Introduction:- Logic is Concerned with methods of reasoning. It provides rules and techniques by which we can determine whether any particular argument is valid or not.

One component of Logic is proposition Calculas, which deals with statements, which has True and False Values and is concerned with analysis of propositions.

The other part is predicate Calculas, which deals with the predicates, which are propositions containing variables.

Propositions

A number of words making a complete grammatical structure, having a sense and meaning and also meant an assertion in logic is called a Sentence. This assertion may be of two types -

1. Declarative
2. Non-Declarative.

Declarative sentences are those, which are either True or False. (i.e. those declare something.)

Ex.1 "The Sun rises in the East." This is a declarative sentence and it is True.

Ex.2 "2+3 = 6" is a declartive sentence and is False.

Non-Deklarative Sentences are those, which does not declare anything. i.e. we are unable to find whether they are True or False.

Ex.3 "Do you speak German?" is a Non-declarative sentence. It is a question.

Ex.4 "$x+3=5$" is not a declarative sentence as we don't know whether it is True or False, as we don't know the values of x.

Although it is a declarative sentence for $x=2$.

A Statement or Proposition is a declarative sentence.

Ex.5 "Three Plus Three equal to six" is a proposition.

If the proposition is True, then we assign a value 'T' to it and if it is False then we assign a value 'F' to it. These values 'T' and 'F' are called as Truth Values of the proposition.

Propositional Variables

Instead of writing the statement(s) repeatedly, it is convinient to denote each of the statement by a unique variable, called propositional variable. These Variables are usually denoted by an English alphabets and can be replaced by $p, q, r, \ldots$ etc. by statements.

Ex.6 p: Jaipur is capital of Rajasthan.

q: Sun rises in the West.

Compound Proposition

A proposition consisting of only a single propositional variable or a single propositional constant is called an <u>atomic</u> or <u>Primary</u> or <u>Primitive</u> proposition or simply Proposition i.e. they can't be further subdivided.

A proposition obtained from the combination of two or more propositions by means of logical operators or connectives of two or more propositions or by negating a single proposition is called as <u>molecular</u> or <u>Composite</u> or <u>Compound</u> Proposition.

Connectives:-

The words or Phrases (or Symbols) use to form compound propositions are called connectives. There are 5 Basic Connectives which are represented in following Table.

Symbol Used.	Connective Word.	Nature of Comp. Prop.	Symbolic Form
$\sim, \neg, \rceil$	not	Negation	$\sim p$
$\wedge$	and	Conjunction	$p \wedge q$
$\vee$	or	Disjunction	$p \vee q$
$\Rightarrow, \rightarrow$	if .... then	Implication (or Conditional)	$p \Rightarrow q$
$\Leftrightarrow, \leftrightarrow$	if and only if	Equivalence (or Bi-Conditional)	$p \Leftrightarrow q$

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Negation:- If p is any proposition, the negation of p denoted by $\sim p$ or $\neg p$ and read as "not p" is a proposition, which is False when p is true and true when p is False.

p	$\sim p$
T	F
F	T

Ex. 7

If p: Paris is in France

$\sim p$: Paris is not in France

Ex. 8 p: All people are intelligent.

q: Every person is intelligent

r: Each person is intelligent

s: Any person is intelligent.

All these propositions have same meaning.

Ex. 9 The negation of the proposition

p: All students are intelligent.

is -

$\sim p$: Some students are not intelligent

$\sim p$: $\exists$ a student who is not intelligent.

$\sim p$: There are students who are not intelligent

$\sim p$: $\exists$ atleast One student is not intelligent.

Negation of the statement.

q: No student is intelligent

is $\sim q$: Some students are intelligent.

Imp. 1. "No student is intelligent" is not the negation of p

2. "All students are intelligent" is not the negation of q.

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Conjunction:- If p and q are two statements then Conjunction of p and q is the compound statement and denoted by $p \wedge q$ and read as "p and q".

$p \wedge q$ is true when both p and q are true otherwise it is False.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex: 10 Form the conjunction of p and q for each of the following -

(a) p: Ram is healthy , q: He has blue eyes.

(b) p: It is Cold , q: It is raining

(c) p: $5x+6=26$, q: $x>3$

Sol:- (a) $p \wedge q$: Ram is healthy and he has blue eyes.

(b) $p \wedge q$: It is Cold and raining

(c) $p \wedge q$: $5x+6=26$ and $x>3$.

Disjunction:- If p and q are two statements, then Disjunction of p and q is the compound statement, denoted by $p \vee q$ and read as "p or q".

$p \vee q$ is true if any one of p or q is True and False if both p and q are False

The English word "or" Can be used in two different senses - as an inclusive ("and/or") or exclusive ("either/or"). For example, Let us

Consider two following statements:-

1. p: He will go to Delhi or Calcutta
2. q: There is something wrong with the bulb or ~~Circuit~~. with the circuit.

In statement (1) the disjunction has been used in exclusive sense (p or q but not both) i.e. the person cannot go both the places.

In statement (2) the disjunction has been used in inclusive sense (either p or q or both) i.e. atleast one of the possibility occured, however both may have occurred.

We shall always use "or" in the inclusive sense unless it is stated.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Ex11 Form the disjunction of p and q, for each of following:-

(i) p: $5 < 5$, q: $5 < 6$

(ii) p: $5 \times 4 = 21$; q: $9 + 7 = 17$

Sol:- (i) $p \vee q$: $5 < 5$ or $5 < 6$ As here p is False but q is true, so $p \vee q$ is True.

(ii) $p \vee q$: $5 \times 4 = 21$ or $9 + 7 = 17$ As here both p and q are False, so $p \vee q$ is False.

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Ex. 12 Make a Truth Table for-
(a) $(\sim p \wedge q) \vee p$ (b) $(\sim p \vee q) \wedge \sim r$

Sol:- (a)

p	q	$\sim p$	$\sim p \wedge q$	$(\sim p \wedge q) \vee p$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

(b)

p	q	r	$\sim p$	$\sim p \vee q$	$(\sim p \vee q) \wedge \sim r$
T	T	T	F	T	F
T	T	F	F	T	T
T	F	T	F	F	F
F	T	T	T	T	F
T	F	F	F	F	F
F	T	F	T	T	T
F	F	T	T	T	F
F	F	F	T	T	T

Implication (or conditional statement)

If p and q are propositions, the compound proposition "if p then q" denoted by $p \Rightarrow q$ or $p \to$ is called a conditional proposition (or statement) or Implication. and the connective is the conditional connective.

The proposition p is called antecedent

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or <u>Hypothesis</u> and the proposition q is
Called <u>Consequent</u> or <u>Conclusion</u>.

<u>Ex.13</u> If Tomorrow is sunday then today is saturday.

Here p: Tomorrow is Sunday
is antecedent. or Hypothesis.

and q: Today is Sunday
is consequent or Conclusion.

The connective <u>if..... then</u> can also be read as-

1. p implies q
2. p is sufficient for q
3. p only if q
4. q is necessary for p
5. q if p.
6. q follows from p
7. q is Consequence of p.

<u>Ex.14</u> Let us analyze an implication-
$p \rightarrow q$: If you wash my car, then I will pay you Rs. 30.

Here p: If you wash my car
q: I will pay you Rs. 30

Now we have following cases.

<u>Case-I</u> <u>when both p and q are True</u>.
If you wash my car and if I pay you Rs. 30, then the implication is True as I kept my words.

<u>Case-II</u> <u>p is True and q is False</u>.
If you wash my car and if I donot pay

you Rs. 30. Then the promise is violated and hence the implication is False.

Case–III p is False

If you do not wash my car, then I may give you Rs. 30 (being generous) or not (not ~~keeping~~ giving any promise). In either case my promise has not been tested and so has not violated. So the implication has not been proved False, so it must be true.

Hence if p is False $p \to q$ is always true by default.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Ex. 15 write the implication $p \to q$, for the following–

p: I am Hungry.

q: I will eat.

Sol:- $p \to q$: If I am hungry then I will eat.

Ex. 16 which of the propositions are True or False.

(a) If Graham Bell invented Telephone, then tigers have wings.

q	$(p \vee \sim q) \rightarrow p$
	T
	T
	T
	F

$\sim(p \wedge q) \vee r$	$(\sim(p \wedge q) \vee r) \rightarrow p$
T	T
F	T
T	T
T	F
T	T
T	F
T	F
T	F

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Converse, Contrapositive and Inverse

There are some related implications that can be formed from $p \to q$. If $p \to q$ is an implication then -

Converse of $p \to q$ is $q \to p$

Contrapositive of $p \to q$ is $\sim q \to \sim p$

Inverse of $p \to q$ is $\sim p \to \sim q$

CP for $\sim q \to p$ is $\sim p \to q$

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Converse

p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Inverse

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Contrapositive

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Ex.18 Find the converse, contrapositive and Inverse of following Implications.-

(i) " If today is Thursday, then I have a test today"

(ii) " If it is raining, then I get wet"

Sol:- (i) $p \rightarrow q$: If today is Thursday, then I have a test today.

p: Today is Thursday

q: I have a test today.

Converse

$q \rightarrow p$: If I have a test today, then today is Thursday.

Contrapositive

$\sim q \rightarrow \sim p$: If I donot have a test today, then today is not Thursday.

Inverse

$\sim p \rightarrow \sim q$: If today is not Thursday, then I donot have a test today.

(ii) $p \rightarrow q$: If it is raining, then I get wet.

p: It is raining

q: I get wet.

Converse

$q \rightarrow p$: If I get wet, then it is raining

Contrapositive

$\sim q \rightarrow \sim p$: If I donot get wet, then it is not raining.

Inverse

$\sim p \rightarrow \sim q$: If it is not raining, then I donot get wet.

Biconditional Statement

If p and q are statements, then the Compound statement "p if and only if q" or "p iff q", denoted by $p \Leftrightarrow q$ or $p \leftrightarrow q$ is called a Biconditional statement and the connective "if and only if" is "Biconditional Connective".

$p \leftrightarrow q$ can also be stated as "p is a necessary and sufficient condition for q" or as "p implies q and q implies p"

Ex. 19 Let us consider ~~the~~ propositions -

p: A new car will be acquired.

q: Additional Funding is available.

and r: A new car will be acquired, if and only if, additional Funding is available.

So (i) r is True when both p and q are true.

(ii) r is False when p is true and q is false or p is False and q is True.

(iii) r is True when both p and q are False.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Tautologies and Contradiction

A compound proposition, that is always true for all possible truth values, is called as Tautology.

A compound proposition, that is always False for all possible truth values, is called as Contradiction.

A proposition that is neither a tautology nor a contradiction is called a Contingency.

Ex.20 Show that the proposition $\sim(p \wedge q) \vee q$ is a Tautology.

Sol:-

p	q	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee q$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	T

As the last column contains only 'T', so it is a Tautology.

Ex.21 Verify that the proposition $p \wedge (q \wedge \sim p)$ is a contradiction.

Sol:

p	q	$\sim p$	$q \wedge \sim p$	$p \wedge (q \wedge \sim p)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

∴ it is a Contradiction.

Ex.22 Verify that the proposition $(p \to q) \wedge (p \vee q)$ is a contingency.

Sol:

p	q	$p \to q$	$p \vee q$	$(p \to q) \wedge (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

$\therefore$ It is a contingency.

Logical Equivalence

If two propositions $P(p, q, r, \ldots)$ and $Q(p, q, r, \ldots)$, where $p, q, r, \ldots$ are propositional variables, have the same truth values in every possible case, then the propositions are called logically equivalent or simply equivalent and denoted by

$$P(p, q, r, \ldots) \equiv Q(p, q, r, \ldots)$$

To test whether two propositions P and Q are logically equivalent, following steps can be followed.

S-1 Construct the Truth table for P

S-2. Construct the Truth table for Q using the same propositional variables.

S-3 Check each combination of truth values of the propositional variables to see whether the truth value of P is same as the truth value of Q. If in each row P and Q have same Truth values, then $P \equiv Q$.

p	q	$p \to q$	$\sim p$	$(\sim p) \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T
		C_3		C_5

As column C_3 and C_5 are identical, so the result is True.

Algebra of Propositions

Propositions satisfies various laws. These Laws are useful to simplify Expression.

1. Idempotent Law

(i) $p \vee p \equiv p$

(ii) $p \wedge p \equiv p$.

2. Commutative Law

(i) $p \vee q \equiv q \vee p$

(ii) $p \wedge q \equiv q \wedge p$.

~~Example~~

p	q	$p \wedge q$	$q \wedge p$	$p \vee q$	$q \vee p$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	F	F	T	T
F	F	F	F	F	F

Associative Law

(i) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(ii) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	F	F
T	F	T	T	T	T	T	F	F
F	T	T	T	T	T	T	F	F
T	F	F	T	F	T	T	F	F
F	T	F	T	T	T	T	F	F
F	F	T	F	T	T	T	F	F
F	F	F	F	F	F	F	F	F

Same   Same.

Distributive Law

(i) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(ii) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$(p \vee q)$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	F	F
F	T	F	F	F	T	T	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

Same.

Similarly for (ii) we can prove.

Identity Law

(i) $p \lor T \equiv T$ (ii) $p \lor F \equiv p$

(iii) $p \land T \equiv p$ (iv) $p \land F \equiv F$

Negation Laws

(i) $\sim(\sim p) \equiv p$ (Involution Law)

(ii) $p \lor (\sim p) = T$

(iii) $p \land (\sim p) = F$

(iv) $\sim T \equiv F$

(v) $\sim F \equiv T$

(Complement Law)

(vi) $\sim(p \lor q) \equiv \sim p \land \sim q$

(vii) $\sim(p \land q) \equiv \sim p \lor \sim q$

(DeMorgan's Law).

Absorption Law

(i) $p \lor (p \land q) \equiv p$

(ii) $p \land (p \lor q) \equiv p$

p	q	$p \land q$	$p \lor (p \land q)$	$p \lor q$	$p \land (p \lor q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	F	T	F
F	F	F	F	F	F

Ex. 24 Show that $\sim(p \lor (\sim p \land q)) \equiv \sim p \land \sim q$

Sol:- $\because \sim(p \lor (\sim p \land q)) = \sim p \land \sim(\sim p \land q)$ [DeMorgan

$= \sim p \land (\sim(\sim p) \lor \sim q)$ [Demorgan Law]

$= \sim p \land (p \lor \sim q)$ [Negation Law]

$= (\sim p \land p) \lor (\sim p \land \sim q)$ [Distributive L

$= F \lor (\sim p \land \sim q)$ [Commutative Law]

$= (\sim p \land \sim q) \lor F$ [Identity Law]

$= \sim p \land \sim q$

Ex.25 Show that $(p \wedge q) \rightarrow (p \vee q)$ is a Tautology.

Sol: $\because$ $p \rightarrow q \equiv \sim p \vee q$

$\therefore$ $(p \wedge q) \rightarrow (p \vee q) \equiv (\sim(p \wedge q)) \vee (p \vee q)$

$\equiv (\sim p \vee \sim q) \vee (p \vee q)$ [Demorgan Law]

$\equiv ((\sim p \vee \sim q) \vee p) \vee q$ [Associative Law]

$\equiv (\sim p \vee (\sim q \vee p)) \vee q$ ["]

$\equiv (\sim p \vee (p \vee \sim q)) \vee q$ [Commutative Law]

$\equiv ((\sim p \vee p) \vee \sim q) \vee q$ [Associative Law]

$\equiv T \vee (\sim q \vee q)$

$\equiv T \vee T \equiv T$

$\therefore$ $(p \wedge q) \rightarrow (p \vee q)$ is a Tautology.

Aliter

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Predicates

Let us consider ~~the~~ two propositions

Ram is a Bachelor.

Shyam is a bachelor.

Both Ram and Shyam has the same property of being bachelor. In the propositional Calculas there is no symbolic presentation of "is a bachelor", as it is not a sentence. The two proposition can be replaced by a single proposition "x is a bachelor". By replacing x by Ram, Shyam or by any other name, we get many propositions.

In Logic, predicates can be obtained by removing any nouns from the statement. A predicate is symbolized by capital letter and the names of individuals or objects are in general by small letters.

The sentence "x is a Bachelor" is symbolized as P(x), where x is a Predicate variable. when concrete Values ~~Values~~ are substituted in place of x (predicate Variable), a statement results. P(x) is also called a Propositional Function, because each choice of x produces a propositionP(x) that is either true or false.

Thus a predicate is a sentence that contains a finite number of variables and becomes a ~~Pred~~ proposition when specific Values are substituted for the Variables.

The domain of a predicate Variable is the set of all possible Values that may be substituted in place of variables.

Ex. The domain for P(x): "x is a bachelor" Can be taken as the set of all human names.

The domain of predicate variable is also Known as "Universe of discourse" or Simply "Universe"

spl. Comment :- In above Example 'x' is the predicate variable and "is a bachelor" is a Predicate.

Ex.26 Let P(x) denote the statement "$x>3$", what are the truth values of P(2) and P(4)

Sol:- The statement P(2) denotes $2>3$, which is False, P(4) denotes $4>3$, which is True.

Ex.27 Let Q(x,y) be the statement "$x=y+3$" what are truth values of the statements Q(1,2) and Q(3,0).

Sol:- Q(1,2) denotes $1=2+3$, which is False.
Q(3,0) denotes $3=0+3$, which is True.

Quantifiers

* Quantification is a technique to create a statement from a Propositional function. There are two types of Quantifications

Universal Quantifier

The Universal quantification of a Predicate P(x) is the statement "P(x) is True for all values of x in the Universe of discourse."

The notation $\forall x P(x)$ ~~$\forall x P(x)$~~ denotes the universal Quantification of P(x). The symbol $\forall$ is called the Universal quantifier. The statement $\forall x P(x)$ can also be stated as "for every x P(x)" or "forall x P(x)"

Existential Quantifiers

The Existential quantification of a Predicate P(x) is the statement "There exists an element x in the Universe of discourse for which P(x) is True."

The notation $\exists x P(x)$ denoted the existential quantification of P(x) and the symbol $\exists$ called the existential quantifier.

The statement $\exists x P(x)$ can also be stated as—
"There is a x such that $P(x)$"
or "There is atleast one x such that $P(x)$"
or "For some x $P(x)$"
or "there exists a x such that $P(x)$"

Ex.28 Express the statement.
"Every student in this class has studied calculus"

Sol:- Let $P(x)$ be the statement "x has studied calculus" and the Universe of discourse consists of all the students in this class. So the given statement can be expressed as $\forall x P(x)$.
Alternatively, this can also be expressed as—
$\forall x(R(x) \rightarrow P(x))$
where $R(x)$ is the statement "x is in this class" and the Universe of discourse is the set of all students.

Table of Quantifiers for One Variable

Statement	When True?	When False
$\forall x P(x)$	$P(x)$ is true for each x.	$\exists$ atleast one x for which $P(x)$ is False.
$\exists x P(x)$	$\exists$ atleast one x for which $P(x)$ is true	$P(x)$ is False for every x.

Ex.29 What is the truth value of $\forall x P(x)$, where $P(x)$: "$x^2 < 10$" and the universe of discourse consists of the positive integers not exceeding 4.

Sol:- $\because$ universe of discourse is $U = \{1, 2, 3, 4\}$ and $\because$ $4 \in U$ is such that $P(x)$ is false. so here $\forall x P(x)$ is False.

Ex.30 Translate the statement
$\forall x (C(x) \vee \exists y (C(y) \wedge F(x,y)))$
in to English, where $C(x)$ is "x has a computer" and the
$F(x,y)$ is "x and y are friends" and the universe of discourse for both x and y is the set of all students in our school.

Sol:- The statement says that, for every student x in our school, x has a computer or there is a student y such that y has a computer and x and y are friends.

In other words, Every student in our school has a computer or has a friend who has a computer.

Ex.31 Express the statement "Everyone has exactly one best friend" as a logical expression using Quantifiers.

Sol:- Let $Q(x,y)$ denotes " y is best friend of x". Now the given statement means that for each person x, there is another unique person y such that y is the best friend of x. it means if z is a person other than y, then z cannot be a best friend of x.

~~$\forall x \exists y (Q(x,y) \wedge (z \neq y)$~~

$\forall x \exists y \forall z (Q(x,y) \wedge ((z \neq y) \rightarrow \sim Q(x,z)))$

L(x) : x is mortal

then express the sentence -

"All men are mortal"

as a logical expression using Quantifiers.

Sol:- $\forall x (K(x) \rightarrow L(x))$

Properties of Quantifiers

The negation of a Quantified statement changes the Quantifier and also negates the given statement as -

(i) $\sim(\forall x P(x)) \equiv \exists x \sim P(x)$ [De-morgan's Law]

(ii) $\sim(\exists x P(x)) \equiv \forall x \sim P(x)$ ["]

(iii) $\exists x (P(x) \rightarrow Q(x)) \equiv \forall x P(x) \rightarrow \exists x Q(x)$

(iv) $\exists x P(x) \rightarrow \forall x Q(x) \equiv \forall x (P(x) \rightarrow Q(x))$

(v) $\exists x (P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$

(vi) $\sim(\exists x \sim P(x)) \equiv \forall x P(x)$

Statement		Negation	
All true	$\forall x P(x)$	$\exists x \sim P(x)$	atleast One False.
Atleast One False	$\exists x \sim P(x)$	$\forall x P(x)$	all true.
All False.	$\forall x \sim P(x)$	$\exists x P(x)$	atleast One true
Atleast One true	$\exists x P(x)$	$\forall x \sim P(x)$	all False.

Ex.33 Let p: It is Cold , q: It is raining
Translate following statement in to English.
(i) $\sim p$ (ii) $p \wedge q$ (iii) $p \vee q$ (iv) $q \vee \sim p$.

Sol:- (i) It is not Cold
(ii) It is Cold and raining
(iii) It is Cold or It is raining
(iv) It is raining or it is not cold.

The construction of truth tables is not practical, if the number of variables involved in a given statement becomes large. In this situation it is very tedious to find whether two expressions P and Q are equivalent or not.

A better method is to transform the expressions P and Q to some standard forms of expressions P' and Q' such that a simple comparision of P' and Q' shows whether $P \equiv Q$. The standard forms are called. <u>Normal Forms</u> or <u>Cannonical Forms</u>.

There are two types of Normal forms <u>Disjunctive Normal Form</u> (DNF) and <u>Conjuctive Normal Form (CNF)</u>. It is convinient to use word Sum and product in place of logical Connectives Disjunction and Conjunction.

<u>Some Basic Terms related to Normal Form</u>

<u>Elementary Product</u> :- A product of propositional Variables and their negations is called an elementary product.

Let p and q are two propositional Variables, then $p, q, p \wedge q$, $p \wedge \sim q$, $\sim p \wedge q$, $\sim p \wedge \sim q$, $\sim p \wedge p$, $\sim q \wedge q$, $p \wedge q \wedge \sim q$, ... are elementary Products.

<u>Elementary Sum</u>: A sum of propositional Variables and their ~~[illegible]~~ negations is called an elementary Sum.

Let p and q are two propositional Variables then p, q, $p \vee q$, $\sim p \vee q$, $p \vee \sim q$, $\sim p \vee \sim q$, $\sim p \vee p$, $\sim q \vee q$, $\sim p \vee p \vee q$, ... are elementary sum.

Factor :- A factor of the given elementary sum or product, is a part of it and is itself an elementary sum or product.

Ex. If p and q are two propositional variables, then $\sim q, p, q$, $p \wedge \sim q$, $\sim q \wedge p$, $\sim q \wedge q$ are the Factors of $\sim q \wedge p \wedge q$.

Minterms :- Let p and q be two propositional variables. All possible formulas which consists of product of p or its negation and q or its negation, but should not contain both the variable and its negation (i.e. p and $\sim p$ does not appear at same time.) in any one of the formula are called minterms of p and q.

If p and q are two variables, then minterms are $p \wedge q$, $p \wedge \sim q$, $\sim p \wedge q$, $\sim p \wedge \sim q$.

* If there are 'n' variables then number of minterms are 2^n

* Each Minterm has the truth value 'T' for exactly one combination of the truth values of the variables p and q.

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge q$	$p \wedge \sim q$	$\sim p \wedge \sim q$
T	T	F	F	T ✓	F	F	F
T	F	F	T	F	F	T ✓	F
F	T	T	F	F	T ✓	F	F
F	F	T	T	F	F	F	T ✓

Maxterm :-

For given variables, the maxterm consists of sum of variables (i.e. Disjunctions) in which each variable or its negation (but not both) appears once.

For two variables p and q the maxterms are - $p \vee q$, $p \vee \sim q$, $\sim p \vee q$, $\sim p \vee \sim q$

* If there are 'n' variables, then there are 2^n maxterms.

* Each maxterm has the truth value F for exactly one combination of the truth values of variables.

p	q	$\sim p$	$\sim q$	$p \vee q$	$p \vee \sim q$	$\sim p \vee q$	$\sim p \vee \sim q$
T	T	F	F	T	T	T	F ✓
T	F	F	T	T	T	F ✓	T
F	T	T	F	T	F ✓	T	T
F	F	T	T	F ✓	T	T	T

Disjunctive Normal Form (DNF)

A statement which consists of a sum of elementary products of propositional variables and is equivalent to the given compound statement is called a DNF of the given statement. ~~Eis~~ This form is not unique for the given statement.

~~Ex~~ The statements $p \vee (q \wedge r)$ and $p \vee (\sim q \wedge r)$ are in DNF but $p \wedge (q \vee r)$ is not in DNF.

$p \vee (q \wedge r)$ is equivalent to ~~$(p \wedge p) \vee (p \wedge q) \vee$~~

$(p \wedge p) \vee (p \wedge q) \vee (p \wedge r) \vee (q \wedge r)$

Procedure to obtain a DNF of a given logical Expression

S-1 Remove all $\rightarrow$ and $\leftrightarrow$ connectives by an equivalent expression containing the connectives $\wedge, \vee, \sim$ only.

S-2 Eliminate $\sim$ before sums and products shift by using double negation or by using De-morgan's Laws.

S-3 Apply the Distributive Law until a sum of elementary product is obtained.

Ex.1 obtain DNF of the statement

$$p \wedge (p \rightarrow q)$$

Sol:- $\because$ $p \wedge (p \rightarrow q) \equiv p \wedge (\sim p \vee q)$ [Distributive Law]

$$\equiv (p \wedge \sim p) \vee (p \wedge q)$$

which is required DNF.

Ex.2 obtain DNF of the statement–

$$p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$$

Sol:- $\because$ $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$

$$\equiv p \vee (\sim p \rightarrow (q \vee (\sim q \vee \sim r)))$$

$$\equiv p \vee (p \vee (q \vee (\sim q \vee \sim r)))$$

$$\equiv p \vee p \vee q \vee (\sim q \vee \sim r)$$

$$\equiv p \vee p \vee q \vee (q \wedge r)$$

$$\equiv p \vee q \vee (q \wedge r) \checkmark$$

$$\cancel{\equiv p \vee ((q \vee q) \wedge (q \vee r))}$$

$$\cancel{\equiv p \vee (q \wedge (q \vee r))}$$

$$\equiv$$

$\sim(p\vee q)\leftrightarrow(p\wedge q) = [\sim(p\vee q)\rightarrow(p\wedge q)]\wedge[(p\wedge q)\rightarrow\sim(p\vee q)]$

$=[(p\vee q)\vee(p\wedge q)]\wedge[\sim(p\wedge q)\vee\sim(p\vee q)]$

$=[\{(p\vee q)\vee(p\wedge q)\}\wedge\sim(p\wedge q)]\vee[\{(p\vee q)\vee(p\wedge q)\}\wedge\sim(p\vee q)]$

$=[((p\vee q)\wedge\sim(p\wedge q))\vee((p\wedge q)\wedge\sim(p\wedge q))]$

$\vee[((p\vee q)\wedge\sim(p\vee q))\vee((p\wedge q)\wedge\sim(p\vee q))]$

$=[((p\vee q)\wedge\sim(p\wedge q))\vee F]\vee[F\vee((p\wedge q)\wedge\sim(p\vee q))]$

$=[(p\vee q)\wedge\sim(p\wedge q)]\vee[(p\wedge q)\wedge\sim(p\vee q)]$

$=[(p\vee q)\wedge\sim(p\wedge q)]\vee[(p\wedge q)\wedge(\sim p\wedge\sim q)]$

$=[((p\vee q)\wedge\sim(p\wedge q))\vee(p\wedge q)]\wedge[((p\vee q)\wedge\sim(p\wedge q))\vee(\sim p\wedge\sim q)]$

$=[((p\vee q)\vee(p\wedge q))\wedge(\sim(p\wedge q)\vee(p\wedge q))]$

$\wedge[(\underbrace{(p\vee q)\vee(\sim p\wedge\sim q)}_{T})\wedge((p\wedge q)\vee(\sim p\wedge\sim q))]$

$=\cancel{p\vee q}\ \cancel{\#}$

$=[((p\vee q)\vee(p\wedge q))\wedge T]\wedge[T\wedge((p\wedge q)\vee(\sim p\wedge\sim q))]$

$=\cancel{\sim(\sim p\wedge q}$

$=[(p\vee q)\vee(p\wedge q)]\wedge[(p\wedge q)\vee(\sim p\wedge\sim q)]$

$=[((p\vee q)\vee(p\wedge q))\wedge(p\wedge q)]\vee[((p\vee q)\vee(p\wedge q))\wedge(\sim p\wedge\sim q)]$

$=[((p\vee q)\wedge(p\wedge q))\vee((p\wedge q)\wedge(p\wedge q))]$

Ex. 3 Obtain DNF of the statement -

$\sim(p \vee q) \leftrightarrow p \wedge q$

Sol:- $\because$ $\sim(p \vee q) \leftrightarrow p \wedge q \equiv [\sim(p \vee q) \rightarrow (p \wedge q)]$
$\wedge [(p \wedge q) \rightarrow \sim(p \vee q)]$

$\equiv [(p \vee q) \vee (p \wedge q)] \wedge [\sim(p \wedge q) \vee \sim(p \vee q)]$

$\equiv [((p \vee q) \vee (p \wedge q)) \wedge \sim(p \wedge q)]$
$\vee [((p \vee q) \vee (p \wedge q)) \wedge \sim(p \vee q)]$

$\equiv [((p \vee q) \wedge \sim(p \wedge q)) \vee ((p \wedge q) \wedge \sim(p \wedge q))] \vee [((p \vee q) \wedge \sim(p \vee q)) \vee ((p \wedge q) \wedge \sim(p \vee q))]$

$\vee$ ... $[((p \vee q) \wedge \sim(p \wedge q)) \vee F]$ $[\because p \wedge \sim p \equiv F$

$\equiv [((p \vee q) \wedge \sim(p \wedge q)) \vee F]$
$\vee [F \vee ((p \wedge q) \wedge \sim(p \vee q))]$ $[\because p \vee F$

$\equiv ((p \vee q) \wedge \sim(p \wedge q)) \vee ((p \wedge q) \wedge \sim(p \vee q))$

$\equiv ((p \vee q) \wedge \sim(p \wedge q)) \vee ((p \wedge q) \wedge (\sim p \wedge \sim q))$

$\equiv ((p \vee q) \wedge \sim(p \wedge q)) \vee ((p \wedge \sim p) \wedge (q \wedge \sim q))$ $[\because p \wedge q = q$

$\equiv ((p \vee q) \wedge \sim(p \wedge q)] \vee F$

$\equiv [(p \vee q) \wedge \sim(p \wedge q)]$

$\equiv (p \vee q) \wedge \sim(p \wedge q)$

$\equiv (p \vee q) \wedge (\sim p \vee \sim q)$

$\equiv (p \wedge (\sim p \vee \sim q)) \vee (q \wedge (\sim p \vee \sim q))$

$\equiv [(p \wedge \sim p) \vee (p \wedge \sim q)] \vee [(q \wedge \sim p) \vee (q \wedge \sim q)]$

$\equiv [(p \wedge \sim p) \vee (p \wedge \sim q)] \vee ((q \wedge \sim p) \vee F)$

$\equiv (F \vee (p \wedge \sim q)) \vee ((q \wedge \sim p) \vee F)$

$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

Ex. 4 Obtain DNF of the statement -

$p \to ((p \to q) \wedge \sim(\sim q \vee \sim p))$

Sol:-

$\because \; p \to ((p \to q) \wedge \sim(\sim q \vee \sim p))$

$\equiv p \to ((p \to q) \wedge (q \wedge p))$ [By Demorgan La

$\equiv p \to ((\sim p \vee q) \wedge (q \wedge p))$

$\equiv \sim p \vee ((\sim p \vee q) \wedge (q \wedge p))$

$\cancel{\equiv \sim p \vee ((\sim p \vee q) \wedge q) \wedge (\sim p \vee q) \wedge p})$

$\equiv \sim p \vee [(\sim p \wedge (q \wedge p)) \vee (q \wedge (q \wedge p))]$

$\equiv \sim p \vee [(\sim p \wedge (p \wedge q)) \vee ((q \wedge q) \wedge p)]$

$\equiv \sim p \vee [((\sim p \wedge p) \wedge q) \vee (q \wedge p)]$

$\equiv \sim p \vee [(F \wedge q) \vee (q \wedge p)]$

$\equiv \sim p \vee [F \vee (q \wedge p)]$

$\equiv \sim p \vee (q \wedge p)$

$\equiv \sim p \vee (p \wedge q)$

Conjunctive Normal Form (CNF)

A statement which consists of a product of elementary sums of propositional variables and is equivalent to the given compound statement, is called CNF of the given statement. This form is not unique for given statement.

The Procedure to obtain CNF is same as for DNF.

p	q	$p \to q$	$p \wedge (p \to q)$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

p	q	$p \wedge q$	$\sim p \wedge q$	$p \wedge \sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	F	T	F	F
F	F	F	F	F	T

Ex.1 Obtain a CNF of the statement-

$$p \wedge (p \rightarrow q)$$

Sol:- $p \wedge (p \rightarrow q) \equiv p \wedge (\sim p \vee q)$

Ex.2 Obtain a CNF of the statement-

$$\sim(p \vee q) \leftrightarrow (p \wedge q)$$

Sol:- $\sim(p \vee q) \leftrightarrow (p \wedge q)$

$\equiv (\sim(p \vee q) \rightarrow (p \wedge q)) \wedge ((p \wedge q) \rightarrow \sim(p \vee q))$

$\equiv ((p \vee q) \vee (p \wedge q)) \wedge (\sim(p \wedge q) \vee \sim(p \vee q))$

$\equiv \cancel{((p \vee q) \vee (p \wedge q)) \wedge \sim((p \wedge q) \wedge (p \vee q))}$

$\equiv \cancel{((p \vee q) \vee p) \wedge ((p \vee q) \vee q) \wedge \sim(p \wedge (p \vee q))}$

$\equiv ((p \vee q) \vee p) \wedge ((p \vee q) \vee q) \wedge ((\sim p \vee \sim q) \vee (\sim p \wedge \sim q))$

$\equiv ((p \vee q) \vee p) \wedge ((p \vee q) \vee q) \wedge [((\sim p \vee \sim q) \vee \sim p) \wedge (\sim p \vee \sim q$

$\equiv (p \vee q \vee p) \wedge (p \vee q \vee q) \wedge [(\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q$

$\equiv (p \vee q \vee p) \wedge (p \vee q \vee q) \wedge (\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q$

$\equiv (p \vee q \vee p) \wedge (p \vee q \vee q) \wedge (\sim p \vee \sim q) \wedge (\sim p \vee \sim q)$

$\equiv (p \vee q) \wedge (p \vee q) \wedge (\sim p \vee \sim q) \wedge (\sim p \vee \sim q)$

$\equiv (p \vee q) \wedge (\sim p \vee \sim q)$

Principal Disjunctive Normal Form (PDNF)

For a given formula, an equivalent formula consisting of Disjunctions of minterms only is known as its PDNF or Sum of products Cannonical Form.

Procedure to obtain PDNF

~~S-1. Const~~

(A) By Truth Table

S-1 Construct a Truth Table for the given Compound Proposition.

S-2. For every truth Value 'T' of the given Proposition, select the minterm, which also has the value T for the same combination of the truth Value of the statement variables.

S-3 The Disjunctive of the minterms selected in S-2 is the required PDNF.

Ex.1. obtain PDNF of - $p \to q$

Sol: S-1. The truth table for given statement is -

p	q	$p \to q$
T	T	$\boxed{T}$ *
T	F	F
F	T	$\boxed{T}$ #
F	F	$\boxed{T}$ #

Minterm Truth Table for p and q is -

S-2

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$\sim p \wedge q$	$\sim p \wedge \sim q$
T	T	F	F	$\boxed{T}$ *	F	F	F
T	F	F	T	F	T	F	F
F	T	T	F	F	F	$\boxed{T}$ #	F
F	F	T	T	F	F	F	$\boxed{T}$ #

S-3 $p \to q \equiv (p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

Ex.2 obtain PDNF of - $q \vee (p \vee \sim q)$

Sol:- The truth table for given statement is -

p	q	$\sim q$	$q \vee (p \vee \sim q)$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	T

By Minterm table for p and q we have:

$$q \vee (p \vee \sim q) \equiv (p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$$

Ex. 3 Find PDNF of the statement

$S \equiv (p \wedge \sim q \wedge \sim r) \vee (q \wedge r)$

Sol:- **S-1** The truth table for given statement

p	q	r	$\sim q$	$\sim r$	$p \wedge \sim q \wedge \sim r$	$q \wedge r$	S
T	T	T	F	F	F	T	$\boxed{T}$
T	T	F	F	T	F	F	F
T	F	T	T	F	F	F	F
F	T	T	F	F	F	T	$\boxed{T}$
T	F	F	T	T	T	F	$\boxed{T}$
F	T	F	F	T	F	F	F
F	F	T	T	F	F	F	F
F	F	F	T	T	F	F	F

Minterm table for p, q & r -

p	q	r	$p \wedge q \wedge r$	$p \wedge q \wedge \sim r$	$p \wedge \sim q \wedge r$	$\sim p \wedge q \wedge r$	$p \wedge \sim q \wedge \sim r$	$\sim p \wedge q \wedge \sim r$	$\sim p \wedge \sim q \wedge r$	$\sim p \wedge$
T	T	T	$\boxed{T}$	F	F	F	F	F	F	F
T	T	F	F	T	F	F	F	F	F	F
T	F	T	F	F	T	F	F	F	F	F
F	T	T	F	F	F	$\boxed{T}$	F	F	F	F
T	F	F	F	F	F	F	$\boxed{T}$	F	F	F
F	T	F	F	F	F	F	F	T	F	F
F	F	T	F	F	F	F	F	F	T	F
F	F	F	F	F	F	F	F	F	F	T

$\therefore (p \wedge \sim q \wedge \sim r) \vee (q \wedge r) \equiv (p \wedge q \wedge r) \vee (\sim p \wedge q \wedge r) \vee (p \wedge \sim q \wedge$

(B) without Truth Table

S-1 Obtain a DNF of the given statement

S-2 Drop elementary products which are contradictions. (such as $p \wedge \sim p$)

S-3 If p_i and $\sim p_i$ are missing in an elementary product α, replace α by $(\alpha \wedge p_i) \vee (\alpha \wedge \sim p_i)$

S-4 Repeat S-3 until all elementary products are reduced to sum of minterms. Identical minterms appearing in the disjunction are deleted and written only one time.

Ex. 4 Obtain the PDNF of $\sim p \vee q$

Sol:- $\because \sim p \vee q \equiv (\sim p \wedge (q \vee \sim q)) \vee (q \wedge (p \vee \sim p))$

$[\because q \vee \sim q = T$ & $p \wedge T = T]$

$\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (q \wedge p) \vee (q \wedge \sim p)$

$\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q)$

$[\because \sim p \wedge q$ & $q \wedge \sim p$ are Identical$]$

which is the required PDNF.

Advantages of PDNF

(a) The PDNF of a given formula is unique.

(b) Two formula are equivalent iff their PDNF coincide.

(c) If the given compound proposition is a Tautology, then its PDNF will contain all possible minterms of its components.

Principal Conjunctive Normal Form (PCNF)

For a given statement, an equivalent statement consisting conjunctions of the maxterms only, is known as its PCNF. or product of sums cannonical form.

Procedure to obtain PCNF of a Statement

(A) By Truth Table

S-1 Construct a Truth table of the given compound proposition (or statement)

S-2 For every truth value F of the given proposition, select maxterms, which also has the value F for the same combination of the truth value of the statement variables.

S-3 The conjunction of the maxterms selected in S-2 is the required PCNF.

Ex.1 obtain PCNF of $p \wedge q$

Sol:- S-1 The truth table for $p \wedge q$ is:-

p	q	$p \wedge q$
T	T	T
T	F	$\boxed{F}$
F	T	$\boxed{F}$
F	F	$\boxed{F}$

S-2 The maxterm table for two variables p and q is as-

p	q	$\sim p$	$\sim q$	$p \vee q$	$p \vee \sim q$	$\sim p \vee q$	$\sim p \vee \sim q$
T	T	F	F	T	T	T	F
T	F	F	T	T	T	$\boxed{F}$	T
F	T	T	F	T	$\boxed{F}$	T	T
F	F	T	T	$\boxed{F}$	T	T	T

<u>S-3</u> Required PCNF is-

$$p \wedge q \equiv (p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q)$$

<u>Ex-2</u> Obtain the PCNF of the statement

$$S \equiv (\sim p \rightarrow r) \wedge (q \leftrightarrow p)$$

<u>Sol</u>:- The truth table for the given statement S is-

p	q	r	$\sim p$	$\sim p \rightarrow r$	$q \leftrightarrow p$	$(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	$\boxed{F}$
F	T	T	T	T	F	$\boxed{F}$
T	F	F	F	T	F	$\boxed{F}$
F	T	F	T	F	F	$\boxed{F}$
F	F	T	T	T	T	T
F	F	F	T	F	T	$\boxed{F}$

The truth table for max terms ~~is as~~ for 3 variables p, q & r is as-

T	T	T	F	F	F	T	T	T	T	T	T	T	F
T	T	F	F	F	T	T	T	T	T	T	F	T	T
T	F	T	F	T	F	T	T	T	T	T	T	$\boxed{F}$	T
F	T	T	T	F	F	T	T	T	T	$\boxed{F}$	T	T	T
T	F	F	F	T	T	T	T	T	$\boxed{F}$	T	T	T	T
F	T	F	T	F	T	T	T	$\boxed{F}$	T	T	T	T	T
F	F	T	T	T	F	T	F	T	T	T	T	T	T
F	F	F	T	T	T	$\boxed{F}$	T	T	T	T	T	T	T

S-3 ∴ The required PCNF is-

$$(\sim p \to r)\wedge(q\leftrightarrow p) \equiv (p\vee q\vee r)\wedge(p\vee\sim q\vee r)\wedge(\sim p\vee q\vee r)\wedge(p\vee\sim q\vee\sim r)\wedge(\sim p\vee q\vee\sim r)$$

(B) without Truth Table

The process for obtaining PCNF is similar to the one followed for PDNF.

Ex.1 Obtain PCNF of -

$$S \equiv (\sim p\to r)\wedge(q\leftrightarrow p)$$

Sol:- $(\sim p\to r)\wedge(q\leftrightarrow p) \equiv (p\vee r)\wedge(q\to p)\wedge(p\to q)$

$$\equiv[(p\vee r)\vee(q\wedge\sim q)]\wedge[(\sim q\vee p)\wedge(\sim p\vee q)]$$

$$\equiv[(p\vee r\vee q)\wedge(p\vee r\vee\sim q)]\wedge[(\sim q\vee p)\vee(r\wedge\sim r)]\wedge[(\sim p\vee q)\vee(r\wedge\sim r)]$$

$$\equiv (p\vee r\vee q)\wedge(p\vee r\vee\sim q)\wedge[(\sim q\vee p\vee r)\wedge(\sim q\vee p\vee\sim r)]\wedge[(\sim p\vee q\vee r)\wedge(\sim p\vee q\wedge\sim r)]$$

$$\equiv (p\vee q\vee r)\wedge(p\vee\sim q\vee r)\wedge(p\vee\sim q\vee r)\wedge(p\vee\sim q\vee\sim r)\wedge(\sim p\vee q\vee r)\wedge(\sim p\vee q\wedge\sim r)$$

$$\equiv (p\vee q\vee r)\wedge(p\vee\sim q\vee r)\wedge(p\vee\sim q\vee\sim r)\wedge(\sim p\vee q\vee r)\wedge(\sim p\vee q\wedge\sim r)$$

which is the required PCNF.

Duality Law

Two statements S and S^* are said to be duals of each other if each of them can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$. The connectives $\wedge$ and $\vee$ are called duals of each other.

If the statement S contains T and F then dual of S, S^* is obtained by replacing T by F and F by T.

Ex.1 Obtain Dual of $(p \vee q) \wedge (q \wedge r) \vee T$

Sol. The dual is - $(p \wedge q) \vee (q \vee r) \wedge F$

Obtaining PCNF from PDNF and PDNF from PCNF

If the PDNF (or PCNF) of a given statement S, containing n variables, is known. Then the PDNF (or PCNF) of $\sim S$ will consists of the disjunction (or conjunction) of the remaining minterms (or maxterms), which are not present in the PDNF (or PCNF) of S.

$\because$ $S = \sim(\sim S)$, so we can obtain the PCNF or (PDNF) of S by applying De-morgan's Laws to the PDNF (or PCNF) of $\sim S$.

Ex.1 Find PCNF of a statement S whose PDNF is - $(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$

Sol:- First we obtain the PDNF of $\sim S$, which is the disjunction of those minterms, those are not in PDNF of S.

$\therefore$ PDNF of $(\sim S)$ is $\equiv (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r)$
$\vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$

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$\therefore$ PCNF of S $\equiv \sim$[PDNF of $(\sim S)$]

$\equiv \sim[(p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)]$

$\equiv (\sim p \vee q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee \sim r) \wedge (p \vee q \vee r)$

Ex.2 Find PDNF of statement S, whose PCNF is
$(p \vee q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r)$

Sol:- First of all Let us find PCNF of $\sim S$, which is the conjunction of those maxterms which are not in PCNF of S:

$\therefore$ PCNF of $(\sim S) \equiv (\sim p \vee \sim q \vee \sim r) \wedge (p \vee q \vee \sim r)$
$\vee (\sim p \vee \sim q \vee r)$

$\therefore$ PDNF of S = $\sim$[PCNF of $(\sim S)$]

$\equiv \sim[(\sim p \vee \sim q \vee \sim r) \wedge (p \vee q \vee \sim r) \wedge (\sim p \vee \sim q \vee r)$

$\equiv (p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge r) \vee (p \wedge q \wedge \sim r)$