# Notes
# The pumping lemma for regular languages

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We'll usually use the lemma on Page 78 and not the one on Page 72.

Consider the lemma on Page 78. Consider the following languages:

1. $L_1 = \left\{0^i 1^i \mid i \geq 0\right\}$ (Page 84)
2. $L_2 = \left\{x \in \{0,1\}^+ \mid n_0(x) = n_1(x) \geq 1\right\}$
3. $L_3 = \left\{x \in \{0,1\}^+ \mid n_0(x) \geq n_1(x) \geq 1\right\}$
4. $L_4 = \left\{x \in \{0,1\}^+ \mid n_0(x) > n_1(x) \geq 1\right\}$

## 1 Remember the following points ...

### 1.1 ... regarding $x$

As is stated on Page 81, we have to choose $x$ very carefully. Remember ...

1. $x \in L$. For $L_1$, choosing $x = 1^n 0^n$ would be pointless. For $L_2$, it would be a good choice.
2. $x$ must always contain $n$ as a superscript. For $L_1$ and $L_2$, choosing $x = 0^{100}1^{100}$ would be pointless.
3. $|x| \geq n$. For $L_1$, choosing $x = 0^{n/10}1^{n/10}$ would be pointless, because the sum of the superscripts is less than $n$. So would $x = 0^{\text{trillion}}1^{\text{trillion}}$, because we don't know what $n$ is.
4. Choose $x$ to be as simple as possible. For $L_1$, choosing $x = 0^{2n}1^{2n}$ might work, but is more complicated than necessary. Even $x = 0^{n+1}1^{n+1}$ is more complicated than necessary. Rather use $x = 0^n 1^n$. In the case of $L_2$, even though you may choose $x$ such that the 0's and 1's are mixed, as in $x = (01)^n$, this will complicate your proof. Rather use $x = 0^n 1^n$ or $x = 1^n 0^n$.

5. Use as few different variables in $x$ as possible. For example, for $L_1$, don't write

$$x = 0^n 1^p, \text{ where } p = n \ ,$$

or, even worse

$$x = 0^k 1^p, \text{ where } k = p \text{ and } k + p \geq n \ .$$

Write $x = 0^n 1^n$.

6. The superscripts must be as close to each other as possible. For example, for $L_3$, choose $x = 0^n 1^n$, even though $x = 0^{n+1} 1^n$ is in $L_3$. For $L_4$, choose $x = 0^{n+1} 1^n$, even though $x = 0^{n+2} 1^n$ is in $L_4$.

### 1.2 **. . . regarding** $m$

Remember $m \geq 0$.

In some proofs you can use either $m = 0$ or $m > 0$ to complete the proof. In some proofs, only $m > 0$ would work. In others, only $m = 0$ would work. It all depends on the language and on your choice of $x$.