GENERAL PHYSICS

PHYSICS

Physics is the scientific study of the natural world around us. It involves the study of matter, energy, and the fundamental laws that govern the behavior of the physical universe.

Things to study in Physics:

- Space
- Time

SPACE is a three-dimensional extent within which entities exist and have a physical relationship to one another. Space is a three-dimensional continuum containing positions and directions.

TIME is defined as the measure of a change in a physical quantity OR a magnitude used to quantify the duration of events. Time is considered absolute and 1-dimensional (it flows at the same rate everywhere).

UNITS are the standards of measurements used to express physical quantities. There are several systems of units, but the most commonly used system in physics is the international system of units (SI).

Basic Units or Fundamental Units: The S.I. system has 7 base units:

1. Metre (m): Unit of length
2. Kilogram (kg): Unit of mass
3. Seconds (s): Unit of Time
4. Kelvin (k): Unit of Temperature
5. Ampere (A): Unit of electric current Mole (mol):
6. Unit of amount of substance **Candela (cd):** Unit of luminous intensity

Derived Units Derived units are the units that can be expressed in terms of base units (combination of two or more basic units). Some examples of derived units include:

1. Newton (N): Unit of force (kgm/s^2)
2. Joule (J): Unit of work done (kgm^2/s^2)
3. Watt (W): Unit of power (kgm^2/s^3)
4. Nm^{-2}: Unit of pressure ($kgm^{-1}s^{-2}$)

Dimension Dimensions are the fundamental characteristics of the physical quantities. There are several dimensions including:

1. Length (L) {Anything measured in meters}
2. Mass (m)
3. Time (T)
4. Temperature (θ)
5. Electric current (I)
6. Luminous intensity (J)

Dimensional Analysis Dimensional Analysis is the process of analyzing the dimensions of physical quantities to determine their relationships. This can be useful for:

1. Checking the validity of equations
2. Deriving new equations
3. Analyzing the behaviour of physical systems

Physics classwork

1. Speed:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{L}{T} = LT^{-1}$$

2. Acceleration:

$$\text{Acceleration} = \frac{\text{Change in Velocity}}{\text{Time}} = \frac{L}{T^2} = LT^{-2}$$

3. Force:

$$\text{Force} = \text{Mass} \times \text{Acceleration} = M \times LT^{-2} = MLT^{-2}$$

1. **Energy (Work Done):**

$$\text{Force} \times \text{Distance}$$

$$\text{mass} \times \text{acceleration} \times \text{distance}$$

$$\text{mass} \times \frac{\text{displacement}}{\text{time}} \times \text{distance}$$

$$\frac{M \times L}{T} \times L = M \cdot L^2 \cdot T^{-2}$$

2. **Pressure:**

$$\frac{\text{Force}}{\text{Area}}$$

$$\frac{M \times L \times T^{-2}}{L^2} = M \cdot L^{-1} \cdot T^{-2}$$

3. **Power:**

$$\text{Force} \times \text{Velocity}$$

$$M \times L \times T^{-2} \times \frac{L}{T} = M \cdot L^2 \cdot T^{-3}$$

1. **Density:**

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{M}{L^3} = M \cdot L^{-3}$$

2. **Viscosity:**

$$\text{Dynamic Viscosity } (\eta) = \frac{\text{Shear Stress}(T)}{\text{Velocity Gradient}(du/dy)}$$

Where T = Force / Area

Force = kg m s^{-2}

Area = m^2

$$T = \frac{\text{kg}\cdot\text{m}}{s^2 \cdot \text{m}^2} = M \cdot L^{-1} \cdot T^{-2}$$

Velocity Gradient = Velocity / Length

$$\text{Velocity} = \frac{m}{s} = T^{-1}$$

$$\text{Therefore, } \eta = \frac{T}{T^{-1}} = \frac{M \cdot L^{-1} \cdot T^{-2}}{T^{-1}} = M \cdot L^{-1} \cdot T^{-1}$$

$$\text{OR Viscosity} = \text{Pressure} \times \text{Time} = \frac{\text{Force}}{\text{Area}} \times \text{Time}$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = M \cdot L^{-1} \cdot T^{-2}$$

Time = T

$$\text{Viscosity} = \frac{M \cdot L^{-1} \cdot T^{-2} \times T}{L^2} = M \cdot L^{-1} \cdot T^{-1}$$

3. **Kinematic Viscosity (ν):**

$$\text{Kinematic Viscosity} = \frac{\text{Dynamic Viscosity}(\eta)}{\text{Density}(\rho)}$$

$\eta = M \cdot L^{-1} \cdot T^{-1}$

$\rho = M \cdot L^{-3}$

$$\text{Kinematic Viscosity} = \frac{M \cdot L^{-1} \cdot T^{-1}}{M \cdot L^{-3}} = L^2 \cdot T^{-1}$$

4. **Surface Tension (γ):**

$$\text{Surface Tension } (\gamma) = \frac{\text{Force}}{\text{Length}}$$

Force = $M \cdot L \cdot T^{-2}$

Length = L

$$\text{Surface Tension} = \frac{M \cdot L \cdot T^{-2}}{L} = M \cdot T^{-2}$$

$$\text{OR: Surface Tension } (\gamma) = \frac{\text{Energy}}{\text{Area}}$$

Energy = $M \cdot L^2 \cdot T^{-2}$

Area = L^2

5. **Strain (ε):**

$$\text{Strain} = \frac{\text{Extension}(\Delta L)}{\text{Original Length}(L)}$$

$$\frac{L}{L} = 1$$

6. **Stress = Force / Area**

$$= \frac{MLT^{-2}}{L^2}$$

$$= ML^{-1}T^{-2} \checkmark$$

Displacement, Velocity, and Acceleration:

1. **Displacement** is the change in position of an object from one point to another. It is a vector quantity which means it has both magnitude and direction. Change in displacement can be represented by Δx\Delta x.
2. **Velocity** is the rate of change of displacement with respect to time. It is also a vector quantity. It is often represented by vv in (m/s)(m/s).
3. **Acceleration** is the rate of change of velocity with respect to time. It is also a vector quantity often represented by the letter "a".

Kinematics is the study of the motion of objects without considering the forces that cause the motion. It involves the use of mathematical equations to describe the motion of objects. Kinematic equations are used to describe the motion of objects in terms of displacement, velocity, and acceleration.

Kinematic Equations:

v = u + at (Velocity, initial Velocity, acceleration, time)

v^2 = u^2 + 2as (Velocity, initial Velocity, acceleration, distance)

s = ut + 1/2at^2 (Distance, initial Velocity, time, acceleration)

Relative Motion

Relative motion refers to the motion of an object with respect to another object or a reference frame. In other words, it is the motion of an object as seen by an observer who is also moving. There are several types of relative motion which include translational, rotational, and circular motion.

Newton's law of motion can be applied to relative motion to describe the motion of objects with respect to each other.

Relative Velocity

The relative velocity of two objects is the velocity of one object with respect to the other.

Relative Acceleration

Relative acceleration involves the acceleration of one object with respect to the other.

Relative motion can be used to solve collision problems where two objects collide and change momentum.

CLASSWORK

1. Two cars are moving in the same direction on a straight road. Car A is moving at a velocity of 80 km/h and car B is moving at a velocity of 60 km/h. What is the relative velocity of car B with respect to car A?

Solution: 80 - 60 = 20 km/h

2. A car is accelerating uniformly to a velocity of 30 m/s in 6 seconds. What is the relative acceleration of the car with respect to the road?

Solution: $v = u + at$ $30 = 0 + 6a$ $a = 5\ m/s^2$ Relative acceleration: $A_c - A_r$ $5 - 0 = 5\ m/s^2$

3. A car of mass 1500 kg is moving at a velocity of 20 m/s when it collides inelastically with a truck of mass 3000 kg moving at a velocity of 10 m/s. What is the relative velocity of the car and truck after the collision?

   $M_1V_1 + M_1U_1 = M_2U_2 + M_1V_2$

   $= (500 \times 20) + (300 \times 10) = 1500V + 300V_2 = 83ms^{-1}$

4. Two cars are moving in opposite direction on a straight road. Car A is moving with a velocity of $60kmh^{-1}$ and car B is moving at $80kmh^{-1}$. RV of the two cars.

60 +80 = $140kmh^{-1}$

5. A boat is moving with a speed of $6ms^{-1}$ in still water. The river has a current of $5ms^{-1}$. Find the resultant velocity of the girl is trying to move perpendicular to the river current.

   $R = \sqrt{(6^2 + 5^2)}$

   $R = \sqrt{34}$

   $R = 5.83ms^{-1}$

CONSERVATION PRINCIPLE

Conservation means the same throughout.

(Conservation of energy, charge, momentum, and angular momentum)

Conservation laws do with preservation of a particular physical quantity during a process. This includes:

A Conservation of Energy

B Conservation of Linear Momentum

C Conservation of Charge

D Conservation of Angular Momentum

CONSERVATIVE FORCES AND NON-CONSERVATIVE FORCES

(Force is anything that can cause a change)

Work is done when force moves an object from one position to another. (It depends on the initial and final position. It does not depend on the path taken - Conservative force)

Non-conservative force path taken (friction) and one covers (G)

CONSERVATIVE forces are those in which the work done is independent of the path taken but only on the initial and final position. (gravitational force, electric force, work done by elastic spring, etc.)

Work done by conservative force has four properties.

1. can be expressed as the difference between the initial and final value of a potential energy function.
2. It is reversible.
3. It is independent of the path taken but only on the starting and final point.
4. When the starting and final points are the same, the work done is zero.

- In the absence of non-conservative forces, the total mechanical energy $E=E_p+E_k$constant.

For non-conservative forces, the work done is path dependent and is mostly irreversible.

Examples: Consider an object of mass 10 kg at position A 100 m above the ground. The object drops to position B 30 m above the ground. Calculate the final speed at position B and show that the work done is conservative. ($g=9.8\ m/s^2$)

Solution:

$$100\text{m} \quad A \quad v = 0 \quad m = 10\text{kg}$$

$$30\text{m} \quad B$$

$$V_f^2 - V_0^2 = -2g(X_f - X_0)$$

$$0 = -2(9.8)(30 - 100)$$

$$a = -10.6(-70)$$

$$V_f^2 = 1372$$

$$V_f = 37.04 \text{ m/s}$$

At Position A

$$E_p = mgh$$

$$= 10 \times 9.8 \times 100 = 9800J$$

$$E_k = \frac{1}{2}mv^2 = 0$$

At Position B

$$E_p = mgh$$

$$= 10 \times 9.8 \times 30 = 2940J$$

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 10 \times (37.04)^2$$

$$= 6860J$$

Total Energy at Postion B

$$E_f = E_p + E_k$$

$$= 2940 + 6860$$

$$= 9800J$$

At Position A

$$E_T = 9800 + 0$$

$$= 9800J$$

The work done is conservative.

$$\Delta ME = 0$$

(There is no change in mechanical energy at both positions, energy is conserved.)

Conservation of Linear Momentum

1. **Quantifying motion**
2. **Product of mass × velocity**
3. **Momentum is a Vector Quantity**

The total momentum remains constant in a closed system of colliding bodies, provided there is no external force. The momentum of the system before collision is equal to the momentum after collision. Momentum is the quantity of motion and is obtained by the product of the mass of an object and its velocity. The basic unit of momentum is kg·m/s or Ns.

Law of Conservation of Linear Momentum

The law of conservation of linear momentum states that in a system of colliding objects, the momentum remains constant, provided there is no external force acting on the system.

$$\sum m_1 u_1 = \sum m_1 v_1$$

For Two Objects

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

For 3-Dimensional Motion

$$x: m_1u_1x + m_2u_2x = m_1v_1x + m_2v_2x$$

$$y: m_1u_1y + m_2u_2y = m_1v_1y + m_2v_2y$$

$$z: m_1u_1z + m_2u_2z = m_1v_1z + m_2v_2z$$

Elastic Collision

In an elastic collision, all objects can return to their original positions. Momentum is conserved.

$$\sum m_1 u_1 = \sum m_1 v_1$$

Elastic Collision

- Momentum is conserved. $\sum ½m_1u_1^2 = \sum ½m_1v_1^2$

2. **Inelastic Collision**

a. Momentum is conserved. $\sum \frac{1}{2}m_1 v_1^2 \neq \sum \frac{1}{2}m_1 v_1^2$
b. Ek is not conserved.

3. **For a completely inelastic collision:** The two bodies stick together after the collision and move as one body.

Coefficient of Restitution:

$$e = \frac{V_2 - V_1}{U_1 - U_2}$$

- For a perfectly elastic collision: e=1
- For a perfectly inelastic collision: e=0

Example 1:

Two bodies of mass 20 kg each collide. The velocities before the collision are U1=15 m/s,and U2=−10 m/s, with e=0.5e = 0.5. Calculate the final velocity V2

MaUa+MbUb=MaVa+MbVb

2(15i +30j) +2(-10i +5j) =2(-51+20j) +2Vb

30i+60j-20i=10j=-10i+40j+2Vb

20i +30j=2 vb

Vb=(10i+15j)

Example 2:

A 10 kg ball moving at 10 m/s collides with a 5 kg ball moving in the opposite direction at 5 m/s. Determine the velocity of the balls after impact if e=0.5

a. Coefficient of restitution=2/5
b. The balls stick together
c. The collision is perfectly inelastic

SOLUTION

$m_1 = 1kg$ $m_2 = 2kg$

$U_1 = 12ms^{-1}$ $U_2 = -24ms^{-1}$

$m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2$

$1(12) + 2(-24) = 1(V_1) + 2V_2$

$12 - 48 = V_1 + 2V_2$

$V_1 + 2V_2 = -36$ —(i)

$e = \frac{V_2 - V_1}{U_1 - U_2}$

$\frac{2}{3} = \frac{V_2 - V_1}{12 - (-24)}$

$\frac{3(V_2 - V_1)}{3} = \frac{72}{3}$

$V_2 - V_1 = 24$

$V_2 = V_1 + 24$ —(ii)

Put in Equation 1.

$-36 = V_1 + 2(V_1 + 24)$

$-36 = V_1 + 2V_1 + 48$

$-36 - 48 = 3V_1$

$3V_1 = -84$

$V_1 = -28ms^{-1}$

$V_2 = -28 + 24 = -4ms^{-1}$

b. i.e collision is perfectly inelastic

$e = 0$

$e = \frac{V_2 - V_1}{U_1 - U_2}$

$\frac{0}{1} = \frac{V_2 - V_1}{12 - (-24)}$

$V_2 - V_1 = 0$

$V_2 = V_1$

from equation (i)

$-36 = V_1 + 2V_1$

$-36/3 = 3V_1/3$ $V_1 = -12ms^{-1}$

$e = 1 = \frac{V_2 - V_1}{U_1 - U_2}$

$1 = \frac{V_2 - V_1}{12 - (-24)}$

$V_2 - V_1 = 36$

$V_2 = 36 + V_1$

Put in eqn (1)

$-36 = V_1 + 2(36 + V_1)$

$-36 = V_1 + 72 + 2V_1$

$-36 - 72 = -3V_1$

$-108 = 3V_1$

$V_1 = -36 ms^{-1}$

$V_2 = 36 + (-36)$

$V_2 = 0$

Moment of a Vector, Torque of Angular Momentum

The moment of a vector **a** about a point **O** is a vector whose magnitude is equal to the product of the magnitude of **a** and the perpendicular distance of **O** from the direction of **O** to the point **B**. The vector is a pseudo vector whose moment is called torque

$T = \vec{r} \times \vec{F}$

T=rfsin∅

∅ is the angle between the direction of F and position vector r

Mechanical Equilibrium

1. Translational Equilibrium

- ΣF = 0

The object is at rest or in motion with constant velocity.

2. Rotational Equilibrium

- ΣT = 0

Example 1

What are the magnitude and direction of the torque about the origin OO on a particle located at coordinates (0,−4m,3m)(0, -4m, 3m) due to force.

1. With component F=20NF = 20N and Fx,Fz=0F_x, F_z = 0.
2. Fx=0,Fy=2N,Fz=4NF_x = 0, F_y = 2N, F_z = 4N

Solution

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \text{or} \quad \tau = rF\sin\theta$$

$$\vec{r} = -4j + 3k$$

$$\vec{F} = 2j + 4k$$

$$\vec{r} \times \vec{F} = \left| \begin{matrix} i & j & k \\ 0 & -4 & 3 \\ 0 & 2 & 4 \end{matrix} \right|$$

$$\vec{r} \times \vec{F} = i(0-0) - j(0-0) + k(0-0)$$

$$\vec{r} \times \vec{F} = 6j + 8k$$

$$|\vec{r} \times \vec{F}| = \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$$

$$|\tau| = 10\,\text{Nm}$$

b. r⃗=r⃗+F⃗

| +   -   + |
| 0 -3   8 |
| 0   2   1 |

$$T = i(-16-6) - j(0-0) + k(0-0)$$

$$T = i(-22)$$

$$T = \sqrt{22^2} = 22Nm$$

Example 3:

Force F=(3.0N)i−(8.0N)kF = (3.0N)i - (8.0N)k

Located at the position P(0.50m,3.0m,2.0m) R, R relative to the origin. What is the resulting torque on the pebble about a point with coordinates (2.0m,0,3.0m)?

Relative r=P−r

Relative $r = -2i + 0.5j + 1k$

$$T = F \times r$$

| I   j   k |
| 2   0   8 |
| 2   0 -3 |

Angular momentum:

Angular momentum of a particle of mass m with linear momentum p⃗=$\overrightarrow{\mathrm{mv}}$

As it passes through point A, which is a position vector $\vec{r}$ with respect to its origin is defined as L⃗=r⃗ + p⃗

Position Vector ×Linear momentum:

The SI unit = kg m^2/s or J s.

Cross Product:

L⃗=rpsinθ

L⃗=rmvsinθ

From 2nd Law:

F⃗=$\frac{d\vec{p}}{dt}$

F⃗net (Total Force)

Tnet=$\frac{dL^{\rightarrow}}{dt}$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = \vec{r} \times (m\vec{v})$$

$$\vec{L} = m(\vec{r} \times \vec{v})$$

$$\frac{d\vec{L}}{dt} = m\left(\frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt}\right)$$

$$\frac{d\vec{L}}{dt} = m(\vec{v} \times \vec{v} + \vec{r} \times \vec{a})$$

$$\frac{d\vec{L}}{dt} = m(\vec{r} \times \vec{a})$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times m\vec{a}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

Rate of change of angular momentum is net torque and of linear momentum is the net force

This means that the vector sum of all the torques acting on a particle is the time rate of change of the angular momentum of that particle

For a system of particles (more than one particle)

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \ \dots + \vec{L}_n$$

$$\vec{L} = \sum_{i=1}^{n} \vec{L}_i$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \tau_{ext,i}$$

This implies that the rate of change of angular momentum of a system of particles is equal to the vector sum of the torque on its individual particles.

Angular momentum of a Rigid Body

A rigid body is one in which the distance between the particles of the body does not change even when the body moves.

Each particle in the body moves in a circle centered at the origin and at each instant its velocity VV is perpendicular to its position vector rr. Hence, θ=90°\theta = 90°.

$$L = rm \cdot v \cdot \sin(\theta)$$

- **A body moving in a circular path will have angular velocity, ω\omega.**

$$V = \omega r$$

$$L = rm(\omega)\sin\theta$$

For the ith particle:

$$L_i = rm(r_i\omega)\sin\theta$$

$$L_i = Mr_i^2(\omega)$$

$$L = I\omega$$

$$I = \sum_n \text{(moment of inertia)}$$

For all particles:

$$L = \sum_n L_i$$

$$L = \sum r_i p_i \sin(\theta_i)$$

$$L = I\omega$$

$$\frac{dL}{dt} = \frac{d(I\omega)}{dt}$$

$$\frac{dL}{dt} = I\frac{d\omega}{dt}$$

$$\frac{dL}{dt} = I\alpha$$

Conservation of Angular Momentum: **The principle states that provided that there is no net external torque acting on a system, the total angular momentum of a system is constant (conserved).**

$$\frac{dL}{dt} = \tau_{\text{net}}$$

$$\frac{dL}{dt} = 0$$

$$dL = 0$$

$$\int dL = \int 0$$

$$L = \text{constant}$$

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

A 2 kg particle has a position vector $\vec{r}$=4i^−2j^ relative to the origin. Its velocity just then is $\vec{v}$=6. 0t^2i^ What are:

a) The particle's angular momentum

b) The torque acting on the particle

c) The angular momentum of the particle about a point with coordinate (-2, -3, 0)

Solution

a) $\vec{L} = \vec{r} \times \vec{p}$ $\vec{p} = m\vec{v}$ $\vec{p} = 2(6.0\hat{\imath} + 2.0\hat{\jmath})$ $\vec{p} = 12.0\hat{\imath} + 4.0\hat{\jmath}$

$\vec{r} = 4\hat{\imath} - 2\hat{\jmath}$ $\vec{p} = 12.0\hat{\imath} + 4.0\hat{\jmath}$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & 0 \\ 12 & 4 & 0 \end{vmatrix}$$

$\vec{L}$=i(0)-j(0)+k(0-24t^2)

$\vec{L}$=24t^2kgm^2s^{-1}

A. **Torque acting on a particle**

$$\vec{T} = \vec{r} \times \vec{F}$$

$$\vec{F} = m\vec{a}$$

$$\vec{a} = \frac{dv}{dt} = -12\hat{\imath}$$

$$\vec{F} = 2(\hat{\imath} + 2t\hat{\imath}) = -24\hat{\imath}$$

$$\vec{r} = \hat{\imath} + \hat{k}$$

$$\vec{T} = \hat{\imath}(0) - \hat{\jmath}(0) + \hat{k}(0 - 14t)$$

$$\vec{T} = 48\hat{k}\ (\text{Nm})$$

B.

$$\vec{T} = \vec{F} \times \vec{r}$$

$$\vec{F} = m\vec{a}$$

$$\vec{a} = \frac{dv}{dt} = -12\hat{i}$$

$$\vec{F} = 2(-12\hat{i}) = -24\hat{i}$$

$$\vec{r} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{r} = \hat{i}(2) - \hat{j}(5) + \hat{k}(0-1)(4)$$

$$\vec{T} = 48\hat{k}\ (N \cdot m)$$

Relative:

$$\vec{P} - (-2, -3, 0) = (\hat{i} - 2\hat{j}) - (-2\hat{i} - 3\hat{j} - 0) = 6\hat{i} + \hat{j}$$

1. **A turbine fan in a jet engine has a moment of inertia of 2.5 kgm² about its axis of rotation. If the turbine is starting up with its angular velocity as a function of time is Wz = (40 rad/s²) t².**

Find:

   a. **The fan's angular momentum as a function of time and its value at t = 3.0 seconds.**
   b. **The net torque acting on the fan as a function of time.**

2. **A solid cylinder of mass 1.0 kg and radius 0.50 m is rotating about its central axis with an angular velocity of 10 rad/s. Find its angular momentum.**

3. **A figure skater is spinning with an angular velocity of 9 rad/s. If she brings her arms closer to her body, reducing her moment of inertia by a factor of two, what is her new angular velocity?**

SOLUTION

1. **I = 2.5 kgm²**

   L = Iω

   = 2.5 x 40 (rad/s)²

$= 100t^2$ (kgm²/s²)

L = 100 (3)² = 900 kgm²/s

b) Tnet = $\frac{dL}{dt} = \frac{d(100t^2)}{dt}$

=200t(Nm^{-1})

2. m = 10kg r = 0.5m ω = 8rad/s

L = Iω

I for a solid cylinder

= L = $\frac{1}{2}$mr²

L = Iω = $\frac{1}{2}$mr² x ω

= $\frac{1}{2}$ x 10 x 0.5² x 8

= 3.75 kgm²/s

3. Given:

- Final angular velocity (W2W_2) = 2 rad/s
- Initial angular velocity (W1W_1) = ?

$$T_2 = \frac{1}{2}T_1$$

Relationship between torques and angular velocities:

$$T_1W_1 = T_2W_2$$

Solve for W_1:

$$W_1 = \frac{T_2}{T_1} \cdot W_2$$

$$W_1 = \frac{(1/2)T_1}{T_1} \cdot 2$$

$$W_1 = 4 \text{ rad/s}$$

POLAR COORDINATES:

- **Rectangular coordinates: (x,y)(x, y)**
- **Polar coordinates: (r,θ)**

When each point on a plane of a two-dimensional coordinate system is specified by a reference point:

$$x = r\cos(\theta)$$

$$y = r\sin(\theta)$$

and an angle taken from a given direction, it is known as polar coordinate system

$$\cos(\theta) = \frac{x}{r}$$

$$x = r\cos(\theta)$$

$$\sin(\theta) = \frac{y}{r}$$

$$y = r\sin(\theta)$$

$$x^2 + y^2 = r^2$$

$$(r\cos(\theta))^2 + (r\sin(\theta))^2 = r^2\cos^2(\theta) + r^2\sin^2(\theta)$$

$$r^2(\cos^2(\theta) + \sin^2(\theta)) = r^2$$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\tan(\theta) = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Examples

Convert the polar coordinate (4, π/2) to a rectangular coordinate

$r = 4$, $\theta = \pi/2$

Solution

$x = r\cos(\theta)$

$= 4\cos(90°)$

$= 0$

$y = r\sin(\theta)$

$= 4\sin(90°)$

$= 4$

Rectangular Coordinates (0, 4)

2. Convert the rectangular coordinates (2, 5) to polar coordinates

$r = \sqrt{(x^2 + y^2)}$

$= \sqrt{(2^2 + 5^2)}$

$= \sqrt{29}$

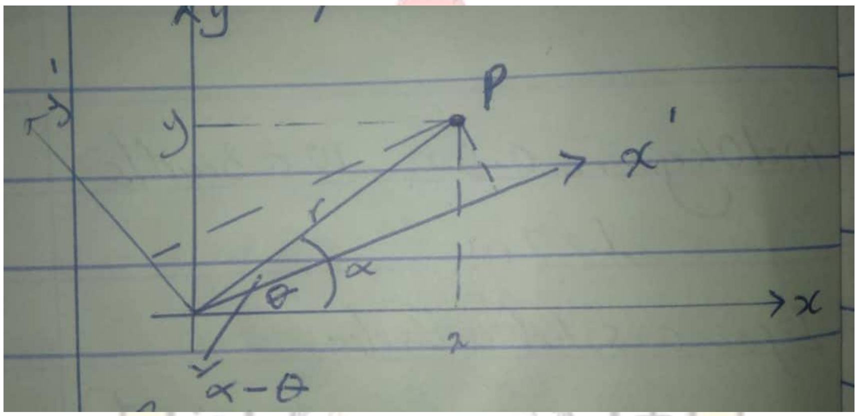
$\theta = \tan^{-1}(y/x)$

$= \tan^{-1}(5/2)$

$= 68.2°$

$\Rightarrow (r, \theta) = (\sqrt{29}, 68.2°)$

Rotation of coordinate axis



Recall that

$$x = r\cos(\theta)$$

$$y = r\sin(\theta)$$

$$x' = x\cos(\alpha - \theta)$$

$$y' = y\sin(\alpha - \theta)$$

$$x' = x(\cos\alpha\cos\theta + \sin\alpha\sin\theta)$$

$$y' = y(\cos\alpha\cos\theta + \sin\alpha\sin\theta)$$

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

$$= y\cos\theta - x\sin\theta$$

$$x' = (\cos\theta \;\; \sin\theta)\begin{pmatrix} x \\ y \end{pmatrix}$$

$$y' = (-\sin\theta \;\; \cos\theta)\begin{pmatrix} x \\ y \end{pmatrix}$$

Moment of Inertia of different shapes and solids

Consider a rigid body made up of a large number of particles with masses $m_1, m_2, m_3, \dots, m_n$ at perpendicular distances $r_1, r_2, r_3, \dots, r_n$ from the axis of rotation. The moment of inertia of the rigid body is defined as

$$I = \sum m_i r_i^2$$

Moment of Inertia of some common shapes and objects: Consider a uniform bar or rod of length L with moment of inertia about various axes.

Case 1: Axis of rotation at distance AB from one end.

$$I_1 = \frac{1}{3} m l^2$$

Case 2: Axis of rotation at the middle

$$I_e = \frac{1}{3} m \left(\frac{l}{2}\right)^3 + \left(\frac{l}{2}\right)^3$$

$$I_e = \frac{1}{3} m \frac{l^3}{8} + \frac{l^3}{8}$$

$$I_e = \frac{1}{3} m \frac{l^3}{8} \cdot 2$$

$$I_e = \frac{1}{3} m \frac{l^3}{4}$$

$$I_e = \frac{1}{12} m l^2$$

Case 3: Axis of rotation about one end edge

$$I_e = \frac{1}{3}m(a + b)$$

For $a = 0$ and $b = l$:

$$I_e = \frac{1}{3}m(0 + l^3)$$

$$I_e = \frac{1}{3}ml^3$$

$$I_e = \frac{1}{3}ml^2$$

$$I_e = \frac{1}{3}m(a + b)$$

For $a = 0$ and $b = l$:

$$I_e = \frac{1}{3}m(0 + l^3)$$

$$I_e = \frac{1}{3}ml^3$$

$$I_e = \frac{1}{3}ml^2$$

$$I_e = \frac{1}{3}m(a + b)$$

For $a = 0$ and $b = l$:

$$I_e = \frac{1}{3}m(0 + l^3)$$

$$I_e = \frac{1}{3}ml^3$$

$$I_e = \frac{1}{3}ml^2$$

3. **Circular ring at its center**

$$I = mr^2$$

4. **Circular Disk with Axis of Rotation through its Center**

$$I = \frac{1}{2}mr^2$$

5. **Hollow Cylinder**

$$I = \frac{1}{2}m(R_i^2 + R_o^2)$$

6. **Solid Cylinder**

$$I = \frac{1}{2}mr^2$$

7. **Thin-walled Hollow Cylinder**

$$I = mr^2$$

8. **Sphere about a Diameter**

$$I = \frac{2}{5}mr^2$$

9. **Thick-walled Hollow Sphere**

$$I = \frac{2}{3}mr^2$$

10. **Rectangular Plate with Axis of Rotation through the Center**

$$I = \frac{1}{12}m(a^2 + b^2)$$

11. **Rectangular Plate with Axis of Rotation about Centroid Only**

$$I = \frac{1}{12}ma^2$$

NEWTON'S LAW OF UNIVERSAL GRAVITATION

A force exists between two objects whose magnitude is given by the product of the two masses, divided by the square of the distance between them. It is expressed as:

$$F_{1,2} = \frac{GM_1M_2}{r^2}$$

where G = gravitational constant (usually 6.67×10^{-11} Nm^2/kg^2

Where F1,2 is the force on object 1 exerted by object 2, and r^\hat{r} is the unit vector that points from object 1 towards object 2.

GRAVITATIONAL POTENTIAL ENERGY

Consider a particle of mass mm displaced between two points ri and rf above the earth's surface, the change in the gravitational potential energy is equal to the displacement is defined as the negative of the work done by the gravitational force during that displacement.

$$\Delta U = U_i - U_f = \int_{r_i}^{r_f} F(r)\,dr \quad — (1)$$

The gravitational force can now be expressed as:

$$F(r) = -\frac{GMm}{r^2} \quad — (2)$$

Substitute equation (2) into equation (1):

$$U_i - U_f = \int_{r_i}^{r_f} \frac{GMm}{r^2}\, dr \quad -(3)$$

$$= GMm \int_{r_i}^{r_f} \frac{1}{r^2}\, dr$$

$$= -GMm\left[\frac{1}{r}\right]_{r_i}^{r_f}$$

$$= -GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Also, the gravitational potential energy for any pair of particles or any pair of particles for masses M and m separated by distance r is

$$Ue = \frac{-Gm_1m_2}{r}$$

This shows that the gravitational potential energy of a body varies as Whereas

$U \propto \frac{1}{r}$, $F \propto \frac{1}{r^2}$.

SATELLITE MOTION AND ORBIT

Consider a satellite of mass m moving with speed \v\ in an orbit around the earth of mass M. The centripetal force required to keep the satellite in orbit is supplied by the gravitational attraction between the satellite and the earth. So the centripetal force is given as:

$$F = \frac{m_1V^2}{r} = \frac{Gm_1m_2}{r^2}$$

where r=Re+ h and \Re \ is the radius of the earth and \h \ is the altitude above the earth's surface.

Then

$$V = \sqrt{\frac{Gm_2}{r}}$$

ESCAPE VELOCITY

If an object of mass m is projected from the earth's surface at point P with an initial speed \v\, it can be to escape from the gravitational influence of the earth. So work done, w=mass* potential difference between infinity and the point.

$$W = M \cdot \frac{Gme}{Re}$$

We have the kinetic energy of the object as

$$K = \frac{1}{2}mv^2 = M \cdot \frac{Gme}{Re}$$

so we have

$$v = \sqrt{\frac{G \cdot m}{r_e}}$$

The gravitational field strength or acceleration due to

$$(g) = \frac{G \cdot m}{R^2}$$

Assignment

What is the difference and similarity between centripetal force and centrifugal force?

PRECESSION OF A GYROSCOPE

A gyroscope is a spinning disc in which the axis of rotation (the axis of spin) is free to assume any orientation.

The precession of a gyroscope can be illustrated by a big open case. If it is placed on a flat surface at an angle to the vertical and it is not spinning, it will fall over. But, if the box is on one of its spinning axes, it will rather than fall over due to its weight, it processes about the vertical.

KEPLER'S LAWS

Kepler discovered three empirical laws that accurately describe the motion of the planets. Kepler's laws state that:

1. Each planet moves in an elliptical orbit with the sun located at one focus of the ellipse.
2. The line joining the sun and the other planet sweeps equal areas in equal time.
3. The square of the period of revolution of the planet is proportional to the cube of their mean distance from the sun. (T^2 ∝ R^3).

Example 1: A 2.0 kg and a 5.0 kg planet 8.0 cm away from each other. Find the mass of the 2.0 kg planet given that the force of gravity between them is 2.0 x 10^-8 N. (G = 6.67 x 10^-11 Nm$^{2/}$kg^{2}).

F = G M1 M2 / r^2

where:

F = 2.0 x 10^-8 N

M2 = 5.0 kg

r = 8.0 cm = 0.08 m

2.0 x 10^-8 = 6.67 x 10^-11 x M1 x 5.0 / (0.08)^2

M1 = 2.0 x 10^-8 x (0.08)^2 / 3.068 x 10^-10 = 5.80 x 10^1 kg

Example 2: Find the mass of a planet given that the acceleration due to gravity on its surface is 6.0 m/s^2. Its radius is 2.0 x 10^6 m and the universal gravitational constant is 6.67 x 10^-11 Nm$^{2/kg}$2.

- g=6.003 m/s2
- r1=7.2 km=7200 m
- G=6.67×10−11 Nm2/kg2

Using the formula for gravitational acceleration:

$$g = \frac{GM_e}{r^2}$$

We can solve for the mass of the Earth (Me):

$$M_e = \frac{gr^2}{G}$$

Substituting the given values:

$$M_e = \frac{6.003 \times 7200^2}{6.67 \times 10^{-11}}$$

$$M_e \approx 5.184 \times 10^{17}\ \text{kg}$$

CENTRE OF MASS (COM)

If a basketball bat is flipped into the air, its motion is complicated than that of, say, a table tennis egg which moves like a particle. Every part of the bat moves in a different way from every other part so that we cannot represent it as a particle; instead, it is a system of particles.

However, a special point of the bat moves in a simple parabolic path just like a particle would move if thrown in the air. This special point is known as the center of mass of the bat. Thus, the Centre of mass of a body or system of bodies is that point that moves as though all of the mass were concentrated.

The center of mass (COM) can be calculated using the formula:

$$X_{\text{com}} = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2}$$

For n particles along the x-axis:

$$X_{\text{com}} = \frac{M_1 x_1 + M_2 x_2 + M_3 x_3 + \ \dots \ + M_n x_n}{M}$$

where:

$$M = M_1 + M_2 + M_3 + \ \dots \ + M_n$$

In summation notation:

$$X_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} M_i x_i \quad \text{(along x-axis)}$$

If the particles are distributed in 3 dimensions, the center of mass can be found for each axis:

$$X_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} M_i x_i$$

$$Y_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} M_i y_i$$

$$Z_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} M_i z_i$$

Two particles of masses m1=2 kg and m2=3 kg are located at positions x1=2 m and x2=4m respectively. Find the force of attraction between masses m1 and m2

$$x_{\text{com}} = \frac{1}{m} \sum M_i x_i \quad \text{where} \quad M_1 = 2\,\text{kg} \quad M_2 = 3\,\text{kg}$$

$$x_1 = 1\,\text{m} \quad x_2 = 4\,\text{m}$$

$$= \frac{1}{2+3}(2 \times 1 + 3 \times 4)$$

$$= \frac{1}{5}(2 + 12)$$

$$= \frac{1}{5} \times 14$$

$$= 2.8\,\text{m}$$

$$x_{\text{com}} = \frac{1}{M}\sum M_i x_i$$

$$= \frac{1}{6}(1 \times 0 + 2 \times 4 + 3 \times 0)$$

$$= \frac{1}{6}(8) = 1.33\,\text{m}$$

$$y_{\text{com}} = \frac{1}{M}\sum M_i y_i$$

$$= \frac{1}{6}(1 \times 0 + 2 \times 0 + 3 \times 3)$$

$$= \frac{1}{6}(9) = 1.5\,\text{m}$$

So, the center of mass is (1.33 m,1.5 m)

CIRCULAR MOTION

During this motion, an object moves in a circular path with a circular speed.

Centripetal force: $F_c = \frac{mv^2}{r}$ **Centripetal acceleration**: $: a_c = \frac{v^2}{r}$ Centripetal means central seeking.

Angular speed: $: w = \frac{\theta}{t}$ **Linear Speed**: $v = wr$

Displacement

$v = wr$ -> Angular Speed $v = v_0 + at$ $v^2 = v_0^2 + 2a(x - x_0)$ $x - x_0 = v_0 t + \frac{1}{2}at^2$

Angular equations:

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Banking of Roads

Diagram:

- A car on a banked road with forces labeled:
  - NN (Normal force)
  - Fcosθ(Component of frictional force)
  - Fsinθ(Component of frictional force)
  - mama (Centripetal force)

$$\mu v^2 = F\cos\theta + N\sin\theta$$

$$\frac{\mu v^2}{r} = N\cos\theta + N\sin\theta$$

Total force along the Ncosθ=Mg+Fsinθ

$$Mg = N\cos\theta - N\sin\theta$$

Equation 2:

$$\mu v^2 = N\cos\theta + N\sin\theta$$

$$\frac{r}{mg} = \frac{N\cos\theta - N\sin\theta}{mg}$$

$$\frac{v^2}{rg} = \frac{N(\cos\theta + \sin\theta)}{N\cos\theta - N\sin\theta}$$

$$\frac{v^2}{rg} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$$

Banking of Roads

$$\frac{\cos\theta + \sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= 1 + \tan\theta$$

$$\frac{\cos\theta - \sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= 1 - \tan\theta$$

$$\frac{N + \tan\theta}{1 - N\tan\theta}$$

$$V^2 = rg\left(\frac{N + \tan\theta}{1 - N\tan\theta}\right)$$ (This is the supposed speed with which one can turn on a banked road)

$$V^2 = rg\tan\theta$$ (In the absence of friction, N = 3)

Examples:

1. **A compact disc uniformly increases angular speed of 400 rpm. Find α**

   Given: $\omega_0 = 0, N = 400$ rpm

   $\omega = \omega_0 + \alpha t$

   $\pi = 0 + (0.5)\alpha$

   $\alpha = 16.76 \text{ rad/s}^2$

   $t = 2.5 \text{ s}$

2. **A microwave oven has a rotative plate which revolves at 900 rpm uniformly to 800 rpm while making a revolution. Find the angular speed and the time required to turn through 50 revolutions.**

Given: $\omega_0 = 900 \times 2 = 30\pi$, $\omega = 300 \times 2 = 10\pi$

$\theta = 50 \text{ revs}$

$50 \times 22 = 100\pi$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$(10\pi)^2 = (30\pi)^2 + 2 \times 100\pi\alpha$

$100\pi^2 = 900\pi^2 + 200\pi\alpha$

$200\pi\alpha = 100\pi^2 - 900\pi^2$

$\alpha = -4\pi \text{ rad/s}^2$

$\theta = (\omega + \omega_0)t$

$t = 5.5 \text{ secs}$

3. **A curved road of radius 20 m is to be banked so that a car can make a turn at a maximum speed of 15 m/s. What must be the angle?**

- Given: $r = 20$ m, $V_{\max} = 15$ m/s, $\mu = 0$
- To find: θ

$$V_{\max}^2 = rg\tan\theta$$

At maximum speed, friction = 20:

$$15 = \sqrt{20 \times 10 \times \tan\theta}$$

$$15 = \sqrt{200\tan\theta}$$

$$225 = 200\tan\theta$$

$$\tan\theta = \frac{225}{200} = 1.125$$

$$\theta = \tan^{-1}(1.125) \approx 48°$$

4. **A curved road is part of a circle with a radius of 150 m. The coefficient of kinetic friction between the road surface and the tires of a car is 0.3. If the road is banked at 12°, calculate the speed with which the car can be driven without skidding.**

Given: $r = 150$ m, $\mu = 0.3$, $\theta = 12°$

To find: V_c

$$V_c = \sqrt{\frac{rg(1 + \mu\tan\theta)}{1 - \mu\tan\theta}}$$

$$V_c = \sqrt{\frac{150 \times 10 \times (1 + 0.3 \times \tan12)}{1 - 0.3 \times \tan12}}$$

$$V_c = \sqrt{\frac{1500 \times 0.5}{0.9}}$$

$$V_c = 30\ \mathrm{m/s}$$

POTENTIAL ENERGY

This is the energy posed by an object or system due to its position from the reference point or arrangement or configuration or condition of parts, it represents stored energy and can be converted to other forms of energy such as kinetic energy (K.E) as an object moves or its condition changes.

Kinetic Energy (K.E) and Work Done in a System of Particles and Bodies of Mass

$$K.E = \frac{1}{2}mv^2$$

where v is the speed (less than the speed of light).

Work: Work is energy transferred to and from an object by a force. Energy transferred to an object is positive work, and energy transferred from an object is negative work.

For a constant force, the work done is equal to:

$$W = Fd\cos\theta$$

Work and Kinetic Energy

The work-kinetic energy theorem states that the net work done on a particle is the change in the kinetic energy of the particle.

$$\text{Work} = \Delta K.E$$

$$\text{Work} = K.E_{\text{final}} - K.E_{\text{initial}}$$

$$\Delta K.E = \frac{1}{2}mv^2 - \frac{1}{2}mv^2$$

Gravitational Potential Energy (GPE)

GPE is the potential energy associated with the Earth and a mass particle. If the particle moves from height y1 to height y2, the change in GPE is:

$$P.E = mgh$$

$$GPE = mg(y_2 - y_1)$$

$$\Delta U = mg\Delta y$$

Elastic Potential Energy

$$\Delta U = \frac{1}{2}kx^2$$

Examples

1. A 2 kg mass hangs 5.0 m above the ground. What is the G.P.E. of the mass-Earth system at:

    (a) Ground level

    (b) A balcony 3 m above the ground

Solution:

For (a):

$$\text{GPE} = mg(y_2 - y_1)$$

Given: $g = 10\,\text{m/s}^2$, $m = 2\,\text{kg}$, $y_1 = 5.0\,\text{m}$, $y_2 = 0$

$$= 2 \times 10 \times (5)$$

$$= 100\,\text{J} = 0.1\,\text{kJ}$$

For (b):

$$\text{GPE} = mg(y_2 - y_1)$$

$$= 2 \times 10 \times (5.0 - 3)$$

$$= 2 \times 10 \times 2$$

$$= 40\,\text{J} = 0.04\,\text{kJ}$$

2. **A ball is sliding across an oily floor through a distance of 2 m while a steady wind pushes the ball with a force of 1 N.**
   A. **How much work does the wind do on the ball?**
   B. **If the ball has a K.E. of 9 J at the beginning of the slide,**
   C. **what is its K.E. at the end of d?**

1.

$$K_{i_z} + W_z = (2.0J - 6.0J) - 3.6m_i$$

$$W_z = 6J$$

2.

$$K_{i_z} = 10J$$

, find K_f:

$$\Delta W = K_f - K_i$$

$$-6J = K_f - 10J$$

$$K_f = 4J$$