Proof

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1 Description

This is the amortised analysis of the operations done on the stack of a typical type

2 Proof

The proof first considers the analysis of all four possibilities where every major time consuming parts are considered in different permutations and then there is a combined analysis

2.1 Push and Push

Here I am considering the length of the array to be n when the stack is completely full and the operation of push is called and the size is made to be double and elements are continuously added till the point the array is again completely filled(I am ignoring pop calls as they would eventually only increase number of $O(1)$ operations and thereby decreasing the average case complexity)

$$O(n) + \sum_{i=1}^{n-1} O(1) = O(2n-1)$$

$$\text{Therefore the avg value} = \frac{O(2n-1)}{(n+1)} = O(1)$$

2.2 Push and Pop

Here I am considering the length of the array to be n when the stack is completely full and the operation of push is called and the size is made to be double and elements are continuously popped till the point the array is $\frac{1}{4}$ filled of the total size(I am ignoring push calls as they would eventually only increase number of $O(1)$ operations and thereby decreasing the average case complexity)

$$O(n) + \sum_{i=1}^{3n/4} O(1) = O(7n/4)$$

$$\text{Therefore the avg value} = \frac{O(7n/4)}{(3n/4+1)} = O(1)$$

2.3 Pop and Push

Here I am considering the length of the array to be n when the stack is only 1 element more than 1/4 filled and the operation of pop is called and the size is made to be double and elements are continuously pushed till the point the array is filled(I am ignoring pop calls as they would eventually only increase number of $O(1)$ operations and thereby decreasing the average case complexity)

$$O(n) + \sum_{i=1}^{3n/4} O(1)= O(7n/4)$$

$$\text{Therefore the avg value}=\frac{O(7n/4)}{(3n/4+1)} = O(1)$$

2.4 Pop and Pop

Here I am considering the length of the array to be n when the stack is 1 element more than being 1/4 filled and the operation of pop is called and the size is made to be double and elements are continuously popped till the point the array is 1/4 filled of the new size(I am ignoring push calls as they would eventually only increase number of $O(1)$ operations and thereby decreasing the average case complexity)

$$O(n) + \sum_{i=1}^{n/2} O(1)= O(3n/2)$$

$$\text{Therefore the avg value}=\frac{O(3n/2)}{(n/2+1)} = O(1)$$

2.5 Combining the cases

Thereby combining the case where all possible sets of the operations are called and the set is partioned at all the points where the size of the stack need to be changed and thereby for each partition the amortised case complexity is $O(1)$ and thereby the overall complexity is hence concluded to be O(1)

3 Conclusion

Thereby from the analysis it can be made sure that the average case complexity in a dynamic stack based on the idea of hysterises is O(1) on operations push and pop