Homework R Markdown Skeleton

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Problem 1

137-HW-3_files/figure-latex/unnamed-chunk-1-1.pdf 137-HW-3_files/figure-latex/unnamed-chunk-1-2.pdf

137-HW-3_files/figure-latex/unnamed-chunk-2-1.pdf 137-HW-3_files/figure-latex/unnamed-chunk-2-2.pdf

Problem 2

2(a)

For MA(1) model:

$$X_t = Z_t + \theta Z_{t-1}, \quad where \;\; Z_t \overset{i.i.d}{\sim} N(0, \sigma^2).$$

For autocovariance:

2)=11.48\ \ $\gamma(1)\& = COV(X\_t, X\_$

\begin{aligned} $\gamma(0)\& = Var(X\_t) = Var(Z\_t{+}\theta Z\_\{t{-}1\}) = Var(Z\_t){+}\theta^{2\text{Var(Z\_\{t-1\})=}\sigma 2(1+\theta}$

$\text{Var}(\overline{X}) = \frac{\tau_n^2}{n}, \quad where$

$\tau_n^2 = \gamma(0) + 2\sum_{h=1}^{n-1}(1 - \frac{h}{n})\gamma(h) = \gamma(0) + 2(1 - \frac{1}{n}\gamma(1)) = (1+\theta^2)\sigma^2 + 2 \cdot \frac{n-1}{n}\theta\sigma^2 = 0.3978947$

$Var(\overline{X}) = \frac{\tau_n^2}{n} = 0.004188366$

2(c)

When $\theta = 0.8$, we get $Var(\overline{X})$=0.2374958.

2(d)

95% confidence interval for $\mu = E(X_t)$ is:

$$(\overline{X} - 1.96\frac{\tau_n}{\sqrt{n}}, \overline{X} + 1.96\frac{\tau_n}{\sqrt{n}})$$

That is, $(37.11824, 41.28176)$.

Problem 3

3(a)

In the case of AR(1) model, i.e.

$$X_t - \mu = \phi(X_{t-1} - \mu) + \epsilon_t$$

For $\gamma(0)$ and $\rho(h)$:

\begin{aligned} &$\gamma(0) = Var(X$_$t) = (1-\phi^{2)}\{-1\}\sigma^{2=19.44444}$\ \ &$\rho(h) = \phi h$\\&$\tau$_$n^{2\approx \gamma(0)[1+2\sum_\{h=1\}\{\infty\}\rho(h)] = \sigma 2/(1-\phi)2 = 2.1604}$

When $\phi = 0.8$, we get $Var(\overline{X}) = 1.842105$.

137-HW-3_files/figure-latex/unnamed-chunk-3-1.pdf 137-HW-3_files/figure-latex/unnamed-chunk-3-2.pdf 137-HW-3_files/figure-latex/unnamed-chunk-3-3.pdf

##4

```
##    AR1.aic  AR2.aic  AR3.aic  AR4.aic  AR5.aic  AR6.aic
## 1 234.5559 235.2847 237.2512 239.1987 240.9296 242.8016
```

```
##
## Call:
## arima(x = data, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.7113     5.6856
## s.e.  0.0807     0.4407
##
## sigma^2 estimated as 1.273:  log likelihood = -114.28,  aic = 234.56
```

```
##       ar1 intercept
## 0.7112991 5.6855723
```

137-HW-3_files/figure-latex/unnamed-chunk-3-4.pdf 137-HW-3_files/figure-latex/unnamed-chunk-3-5.pdf

```
##
##  Box-Ljung test
##
## data:  AR1$residuals
## X-squared = 2.4072, df = 10, p-value = 0.9922
```

```
##  [1] 3.750000 6.050000 5.208333 3.283333 3.025000 2.925000 5.591667 4.366667
##  [9] 4.125000 4.300000 6.841667 5.450000 5.541667 6.691667 5.566667 5.641667
## [17] 5.158333 4.508333 3.791667 3.841667 3.558333 3.491667 4.983333 5.950000
## [25] 5.600000 4.858333 5.641667 8.475000 7.700000 7.050000 6.066667 5.850000
## [33] 7.175000 7.616667 9.708333 9.600000 7.508333 7.191667 7.000000 6.175000
## [41] 5.491667 5.258333 5.616667 6.850000 7.491667 6.908333 6.100000 5.591667
## [49] 5.408333 4.941667 4.500000 4.216667 3.966667 4.741667 5.783333 5.991667
## [57] 5.541667 5.083333 4.608333 4.616667 5.800000 9.283333 9.608333 8.933333
## [65] 8.075000 7.358333 6.158333 5.275000 4.875000 4.358333 3.891667 3.675000
```

```
##   ARn1.aic ARn2.aic ARn3.aic ARn4.aic ARn5.aic ARn6.aic
## 1 213.3339 210.2611 211.9065 213.8531 215.4704 217.1695
```

```
##        ar1        ar2  intercept
##  0.9934626 -0.2690977  5.6128301
```

```
##
## Please cite as:
```

```
##  Hlavac, Marek (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables.
```

```
##  R package version 5.2.2. https://CRAN.R-project.org/package=stargazer
```

```
##
## ==============================================
##                      Dependent variable:
##                  ----------------------------
##                        data         newdata
##                        (1)            (2)
## ----------------------------------------------
## ar1                  0.711***      0.993***
##                      (0.081)        (0.116)
##
## ar2                                -0.269**
##                                     (0.117)
##
## intercept            5.686***      5.613***
##                      (0.441)        (0.412)
##
```

```
## -----------------------------------------------
## Observations            74            72
## Log Likelihood       -114.278      -101.131
## sigma2                1.273          0.957
## Akaike Inf. Crit.    234.556        210.261
## ===============================================
## Note:               *p<0.1; **p<0.05; ***p<0.01
```

```
## [1] 4.185842
```

```
## [1] 4.75126
```

```
## $pred
## Time Series:
## Start = 73
## End = 73
## Frequency = 1
## [1] 4.15083
##
## $se
## Time Series:
## Start = 73
## End = 73
## Frequency = 1
## [1] 0.9782596
```

```
## $pred
## Time Series:
## Start = 73
## End = 74
## Frequency = 1
## [1] 4.150830 4.681853
##
## $se
## Time Series:
## Start = 73
## End = 74
## Frequency = 1
## [1] 0.9782596 1.3789533
```

Code Appendix

```r
knitr::opts_chunk$set(echo = FALSE,warning=FALSE)
library(tidyverse)
library(ggplot2)
#Problem 1(a)
set.seed(123)
simMA1.1a<-arima.sim(n=275,model=list(c(ma=0.7)),sd=1)

plot.ts(simMA1.1a)
acf(simMA1.1a,lag.max = 10)
```

```r
#Problem 1(b)
set.seed(123)
simMA2.1b<-arima.sim(n=275,model=list(ma=c(1.1,0.7)),sd=1)
plot.ts(simMA2.1b)
acf(simMA2.1b,lag=10)
library(readxl)
Unemp1948.2021 <- read_excel("Unemp1948-2021.xls",skip = 10)
tm <- 1:74
plot1 = plot.ts(Unemp1948.2021$UNRATE)
acf(Unemp1948.2021$UNRATE,main = "acf plot")
pacf(Unemp1948.2021$UNRATE,main = " PACF")
data = Unemp1948.2021$UNRATE
AR1 = arima(data,order = c(1,0,0))
AR2 = arima(data,order = c(2,0,0))
AR3 = arima(data,order = c(3,0,0))
AR4 = arima(data,order = c(4,0,0))
AR5 = arima(data,order = c(5,0,0))
AR6 = arima(data,order = c(6,0,0))
aic_table = data.frame(AR1$aic,AR2$aic,AR3$aic,AR4$aic,AR5$aic,AR6$aic)
aic_table
# choose AR 1 because smallest AIC value
AR1
AR1$coef
residual_plot = plot(AR1$residuals) # residuals
acf(AR1$residuals, main = " acf plot of residuals")
test <- Box.test(AR1$residuals, lag = 10, type = "Ljung-Box")
test
# We do not rehect the null hypothesis that residuals are i.i.d. or all the aurocorrelations are zero.
newdata = Unemp1948.2021$UNRATE[1:72]
newdata
ARn1  = arima(newdata,order = c(1,0,0))
ARn2  = arima(newdata,order = c(2,0,0))
ARn3  = arima(newdata,order = c(3,0,0))
ARn4  = arima(newdata,order = c(4,0,0))
ARn5  = arima(newdata,order = c(5,0,0))
ARn6  = arima(newdata,order = c(6,0,0))
aic_table2 = data.frame(ARn1$aic,ARn2$aic,ARn3$aic,ARn4$aic,ARn5$aic,ARn6$aic)
aic_table2
# we choose AR2 now because smallest aic value
ARn2$coef
u = mean(newdata)
library(stargazer)
stargazer(AR1,ARn2,type = "text")
forecast_2020 = 0.9927 * (newdata[72]-u) - 0.2691 * (newdata[71] - u) + u
forecast_2020
forecast_2021 = 0.9927 * (forecast_2020-u) -0.2691 * (newdata[72] - u) + u
forecast_2021
predict(ARn2,n.ahead = 1)
predict(ARn2,n.ahead = 2)
```