5292/KM6/PMAT07 **NOVEMBER 2018**

ANALYSIS — III

Time : Three hours Maximum : 100 marks

SECTION A — $(4 \times 10 = 40$ marks)

Answer any FOUR questions.

1. Prove that $f \in \Re(\alpha)$ on $[a, b]$ if and only if for every $\in > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

2. Prove that there exists a real continuous function on the real line which is nowhere differentiable.

3. Suppose Σc_n converges. Put $f(x) = \sum_{n=0}^{\infty} c_n x^n \; (-1 < x < 1)$. Then prove that $\lim_{x \to 1} f(x) = \sum^{\infty} n = 0 c_n$.

4. Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then prove that $\dim X \leq r$.

5. Prove that a linear operator A on R^n is invertible if and only if $\det[A] \neq 0$.

6. Prove that every interval is measurable.

7. Prove that not every measurable set is a Borel set.

8. State and prove Lebesgue's Monotone Convergence theorem.

SECTION B — ($4 \times 15 = 60$ marks)

Answer any FOUR questions.

9. Suppose $f \in \mathfrak{R}(\alpha)$ on $[a,b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$, and $h(x) = \phi(f(x))$ on $[a, b]$. Then prove that $h \in \mathfrak{R}(\alpha)$ on $[a, b]$

10. Let $\mathcal{A}$ be an algebra of real continuous functions on a compact points on K. If $\mathcal{A}$ separates points on K and if $\mathcal{A}$ vanishes at no point of K, then prove that the uniform closure $\mathfrak{B}$ of $\mathcal{A}$ consists of all real continuous functions on K.

11. State and prove Parseval's theorem.

12. If X is a complete metric space, and if ϕ is a contraction of X into X, then prove that there exists one and only one $x \in X$ such that $\phi(x) = x$.

13. State and prove the implicit function theorem.

14. Show that the outer measure of an interval equals its length.

15. Show that the derivatives of a continuous function are measurable.

16. State and prove Lebesgue's Differentiation theorem.
