COURSE OUTLINE

TUE DEC 10

Mr. Ademola

1. Indices, Logarithm and Surd
2. Sets, subsets, Union, Intersection, complement and use of venn diagram
3. Quadratic Equation
4. Trigonometric Functions
5. Identify ~~using~~ various types of numbers
6. Binomial Theorem

- Mathematical Induction
- Complex Numbers

MTH 101

Tue, 10th Dec. 2024

INDICES

$3 \times 3 \times 3 \times 3 = 3^4$ → exponent/power

↳ Base

Laws of Indices

- $a^x \times a^y = a^{x+y}$
- $a^x \div a^y = a^{x-y}$
- $(a^m)^n = a^{mn}$
- $a^{1/n} = \sqrt[n]{a}$
- $a^{m/n} = a^{1/n \times m}$
- $a^0 = 1$
- $a^{-n} = 1/a^n$

Ex 1. Simplify $\dfrac{a^{-2} b^3 c^{-4}}{6} \times \dfrac{9}{a^3 b^{-3} c^4}$

$\dfrac{9}{6} \times \dfrac{a^{-2} b^3 c^{-4}}{a^3 b^{-3} c^4}$

$\dfrac{9}{6} \times a^{-2-3}\, b^{3+3}\, c^{-4-4}$

$9/6 \times a^{-5} b^6 c^{-8}$

$3/2 \times a^{-5} b^6 c^{-8}$

$\dfrac{3}{2} \times \dfrac{1}{a^5} \times b^6 \times \dfrac{1}{c^8}$

$= \dfrac{3b^6}{2a^5c^8}$

2. Simplify $\frac{y^{m+2x} \times y^{3m-8x}}{y^{5m-6x}}$

$= \frac{y^{(m+2x)+(3m-8x)}}{y^{5m-6x}}$

$= \frac{y^{4m-\cancel{6x}}}{y^{5m-6x}}$

$= y^{(4m-\cancel{6x})-(5m-6x)}$

$= y^{4m-\cancel{6x}-5m+6x}$

$= y^{-m}$

$= \frac{1}{y^m}$

EXPONENTIAL EQUATION

Solve:

(i) $3^{x^2+2} = 27^x$

$3^{x^2+2} = 3^{3x}$

$x^2+2 = 3x$

$x^2 = 3x - 2$

$x^2 - 3x + 2 = 0$

$(x^2 - x)(-2x+2)$

$x(x-1) - 2(x-1) = 0$

$(x-2)(x-1)$

$\therefore x = 2$ or 1

2. (ii) $2^{x^2-2} = 16(2^{5x})$

$2^{x^2-2} = 2^4(2^{5x})$

$2^{x^2-2} = 2^{4+5x}$

$x^2 - 2 = 4 + 5x$

$x^2 - 5x - 2 - 4 = 0$

$x^2 - 5x - 6 = 0$

$(x^2 + x)(-6x - 6) = 0$

$x(x+1) - 6(x+1)$

$(x-6)(x+1)$

$x = 6$ or $x = -1$

(iii) $3^{2x-3} - 4(3^{x-2}) + 1 = 0$

$3^{2x-3} - 2^2(3^{x-2}) + 1 = 0$

$3^{2x} \times 3^{-3} - 2^2(3^x \times 3^{-2}) + 1 = 0$

$(3^x)^2 \times 3^{-3} - 4(3^x \times 3^{-2}) + 1 = 0$

Let $3^x = y$

$\therefore y^2 \times 3^{-3} - 4(y \times 3^{-2}) + 1 = 0$

$\frac{y^2}{3^3} - 4(y \times \frac{1}{3^2}) + 1 = 0$

$\frac{y^2}{3^3} - \frac{4y}{3^2} + 1 = 0$

$\frac{y^2}{27} - \frac{4y}{9} + 1 = 0$

Multiply zru by 27

$27\frac{(y^2)}{27} - \frac{27(4y)}{9} + 27(1) = 27(0)$

$y^2 - 12y + 27 = 0$

$(y^2 - 3y)(-9y + 27) = 0$

$y(y-3) - 9(y-3) = 0$

$(y-9)(y-3)$

$y = 9$ or 3

Recall $y = 3^x$

$\therefore 9 = 3^x$

$3^2 = 3^x$

$x = 2$

$3 = 3^x$

$3^1 = 3^x$

$x = 1$

$\therefore x = 1$ or $x = 2$

(iv) Solve the equation

$8(4^x - 2^x) + 2 = 2^{x+1}$

$8(2^{2x} - 2^x) + 2 = 2^x \times 2^1$

~~8((2^x) 2^x - (2^x)) + 2 = 2^x × 2^1~~

$8((2^x)^2 - 2^x) + 2 = 2^x \times 2^1$

Let $2^x = P$

$\therefore 8(P^2 - P) + 2 = P \times 2$

$8P^2 - 8P + 2 = 2P$

$8P^2 - 10P + 2 = 0$

Divide all by 2

$\therefore 4P^2 - 5P + 1 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ using Quad. formula

$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(1)}}{2(4)}$

$\frac{5 \pm \sqrt{25 - 16}}{8}$

$\frac{5 \pm \sqrt{9}}{8}$

$\Rightarrow \frac{5 + \sqrt{9}}{8}$ or $\frac{5 - \sqrt{9}}{8}$

$\Rightarrow \frac{5+3}{8}$ or $\frac{5-3}{8}$

$\Rightarrow \frac{8}{8}$ or $\frac{2}{8}$

$\Rightarrow P = 1$ or $1/4$

Recall $2^x = P$

$2^x = 1$

$2^x = 2^0$

$x = 0$

$\Rightarrow 2^x = 1/2^2$

$2^x = 2^{-2}$

$x = -2$

$\therefore x = 0$ and $x = -2$

ASSIGNMENT

1. Solve, $x = 9\sqrt{9x^{1/2}}$
2. Solve, $3^x - 2^{y+2} = 10$
3. Simplify $\frac{16(32)^n - 2^{3m-2} \times 4^{m+1}}{15(2^{m-1})(16)^n} - \frac{5^m}{\sqrt{5^{2m}}}$
4. Simplify $\frac{3^{-3} \cdot 6^2 \sqrt{48}}{5^2 \times \sqrt[3]{1/25} \times (15)^{-4/3} \times (3)^{1/3}}$

LOGARITHM

If $b^y = N$, then y is called logarithm of N to base b, and is written as $\log_b N$.

Thus, $b^y = N$, $y = \log N$

LAWS OF LOGARITHM

1. $\log_a XY = \log_a X + \log_a Y$
2. $\log_a x/y = \log_a x - \log_a y$
3. $\log_a x^n = n\log_a x$
4. $\log_a a = 1$
5. $b^y = N$, $y = \log_b N$

Ex: If $a^2 + b^2 = 23ab$; show that $\log a = 2\log\left[\frac{a+b}{5}\right]$

$a^2 + b^2 = (a+b)^2 - 2ab$

Since $a^2 + b^2 = 23ab$

$\therefore (a+b)^2 - 2ab = 23ab$

$(a+b)^2 - 25ab = 0$

$(a+b)^2 = 25ab$

$$ab = \frac{(a+b)^2}{25}$$

$$ab = \left(\frac{a+b}{5}\right)^2$$

Taking log of both sides

$$\log ab = \log\left(\frac{a+b}{5}\right)^2$$

$$\log a + \log b = 2\log\left(\frac{a+b}{5}\right)$$

ASSIGNMENT SOLUTION

1. $x = 9\sqrt{9x^{1/2}}$

$x = 9(9x^{1/2})^{1/2}$

$x = 9(9^{1/2} \cdot x^{1/4})$

$x = 9(3 \cdot x^{1/4})$

$x = 9(3x^{1/4})$

$x = 27x^{1/4}$

$x - 27x^{1/4} = 0$

x . WRONG!

2. $3^x - 2^{y+2} = 10$; $\boxed{2^y + 3^{x-2} = 2}$

Let $3^x = P$, and $2^y = Q$ $\quad 2^y + \frac{3^x}{3^2} = 2$

$3^x - 2^y \times 2^2 = 10$

$P - 4Q = 10$ ——— eqn(i)

$Q + P/9 = 2$ ——— eqn(ii)

$P = 9(2-Q)$

first...

$9(2-Q) - 4Q = 10$

$18 - 9Q - 4Q = 10$

$-13Q = -8$

$Q = 8/13$

$P - 4(8/13) = 10$

$P = \frac{10}{1} + \frac{32}{13}$

$P = \frac{130+32}{13}$

$P = \frac{162}{13}$

P/F

$3^x = \frac{162}{13}$; $2^y = 8/13$

Hmmm...

3. $\frac{16(32)^n - 2^{3m-2} \times 4^{m+1}}{15(2^{m-1})(16^n)} - \frac{5^m}{\sqrt{5^{2m}}}$

$\frac{2^4(2^5)^n - 2^{3m-2} \times 2^{2(m+1)}}{15(2^{m-1})(2^{4n})} - \frac{5^m}{\sqrt{5^{2m}}}$

$\frac{2^4(2^{5n}) - 2^{3m-2} \times 2^{2m+2}}{15(2^{m-1})(2^{4n})} - \frac{5^m}{\sqrt{5^{2m}}}$

$2^{4+5n - (3m-2) + 2m+2}$

$\frac{2^{4+5n} - 2^{3m-2+(2m+2)}}{15(2^{4n+m-1})} - \frac{5^m}{\sqrt{5^{2m}}}$

$\frac{2^{4+5n} - 2^{5m}}{15(2^{4n+m-1})} - \frac{5^m}{\sqrt{5^{2m}}}$

$\frac{2^{4+5n} - 2^{5m}}{15(2^{4n+m-1})} - \frac{5^m}{(5^{2m})^{1/2}}$

$\frac{2^{4+5n} - 2^{5m}}{15(2^{4n+m-1})} - \frac{5^m}{5^m}$

$= \frac{2^{4+5n} - 2^{5m}}{15(2^{4n+m-1})} - 1$

4. $\frac{3^{-3} \cdot 6^2 \sqrt{48}}{5^2 \cdot \sqrt[3]{1/25} \cdot (15)^{-4/3} \cdot (3)^{1/3}}$

$\frac{3^{-3} \cdot 6^2 \cdot 4\sqrt{3}}{5^2 \cdot (1/25)^{1/3} \cdot (15)^{-4/3} \cdot (3)^{1/3}}$

NUMERATOR:

$3^{-3} \cdot 6^2 \cdot 4\sqrt{3}$

$1/27 \times 36 \times 4\sqrt{3}$

$1/3 \times 4 \times 4\sqrt{3}$

$= \frac{16\sqrt{3}}{3}$

DENOMINATOR:

$5^2 \cdot (1/25)^{1/3} \cdot (15)^{-4/3} \cdot (3)^{1/3}$

$5^2 \cdot (1/25)^{1/3} \cdot (5 \times 3)^{-4/3} \cdot (3)^{1/3}$

$5^2 \cdot (5^{-2})^{1/3} \cdot (50625)^{-1/3} \cdot 3^{1/3}$

$5^2 \cdot (5^{-2})^{1/3} \cdot (5^4 \times 3^4)^{-1/3} \cdot 3^{1/3}$

$5^2 \cdot (5^{-2})^{1/3} \cdot (\sqrt[3]{1/5^4} \times \sqrt[3]{1/3^4}) \cdot 3^{1/3}$

$5^2 \cdot 5^{-2/3} \cdot \sqrt[3]{1/5^4} \times \sqrt[3]{1/3^4} \cdot 3^{1/3}$

$5^2 \cdot 5^{-2/3} \cdot 5^{-4/3} \times 3^{-4/3} \cdot 3^{1/3}$

$5^{2+2/3+(-4/3)} \times 3^{-4/3+1/3}$

$5^{\frac{6+2-4}{3}} \times 3^{\frac{-4+1}{3}}$

$5^{4/3} \times 3^{-3/3}$

$5^{4/3} \times 3^{-1}$

$5^{4/3} \times 1/3$

$\Rightarrow \frac{5^{4/3}}{3}$

$\frac{16\sqrt{3}}{\cancel{3}} \times \frac{\cancel{3}}{5^{4/3}}$

$\frac{16\sqrt{3}}{5^{4/3}}$

$3^{x} - 2^{y+2} = 10$

$2^y + 3^{x+2} = 2$

$3^x - 2^y \times 2^2 = 10$

$3^x \times 3^2 + 2^y = 2$

Let $3^x = P$; $2^y = Q$

$P - Q \times 4 = 10$

$P \times 9 + Q = 2$

$P - 4Q = 10$

$9P + Q = 2 \quad \times 4$

$P - 4Q = 10$

$\underline{36P + 4Q = 8}$

$37P \quad = 18$

$P = \frac{18}{37}$

$P - 4Q = 10$

$18/37 - 4Q = 10$

$-4Q = 10 - 18/37$

$-148Q = 370 - 18$

$-Q = \frac{352}{148}$

$-Q = \frac{88}{37}$

$Q = -88/37$

$3^x = P$

$3^x = 18/37$

Taking log of both sides

$\log_3 x = \log 18/37$

$\log_3 x = \log 18 - \log 37$

$\log_3 x = 1.2553 - 1.5682$

$\log_3 x = -0.$

$\log 3^x = \log 18/37$

$x \log 3 = -0.3129$

$0.4471x = -0.3129$

$x = -0.6558$

1. $x = 9\sqrt{9x^{1/2}}$

$x = 9(9x^{1/2})^{1/2}$

$x = 9(3^2)^{1/2} \cdot (x^{1/2})^{1/2}$

$x = 3^2(3^2)^{1/2} \cdot x^{1/4}$

$x = 3^2 \cdot x^{1/4}$

$\frac{x}{x^{1/4}} = 3^2$

$x^{1-1/4} = 3^2$

$x^{3/4} = 3^2$

Sq

$x^{3/4 \times 4} = 3^{2 \times 4}$

$x^3 = 3^8$

Cube root both sides

$\sqrt[3]{x^3} = 3^{8 \times 1/3}$

$x = 3^{8/3}$ ←

FURTHER:

$x = 6561^{1/3}$

$x = \sqrt[3]{27} \times \sqrt[3]{27} \times \sqrt[3]{9}$

$x = 3 \times 3 \times 3\sqrt{3}$

$\boxed{x = 27\sqrt{3}}$

Surds

Surds are number of the form; $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \ldots$ They are numbers whose squareroots cannot be easily gotten

$\sqrt{4} = \sqrt{2^2} = (2^2)^{1/2} = 2$

LAWS OF SURDS

1. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
2. $\sqrt{a/b} = \sqrt{a}/\sqrt{b}$
3. $\sqrt{a} \times \sqrt{a} = a$

Ex: $\sqrt{72} + \sqrt{8}$

$\sqrt{36 \times 2} + \sqrt{4 \times 2}$

$6\sqrt{2} + 2\sqrt{2}$

$= 8\sqrt{2}$

2. $\sqrt{80} + \sqrt{20} - \sqrt{45}$

$\sqrt{16 \times 5} + \sqrt{4 \times 5} - \sqrt{9 \times 5}$

$4\sqrt{5} + 2\sqrt{5} - 3\sqrt{5}$

$6\sqrt{5} - 3\sqrt{5}$

$= 3\sqrt{5}$

Rationalisation of the surdic - denominator

$\frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}}$ LAW 3 CONCEPT

$= \frac{a\sqrt{b}}{b}$

3. Simplify $\frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{2}}$

$\frac{3+1}{3\sqrt{2}} = \frac{4}{3\sqrt{2}}$

$\frac{4}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{4\sqrt{2}}{3(2)} \Rightarrow \frac{4\sqrt{2}}{6}$

$= \frac{2\sqrt{2}}{3}$

4. $\frac{\sqrt{162} + \sqrt{32} - \sqrt{98}}{\sqrt{8}}$

$\frac{\sqrt{81 \times 2} + \sqrt{16 \times 2} - \sqrt{49 \times 2}}{\sqrt{4 \times 2}}$

$\frac{9\sqrt{2} + 4\sqrt{2} - 7\sqrt{2}}{2\sqrt{2}}$

$\frac{13\sqrt{2} - 7\sqrt{2}}{2\sqrt{2}}$

$\frac{6\sqrt{2}}{2\sqrt{2}} = 3$

Simplify $\frac{3\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}}$

$3\sqrt{5}$

Conjugate of surds

Given that the surd expression: $a + \sqrt{b}$, then, $a - \sqrt{b}$ is called its conjugate.

Similarly, if $a - \sqrt{b}$ is a surd, then $a + \sqrt{b}$ is its conjugate.

Rationalisation of the surd of the form $\frac{a}{b + \sqrt{c}}$ is to multiply the numerator and the denominator of the given surd with the conjugate of the denominator.

$\therefore \frac{a}{b + \sqrt{c}} \times \frac{b - \sqrt{c}}{b - \sqrt{c}}$

$$\frac{ab - a\sqrt{c}}{b^2 - c} = \frac{a(b-\sqrt{c})}{b^2-c}$$

5. $\frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

$\frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$\frac{3(2) - 3\sqrt{6} + \sqrt{6} - 3}{2 - 3}$

$\frac{6 - 2\sqrt{6} - 3}{2-3}$

$\frac{3 - 2\sqrt{6}}{-1}$

$= -3 + 2\sqrt{6}$

$= 2\sqrt{6} - 3$

$\frac{7+3\sqrt{5}}{3+\sqrt{5}}$

$\frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$

$\frac{21 - 7\sqrt{5} + 9\sqrt{5} - 3(\sqrt{5})}{9-5}$

$\frac{6+2\sqrt{5}}{4}$

$\frac{2(3+\sqrt{5})}{4}$

$= \frac{3+\sqrt{5}}{2}$

~~3 + 2√5~~

~~21 − 7√5 + 9√5 − 15~~

$\frac{7+3\sqrt{5}}{3+\sqrt{5}} + \frac{7-3\sqrt{5}}{3-\sqrt{5}}$

$\frac{(3-\sqrt{5})(7+3\sqrt{5}) + (3+\sqrt{5})(7-3\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$

$\frac{21 - 9\sqrt{5} - 7\sqrt{5} - 15 + 21 - 9\sqrt{5} + 7\sqrt{5}}{9-5}$ -15

$\frac{42 - 30 - 18\sqrt{5}}{4}$

$\frac{-6\sqrt{5}}{4}$

$-\frac{3\sqrt{5}}{2}$ WRONG!!

SEE ANSWER ON LAST PG

Task! $\frac{3+\sqrt{6}}{5\sqrt{3} - 3\sqrt{2} - \sqrt{32} + \sqrt{5}}$

$\rightarrow \frac{\sqrt{10}+\sqrt{5}-\sqrt{3}}{\sqrt{3}-\sqrt{10}-\sqrt{5}}$

$\rightarrow \frac{3\sqrt{5}-\sqrt{3}}{2\sqrt{5}+3\sqrt{3}}$ in form $a\sqrt{15}+b$

Solution

~~5√3 − 3√2 − 4√2 + √5~~

i. $\frac{3+\sqrt{6}}{5\sqrt{3} - 3\sqrt{2} - 4\sqrt{2} + \sqrt{5}}$

$\frac{3+\sqrt{6}}{5\sqrt{3} - 7\sqrt{2} + \sqrt{5}}$ TO BE CONTINUED!!

An Integer is a positive or negative whole number.

VARIABLES

CONSTANTS : A

Arbitrary constant

Fixed constant

pi $\pi = 22/7$

All real numbers are correct

MR. IDOWU

SET THEORY

A set is the collection of objects or things that are well-defined.

Examples of sets!

Real numbers,

SOLUTION

1. $\dfrac{3+\sqrt{6}}{5\sqrt{3}-3\sqrt{2}-\sqrt{32}+\sqrt{5}}$

$\dfrac{3+\sqrt{6}}{5\sqrt{3}-3\sqrt{2}-4\sqrt{2}+\sqrt{5}}$

$\dfrac{3+\sqrt{6}}{5\sqrt{3}-7\sqrt{2}+\sqrt{5}} \times \dfrac{5\sqrt{3}+7\sqrt{2}+\sqrt{5}}{5\sqrt{3}+7\sqrt{2}+5}$

$\dfrac{15\sqrt{3}+21\sqrt{2}+3\sqrt{5}+5\sqrt{18}+7\sqrt{12}+\sqrt{30}}{25(3)+\cancel{35\sqrt{6}}+\cancel{25\sqrt{3}}-35\sqrt{6}-49\sqrt{4}-35\sqrt{2}-35\sqrt{3}-7\sqrt{10}+5\sqrt{5}}$

$\dfrac{15\sqrt{3}+21\sqrt{2}+3\sqrt{5}+15\sqrt{2}+14\sqrt{3}+\sqrt{30}}{75+49(2)-35\sqrt{2}-7\sqrt{10}+5\sqrt{5}}$

$\dfrac{29\sqrt{3}+36\sqrt{2}+3\sqrt{5}+\sqrt{30}}{75-98-35\sqrt{2}-7\sqrt{10}+5\sqrt{5}}$

$\dfrac{29\sqrt{3}+36\sqrt{2}+3\sqrt{5}+\sqrt{30}}{-23-3\sqrt{2}-7\sqrt{10}+5\sqrt{5}}$

II PAUSE

2. $\dfrac{\sqrt{10}+\sqrt{5}-\sqrt{3}}{\sqrt{3}-\sqrt{10}-\sqrt{5}}$

$\dfrac{\sqrt{10}+\sqrt{5}-\sqrt{3}}{\sqrt{3}-\sqrt{10}-\sqrt{5}} \times \dfrac{\sqrt{3}+\sqrt{10}-\sqrt{5}}{\sqrt{3}+\sqrt{10}-\sqrt{5}}$

$\dfrac{\sqrt{30}+10-\sqrt{15}+\sqrt{15}+\sqrt{50}-5-9-\cancel{\sqrt{30}}+\cancel{\sqrt{15}}}{3+\sqrt{30}-\sqrt{15}-\sqrt{30}+10+\sqrt{50}-\sqrt{15}-\sqrt{50}+5}$

$\dfrac{-4+\sqrt{15}+5\sqrt{2}}{-2-2\sqrt{15}} \times \dfrac{-2+2\sqrt{15}}{-2+2\sqrt{15}}$

$\dfrac{8-8\sqrt{15}-2\sqrt{15}+2(15)+10\sqrt{2}+10\sqrt{30}}{4-4\sqrt{15}+4\sqrt{15}-4(15)}$

$\dfrac{38-10\sqrt{15}+10\sqrt{30}-10\sqrt{2}}{-56}$

$\dfrac{2(19-5\sqrt{15}+5\sqrt{30}-5\sqrt{2})}{2(-28)}$

$= \dfrac{19-5\sqrt{15}+5\sqrt{30}-5\sqrt{2}}{-28}$

$= -28$

3. $\dfrac{3\sqrt{5}-\sqrt{3}}{2\sqrt{5}+3\sqrt{3}}$ in form $a\sqrt{15}+b$

$\dfrac{3\sqrt{5}-\sqrt{3}}{2\sqrt{5}+3\sqrt{3}} \times \dfrac{2\sqrt{5}-3\sqrt{3}}{2\sqrt{5}-3\sqrt{3}}$

$\dfrac{6(5)-9\sqrt{15}-2\sqrt{15}+3(3)}{4(5)-\cancel{6\sqrt{15}}+\cancel{6\sqrt{15}}-9(3)}$

$\dfrac{30+9-11\sqrt{15}}{20-27}$

$= \dfrac{39-11\sqrt{15}}{-7}$

$= \dfrac{-11\sqrt{15}+39}{-7}$

$= \dfrac{11\sqrt{15}-39}{7}$

$= \frac{11}{7}\sqrt{15}-39$

JAN TUE 14 '25

SET THEORY

A set is a collection of objects, or things that are well defined

Examples
- A collection of students in 100L, 2025/2026 academic session
- Letters of the English alphabet
- The numbers, 2, 3, 5, 7, 11 (Prime numbers)
- A collection of all positive numbers
- ~~A~~ The content of a student's bag

The concept of a set is very important because set is now used as an official mathematical language. A good knowledge of this concept is therefore necessary for mathematics to be meaningful to its users.

~~Notation~~ NOTATIONS

A set is usually denoted by capital letters, while the objects comprising the set are written in small letters. These objects are called members or elements of a set.

E.g. Set A has members a, b, c, d

CONVENTION

The listing of ~~set~~ A ~~has~~ as a, b, c, d; is not an acceptable mathematical specification of a set. The correct representation of a set is to write the elements, separated by commas, and enclosed between braces or curly ~~braces~~ brackets. E.g. Set $A = \{a, b, c, d\}$.

The statement "b is an element of set A or b belongs to A" is written in the manner of $\boxed{b \in A}$. The contrary statement - that b doesn't belong to A, is written as $\boxed{b \notin A}$.

There are two ways of specifying a set - by listing the element in the set, such as $A = \{a, b, c, d\}$; and by stating the rules or properties which characterise the set. E.g. $B = \{x \mid 2 < x < 5\}$ or $B = \{x : 2 < x < 5\}$

NB: The stroke or colon ($|$) ($:$) can be used interchangeably with each of these read as", "such as".

$\therefore B = \{3, 4\}$

FINITE & INFINITE SETS

A finite set is one whose members are countable. For example, the set of students in CBS, 100L, 2024/2025 academic session.

- The whole numbers lying between 1 and 10
- Members of a football team
- An array/list of grocery items in a grocery list.

An Infinite set is one whose elements are uncountable. Examples:

- Real numbers
- Rational numbers
- Positive even numbers
- Complex numbers

The difference between a finite set and infinite set is that, a finite set has a defined beginning and a defined end, while an infinite set has a defined beginning and no defined end, or viceversa, or no defined beginning or end.

$A = \{1, 2, 3, 4\}$ — finite

$B = \{\ldots, 1, 2, 3, 4\}$

$C = \{\ldots, 1, 2, 3, 4, \ldots\}$ — Infinite

SUBSETS

Suppose:

$P = \{a, b, c, d, e, f\}$

$Q = \{c, d, e\}$

Then, it can be said that Q is contained in P; and the symbol, "$\subset$" is used to denote, "is contained in" or "is a subset of". Thus, Q is a subset of P $\Rightarrow Q \subset P$

THE QUALITY OF A SET

Two sets, X and Y are equal, if and only if X is a subset of Y, and Y is a subset of X. $[X \subset Y$, and $Y \subset X]$

Suppose $X = \{1, 2, 3\}$, and $Y = \{3, 1, 2\}$; then $X = Y$

NOTE: The rearrangement of the set does not alter the set

TYPES OF SETS

1. Null or Empty Sets : Null means void, therefore a null set is an empty set or a set that has no members or elements. The Null set is usually denoted by ϕ or { }.

<u>NB:</u> {0} is not an empty set, for 0 is an element.

2. Singleton Sets: Any set which has only one member is called "SINGLETON SET". E.g. $A = \{0\}$ or $B = \{a\}$

3. Universal Set: Every set is a subset of a larger or equivalent fixed set. This larger set is called the 'Universal set'. The symbols 'U' or 'ξ' are used to denote the universal set.

4. Power set: The collection of all the subsets of any set is called the power set. If a set has "n" members, where "n" is finite; then the total number of subset of S is 2^n. Occasionally, the power set, S, is denoted by 2^S or 2^n. For example:

Let $A = \{a, b, c\}$

$S = 2^n$; $n = 3$ (elements in set A)

$S = 2^3 = \boxed{8}$ → There are 8 subsets of the set A

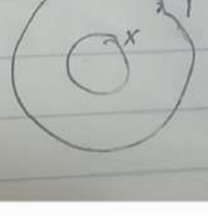
Subsets:

{a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}, { }

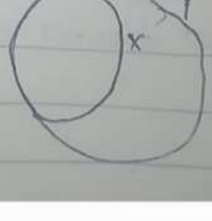
EULER-VENN DIAGRAM

The Theory of a set can be better understood if the venn diagram is implemented. The Venn-diagram is an ~~illust~~ instructive illustration, which shows relationship between the sets.

Suppose $X \subset Y$ and $X \neq Y$; we can represent this statement in Venn diagram as follows:

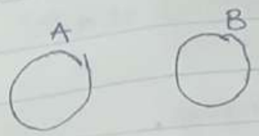


or



Disjoint set: Two sets A and B are said to be disjoint if neither A or B has elements in common.

The venn Diagram for disjoint sets, A and B, can be seen below



SET OPERATIONS

In set, the symbol, "$\cup$" denotes "union", and "$\cap$" denotes "Intersection".

- UNION OF A SET:

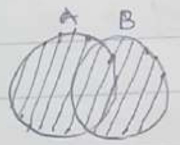
The union of sets A and B is the set of all elements which belongs to A or B, or to all elements. It is usually written as:

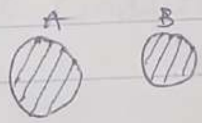
$A \cup B$

In set language, $A \cup B$ can be defined as:

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

The shaded portion in the venn diagram below, shows the relationship between A and B

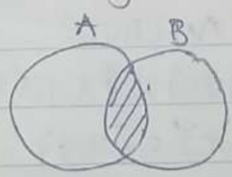




- INTERSECTION OF A SET:

The Intersection of a set A and B is the set of elements which belong to both A and B; and is written as $A \cap B$.

The venn diagram below shows the shaded portion of $A \cap B$.

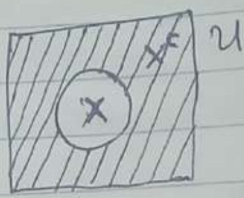


In set language; $A \cap B = \{x : x \in A \text{ and } x \in B\}$

- COMPLEMENT OF A SET

The complement of a set X is the set of elements which do not belong to X, but belongs to the Universal set. It is represented by X^c or X'

The diagram below ~~shows a~~ is a representation of ~~a~~ X'



In set language:

$X^c = \{x : x \in U, x \notin A\}$

Example:

Given that A, B, and C are subsets of the Universal set, Each of which are defined as follows:

$\xi = \{x : 2 \le x < 12, x \text{ is an integer}\}$

$A = \{x : 3 < x < 6\}$

$B = \{x : \{2 < x \le 5\} \cup \{9 < x < 12\}\}$

$C = \{x : 4 \le x \le 8\}$

(a.) List the members of A, B, and C, and Universal set.

(b.) Find:

(i) $(A \cup B) \cup C$ (ii) $A \cup (B \cap C)$ (iii) $A \cap (B \cup C)'$

Solution

(a.)

$\xi = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$A = \{4, 5\}$

$B = \{3, 4, 5, 10, 11\}$

$C = \{4, 5, 6, 7, 8\}$

(b.) (i) $(A \cup B) \cup C$

$(A \cup B) = \{3, 4, 5, 10, 11\}$

$(A \cup B) \cup C = \{3, 4, 5, 6, 7, 8, 10, 11\}$

(ii) $A \cup (B \cap C)$

$B \cap C = \{4, 5\}$

$A \cup (B \cap C) = \{4, 5\}$

(iii) $A \cap (B \cup C)'$

$(B \cup C) = \{3, 4, 5, 6, 7, 8, 10, 11\}$

$(B \cup C)' = \{2, 9\}$

$A \cap (B \cup C)' = \phi$

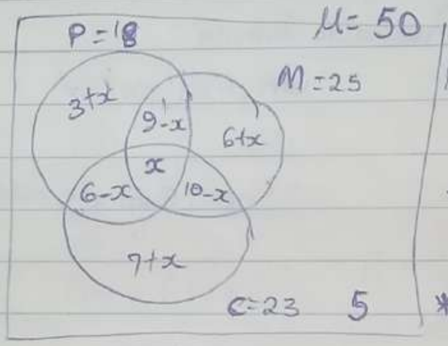
JAN TUE 21 25

SET THEORY

In a chemistry class, 18 students read Physics, 25, mathematics, 23, chemistry, 9, Physics and mathematics, 10, Maths. and Chemistry; and 6 read Physics and Chemistry. If there were 50 students altogether, and 5 students didn't read any of the three subjects, how many students read:
(i) All 3 subjects (ii) Only Mathematics (iii) Chemistry, but not maths
(iv) Physics and chemistry, but not mathematics

Solution-

Let P, M, and C; denote the students who read Physics, Mathematics, and Chemistry; in the following Venn diagram. Let x denote those who offer all 3 subjects, and $\mathcal{U}$, the Universal set.



* $n(P \cap M' \cap C') = (18 - (6-x) - (9-x) - x)$
$= 3+x$

* $n(M \cap P' \cap C') = (25 - (10-x) - (9-x) - x)$
$= 6+x$

* $n(C \cap P' \cap M') = (23 - (10-x) - (6-x) - x)$
$= 23 - 10 + x - 6 + \cancel{x} - \cancel{x}$
$= 7+x$

Total number of student who read $= 50 - 5$
$\Rightarrow 45$

(i) All three subjects

$(P \cup M \cup C)$

$3+\cancel{x} + 9 - \cancel{x} + \cancel{x} + 6 - \cancel{x} + 6 + \cancel{x} + 10 - \cancel{x} + 7 + x = 45$

$41 + x = 45$

$x = 4$

(ii) Only mathematics

$6+x \Rightarrow 6+4 = 10$

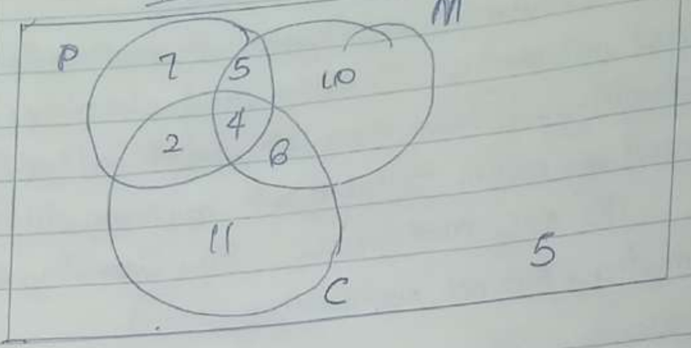
(iii) Chemistry, but not mathematics

$11 + 2 = 13$

(iv) Physics and Chemistry, but not mathematics

$= 2$

Updated Venn-diagram



$\mu = 50$

JAN TUE 20, '25

QUADRATIC EQUATIONS

There are four methods by which quadratic equations can be solved:

- Factorisation
- Completing the squares
- Quadratic formula
- Graphical method

Linear, Quadratic, cubic, quartic.

FACTORISATION

$x^2 + 5x - 6 = 0$

$(x^2 - x)(+6x - 6) = 0$

$x(x-1) + 6(x-1) = 0$

$(x+6)(x-1) = 0$

$\therefore x = -6$ or 1

COMPLETING THE SQUARES

Given that:

$ax^2 + bx + c = 0$ is the quadratic equation. Find the root of the equation.

- Rewrite the equation by eliminating 0

$ax^2 + bx = -c$

- Divide both sides by the coefficient of x^2

$\frac{ax^2}{a} + \frac{bx}{a} = \frac{-c}{a}$

$x^2 + \frac{bx}{a} = -\frac{c}{a}$

Add $(1/2)^2$ of coefficient of x to both sides

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{}\right)^2$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Root both sides

$$\sqrt{\left(x + \frac{b}{2a}\right)^{\cancel{2}}} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \Longleftarrow \text{Quadratic formula}$$

Root of the equation:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

SYMMETRIC PROPERTIES

Symmetric properties of the roots:

Let α, β be the root of the quadratic equation, $ax^2 + bx + c = 0$, therefore:

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$\sqrt{b^2 - 4ac}$ I ⟶ Discriminant: Used to ~~test~~ check for perfect squares

Let $D = b^2 - 4ac$

$$\therefore \alpha = \frac{-b + \sqrt{D}}{2a}, \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

THE SUM OF THE ROOTS $(\alpha + \beta)$

$$\alpha + \beta = \frac{-b + \sqrt{D}}{2a} + \frac{-b - \sqrt{D}}{2a}$$

$$= \frac{-b + \cancel{\sqrt{D}} - b - \cancel{\sqrt{D}}}{2a}$$

$$= \frac{-\cancel{2}b}{\cancel{2}a}$$

$$\boxed{\alpha + \beta = \frac{-b}{a}} \Longrightarrow \text{Sum of the quadratic roots}$$

THE PRODUCTS OF THE ROOTS $(\alpha\beta)$

$$\alpha\beta = \frac{-b + \sqrt{D}}{2a} \times \frac{-b - \sqrt{D}}{2a}$$

$$= \frac{b^2 \cancel{+ b\sqrt{D}} - \cancel{b\sqrt{D}} - D}{4a^2}$$

$$= \frac{b^2 - D}{4a^2}$$

$$= \frac{b^2 - (b^2 - 4ac)}{4a^2}$$

$$= \frac{\cancel{b^2} - \cancel{b^2} + \cancel{4a}c}{\cancel{4a^2}}$$

$$\boxed{\alpha\beta = \frac{c}{a}} \Longrightarrow \text{Products of the roots}$$

REVISION OF QUADRATICE EQUATION

If α, β are the roots of the equation $2x^2 + 3x - 7 = 0$; Find:

TUE JAN 28, '25

(i) $\alpha^2 + \beta^2$

(ii) $(\alpha - \beta)^2$

(iii) $\alpha - \beta$

Solution

(i) $\alpha^2 + \beta^2$

$2x^2 + 3x - 7 = 0$

$\alpha + \beta = -3/2$; $\alpha\beta = -7/2$

$a = 2$; $b = 3$; $c = -7$

$(\alpha^2 + \beta^2) = (\alpha^2 + \beta)^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{-3}{2}\right)^2 - 2\left(\frac{-7}{2}\right)$

$\frac{9}{4} + \frac{14}{2} = \frac{9+28}{4}$

$= \frac{37}{4}$

(ii) $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= \frac{9}{4} - 4\left(-\frac{7}{2}\right)$

$= \cancel{37}\ \frac{9}{4} + \frac{28}{2}$

$= \frac{9+56}{4} = \frac{65}{4}$

(iii) $\alpha-\beta$

$\alpha-\beta = \sqrt{(\alpha-\beta)^2}$

$\alpha-\beta = \sqrt{\frac{65}{4}}$

$\alpha-\beta = \frac{\sqrt{65}}{2}$

SIMULTANEOUS EQUATIONS

There are 4 ways of solving simultaneous equation

- Substitution method
- Elimination method
- Graphical method
- Matrix method.

ELIMINATION METHOD:

$ax^2 + c = 0$ → Quad

$bx + e = 0$ → Linear

→ Substitution Method

Ex: $3x+2y=8 \quad \times 2$

$2x+3y=7 \quad \times 3$

$6x+4y=16$; $6x+9y=21$

$6x + 4y = 16$

$~~6x+6~~ - (6x + 9y = 21)$

$-5y = -5$

$\therefore y = -5/5$

$y = 1$

Subst. the value of y for

$3x + 2y = 8$

$3x + 2(1) = 8$

$3x = 8 - 2$

$x = 6/3$

$x = 6$

$\therefore x = 6, y = 1$

SUBSTITUTION METHOD

2: Solve the simultaneous equations

$3x - 2y = 5 \qquad x^2 + y^2 = 13$

$\Rightarrow$

$3x - 2y = 5$

$3x = 5 + 2y$

$x = \frac{5+2y}{3}$

$x^2 + y^2 = 13$

$\left(\frac{5+2y}{3}\right)^2 + y^2 = 13$ — Eq 4

From Equation 4

$\frac{25 + 4y^2}{9} + y^2 = 13$

$20y + 25 + 4y^2 + 9y^2 = 117$

$13y^2 + 20y - 92 = 0 \qquad \therefore a = 13;\ b = 20;\ c = -92$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{-20 \pm \sqrt{20^2 - 4(13)(-92)}}{2(13)}$

$\frac{-20 \pm \sqrt{400 + 4784}}{26}$

$\frac{-20 \pm \sqrt{5184}}{26}$

$\frac{-20 \pm 72}{40}$

$\therefore y = \frac{-20+72}{\cancel{40}}$ or $\frac{-20-72}{\cancel{40}}$

$y = +\frac{\cancel{13}}{\cancel{10}}$ ~~or~~ $-\frac{\cancel{23}}{\cancel{10}}$ $\quad 2$ or $-\frac{46}{13}$

$3x - 2y = 5$

$\therefore 3x - 2\left(\frac{-13}{10}\right) = 5$

$3x + \frac{26}{10} = 5$

$30x + 26 = 50$

$30x = 50 - 26$

$x = 24/30$ WRONG!

$\Rightarrow x = \frac{5+2y}{3}$

$\therefore$ ~~same~~ when $y = 2$

$x = \frac{5+2(2)}{3} = 9/3$

$x = 3$

when $y = -46/13$

$x = \frac{5 + 2\left(\frac{-46}{13}\right)}{3}$

$x = \frac{5 - \frac{92}{13}}{3}$

$x = \frac{\frac{65-92}{13}}{3}$

$x = \frac{-27}{13} \div 3$

$x = \frac{-27}{13} \times \frac{1}{3}$

$\therefore x = -\frac{9}{13}$

when

$\Rightarrow x = 3 \; ; \; y = 2$

when $x = -9/13$; $y = -46/13$

TUE 04 FEB 25

SEQUENCE & SERIES

A sequence is an ordered set of objects formed by a particular rule which are functions of natural numbers

23, 33, 43, 53...

7, 14, 21,...

5, 3, 1, -1, ...

$1, 3/2, 2, 5/2, \ldots$

2, 11, ~~5~~, 4, 3 (Not a sequence)

SERIES:

The sum of the terms in a sequence.

7+14+21+... (series)

2+11+5+4+3... (Not a series, Because the terms are not a sequence)

A sequence with first term, a, and n-th term is called finite sequence.

7, 14, 21, 28   1, 3, 5, 7, 9.

a   L or nth   18, 16, 14, 12, 10.

On the other hand, a sequence that has no last term or tends to infinity (∞) is called INFINITE SEQUENCE

~~1/2, 1~~ 0.5, 1.0, 1.5, 2.0, 2.5...

1, 4, 9, 16, 25, 36, 49...

ARITHMETIC SEQUENCE / PROGRESSION

This refers to any sequence in which each term after the first term is obtained by adding a fixed number, called common difference, d, to the preceding term.

Ex:

$\therefore T_n = a+(n-1)d$

23, 33, 43, 53

$a, a+d, a+2d, a+3d, \ldots a+(n-1)d$

Ex: 1: The 5th term of an AP is 19 and the 14 term is 55. Find:
(i) a and common difference (ii) The 31st term

Solution

$T_5 = a+4d$ $T_{14} = a+13d$

(i)
$19 = a+4d$
$55 = a+13d$
$-36 = -9d$
$d = 4$

$19 = a+4d$
$19 - 4(4) = a$
$a = 3$

$\therefore a = 3; d = 4$

(ii) $T_{31} = a+30d$
$\therefore T_{31} = 3+30(4)$
$T_{31} = 123$

2: The sum of the first three terms of an AP is 21. If the different between the 3rd term and 1st term is 4, find the sum of the next 4 terms of the sequence.

soln.

$T_1+T_2+T_3 = 21$
$a+a+d+a+2d = 21$
$3a+3d = 21$
$\therefore a+d = 7 \rightarrow$ eqn ③

$T_3 - T_1 = 4$
$a+2d - a = 4$
$2d = 4$
$d = 2$

$\therefore$ Substituting d in eqn ③
$a+2 = 7$
$a = 5$

$\therefore$ ~~$T_4 = a+3d$ $= 5+3(2)$ $= 11$~~ ~~$T_5 = a+4d$ $= 5+4(2)$ $= 13$~~ ~~$T_6 = a+5d$~~

Terms of the AP = $5, 7, 9, \underbrace{11, 13, 15, 17}_{\text{next} \rightarrow 4 \text{ terms}}$
$\therefore 11+13+15+17$
$T_4+T_5+T_6+T_7 \rightarrow 56$

3. Given a sequence: 2, 4, 6, 8, ... ; Find the 22nd term.

$T_{22} = a + 21d$

$= 2 + 21(2)$

$= 2 + 42$

$= 44$

4. Find the value of k, given that $k+1, 2k,$ and $2k+3$ are consecutive terms of an AP.

Finding Common difference:

$2k - (k+1) = 2k + 3 - 2k$

$k - 1 = 3$

$k = 4$

5. ARITHMETIC MEAN

Arithmetic mean of two numbers P, R, is a number Q, between P and R, such that P, Q, R are Arithmetic Progression.

P Q R → AM (Arithmetic Mean)

→ Common Difference

$d = Q - P = R - Q$

$Q - P = R - Q$

$2Q = R + P$ → Collect Like Terms

$\therefore \boxed{Q = \frac{R+P}{2}}$ ⇐ Hence, mean is sum of two terms divided by 2

Ex:

Find the values of x if the Arithmetic Mean of:

(i) $x+4$ and $4x+5$ is 12 (ii) $x-5$ and $2x+1$ is $x+2$

Soln.

(i) $12 = \frac{x+4+4x+5}{2}$

$5x + 9 = 24$

$x = 3$

(ii) $x+2 = \frac{x-5+2x+1}{2}$

$3x - 4 = 2x + 4$

$x = 8$

SUM OF AN ARITHMETRIC PROGRESSION

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Ex: When last term is given, l:

$$S_n = \frac{n}{2}\left[a + l\right]$$

Ex: The first term of an AP is 7, and the last term is 70, and sum of the series is 385. Find the number of terms and common difference.

$385 = \frac{n}{2}(7+70)$

$770 = 77n$

$n = 10$

~~$a = 7$; $d = 13$~~

$a + a+d + a+2d + a+3d + a+4d + a+5d + a+6d + a+7d + a+8d + a+9d = 385$

$10 + 45d = 385$

$45d = 375$ - WRONG!!

$T_n = a + (n-1)d$

$T_{10} = 7 + (10-1)d$

$70 = 7 + 9d$

$63 = 9d$

$d = 7$ ✓

GEOMETRIC PROGRESSION

The sequence ~~an~~ a_n whose first term is $a_1 = a$; and for every n, the $n+1$ term is given as:

$a_{n+1} = ar^n$ for any fixed, non-zero constant a, and r, is known as Geometric Progression.

The number r, is called the common ratio of the GP, i.e. $a, ar, ar^2, ar^3, \ldots ar^{n-2}, ar^{n-1}$; such that $a_1 = a$; $a_2 = ar$; $a_3 = ar^2$; $a_4 = ar^3$.

We occasionally call geometric progress an EXPONENTIAL PROGRESSION

$\frac{1}{2}, 1, 2, 4 \ldots$ $\qquad 1, 3x, 9x^2, 27x^3$

$2, 8, 32, 128 \ldots$

Ex: The second term of a GP is 35, and the 4th term is 875. Find:
(i) a (ii) 5th term

$T_2 = ar$
$T_4 = ar^3$ $\rightarrow$ $\frac{875}{35} = \frac{ar^3}{ar}$

$\sqrt{r^2} = \sqrt{25}$

$r = 5$

$35 = a(5)$

$a = 7$

(ii) $T_5 = ar^4$

$T_5 = 7(5)^4$

$T_5 = 7 \times 625$

$T_5 = 4375$

OR.

$T_4 = 875$

$T_5 = 875 \times 5$

$= 4375$

Sum of terms of G.P.

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad r > 1$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad r < 1$$

Sum to Infinity

$$S_\infty = \frac{a}{1 - r}$$

Ex: Find the sum of the first 6 terms of the exponential sequence 18, 6, 2, ...

$$S_6 = \frac{18(1 - 1/3^6)}{1 - 1/3}$$

$$= \frac{18(1 - 1/729)}{2/3}$$

$$= \frac{18\left(728/729\right)}{2/3}$$

$$= \frac{\cancel{1461}}{\cancel{81}} \div \frac{\cancel{2}}{\cancel{3}} \qquad 18\left(\frac{728}{729}\right) \div \frac{2}{3}$$

$$= \frac{\cancel{1461}}{\cancel{81}} \times \frac{\cancel{3}}{\cancel{2}} \qquad = 18\left(\frac{728}{729}\right) \times \frac{3}{2}$$

$$= \frac{\cancel{1461}}{\cancel{27}} \times \frac{\cancel{1}}{\cancel{2}} \qquad = \frac{728}{81} \times 3$$

$$= \frac{\cancel{461}}{\cancel{18}} \qquad = \frac{728}{27}$$

$$= \cancel{27.086} \qquad = 26.96$$

$\hookrightarrow 27.056$

2: Find the sum to infinity of the sequence : 1, 1/4, 1/16, 1/64

$a = 1$; ~~mf~~ $r = 4$

$$\frac{1}{64} \div \frac{1}{16} = \frac{1}{16} \div \frac{1}{4}$$

$$= 4 \qquad = 4$$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{1}{1-4}$$

$$S_\infty = -1/3$$

ASSIGNMENT

1. Find the 8th term and sum of 1st 8 terms of the sequence:
   1/2, −1, 2, −4

2. The 3rd term of a G.P is 63 and the 5th term is 567. Find the sum of the 1st 6 terms of the progression.

3. An exponential progression is such that the 3rd term minus the 1st term is 48. The 4th term minus the second term is 144. Find:

(i) The common ratio
(ii) The 1st term
(iii) The sixth term of the sequence

middle
x in decrease

4. Find the 8th, 9th, and 10th terms of each of the following sequence:

(i) 5, 8, 11, 14, ...

(ii) 3, 5, 7, 9, ...

(iii) -1, 2, 5, 8, ...

(iv) 4, -1, 6, -11, ...

(v) 2, $3\frac{1}{2}$, 5, $6\frac{1}{2}$, ...

DONE IN SHEET OF PAPER

Polynomials & Partial Fractions

FEB 25 25

Let $P_n(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} + \dots + a_1 x^1 + a_0$

i.e. $P(x) = \sum_{i=0}^{n} a_i x_i$

The degree of polynomial is the highest power of x in the polynomial. The degree of the term, $a_i x_i$, is defined as i, the leading coefficient is the coefficient of the term with the highest power, for example above, the leading coeffiecient is a_n and the constant term is the term with no power of x in it, wher a_0 is the constant term.

TYPES OF POLYNOMIALS

1. Linear Polynomial
2. Quadratic "
3. Cubic "

1. LINEAR: When the power of x is 1.

i. $a_1 x + a_0$

E.g. = $2x+5$

2. QUADRATIC: Highest power of x is 2.

$a_2 x^2 + a_1 x + a_0$

E.g. $5x^2 + 2x + 1$

3. CUBIC: Highest power of x is 3.

$a_3 x^3 + a_2 x^2 + a_1 x + a_0$

Eg.

MISCELLANEOUS: Polynomials with power greater than three (3).

Degree 5: Highest power of x is 5

$a_5 x^5 + a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$

...ple: If $P(x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

(i) Leading coefficient (iii) Coefficient of term involving x^3

(ii) Degree of polynomial (iv) The constant term

Leading coefficient = 1 (iii) Coefficient of x^3 = 10

(iii) Degree => Degree 5 (iv) Constant term => 1

OPERATIONS IN POLYNOMIAL

1. Addition and Subtraction of Polynomial
2. Multiplication
3. Division

ADDITION & SUBTRACTION

Ex: $P_1(x) = 3x^4 + 5x^3 + 11x^2 - x + 6$ — find (i) $P_1(x) + P_2(x)$

$P_2(x) = 6x^3 + 5x^2 - 3x + 7$ (ii) $2P_2(x) - P_1(x)$

[SOLN]

(i) $P_1(x) + P_2(x) = (3x^4 + 5x^3 + 11x^2 - x + 6) + (6x^3 + 5x^2 - 3x + 7)$

$= 3x^4 + 11x^3 + 16x^2 - 4x + 13$

(ii) $2P_2(x) - P_1(x)$

$2P_2(x) = 2(6x^3 + 5x^2 - 3x + 7)$

$= 12x^3 + 10x^2 - 6x + 14$

$2P_2(x) - P_1(x) = (12x^3 + 10x^2 - 6x + 14) - (3x^4 + 5x^3 + 11x^2 - x + 6)$

$= 12x^3 + 10x^2 - 6x + 14 - 3x^4 - 5x^3 - 11x^2 + x - 6$

$= -3x^4 + 7x^3 - x^2 - 5x + 8$

$P_1(x) = x^5 + 3x^4 + 2x^3 - x + 5$

MULTIPLICATION $P_2(x) = 3x^2 - x$

$(P_1(x))(P_2(x)) = (3x^4 + 5x^3 + 11x^2 - x + 6)(6x^3 + 5x^2 - 3x + 7)$

$= 18x^7 + 15x^6 - 9x^5 + 21x^4 + 30x^6 + 25x^5 - 15x^4 + 35x^3 + 66x^5 +$
$55x^4 - 33x^3 + 77x^2 - 6x^4 - 5x^3 + 3x^2 - 7x + 36x^3 + 30x^2 - 18x + 42$

WRONG VALUES P_1, P_2

$(P_1(x))(P_2(x)) = (x^5 + 3x^4 + 2x^3 - x + 5)(3x^2 - x)$

$= 3x^7 - x^6 + 9x^6 - 3x^5 + 6x^5 - 2x^4 - 3x^3 + x^2 + 15x^2 - 5x$

$= 3x^7 + 8x^6 + 3x^5 - 2x^4 - 3x^3 + 16x^2 - 5x$

DIVISION

Ex: Divide $3x^3 - 2x^2 + x + 1$ by $x-1$

Representation of a polynomial:

$\boxed{f(x) = q(x)\,d(x) + r(x)}$

$$
\begin{array}{r|l}
 & 3x^2 + x + 2 \\
\hline
(x-1) & 3x^3 - 2x^2 + x + 1 \\
 & -(3x^3 - 3x^2) \\
\hline
 & \quad x^2 + x \\
 & \quad -(x^2 - x) \\
\hline
 & \qquad 2x + 1 \\
 & \qquad -(2x - 2) \\
\hline
 & \qquad\quad 3
\end{array}
$$

$\because$ Quotient $= 3x^2 + x + 2$

Remainder $= 3$

QDR:

$\Rightarrow (3x^2 + x + 2)(x-1) + 3$

(ii)

$$
\begin{array}{r|l}
 & 2x^2 - 7x + 16 \\
\hline
x+3 & 2x^3 - x^2 - 5x + 1 \\
 & -(2x^3 + 6x^2) \\
\hline
 & \quad -7x^2 - 5x \\
 & \quad -(-7x^2 - 21x) \\
\hline
 & \qquad 16x + 1 \\
 & \qquad -(16x + 48) \\
\hline
 & \qquad\quad -47
\end{array}
$$

QDR:

$\Rightarrow (2x^2 - 7x + 16)(x+3) \{-47\}$

(iii) $x^3 - 2x^2 - 5x + 6$ divided by $x-1$

$$
\begin{array}{r|l}
 & x^2 - x - 6 \\
\hline
x-1 & x^3 - 2x^2 - 5x + 6 \\
 & -(x^3 - x^2) \\
\hline
 & \quad -x^2 - 5x \\
 & \quad -(-x^2 + x) \\
\hline
 & \qquad -6x + 6 \\
 & \qquad -(-6x + 6) \\
\hline
 & \qquad\quad 0
\end{array}
$$

$f(x) =$

Remainder $= 0$

$(x^2 - x - 6)(x-1)$

Hence! $x-1$ is a factor of $x^3 - 2x^2 - 5x + 6$

*Quadratic Divisors

REMAINDER THEOREM

Find the remainder when:

(i) x^3+3x^2-4x+2 is divided by $x-1$

$$\begin{array}{r|l} & x^2+4x \\ x-1 & x^3+3x^2-4x+2 \\ & -(x^3-x^2) \\ & \quad 4x^2-4x \\ & \quad -(4x^2-4x) \\ & \qquad 0 \quad \boxed{+2} \end{array}$$

Remainder Theorem:

$f(x) = x^3+3x^2-4x+2$

Let $x-1=0 \Rightarrow x=1$

$f(1) = 1^3+3(1)^2-4(1)+2$

$f(1) = 1+3-4+2$

$f(1) = 6-4$

$= 2$

~~x^3+3x^2-4x+2~~ $(x^2+4x)(x-1)+2$

(ii) x^3+3x^2+3x+1 divided by $x+2$

Let $x+2=0 \Rightarrow x=-2$

$f(-2) = (-2)^3+3(-2)^2+3(-2)+1$

$f(-2) = -8+12-6+1$

$= 13-14$

Remainder $= -1$

Factorize the following:

(i) x^3-2x^2-5x+6

(ii) $x^4-5x^3+5x^2+5x-6$

~~(iii)~~ soln.

(i) When $(x-1)$; $x-1=0 \Rightarrow x=1$

$f(1) = (1)^3-2(1)^2-5(1)+6$

$= 1-2-5+6$

$= 0$

$\therefore (x-1)$ is a factor

Quotient, when divided by $(x-1)$

$= (x^2-x-6)$ is also a factor

$\to (x-3)(x+2)$

(ii) $x^4-5x^3+5x^2+5x-6$

$f(1) = 1^4-5(1)^3+5(1)^2+5(1)-6$

$= 1-5+5+5-6$

$= 0$

$\therefore (x-1)$ is a factor

Quotient, when divided by $x-1$

$= x^3+4x^2$

$$
\begin{array}{r|l}
 & x^3 + 4x^2 + x + 6 \\
\hline
x-1 & x^4 - 5x^3 + 5x^2 + 5x - 6 \\
 & -(x^4 - x^3) \\
\hline
 & -4x^3 + 5x^2 \\
 & -(4x^3 - 4x^2) \\
\hline
 & x^2 + 5x \\
 & -(x^2 - x) \\
\hline
 & 6x - 6 \\
 & -(6x - 6) \\
\hline
 & 0
\end{array}
$$

PARTIAL FRACTIONS

Find the values of A, B, & C

(i) $22 - 4x - 2x^2 = A(x-1)^2 + B(x-1)(x+3) + C(x+3)$

(ii) $5x + 31 = A(x+2)(x+1) + B(x-1)(x-5) + C(x-5)(x+2)$

$-(x)(2x$

(i) Let $x = 1$ ~~and~~

$22 - 4(1) - 2(1)^2 = A(1-1)^2 + B(1-1)(1+3) + C(1+3)$

~~22~~ $16 = 4C$

$C = 4$

Let $x = -3$

$22 - (4(-3)) - 2(-3)^2 = A(-3-1)^2 + B(-3-1)(-3+3) + C(-3+3)$

~~22+12-2(9)~~ $22 + 12 - 18 = A(-4)^2$

$16 = 16A$

$A = 1$

Let $x = 0$

$22 - 4(0) - 2(0)^2 = A(0-1)^2 + B(0-1)(0+3) + C(0+3)$

$22 = A + B(-1)(3) + 3C$ $\qquad A = 1;\ C = 4$

$22 = A - 3B + 3C$

$22 = 1 - 3B + 3(4)$

$22 = -3B + 13$

$-3B = 22 - 13$

$B = -\frac{9}{3}$

$B = -3$

$\therefore A = 1,\ B = -3;\ C = 4$

(iii) Let $x = -2$

$5(-2) + 31 = A(-2+2)(-2+1) + B(-2-1)(-2-5) + C(-2-5)(-2+2)$

$21 = B(-3)(-7)$

$21 = -21B$

$B = -1$

Let $x = 5$

$5(5) + 31 = A(5+2)(5+1) + B(5-1)(5-5) + C(5-5)(5+2)$

$25 + 31 = A(7)(6)$

$56 = 42A$

$A = 4/3$

Let $x = 0$

$5(0) + 31 = A(0+2)(0+1) + B(0-1)(0-5) + C(0-5)(0+2)$

$31 = A(2)(1) + B(-1)(-5) + C(-5)(2)$

$31 = 2A + 5B - 10C$ $\quad A = 4/3;\ B = -1$

$31 = 2(4/3) + 5(-1) + 10C$

$31 = 2(4/3) + 5B - 10C$

$31 = 8/3 + 5 - 10C$

$31 - 23/3 = -10C$

$\frac{70}{3} = \frac{-10C}{1}$

$-30C = 70$

$C = -7/3$

$\therefore A = 4/3;\ B = -1;\ C = -7/3$

(iii) Express as partial fraction:

$$\frac{4}{x^2-4}$$

$$\frac{4}{(x-2)(x+2)}$$

when $x=2$

$$\frac{4}{0} = \text{undefined}$$

when $x=-2$

$$\frac{4}{(-2-2)(-2+2)} = 0.$$

$$\frac{4}{(x-2)(x+2)} = \frac{A}{(x-2)} + \frac{B}{(x+2)}$$

$$\frac{4}{(x-2)(x+2)} = \frac{A(x+2)+(x-2)B}{(x-2)(x+2)}$$

Since, the ~~...~~ denominators are the same; they cancel out

Comparing the numerators:

$4 = A(x+2) + B(x-2)$

Let $x=2$

$4 = A(2+2)$

$4 = 4A$

$A = 1$

Let $x=-2$

$4 = B(-4)$

$4 = -4B$

$B = -1$

Subst. the values of $A, \& B$

$$\therefore \frac{4}{(x-2)(x+2)} = \frac{1}{x-2} - \frac{1}{x+2}$$

~~ASSIGNMENT~~

TUE 04 ~~MAR~~ 25

COMPLEX NUMBERS $\boxed{Z = a+bi}$

Complex numbers that have real part and imaginary part.

e.g. $\sqrt{-20} = \sqrt{-1} \times \sqrt{20}$

$= \sqrt{-1} \times 2\sqrt{5}$

$= i \times 2\sqrt{5}$

$= 2i\sqrt{5}$

$i = \sqrt{-1}$

$i^2 = -1$

$Z = \underbrace{a}_{\text{Real part}} + \underbrace{ib}_{\text{Imaginary part}}$

Conjugate of a Complex Number: Changing of signs.

e.g. $Z = a + ib$; $Z = a - ib$

$\bar{Z} = a - ib$; $\bar{Z} = a + ib$

ADDING & SUBTRACTION OF COMPLEX NUMBER

Let $z_1 = x_1 + iy_1$; $z_2 = x_2 + iy_2$

$z_1 + z_2 = x_1 + iy_1 + x_2 + iy_2$

$= x_1 + x_2 + iy_1 + iy_2$

$$\boxed{z_1 + z_2 = \underbrace{x_1 + x_2}_{\text{real}} + \underbrace{i(y_1 + y_2)}_{\text{imaginary}}}$$

Ex:

If $z_1 = 4 + 6i$; $z_2 = 3 - 5i$; $z_3 = 5 - i$; Find:

(i) $z_1 + z_2$ (ii) $2z_1 + 4z_2 - 5z_3$

soln.

(i) $z_1 + z_2 = 4 + 6i + 3 - 5i$

$= 7 + i$

(ii) $2z_1 + 4z_2 - 5z_3 = 2(4 + 6i) + 4(3 - 5i) - 5(5 - i)$

$= 8 + 12i + 12 - 20i - 25 + 5i$

$= -5 - 3i$

MULTIPLICATION OF A COMPLEX NUMBER

$z_1 = x_1 + iy_1$; $z_2 = x_2 + iy_2$

$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$

$= x_1x_2 + x_1iy_2 + x_2iy_1 + iy_1iy_2$

$= x_1x_2 + i(x_1y_2 + x_2y_1) + i^2(y_1y_2)$

Find:

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = i^2 \times i^1 \Rightarrow -1\sqrt{-1}$

$i^4 = i^2 \times i^2 \Rightarrow -1 \times -1 = 1$

$i^5 = i^2 \times i^2 \times i = (-1)(-1)(\sqrt{-1}) \Rightarrow \sqrt{-1}$

$i^6 = i^2 \times i^2 \times i^2 = (-1)(-1)(-1) = -1$

$i^7 = i^4 \times i^3 = 1 \times (-1\sqrt{-1}) = -1\sqrt{-1}$

$i^8 = i^4 \times i^4 = 1 \times 1 = 1$

$i^9 = i^4 \times i^5 = 1 \times \sqrt{-1} = \sqrt{-1}$

$i^{10} = i^5 \times i^5 = \sqrt{-1} \times \sqrt{-1} = -1$

Ex:

$Z_1 = 4+6i,\ Z_2 = 3-5i;\ Z_3 = 5-i$

(i) $Z_2 Z_3$ (ii) $Z_3 \bar{Z}_1$

Soln:

(i) $Z_2 Z_3 = (3-5i)(5-i)$

$15 - 3i - 25i + 5i^2$

$15 - 28i + 5(-1)$

$Z_2 Z_3 = 10 - 28i$

(ii) $Z_3 \bar{Z}_1 = (5-i)(4-6i)$

$= 20 - 30i - 4i + 6i^2$

$= 20 - 34i + 6(-1)$

$= 14 - 34i$

DIVISION OF COMPLEX NUMBERS

Let $z_1 = x_1 + iy_1;\ z_2 = x_2 + iy_2$

$\cancel{\frac{Z_1}{Z_2} = \frac{x_1+iy_1}{x_2+iy}}\ \frac{Z_1}{Z_2} \times \frac{Z_1}{Z_2} \times \frac{\bar{Z}_2}{\bar{Z}_2}$

$\frac{Z_1}{Z_2} = \frac{x_1+iy_1}{x_2+iy_2} \times \frac{x_2-iy_2}{x_2-iy_2}$

$\frac{x_1^2 - i^2y_1^2}{x_2^2 - x_2iy_2 + x_2iy_2 - i^2y_2^2} \Rightarrow \frac{x_1^2 - (-y_1^2) - x_1iy_1 + x_2iy_2}{x_2^2 - (-y_2^2)}$

$$\frac{x_1x_2 + y_1y_2 + i(x_1y_2 + x_2y_1)}{x_2^2 + y_2^2}$$

Ex:

$Z_1 = 4+6i$, $Z_2 = 3-5i$; $Z_3 = 5-i$

Evaluate: (i) Z_2/Z_1 (ii) Z_1/Z_3

soln

(i)
$$\frac{Z_2}{Z_1} = \frac{3-5i}{4+6i} \times \frac{4-6i}{4-6i}$$
$$= \frac{12-18i-20i+30i^2}{16 \quad -36i^2}$$
$$= \frac{12-38i-30}{16-36i^2}$$
$$= \frac{18-38i}{16+36}$$
$$= \frac{18-38i}{52}$$
$$= \frac{2(9-19i)}{2(26)}$$
$$= \frac{9-19i}{26}$$

(ii)
$$\frac{Z_1}{Z_3} = \frac{4+6i}{5-i} \times \frac{5+i}{5+i}$$
$$= \frac{20+4i+30i+6i^2}{25+\cancel{5i}-\cancel{5i}-i^2}$$
$$= \frac{20+34i+6i^2}{25-i^2}$$
$$= \frac{20+34i-6}{25-(-1)}$$
$$= \frac{14+34i}{26}$$
$$= \frac{2(7+17i)}{2(13)}$$
$$= \frac{7+17i}{13}$$

MATHEMATICAL INDUCTION

$\boxed{\sum_{i=1}^{n} i^2}$

An assumption which follows a principle.

$\rightarrow n = 1, 2 \ldots k + (k+1)$

$k =$ any constant

Principle:

When $n=1$, LHS = RHS

$n=2$, LHS = RHS

$n=k$, LHS = RHS → Add $k+1$ to both sides, depending on the series)

~~$n=k+1$ LHS = RHS~~

Neglect the LHS, and signify the RHS to the SIMPLEST level.

Ex!

1. Prove by mathematical Induction that:

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

L.H.S:

When $n=1$

$1^2 = 1$ since LHS = RHS;

$\therefore$ It is true for when $n=1$

When $n=2$

1^2+2^2

$= 5$ ~~since LHS = RHS;~~

~~$\therefore$ It is t~~

When $n=k$

k^2

$= k^2$

R.H.S:

when $n=1$

$$\frac{1(1+1)(2(1)+1)}{6} = \frac{1(2)(3)}{6} = \frac{6}{6} = 1$$

when $n=2$

$$\frac{2(2+1)(2(2)+1)}{6} = \frac{2(3)(5)}{6} = \frac{30}{6} = 5$$

Since LHS = RHS; therefore it is true for when $n=2$

when $n=k$

$$\frac{k(k+1)(2(k)+1)}{6}$$

Add $(k+1)$ to both sides

LHS: $k^2+(k+1)^2 \Rightarrow k^2+k^2+2k+1$ ~~$+k^2$~~

$\hookrightarrow 2k^2+2k+1$

RHS:

$$\frac{k(k+1)(2k+1)}{6}+(k+1)^2$$

WRONG!!

$$= \frac{k(2k^2+k+2k+1)}{6}+(k+1)^2 \Rightarrow \frac{k(2k^2+k+2k+1)}{6}+(k+1)^2$$

$$= \frac{2k^3+k^2+2k^2+k}{6}+(k+1)^2 = \frac{(2k^3+3k^2+k)+(k^2+2k+1)}{6}$$

$$= \frac{2k^3+3k^2+k}{6}+\frac{(k+1)^2}{1} = \frac{2k^3+3k^2+k+6k^2+12k+6}{6}$$

$$= \frac{\cancel{2k^3+3k^2+k+6k+6}}{6}\times\cancel{(k+1)} = \frac{2k^3+9k^2+13k+6}{6}$$

$$= \frac{\cancel{2k^3+3k^2+7k+6}}{\cancel{6}}$$

FLASHBACK! $\Longleftarrow$ REROUTED!!

$$\frac{k(k+1)(2k+1)}{6}+(k+1)^2$$

$$k+1\left[\frac{k(2k+1)+6k+6}{6}\right] \Rightarrow k+1\left[\frac{2k^2+k+6k+6}{6}\right]$$

$$k+1\left(\frac{2k^2+7k+6}{6}\right)$$

$$\frac{k+1}{6}(2k^2+7k+6)$$

$$\frac{k+1}{6}\left(2k^2+4k+3k+6\right)$$

$$\cancel{\frac{k+1}{6}\left(2k(2+2\right.} \qquad \frac{k+1}{6}\left(2k(k+2)+3(k+2)\right)$$

$$\frac{k+1}{6}\left((2k+3)(k+2)\right) \Leftarrow \quad \text{Simplest form}$$

$$\frac{(k+1)(2k+3)(k+2)}{6}$$

In conclusion, since the formula is true for when $n=1$, $n=2$, $n=k$, and $n=k+1$; therefore it is true for all positive values of n

2. Prove by mathematical induction that:

$$2^1+2^2+2^3+\ldots+2^n = 2(2^n-1)$$

When $n=1$ RHS: when $n=1$

$2^1 = 2$ $2(2^1-1) = 2(1) = 2$

Since LHS = RHS; it is true for when $n=1$

When $n=2$ when $n=2$

2^1+2^2 $2(2^2-1) = 2(3)$

$= 6$ $= 6$

Since LHS = RHS; it is true for when $n=2$

When $n=k$ When $n=k$

$\cancel{2^k}\ 2^1+2^2+2^3+\ldots+2^k$ $2(2^k-1) = \cancel{2(2^k-2^0)} =$

$\Rightarrow \cancel{2^k+2^0}$

Since LHS = RHS; it is true for when $n=k$

Add ~~k+1~~ $n=k+1$ to both sides

~~When $n=k+1$~~   ~~When $n=k+1$~~

$\cancel{2^k+(k+1)^2}$ $\cancel{2(2^k-1)+(k+1)^2}$

$2^1+2^2+2^3+\ldots+2^k+\cancel{(k+1)}^2$

Add $n=k+1$ to both sides

$2^1+2^2+2^3+\ldots+2^k+2^{k+1} = 2(2^k-1)+2^{k+1}$

RHS:

#1 $2(2^k-1)+2^{k+1}$

$2(2^k-1)+2^k+2^1$

Let $2^k = p$

$\therefore 2(p-1)+p+2^1$

$2p-2+p+2$

$2p+p-2+2$

$3p$

$\therefore 3(2^k)$

→ Wrong!

#2. $2(2^k-1)+2^{k+1}$

$2(2^k-1)+2^k \times 2$

$2[(2^k-1)+2^k]$

$2(2^k-1+2^k)$

$2(2^k+2^k-1)$

$2(4^k-1)$

RIGHT!

In conclusion, since LHS = RHS, and is true ~~thus true for~~ for when $n=1$, $n=2$, $n=k$, and $n=k+1$; it is true for all positive values of n