Shape and Balance in Police Districting

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Chapter 14
Shape and Balance in Police Districting

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14.1 Introduction

Districting is a classic design problem when attempting to provide an efficient service to a geographically dispersed demand. There exists a natural trade-off between aggregation, which allows the pooling of resources, and individualization, where each individual demand has its own resources and response is as efficient as possible. Police and security providers are no strangers to this phenomenon. Having a single set of resources to satisfy the demand over an entire service area avoids the duplication of resources, coordination problems, and uneven workloads that can occur in districting. However, when the service area is large, service times at certain locations can exceed acceptable levels, making it more attractive to service this demand from distributed resources. Furthermore, districting allows for specialization of the resources to efficiently service a diverse demand in different areas. Such specialization causes additional complexity if managed from a centralized pool of resources. This creates the basic problem of separating a demand area into subregions to organize the service process.

The problem of districting in general, and police districting in particular, consists of dividing a geographical region in subregions (also referred to as districts or quadrants) in a way that improves some objective measure. Perhaps the most well known version of this problem is the political districting problem, where due to population changes political districts are frequently reshaped, sparking intense debate (Hess et al. 1965; Fleischmann and Paraschis 1988; Hojati 1996). There are however a number of other applications where districting is regularly used, including territory

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design for sales and services, health care districting, police and emergency service districting, and districting in logistics operations (Kalcsics et al. 2005).

Important considerations when designing districts include balancing the demand for resources and creating districts that have geographical contiguity and compactness (Wright et al. 1983). Existing literature has addressed the need for demand balance constraints in districting models. In particular, not considering demand balance in the police districting problem has been shown to create difficulties such as lack of indicators to compare the performance of the different districts by police management, staff imbalance, and morale problems, among others Kistler (2009). Compactness of a district is important because response time, a key measure of quality of service, is related to the distance that has to be traversed. This makes it desirable that all points in a district be close together, as measured by travel time, to be serviced within a small response time. Therefore, travel time can be used to measure the compactness of a given district. Finally, contiguity is desirable because in addition to the need for fast response times simple districts are easier to administrate.

A natural mathematical model of the districting problem can be built on a graph representation of the geographical area and, as we describe in the literature review section, previous work has introduced such models before. However, although the requirements of balanced, contiguous and compact districts have been dealt with before, these are rarely combined in a single model. In this work we introduce such a mathematical model for a specific police districting problem of the Chilean national police force in urban areas, including the requirements of balanced, contiguous, and compact districts.

To build this mathematical model we consider a graph where nodes represent a city block (or groups of city blocks) and arcs connect adjacent nodes (blocks), or nodes that can be accessed thought the road network, see for example Fig. 14.1. In addition, this network should consider information regarding the demand of police resources at every node and information regarding distances (or travel times) on arcs between nodes. On this network structure, the districting problem becomes a graph partitioning problem. The demand balance constraints can be represented as capacity constraints on the total demand on each district, where the lower and upper bounds are calculated as a percentage deviation respect to the average demand of resources or workload. The information on distances between nodes can be used to characterize compactness and contiguity. Hence, the proposed graph partitioning model at the heart of a districting problem can take into account the features of balance, contiguity and compactness of each district.

Regrettably, introducing the requirements of balance, contiguity and compactness makes the districting problem more difficult to solve. For instance, consider the example in Fig. 14.2. This example shows two districting solutions, one for a p-median problem without balance, contiguity or compactness constraints and one for a p-median problem with balance constraints. For this problem which defines 9 districts out of 407 nodes, the solution to the p-median problem on the left is obtained in a few minutes, while the problem with balance constraints can take hours to solve exactly.

We note that the p-median districting problem results in districts that seem reasonably compact and contiguous. The reason for this is because the p-median problem forms clusters around centers so that the total distance of nodes to these

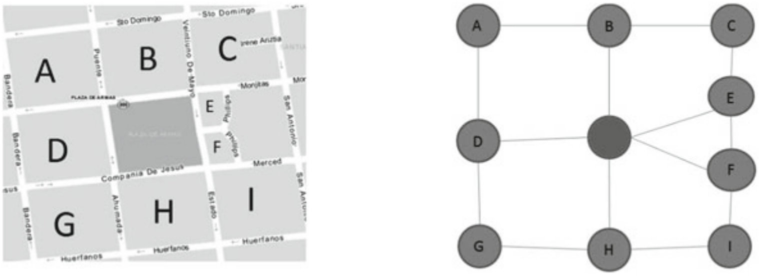

Fig. 14.1 Adjacency graph

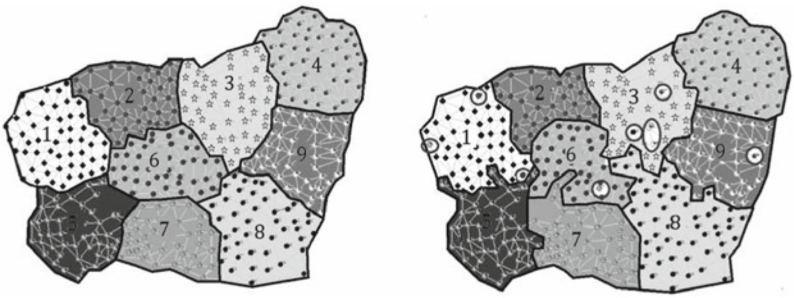

Fig. 14.2 Comparison between the results of a p-median problem (*left*) and a p-median with balance constraint of 1 % of deviation with respect to the average workload (*right*)

centers is minimized, see Daskin (2013).This total distance objective is minimized when clusters are compact and contiguous around selected centers. If the p-median problem has balance constraints, however, then it creates districts that are not contiguous. For example districts 4, 8 and 9 include nodes that are in the interior of district 3. It also creates districts with branches that protrude into neighboring districts, see for instance the branches out of districts 5, 7 and 8. Districts that are not contiguous or have long boundaries tend to be less compact according to existing definitions (Taylor 1973; Horn 1995).

It therefore becomes important to explicitly include contiguity and compactness constraints in optimization models where balance requirements are enforced, to obtain acceptable districts. This is particularly relevant in the police districting problem considered, because the current districts are built with a detailed manual procedure that immediately discards solutions that significantly violate contiguity and compactness requirements.

The main objective of this chapter is to present a mathematical model of a realistic police districting problem. We are inspired by the urban districting problem faced by Chile's national police force (Carabineros de Chile) in their Preventive Security Quadrant Plan (*PCSP* for its acronym in Spanish). This districting problem has been gradually implemented in Chile since 2001. By the year 2013 it had been implemented in the 120 most populous municipalities. In this work we propose a mathematical optimization formulation of the districting problem that Chile's national police force solves as part of the *PCSP*. This optimization problem also takes into consideration balance, contiguity and compactness constraints.

In the next section we present a brief literature review on previous related work on districting problems. In Sect. 14.3 we describe the general characteristics of Chile's national police force *PCSP* program and in Sect. 14.4 we present a mathematical formulation of this particular districting problem. Section 14.5 presents our computational results evaluating the tradeoffs between the different model objectives and shows how the districting solution of the proposed model can improve current practice for an example corresponding to a real municipality in Santiago.

14.2 Literature Review

The police districting problem can be considered as a particular case of the territory design problem, which consists in grouping small geographic areas (such as city blocks) into larger geographic clusters or districts (in our case called quadrants) in such a way the clusters are acceptable according to relevant planning criteria (Kalcsics et al. 2005). Some important applications of the territory design problem are political districting (Hess et al. 1965; Fleischmann and Paraschis 1988; Hojati 1996), territory design for sales and services (Hess and Samuels 1971), and the assignment of students to public schools (Ferland and Guénette 1990; Caro et al. 2004), among others. This previous literature on territory design usually does consider at least one of the following three concepts: balance of demand, geographical contiguity, and compactness. We separate this literature review in describing how these three concepts have been addressed by previous work on territory design and in previous work on police districting. We begin by pointing out that the concepts of balance, contiguity, and compactness are present in political districting problems and touched on by the practice of "gerrymandering", which involves redesigning districts to make electoral gains (Erikson 1972; Vickrey 1961).

The concept of balance in territory planning refers to building quadrants where geographic measures or characteristics are similar across the quadrants formed. These measures or characteristics could be population (important, for instance, in political and school districting), workload or demand (important in services such as police districting), number of buildings or linear kilometers of roads, to mention some possibilities. Related to police districting, the work in Kistler (2009) considers a combined measure that takes into account the amount of historic calls, the average response time, the linear kilometers of roads, the total district area, and the population.

Geographical contiguity means that every quadrant is geographically connected. Optimal solutions to standard p-median problems with a metric or distance function between locations have contiguous quadrants. Indeed, since the objective of a p-median problem minimizes the sum of distances to each center, each node is assigned to the center that it is closest to. Therefore, the triangular inequality implies that the line between the point and the center it is assigned to must also belong to the same quadrant, making it contiguous (Daskin 2013). This is not necessarily true for every objective and in particular, optimal solutions to the *p*-center problem, where the objective is to minimize the maximal distance to the corresponding center, need not be contiguous. This because nodes whose distance to more than one center is smaller than the maximal distance could be assigned to either without changing the objective. Partitions of an area formed by having each node/block assigned to the center it is closest to, for some distance metric, are known as Voronoi tessellations. As discussed above, such partitions form naturally contiguous quadrants. However, even without using a distance metric, the geographical contiguity can be enforced by explicit linear constraints. These constraints ensure that for every node assigned to a district, an entire path of nodes to that district's center are assigned to the same district. This is done for example in Williams (2002) for a land acquisition problem and in Drexl and Haase (1999) for a sales force deployment.

There are a number of different notions of compactness that are used in territory planning, which do not refer to the standard topological definition. Some of the compactness measures proposed include the ratio of the perimeter to the area, comparisons to a related compact shape such as a rectangle or circle, or moment of inertia. See Li et al. (2013) for a recent review of different compactness measures. To illustrate the variety in measures of compactness we review a few definitions that arise in some political districting examples.

For instance, Niemi et al. (1990) investigate how different measures of compactness are related to racial and partisan political discrimination in a political districting problem. These different compactness measures are treated as a multidimensional property of a district capturing: the *dispersion* and the *perimeter* of the district, and the geographic dispersion of the *population*. The *dispersion* of a district was quantified by one of four measures: the value $|L_i - W_i|$ or L_i/W_i, where L_i and W_i are the maximum length and width of district i, respectively, a comparison between the area of the district and the area of a compact shape circumscribing the district (such as square, circle or hexagon) and a measure of the distances within a district, given by the sum of the distances of every block to the center of gravity of its district. Two *perimeter* measures of a district are considered: the length of the boundary of the district (Horn 1995), and the difference of the perimeter of the district with a circumference with equal area, or conversely, the difference between the area of a district with the area of a circumference of equal perimeter. Finally, compactness measures that considered the *population* are: the moment of inertia of the population (Papayanopoulos 1973), and the ratio between the population of the district and the population existing in the maximum convex figure or circle circumscribed in the district.

A different compactness measure of a district, proposed in Theobald (1970), is the absolute deviation of the district area with respect to the average area. Young (1988) proposes eight measures of compactness of legislative districts. Many of these measures are similar to the ones listed above, except for two: a visual test, in which a human evaluator gauges the compactness of a district, and the Taylor's test

(Taylor 1973). The Taylor's test quantifies the indentations of the districts by the difference between reflexive angles and non-reflexive angles on the border divided by the total number of angles. For example a convex figure has ratio 1 (compact), and a five pointed star has ratio 0 (non compact).

Summarizing, there are many definitions of compactness measures which can be used, depending on the application considered. To the best of our knowledge, to date there is no previous work that builds compactness measures for territory planning from axioms that the measure should satisfy, even for a specific application. Identifying desirable first principles for compactness measures is an interesting area of future research that could help simplify these definitions. Compactness measures in use focus on different characteristics of districts that should be taken into account. Which characteristic is most important depends on the application being considered. For example, in political districting measures of how population is distributed in the district are important, while in territory planning for emergency service applications a compactness measure that bounds the longest distance, such as a deviations from a circle of the same perimeter, can be a desirable objective. Another consideration is the difficulty in computing the compactness measure from geographical information. Recent work has shown that a moment of inertia measure of compactness can be more efficiently computed than isoperimetric ratios (Li et al. 2013).

A different application where the shape and characteristic of an area is important is the problem of designing habitat reserves, see Marianov et al. (2008) or Church (2015) in this volume. Habitats that are suitable to hosting a given species of animals must satisfy certain shape constraints that depend on the species considered. These shape constraints can impose restrictions on compactness, size, and connectivity. Similar issues arise in the problem of harvest scheduling for environmental reasons in the forestry industry (Barahona et al. 1992).

Previous work on police districting has already considered the issues of demand balance and districts shape constraints. For instance, in Mitchell (1972) an optimization model is proposed that aims to minimize the moment of inertia of the expected workload of each area subject to balance constraints. An applied police districting model for the Buffalo Police Department is presented in Sarac et al. (1999). The paper discusses a set partitioning problem formulation and a practical approach based on census tacks. The optimization approach was abandoned because the multiple objectives (with regards to demand balance, contiguity and compactness of the districts) made the problem computationally challenging for problems of real size. This difficulty is surmounted by limiting the flexibility of the districts to existing census tracks. In D'Amico et al. (2002) the problem of police districting is considered as a graph partitioning problem subject to compactness, contiguity, convexity, and size constraints. The problem also considered a constraint which forced an upper bound on the average response time in each quadrant. This consideration leads to a non-linear approach, which is solved by local search techniques such as simulated annealing. The Constraint-Based Polygonal Spatial Clustering (*CPSC*) method is another heuristic solution method used in police districting (Joshi et al. 2009). The *CPSC* consists of designing quadrants by adding blocks to a seed block until an objective, such as a compactness score, is met. This districting method has been used for the Charlottesville Police Department and evaluated with an agent-based model (Zhang and Brown 2013).

Another optimization model is used in Curtin et al. (2010) to determine optimal police patrol areas. This work considers a maximum covering model (Church and ReVelle 1974) to formulate a first phase problem. Then, a second phase problem determines how to distribute resources to meet a set of options with backup coverage, i.e. maximizing the blocks that are covered two or more times without considering any geographic issue or balance. In Verma et al. (2010) Voronoi Tessellations are used to design police districts testing the incremental changes in the improvement of measures such as balance of workload, 911 response time, geographical area fit, and citizen satisfactions among others.

The police districting problem considered in this chapter is represented as a modified p-median problem with multiple objectives and balance constraints. One of the objectives is related to the police's definition of preventive policing, which is non-linear in the quadrants formed. The combination of balance constraints and this non-linear objective cause the p-median problem to provide solutions that are not contiguous or compact. We therefore include a compactness measure that aims to minimize the quadrant boundaries (or nodes adjacent to a different quadrant) similar to minimizing the length of the boundary in (Horn 1995). As noted in Sarac et al. (1999), this multiple objective problem is challenging to solve for real size instances, we therefore adapt a location-allocation heuristic (Teitz and Bart 1968) to solve this problem.

14.3 The Chilean Police Quadrant Plan

The Chilean national police force, Carabineros de Chile, is the principal internal police force in Chile. Its mission includes maintaining and re-establishing order and security throughout the country and patrolling the borders. Starting in 2001 Carabineros de Chile developed its own police districting methodology for urban areas, known as the Preventive Security Quadrant Plan (*PCSP*). The objective of this plan is to define smaller districts within police precincts in order to (1) quantify the need for policing resources in each district, (2) establish a closer connection with the population by providing a point of contact and information for each district to residents, (3) organize and facilitate deployment of police resources to the community.

The process of implementing the *PCSP* in a municipality begins with the division of the precincts being analyzed into quadrants. This division is currently performed by the expert judgment of the police officers conducting the analysis. For each of the quadrants identified, different requirements for police resources are tallied and converted to a uniform policing unit, referred to as an Equivalent Unit of Vigilance (or *UVE* in Spanish). This exercise helps organizing the expected policing activities that have to be conducted in each quadrant, and dimensioning the need for policing resources in the area under consideration. The *PCSP* also considers a number of novel policing practices aimed at establishing a closer connection between the police and area residents, reduce response time, as well as institutionalizing improvement practices.

Our work seeks to automate the design process of the *PCSP* in a given area. In particular we aim to develop mathematical models that will form optimized quadrants that can improve the utilization of the police resources. Having an aid in the planning of *PCSP* can help streamline the territory design process allowing a more frequent revision of the information in different municipalities where this districting methodology is deployed. We begin by detailing different aspects that are considered by Carabineros de Chile in the *PCSP* methodology as described in the *PCSP* manual Dirección General de Carabineros de Chile (2010). In particular, the process of dividing a precinct into quadrants takes into account the following four criteria:

- **Patrolling constraints**: There should be enough quadrants so that each quadrant can be patrolled in one shift. The police has estimated that a police vehicle patrolling a neighborhood can traverse 82 km during an 8 h shift. Therefore the number of quadrants has to be at least the total linear kilometers in the region divided by 82.
- **Geographical considerations**: Quadrants should not be divided by avenues, roads, train lines, rivers, mountains or other elements that can make it difficult to patrol.
- **Activities related considerations**: It is desirable that the design of quadrants does not partition areas of activities, such as commercial or civic districts, residential neighborhoods, etc.
- **Concentric radial design**: It is also desired that quadrants be formed following a concentric distribution around a civic quadrant or central plaza.

Of these, the first two considerations are strictly enforced by experts during the design of quadrants, while the last two are desirable features that may not necessarily be implemented.

Over each of these quadrants the *PCSP* methodology determines the amount of demand for police resources in a standardized unit of *UVEs*. Each *UVEs* corresponds to the amount of patrolling that a police car with three police officers is able to conduct in one 8 h shift. The demand for police resources in each quadrant is divided in two components: a reaction component and a prevention component. The reaction component measures the procedures and other operational functions that are fulfilled by Carabineros on average during an 8 h shift on the quadrant. This quantity is made up of four factors:

- **Deployments**: Quantifies an expected number of deployments to service events such as theft, injury, damage to property, alcohol law, disorder on public streets, drugs, traffic accidents.
- **Court orders**: The expected number of activities due to court orders, such as: arrests, detentions, notices, citations, closures, protective measures, injunctions and evictions.
- **Monitoring Establishments**: Includes all resources needed to monitor establishments such as liquor stores, restaurants, banks, gas stations, nightclubs, among others.

Table 14.1 Parameters for preventive patrolling in *PCSP*

Factor	Constant	Units
Km	82	km/*UVE*
RC	23,500	Yearly reported crime/*UVE*
Pop	50,000	People/*UVE*

- **Extraordinary services**: This factor is caused by special events that generate a non-repeated large request for police resources such as: sporting events, concerts, protests, and visits of foreign dignitaries or other authorities.

The prevention component of the demand for police resources is defined as the maximum of three factors over the quadrant: a measure of the criminal activity (amount of reported crime *RC*), the population (*Pop*), and the total kilometers of roads in the quadrant (*Km*). Each of these factors influences the need for police resources used for preventive patrols. For instance: reported crime can be used as proxy for possible future criminal activity, the population can be proportional to the level of criminal activity, and finally the total linear kilometers in a quadrant is related to the number of resources needed to patrol this distance in a fixed period. The methodology, described in the Dirección General de Carabineros de Chile (2010), considers normalizing constants, given in Table 14.1, to translate these factors in terms of *UVEs*.

Therefore, the demand for police resources for prevention according to the PCSP is given by

$$D = \max\left\{\frac{1}{82}Km, \frac{1}{23500}RC, \frac{1}{50000}Pop\right\}. \tag{14.1}$$

This demand is defined as the maximum because any one of these three factors can be the cause for additional police resources for prevention. If a quadrant has a very large population, or reported crime, or total kilometers of roads, it can require additional resources to conduct adequate preventive patrolling. For example, a large, sparsely populated district with little reported crime can have a demand for preventive police resources determined by the *Km*, while a small, densly populated district with high reported crime will have the number of police resources for prevention determined by either *RC* or *Pop*.

An automatic method of constructing quadrants would naturally assign the information regarding demand for police resources to each block, then depending on the resulting quadrants, compute the reactive and preventive components of demand over them. We note that the reactive component of demand is defined as the sum of the demand of each of the blocks. Therefore over the whole area the reactive demand remains constant, independent of how the quadrants are constructed. The same is not true for the preventive component of demand for police resources. Since this quantity is defined by a maximization over three factors, the total number of resources needed to cover a district depends on the shape of the quadrants the district is divided on.

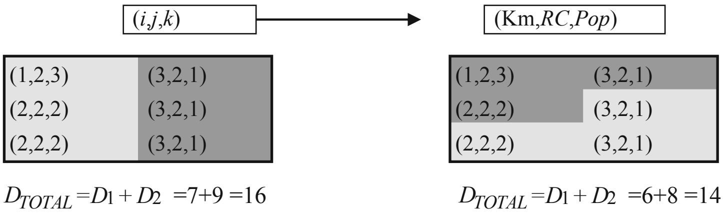

Fig. 14.3 Two ways of splitting an area with 6 nodes showing differences in the prevention demand component

To illustrate this consider Fig. 14.3. This example considers a region formed of six building blocks that can be organized in two quadrants. Each cell represents a building block, and each number represents the normalized amount of Kilometers, Reported Crime and Population. Two ways of splitting the region are represented, each district is identified by the different shades of gray and the demand for each district is calculated as the maximum between the sums of each of the three components of demand prevention. The split of the area on the left achieves a total prevention demand component of 16, while the one on the right equals 14. The example shows that the total number of resources required over both quadrants depends on how the quadrants are formed.

The demand of the district p is

$$D_p = \max\left\{\sum_{\ell \in P} Km_\ell, \sum_{\ell \in P} RC_\ell, \sum_{\ell \in P} Pop_\ell\right\}.$$

This example shows that, in addition to automating the police districting problem for the *PCSP* methodology, it is possible to optimize the amount of resources used to provide the preventive component of demand by adjusting the shape of the quadrants formed. This is a key component of the mathematical model of the *PCSP* methodology presented in the next section and a unique feature of this districting problem.

14.4 A Police Districting Model

The model is based on a p-median model with a balance constraint in the demand of police resources. Given a set of blocks I and a set of candidates to be centers of districts $J \subseteq I$. The p-median model locates p facilities and assigns blocks to each facility, so as to minimize the sum of the distances of each block to its center. We refer to this measure as the total distance to centers. The p-median model with balance constraints considers the previous model with lower and upper bound constraints on the demand allocated to each quadrant.

In order to avoid non contiguities and "bad shapes," we consider the compactness as a two dimensional factor which minimizes the total distance to centers and the length of the boundary of each district. To minimize the boundary of each district we define an adjacency graph in which each node represents a block, and an edge (i, j) exists if the block i share a piece of boundary with the block j, in this case we say that $i \in N(j)$ and vice-versa. We call the set $N(i)$ as the neighborhood of i. Also, in this model, constraints could be easily added that avoid dividing conflictive areas, e.g., hot spots, or quadrants being crossed by rivers or main roads. Finally we include the preventive demand component that minimizes the sum of the maximum between the reported crime level, the population, and the total kilometers of streets in each quadrant.

We use the variables of the classical p-median problem: x_j is a location binary variable which takes the value 1 if the center $j \in J$ is assigned and 0 otherwise, and y_{ij} are variables which take the value 1 if the block $i \in I$ belongs to the center is $j \in J$ and 0 otherwise. Also we use variables z_{ik} which take value 1 if the block $i \in I$ is allocated to a different center than $k \in N(i)$. This $N(i)$. This $N(i)$. This variable allows to count the length of the boundary of each district. Finally we use a continous variable D_j which takes the value of the prevention component of the demand. With these definitions the districting model is as follows:

$$\text{Min } \theta_1 \sum_{i\in I, j\in J} \ell_{ij} y_{ij} + \theta_2 \sum_{i\in I} \sum_{m=1}^{|N(i)|} \kappa_m u_{im} + \theta_3 \sum_{j\in J} D_j \tag{14.2}$$

$$\text{s.t.} \sum_{j\in J} x_j = p \tag{14.3}$$

$$\sum_{j\in J} y_{ij} = 1,\ i \in I \tag{14.4}$$

$$x_j \geq y_{ij},\ i \in I,\ j \in J \tag{14.5}$$

$$x_j L \leq \sum_{i\in I} y_{ij} dem_i \leq x_j U,\ j \in J \tag{14.6}$$

$$z_{ik} \geq y_{ij} - y_{kj},\ i \in I,\ k \in N(i),\ j \in J \tag{14.7}$$

$$z_{ik} \geq -y_{ij} + y_{kj},\ i \in I,\ k \in N(i),\ j \in J \tag{14.8}$$

$$u_{im} \leq 1,\ i \in I,\ m \in \{1, \cdots, |N(i)|\} \tag{14.9}$$

$$\sum_{m=1}^{|N(i)|} u_{im} \geq \sum_{k\in N(i)} z_{ik},\ i \in I \tag{14.10}$$

$$D_j \geq f_{pop} \sum_{i\in I} y_{ij} pop_i,\ i \in J \tag{14.11}$$

$$D_j \geq f_{km} \sum_{i\in I} y_{ij} km_i,\ j \in J \tag{14.12}$$

$$D_j \geq f_{rc} \sum_{i \in I} y_{ij} rc_i, \;\; j \in J \tag{14.13}$$

$$x_j, \; y_{ij} \in \{0,1\}, i \in I, j \in J \tag{14.14}$$

$$D_j, z_{ik}, u_{im} \geq 0, \;\; i \in I, \;\; j \in J, \;\; k \in N(i), \;\; m \in \{1, \ldots, |N(i)|\} \tag{14.15}$$

The objective (14.2) minimizes the multi-objective weighted function, where θ_1 is the weight of the total distance to centers (where ℓ_{ij} represents the distance of block $i \in I$ to its allocated center $j \in J$), θ_2 is the weight of the penalty function of size of the boundary and θ_3 weights the prevention demand component.

The Eq. (14.3) states the number of centers or quadrants. The set of constraints (4) states that each block must be assigned to a quadrant. The set of inequalities (5) establishes that a block could be assigned to a center only if the center is activated. The set of constraints (6) balances the demand between the quadrants. This means that, denoting dem_i the demand from block i, the total demand assigned to each center cannot be more than an upper bound $U = (1+\alpha)\sum_{i \in I} dem_i/p$ and cannot be less than the lower bound $L = (1-\alpha)\sum_{i \in I} dem_i/p$. Here, α is the maximum acceptable percentage of deviation from the average of the demand of the reaction component (e.g., 5 %). Expressions (7) and (8) define z_{ik} as the absolute value of the difference between y_{ij} and y_{kj} for every pair of adjacent blocks. By defining auxiliary variables u_{im}, the sets of constraints (9) and (10) allow to express $\sum_{i \in I} \left(\sum_{k \in N(i)} z_{ik}\right)^{\beta}$ as a piecewise linear approximation $\sum_{i \in I} \sum_{m=1}^{|N(i)|} \kappa_m u_{im}$, where $\kappa_m = \kappa_m(\beta)$ depends of the convexity of the function. These variables basically count the number of adjacent blocks not assigned to the same median, so disconnected blocks can be penalized. The set of constraints (11–13) defines D_j as the maximum of three factors: the linear kilometers km_i, the amount of reported crime for the block i, rc_i, and the population of the block, pop_i. All of them are normalized by a factor f_w. Finally, expressions (14) and (15) define the domain of the variables.

Realistic instances of this optimization problem can be difficult to solve exactly. If we consider a problem on n nodes, where each node has at most ρneighbors, then this optimization problem would have $O(n^2)$ binary variables, $O(n\rho)$ non-negative continuous variables, and $O(n^2\rho)$ inequality constraints. The realistic case study presented in this chapter considers $n = 1266$ blocks and up to $\rho = 7$ neighbors, which gives a problem with millions of constraints and binary variables. Even reducing the problem size by aggregating variables or limiting the possible centers and possible block to center assignments we obtain problems whose size are a challenge for existing exact solution methods. We therefore consider a heuristic solution algorithm based on a Location-Allocation heuristic (Teitz and Bart 1968) to find good solutions to this problem efficiently. This heuristic implemented is as follows:

1. *Step 1:* Generate an initial set of p centers. Any rule can be used to generate this set, for instance random selection or solving the p-median problem.
2. *Step 2:* (*Allocation phase*): Solve the mixed integer optimization problem that allocates every block to a given set of centers, so that the balance constraints hold, and the objective is minimized.
3. *Step 3*: (*Location phase*): For every quadrant generated, find the best geometrical center, and go back to the step 2 until there are no changes.

We repeat this heuristic with multiple random seeds or initial solutions to further search the solution space. One of the best features of this heuristic is that in every step a feasible solution is obtained. This gives several districting plans within a reasonable computational time that could be evaluated by the decision maker with other criteria, as desired. We note that this heuristic can be accelerated by solving the linear relaxation in the allocation phase, assigning the fractional blocks using a heuristic rule. This acceleration makes it more difficult to satisfy the balance constraints.

In the next section we show how this model with the heuristic and the adequate parameters of the objective function (θ, κ) gives compact and balanced districts for a real instance.

14.5 A Case Study

In this section we show the most important results of the model applied to a real instance in Ñuñoa, Santiago de Chile. The current districts have a configuration as shown in Fig. 14.4. The data of each block was extracted from demographic census data and historical data of Carabineros de Chile. This instance has a size of 1266 blocks. The current districting plan has a maximum deviation of the reactive component of demand for police resources that deviates 77 % from the average value in this region (Bustamante 2011).

This map, after a clustering of some blocks was represented in an adjacency graph as shown in Fig. 14.5. The clustering was performed with a grid of 200×200-m cells. Each node represents a cell of the grid that contains a block of the original map. After this clustering the graph has a size of 407 nodes.

If a p-median is used to find quadrants, the results would be nice and compact shapes. However, the maximum deviation from the workload balance could reach large figures, similar to the 77 % shown by the current quadrants. On the other hand, the p-median with balance constraints will find an optimal solution, but the shape of the districts will be irregular forming non compact quadrants. Even non contiguities could be observed in this graph.

We applied the model and the heuristic for different values of the parameters of the objective function ($\theta = (\theta_1, \theta_2, \theta_3)$) and found that the quality of solutions and the performance of the heuristic are very sensitive in this parameter. For instance, Fig. 14.6 shows the output for different parameters θ.

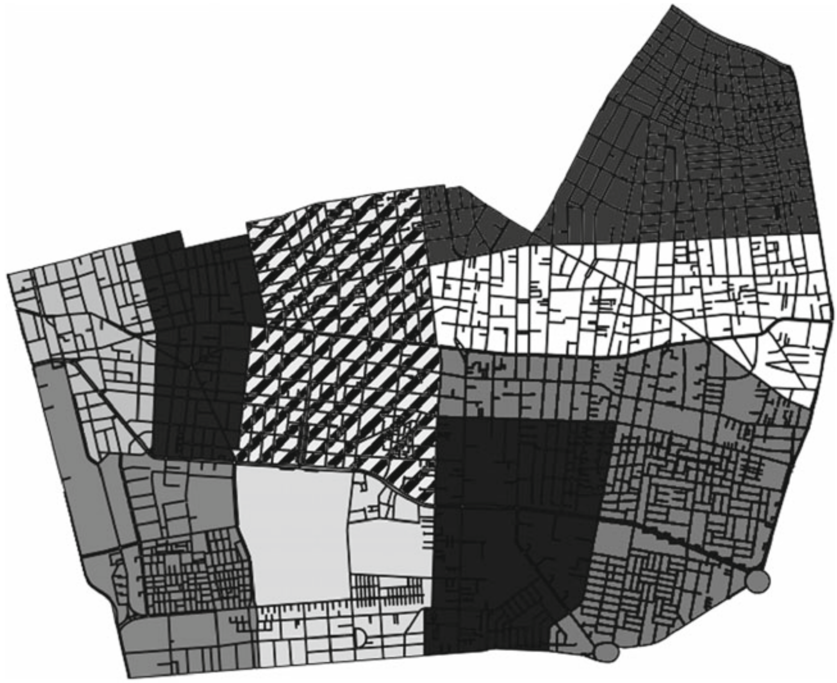

Fig. 14.4 Current quadrants

Figure 14.6a shows the result when the relative importance of the three factors in the objective function is the same, i.e., all three are weighted uniformly. In this case non-contiguities are observed and the shape of the districts is irregular. This situation is improved when the total distance to centers is weighted with greater relative importance, which gives a solution closer to the *p*-median problem as in the case of Fig. 14.6b. In that graph each district generated has contiguous blocks. The case of Fig. 14.6c shows the result where the size of the boundary has greater relative importance in the objective function. Due to the symmetries in the optimization model, using the proposed objective function, the allocation integer optimization problem in the heuristic becomes quite difficult to solve. We therefore fixed a 300 s limit for each iteration of the heuristic. Figure 14.6d shows the graph generated by the heuristic where the most important factor is the prevention demand. As expected, non contiguities and non compactness are present in the result. Finally we weighted with greater relative importance both the total distance to centers and size of the boundary. Figure 14.6e shows better shapes, while the balance between the quadrants is maintained. In particular the improvement of this solution over the one depicted in Fig. 14.6b is that because of the greater weight on the boundary, nodes which have many neighbors tend not to be on the boundary.

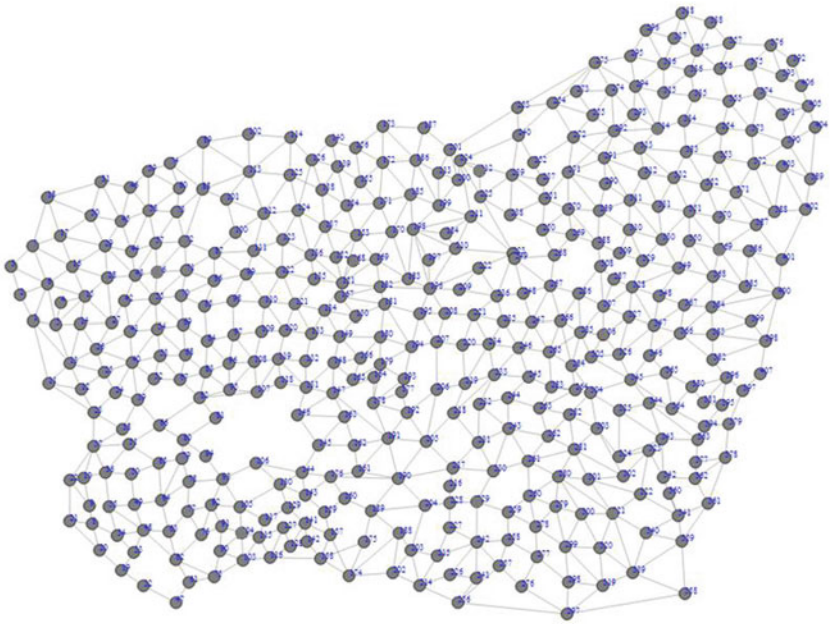

Fig. 14.5 Adjacency graph that represents Ñuñoa

We analyze these solutions further with the results presented in Table 14.2. This table shows the average solution time per iteration of the heuristic and the contributions to each part of the objective (distance to centers, boundary, prevention demand) of the five solutions presented in Fig. 14.6. We note that the total distance to centers component of the objective is not very sensitive to changes in the weights and varies at most 6 % of the mean value, while the objective components of boundary and prevention demand vary greatly (with a maximal variation of 60 % and 44 % with respect to the mean value respectively). Another important aspect is the difficulty of solving each iteration in the heuristic. When the objective is primarily driven by the size of the boundary we observe that the heuristic reaches the run time limit. This is due to the difficulty of solving the allocation step when the contribution of this non-linear objective is significant. It is interesting to note that, for the best shapes found (experiment e), the time required was just two seconds. This solution method also provides many feasible solutions (one during each iteration) which can be evaluated afterwards by decision makers.

Finally, in Fig. 14.7 we show a new districting plan which is the output of the heuristic where the parameters of the objective function θ weight most importantly the geometrical aspects as in the experiment (e). Notice that this districting solution has a demand balance within 5 % of the mean demand value and forms contiguous and compact districts.

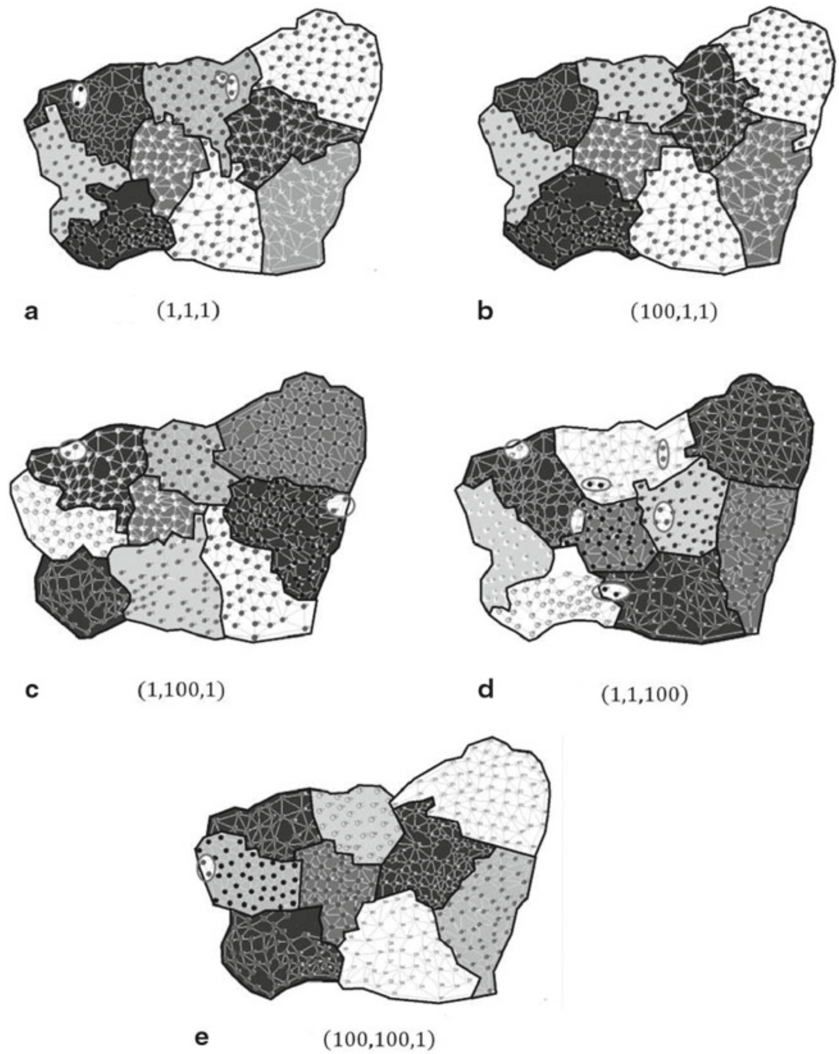

Fig. 14.6 Comparison for the output of the model for differents values of $(\theta_1, \theta_2, \theta_3)$

Table 14.2 Average resolution time and contributions of each objective component for each experiment

Experiment label	Parameters			Time [sec]	Distance to Centers $\sum d_{ij} y_{ij}$	Boundary Component $\sum \kappa_m u_{im}$	Prevention Demand $\sum \gamma_j$
	θ_1	θ_2	θ_3				
a	1	1	1	5	229,528	4308	185,895
b	100	1	1	3	222,551	4173	288,158
c	1	100	1	300	235,451	2131	221,071
d	1	1	100	18	229,635	4186	185,895
e	100	100	1	2	222,639	3350	288,326

Fig. 14.7 Districting plan given for the heuristic using $\theta = (100, 100, 1)$ and $\beta = 3$

14.6 Conclusions

Districting is a difficult problem arising in many contexts where resources must be allocated and divided to provide an efficient service to a distributed demand. In particular, police institutions are no stranger to this challenge, where the demand for police resources must accomplish both preventive and reactive actions to provide security to an area.

In this work, done in collaboration with Chile's national police force, we formulated an optimization problem to represent Carabineros' *PCSP* police districting problem. This leads to a modified p-median model with a preventive demand non-linear objective, balance constraints on the reactive demand component, and an objective of minimizing the boundary. The balance requirements and non-linear preventive demand objective not only make the districting problem much harder, but give a problem that has optimal solutions that do not have contiguous or compact quadrants. Adding the compactness measure that minimizes the boundary to this multi-objective problem helps provide solutions that achieve efficient objectives with acceptable quadrant shapes.

The proposed model and location-allocation heuristic provide several efficient solutions that a planner can compare to trade-off additional less tangible criteria in selecting the optimal districting plan. We applied the heuristic to a real case in Santiago, Chile, and obtain districts with a reactive demand imbalance of at most 5 % from the average quadrant demand, as opposed to an imbalance of 77 % obtained by the current manual solution. This benefit comes at the expense of districts which have more complicated boundaries, which become operationally more challenging to manage. Improvements of this model should include additional constraints on the boundaries to achieve quadrant shapes that are simple to patrol. Ongoing work includes evaluating the use of this methodology in other precincts of Santiago with real police data and adding geographical and operational considerations of what nodes can be used as boundaries of quadrants. Lieutenant Colonel Bassaletti, who participated in this work, has stated that this model and solution method are part of the techniques that the Chilean national police force is taking into account in their ongoing revision of the *PCSP* districting plan.

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