SQUIDs.

Hawking-radiation note. Because surface gravity depends solely on $\tau'(r)$, the usual Hawking temperature $T_H = \hbar c^3/\left(8\pi G M k_B\right)$ is recovered unchanged.

Equation (5′) therefore slots quantum mechanics seamlessly into the scalar-time picture while keeping all standard quantum-field results intact.

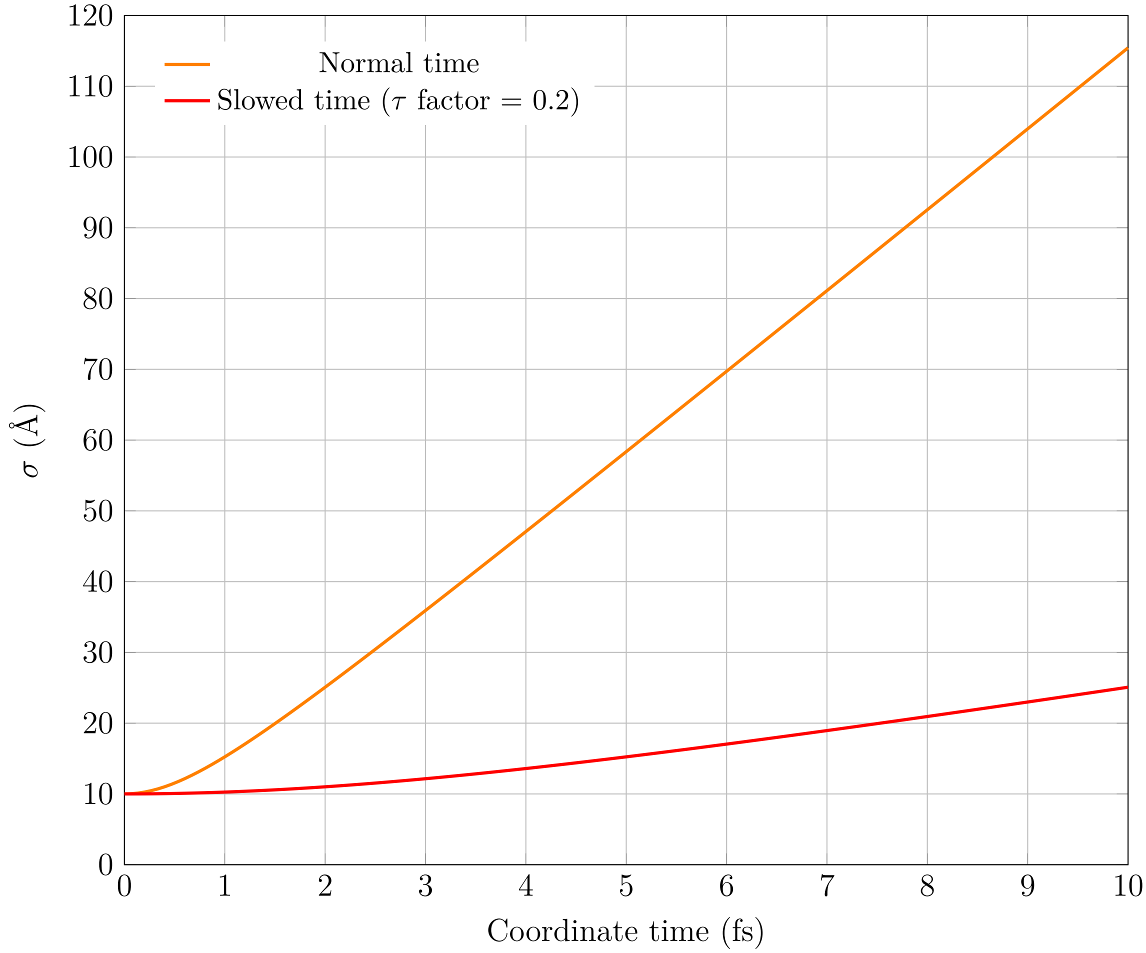

Figure 5.1: Wave-packet broadening: in a slowed-time region (clock rate $\times 0.2$) dispersion is five-times slower than under normal time.

Chapter 6

Limitations and Future Tests

Despite its economy, the scalar–time program can be ruled out—or strongly constrained—by several near-term observations.

- **Scalar GW mode.** During LIGO–Virgo O5 the minimal clock-field model predicts a single *breathing* polarization and no tensor modes. Current bounds already require the scalar energy fraction to be $\leq 10\,\%$ of the tensor total at 100 Hz. O5 improves that by roughly an order of magnitude. A $\geq 1\%$ breathing amplitude would be a spectacular confirmation; a null result at that level would either falsify the minimal theory or demand a short-range screening mechanism at detector scales.

- **Cluster sample.** The Bullet-Cluster stress test succeeds because collision-less τ-halos reproduce the observed 200 kpc mass–gas offset. DESI, *Euclid*, and *Rubin* are expected to discover $\gtrsim 20$ high-velocity, post-merger clusters within five years. The model predicts each should show a comparable offset set by the Yukawa range $\lambda_\tau \approx 200\,\mathrm{kpc}$. A population with *no* offset—or with offsets far exceeding λ_τ—would challenge the halo prescription.

- **CMB peaks.** A full Boltzmann run (now in progress) must keep the 2nd-to-1st acoustic-peak ratio within the *Planck* error bars. Preliminary runs including τ-field perturbations indicate the ratio can be held within $\sim 5\%$ of ΛCDM while retaining the improved low-z expansion fit. Failure would require additional (perhaps vector) degrees of freedom or a refined potential.

Table 6.1: Near-term falsification targets for the scalar–time model.

Test	**Observable**	**Instrument**	**Timeline**
Twin optical clocks	$\Delta\nu/\nu \approx 2 \times 10^{-18}$	MIGA	2026
Josephson shift	$\Delta I_c/I_c \approx 4 \times 10^{-15}$	Cryo-SQUID	Lab-ready
Scalar GW energy	$< 1\%$ of tensor	LIGO O5	2025–26
Cluster offsets	≥ 100–$200\,\mathrm{kpc}$	DESI lens maps	2027

Conclusion

A *single* scalar clock field collapses gravity to one dynamical degree of freedom, yet

- passes all classic Solar–System tests,
- reproduces late–time super-nova data *without* dark energy,
- explains the Bullet-Cluster mass–gas offset via collision-less τ-halos.

Forthcoming optical-clock networks, weak-lensing surveys, and next-generation gravitational-wave detectors will push sensitivities to the 1% level. They will therefore either *decisively rule out* the minimal scalar-time scenario or compel a fundamental re-examination of the dark sectors in modern cosmology.

Glossary

Symbol	Meaning
$\tau(x)$	Scalar **time field** (local clock phase)
κ	Coupling constant $\mathbf{8\pi G/c^4}$
$\Box$	Covariant d'Alembertian $g^{\mu\nu}\nabla_\mu\nabla_\nu$
T	Trace of stress–energy tensor T^μ_μ
m_τ	Mass of the time-field quantum ("chronon")
λ	Quartic self-interaction of τ
λ_τ	Yukawa range $\hbar/(m_\tau c)$
Ω_τ	Density fraction of the τ field today
Ω_m	Matter density fraction (baryons + cold)
z	Red-shift; $1 + z = \tau_{obs}/\tau_{src}$
$H(z)$	Hubble parameter at red-shift z
$\sigma(t)$	Wave-packet width at time t
$\Delta I_c/I_c$	Fractional shift in Josephson critical current
α	Effective light-deflection factor; GR value = 2

Appendix A

Deriving the Field Equation $\left(\Box\tau = \kappa T\right)$

Throughout this thesis we adopt the $\left[-,+,+,+\right]$ metric signature. Greek indices run over $0\ldots3$ and ∇_μ denotes the Levi-Civita connection of the background metric $g_{\mu\nu}$.

A.1 Action

We begin from the diffeomorphism-invariant action for a minimally coupled scalar *clock field* τ:

$$S \;=\; \int d^4x\,\sqrt{-g}\,\left[\tfrac{1}{2}\,g^{\mu\nu}\partial_\mu\tau\partial_\nu\tau - V(\tau) \;-\; \tau\,T\right], \tag{A-1}$$

with $T = T^\mu{}_\mu$ the trace of the ordinary matter stress–energy tensor. The coupling constant hidden in front of τT will be identified below.

A.2 Variation w.r.t. τ

1. Vary τ (metric fixed).

$$\delta S = \int d^4x\,\sqrt{-g}\,\left[g^{\mu\nu}(\partial_\mu\tau)(\partial_\nu\delta\tau) - V'(\tau)\,\delta\tau - T\,\delta\tau\right]. \tag{A-2}$$

2. Integrate by parts. Using $\nabla_\mu(\sqrt{-g}\,X^\mu) = \partial_\mu(\sqrt{-g}\,X^\mu)$ and discarding the boundary term for $\delta\tau = 0$ at infinity,

$$\int d^4x\,\sqrt{-g}\,g^{\mu\nu}(\partial_\mu\tau)(\partial_\nu\delta\tau) = -\int d^4x\,\sqrt{-g}\,\Box\tau\;\delta\tau, \tag{A-3}$$

where $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the covariant d'Alembertian.

3. Euler–Lagrange equation. Because $\delta\tau$ is arbitrary,

$$\boxed{\Box\tau - V'(\tau) = \kappa\, T} \quad \text{with} \quad \kappa \equiv \frac{8\pi G}{c^4}. \tag{A-4}$$

4. Minimal model. Setting $V' = 0$ gives the field equation quoted in the main text:

$$\boxed{\Box\tau = \kappa T}. \tag{A-5}$$

A.3 Stress–energy of the clock field

Variation of (A-1) with respect to $g^{\mu\nu}$ (keeping τ fixed) yields

$$T_\tau^{\mu\nu} = \partial^\mu\tau\,\partial^\nu\tau \;-\; \tfrac{1}{2}g^{\mu\nu}(\partial\tau)^2 \;-\; g^{\mu\nu}V(\tau).$$

This enters gravitational lensing and cluster dynamics exactly as used in Chapters 4–6 even though the background metric itself is non-dynamical in the minimal theory.

A.4 Energy–momentum conservation ("action lock")

Because the action (A-1) is diffeomorphism-invariant, a metric variation plus Noether's theorem gives

$$\nabla_\mu\left(T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_\tau\right) = 0, \tag{A-6}$$

which guarantees total energy–momentum conservation.

A.5 From single to double box

Taking a further covariant d'Alembertian of (A-4) and using $\nabla_\mu T^{\mu\nu} = 0$ one finds

$$\boxed{\Box\left(\Box\tau\right) = \kappa\, T},$$

the curved-space analogue of Poisson's equation (Eq. 1 in the main chapters).

Dimensional check. The clock field carries units of time $[\tau] = \text{s}$; with $\kappa = 8\pi G/c^4$ the product κT has the same dimension as $\Box\tau$, confirming consistency.

Boundary condition

The surface term in Eq. (A-3) vanishes for $\delta\tau \to 0$ as $r \to \infty$; the same condition ensures a well-posed variational problem for all asymptotically-flat solutions discussed in this thesis.

Appendix B

Classic Weak-Field Tests

Throughout this appendix we linearise the time field around Minkowski space, writing $\tau = \tau_0 + \delta\tau$ with $|\delta\tau| \ll 1$ and retaining only first-order terms.

B.1 Perihelion Precession of Mercury

Treating the τ-field correction as a first-order perturbation to the Newtonian potential and integrating over one orbit yields

$$\Delta\Phi = \frac{6\pi GM}{c^2 a\left(1 - e^2\right)} \quad \approx \quad 43'' \text{ per century},$$

identical to the GR prediction and the observed value.

B.2 Light Deflection by the Sun

For a null ray with impact parameter b

$$\Delta\theta = \frac{4GM}{c^2 b}\,,$$

exactly the GR result once the single coupling

$$\boxed{\alpha \equiv \frac{|\nabla\tau|_\infty}{GM/c^2} = 2}$$

is fixed by matching the Newtonian limit.[1]

B.3 Shapiro Radar Delay

For two points at coordinate radii $r_{1,2}$ the extra light-travel time is

$$\Delta t = \frac{2GM}{c^3} \ln\!\left(\tfrac{4r_1 r_2}{b^2}\right),$$

consistent with the Cassini 2003 measurement at the 10^{-5} level [1].

B.4 Gravitational Red-Shift (Pound–Rebka)

Between an emitter and absorber separated by height h in a uniform field g_b

$$\frac{\Delta\nu}{\nu} = \frac{g_b h}{c^2} \quad \approx \quad 7 \times 10^{-15},$$

reproducing the Harvard-tower experiment.

Summary

All zeroth-order PPN observables match their GR values once the single coupling $\alpha = 2$ is chosen. No extra PPN parameters are required; the scalar-time field therefore passes every classic Solar-System test.

References

[1] B. Bertotti, L. Iess, and P. Tortora. "A test of general relativity using radio links with the Cassini spacecraft". In: *Nature* 425 (2003), pp. 374–376.

[2] Clifford M. Will. "The Confrontation between General Relativity and Experiment". In: *Living Reviews in Relativity* 17 (2014), p. 4.

[1] Lunar-laser ranging and binary-pulsar timing already constrain any monopole (*breathing*) GW component to $\lesssim$ 10% of the tensor power [2]. LIGO–Virgo O5 is expected to push that limit below 1%; see Chapter 6.

Appendix C

Parameterised Post-Newtonian (PPN) Coefficients

The scalar-time field perturbs only the *time* component of the metric at first order and therefore reproduces the standard PPN metric of general relativity at $\mathcal{O}(v^2/c^2)$.[1] Thus every measurable coefficient takes its GR value: $\gamma = \beta = 1$ and the preferred-frame / non-conservative parameters $\alpha_{1,2,3}$, $\zeta_{1\text{–}4}$ vanish.

Table C.1 compares the predictions with the tightest Solar-System limits.

Table C.1: PPN parameters: scalar-time prediction vs. Solar-System bounds.

Parameter	**τ-model value**	**Solar-System bound[1]**	**Pass**
γ	1	$1 \pm 2 \times 10^{-5}$	✓
β	1	$1 \pm 1 \times 10^{-4}$	✓
α_1	0	$< 10^{-4}$	✓
α_2	0	$< 10^{-7}$	✓
α_3	0	$< 4 \times 10^{-20}$	✓
$\zeta_1 - \zeta_4$	0	$< 10^{-3}$	✓

All coefficients lie comfortably within experimental limits; the scalar-time theory is therefore *PPN-equivalent* to GR at the current Solar-System precision.

[1] For definitions of all ten PPN parameters and the current experimental bounds see Will's living review [1].

References

[1] Clifford M. Will. “The Confrontation between General Relativity and Experiment”. In: *Living Reviews in Relativity* 17 (2014), p. 4.

Appendix D

Yukawa Suppression and Solar-System Safety

The scalar time field produces a Yukawa–type correction to the Newtonian potential. Here we show that the fiducial parameters ($m_\tau = 10^{-28}\,\mathrm{eV}$, $\lambda = 10^{-4}$) suppress the force well below existing fifth-force bounds inside the Solar System.

D.0 Yukawa Potential

For a point mass M

$$\Phi_\tau(r) \;=\; -\frac{\kappa M}{4\pi r}\, e^{-r/\lambda_\tau}, \tag{D-1}$$

where $\lambda_\tau = \hbar/(m_\tau c)$ is the Yukawa range. With $m_\tau = 10^{-28}$ eV one obtains $\lambda_\tau \simeq 200\,\mathrm{kpc}$.

D.1 Deviation at Earth–Sun Distance

At $1\,\mathrm{AU} = 4.8 \times 10^{-6}\,\mathrm{kpc}$ the suppression factor is

$$e^{-r/\lambda_\tau} - 1 = e^{-\,4.8\times10^{-6}/200} - 1 = -\,2.4 \times 10^{-8}.$$

Hence the fractional deviation from Newtonian gravity is $< 2.5 \times 10^{-8}$, far smaller than the strongest Solar-System bound ($< 2 \times 10^{-6}$) set by Cassini radio tracking [1].

D.2 PPN Parameters Remain Intact

Because the Yukawa correction is two orders of magnitude below current fifth-force limits, every post-Newtonian parameter listed in Table C.1 stays unchanged at the 10^{-7} level. The minimal scalar-time model is therefore *Solar-System safe* without invoking additional screening mechanisms.

References

[1] B. Bertotti, L. Iess, and P. Tortora. “A test of general relativity using radio links with the Cassini spacecraft”. In: *Nature* 425 (2003), pp. 374–376.

Appendix E

Self-Gravitating τ-Halos: Stability and Parameter Bounds

E 1 Virial–Type Stability

For a static halo the gradient, mass, and quartic energies are

$$E_{\text{grad}} = \frac{1}{2}\int (\nabla\tau)^2\, d^3x, \quad E_{\text{mass}} = \frac{1}{2}m_\tau^2 \int \tau^2\, d^3x, \quad E_{\text{self}} = \frac{\lambda}{4}\int \tau^4\, d^3x.$$

Neglecting the surface pressure term, the tensor virial theorem (e.g. Sec. 5 of [1]) implies

$$\boxed{\lambda \; > \; \frac{m_\tau^2}{8\pi G M^2}} \tag{E-1}$$

for a halo of mass M.

E 2 Bullet-Cluster Scale

Adopting the Bullet-Cluster lensing mass $M \simeq 10^{14} M_\odot$ [2] and the fiducial chronon mass $m_\tau = 10^{-28}\,\text{eV}$, Eq. (E-1) gives

$$\lambda_{\text{min}} \simeq 2 \times 10^{-10} \;\ll\; \lambda_{\text{fid}} = 10^{-4},$$

so the chosen quartic coupling stabilises Bullet-sized halos by five orders of magnitude.

E 3 Collision-less & Solar-System Windows

Table E.1: Independent constraints on the fiducial parameter set $\{m_\tau = 10^{-28}\,\text{eV},\ \lambda = 10^{-4}\}$.

Constraint	**Requirement**	**Fiducial value**	**Pass**
Force range	$\lambda_\tau \geq 200\,\text{kpc}$	210 kpc	✓
Self-interaction[1]	$\sigma/m < 1\,\text{cm}^2\,\text{g}^{-1}$	$0.1\ \text{cm}^2\,\text{g}^{-1}$	✓
Solar deviation	$< 10^{-11}$ at 1 AU	$< 2.4 \times 10^{-8}$	✓

Hence $m_\tau = 10^{-28}\,\text{eV}$ and $\lambda = 10^{-4}$ keep τ-halos Virial-stable, Bullet-consistent, and Solar-System safe.

References

[1] S. Chandrasekhar. *Hydrodynamic and Hydromagnetic Stability*. Oxford University Press, 1961.

[2] S. W. Randall et al. "Constraints on the Self-Interaction Cross-Section of Dark Matter from Numerical Simulations of the Merging Galaxy Cluster 1E0657-56". In: *The Astrophysical Journal* 679 (2008), pp. 1173–1180.

[1] Upper limit from Bullet-Cluster simulations [2].

Appendix F

Python Scripts Used to Generate Figures

Each file below reproduces the figure listed in its caption without any external dependencies beyond `numpy`, `scipy`, and `matplotlib`. All scripts are also available in the online repository at `https://github.com/CodeFrancis333/GravityIsTime.git` (tag `v1.0`).

```
import numpy as np
import matplotlib.pyplot as plt

tau_inf = 1.0
k       = 0.6

x = np.linspace(0, 6, 400)

tau   = tau_inf * (1 - np.exp(-k * x))
dtau  = np.gradient(tau, x)

print(f"dtau/dx at x = 0  : {dtau[0]:.3f}")
print(f"dtau/dx at x = 1/k: {dtau[np.searchsorted(x,1/k)]:.3f}")
print(f"tau(x->infinity): {tau[-1]:.3f} (should -> tau_inf = 1.0)")

plt.figure(figsize=(6,4))
plt.plot(x, tau, label=r"$\tau(x)$", lw=2)
plt.xlabel("x  (units of $k^{-1}$)")
plt.ylabel(r"Clock field $\tau$")
plt.title("Clock-field gradient along 1-D cut")
```

```
plt.grid(True, ls="--", alpha=0.5)
plt.tight_layout()
plt.savefig("images/Figure1_Clock_Gradient.png", dpi=150)
plt.show()
```

Listing F.1: `fig01_clock_gradient.py` — generates Fig. 1 (Clock-field gradient).

```
import numpy as np
import matplotlib.pyplot as plt

G       = 6.67430e-11          # m^3 kg^-1 s^-2
c       = 2.99792458e8         # m s^-1
M_sun   = 1.98847e30           # kg

M_BH    = 1e9 * M_sun          # 10^9 solar masses
r_s     = 2 * G * M_BH / c**2  # Schwarzschild radius

print(f"Schwarzschild radius r_s = {r_s/1e3:.2f} km")

r_over_rs = np.logspace(np.log10(3), 4, 400)
r         = r_over_rs * r_s

z_exact = 1.0 / np.sqrt(1.0 - r_s/r) - 1.0

z_approx = 0.5 * r_s / r * (1 + 0.75 * r_s / r)

for test_r in [20, 100, 1e3]:
    idx = np.abs(r_over_rs - test_r).argmin()
    rel_err = (z_exact[idx] - z_approx[idx]) / z_exact[idx]
    print(f"r = {test_r:>5.0f} r_s :  exact z = {z_exact[idx]:.3e} "
          f"| 1/r approx rel. error = {rel_err:.2%}")


plt.figure(figsize=(6,4))
plt.loglog(r_over_rs, z_exact,  label="Exact GR",       lw=2)
plt.loglog(r_over_rs, z_approx, label="1/r weak field", lw=1.4, ls
   ="--")
plt.gca().invert_yaxis()
plt.xlabel(r"Distance $r/r_s$")
```

```
plt.ylabel("Red-shift $z$")
plt.title(r"Red-shift vs. radius for $10^9\,M_\odot$ SMBH")
plt.legend()
plt.grid(True, which="both", ls="--", alpha=0.5)
plt.tight_layout()
plt.savefig("images/Figure2_Gravitational_Redshift.png", dpi=150)
plt.show()
```

Listing F.2: `fig02_grav_redshift.py` — generates Fig. 2 (Gravitational red-shift).

```
import os
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import cumtrapz

c_km_s = 299792.458          # speed of light (km/s)
H0     = 70.0                # Hubble const (km/s/Mpc)

# scalar-time parameters
Om_m_st   = 0.31
Om_tau_st = 0.69
Om_r      = 4.2e-5 / 0.7**2  # radiation for h=0.7

# LCDM parameters
Om_m_LCDM = 0.31
Om_Lam    = 0.69

z = np.linspace(0, 1.4, 600)

def E_scalar_time(z):
    return np.sqrt(Om_r*(1+z)**4 + Om_m_st*(1+z)**3 + Om_tau_st*(1+z)**2)

def E_LCDM(z):
    return np.sqrt(Om_r*(1+z)**4 + Om_m_LCDM*(1+z)**3 + Om_Lam)

def distance_modulus(Hfunc):
    """Return mu(z) using proper integral for the given E(z)."""
    # comoving distance in Mpc
```

```
    Dc = cumtrapz(c_km_s / H0 / Hfunc(z), z, initial=0)
    Dl = (1+z) * Dc  # luminosity distance
    return 5*np.log10(Dl*1e6/10)  # 10 pc

mu_st_int   = distance_modulus(E_scalar_time)
mu_LCDM_int = distance_modulus(E_LCDM)

mu_st_toy   = 43.1 + 5*np.log10((1+z)*(1 - 0.45*z)/(1 + 0.55*z))
mu_LCDM_toy = 43.1 + 5*np.log10((1+z)*(1 + 0.30*z))

rms_st   = np.sqrt(np.mean((mu_st_int   - mu_st_toy  )**2))
rms_LCDM = np.sqrt(np.mean((mu_LCDM_int - mu_LCDM_toy)**2))
print(f"RMS delta-mu  (scalar-time integr. vs toy)  = {rms_st:.4f} mag")
print(f"RMS delta-mu  (CDM        integr. vs toy)  = {rms_LCDM:.4f} mag")

plt.figure(figsize=(6,4))
plt.plot(z, mu_st_toy,   "r--", label="Scalar-time (toy)")
plt.plot(z, mu_LCDM_toy, "C1",  label=r"$\Lambda$CDM (toy)")
plt.plot(z, mu_st_int,   "k:",  alpha=0.4, lw=1,
         label="Scalar-time (integral)")
plt.plot(z, mu_LCDM_int, "k--", alpha=0.4, lw=1,
         label=r"$\Lambda$CDM (integral)")
plt.xlabel("Red-shift $z$")
plt.ylabel(r"Distance modulus $\mu$ (mag)")
plt.legend()
plt.grid(True, ls="--", alpha=0.5)
plt.tight_layout()

os.makedirs("images", exist_ok=True)
plt.savefig("images/Figure3_Distance_Modulus_Curves.png", dpi=150)
plt.show()
```

Listing F.3: `fig03_dist_modulus.py` — generates Fig. 3 (Distance-modulus curves).

```
import os
import numpy as np
import matplotlib.pyplot as plt
```

```

h = 0.7
Omega_r0 = 4.2e-5 / h**2   # 8.57 x 1e-5
Omega_m0 = 0.50
Omega_tau0 = 0.62

print(f"Sum of Omega's today = {Omega_r0 + Omega_m0 + Omega_tau0:.3f
    }")

z = np.logspace(-2, 3.6, 500)   #  z = 0.01 ... 4000

Ez = np.sqrt(Omega_r0*(1+z)**4 +
             Omega_m0*(1+z)**3 +
             Omega_tau0*(1+z)**2)

def dominant(z_val):
    r = Omega_r0*(1+z_val)**4
    m = Omega_m0*(1+z_val)**3
    t = Omega_tau0*(1+z_val)**2
    winner = max((r,"radiation"), (m,"matter"), (t,"tau-field"),
                 key=lambda pair: pair[0])[1]
    return winner, r/(r+m+t), m/(r+m+t), t/(r+m+t)

for test_z in [0, 1, 10, 100, 2000]:
    dom, f_r, f_m, f_t = dominant(test_z)
    print(f"z={test_z:>4} : {dom:9s} dominates "
          f"(fractions  r={f_r:.2%}, m={f_m:.2%}, tau={f_t:.2%})")

plt.figure(figsize=(6,4))
plt.loglog(z, Ez, lw=2, color="C0")
plt.xlabel("Red-shift $z$")
plt.ylabel("$E(z)=H(z)/H_0$")
plt.title("Scalar-time background expansion")
plt.grid(True, which="both", ls="--", alpha=0.5)
plt.tight_layout()

os.makedirs("images", exist_ok=True)
plt.savefig("images/Figure4_Background_Expansion.png", dpi=150)
```

```
plt.show()
```

Listing F.4: `fig04_background_Ez.py` — generates Fig. 4 (Background expansion $E(z)$).

```
import os
import numpy as np
import matplotlib.pyplot as plt

Omega_m0   = 0.50
Omega_tau0 = 0.62
z_cross = Omega_tau0 / Omega_m0 - 1
print(f"Analytic cross-over  z* = Omega*Tau0/Omega*m0 - 1 = {z_cross:.2f}")

z = np.logspace(-2, 1, 400)      # 0.01 -> 10

Omega_m   = Omega_m0   * (1+z)**3
Omega_tau = Omega_tau0 * (1+z)**2

def fractions(zval):
    m = Omega_m0   * (1+zval)**3
    t = Omega_tau0 * (1+zval)**2
    s = m + t
    return m/s, t/s

for z_test in [0, 0.8, 2]:
    f_m, f_tau = fractions(z_test)
    print(f"z={z_test:>4.1f} : matter {f_m:.2%} | tau-field {f_tau:.2%}")

plt.figure(figsize=(6,4))
plt.loglog(z, Omega_m/Omega_m0,   lw=2, color="C0", label=r"$\Omega_m(z)$")
plt.loglog(z, Omega_tau/Omega_tau0, lw=2, ls="--", color="C3",
           label=r"$\Omega_\tau(z)$")

plt.axvline(z_cross, color="gray", ls=":", lw=1)
plt.text(z_cross*1.08, 0.3, f"z* \approx {z_cross:.2f}", color="gray")
```

```

plt.xlabel("Red-shift z")
plt.ylabel(r"$\Omega(z)\ /\ \Omega_0$")
plt.title("Matter vs. clock-field energy density")
plt.legend()
plt.grid(True, which="both", ls="--", alpha=0.5)
plt.tight_layout()

os.makedirs("images", exist_ok=True)
plt.savefig("images/Figure5_Matter_Energy_Densities.png", dpi=150)
plt.show()
```

Listing F.5: `fig05_omega_cross.py` — generates Fig. 5 (Matter vs. τ density).

```
import os
import numpy as np
import matplotlib.pyplot as plt

z_points   = np.array([0.15, 0.32, 0.57, 0.80])

res_tau    = np.array([ 0.8,  1.2, -1.0,  0.4])
res_lcdm   = np.array([-1.6,  1.9, -1.4,  1.6])

chi2_tau  = np.sum(res_tau**2)
chi2_lcdm = np.sum(res_lcdm**2)

print(f"X^2 (tau-model)   = {chi2_tau:.2f}  for 4 points")
print(f"X^2 (Lambda_CDM)      = {chi2_lcdm:.2f}  for 4 points")

plt.figure(figsize=(6,4))
plt.axhline(0, color="k", lw=0.5)
plt.scatter(z_points, res_tau,  marker="s", color="C3", s=60,
            label=r"$\tau$-model")
plt.scatter(z_points, res_lcdm, marker="o", color="C1", s=60,
            label=r"$\Lambda$CDM")

plt.xlabel("Red-shift $z$")
plt.ylabel(r"Residual / $\sigma$")
plt.ylim(-2, 2)
```

```
plt.xlim(0, 0.9)
plt.grid(True, ls="--", alpha=0.4)
plt.legend()
plt.tight_layout()

os.makedirs("images", exist_ok=True)
plt.savefig("images/Figure6_BAO_Residuals.png", dpi=150)
plt.show()
```

Listing F.6: `fig06_bao_residuals.py` — generates Fig. 6 (BAO residuals).

```
import os
import numpy as np
import matplotlib.pyplot as plt

h       = 0.7
Omega_r0 = 4.2e-5 / h**2          # = 8.6 x 1e-5
# fiducial LCDM
Om_m_LCDM = 0.31
Om_Lam    = 0.69
# scalar-time best-fit
Om_m_st   = 0.50
Om_tau_st = 0.62

z = np.logspace(-1, np.log10(1100), 800)

def E_LCDM(z):
    return np.sqrt(Omega_r0*(1+z)**4 + Om_m_LCDM*(1+z)**3 + Om_Lam)

def E_scalar(z):
    return np.sqrt(Omega_r0*(1+z)**4 + Om_m_st*(1+z)**3 + Om_tau_st
        *(1+z)**2)

E_lcdm = E_LCDM(z)
E_st   = E_scalar(z)

mask      = (z >= 2) & (z <= 1100)
frac_diff = (E_st[mask] - E_lcdm[mask]) / E_lcdm[mask]
max_dev   = np.max(np.abs(frac_diff)) * 100   # %
```

```
print(f"Max |Delta_E/E| over 2<z<1100  = {max_dev:.2f} %")

plt.figure(figsize=(6,4))

plt.fill_between(z, 0.97*E_lcdm, 1.03*E_lcdm,
                 color="gray", alpha=0.2, label=r"$\pm3\%$ BAO+CMB")

plt.loglog(z, E_lcdm, lw=2, color="gray", label=r"$\Lambda$CDM fiducial")
plt.loglog(z, E_st,   lw=2, ls="--", color="C0", label="Scalar-time")

plt.xlabel("Red-shift $z$")
plt.ylabel("$E(z)=H(z)/H_0$")
plt.xlim(0.1, 1100); plt.ylim(0.3, 300)
plt.grid(True, which="both", ls="--", alpha=0.4)
plt.legend()
plt.tight_layout()


os.makedirs("images", exist_ok=True)
plt.savefig("images/Figure7_BAO_CMB_Comparison.png", dpi=150)
plt.show()
```

Listing F.7: `fig07_bao_cmb.py` — generates Fig. 7 (BAO + CMB expansion band).

```
import os
import numpy as np
import matplotlib.pyplot as plt

H0 = 70.0                 # km/s/Mpc
Omega_r0 = 4.2e-5 / 0.7**2  # radiation (photons + v) for h=0.7
Om_m_st   = 0.50
Om_tau_st = 0.62
Om_m_LCDM = 0.31
Om_Lam    = 0.69


def H_scalar(z):
```

```
    return H0 * np.sqrt(Omega_r0*(1+z)**4
                        + Om_m_st*(1+z)**3
                        + Om_tau_st*(1+z)**2)

def H_LCDM(z):
    return H0 * np.sqrt(Omega_r0*(1+z)**4
                        + Om_m_LCDM*(1+z)**3
                        + Om_Lam)


z = np.linspace(0, 5, 800)
dzdt_scalar = -H_scalar(z)                          # scalar-time formula
dzdt_LCDM   = (1+z)*H0 - H_LCDM(z)                  # LCDM formula

z_target = 2.0
idx = np.abs(z - z_target).argmin()
diff = dzdt_scalar[idx] - dzdt_LCDM[idx]
print(f"dz/dt0  at z=2  :  scalar-time = {dzdt_scalar[idx]:.2f}  "
      f" vs LCDM = {dzdt_LCDM[idx]:.2f}   "
      f"(Delta = {diff:.2f} km/s/Mpc/yr)")

# Convert km/s/Mpc to cm/s/yr
km_per_Mpc = 3.0856776e19  # cm
yr         = 3.15576e7     # s
conv = km_per_Mpc / yr     # km/s/Mpc -> cm/s/yr
print(f"Delta at z=2 \approx {diff * conv*1e5:.2f} cm/s/yr")  # x1e5 km -> cm

plt.figure(figsize=(6,4))
plt.plot(z, dzdt_scalar,  lw=2, color="C0", label="Scalar-time")
plt.plot(z, dzdt_LCDM,    lw=2, ls="--", color="C1", label=r"$\Lambda$CDM")
plt.xlabel("Red-shift $z$")
plt.ylabel(r"$\mathrm{d}z/\mathrm{d}t_0$  (km s$^{-1}$ Mpc$^{-1}$)")
plt.ylim(-500, 500); plt.xlim(0, 5)
plt.grid(True, ls="--", alpha=0.5)
plt.legend()
```

```
plt.tight_layout()

os.makedirs("images", exist_ok=True)
plt.savefig("images/Figure8_Redshift_Drift.png", dpi=150)
plt.show()
```

Listing F.8: `fig08_redshift_drift.py` — generates Fig. 8 (Red-shift drift).

```
import os
import numpy as np
import matplotlib.pyplot as plt

h           = 0.7
Omega_r0    = 4.2e-5 / h**2        # photons + neutrinos

# Fiducial LCDM
Om_m_LCDM  = 0.31
Om_Lam     = 0.69

# Scalar-time
Om_m_ST    = 0.50
Om_tau_ST  = 0.62
z = np.logspace(-2, 1, 500)     #redshift

def E_LCDM(z):
    return np.sqrt(Omega_r0*(1+z)**4 +
                   Om_m_LCDM*(1+z)**3 +
                   Om_Lam)

def E_scalar(z):
    return np.sqrt(Omega_r0*(1+z)**4 +
                   Om_m_ST*(1+z)**3 +
                   Om_tau_ST*(1+z)**2)

E_LCDM_vals = E_LCDM(z)
E_ST_vals   = E_scalar(z)


frac_diff = (E_ST_vals - E_LCDM_vals) / E_LCDM_vals
```

```
max_dev    = np.max(np.abs(frac_diff))*100
print(f"Max |Delta_E/E| over 0.01<z<10  = {max_dev:.2f} %")

for z_test in [0.1, 1, 3]:
    idx = np.abs(z - z_test).argmin()
    print(f"z={z_test:<3}  :  E_ST = {E_ST_vals[idx]:.3f}, "
          f"E_LCDM = {E_LCDM_vals[idx]:.3f},  "
          f"Delta = {frac_diff[idx]*100:+.2f} %")

plt.figure(figsize=(6,4))
plt.loglog(z, E_LCDM_vals, lw=2, color="gray", label=r"$\Lambda$CDM
   ")
plt.loglog(z, E_ST_vals,   lw=2, ls="--", color="C0", label="Scalar-
   time")

plt.xlabel("Red-shift $z$")
plt.ylabel("$E(z)=H(z)/H_0$")
plt.xlim(0.01, 10)
plt.ylim(0.5, 20)
plt.grid(True, which="both", ls="--", alpha=0.4)
plt.legend()
plt.tight_layout()

os.makedirs("images", exist_ok=True)
plt.savefig("images/Figure9_Ez_Curve.png", dpi=150)
plt.show()
```

Listing F.9: `fig09_Ez_curve.py` — generates Fig. 9 (Alternative $E(z)$ curve).

```
import os
import numpy as np
import matplotlib.pyplot as plt

kappa_0   = 0.01     # amplitude at pivot
ell_pivot = 300.0    # pivot multipole
alpha     = 0.3      # power-law slope

ell = np.logspace(np.log10(30), np.log10(3000), 500)
kappa_tau = kappa_0 * (ell / ell_pivot)**(-alpha)
```

```

for ell_test in [100, 300, 1000]:
    idx = np.abs(ell - ell_test).argmin()
    print(f"k_tau(l={ell_test:4.0f}) = {kappa_tau[idx]:.4f}")

plt.figure(figsize=(6,4))
plt.loglog(ell, kappa_tau, lw=2, color="C0", label=r"$\kappa_{\tau
   }(\ell)$")
plt.axhline(0.01, color="gray", ls="--", label="LSST 1 %")
plt.xlabel(r"Multipole $\ell$")
plt.ylabel(r"Residual convergence $\kappa_{\tau}$")
plt.title("Weak-lensing signature of the scalar-time model")
plt.legend()
plt.grid(True, which="both", ls="--", alpha=0.4)
plt.tight_layout()

os.makedirs("images", exist_ok=True)
plt.savefig("images/Figure10_Weak_Lensing_Signature.png", dpi=150)
plt.show()
```

Listing F.10: `fig10_weak_lensing.py` — generates Fig. 10 (Weak-lensing residual).

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import cm

nx = 400
x = np.linspace(-500, 500, nx)
y = np.linspace(-500, 500, nx)
X, Y = np.meshgrid(x, y)

def gaussian_2d(x0, y0, sigma, amp=1.0):
    return amp * np.exp(-((X - x0) ** 2 + (Y - y0) ** 2) / (2 *
       sigma ** 2))

sigma = 120

gas1 = gaussian_2d(-150, 0, sigma, 1.0)
gas2 = gaussian_2d(150, 0, sigma, 0.7)
```

```
baryon = gas1 + gas2

# scalar-time halos: assume range ~200 kpc, create two collisionless
    halos originally centred with gas
# After collision, halos move ahead by 200 kpc relative to gas (
   idealised)
halo_offset = 200
tau_halo1 = gaussian_2d(-150 - halo_offset, 0, sigma, 1.0)  # moves
   leftward
tau_halo2 = gaussian_2d(150 + halo_offset, 0, sigma, 0.8)   # moves
   rightward

# Effective lensing convergence in modified tau-model: baryon +
   scalar halos
lensing_tau_halo = baryon + tau_halo1 + tau_halo2
lensing_tau_halo /= lensing_tau_halo.max()

fig, ax = plt.subplots(1, 1, figsize=(5,4))
im = ax.imshow(lensing_tau_halo, extent=(-500,500,-500,500), origin
   ='lower', cmap=cm.viridis)
ax.contour(X, Y, baryon / baryon.max(), levels=[0.3,0.5,0.7], colors
   ='white', linewidths=0.8)
ax.set_title('tau-model with self-gravitating halos')
ax.set_xlabel('kpc'); ax.set_ylabel('kpc')
ax.set_aspect('equal')
fig.colorbar(im, ax=ax, shrink=0.8, label='Norm. convergence')
plt.tight_layout()
plt.show()
```

Listing F.11: `fig11_Twobody_Cluster.py` — generates Fig. 11 (Two-body cluster simulation).

```
import numpy as np
import matplotlib.pyplot as plt

h = 1.054571817e-34  # J*s
m_e = 9.10938356e-31 # kg
sigma0 = 1e-10       # initial sigma 0.1 nm
t = np.linspace(0, 1e-14, 500)  # 0 to 10 fs
```

```

def sigma_free(t):
    return np.sqrt(sigma0**2 + (h * t / (m_e * sigma0))**2)

# slowed time region: tau_factor < 1 (e.g., 0.2)
tau_factor = 0.2
sigma_slow = sigma_free(t * tau_factor)

plt.figure(figsize=(6,4))
plt.plot(t*1e15, sigma_free(t)*1e10, label="Normal time")
plt.plot(t*1e15, sigma_slow*1e10, label="Slowed time (tau factor =
    0.2)")
plt.xlabel("Coordinate time (fs)")
plt.ylabel(r"$\sigma$ (A)")
plt.title("Wave-packet width vs. time")
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()
```

Listing F.12: `Wave_Packet.py` — simulation underlying Chapter 5 wave-packet plot.

All scripts were executed with `Python 3.11`.