

Algorithms: Merge sort

Relevel
by Unacademy



Introduction

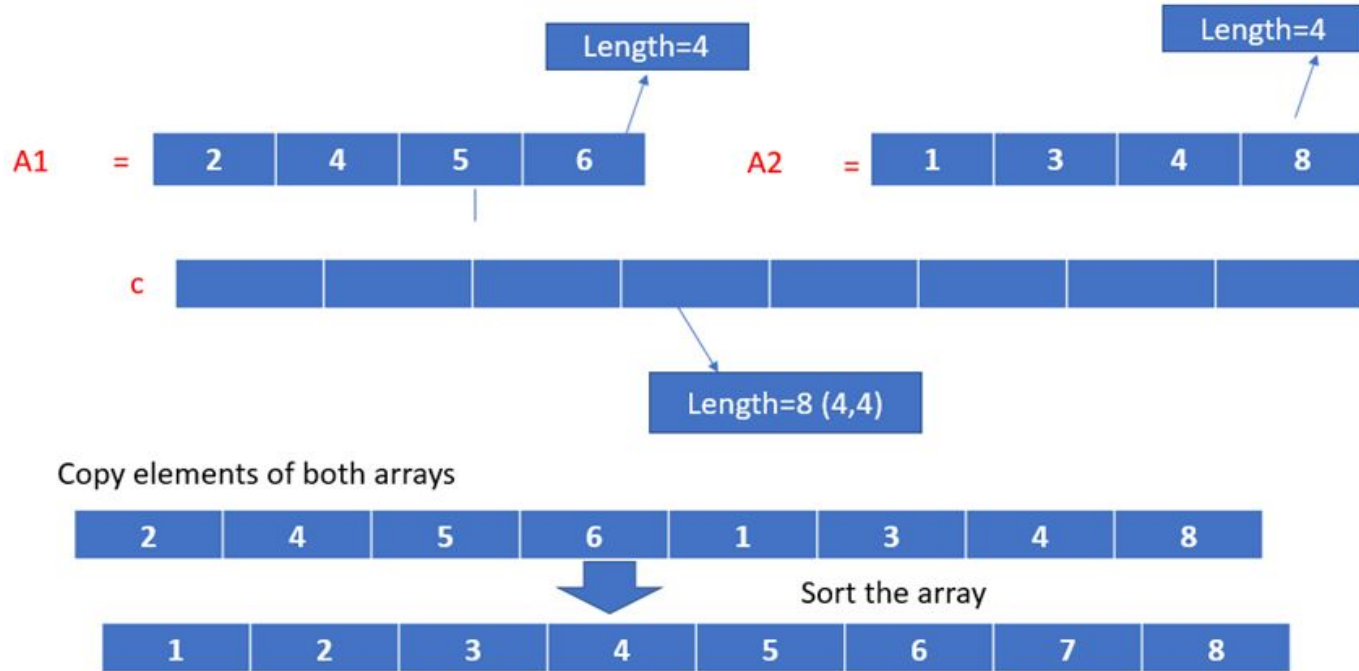
- This document contains a detailed course on Merge Sort and its application. After reading this document candidates would be versed with the merge sort algorithm and how it can be used to solve various interview problems.
- Here in this course, we would be going through the following table of contents. Secondly, we would dive deep into the crux of the algorithm.

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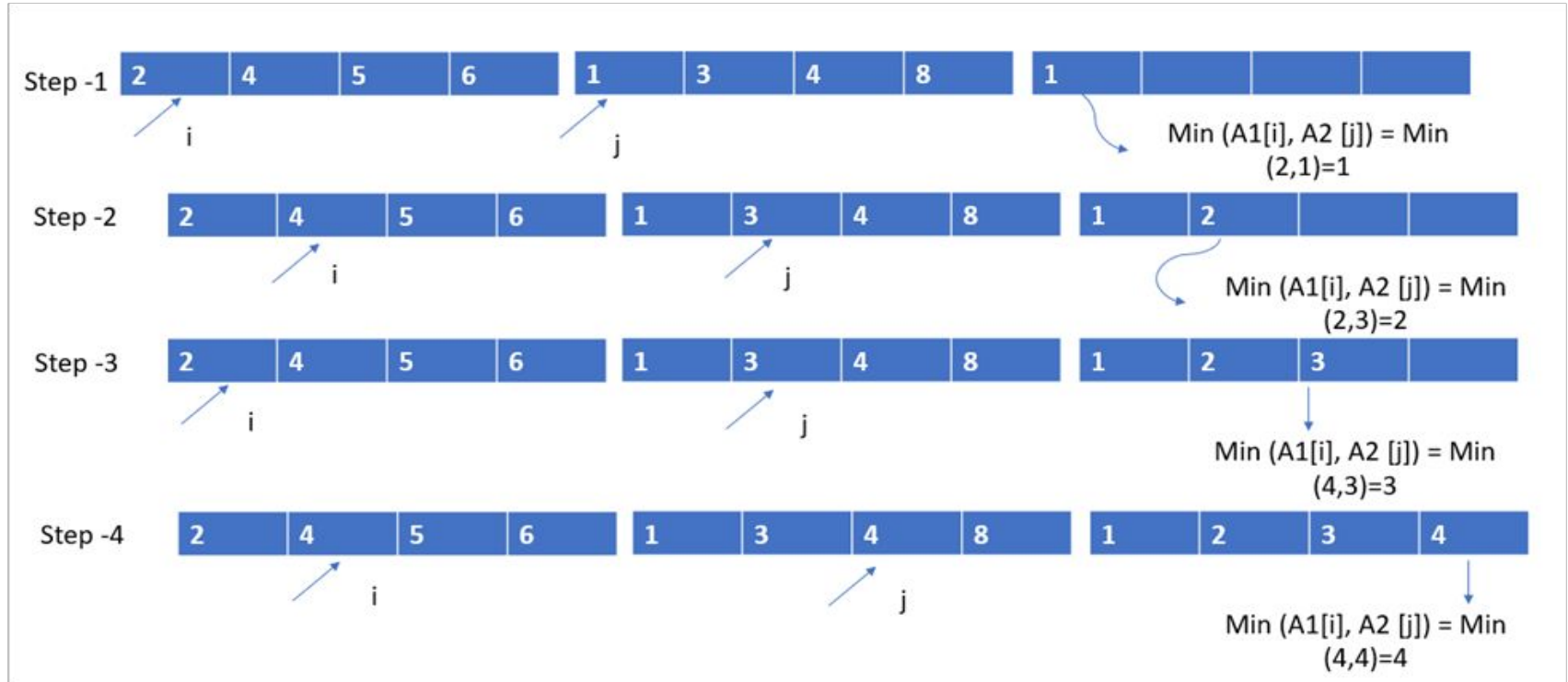
Why do we need another sorting algorithm if we already have bubble, selection and insertion sort ?

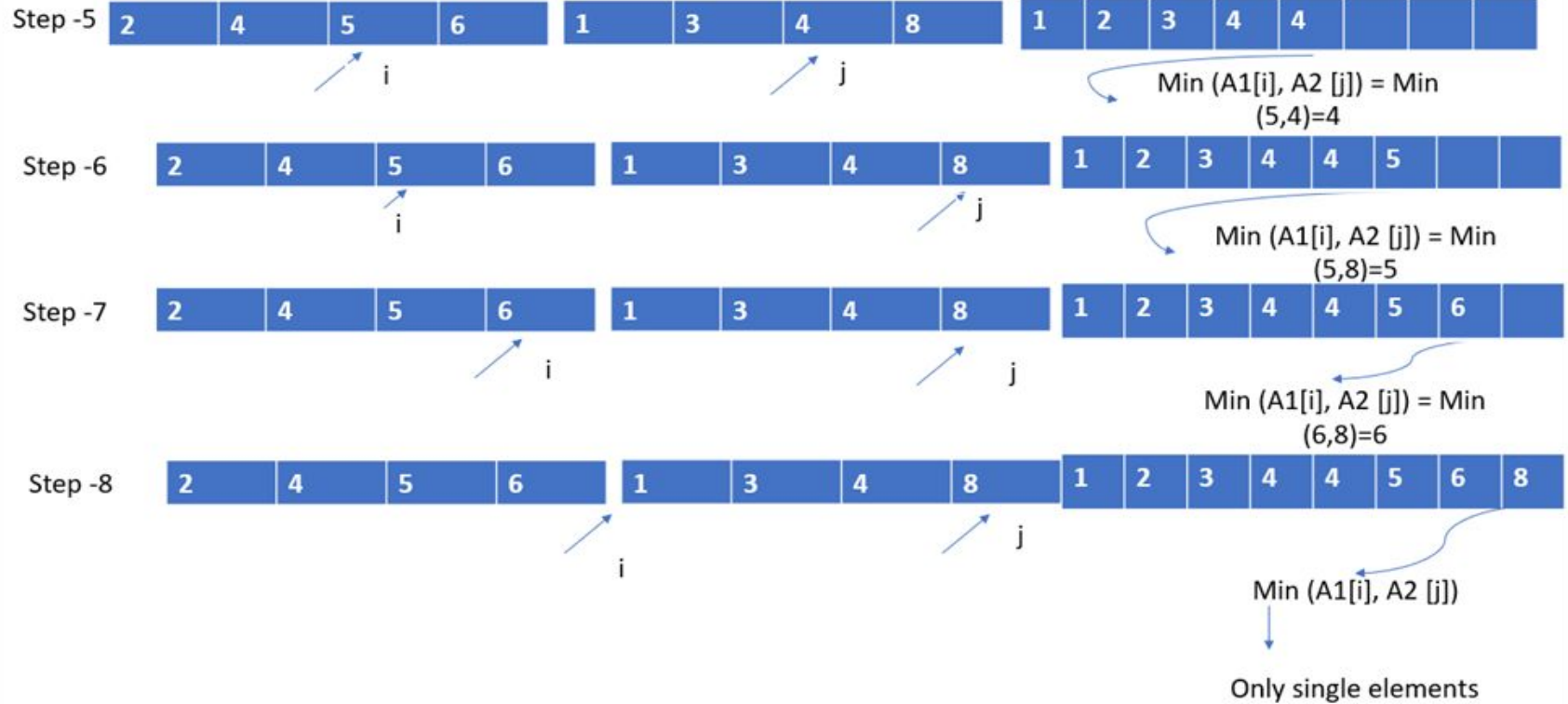
- Merge Algorithm
- Merge Sort Algorithm
- Stability and Inplace property of merge sort
- Time complexity analysis of Merge Sort
- Space Complexity analysis Merge Sort
- What is Divide And Conquer and how Merge Sort is following it ?
- Some important questions on the way merge sort works
- Applications Of Merge Sort
- Problem Solving On Merge Sort

Merge Algorithm



Can we optimise the solution ?





Coding Merge Algorithm

```
function mergeTwo(arr1, arr2) {
  let merged = [];
  let index1 = 0;
  let index2 = 0;
  let current = 0;

  while (current < (arr1.length + arr2.length)) {

    let isArr1Depleted = index1 >= arr1.length;
    let isArr2Depleted = index2 >= arr2.length;

    if (!isArr1Depleted && (isArr2Depleted || (arr1[index1] < arr2[index2]))) {
      merged[current] = arr1[index1];
      index1++;
    } else {
      merged[current] = arr2[index2];
      index2++;
    }

    current++;
  }

  return merged;
}

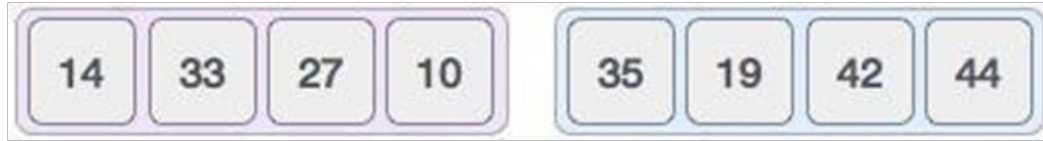
mergeTwo(arr1, arr2);
```

What is Divide And Conquer and how Merge Sort is following it ?

Consider the following given array –



Merge sort first will divide the whole array into equal halves, We see here that an array of 8 values is divided into two arrays of size 4 values each



Now we divide these two arrays into halves.

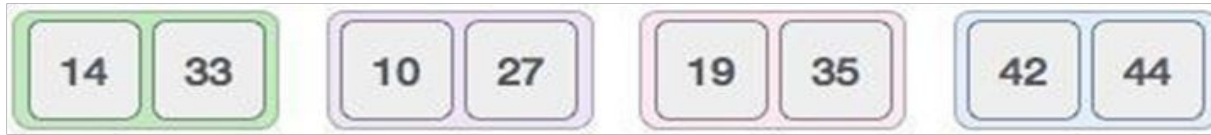


We further divide these arrays and we achieve single length array which can no longer be divided.



Now, we combine them in using merge algorithm.

We first compare the values from each array and then merge them into another array in a sorted fashion. We see that 14 and 33 are in sorted positions. We compare 27 and 10 and in the target list of 2 values we put 10 first, followed by 27. We change the order of 19 and 35 whereas 42 and 44 are placed sequentially.

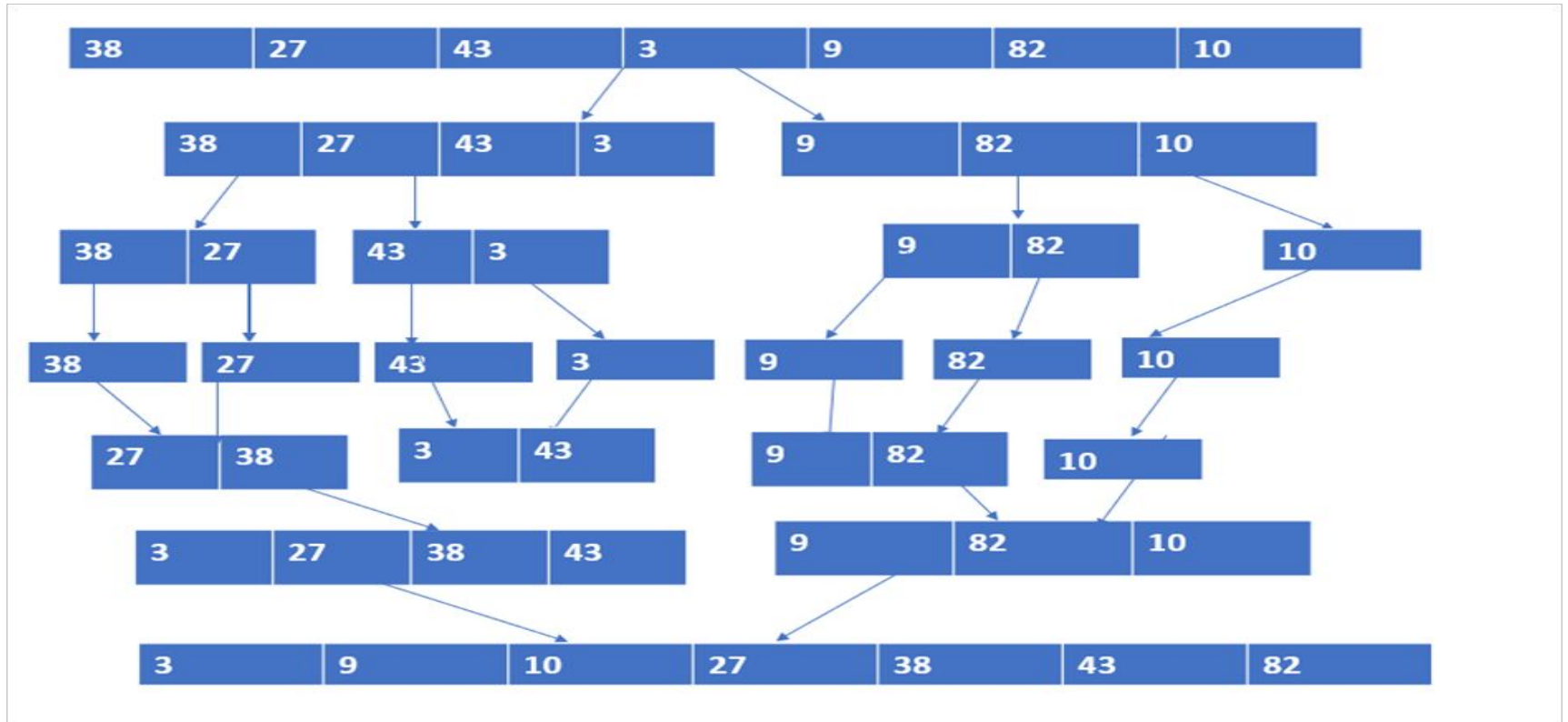


In the next iteration, we compare array values and merge them into the new array, in sorted order.



After the final merging, the final array should be–



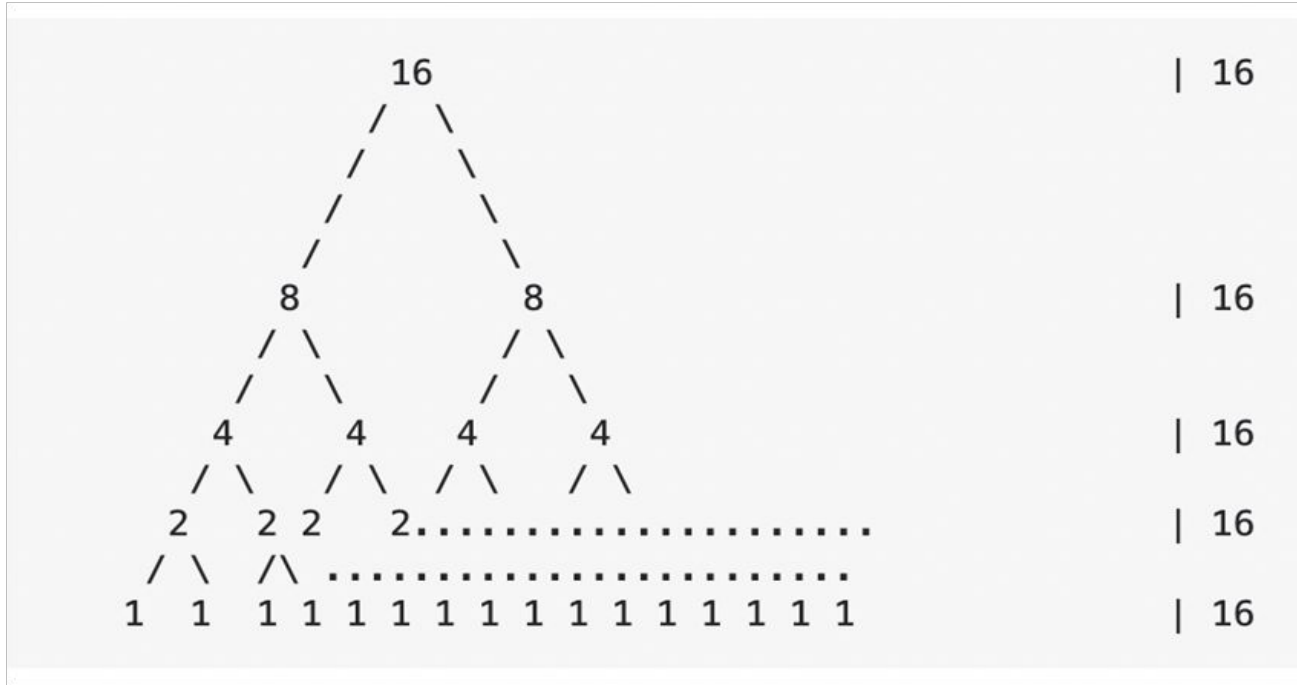


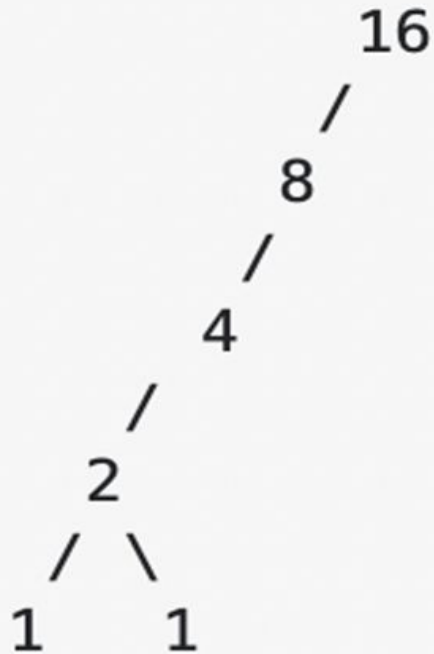
Time Complexity

$$\begin{aligned} & \sum_{i=0}^{\log_2 n} 2^i O\left(\frac{n}{2^i}\right) \\ &= \sum_{i=0}^{\log_2 n} O(n) \\ &= (1 + \log_2 n) O(n) \\ &= O(n \log n) \end{aligned}$$

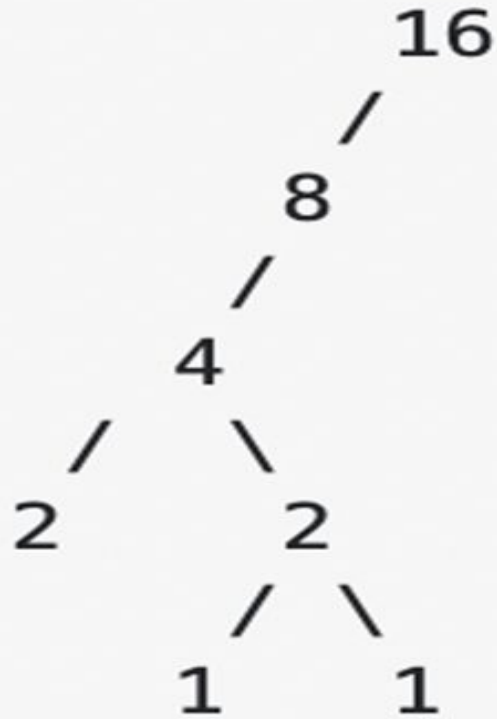
Space Complexity

If you draw the merge sort space tree it gives a hint of $O(n \log n)$

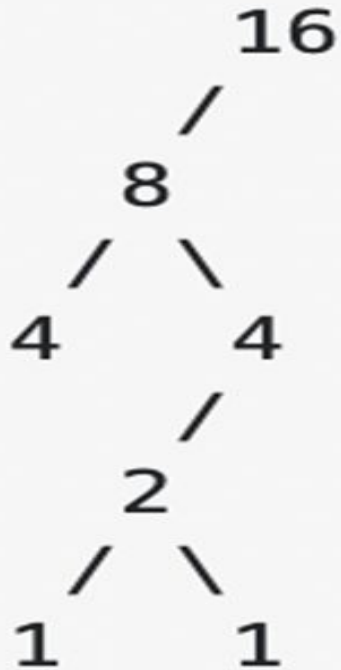




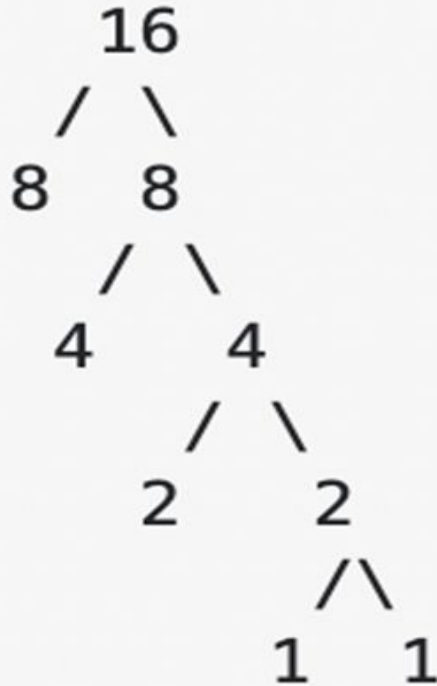
how number of space used is 32 $(16 + 8 + 4 + 2 + 1 + 1) = 2n = 2 \cdot 16 < 3n$
Then it merge upwards



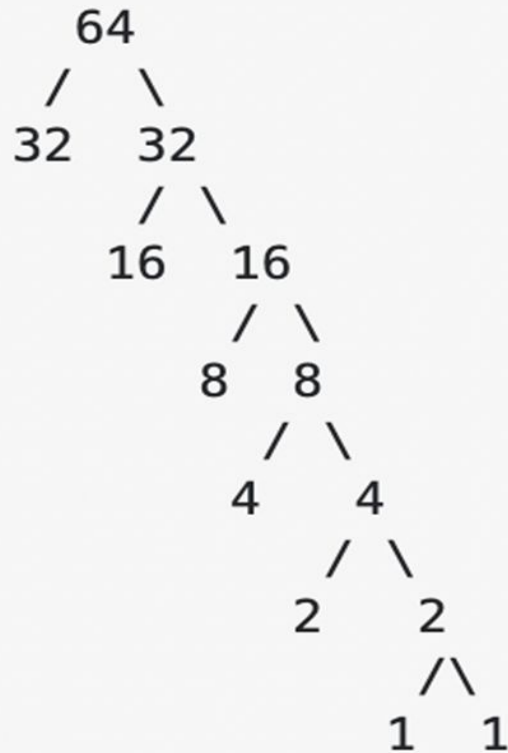
which is $34 (16 + 8 + 4 + 2 + 2 + 1 + 1) < 3n$. Then it merge upwards



$36 (16 + 8 + 4 + 4 + 2 + 1 + 1) < 16 * 3 = 48$
then it merge upwards



$46 (16 + 8 + 8 + 4 + 4 + 2 + 2 + 1 + 1) < 3 \cdot n = 48$
in a larger case, $n = 64$



Applications of Merge Sort

- Merge Sort is useful for sorting linked lists in $O(n \log n)$ time
- Merge sort can be implemented without extra space for linked lists
- Merge sort is used for counting inversions in a list
- Library Sort functions of many languages use merge sort

Practice Problem:

Q: Can we devise a modification to merge sort where we can make it inplace.

Q. Using D & C, find maximum and minimum element from given array.

Upcoming Teaser

- Quick Sort
- Analysis
- Applications
- Quick select algorithm
- Solving order statistics problem
- Median of medians vs quick select
- Merge sort vs quick sort

Thank You!