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Dynamic prediction of the National Hockey League draft with rank-ordered logit models ☆

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ABSTRACT

The National Hockey League (NHL) Entry Draft has been an active area of research in hockey analytics over the past decade. Prior research has explored predictive modelling for draft results using player information and statistics as well as ranking data from draft experts. In this paper, we develop a new modelling framework for this problem using a Bayesian rank-ordered logit model based on draft ranking data from industry experts between 2019 and 2022. This model builds upon previous approaches by incorporating team tendencies, addressing within-ranking dependence between players, and solving various other challenges of working with rank-ordered outcomes, such as incorporating both unranked players and rankings that only consider a subset of the available pool of players (e.g., North American skaters, European goalies, etc.).

1. Introduction

Over the past two decades, the National Hockey League (NHL) has imposed a hard salary cap which limits player salaries and controls a team's ability to retain and add talented players. This may be seen as an effort to enforce competitive balance throughout the league. Consequently, teams have become increasingly savvy in their allocation of resources. The NHL has three main outlets where a team can add, lose or maintain talent: free agency, trades, and the entry draft. Acquiring players through free agency or trades can often be expensive, costing valuable cap dollars or assets. On the other hand, the entry draft is a low-cost, high-upside way to find and develop NHL-level talent.

The entry draft is an annual event held by the NHL where teams take turns selecting the world's best young, prospective hockey players to disperse incoming talent throughout the league. The order of the draft is predominantly determined by the reverse order of the standings to ensure that the worst teams from the previous season have the best chance at improving their roster. The draft lasts seven rounds, each with 32 picks - one for each team. However, many wrinkles may affect the draft order, such as trades, playoff results, compensatory picks, and the NHL draft lottery. Knappe (2022) provides a primer for the rules and setup of the NHL draft.

Every NHL team employs a department of scouts to identify and evaluate the top draft-eligible players each season. Teams make assumptions about how long a player will remain unselected during the upcoming draft to strategize and obtain the players they desire. They must dynamically consider how the observed selections and the selectors affect the chances that a coveted player will remain available by their next pick.

This project estimates the probability that any draft-eligible player will be selected at any upcoming draft pick. Additionally, the proposed modelling framework can go beyond this to answer various questions related to the

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NHL draft. For example, how does a player's probability of selection change as each new draft pick is made? Which qualities are valued by NHL teams and draft experts, and how do these assessments differ between teams and draft experts? Which expert rankings best align with the tendencies of NHL teams?

Previous research related to the NHL entry draft has predominantly focused on the success of draft selections at the NHL level, either through retrospective analysis of past draft selections or by building predictive models to estimate the level of success prospects will have in their careers. For example, Liu (2018) assesses draft prospects using a tree learning approach, and Schuckers (2016) does likewise using a Poisson generalized additive model based on demographic and performance covariates. Tingling (2017) provides a brief history of drafts in major league sports and a review of research related to the NHL draft. Schuckers (2011) and Tulsky (2013) use historical draft results, player statistics, and draft pick trades to estimate relative pick value. Both find that the values of draft picks have an exponential decaying pattern in the NHL draft (e.g., the decrease in value from the 1st pick to the 2nd pick is much greater than the decrease from the 101st pick to the 102nd pick). Nandakumar (2017) conducts a retrospective analysis of how well teams performed in the draft by comparing their realized draft results to what a 'perfect draft' would look like given the draft capital they had available to them and the future NHL performance of players available in the draft class. Li (2019) uses data from the NHL Combine - an annual fitness assessment of draft-eligible players - to look at the relationship between combine results, draft stock, and success at the NHL level. The models found positive relationships between lower-body strength and draft stock, as well as aerobic and anaerobic fitness and NHL success.

A subset of the research surrounding major league sports drafts has explored predictive modelling approaches for the outcome of the draft to answer some of the questions we have proposed in this project. ESPN Analytics (2022) have developed a model to predict the probability that a player is available at a given pick in the National Football League (NFL) draft; however, their methodology remains private. Burke (2014) and Sprigings (2016) discuss this model's foundations at a high level. It is a Bayesian model based on consensus player rankings and projections from individual experts with weightings for each expert based on historical accuracy. Robinson (2020) uses a Bayesian gamma regression based on mock draft data from draft experts, fans, and media in the NFL to estimate the probability that a player will be available by a given pick.

This project looks to build upon these previous approaches with a Bayesian rank-ordered logit (ROL) model. The ROL model is a product of conditional multinomial logit (MNL) densities used to model rank-ordered outcomes. The proposed framework addresses within-ranking dependence, allowing updated selection probabilities after each new pick. Additionally, it provides natural solutions for incorporating unranked players and ranking sets that contain a subset of the available pool of draft-eligible players such as NHL Central Scouting - which breaks down their rankings into four categories: North American skaters, North American goalies, European skaters, and European goalies. Finally, it accounts for differences between NHL teams and draft experts by leveraging previous years' expert rankings and draft results to estimate team and expert-specific effects. This provides key strategic insights into other teams' draft tendencies and adjusts predictions to align with the tendencies of NHL teams as opposed to draft experts.

Although scarcely applied to hockey, Multinomial logit models have had many applications within sports and other domains. Tea and Swartz (2022) use a Bayesian MNL model to estimate probabilities of serve placements in tennis. Gerber and Craig (2021) also use a Bayesian MNL model to predict plate appearance outcomes and project batter performance over a season in Major League Baseball (MLB).

The rank-ordered logit model - also referred to as the Plackett-Luce or exploded logit model - was originally developed by Plackett (1975), who leveraged the multinomial choice logit model proposed by Luce (1959) to model rank-ordered outcomes. Many of the applications of ROL models in sports have involved racing sports, including horse racing (Ali, 1998; Bolton & Chapman, 1986; Johnson et al., 2010; Lo & Bacon-Shone, 1994) and automobile racing (Anderson, 2014; Graves et al., 2003; Guiver & Snelson, 2009). More recently, the focus of ROL models in sports has accounted for changes in player abilities over time. Glickman and Hennessy (2015) developed a stochastic ROL model to predict the outcome of multi-competitor competitions while allowing for changes in competitor abilities over time with applications to women's alpine data.

Section 2 describes the data collected for this project, defines important terminology related to our model, and reviews the multinomial logit model. In Section 3, we define our ROL model specifically designed for the case of the NHL draft. We define a base ROL model and propose enhancements for unranked players, subset rankings, and team tendencies. Section 4 explores the results of applying our model to the 2019 to 2022 NHL drafts. We demonstrate the effect of incorporating team and expert tendencies into the model and explore differences between teams and experts. As a specific example, we examine how much of a risk the Dallas Stars took by trading down from pick 15 to 23 in acquiring Wyatt Johnston in the 2021 draft. Additionally, we investigate model evaluation techniques to compare our work with previous approaches. In Section 5, we conclude with a short discussion of the implications of our work and future research directions.

2. Background

2.1. Data

In multi-competitor sports, athletes typically compete in many events, which can be used to predict future events. On the other hand, the NHL draft only occurs once per year, and each draft involves an entirely different pool of players available to be selected.

Table 1
List of agencies (who provide ranking sets) and corresponding abbreviations.

Agency	Abbreviation
The Athletic - Corey Pronman	TA-CP
The Athletic - Scott Wheeler	TA-SW
Daily Faceoff - Chris Peters	DF-CP
Dobber Prospects - Full Staff	DP
The Draft Analyst - Steve Kournianos	TDA-SK
Draft Prospects Hockey - Full Staff	DPH
Elite Prospects - Full Staff	EP
Elite Prospects - Cam Robinson	EP-CR
Future Considerations Hockey - Full Staff	FCH
The Hockey News - Ryan Kennedy	THN-RK
The Hockey Writers - Peter Baracchini	THW-PB
The Hockey Writers - Andrew Forbes	THW-AF
The Hockey Writers - Matthew Zator	THW-MZ
McKeen's Hockey - Full Staff	MKH
NHL Central Scouting - Full Staff	NHLCS
Recruit Scouting - Full Staff	RS
Scouching - Will Scouch	SCO-WS
Smaht Scouting - Full Staff	SS
Sportsnet - Sam Cosentino	SN-SC
TSN - Craig Button	TSN-CB
TSN - Bob McKenzie	TSN-BM

Due to the draft's popularity among NHL fans, there exists a market for draft experts who spend hours producing media content related to the NHL draft, including ordered lists that rank the top draft-eligible players. These draft experts are typically employed by sports media networks such as TSN and Sportsnet or websites that produce hockey-related content such as Elite Prospects, Dobber Hockey, and The Hockey News. We leverage these expert rankings along with NHL draft results from previous years to compensate for the fact that we have no true realizations of the draft results until the actual NHL draft occurs.

From now on, we will refer to the draft experts from media networks and hockey websites that provide rankings for the NHL draft as 'agencies'. Additionally, from now on, we will refer to each set of ranked players provided by an agency as a 'ranking set' and the observed results from the NHL draft in a given year as the 'draft results'.

The data used to fit our model includes draft results and ranking sets from the 2019–2022 NHL drafts, as well as player information for every draft-eligible player that appears in at least one ranking set or draft result. Each ranking set contains the rank ordering of players provided by the agency, along with the full set of players available to be ranked, regardless of whether the agency ranked them. We obtained this data through a mixture of web scraping with Python Selenium and manual data entry. There were 21 agencies (see Table 1) for which we gathered ranking sets, most having their own websites. Some of the websites were eliteprospects.com, tsn.ca and sportsnet.ca.

The draft is typically held each year between June 20th and 30th. We only consider the final ranking set by each agency prior to the draft. This gives us 14, 17, 22, and 22 ranking sets for each draft year from 2019 to 2022.

To get a sense of the variability in the rankings, Table 2 provides the top 10 draft picks of the 2022 NHL draft along with the rankings provided by TA-CP, EP, THN-RK, NHLCS and TSN-BM. Although more variability is found further down the list, we observe that there is greater consistency of opinion involving the top picks.

2.2. Notation

The notation required to express the ROL model proposed in this project involves many nested subscripts. To make the model easier to digest, we use two different types of notation throughout the paper for different purposes. To illustrate how the model is structured, we use notation in terms of a single ranking set for a particular draft year, as outlined in Table 3. Alternatively, when we wish to expand the model to encompass all ranking sets over all draft years, we use expanded notation that specifies the ranking set and year as outlined in Table 4. Notice that most of the notation stays consistent between both cases.

2.3. Latent variable multinomial logit models

Before laying out the methodology of the ROL model, we first provide a brief overview of the multinomial logit model applied to the NHL draft. MNL models are the main building blocks defining the rank-ordered logit model in Section 3.1. They are models used in statistics to classify observations into categories.

Let's consider a special case of the MNL model where we wish to predict the 1st overall draft pick with N total draft-eligible players. The list of draft-eligible players which leads to the determination of the constant N is obtained by compiling a list of all unique players who appeared in at least one ranking set or draft result. The goal of the MNL model is to estimate the probability of being selected first for each of the available players. In other words, we wish to estimate probabilities $\boldsymbol{\pi}_1 = [\pi_{11}, \pi_{21}, \dots, \pi_{N1}]$ such that π_{i1} is the probability that player i will be selected with the 1st overall pick subject to the constraint that $\sum_{j=1}^{N} \pi_{j1} = 1$.

We assume that the ith player has a true ability parameter θ_i which is unknown. Further, an agency interprets the ith player's ability through a latent (i.e. unobserved) rating $Y_i = \theta_i + \epsilon_i$ where the errors ϵ_i are independent and $\epsilon_i \sim \text{Gumbel}(0, 1)$, $i = 1, \dots, N$. We also assume that the agency will select the player they assign the highest rating to out of all available players. Thus, π_{i1} is considered the probability that Y_i is greater than Y_j, $\forall j \in \{1, \dots, N\} \setminus \{i\}$. In this context, we obtain

$$\pi_{i1} = P(Y_i > Y_j, \forall j \in \{1, \dots, N\} \setminus \{i\}) = \frac{\exp(\theta_i)}{\sum_{j=1}^{N} \exp(\theta_j)} \quad (1)$$

where the equivalency of the right-hand side of Eq. (1) is derived by Train (2009). In this specification of the MNL model, we wish to estimate the player ability parameters $\theta_1, \dots, \theta_N$, and use (1) to estimate the probability that player i will be selected 1st overall.

The reasoning behind the Gumbel (1958) distribution assumption is made clear by Luce and Suppes (1965), who provide the original derivation of the multinomial choice probability (1). Furthermore, McFadden (1974) shows that

Table 2
The top 10 draft picks from the 2022 NHL draft with five sets of rankings. For the NHL Central Scouting Rankings (NHLCS), E refers to the European Skater category, and NA refers to the North American Skater category.

Draftee	TA-CP	EP	THN-RK	NHLCS	TSN-BM
1. J. Slafkovsky (MTL)	1	3	2	1 (E)	1
2. S. Nemec (NJ)	5	6	4	3 (E)	4
3. L. Cooley (ARI)	3	4	3	2 (NA)	3
4. S. Wright (SEA)	2	1	1	1 (NA)	2
5. C. Gauthier (PHI)	6	11	6	3 (NA)	5
6. D. Jiricek (CBJ)	4	2	5	4 (E)	6
7. K. Korchinski (CHI)	16	19	13	7 (NA)	11
8. M. Kasper (DET)	9	12	10	5 (E)	10
9. M. Savoie (BUF)	8	9	9	4 (NA)	9
10. P. Mintyukov (ANA)	18	7	18	6 (NA)	12

Table 3
The list of variables used to describe the ROL model for a single ranking set.

Variable	Description
n	The number of picks made in the ranking set
N	The number of draft-eligible players considered in the ranking set
s_i	The ith ranked player in the ranking set
S'_i	The set of available players prior to the ith ranking in the ranking set

Table 4
The list of variables used to describe the ROL model when considering all ranking sets in all draft years between 2019–2022.

Variable	Description
R	The set of draft years considered in the model. Here $R = \{2019, 2020, 2021, 2022\}$
K_r	The number of ranking sets or draft results in year r where $r \in R$
n_r	The number of picks made in the draft results of year r where $r \in R$
N_r	The number of draft-eligible players considered in year r where $r \in R$
n_{rk}	The number of players ranked in the kth ranking set or draft result of year r
N_{rk}	The number of players available to be ranked in the kth ranking set or draft result of year r
s_{irk}	The ith ranked player in the kth ranking set or draft result of year r
S'_{irk}	The set of available players before the ith rank is made in the kth ranking set or draft result of year r
a_{irk}	The agency or team that selected the ith ranked player in the kth ranking set or draft result of year r

if (1) is true, then the errors of Y_i must follow the iid standard Gumbel distribution. The assumption of iid standard Gumbel errors is analogous to the assumption of iid standard normal errors, with extreme value distributions such as the Gumbel having slightly fatter tails (Train, 2009). However, the latent formulation of the MNL model with Gumbel errors is much more convenient to work with than the equivalent formulation of the multinomial probit (MNP) model with normal errors. Agresti (2019) provides further details on the theoretical framework behind multinomial logit models, along with various examples of MNL models applied to real-world scenarios.

3. Rank-ordered logit models

3.1. A baseline rank-ordered logit model

The MNL model provides a simple framework for describing the probability that a player is selected with the 1st pick in the draft. However, there are still many questions that this model cannot address, such as: What is the probability of a player being drafted 2nd, 3rd or beyond? How do these probabilities differ depending on which player(s) are previously selected? If a player is consistently ranked in the top five but is never ranked 1st by an agency, would his probability of being selected 1st be the same as a player rarely ranked in the top 200?

These questions can be addressed using a rank-ordered logit model. The ROL model can be considered a product of conditional multinomial logit densities. The 1st overall pick is modelled as an MNL model, with the pick taken from the pool of all draft-eligible players. The 2nd pick is modelled as an MNL model with a single pick taken from all draft-eligible players excluding the player selected 1st, and so on, until the last pick. The ROL model provides a full rank ordering from the 1st pick to the last pick.

Recall the latent variable formulation of the MNL model introduced in Section 2.3. We defined Y_i as the latent rating of player i's ability by the ranking agency and assumed that $Y_i = \theta_i + \epsilon_i$ where $\epsilon_i \sim$ Gumbel(0, 1) are iid error terms, $i = 1, \ldots, N$.

We can generalize Eq. (1) to obtain multinomial choice probabilities π_{im} for the mth draft pick, $m = 1, \ldots, N$ conditional on the results of the previous $m-1$ draft selections. Recall that S'_m is the set of players that remain available at pick m. We express the probability (Train, 2009) that player i, $i \in S'_m$, will be selected with the mth pick given the previous $m-1$ picks as

$$\pi_{im} = P(Y_i > Y_j, \forall j \in S'_m \setminus \{i\}) = \frac{\exp\{\theta_i\}}{\sum_{j \in S'_m} \exp\{\theta_j\}} . \quad (2)$$

The likelihood of a full ranking set is proportional to the probability of obtaining the exact ordering provided by the ranking set. This probability can be expressed using the multinomial choice probabilities from (2) as

$$L(\boldsymbol{\theta}) = P(Y_{s_1} > Y_{s_2} > \cdots > Y_{s_N} \mid \boldsymbol{\theta}) = \prod_{i=1}^{N} \pi_{s_i i} = \prod_{i=1}^{N} \frac{\exp\{\theta_{s_i}\}}{\sum_{j=i}^{N} \exp\{\theta_{s_j}\}} . \quad (3)$$

Note that (3) is the joint probability that player s_1 is ranked 1st, player s_2 is ranked 2nd given that player s_1 is no longer available, and so on down to player s_N ranked last given that players $s_1, \ldots, s_{N-1}$ are no longer available. Recall that s_i is the index of the player ranked ith by the ranking agency.

Note that we only observe the *ordering* of the agencies' player ratings through the ranking sets. Agencies typically do not release player ratings and might not use a quantitative rating system to formulate their rankings. Thus, the agencies' ratings Y_i are unobserved.

The likelihood given by (3) is the baseline ROL model for a single ranking set. We expand the model in Sections 3.2 and 3.3 to address issues modelling the NHL draft. Beyond that, Sections 3.4 and 3.5 describe the full model and the estimation methods.

3.2. Unranked players and subset rankings

The model likelihood proposed in (3) considers a ranking set where all N draft-eligible players are both ranked and available to be ranked. However, this is not always the case in practice. Typically, an agency provides a ranking set up to some number of picks $n < N$. In this case, (3) does not reflect the agency's ordering of the remaining $N - n$ players who were unranked. To address this, we adjust the likelihood based on the work of Fok et al. (2010), which takes the product of multinomial choice probabilities up until the nth pick as follows

$$P(Y_{s_1} > Y_{s_2} > \cdots > Y_{s_n} > \max\{Y_{s_{n+1}}, \ldots, Y_{s_N}\} \mid \boldsymbol{\theta}) = \prod_{i=1}^{n} \frac{\exp\{\theta_{s_i}\}}{\sum_{j=i}^{N} \exp\{\theta_{s_j}\}} . \quad (4)$$

Note that the updated model likelihood (4) still considers all N players to be available to the agency even though only n players are ranked. This is an important distinction as it incorporates the unranked players into the likelihood and recognizes that the latent rating for each unranked player is less than the latent rating of the last ranked player.

There are also ranking sets that only consider a subset of the total pool of players. A popular example of this is the NHL Central Scouting Agency, who divide their ranking sets into four categories: North American skaters, North American goalies, European skaters and European goalies. Fyffe (2011) integrates the four Central Scouting ranking sets to construct a single Central Scouting ranking set referred to as Cescin. Schuckers and Butterfield (2020) provide more details on the Cescin approach and update the Cescin conversion factors using more recent data. Alternatively, we allow N to vary for each Central Scouting ranking set. No change is required in the likelihood (4) for a single ranking set; however, we address this in Section 3.4 when we state the full likelihood by letting n_{rk} and N_{rk} be the number of players ranked and number of players available for ranking set k in draft year r where $r \in R$ and $k = 1, \ldots, K_r$, respectively.

3.3. Agency and team tendencies

The proposed ROL model has assumed a linear predictor $\eta_{ij} = \theta_i$ for agency or team j's latent rating of player i where θ_i is player i's ability parameter where all other variations in player ratings are due to noise from an iid standard Gumbel error term. However, the agency or team can majorly influence the player ratings.

For example, consider the 2020 draft selections made by the Toronto Maple Leafs and the Ottawa Senators. That year, the Maple Leafs' draftees had an average height of approximately 5'10.5", while the Senator's draftees averaged approximately 6'2". Based on the disparity between the two teams, it is likely that the Senators' management team valued tall players more than the Maple Leafs.

To address this, we define a general linear predictor for player i being ranked by agency or team j as

$$\eta_{ij} = \theta_i + \beta_{j1}x_{i1} + \beta_{j2}x_{i2} + \cdots + \beta_{jp}x_{ip} \quad (5)$$

where θ_i is the player ability parameter, x_{ik} is a covariate with player-specific information and β_{jk} is an agency or team-specific parameter corresponding to x_{ik}, $k = 1, \ldots, p$.

By including the updated linear predictor (5) that varies with the ranking agency or team, the likelihood (4) is updated as

$$\prod_{i=1}^{n} P\left(Y_{s_i a_i} > Y_{j a_i}, \forall j \in S_i' \setminus \{s_i\} \mid \boldsymbol{\theta}, \boldsymbol{\beta}\right) = \prod_{i=1}^{n} \frac{\exp\{\eta_{s_i a_i}\}}{\sum_{j=i}^{N} \exp\{\eta_{s_j a_i}\}} \quad (6)$$

where $\boldsymbol{\beta}$ is an $A \times p$ matrix with rows representing the A total agencies and teams and columns representing each of the p covariates.

Since we are now assuming that the linear predictor is dependent on the agency or team, the latent rating $Y_{ij} = \eta_{ij} + \epsilon_{ij}$, will also be dependent on the agency or team where the ϵ_{ij} remain iid standard Gumbel errors.

3.4. Final ROL model

We have defined a ROL model for the NHL draft with respect to a single ranking set in Section 3.1 and built upon it to address draft-related issues in Sections 3.2 and 3.3. Now, we extend the model to all ranking sets over all draft years. Recall that Table 4 provides a guide to the notation used for the final ROL model.

We let $Y_{ijrk} = \eta_{ij} + \epsilon_{ijrk}$ be the latent rating by agency j for player i in the kth ranking set of year r where ϵ_{ijrk} remains an iid standard Gumbel error term $\forall i, j, r, k$. The full model likelihood can now be stated as the product of (6) taken over all ranking sets in all draft years as follows

$$L(\boldsymbol{\theta}, \boldsymbol{\beta}) = \prod_{r \in R} \prod_{k=1}^{K_r} \prod_{i=1}^{n_{rk}} \frac{\exp\{\eta_{s_{irk} a_{irk}}\}}{\sum_{j=i}^{N_{rk}} \exp\{\eta_{s_{jrk} a_{irk}}\}} \quad (7)$$

where s_{irk} and a_{irk} are the ith ranked player and the agency or team that ranked the ith player in the kth ranking set or draft result in year r, respectively. Note that the denominator in (7) has a differing subscript for the player being considered, s_{jrk}, as compared to the numerator, s_{irk}. In contrast, the agency, a_{irk}, remains consistent in both the numerator and denominator. When we are considering the ith selection in the ranking set, the linear predictor must always be taken with respect to the agency or team making the ith selection, a_{irk}. On the other hand, the index for the player varies in the denominator since we are considering the linear predictor for other players that are available for selection.

The model defined in (5) and (7) is our full ROL model proposed for the NHL draft. This triple product represents the probability of observing all selections over all ranking sets and draft results in all draft years.

Note that in the case of draft results, we only observe a team's selection among the remaining pool of players whenever they make a draft pick as opposed to their ranking set of the entire pool of players. As a result, we assume that the team's latent rating of the player they selected is greater than their latent rating of all other available players, and we infer team tendency parameters accordingly.

3.5. Parameter estimation

The ROL model we proposed for the NHL draft typically involves estimating hundreds of parameters. For estimation in this high-dimensional setting, we turn to Bayesian methods. In the Bayesian setting, we express the posterior density of parameters $\boldsymbol{\theta}, \boldsymbol{\beta}$ as proportional to the model likelihood stated in (7) multiplied by the prior distribution, $\pi(\boldsymbol{\theta}, \boldsymbol{\beta})$, as follows

$$\pi(\boldsymbol{\theta}, \boldsymbol{\beta} \mid \boldsymbol{Z}) \propto \prod_{r\in R}\prod_{k=1}^{K_r}\prod_{i=1}^{n_{rk}} \frac{\exp\{\eta_{s_{irk}a_{irk}}\}}{\sum_{j=i}^{N_{rk}}\exp\{\eta_{s_{jrk}a_{irk}}\}}\ \pi(\boldsymbol{\theta}, \boldsymbol{\beta})$$

where $\boldsymbol{Z}$ is the ranking set data.

Here, we are interested in estimating the unknown parameters $\boldsymbol{\theta}, \boldsymbol{\beta}$, which quantify each player's ability and the level at which agencies and teams value particular player traits, respectively.

3.5.1. Prior distribution

We assume a $\sum_{r\in R} N_r$ dimensional multivariate normal prior on the ability parameters with covariance matrix $\Sigma = \sigma_\theta^2 \boldsymbol{I}$ and mean vector $\boldsymbol{\mu}_\theta = 0$. Thus, our prior distribution is $\boldsymbol{\theta} \sim \text{MVN}(\boldsymbol{\mu}_\theta, \sigma_\theta^2\boldsymbol{I})$ where we place a Inv-Gamma(1, 1) hyperprior on σ_θ^2. An alternative empirical Bayes procedure for setting $\boldsymbol{\mu}_\theta$ is described in Kumagai (2022); it leads to faster convergence times for the resultant Markov chain Monte Carlo (MCMC) computations.

We assume a hierarchical structure on each $\boldsymbol{\beta}_k$ in the parameter matrix $\boldsymbol{\beta} = [\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_p]$. That is, $\beta_{jk} \sim N(\mu_{\beta k}, \sigma^2_{\beta k})$ where we set standard hyperpriors $\mu_{\beta k} \sim N(0, 1)$ and $\sigma^2_{\beta k} \sim$ Inv-Gamma(1, 1), $k = 1, \ldots, p$ where p is the total number of player information covariates in the model.

From (5) it is evident that $\boldsymbol{\theta}$ and $\boldsymbol{\beta}_k$ are identifiable up to an additive constant. We, therefore, impose a constraint on the model such that all player ability parameters in a given draft year sum to zero. We also impose the same constraint for each $\boldsymbol{\beta}_k$, $k = 1, \ldots, p$. As a result, zero represents the average value of a parameter which aids interpretation.

3.5.2. Computation

The model is fit with Stan (Stan Development Team, 2022a), an open-source software that uses Hamiltonian Monte Carlo (HMC) methods to obtain draws from the posterior distribution of model parameters. We access Stan through RStudio and the R package 'rstan' (Stan Development Team, 2022b), which provides an interface for integrating Stan code into the R programming language.

We run the model on four chains with 4500 iterations for each chain, 2000 of which are used as the 'burn-in' stage. Thus, we obtain 10000 total posterior draws for our model parameters. We denote $\boldsymbol{\theta}^{(t)}, \boldsymbol{\beta}^{(t)}$ as a posterior draw provided from Stan's HMC simulations for our model where $t = 1, \ldots, 10000$. Thus, posterior means for model parameters can be expressed as $\hat{\boldsymbol{\theta}} = \frac{1}{10000}\sum_{t=1}^{10000}\boldsymbol{\theta}^{(t)}$ and $\hat{\boldsymbol{\beta}} = \frac{1}{10000}\sum_{t=1}^{10000}\boldsymbol{\beta}^{(t)}$.

Under this specification, it took approximately 36 h to run on a 16-core MacBook Pro with 64 GB of RAM. The posterior samples from our model are stored and used for forecasting the draft as described in Section 3.5.3.

3.5.3. Predictive distributions

The desired output of the ROL model for the NHL draft is an estimate of the joint probability distribution of future draft picks. To do this, we wish to obtain the predictive distribution, which averages the density of future data over the posterior densities of the unknown model parameters.

Recall that the posterior distribution, $\pi(\boldsymbol{\theta}, \boldsymbol{\beta} \mid \boldsymbol{Z})$, corresponds to the ability parameters and team and agency preferences based on past ranking sets and draft results. The data $\boldsymbol{Z}$ corresponds to all previous ranking sets and draft results, as well as the results of the current draft up to the time of interest. We formally define $\boldsymbol{Z}$ as $\boldsymbol{Z} = \{\boldsymbol{S}, \boldsymbol{A}, \boldsymbol{X}\}$ where $\boldsymbol{S} = \{s_{irk}, \forall i, r, k\}$ represents the set of all players ranked across all ranking sets and draft results, $\boldsymbol{A} = \{a_{irk}, \forall i, r, k\}$ represents the set of all agencies or teams that provided the selections for each ranking across all ranking sets and draft results, and $\boldsymbol{X}$ represents the $M \times p$ design matrix which has elements x_{ij} corresponding to player i's value of covariate j where $M = \sum_{r\in R} N_r$. Fortunately, the Bayesian framework provides a convenient approach for prediction of future draft results, $\tilde{\boldsymbol{Z}}$. The predictive density of $\tilde{\boldsymbol{Z}}$ is given by

$$p(\tilde{\boldsymbol{Z}} \mid \boldsymbol{Z}) = \iint p(\tilde{\boldsymbol{Z}} \mid \boldsymbol{Z}, \boldsymbol{\theta}, \boldsymbol{\beta})\pi(\boldsymbol{\theta}, \boldsymbol{\beta} \mid \boldsymbol{Z})\, d\boldsymbol{\theta} d\boldsymbol{\beta} \tag{8}$$

where $p(\tilde{\boldsymbol{Z}} \mid \boldsymbol{Z}, \boldsymbol{\theta}, \boldsymbol{\beta})$ is the joint probability mass function across all remaining draft picks, given $\boldsymbol{Z}, \boldsymbol{\theta}, \boldsymbol{\beta}$.

As a by-product of MCMC, we generate realizations of the predictive distribution by first generating $(\boldsymbol{\theta}^{(t)}, \boldsymbol{\beta}^{(t)})$, $t = 1, \ldots, 10000$ from the posterior distribution, and then generating $\tilde{\boldsymbol{Z}}$ from $p(\tilde{\boldsymbol{Z}} \mid \boldsymbol{Z}, \boldsymbol{\theta}^{(t)}, \boldsymbol{\beta}^{(t)})$. This is done by

Table 5
List of teams and corresponding abbreviations.

Team	Abbreviation	Team	Abbreviation
Anaheim Ducks	ANA	Nashville Predators	NSH
Arizona Coyotes	ARI	New Jersey Devils	NJ
Boston Bruins	BOS	New York Islanders	NYI
Buffalo Sabres	BUF	New York Rangers	NYR
Calgary Flames	CGY	Ottawa Senators	OTT
Carolina Hurricanes	CAR	Philadelphia Flyers	PHI
Chicago Blackhawks	CHI	Pittsburgh Penguins	PIT
Colorado Avalanche	COL	San Jose Sharks	SJ
Columbus Blue Jackets	CBJ	Seattle Kraken	SEA
Dallas Stars	DAL	St. Louis Blues	STL
Detroit Red Wings	DET	Tampa Bay Lightning	TB
Edmonton Oilers	EDM	Toronto Maple Leafs	TOR
Florida Panthers	FLA	Vancouver Canucks	VAN
Los Angeles Kings	LA	Vegas Golden Knights	VGK
Minnesota Wild	MIN	Washington Capitals	WSH
Montreal Canadiens	MTL	Winnipeg Jets	WPG

repeatedly simulating with a diminishing choice set that removes players as they are ‘selected’ in the simulation. Thus, we do not directly solve integral (8) but rather approximate it using posterior samples.

Specifically, let $\pi_{ijm}^{(t)} = \exp\{\eta_{ij}^{(t)}\}/\sum_{\ell=i}^{N_r}\exp\{\eta_{\ell j}^{(t)}\}$ be the probability that player i is selected by team j with the mth pick in the draft, given that the previous $m-1$ picks are known where $\eta_{ij}^{(t)}$ is the linear predictor as defined in (5) under the tth draw from the posterior. Suppose we wish to predict the remainder of the draft beginning with draft pick m, where $m = 1, \ldots, n_r$ and S'_m is the set of size $N_r - (m-1)$ players that have not been selected. We obtain a single iteration of the remaining draft by (i) selecting a player according to the set of multinomial choice probabilities, $(\pi_{ijm}^{(t)})_{i\in S'_m}$, (ii) removing the selected player from S'_m and adding him to the selected set of players $\tilde{\boldsymbol{Z}}^{(t)}$, and (iii) repeating (i) and (ii) until n_r draft selections are made. By repeating this process, $t = 1, \ldots, 10000$, we can estimate the probability of the remaining draft $\tilde{\boldsymbol{Z}}$ as expressed in (8). Despite the slow computational time fitting the model, it takes approximately 12 s to run 10000 simulations of the next 32 picks and 25 s to run 10000 simulations on the entire draft.

4. Results

We now explore the results of the ROL model applied to the ranking set and draft results data from the 2019 to 2022 NHL drafts as described in Section 2.1. All models were fit by modifying code published by Stokes (2022). Tables 1 and 5 provide agencies and teams, respectively, considered in the model and corresponding abbreviations.

4.1. Without agency and team tendencies

We begin exploring the ROL model *without* considering agency and team tendencies. Here, we use the likelihood derived in (7) with linear predictor $\eta_{ij} = \theta_i$ rather than (5). Thus, we estimate the posterior distribution of the player ability parameters without considering agency or team tendencies. Since each draft class contains a completely new pool of players, we independently fit the predictive models for each draft class. This section focuses primarily on the 2022 draft, with the 2021 draft used to demonstrate how our model can identify differences across draft classes.

Fig. 1 displays posterior summaries for the ability parameters of the top 32 players in the 2022 NHL draft. The posterior draws are used to form the 95% credible intervals. We obtain a consolidated draft ranking by ordering players by the posterior mean of their ability parameter.

We observe large gaps between the posterior means of Shane Wright and Logan Cooley, as well as David Jiricek and Joakim Kemell. These are due to the natural tiers that form in the draft when ranking sets consistently rank a group of players above those below them. For example, agencies consistently ranked Wright, Cooley, Slafkovsky, Nemec and Jiricek as the top five in the draft, while Kemell and Savoie rarely achieved this feat. Thus, despite being ranked 6th and 7th, respectively, the model estimates a relatively large gap between them and the top five.

Figs. 2(a) and 2(b) illustrate the estimated predictive probabilities that a player is selected at any pick as described in Section 3.5.3. Here, we look at the top 32 players and 32 picks in the 2021 and 2022 drafts immediately prior to the drafts. We refer to this visual as the player-pick probability mass function (pmf) since it represents the joint pmf of players’ draft positions. The ith row is the pick pmf for player i up until pick 32, which describes the marginal pmf for player i’s selection position for any pick in the top 32.

We observe that each draft class displays unique trends. In 2021, there was uncertainty towards the top of the draft as there was variation among agencies in ranking the top six players. In 2022, the majority of agencies ranked Shane Wright 1st; consequently, Wright had a very high probability of being selected 1st overall. Additionally, we observe the same tiers in the 2022 draft on the pick probability scale compared to Fig. 1 depicting the ability parameters. This feature allows teams to understand where large perceived drops in pick value occur and strategize accordingly.

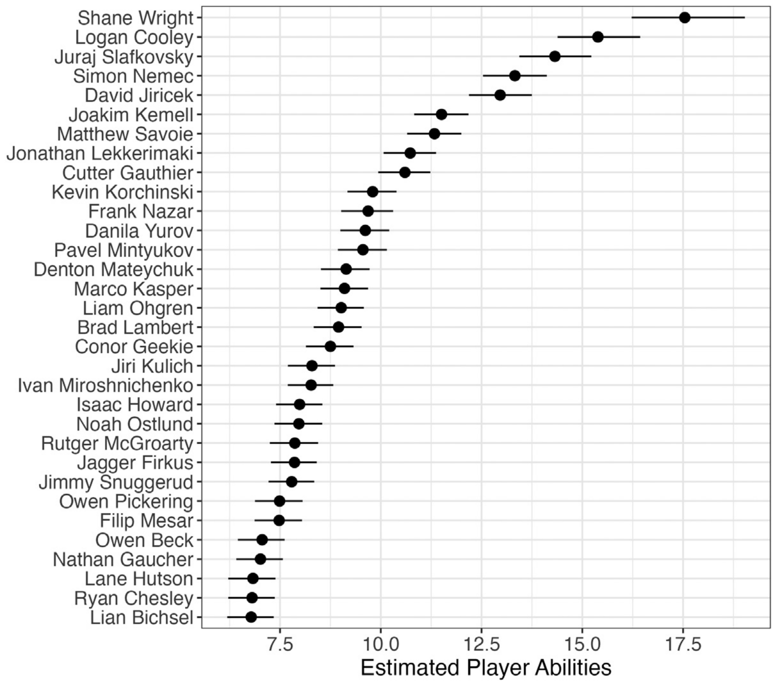


Fig. 1. Posterior summaries of the player ability parameters $\boldsymbol{\theta}$, for the top 32 of the 601 draft-eligible players in the 2022 NHL draft. Players are listed in descending order based on posterior mean estimates of $\boldsymbol{\theta}$. Points represent the posterior means of the θ_i; lines represent the corresponding 95% credible intervals.

Once the first $m-1$ picks are known, we can use the same process to simulate forward from the mth pick and obtain the updated probabilities conditional on the known picks. Figs. 3(a) and 3(b) display the player-pick pmfs of the 2022 draft after the first three picks in two different scenarios: (a) the actual observed NHL draft results of Slafkovsky, Nemec, and Cooley and (b) the most likely draft result by this model after three hypothesized picks of Wright, Cooley, and Slafkovsky.

In the 5th row of Figs. 3(a) and 3(b) we see the pick pmf of David Jiricek. Notice the change in Jiricek's pick pmfs in the two different scenarios. Under the observed draft order with Shane Wright still available, Jiricek has less than a 1% chance of being selected 4th. However, if Wright had been selected (Fig. 3(b)), and the best remaining player was Nemec, then Jiricek has approximately a 22% chance of being selected 4th. This example shows how the ROL model can dynamically update selection probabilities to account for changes in the pool of available players.

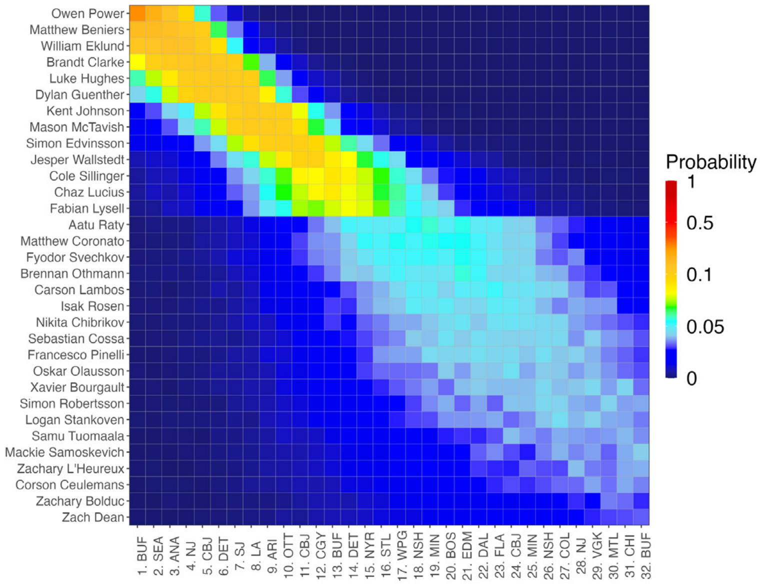
The player-pick pmfs can be used to provide teams with data-driven information when time-sensitive decisions must be made. As mentioned in Section 3.5.3, it takes approximately 12 s to obtain player-pick pmfs based on 10000 simulations of the next 32 picks in the draft, while each team has three minutes to make their selection.

4.2. With agency and team tendencies

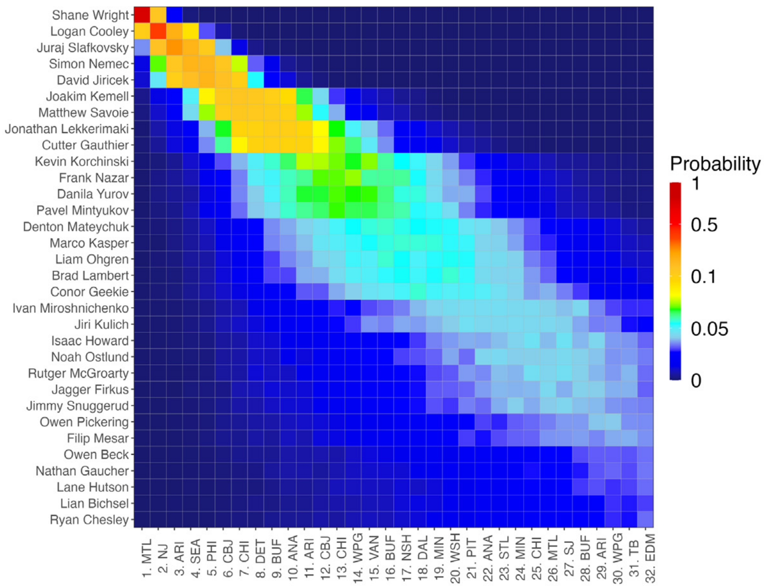
We now generate draft simulations based on posterior draws from the full model with agency and team tendencies as described in (5) and (7) where each parameter is constrained to zero across player or agency and team. Thus, we can leverage the same insights and features detailed in Section 4.1. Beyond that, we can also gain insights into the draft tendencies of teams and obtain more accurate estimates of pick probabilities.

We estimate agency and team tendency parameters for three covariates of interest:

- x_{i1}: an indicator for whether or not player i is an overager in the current draft year. An overager in the NHL draft is a player who was eligible but not selected in a prior NHL draft and is thus re-entering the draft as an 'overaged' player
- x_{i2}: proportion of games by player i in professional men's hockey during draft year (More specifically, x_{i2} is the weighted proportion of games played by player i in leagues with no age restriction during the draft year. This value is calculated by adding the proportion of games played in leagues with no age restriction and 0.5 times the proportion of games played in college hockey leagues such as NCAA and U-Sports where there is no hard age restriction, but the age distribution is skewed towards players in their late teens or early twenties.)



(a) 2021 NHL Draft



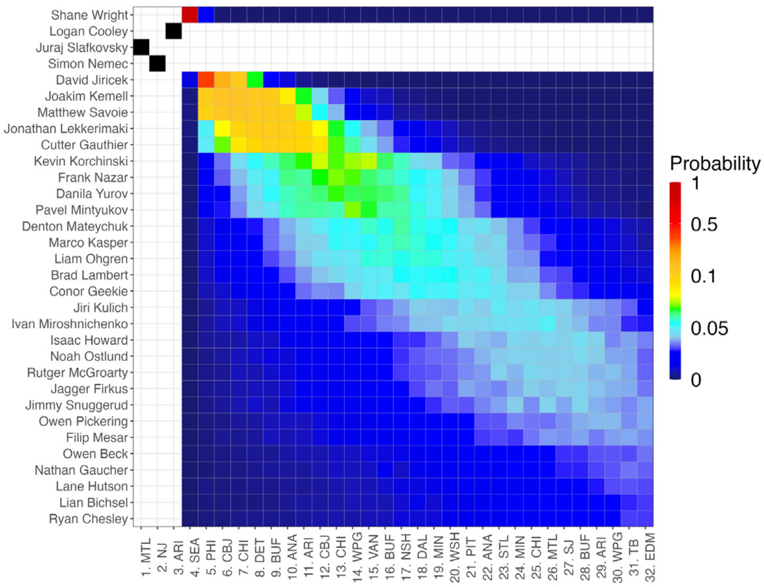
(b) 2022 NHL Draft

Fig. 2. Player-pick pmfs based on draft simulations immediately before the draft without considering agency and team tendencies. The horizontal axis (pick number) also indicates the team allocated the pick. Players are listed in descending order based on ability parameters $\boldsymbol{\theta}$.

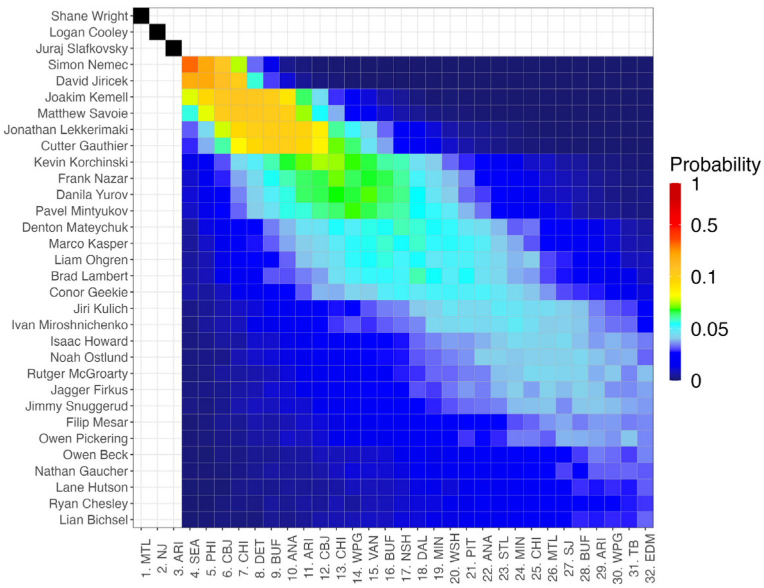
- x_{i3}: height of player i, z-scored by position and draft year

In Fig. 4, we display the posterior means of the tendency parameters for each agency (blue) and team (red) along with horizontal lines to represent the average of the posterior means for both types. The plots identify differences in draft tendencies between agencies and teams as well as highlight notable teams with respect to each covariate. From these plots, we observe that teams value height more than agencies, agencies value professional experience slightly more than teams, and overaged players are less valued by agencies than teams.

Fig. 5 shows the player-pick pmfs after incorporating agency and team tendencies for the beginning of the 2022 draft. The probabilities are not as smooth as in Fig. 2. However, they adjust for past draft tendencies that other teams have shown. For example, the probability that Lian Bichsel is selected spikes at Chicago's 13th and 25th overall picks. In Fig. 4, we see that Chicago values height and experience tendency parameters. Notably, Lian Bichsel is



(a) Using the observed outcome for the top three picks



(b) Using the most likely outcome for the top three picks

Fig. 3. Player-pick pmfs based on draft simulations from the ROL draft model without agency and team tendencies after the first three picks of the draft. The horizontal axis (pick number) also indicates the team allocated the pick.

a 6'4'' defenceman who played 72.5% of his games in the Swedish Hockey League, Sweden's top professional league.

4.3. Wyatt johnston: A star in the making?

In the 2021 draft, the Dallas Stars took a risk. Although they coveted Wyatt Johnston (Nill, 2021), believing that he may develop into a star, Dallas traded their 15th pick to the Detroit Red Wings in exchange for the 23rd pick, the 48th pick and the 138th pick. Dallas gambled that Johnston would be available for selection with the 23rd pick, and he was. At the time of the draft, NHL Central Scouting had ranked Johnston 16th among draft-eligible North American skaters. According to Fyffe (2011), Johnstone would have been ranked 21st overall.

Using the approach described in Section 3.5.3 and recognizing the trade, we can estimate Wyatt Johnston's probability of selection for any specified pick given the information available immediately prior to the 15th pick.

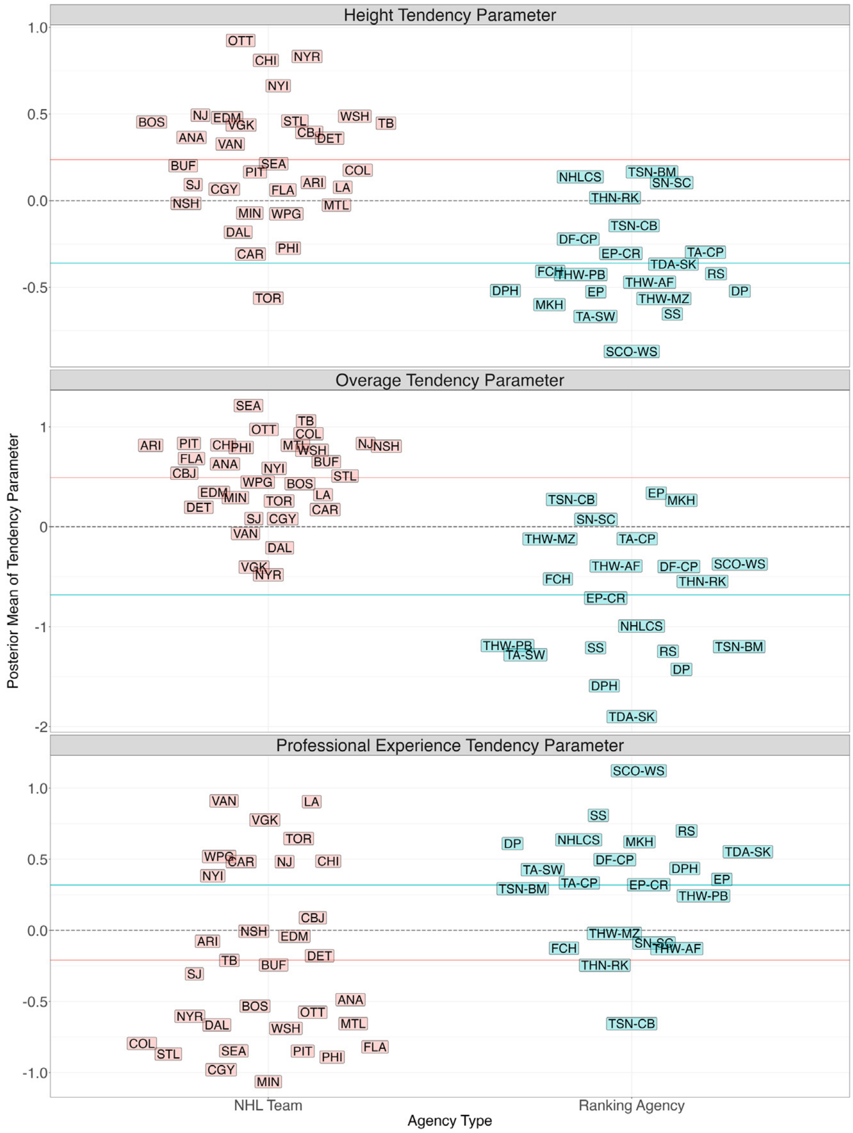


Fig. 4. A visualization of the posterior means for agency and team tendency parameters; top, middle and bottom represent the height, overager, and professional experience tendency parameters, respectively. The *y*-axis represents the posterior means of parameters, and the *x*-axis differentiates teams and agencies with meaningless jitter to separate individual teams and agencies. The red lines represent the average of the posterior means for teams, while the blue lines represent the average of the posterior means for agencies. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The estimated probabilities are conditional on all picks that have already been observed but unconditional on all picks that occur from the current pick-up to the pick of interest. Thus, by summing the probabilities that he is selected in all prior picks, we can estimate the probability that Johnston is available by the specified pick.

Fig. 6 provides the pick cumulative distribution function (cdf) for Johnston over the first three rounds of the NHL draft, given the first 14 known picks. If Dallas had not made the trade with Detroit, there was a 100% chance that Johnston was available. However, given that he was traded, the next draft opportunity for Dallas was the 23rd pick, where there was a 97.9% chance that Johnston was available. Had Dallas waited further to their 47th pick to select Johnston, there remained a good chance (81.7%) that Johnston was available. Assuming Johnston was the best player on their draft board at pick 15 as the Stars' general manager Jim Nill suggested (Nill, 2021) - the Stars acquired picks 48 and 138 for only an estimated 2.1% drop in probability of acquiring Wyatt Johnston. It seems that Dallas was clever and fortunate in their decision-making.

4.4. Model evaluation

To evaluate the various ROL models, we compare the accuracy of the 2022 NHL draft predictions to a pair of notable ranking sets:

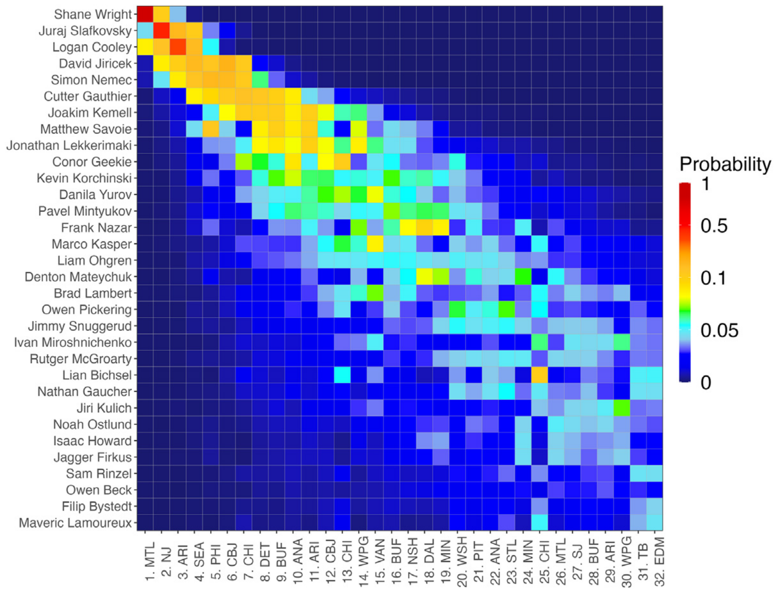


Fig. 5. Player-pick pmfs based on draft simulations from the ROL draft model with agency and team tendencies at the start of the 2022 NHL draft. The horizontal axis (pick number) also indicates the team that was allocated the pick.

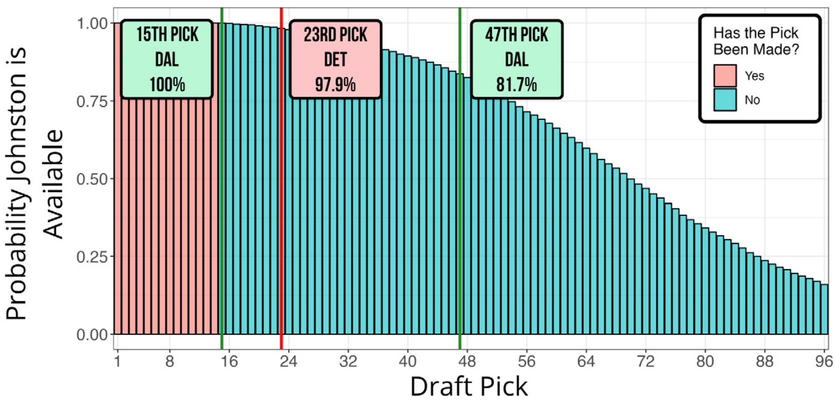


Fig. 6. The pick cdf for Wyatt Johnston during the 2021 NHL draft. Markers represent the probability that Johnston is available at the current pick (15), the 23rd pick and the 48th pick.

- TSN Bob McKenzie's final ranking set (TSN): McKenzie's list is highly regarded in the hockey community for its accuracy in predicting the draft results. This ranking set contains 90 players.
- SB Nation Jared Book's consolidated ranking set (SBN)[1]; Book obtains average rankings for draft-eligible players from a collection of 15 different sources. This ranking set contains 154 players.

We obtain a rank ordering from the ROL models by calculating the expected value based on each player's pick pmf and then ranking all players according to the expected value. We do this for two ROL models: with agency and team tendencies (ROL-WT) and without agency and team tendencies (ROL-NT).

We then compute the Spearman (Spearman, 1904) and Kendall (Kendall, 1955) correlation coefficients for the observed 2022 NHL draft results versus each of the four rank orderings (TSN, SBN, ROL-WT, ROL-NT). For this analysis, we consider the players who formed the first $n = 64$ and $n = 96$ draft positions in the NHL 2022 draft. We note that TSN and SBN did not provide extensive ranking sets. Therefore, we assign tied ranks to drafted players unranked by TSN or SBN, where the tied rank is the max N_r of those evaluated.

Table 6 provides the correlation results for the four ranking sets for the two choices of n. With $n = 64$, the ROL model with agency and team tendencies appears to provide a slight edge in predictive performance. On the other hand, the ROL model without agency and team tendencies does not perform as well as the agency rankings. With $n = 96$, there appears to be a preference towards McKenzie's TSN rankings.

1 We do not use Book's consolidated rankings in the model fitting process due to its clear dependence on other ranking sets.

Table 6
The Spearman and Kendall rank correlation coefficients between the first n picks in the 2022 NHL draft and the ROL model with agency and team tendencies (ROL-WT), the ROL model without agency and team tendencies (ROL-NT), TSN Bob McKenzie's final rankings (TSN), and SB Nation Jared Book's consolidated rankings (SBN).

n	Ranking set	Spearman	Kendall
$n = 64$	ROL-WT	0.872	0.687
$n = 64$	ROL-NT	0.818	0.619
$n = 64$	TSN	0.868	0.686
$n = 64$	SBN	0.808	0.615
$n = 96$	ROL-WT	0.862	0.690
$n = 96$	ROL-NT	0.826	0.643
$n = 96$	TSN	0.884	0.718
$n = 96$	SBN	0.842	0.658

The increase in correlation obtained by incorporating agency and team tendencies into the model with three simple covariates provides us with a boost beyond Bob McKenzie's highly-regarded ranking set and the consolidated rankings. This model will help teams gain more accurate and transparent estimates of how long a player will remain available in the NHL draft and, in turn, can help teams gain a competitive edge during the draft.

5. Discussion

In this paper, we introduce a novel framework for modelling the outcome of the NHL draft that overcomes many obstacles not tackled by previous approaches. We leverage a Bayesian rank-ordered logit model obtained as a product of multinomial logit densities. The primary goal is the estimation of the probabilities of player selection at specified draft picks. This is accomplished via simulation. We obtain real-time probabilities within a draft conditional on previous picks that have been observed, address unranked players, subset rankings into the model likelihood, and incorporate team and agency tendencies. The work provides insights for strategies involving draft selection.

There are still areas for improvement that may enhance model performance. For example, future work may consider an investigation into both the prior specification and the determination of model covariates. Specifically, we may consider covariates for team needs and changes in team personnel. We may also investigate the dependence between ranking sets and consider alternative model evaluation criteria which involve more years of data. Applying the ROL model framework could also be extended to other sports involving drafts, such as the NFL, NBA and MLB. However, the two main limitations of the proposed framework are computational feasibility and assumptions relating to team tendencies. We currently require over 24 h of computation to fit the model with a sufficient number of iterations. Therefore, the investigation of model improvements can be time-consuming. Additionally, the model assumes that team tendencies will remain the same throughout the draft. This is not necessarily the case, as teams will likely adapt their preferences as they satisfy team needs. For example, if a team drafts a goalie with their 1st round pick, we suspect they are less likely to select a goalie with future picks.

We view the proposed ROL model as a state-of-the-art approach for modelling sports drafts. Teams that implement the framework can gain insights into their opponent's draft strategies and have a data-driven approach for estimating a player's pick selection number.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Yes, I (Tim Swartz) serve as an AE for IJF.

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