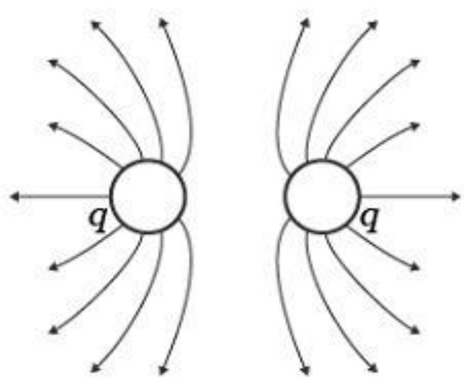
PHY 102

GENERAL PHYSICS II

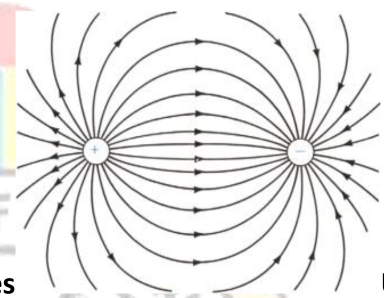
Electrostatics

Electrostatics is the study of static charges. There are two kinds of electric charges:
(i) Positive Charge
(ii) Negative Charge

Like charges repel each other, unlike charges attract each other.



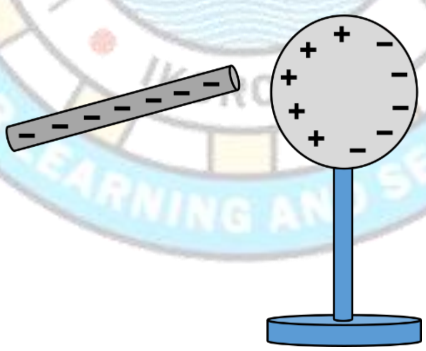
Like Charges



Unlike Charges

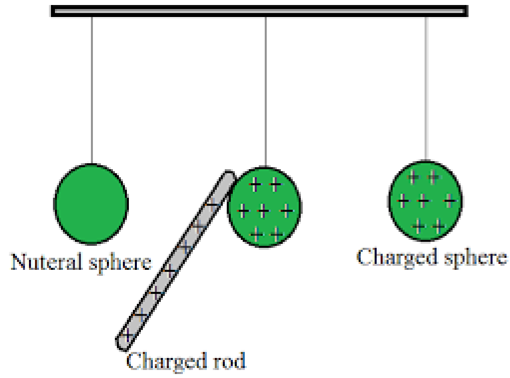
Charging By Induction

A positively charged rod is brought close to a neutral metal rod without touching it. The free electrons of the neutral metal rod will move within the metal towards the more external positive charge while the positive charges are left at the opposite end of the rod. In this way, a charge is induced at the two ends of the metal rod.

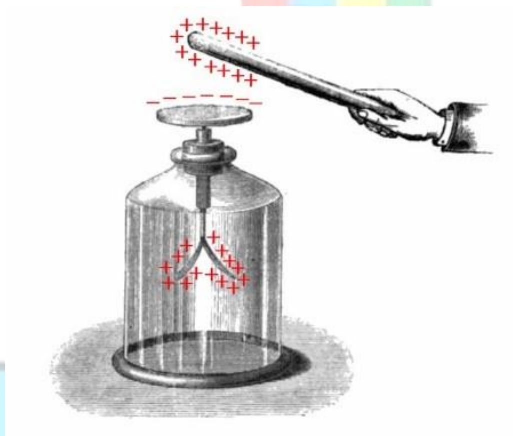


Charging By Conduction

This is also known as charging by contact, the charged metal rod It is close to touch the uncharged metal object. When a positively charged metal object touches a neutral metal rod, the free electrons in the neutral rod are attracted to the positively charged object.



Electroscope (Gold Leaf Electroscope)
This is a device that can be used for detecting the presence and magnitude of charge. It consists of a metal leaf, often made of gold, connected to a metal disk.



Coulomb's Law
It states that in free space (vacuum), oppositely charged bodies attract each other while similarly charged bodies repel each other. The force varies directly as the product of the magnitudes of each charge and varies inversely as the square of the distance between them.

If two particles q_1 and q_2 are separated by a distance r in a vacuum, the electrostatic force F_{21} exerted by q_1 on q_2 is given by:

$$F_{21} = \frac{K q_1 q_2}{r^2}$$

where K is the electrostatic constant (constant of proportionality):

$$K = 9 \times 10^9 \, \mathrm{Nm}^2/\mathrm{C}^2$$

Alternatively,

$$K = \frac{1}{4\pi\varepsilon_0}$$

where $\varepsilon_\circ$ is the **permittivity of free space**.

Assignment: Explain the relationship between the constant of proportionality and the permittivity of free space, and state the constant value of the permittivity of free space.

Superposition

If there are several charges present, the net force (total force) on any charge will be the vector sum of the forces due to each of the other charges. This is known as superposition.

For example, the net force on charge q_o is given as:

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03}$$

$$\vec{F}_0 = k\frac{q_0 q_1}{|\vec{r}_{01}|^2}\hat{r}_{01} + k\frac{q_0 q_2}{|\vec{r}_{02}|^2}\hat{r}_{02} + k\frac{q_0 q_3}{|\vec{r}_{03}|^2}\hat{r}_{03}$$

Examples:

A 100 µC charge q0 is placed at the origin of a Cartesian plane.

A −150 µC charge is placed on the x-axis, 2 m from the origin in the positive x direction, while a 300µC charge is placed 1 m from the origin in the negative x direction.

- What is the net force acting on the 100 µC charge?
- What is the net force acting on the −150 µC charge?

Charge arrangement diagram:

$$(300\ \mu C\ (q_3)) \longleftarrow\ 1\text{ m}\ \longrightarrow (100\ \mu C\ (q_0)) \longrightarrow\ 2\text{ m}\ \longrightarrow (-150\ \mu C\ (q_1))$$

$$\vec{F}_{01}\ (\text{Force on } q_0 \text{ due to } q_1) \longrightarrow\ (\text{positive x direction})$$

$$\vec{F}_{03}\ (\text{Force on } q_0 \text{ due to } q_3) \longleftarrow\ (\text{negative x direction})$$

First, calculate the net force:

$$F_{\text{net}} = F_{12} + F_{13}$$

Calculate F_{12}:

$$F_{12} = k\frac{q_1 q_2}{r} \quad \text{where} \quad r = \frac{9 \times 150 \times 100 \times 10^{-12}}{25 \times 10 \times 10^{-12}} \div 4$$

$$F_{12} = 1.35 \times 10^2 \div 4$$

$$F_{12} = 33.75\,\text{N}$$

Calculate F_{13}:

$$F_{13} = k\frac{q_1 q_3}{r} \quad \text{where} \quad r = \frac{9 \times 100 \times 300 \times 10^{-12}}{45 \times 10 \times 10^{-12}} \div 16$$

$$F_{13} = 2.7 \times 10^2 \div 16$$

$$F_{13} = 16.875\,\text{N}$$

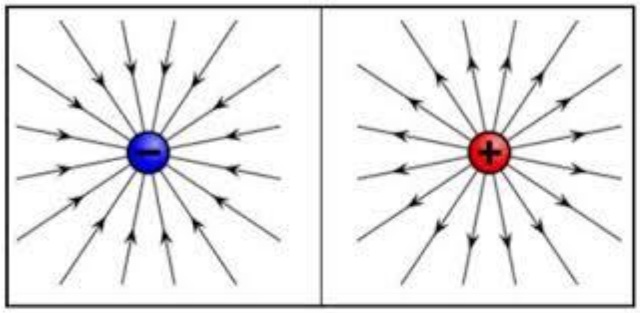
Find the net force:

$$F_{\text{net}} = 33.75 + 16.875$$

$$F_{\text{net}} = 50.625\,\text{N}$$

ELECTRIC FIELD

An electric field can be defined as a region where a charged particle experiences an electric force. It can be mapped out by the lines of force, which are imaginary lines representing a field of force.



PROPERTIES OF ELECTRIC LINES OF FORCE

1. The electric lines of force are drawn in such a way that the magnitude of the electric field is proportional to the number of lines crossing a unit area perpendicular to the line.
2. The lines of force are continuous and start on positive charges and end only on negative charges.
3. Lines of force do not touch or intersect one another.
4. The tangent to the lines of force at any point gives the direction of the field at that point.

ELECTRIC FIELD STRENGTH

The electric field strength at any point is the force per unit it exerts at a point. It is given as

$$E = \frac{F}{q}$$

, where q=test charge placed at that point

Relationship Between Electric Field (E) and Electric Potential (V)

The electric field (E) at a point due to a charge is given as:

$$E = \frac{F}{q}$$

where (q) is the test charge placed at that point.

The expression relates the electric field (E) and electric potential (V):

$$V_B - V_A = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{l}$$

For an infinitesimal change:

$$\Delta V = -\mathbf{E} \cdot \Delta \mathbf{L} = -E \Delta L \cos \theta$$

Rewriting in proportional unit form:

$$E = \frac{\Delta V}{\Delta L}$$

In Cartesian coordinates, the electric field components are:

$$E_x = -\frac{\Delta V}{\Delta x}, \quad E_y = -\frac{\Delta V}{\Delta y}, \quad E_z = -\frac{\Delta V}{\Delta z}$$

Electric Fields of Electric Dipoles

Diagram:

- Two charges, +q and -q, separated by a distance **2d**.
- **Point P** is the point of reference.

Electric field components:

- $$E_x = E\cos(\theta) + E\cos(\theta)$$
- $$E_y = E\sin(\theta) - E\sin(\theta) = 0$$

Net Electric Field Equation:

$$E_{\text{net}}^2 = E_x^2 + E_y^2$$

Explanation:
The net electric field intensity **(E_{\text{net}})** at point P is the vector sum of the electric field components in the horizontal and vertical directions. Since the vertical components cancel out, only the horizontal components contribute to (E_{\text{net}}).

Additional Diagram Details:

- **Point P** (point of reference)
- **Distance (r)** from the charges to Point P
- **Distance (d)** between the charges
- **Charges (+q) and (-q)** separated by distance (2d)
- **Axial line (2d) and Equatorial line (xz)**

Sum of Horizontal Component:

$$E_x = E\cos(\theta) + E\cos(\theta)$$

Sum of Vertical Component

$$E_y = E_1 \sin\theta - E_2 \sin\theta$$

Sum of Vertical Components

We start with:

$$E_y = E_1 \sin\theta - E_2 \sin\theta$$

But since $E_1 = E_2$, the vertical components cancel:

$$E_y = 0$$

Sum of Horizontal Components

$$E_x = E_1 \cos\theta + E_2 \cos\theta = 2E_1 \cos\theta$$

We now express E_1 and $\cos\theta$.

Electric Field Magnitude from One Charge

$$E_1 = \frac{kq}{(x^2 + d^2)}$$

Assuming the point is along the axis of the dipole and $r^2 = x^2 + d^2$, where d is the perpendicular distance of each charge from the x-axis.

Cosine of the Angle

$$\cos\theta = \frac{d}{\sqrt{x^2 + d^2}}$$

Substitute into Equation (2):

$$E_x = 2 \cdot \frac{kq}{x^2 + d^2} \cdot \frac{d}{\sqrt{x^2 + d^2}} = \frac{2kqd}{(x^2 + d^2)^{3/2}}$$

Introduce the Electric Dipole Moment

The dipole moment is defined as:

$$p = 2qd$$

Substitute into (5):

$$E_x = \frac{kp}{(x^2 + d^2)^{3/2}}$$

Far-Field Approximation (x≫dx)

When the point of observation is far from the dipole (x≫dx), we can approximate:

$$x^2 + d^2 \approx x^2 \Rightarrow (x^2 + d^2)^{3/2} \approx x^3$$

$$E_x \approx \frac{kp}{x^3}$$

$$E_x = \frac{kp}{x^3}$$

MOTION OF A CHARGED PARTICLE IN A UNIFORM ELECTRIC FIELD

From Newton's second law, we recall that the force **F** with which a body of mass **m** accelerates with an acceleration **a** is given as:

F = ma ---------(1)

Electric field intensity, expressed as force per unit charge, is **E = F/q**. So, **F = qE** --------(2)

Equating equations (1) and (2), we have:

ma = qE

Hence, **a = qE/m**

Example 1:

An electron experiences a force pushing it from west to east after being placed in a uniform electric field. The value of the electric field is 0.1V/m, and the electron has a charge of 6C and a mass of 9.1×10^{-31} kg. In what direction is the electric field? Then, what is the value of the force causing the electron to accelerate, and what is its acceleration?

1. The direction of an electron is usually opposite the direction of the electric field, so the direction of the electric field is from east to west.
2. F = eq
   $e = 0.1 \times 6 \times 10^{-6}$
   $= 6 \times 10^{-7}$ N
3. $a = eq/m = 6 \times 10^{-7} / 9.1 \times 10^{-31}$
   $= 6 \times 10^{28}\ ms^{-2}$

CLASSWORK

1. An electric field of $60NC^{-1}$ has a unit positive and unit negative charge placed in it at rest. They have masses of 1.6×10^{-27} kg and 9.1×10^{-31} kg, and both have the same charge of magnitude 1.6×10^{-19} C. What is the acceleration of each particle after they must have been released?

   $E = 60NC^{-1}$
   $m_1 = 1.6 \times 10^{-27}$ kg

$m_2 = 9.1 \times 10^{-31}$ kg

$q = 1.6 \times 10^{-19}$ C

1.

$$a_1 = \frac{60 \times 1.6 \times 10^{-19+27}}{1.6}$$

$$a_1 = \frac{60 \times 1.6 \times 10^{8}}{1.6}$$

$$a_1 = 60 \times 10^{8} = 6.0 \times 10^{9}$$

2.

$$a_2 = \frac{60 \times 1.6 \times 10^{-19+31}}{9.1}$$

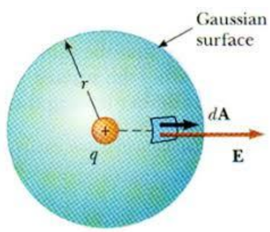
$$a_2 = \frac{96 \times 10^{12}}{9.1}$$

$$a_2 \approx 1.055 \times 10^{13}$$

GAUSS' LAW

Gauss' law states that the net flux around any closed surface is:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$$



Where

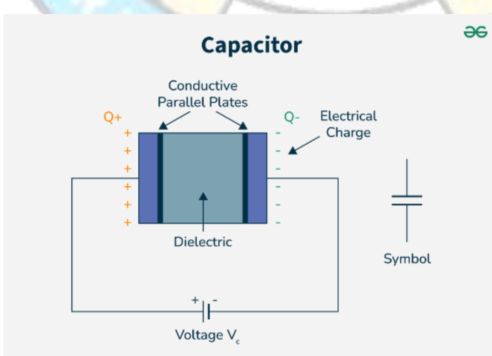
- (Q_{in}) represents the net charge inside the surface,
- (E) represents the electric field at any point on the surface, and
- ε_o is the permittivity of free space.

CAPACITANCE OF A CAPACITOR

Capacitors are used to store electric charge.

Capacitance refers to the ability of a capacitor to store charge. The unit of capacitance is coulomb per volt C.V $^{-1}$which is called a **Farad**.

Capacitors consist of two charged metal plates separated by an insulator, called a **dielectric**.



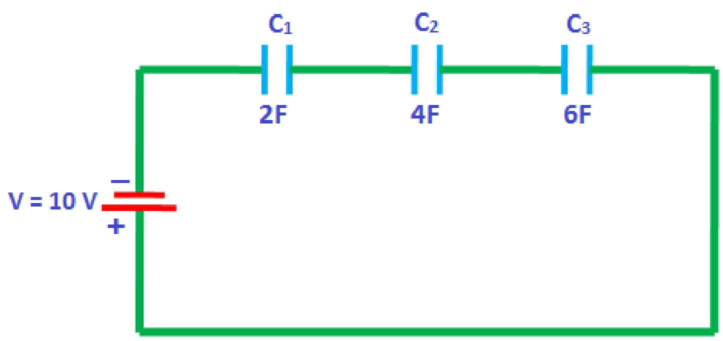
It is found that the charge of each capacitor is proportional to the potential difference of each conductor.

CAPACITORS IN SERIES AND IN PARALLEL

In an electric circuit, capacitors can be connected together in various ways. The two common methods are **series** and **parallel** connections.

- Capacitors connected in **series** have the same charge.
- Capacitors connected in **parallel** have the same potential charge or voltage.

CAPACITORS CONNECTED IN SERIES



Series capacitor circuit

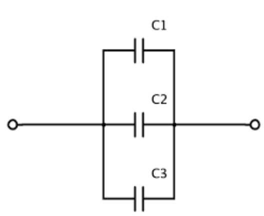
Physics and Radio-Electronics

- **Capacitors in Series:**
- When capacitors are connected end-to-end, the total or equivalent capacitance $Ceq \cdot C_{eq} \cdot Ceq$ decreases.
- The reciprocal of the total capacitance is the sum of the reciprocals of the individual capacitances:

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

- The charge Q on each capacitor in series is the same.
- The total voltage across the series combination is the sum of the voltages across each capacitor.

CAPACITORS CONNECTED IN PARALLEL



- When capacitors are connected with all their terminals joined respectively, the total or equivalent capacitance increases.
- The total capacitance is the sum of the individual capacitances:
- $$C_{eq} = C_1 + C_2 + C_3 + \cdots$$ The voltage V across each capacitor in parallel is the same.
- The total charge stored is the sum of the charges on each capacitor.

Energy Stored in Capacitors

A charged capacitor stores electrical energy. The energy stored in a capacitor is equal to the work done in charging the capacitor. The work done to increase the charge on a charged capacitor is (QV). The net voltage across the plates is:

$$U = \int_0^Q V \, dQ$$

$$U = \frac{1}{C} \int_0^Q q \, dq$$

$$= \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q$$

$$= \frac{1}{C} \cdot \frac{Q^2}{2}$$

$$= \frac{Q^2}{2C}$$

Faraday's Law of Induction

When a changing magnetic flux through a closed loop induces an electromotive force (EMF) in the loop, it is proportional to the rate of change of the magnetic flux.

$$\mathcal{E} = -N \frac{d\Phi}{dt}$$

Lenz's Law

The direction of the induced current is such that it opposes the change in the magnetic flux that induced it. This means the induced current will generate a magnetic field that opposes the change in the original magnetic field and tries to maintain the original magnetic flux.

Relationship between Faraday and Lenz's Law

Lenz's law provides the direction of induced current, while Faraday's law provides the magnitude of induced EMF.
The two laws are crucial for designing and analyzing generators, transformers, and induction motors.

Self-Inductance and Mutual Inductance

Self-inductance is the property of a coil or circuit that opposes changes in the current flowing through it. When the current changes, a magnetic field is generated, inducing an emf that opposes the change in current.

Mutual inductance is the property of two or more coils or circuits that are magnetically coupled. When the current in one coil changes, it induces an emf in the other coil.

For self-inductance, we have a coil's ability to oppose change in its own current. Mutual inductance, on the other hand, is the ability of one coil to induce an emf in another coil.

Magnetic Coupling:

The interaction between the magnetic fields of two or more coils.

The unit of self-inductance and mutual inductance is **Henry**.

Assignment:

1. A coil with a self-inductance of 10 henry has a current of 2A going through it. What is the energy stored in the coil?
2. Two coils have a mutual inductance of 6 henry. If the current in one coil changes at the rate of 2A per second, what is the induced emf in the other coil?
3. What is the purpose of using a ferromagnetic core in a coil?
4. What is the energy stored in the coil for which a density function holds?

Answers

1. Current $I=2$ A, Inductance $L=10$ H
   Energy stored in an Inductor

$$E_c = \frac{1}{2} L I^2$$

Substitute Values

$$E_c = \frac{1}{2} \times 10 \times (2)^2$$

$$E_c = \frac{1}{2} \times 10 \times 4$$

$$E_c = 5 \times 4 = 20\,J$$

2.

$$E_c = M\frac{dI}{dt}$$

$$M = 5\,\mathrm{H}$$

$$\frac{dI}{dt} = 2\,\mathrm{A/s}$$

$$E_c = 5 \times 2 = 10\,\mathrm{V}$$

3.

- Higher Inductance
- Increased magnetic permeability
- Stronger magnetic field
- Improved coupling

ELECTROMOTIVE FORCE

EMF is the energy per unit charge that a device such as a battery or generator can provide to a circuit. It is measured in volts (V).

Electromotive force has its applications in power generation or energy storage and electrical circuits.

Assignment 2

1. A device has an emf of 12 volts and an internal resistance of 2Ω. What is the terminal voltage when a current of 2A flows through it?
2. Differentiate between emf and voltage.
3. Differentiate between magnetic field and magnetic flux.

Answers:

1.

$$V = \text{terminal voltage}$$
$$E = 12\,\text{V} \quad (\text{emf})$$
$$I = 2\,\text{A}$$
$$r = 2\,\Omega \quad (\text{internal resistance})$$

$$V = E - Ir$$
$$V = 12 - (2 \times 2)$$
$$V = 12 - 4 = 8\,\text{V}$$

2.

E.M.F

- Energy provided per coulomb of charge by a source (like a battery or generator) to move charges through an entire circuit.

Voltage

- The difference in electric potential between any two points in a circuit.

3. **Magnetic Field**

- A region around a magnetic material or current-carrying wire where magnetic force is experienced.
- Vector Quantity
- SI unit is Tesla (T)
- Formula:

$$F = qvB\sin\theta$$

Magnetic Flux

- The total magnetic field passing through a given surface.
- Scalar Quantity
- SI unit is Weber (Wb)
- Formula:

$$\Phi = B \cdot A\cos\theta$$

TRANSFORMER

A transformer is an electrical device that transfers electrical energy from one circuit to another through electromagnetic induction. It consists of two coils of wire known as the primary and secondary coils, wrapped around a common magnetic core.

STEP UP TRANSFORMER

A step-up transformer increases the voltage of an AC from the primary coil to the secondary coil. This is achieved by having more turns in the secondary coil than in the primary coil: **Ns > Np**.

STEP DOWN TRANSFORMER

A step-down transformer decreases the voltage of an AC alternating current from the primary coil to the secondary coil. This is achieved by having more turns in the primary coil than in the secondary coil: **Ns < Np**.

Transformers have numerous applications in electronic devices and power transmission and distribution.

Question 1:

A step-up transformer has 100 turns in the primary coil and 50 turns in the secondary coil. If the primary voltage is 120V, calculate the secondary voltage.

$$\frac{N_p}{N_s} = \frac{U_p}{U_s}$$

$$\frac{100}{50} = \frac{120}{U_s}$$

$$100 \times U_s = 50 \times 120$$

$$100U_s = 6000$$

Question 2:

A step-down transformer has 500 turns in the primary coil and 100 turns in the secondary coil. If the primary voltage is 240V, calculate the secondary voltage.

$$\frac{N_p}{N_s} = \frac{U_p}{U_s}$$

$$\frac{500}{100} = \frac{240}{U_s}$$

$$500 \times U_s = 100 \times 240$$

$$500U_s = 24000$$

$$U_s = \frac{24000}{500} = 48\,\mathrm{V}$$

Maxwell's Equations

Maxwell's equations are a set of four fundamental equations that describe the behavior of electric and magnetic fields.

1. **Gauss's Law of Electric Fields:**

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon}$$

2. **Gauss's Law of Magnetic Fields:**

$$\nabla \cdot \mathbf{B} = 0$$

3. **Faraday's Law of Induction:**

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

4. **Ampere's Law with Maxwell's Addition:**

$$\nabla \times \mathbf{B} = \mu_0 NI + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Maxwell's equations unify electricity and magnetism. They predict electromagnetic waves and describe light as an electromagnetic wave.

Electromagnetic Oscillations and Waves

These describe the behavior of electric and magnetic fields.

- **Electromagnetic Oscillation:**
  Electromagnetic oscillation occurs when electric and magnetic fields oscillate at a specific frequency, often in a resonant cavity or aircraft.

- **Electromagnetic Waves:**
  Electromagnetic waves propagate through space, carrying energy and information.

  Types of Electromagnetic Waves

  - Radiowaves
  - Microwaves
  - Infrared radiation
  - Visible light
  - Ultraviolet radiation

- X-rays
- Gamma rays

Electromagnetic waves have several properties:

1. Frequency (f)
2. Wavelength (λ)
3. Speed ($c = \lambda f$)
4. Amplitude

An electromagnetic wave has a frequency of 100 megahertz. The calculated wavelength, based on the speed of light (3×10^8 m/s), is:

Formula:
$c = \lambda f$

Solving for wavelength (λ):
$\lambda = c/f$
$\lambda = (3 \times 10^8) / (100 \times 10^6)$
$\lambda =$ **3 meters**

Voltage and Current Application to Inductors, Capacitors, and Resistors

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Z is the **impedance** in an AC circuit,

R is the **resistance**,

X_L is the **inductive reactance**,

X_c is the **capacitive reactance**.

The inductive reactance increases with frequency, meaning that inductance upholds high-frequency signals more than low-frequency signals.

2. Voltage and Current Relationship

$$V = L\frac{dI}{dt}$$

Where:

- (V) = Voltage
- (I) = Current

The voltage leads the current by 90°, meaning that the voltage peaks before the current peaks.

Energy Storage

Inductors store energy in a magnetic field when current flows through them. The energy stored is proportional to the square of the current.

$$E \propto I^2$$

Capacitive Reactance

$$X_c = \frac{1}{2\pi f C}$$

where:
(c) = capacitance

(f) = frequency
(Xc) = Capacitive Reactance

Capacitors oppose low-frequency signals more than high-frequency signals.

Voltage and Current Relationship

$$I = C\frac{dv}{dt}$$

where:
(I) = current
(V) = voltage

The current leads the voltage by **90°**, meaning that the current peaks before the voltage peaks.

Capacitors store energy in an **electric field** when there is voltage across the capacitor.

Inductors store energy in a **magnetic field**. The energy stored is proportional to the square of the voltage.

Resistors oppose current flow, converting some energy into heat.

For resistors, (V) and (I) are **in phase**, meaning that the peaks and troughs align.

Energy Dissipation
Resistors dissipate energy as heat:

$$P = \frac{V^2}{R}$$

$$P = IV$$

$$P = I^2R$$