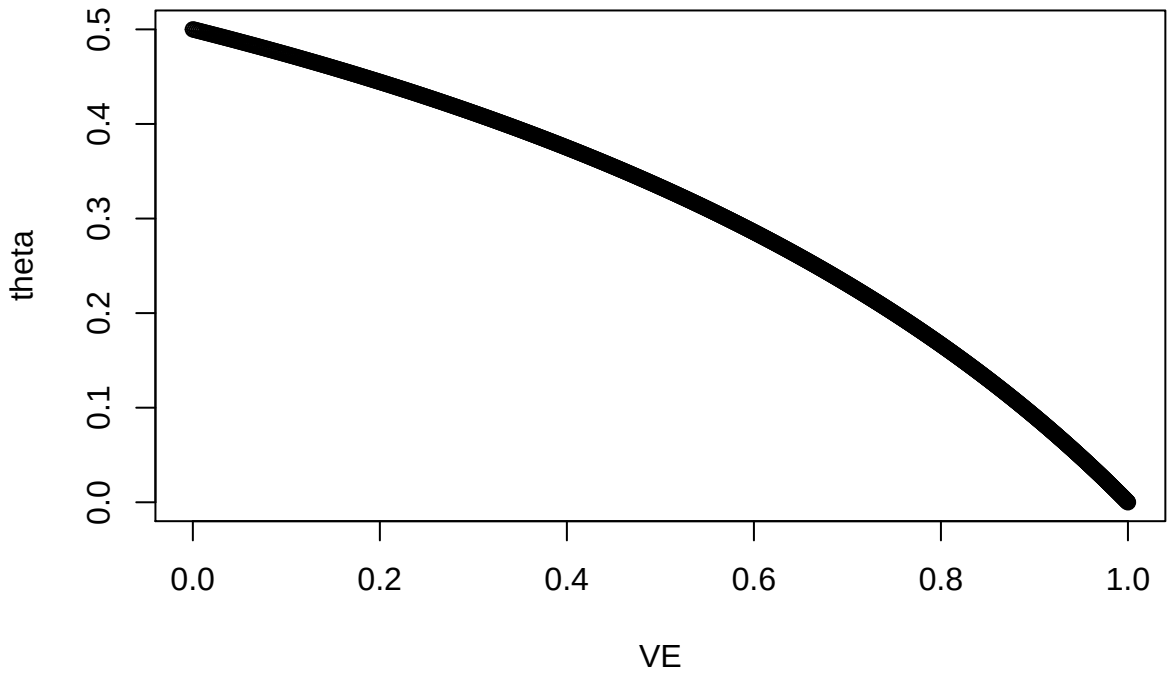
Midterm 1 - Ishita Dutta

```r
# From midterm prep
theta<-seq(0,0.5,len=1000)

VE<-(2*theta-1)/(theta-1)

plot(VE,theta)
```




```r
theta<-(1-0.3)/(2-0.3)



#=================================================================
# VE versus theta
#=================================================================
```

```r
#=====================================================================
# Prior elicitation
#=====================================================================

E.theta<-0.4118



 f=function(a){
    E.theta<-0.4118
    cumulative<-pbeta(0.964,a,((1-E.theta)/E.theta)*a)-pbeta(0.005,a,((1-E.theta)/E.theta)*a)-0.95
    return(cumulative)
             }

 a = uniroot(f,lower=0.01,upper=4)$root

 b = ((1-E.theta)/E.theta)*a



E.theta<-a/(a+b)

E.theta
```

```
## [1] 0.4118
```

```r
a1 <- 1
b1 <- 1

a1
```

```
## [1] 1
```

```r
b1
```

```
## [1] 1
```

```r
w1 = a1 + b1


w1
```

```
## [1] 2
```

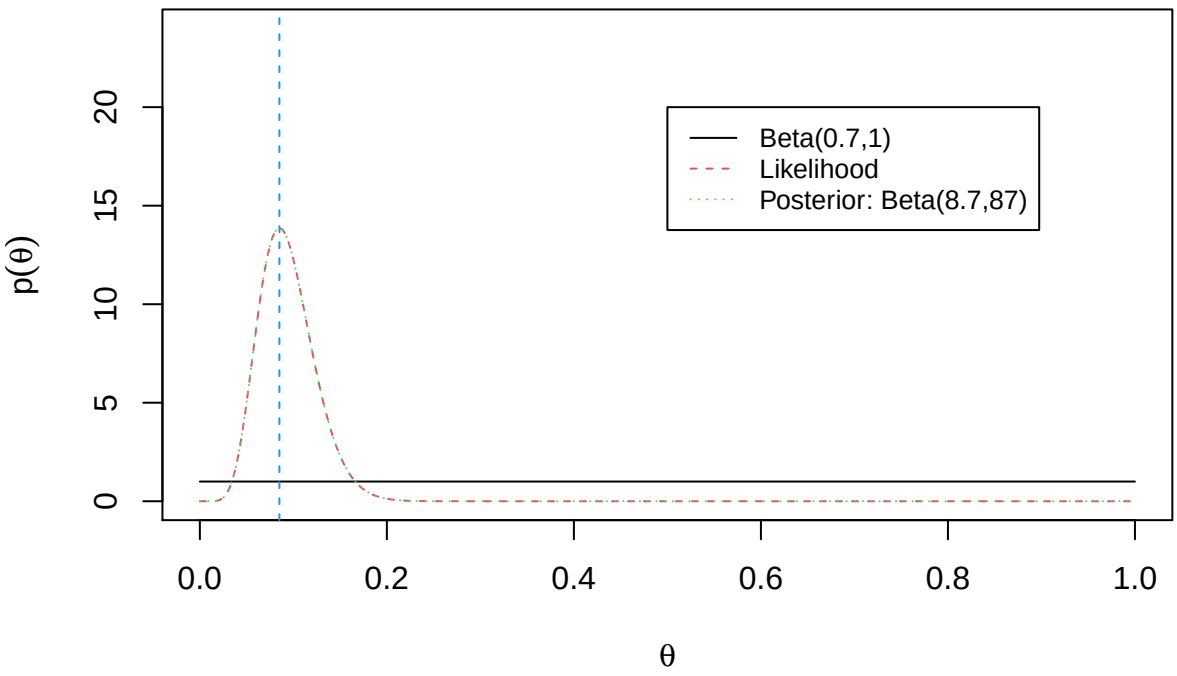
a) Prior likelihood posterior distributions (prior elicitation 1)

```r
# Data
```

```r
y<-8
n<-94



theta<-seq(0,1,len=1000)


# Prior --> Following Uniform Dist
## a and b are both 1 on a beta distribution following the prior elicitation
## combine this with the definition of a uniform dist on a beta formula (?)

E_theta_prior <- 1/(b - a) # following prior elicitaion




plot(theta,dbeta(theta,a1,b1),type="l",xlab=expression(theta),
ylab=expression(p(theta)),main="Beta/Binomial model with Uniform Prior, where E(theta)=0.4118, VE=30%",y

# Likelihood

# MLE


MLE<-y/n

MLE
```

```
## [1] 0.08510638
```

```r
lines(theta,dbeta(theta,y+1,n-y+1),type="l",col=2,lty=2)

# theta^(y)*(1-theta)^(n-y)

a.p1 = a1 + y
b.p1 = b1 + n - y


# Posterior


lines(theta,dbeta(theta,a.p1,b.p1),type="l",col=3, lty=3)


legend(0.5,20,paste(c("Beta(0.7,1)","Likelihood", "Posterior: Beta(8.7,87)")),lty=c(1,2,3),col=c(1,2,3)
abline(v=MLE,col=4,lty=2)
```



b)

```
# Posterior expectation

E_theta_posterior1 <- a.p1/(a.p1 + b.p1)
E_theta_posterior1
```

```
## [1] 0.09375
```

```
w <- a1 + b1

eta<- w/(w+n)

E.theta <- a1/(a1 + b1)

eta*E.theta + (1-eta)*MLE
```

```
## [1] 0.09375
```

```
# Posterior probability theta>0.4118
(1-pbeta(0.4118, a.p1, b.p1))*100 # is smaller than 2.5%.
```

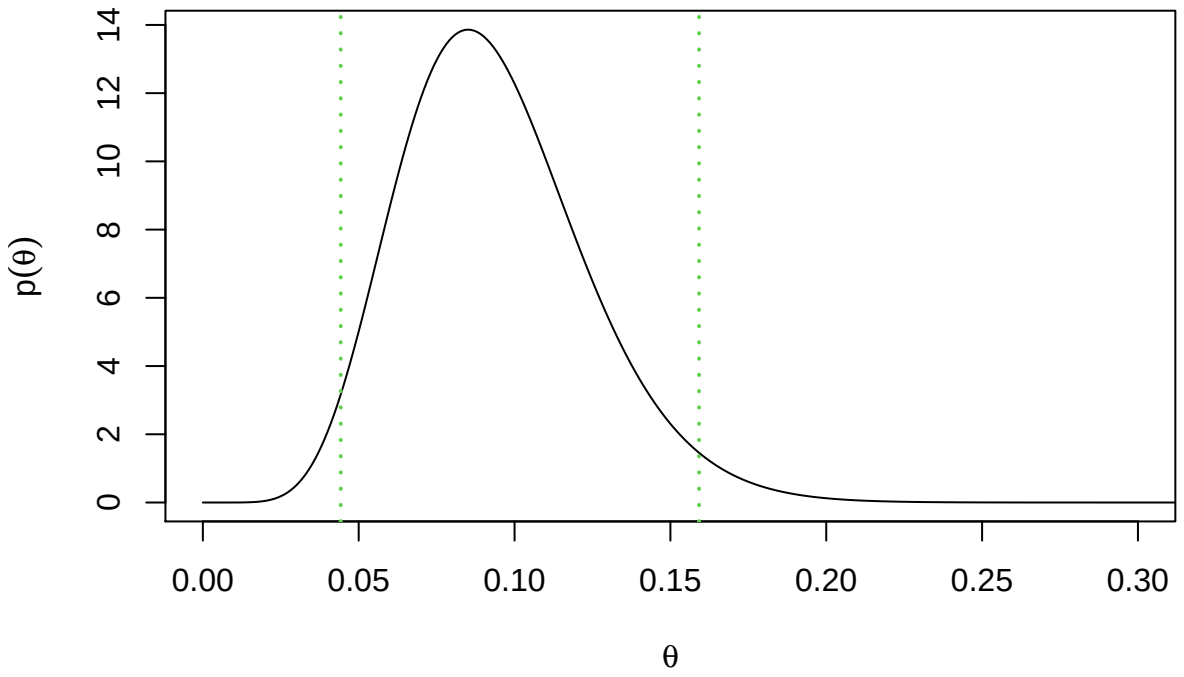
```
## [1] 1.023515e-10
```

c)

```
theta.credible.interval = qbeta(c(0.025,0.975),a.p1, b.p1)
round(theta.credible.interval,1)
```

```
## [1] 0.0 0.2
```

```
plot(theta,dbeta(theta,a.p1,b.p1),type="l",xlab=expression(theta),
ylab=expression(p(theta)),xlim=c(0,0.3))
abline(v=theta.credible.interval[1],col=3,lwd=2, lty=3)
abline(v=theta.credible.interval[2],col=3,lwd=2, lty=3)
```



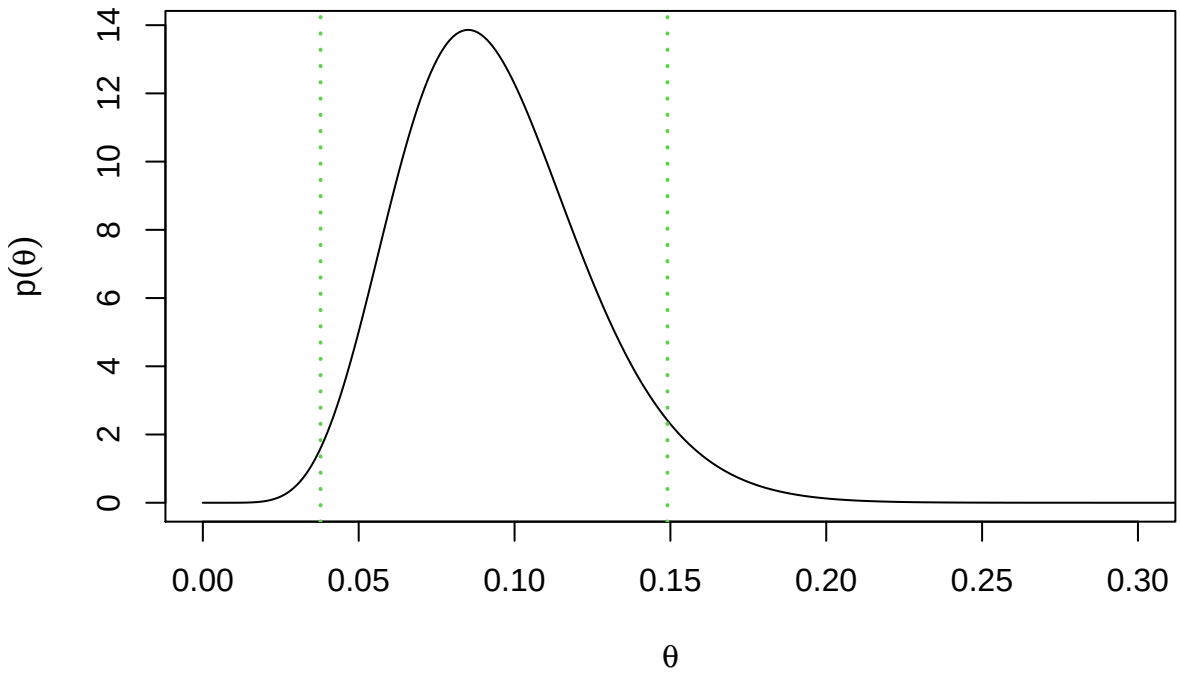

```
VE.credible.interval = rev((1-2*theta.credible.interval)/(1-theta.credible.interval))
round(VE.credible.interval*100,1)
```

```
## [1] 81.1 95.4
```

```
library(TeachingDemos)

hpd(qbeta, shape1=a.p1, shape2= b.p1)
```

```
## [1] 0.03982654 0.15262764
```

```
plot(theta,dbeta(theta,a.p1,b.p1),type="l",xlab=expression(theta),
ylab=expression(p(theta)),xlim=c(0,0.3))
abline(v=0.03776744,col=3,lwd=2, lty=3)
abline(v=0.14905619,col=3,lwd=2, lty=3)
```




```
VE.credible.interval = rev((1-2*hpd(qbeta, shape1=a.p1, shape2= b.p1))/(1-hpd(qbeta, shape1=a.p1, shape1
round(VE.credible.interval*100,1)
```

```
## [1] 82.0 95.9
```

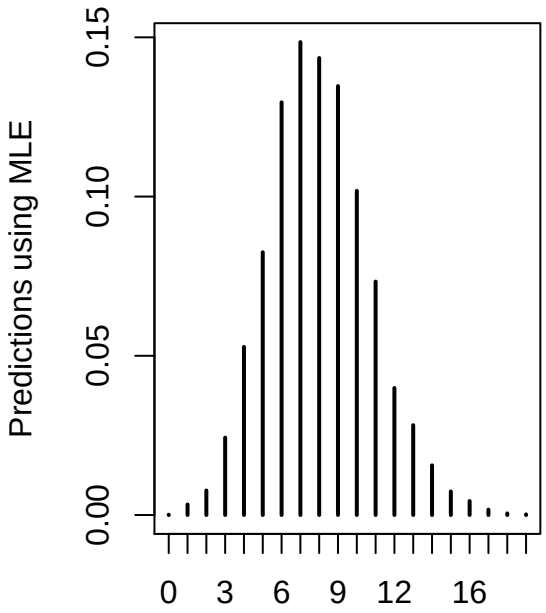
```
c(MLE-1.96*sqrt(MLE*(1-MLE)/n),MLE+1.96*sqrt(MLE*(1-MLE)/n))
```

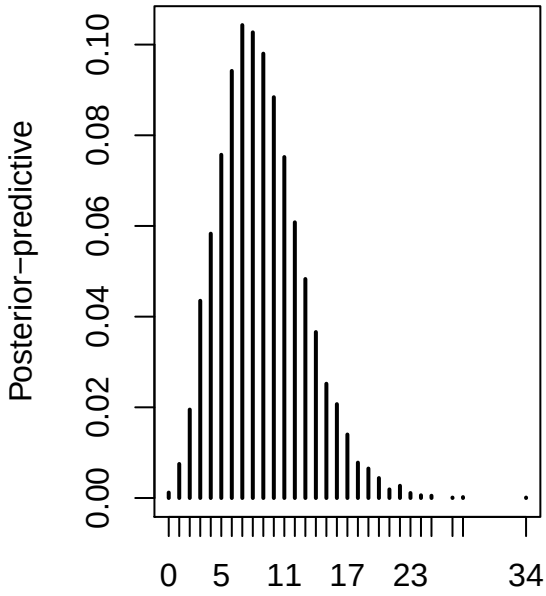
```
## [1] 0.02869607 0.14151670
```

d) Posterior-predictive distribution

```
par(mfrow=c(1,2))

# TailRank package  for Beta-binomial distribution,
#install.packages("extraDistr")
library(extraDistr)
```

```
m <- 94
y <- rbinom(10000, m, MLE)
y_m <- rbbinom(10000, m, a.p1, b.p1)
plot(table(y)/10000, xlab="" , ylab="Predictions using MLE")
plot(table(y_m)/10000, xlab="" , ylab="Posterior-predictive")
```






```
Posterior_mean <- (a.p1)/(a.p1+ b.p1)

Posterior_mean
```

```
## [1] 0.09375
```

```
mean_predictive <- m*(a.p1)/(a.p1+ b.p1)

mean_predictive
```

```
## [1] 8.8125
```