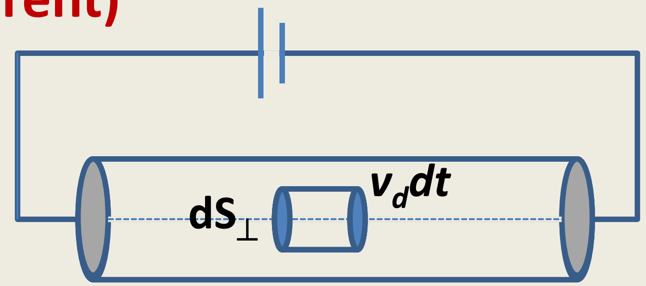
Magnetostatics

Eelectrostatics : charges at rest

Magnetostatics : ??????

Magnetic field is produced by charges in flow (current)



n : number density; no. of charge carriers per unit volume

v_d : additional velocity of charge carriers due to applied p.d.

$dS_\perp$: area of the pillbox $\perp$ v

$v_d\, dt$: length of pillbox

Charge in the pillbox $dQ = qn(v_d dt\, dS_\perp)$

Current flowing in the pillbox $i = \frac{dQ}{dt} = qnv_d dS_\perp$

J : current density

Current density $J = \frac{i}{dS_\perp} = qnv_d = \rho v_d$

ρ = volume charge density (number of charges per unit volume

J : volume current density = dI/dS$_\perp$

$$I = \int \vec{J}.\hat{n}\, dS$$

$\hat{n}$ unit vector normal to the surface.

A current carrying wire with current density $\vec{J} = C\, r\, \hat{z}$ (i.e, current along z direction)

C : constant,
r : radial distance from the axis of the cylinder.

$$I = \int \vec{J}.\hat{n}\, dS = \int_0^R Cr\,(r dr d\theta) = \frac{2\pi C R^3}{3}$$

Surface current density, $K = \dfrac{dI}{dl_\perp} = \sigma v$

Similarly, $I = \lambda v$

Magnetic force on a current carrying wire in a magnetic field *B*

$$F_{mag} = \int (\lambda\, dl)(\vec{v} \times \vec{B}) = \int (\vec{I} \times \vec{B})\, dl$$

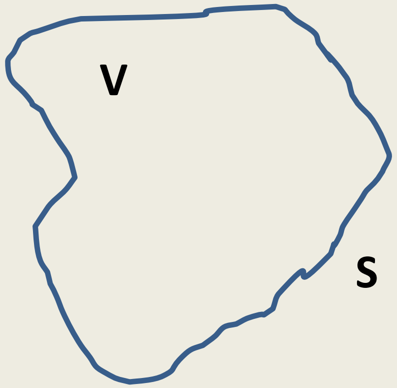
$$= \int I(\vec{dl} \times \vec{B}) = I \int (\vec{dl} \times \vec{B})$$

$$F_{mag} = \int (\vec{K} \times \vec{B})\, dS$$

$$F_{mag} = \int (\vec{J} \times \vec{B})\, d\tau$$

Continuity Equation and conservation of charge

Surface **S** encloses volume **V** with total charge ***Q***



Rate at which charges flow out = Rate at which charges deplete in the volume

$$\oiint_S \vec{J}.\hat{n}\, dS = -\frac{d}{dt} Q_{\text{inside}} = -\frac{d}{dt}\int_\tau \rho\, d\tau$$

$$\int_\tau (\vec{\nabla}\cdot\vec{J})\, d\tau = -\int_\tau \frac{d\rho}{dt} d\tau \qquad \vec{\nabla}\cdot\vec{J} = -\frac{d\rho}{dt}$$

Continuity equation
Conservation of charge

$$\text{IF } \frac{d\rho}{dt} = 0, \quad \vec{\nabla}\cdot\vec{J} = 0 \Longrightarrow$$ **Steady currents**

$\Longrightarrow$ **Magneto-statics**

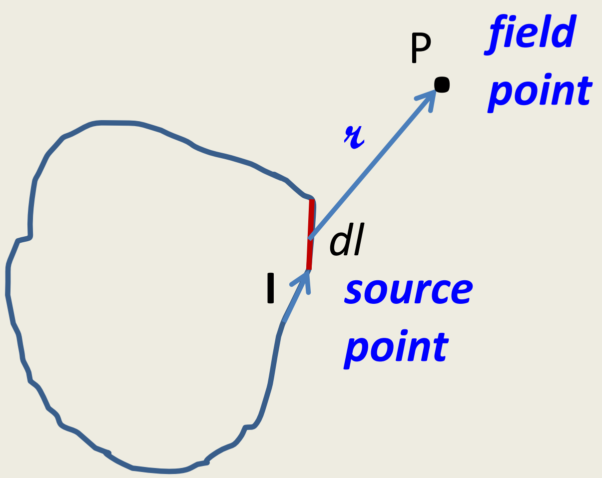
Biot-Savart's Law

For steady currents,

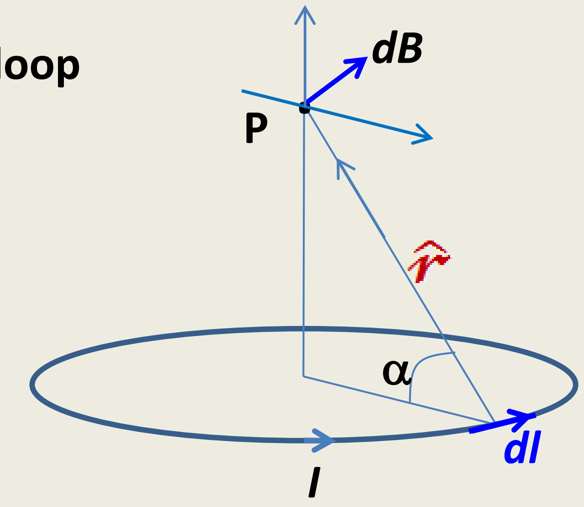
$$\vec{B}(P) = \frac{\mu_0 \, I}{4\pi} \int_L \frac{\vec{dl} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{B}(P) = \frac{\mu_0}{4\pi} \int_L \frac{\vec{K} \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, \mathrm{ds}$$

$$\vec{B}(P) = \frac{\mu_0}{4\pi} \int_L \frac{\vec{J} \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, \mathrm{d}\tau$$



Magnetic field along the axis due to current carrying circular loop



$$\hat{r} = -\hat{\rho} + \hat{k}$$

$$\vec{dl} \times \hat{r} = dl(\hat{k} + \hat{\rho})$$

$$\vec{dl} = dl\,\hat{\phi}$$

$$\vec{B}(P) = \frac{\mu_0\, I}{4\pi} \int_L \frac{\vec{dl} \times \hat{r}}{r^2} = \hat{k}\,\frac{\mu_0\, I}{4\pi}\,\frac{\cos\alpha}{r^2} \int_L dl$$

$$= \hat{k}\,\frac{\mu_0\, I}{2}\,\frac{R^2}{r^3} = \hat{k}\,\frac{\mu_0\, I}{2}\,\frac{R^2}{(R^2 + z^2)^{\frac{3}{2}}}$$

Magnetic field at the centre of the ring

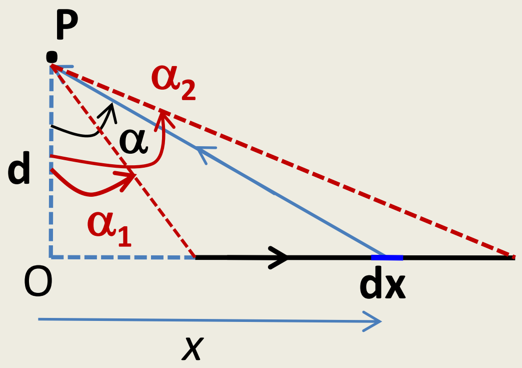
$$\vec{B} = \hat{k}\,\frac{\mu_0\, I}{2R}$$

Magnetic field due to current carrying wire (finite length)

$$\vec{dl} = dx\,\hat{\imath}$$

$$\hat{r} = -\hat{\imath} + \hat{\jmath}$$

$$\vec{dl} \times \hat{r} = dx\,\hat{k}$$



$$\vec{B}(P) = \frac{\mu_0\, I}{4\pi} \int_L \frac{\vec{dl} \times \hat{r}}{r^2} = \hat{k}\,\frac{\mu_0\, I}{4\pi} \int_L \frac{dx}{x^2 + d^2}$$

$$\vec{B}(P) = \hat{k}\,\frac{\mu_0\, I}{4\pi d} \int_{\alpha_1}^{\alpha_2} \cos\alpha\, d\alpha = \hat{k}\,\frac{\mu_0\, I}{4\pi d}(\sin\alpha_2 - \sin\alpha_1)$$

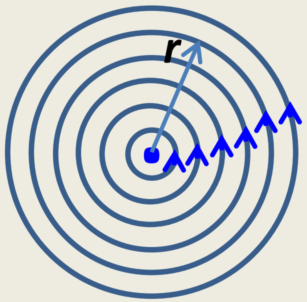
For an infinite wire,

$$\alpha_2 = \frac{\pi}{2}, \quad \alpha_1 = -\frac{\pi}{2} \qquad \vec{B}(P) = \hat{k}\,\frac{\mu_0\, I}{2\pi d}$$

direction : right hand thumb rule

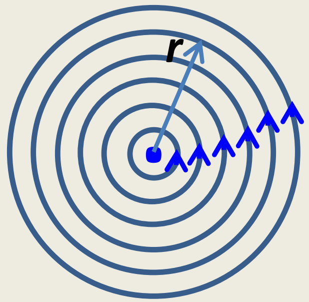
$$\vec{B} = \hat{\phi}\,\frac{\mu_0\, I}{2\pi d}, \quad \text{IF} \quad I = I\,\hat{k}$$

Take $\oint \vec{B} \cdot \vec{dl}$ along a circular loop of radius r



$\vec{B} = \hat{\phi}\,\frac{\mu_0\, I}{2\pi d}$, IF $I = I\,\hat{k}$

Take $\oint \vec{B} \cdot \vec{dl}$ along a circular loop of radius r



$$\oint \vec{B} \cdot \vec{dl} = \oint \frac{\mu_0\, I}{2\pi r}\,\hat{\phi} \cdot dl\hat{\phi} = \frac{\mu_0\, I}{2\pi r}\,2\pi r = \mu_0\, I$$

Need not be a circular loop as long as the loop goes around the current

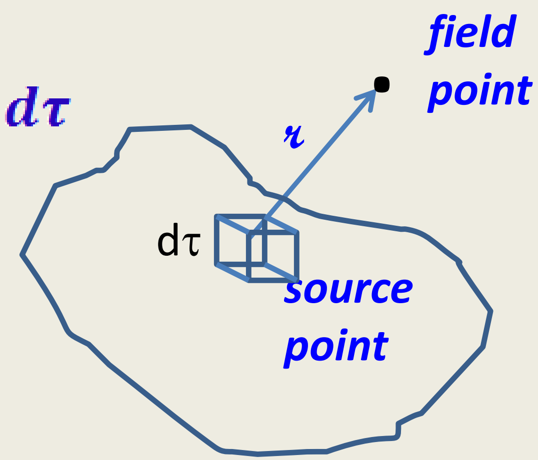
$$\oint \vec{B} \cdot \vec{dl} = \oint \frac{\mu_0\, I}{2\pi r}\,\hat{\phi} \cdot (dr\hat{r} + rd\phi\hat{\phi} + dz\hat{z})$$

$$= \frac{\mu_0\, I}{2\pi}\,2\pi = \mu_0\, I_{\text{encl}}$$

$$\oint (\vec{\nabla} \times \vec{B}) \cdot \vec{dS} = \mu_0 \iint \vec{J} \cdot \vec{dS} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

What about other current configurations?

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}\times\frac{\mu_0}{4\pi}\int_\tau \frac{\vec{j}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau = \frac{\mu_0}{4\pi}\int_\tau \vec{\nabla}\times\left(\vec{j}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)d\tau$$



$$B = B(x, y, z) \qquad J = J(x', y', z')$$

$$d\tau = dx'dy'dz'$$

$$\vec{\mathscr{r}} = (x - x')\hat{\imath} + (y - y')\hat{\jmath} + (z - z')\hat{k}$$

$$\vec{\nabla}\times\left(\vec{j}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = \vec{j}\underbrace{\left(\vec{\nabla}\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)}_{4\pi\delta^3(\mathscr{r})} - \cancelto{0}{(\vec{j}\cdot\vec{\nabla})\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}}$$

$$\vec{\nabla}\times\vec{B} = \frac{\mu_0}{4\pi}\int_\tau J(\mathscr{r}')4\pi\delta^3(\mathscr{r} - \mathscr{r}')\ d\tau = \mu_0 J(\mathscr{r})$$

$$\vec{\nabla}\cdot\left(\vec{j}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\cdot\cancelto{0}{(\vec{\nabla}\times\vec{j})} - \vec{j}\cdot\cancelto{0}{\left(\vec{\nabla}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)} \qquad \vec{\nabla}\cdot\vec{B} = 0$$

Ampere's Law : Applications

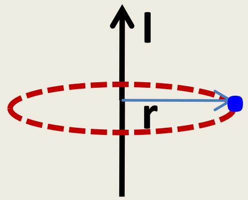
Magnetic field at a distance r from the current carrying wire (infinite length)

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \qquad \oint \vec{B} \cdot \vec{dl} = \mu_0 \, I_{\text{encl}}$$

$$\vec{B} = B\hat{\phi}, \quad \vec{dl} = dl \, \hat{\phi}$$

$$\oint \vec{B} \cdot \vec{dl} = B \oint dl = B \, 2\pi r = \mu_0 \, I_{\text{encl}}$$

$$\vec{B} = \frac{\mu_0 \, I}{2\pi r} \hat{\phi}$$



Amperian loop

Symmetries involved:

Straight line currents
Cylinder
Plane sheets
Solenoids and torroids

Ampere's Law : Applications

A current carrying wire with current density $J = Cr$

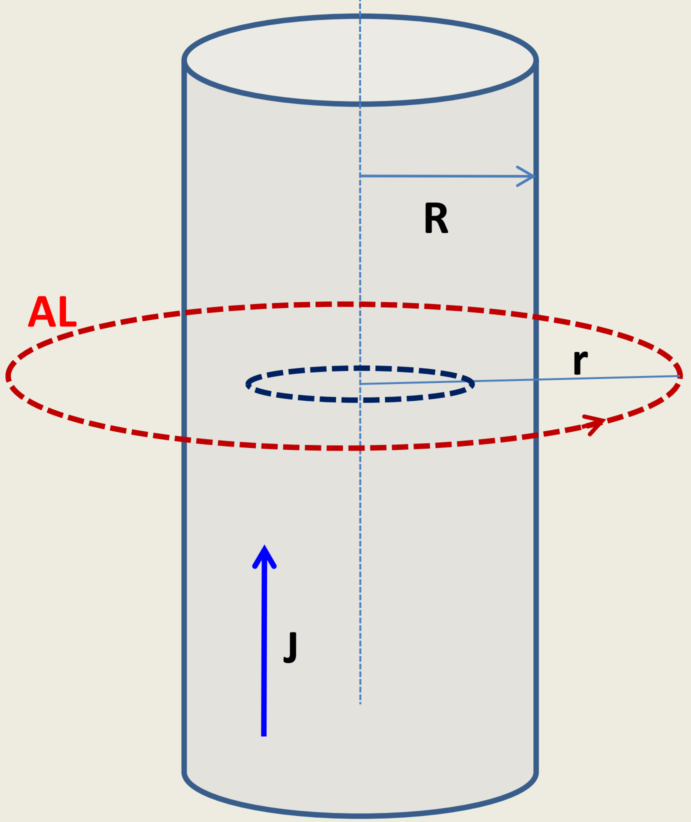
C : constant,
r : radial distance from the axis of the cylinder.

$$\oint \vec{B} \cdot \vec{dl} = B \oint dl = B\, 2\pi r = \mu_0\, I_{\text{encl}}$$

$$I = \int \vec{J}.\hat{n}\, dS = \int_0^R Cr\,(r dr d\theta) = \frac{2\pi C R^3}{3}$$

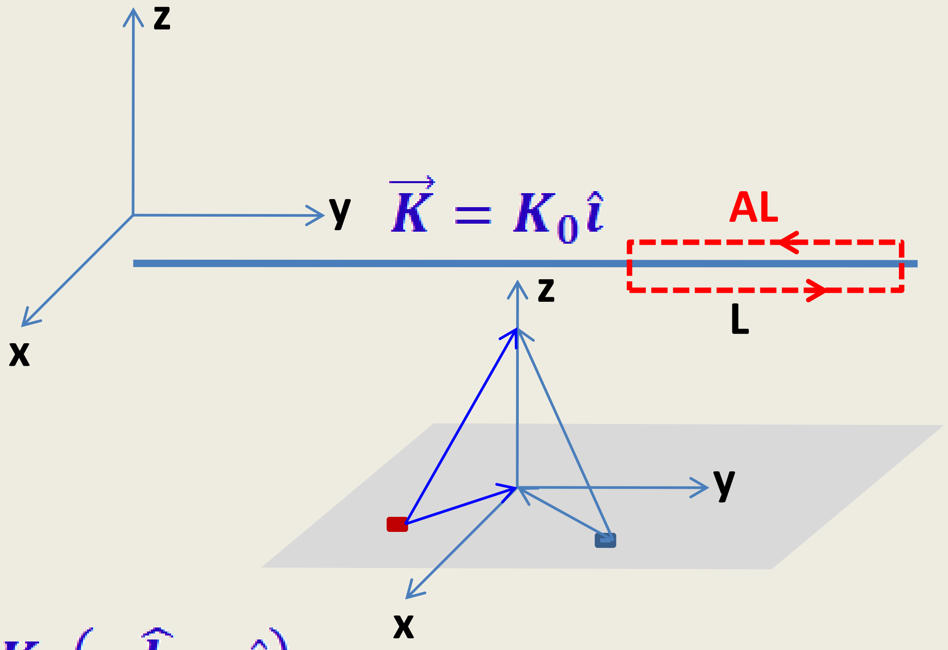
$$B\, 2\pi r = \mu_0 \frac{2\pi C R^3}{3} \qquad \vec{B}_{\text{out}} = \frac{\mu_0 C R^3}{3r}\hat{\phi}$$

$$B_{\text{in}}\, 2\pi r = \mu_0 \frac{2\pi C r^3}{3} \qquad \vec{B}_{\text{in}} = \frac{\mu_0 C r^2}{3}\hat{\phi}$$



Ampere's Law : Applications

Sheet current (surface current)



$$\vec{B}(P) = \frac{\mu_0}{4\pi} \int_L \frac{\vec{K} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\hat{\mathscr{r}} = -\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\vec{K} \times \hat{\mathscr{r}} = K_0 \hat{\imath} \times (-\hat{\imath} - \hat{\jmath} + \hat{k}) = K_0(-\hat{k} - \hat{\jmath})$$

$$\hat{\mathscr{r}} = -\hat{\imath} + \hat{\jmath} + \hat{k}$$

$$\vec{K} \times \hat{\mathscr{r}} = K_0 \hat{\imath} \times (-\hat{\imath} + \hat{\jmath} + \hat{k}) = K_0(+\hat{k} - \hat{\jmath})$$

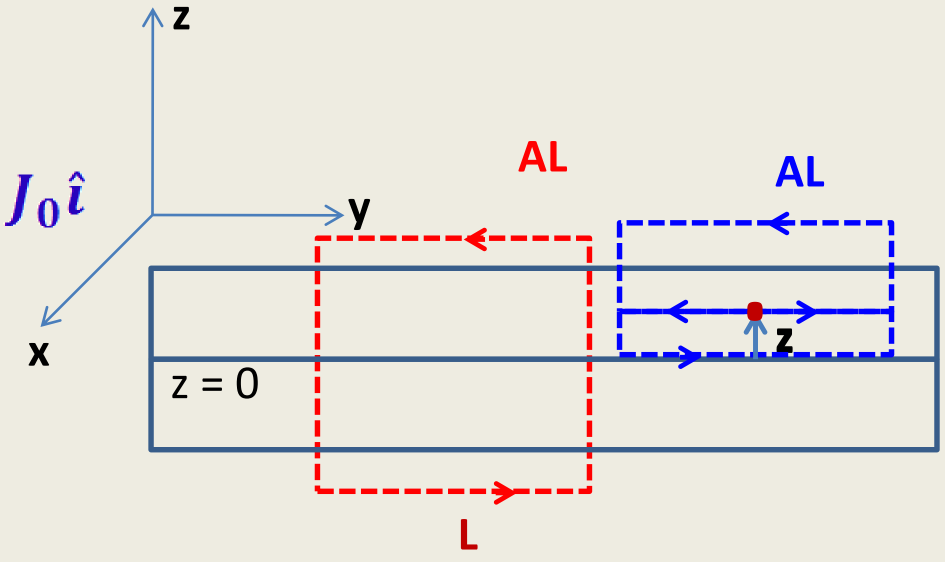
$$\oint \vec{B} \cdot \vec{dl} = B_{\text{above}} L + B_{\text{below}} L = \mu_0 K_0 L \qquad B_{\text{above}} = B_{\text{below}} = B$$

$$B_{\text{above}} = \frac{\mu_0 K_0}{2} (-\hat{\jmath}) \qquad B_{\text{below}} = \frac{\mu_0 K_0}{2} (+\hat{\jmath})$$

Ampere's Law : Applications

current carrying slab, thickness $2d$

$$\vec{J} = J_0 \hat{i}$$



B-outside is constant;
–y direction above and
+y direction below

$$\oint \vec{B} \cdot \overrightarrow{dl} = B\,2L = \mu_0 J_0 (2dL)$$

$$B_{\text{above}} = \mu_0 J_0 d(-\hat{j}) \qquad B_{\text{below}} = \mu_0 J_0 d(+\hat{j})$$

From symmetry, on z = 0 plane, B = 0.
–y direction above z = 0 and +y direction below z = 0

$$B_{\text{above}} L - BL = \mu_0 J_0 (d - z) L$$

$$\mu_0 J_0 d + B = \mu_0 J_0 (d - z)$$

$$B = \mu_0 J_0 z$$

Magnetic vector potential

Electrostatics

$$\vec{\nabla} \times \vec{E} = 0 \quad \Rightarrow \quad \vec{E} = -\vec{\nabla} V$$ scalar potential V

Magnetostatics

$$\vec{\nabla} \times \vec{B} \neq 0 \quad \Rightarrow \quad \vec{B} \neq -\vec{\nabla} ?$$ **NO scalar potential**

However,

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = \vec{\nabla} \times \vec{A}$$ $\vec{A}$ vector called Magnetic Vector Potential

Not as useful as *V*, since *A* is also a vector

However, much more useful quantity than *B* itself in em theory

$$\vec{\nabla} \times \vec{B} = \vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 J$$

To define $\vec{A}$ completely, we need to define $\vec{\nabla} \cdot \vec{A}$ also

For magnetostatics, we take $\vec{\nabla} \cdot \vec{A} = 0$ without affecting *B* : Coloumb gauge

In electrodynamics, Lorenz gauge, $\vec{\nabla} \cdot \vec{A} = -\mu_0 \varepsilon_0 \frac{\partial V}{\partial t}$

$\nabla^2 \vec{A} = -\mu_0 J$ three equations!

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \Rightarrow V = \frac{1}{4\pi\varepsilon_0} \int_\tau \frac{\rho d\tau}{r}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int_\tau \frac{\vec{J} d\tau}{r}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int_S \frac{\vec{K} dS}{r} \qquad \vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{\vec{dl}}{r}$$

Non – uniqueness of $\vec{A}$ Adding a scalar constant to *A* does not affect *B*

Assume vector potentials A and A' gives same B

$$\vec{\nabla} \times \vec{A} = \vec{B} = \vec{\nabla} \times \vec{A}'$$

$$\vec{A}' = \vec{A} + \vec{\nabla}\lambda$$

$$\vec{\nabla} \times (\vec{A}' - \vec{A}) = 0 \Rightarrow \vec{A}' - \vec{A} = \vec{\nabla}\lambda$$

Eg: $\vec{B} = B_0\hat{k}$

$$B_x = 0 \quad \Rightarrow \quad \frac{\partial A_z}{\partial y} = \frac{\partial A_y}{\partial z}$$

$$B_y = 0 \quad \Rightarrow \quad \frac{\partial A_x}{\partial z} = \frac{\partial A_z}{\partial x}$$

$$B_z = B_0 \quad \Rightarrow \quad \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = B_0$$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ A_x & A_y & A_z \end{vmatrix}$$

Solutions

$$A_z = 0, \qquad A_y = 0, \quad A_x = -B_0 y$$

$$A_z = 0, \qquad A_x = 0, \quad A_y = B_0 x$$

$$A_z = 0, \qquad A_y = \frac{1}{2}B_0 x, \quad A_x = -\frac{1}{2}B_0 y$$

OR a linear combination of these solutions!

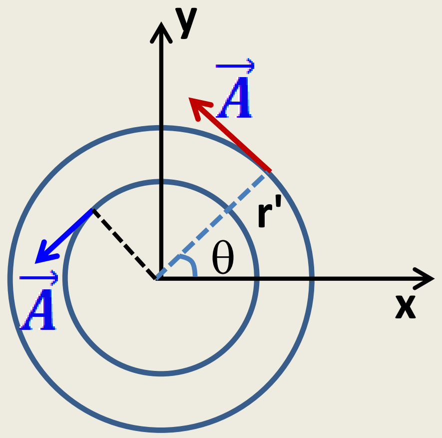
Even though A is not unique, as long the two conditions are satisfied,

$$\vec{\nabla} \times \vec{A} = \vec{B}$$
$$\vec{\nabla} \cdot \vec{A} = 0$$

we can make use of the solution

Consider the solution

$$A_z = 0, \qquad A_y = \frac{1}{2}B_0 x, \quad A_x = -\frac{1}{2}B_0 y$$



$$\vec{A} = (-\hat{\imath})\frac{1}{2}B_0 y + (\hat{\jmath})\frac{1}{2}B_0 x$$

$$= \frac{1}{2}B_0 r'(\sin\theta\,(-\hat{\imath}) + \cos\theta\,(\hat{\jmath}))$$

$$|\vec{A}| = \frac{1}{2}r' B_0 \quad \text{AND} \quad A \perp r' \quad \Rightarrow \vec{A} = \frac{1}{2}\vec{B} \times \vec{r'}$$

$$\text{IF } \vec{B} = B_0\hat{k}, \quad \text{THEN} \quad \vec{A} = A_0\hat{\phi}$$

A rotates about the z-axis

We can make use of this situation. If *B* is uniform along a particular direction, then *A* circulates around that direction

$$\oint \vec{A}\cdot\vec{dl} = \int_S (\vec{\nabla}\times\vec{A})\cdot\vec{dS} = \int_S \vec{B}\cdot\vec{dS} = \Phi$$

Ampere's law for vector potential

If *B* is uniform along a particular direction, circulation of *A* around a closed loop = flux of *B* enclosed by the loop

$$A\,2\pi r' = B_0\,\pi r'^2 \;\Rightarrow\; A = \frac{1}{2}B_0 r'$$

Vector potential for a long solenoid

Magnetic field inside and outside

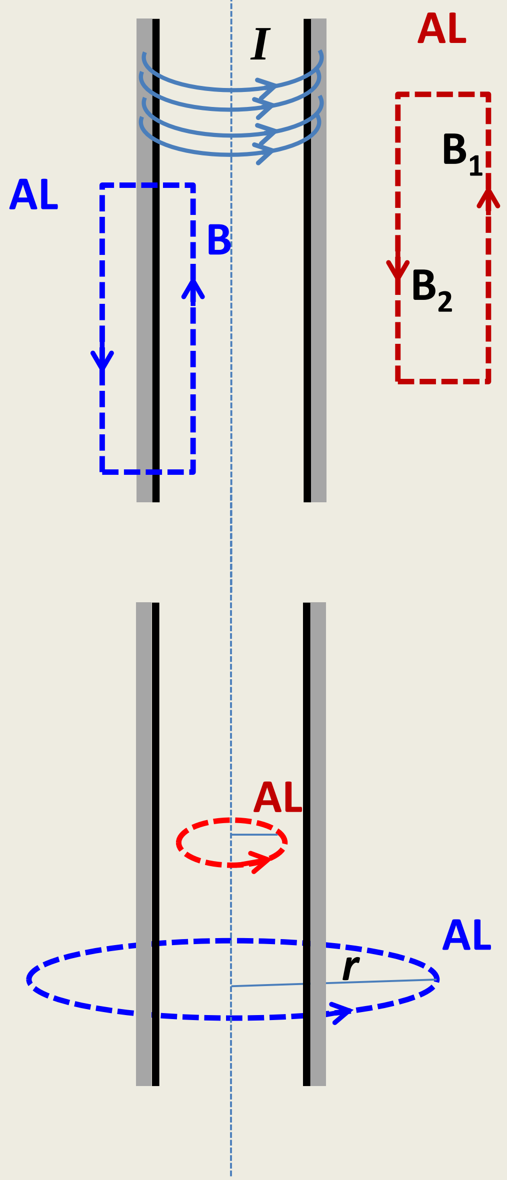
$$B_1 L - B_2 L = 0 \Rightarrow B_1 = B_2$$

$$r \to \infty, B = 0 \Rightarrow B_1 = B_2 = 0$$

$B_{out} = 0$

$$BL = \mu_0 NIL \Rightarrow \quad B = \mu_0 NI$$

B_{in} = constant



Vector potential

$$\oint \vec{A} \cdot \vec{dl} = \Phi \qquad A\, 2\pi r = B\, \pi r^2 = \mu_0 NI\, \pi r^2$$

$$\Rightarrow \quad \vec{A}_{in} = \frac{1}{2}\mu_0 NI r \hat{\phi}$$

$$A\, 2\pi r = \mu_0 NI\, \pi R^2$$

$$\Rightarrow \quad \vec{A}_{out} = \frac{\mu_0 NIR^2}{2r}\hat{\phi}$$

$$\vec{A}_{\text{in}} = \frac{1}{2}\mu_0 N I r \hat{\phi}$$

$$\vec{A}_{\text{out}} = \frac{\mu_0 N I R^2}{2r}\hat{\phi}$$

$$\vec{\nabla} \times \vec{A} = \frac{1}{r}\left(\frac{\partial}{\partial r} r A_\phi\right)\hat{k}$$

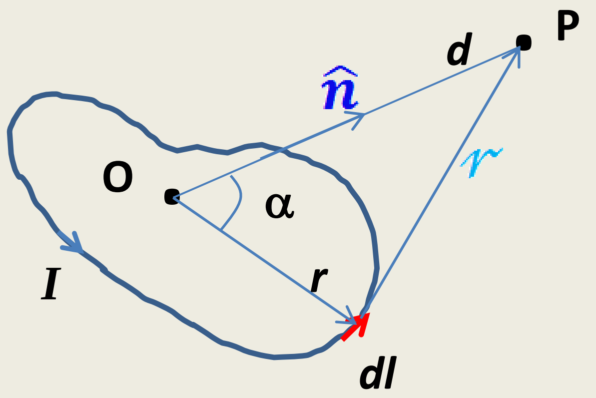
$$= \frac{1}{r}\left(\frac{\partial}{\partial r} r \frac{1}{2}\mu_0 N I r\right)\hat{k} = \mu_0 N I \hat{k}$$

$$= \frac{1}{r}\left(\frac{\partial}{\partial r} r \frac{\mu_0 N I R^2}{2r}\right)\hat{k} = 0$$

$$\vec{\nabla} \cdot \vec{A} = 0$$

$$\vec{\nabla} \cdot \vec{A} = 0$$

Multipole expansion for Vector potential



$$\frac{1}{r} = \frac{1}{\sqrt{r^2 + d^2 - 2dr\cos\alpha}}$$
$$= \frac{1}{d} + \frac{1}{d^2} r\cos\alpha + \frac{1}{d^3} r^2 \left(\frac{3}{2}\cos^2\alpha - \frac{1}{2}\right)$$
$$+ \cdots$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{\vec{dl}}{r} \quad = \frac{\mu_0 I}{4\pi} \left\{ \frac{1}{d} \oint \vec{dl} + \frac{1}{d^2} \oint r\cos\alpha\, \vec{dl} + \dots\dots \right\}$$

$$\oint \vec{dl} = 0$$

1st term (Monopole term) is ZERO

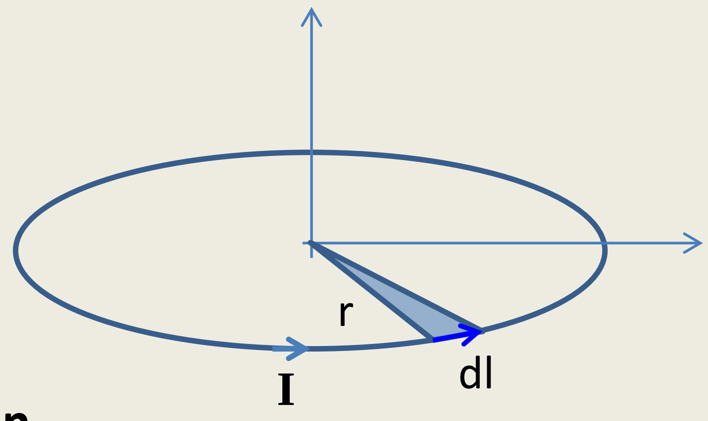
Dipole term

$$\vec{A}_{\text{dipole}} = \frac{\mu_0 I}{4\pi d^2} \oint r\cos\alpha\, \vec{dl} = \frac{\mu_0 I}{4\pi d^2} \oint (\vec{r}\cdot\hat{n})\vec{dl}$$

$$= \frac{\mu_0 I}{4\pi d^2} \left\{ -\frac{1}{2} \hat{n} \times \oint \vec{r} \times \vec{dl} \right\} = \frac{\mu_0}{4\pi d^2} (\vec{m} \times \hat{n})$$

Magnetic dipole moment

$$\vec{m} = \frac{I}{2}\oint \vec{r} \times \vec{dl}$$

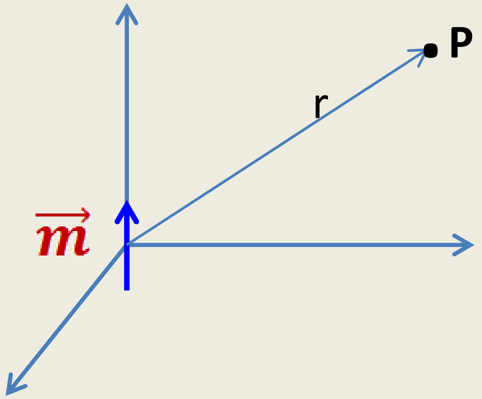


Special case for a plane loop

$\frac{1}{2}(\vec{r} \times \vec{dl})$ becomes the area of the shaded portion

$$\vec{m} = \frac{I}{2}\oint \vec{r} \times \vec{dl} = I\vec{a}$$

Any plane current loop can be replaced by a magnetic moment $\vec{m}$



$$\vec{A}_{\text{dipole}} = \frac{\mu_0}{4\pi d^2}(\vec{m} \times \hat{n}) = \frac{\mu_0 m \sin\theta}{4\pi r^2}\,\hat{\phi}$$

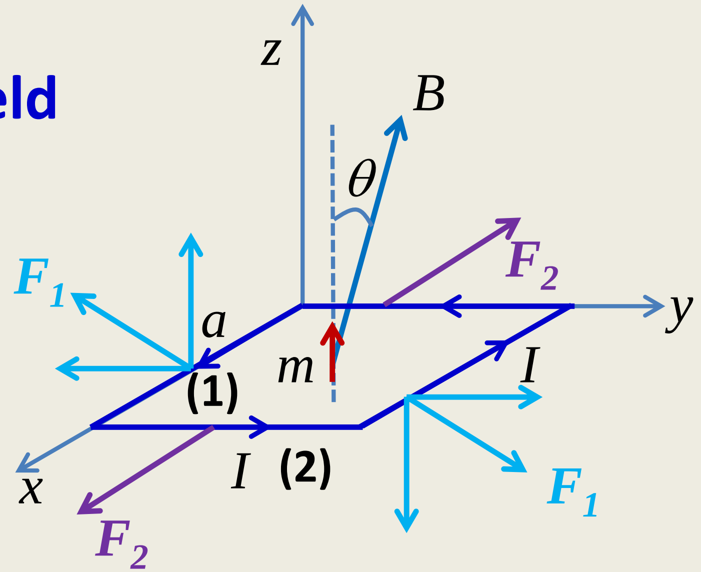
$$\vec{B} = \vec{\nabla} \times \vec{A} = \frac{\mu_0 m}{4\pi r^3}\{2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\}$$

$$= \frac{\mu_0}{4\pi r^3}\{3(\vec{m} \cdot \hat{r}) - \vec{m}\}$$

Magnetic Dipole in a UNIFORM magnetic field

Replace the magnetic dipole by a square current loop in the *x-y* plane

$$\vec{m} = Ia^2\,\hat{k}$$



Let $\vec{B} = B_0(\cos\theta\,\hat{k} + \sin\theta\,\hat{j})$

Force experienced by side (1)

$$\vec{F}_1 = I\int \vec{dl}\times\vec{B} = IB_0\int dx\,\{\hat{\imath}\times(\cos\theta\,\hat{k} + \sin\theta\,\hat{j})\}$$

$$= IB_0a(\cos\theta\,(-\hat{j}) + \sin\theta\,\hat{k})$$

Force experienced by side (2)

$$\vec{F}_2 = IB_0a(\cos\theta\,(\hat{\imath}))$$

F_1 produces a Torque

$$\vec{\tau} = \vec{r}\times\vec{F} = (Ia^2)B_0\sin\theta\,(-\hat{\imath})$$

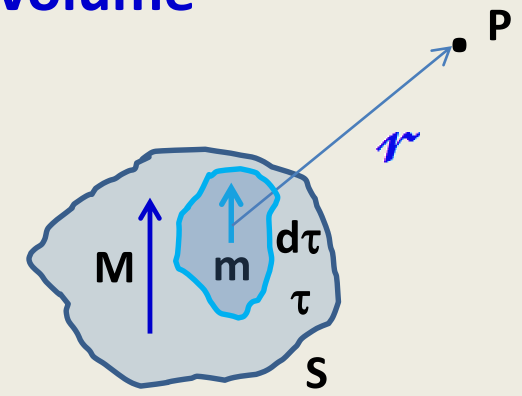
$$= \vec{m}\times\vec{B}$$

If a magnetic dipole is kept in a uniform magnetic field, it experiences a torque, resulting in a ROTATION to align along the field

What happens if a material is kept in a magnetic field?

Material becomes magnetized; tiny magnetic dipoles are created which align along some direction

M = magnetic dipole moment per unit volume



Magnetic field produced by a "Magnetized object"

Vector potential due to a single dipole m

$$d\vec{A}_{\text{dipole}} = \frac{\mu_0}{4\pi}\frac{(\vec{m}\times\hat{r})}{r^2} \qquad d\vec{A} = \frac{\mu_0}{4\pi}\frac{(\vec{M}\times\hat{r})}{r^2}d\tau$$

$$\vec{\nabla}\left(\frac{1}{r}\right) = \frac{\hat{r}}{r^2}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_\tau \frac{(\vec{M}\times\hat{r})}{r^2}d\tau$$

(differentiation is w.r.t. source coordinates)

$$\vec{A} = \frac{\mu_0}{4\pi}\int_\tau \vec{M}\times\vec{\nabla}\left(\frac{1}{r}\right)d\tau$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_\tau \vec{M} \times \vec{\nabla}\left(\frac{1}{r}\right) d\tau$$

$$\vec{A} = \frac{\mu_0}{4\pi}\left\{\int_\tau \frac{1}{r}(\vec{\nabla} \times \vec{M}) d\tau - \int_\tau \vec{\nabla} \times \left(\frac{1}{r}\vec{M}\right) d\tau\right\}$$

Problem 1.61(b) in Griffiths

$$\int_\tau (\vec{\nabla} \times \vec{v}) d\tau = -\oint_S (\vec{v} \times \vec{dS})$$

$$\vec{A} = \frac{\mu_0}{4\pi}\left\{\int_\tau \frac{1}{r}(\vec{\nabla} \times \vec{M}) d\tau + \oint_S \frac{1}{r}(\vec{M} \times \vec{dS})\right\}$$

$$\vec{\nabla} \times \vec{M} = \vec{J}_b \qquad \vec{M} \times \hat{n} = \vec{K}_b$$

Bound currents

volume current density J_b

Surface current density K_b

$$\vec{A} = \frac{\mu_0}{4\pi}\int_\tau \frac{\vec{J}_b}{r} d\tau + \frac{\mu_0}{4\pi}\oint_S \frac{\vec{K}_b}{r} dS$$

$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$

$$\sigma_b = \vec{P} \cdot \hat{n}$$