Name of the Course : Generic Elective

Unique Paper Code : 32355301

Name of the Paper : **GE-3 Differential Equations**

Semester : III

Duration : 3 Hours

Maximum Marks : 75 Marks

Attempt any four questions. All questions carry equal marks.

1. (i) Solve

$$xdy - ydx = \sqrt{x^2 + y^2} dx.$$

(ii) Solve the initial value problem

$$(x^2 + y^2 + x)dx + xy dy = 0,$$
 $y(1) = 1.$

(iii) Solve the initial value problem

$$x\frac{dy}{dx} + y = y^2 \log x, \qquad y(1) = -1.$$

- 2. (i) Find the orthogonal trajectories of the family of curves $3xy = x^3 a^3$, a being parameter of the family.
 - (ii) Find a family of oblique trajectories that intersect the family of circles $x^2 + y^2 = c^2$ at angle 45°.
 - (iii) Solve

$$\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y} .$$

3. (i) Solve by method of variation of parameters

$$y'' + y = \cos ec x.$$

(ii) Solve by method of undetermined coefficients

$$y'' + 1.44y = 24\cos 1.2 x.$$

(iii) Solve

$$y''' - 2y'' + 4y' - 8y = 0$$
, $y(0) = -1$, $y'(0) = 30$, $y''(0) = 28$.

4. (i) Show that $\{e^{-x}, e^{3x}, e^{4x}\}$ forms a basis of the solution set of the equation

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0.$$

(ii) Solve the initial value problem

$$x^2 y'' - 2x y' - 10y = 0$$
, $y(1) = 5$, $y'(1) = 4$.

(iii) Solve the linear system

$$y_1' = 2y_1 + 5y_2$$

 $y_2' = 5y_1 + 12.5y_2$

5. (i) Find the partial differential equation arising from the surface

$$z = xy + f\left(x^2 + y^2\right).$$

(ii) Find the general solution of the partial differential equation

$$u_x + 2xy^2 u_y = 0.$$

(iii) Apply the method of separation of variables u(x, y) = f(x)g(y) to solve

$$yu_x + xu_y = 0$$
 on $u(0, y) = y^2$.

- **6.** Reduce each of the following equations into canonical form and find the general solution:
 - (i) $u_x u_y = u$, $u(x,0) = 4e^{-3x}$.
 - (ii) $u_{xx} + 6u_{xy} + 9u_{yy} + 3yu_y = 0.$
 - (iii) $u_{xx} 3u_{xy} + 2u_{yy} = 0.$