1. (30 pts)

   (a) a)

$$D = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

   (b) b)

$$\mathbf{q} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

   (c) c) With 1-norm, you get:

$$\mathbf{qd}_1 = \begin{bmatrix} 3 & 5 & 2 & 1 & 3 & 3 & 3 & 3 & 3 & 2 & 3 \end{bmatrix}$$

   2-norm basically taking the root for all of these numbers:

$$\mathbf{qd}_2 = \begin{bmatrix} \sqrt{3} & \sqrt{5} & \sqrt{2} & \sqrt{1} & \sqrt{3} & \sqrt{3} & \sqrt{3} & \sqrt{3} & \sqrt{3} & \sqrt{2} & \sqrt{3} \end{bmatrix}$$

   Max-norm:

$$\mathbf{qd}_{\max} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

   Using these numbers I find documents 4, 3, and 10 (in order) most useful for this seach. I would use the 1-norm method as it is most simple as well as properly informative in this case.

2. (15 pts)

   (a)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1\mathrm{n}} \\ a_{21} & a_{22} & \cdots & a_{2\mathrm{n}} \\ \vdots & \ddots & \vdots & \\ a_{\mathrm{m}1} & a_{\mathrm{m}2} & \cdots & a_{\mathrm{mn}} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1\mathrm{m}} \\ b_{21} & b_{22} & \cdots & b_{2\mathrm{m}} \\ \vdots & \ddots & \vdots & \\ b_{\mathrm{n}1} & b_{\mathrm{n}2} & \cdots & b_{\mathrm{nm}} \end{bmatrix}$$

$$AB = \begin{bmatrix} \mathbf{a}_{1.} * \mathbf{b}_{.1} & \mathbf{a}_{1.} * \mathbf{b}_{.2} & \cdots & \mathbf{a}_{1.} * \mathbf{b}_{.\mathrm{m}} \\ \mathbf{a}_{2.} * \mathbf{b}_{.1} & \mathbf{a}_{2.} * \mathbf{b}_{.2} & \cdots & \mathbf{a}_{2.} * \mathbf{b}_{.\mathrm{m}} \\ \vdots & \ddots & \vdots & \\ \mathbf{a}_{\mathrm{m}.} * \mathbf{b}_{.1} & \mathbf{a}_{\mathrm{m}.} * \mathbf{b}_{.2} & \cdots & \mathbf{a}_{\mathrm{m}.} * \mathbf{b}_{.\mathrm{m}} \end{bmatrix}$$

$$BA = \begin{bmatrix} \mathbf{b}_{1.} * \mathbf{a}_{.1} & \mathbf{b}_{1.} * \mathbf{a}_{.2} & \cdots & \mathbf{b}_{1.} * \mathbf{a}_{.\mathrm{n}} \\ \mathbf{b}_{2.} * \mathbf{a}_{.1} & \mathbf{b}_{2.} * \mathbf{a}_{.2} & \cdots & \mathbf{b}_{2.} * \mathbf{a}_{.\mathrm{n}} \\ \vdots & \ddots & \vdots & \\ \mathbf{b}_{\mathrm{n}.} * \mathbf{a}_{.1} & \mathbf{b}_{\mathrm{n}.} * \mathbf{a}_{.2} & \cdots & \mathbf{b}_{\mathrm{n}.} * \mathbf{a}_{.\mathrm{n}} \end{bmatrix}$$

$$tr(AB) = \mathbf{a}_{1.} * \mathbf{b}_{.1} + \mathbf{a}_{2.} * \mathbf{b}_{.2} + ... + \mathbf{a}_{\mathrm{m}.} * \mathbf{b}_{.\mathrm{m}}$$

$$tr(BA) = \mathbf{b}_{1.} * \mathbf{a}_{.1} + \mathbf{b}_{2.} * \mathbf{a}_{.2} + ... + \mathbf{b}_{\mathrm{n}.} * \mathbf{a}_{.\mathrm{n}}$$

In the end, we would be multiplying the same numbers in a similar order then summing them up, which makes it so that tr(AB) = tr(BA).

(b) b) tr(AB) = 1 + 4 + 9 + 16 + 25 = 55

3. (50 pts)

4. (50 pts)

(a) a)

$$\begin{aligned} T(\mathbf{v}) &= A(\mathbf{v}) \\ &= \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 4 & 8 & 4 \\ 4 & 6 & 3 \end{bmatrix} * \begin{bmatrix} v1 \\ v2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 \\ 0 & -2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} * \begin{bmatrix} v1 \\ v2 \end{bmatrix} \\ &= \begin{bmatrix} v1 & 2v2 \\ 0 & -2v2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

(b) b)

$$C(A) = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

(c) c) The nullspace is the inverse image of the zero vector of A; the set of vectors such that Tv = 0 for all v in null(T).

(d) d)

$$C(A) = \begin{bmatrix} 0 \\ -1/2 \\ 1 \end{bmatrix}$$

(e) e) The row space of a matrix is the collection of all of its linear combinations of the rows. Essentially separate the matrix on the rows, and each row is its separate vector to get the row space of a matrix. Replace row with column for the column space.

(f) f)

$$C(A^T) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/2 \end{bmatrix}$$

(g) g) The left nullspace is the inverse image of the zero vector of A(T), the set of vectors such that A(T)v = 0.

(h) h)

$$N(A^T) = \begin{bmatrix} 1 & 0 & -1/4 & 0 \\ 0 & 1 & 1/4 & -1 \end{bmatrix}$$

(i) i)

$$rank(A) = 2$$

5. (25 pts)

(a) a) A linearly independent set must have no free rows when reduced to row echelon form. This means that after reaching row echelon form, there should not be any rows where the row is:

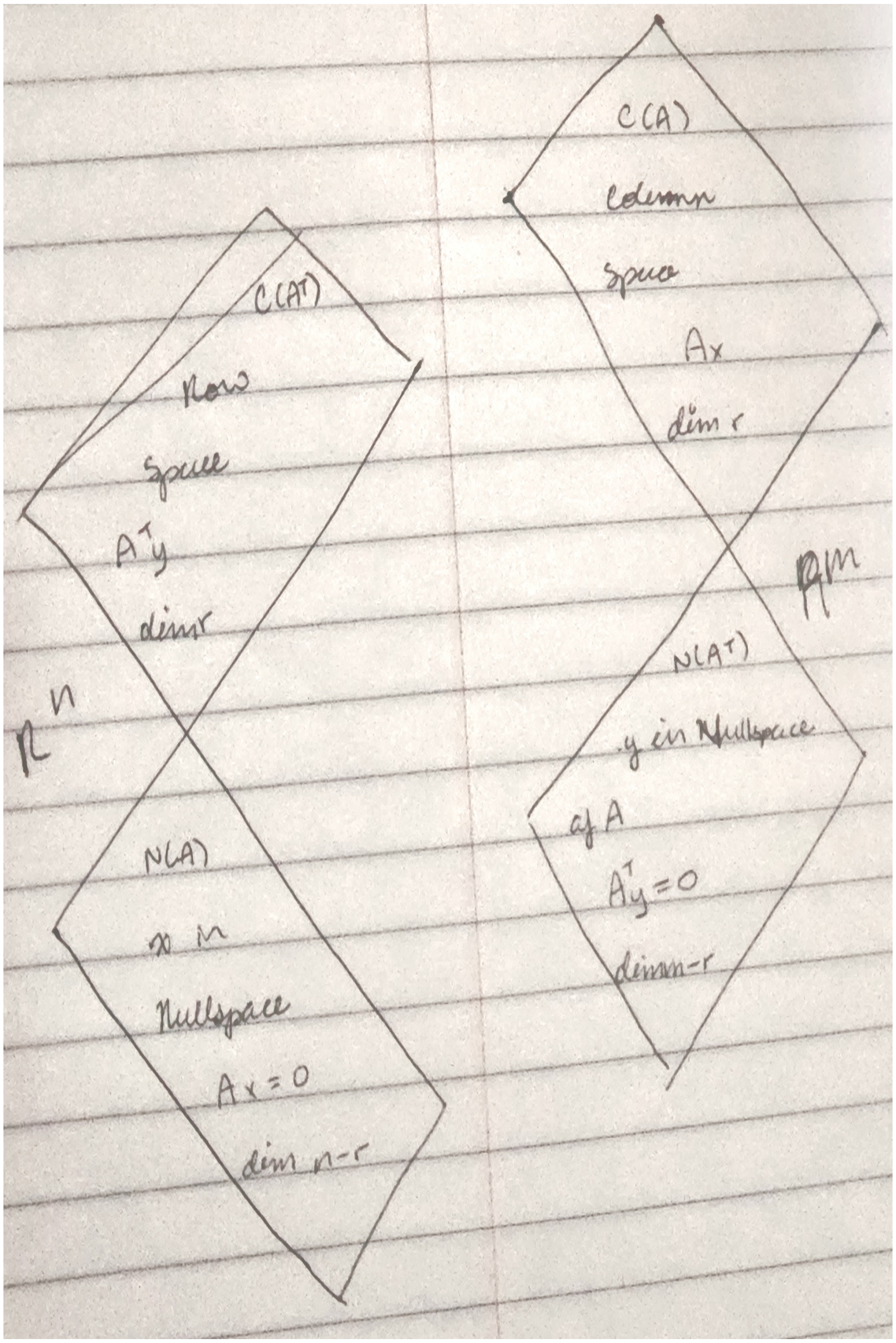
$$\mathbf{r}_x = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}$$

(b) Based on the definition above:

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

Because there are no rows where the entire row is all 0s, we can conclude this set is linearly independent.

6. (70 pts)


C(A)
column
space
Ax
dim r
C(A^T)
Row
Space
A^T y
dim r
R^n
R^m
N(A^T)
y in Nullspace
of A
A^T y = 0
dim m-r
N(A)
x in
Nullspace
Ax = 0
dim n-r