PH 108: Basics of Electricity and Magnetism

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Attendance: 80% Mandatory as per Institute Rules

Recommended Textbooks

- Introduction to Electrodynamics, by David Griffiths (Prentice Hall, 4th Indian Edition available in Campus Bookstore, *Primary Text*)
- E&M, Berkley Physics Course, by Edward Purcell (2nd Indian Edition, Tata McGraw Hill)
- Classical Electromagnetism, by Jerrold Franklin (Pearson/Addison Wesley Publication)
- Feynman Lectures, Vol 2 (Wonderful for new insights, conceptual understanding)

Syllabus

- Review of vector calculus: Spherical polar and cylindrical coordinates; gradient, divergence and curl; Divergence and Stokes` theorems; Divergence and curl of electric field
- Electric potential, properties of conductors; Poisson's and Laplace's equations, uniqueness theorems, boundary value problems, separation of variables, method of images, multipole expansion

Syllabus Continued…

- Polarization and bound charges, Gauss` law in the presence of dielectrics, Electric displacement D and boundary conditions, linear dielectrics
- Divergence and curl of magnetic field, Vector potential and its applications; Magnetization, bound currents, Ampere`s law in magnetic materials,

Syllabus Continued…

- Magnetic field H, boundary conditions, classification of magnetic materials; Faraday's law in integral and differential forms, Motional emf, Energy in magnetic fields, Displacement current, Maxwell's equations
- Electromagnetic (EM) waves in vacuum and media, Energy and momentum of EM waves, Poynting`s theorem; Reflection and transmission of EM waves across linear media

Fundamental forces of Nature

Experiments show that <u>all interactions found in nature</u> can be described in terms of four *Fundamental Interactions*:

- Gravity
- *Electromagnetism*
- Strong force (responsible for nuclear binding)
- Weak force (responsible for certain decays, like nuclear beta decay)

Classical Electromagnetism

- Greeks and other early investigators knew about electrified objects which attracted and repelled; Lightening, Loadstone attracting Iron etc.
- Had little idea about the underlying laws governing Electric and Magnetic phenomena—which seemed different from each other
- Governing laws of EM discovered in bits and pieces by Franklin, Coulomb, Oersted, Ampere, Faraday, Biot, Savart, *Maxwell*…

Classical Electrodynamics ....

- Experiments showed that Electric and Magnetic phenomena are strongly intertwined
- James Clerk Maxwell developed EM into a compact and consistent theory—unified ElectroMagnetic theory
- Classical EM of Maxwell deals with electric charges and currents and their interactions
- Maxwell's eqns. formulated as compact set of four (partial) Differential eqns. for the **E** and **B** fields

- Maxwell's eqns. Fantastic achievement of 19th century theoretical and experimental physics
- Correctly describes *all know electric and magnetic phenomena* (leaving aside quantum effects)
- Along with gravity, EM dominant and ubiquitous in everyday life
- EM describes atomic, molecular and nuclear structure; binding of atoms into larger structures—metals, wood, living organisms
- Light *is* an EM effect (EM radiation)
- EM correct up to cosmological distances

- Maxwell’s eqns. describes Electric and Magnetic phenomena correctly down from microscopic sub-atomic to cosmological distances!

  (Small corrections to Coulomb’s law due to Quantum effects, at electron Compton wavelength or less)
- Basis of all Electrical engineering, electronics, semiconductors, photonics, metallurgy and material science—indeed of life itself!
- Two approaches to learn EM:

  Deductive: Start from Maxwell’s eqns. And derive all EM phenomena systematically

✓ **Inductive**: Start from experimentally observed phenomena and work up to Maxwell’s theory

- Historically Electromagnetic and Gravitational Interactions were studied extensively
- *Quantum Mechanics* developed rapidly in the 20th century and was applied to atomic, molecular, solid state and nuclear physics
- Finally Quantum concepts were *applied to the Electromagnetic field itself,* to create **Quantum Electrodynamics (QED)** (by Dirac, Pauli, Heisenberg, Jordan, Schwinger, Tomonaga, Feynman and others)

- QED is a theory of fantastic precision-the most precise physical theory which has been tested experimentally
- Feynman compared the precision of QED to measuring the distance between New York and Los Angeles to *within the breadth of a human hair!*
- Strong and Weak interactions are more complicated generalizations of electrodynamics
- Have been given a quantum formulation from the start

The Field Concept

- Fundamental problem of EM: What are the forces that electrical charges, currents and magnets exert on each other?
- These forces do *not act instantaneously* (``No action at a distance”)
- Charges and currents produce vector *Electric (**E**) and Magnetic (**B**) fields* which permeate the space around them—abstract concept
- These fields act on other charges

The Field Concept ⋯

- Why not give the force exerted by one charged particle on another directly?
- Answer: Force law complicated!
- Depends not only on position, but also velocity, acceleration of charges
- Not easy to use and apply
- It is best to formulate EM theory as a set of compact *differential eqns*. for **E** and **B**

The Field Concept...

- When a charge accelerates, a portion of the fields detach themselves, travelling at the speed of light carrying energy, momentum and angular momentum
- Can thus regard the **E** and **B** fields as real entities—as real as particulate matter
- **E** and **B** fields can exist independent of the charges and currents that produce them

Electric Charge

- Charge is a scalar quantity—a number, which can be positive or negative—which is the source of the **E** and **B** fields
- Charge is conserved – total amount of charge in the universe is cannot change (*global conservation* of charge)
- *Stronger statement*: Charge is also conserved *locally*--it cannot suddenly disappear in one place and instantly appear at another point—it has to flow along a continuous path—this *continuity eqn.* is built into Maxwell's eqns.
- Charge conservation consequence of ``Gauge invariance"!! (Gauge invariance will be studied later)

Electric Charge ....

- Charge is quantized– it occurs only in *integral* units of the proton charge $e = 1.6 \times 10^{-19}$ C
- Since *e* is so small we can regard macroscopic charge distributions as *continuous*
- Nothing in EM theory *requires* that charge be quantized
- Dirac showed that *if* magnetic monopoles existed, charge quantization could be explained—but magnetic monopoles have not been found so far

Maxwell's Equations
(Entire Foundation of EM Theory)

$$\nabla \cdot \mathrm{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial t}$$

$$\nabla \cdot \mathrm{B} = 0 \qquad \nabla \times \mathrm{B} = \mu_0 \mathrm{J} + \mu_0 \varepsilon_0 \frac{\partial \mathrm{E}}{\partial t}$$

$$\mathrm{F} = q(\mathrm{E} + \mathrm{v} \times \mathrm{B}) \text{ (Lorentz force law)}$$

Symmetries of Maxwell's Eqns.

- How do Maxwell's eqns. change under:
- *Translation* of co-ordinate system?
- *Rotation* of co-ordinate system?
- *Space Inversion,* i.e, $x \to -x, y \to -y, z \to -z$
- *Time inversion*, i.e, $t \to -t$
- *Velocity transformations* ("Boosts") (Best studied after understanding relativity)

Coulomb's law

Interaction of electric Charges at rest:
The Forces between two point charges q_1 and q_2 are central :

$$\vec{F}_2 \propto \frac{q_1 q_2}{r_{21}^2}\widehat{r_{21}}$$

Here $\vec{F}_2$ (=- $\vec{F}_1$) is the force on charge 2, q_1 and q_2 are scalars- giving sign and magnitude of the respective charges and $\widehat{r_{12}}$ is the unit vector in the direction from 1 to 2

$\Rightarrow$ force is parallel to line joining the charges--
isotropy of empty space

$$\vec{F}_2 = k\frac{q_1 q_2}{r_{21}^2}\widehat{r_{21}}$$

Here k is constant of proportionality. In SI units

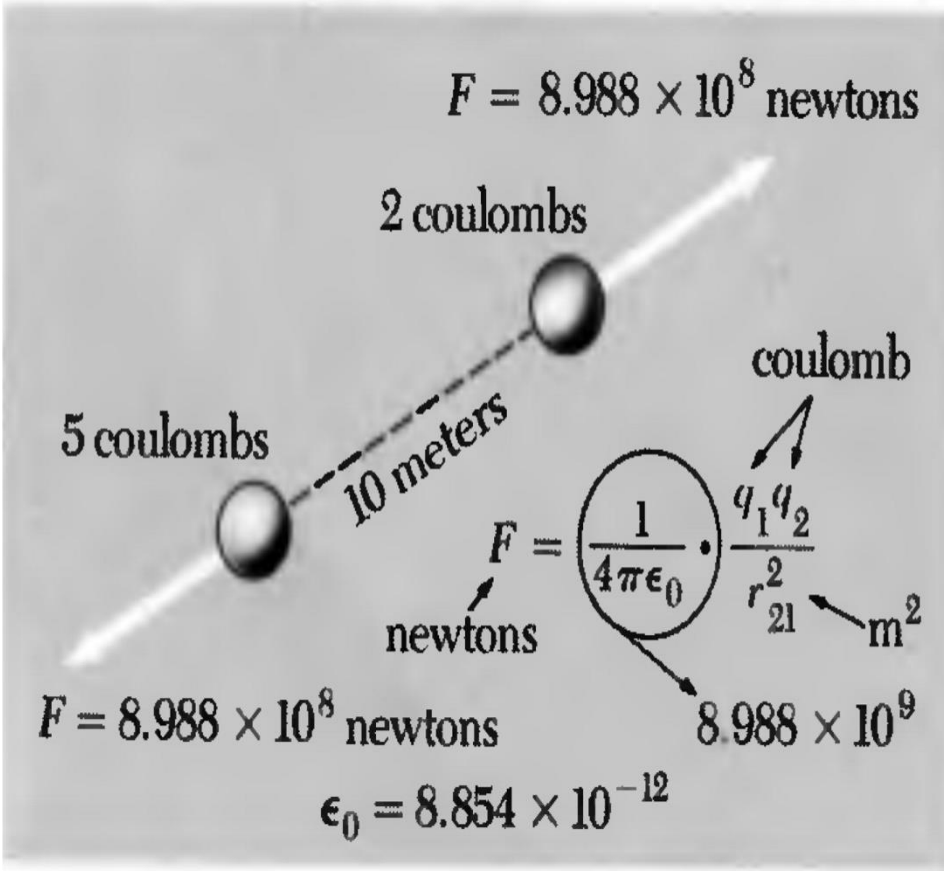
$k = \frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9$

And

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{21}^2}\widehat{r_{21}}$$

Coulomb's law implicitly provides definition of charges as various quantities can be varied

Note: Both charges localized into region<< r_{21} otherwise r_{21} can not be defined precisely



Principle of superposition

- Implicitly provides definition of electric charge as various quantities can be varied
- Only way of detecting and measuring the charges is by observing their interaction.

Significant physical content of coulomb's law:

•Inverse square dependence:

PHYSICAL REVIEW D VOLUME 2, NUMBER 3 1 AUGUST 1970

Experimental Test of Coulomb's Law*

D. F. Bartlett, P. E. Goldhagen, and E. A. Phillips†
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08540

(Received 17 April 1970)

One of the classic "null experiments" tests the exactness of the electrostatic inverse-square law. The outer shell of a spherical capacitor is raised to a potential V with respect to a distant ground, and the potential difference ΔV induced between the inner and outer shells is measured. If this induced potential difference is not zero, Coulomb's law is violated. For example, if we assume that the force between charges varies as r^{-2+q}, then $\Delta V/V$ is approximately a tenth of q. In our experiment five concentric spheres are used. A potential difference of 40 kV at 2500 Hz is impressed between the outer two spheres. A lock-in detector with a sensitivity of about 0.2 nV measures the potential difference between the inner two spheres. We find $|q| \leq 1.3\times10^{-13}$. We also find comparable limits on the detected signal when the operating frequency is 250 Hz, and when the detector is synchronized with the charging current rather than with the charge itself.

- PRL, **26**, 721(1971)

New Experimental Test of Coulomb's Law: A Laboratory Upper Limit on the Photon Rest Mass

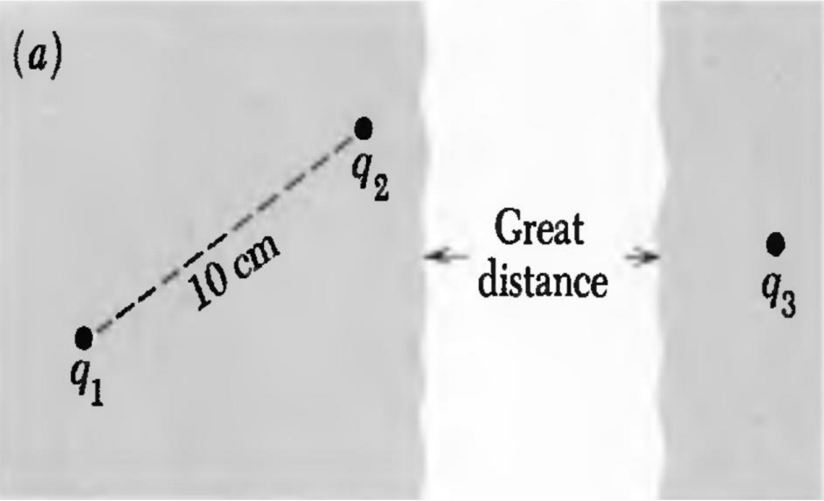
E. R. Williams,* J. E. Faller, and H. A. Hill
Department of Physics, Wesleyan University, Middletown, Connecticut 06457
(Received 22 January 1971)

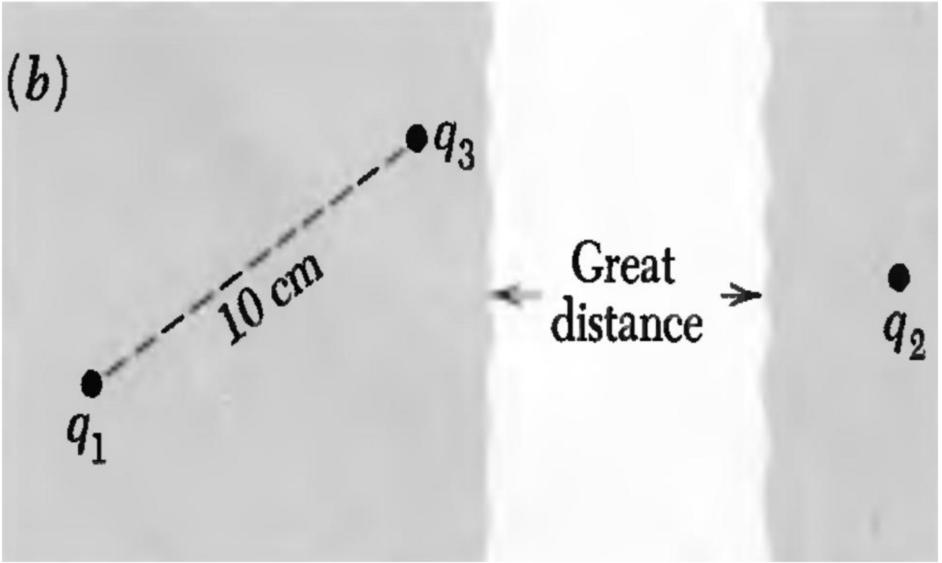
A high-frequency test of Coulomb's law is described. The sensitivity of the experiment is given in terms of a finite photon rest mass using the Proca equations. The null result of our measurement expressed in the form of the photon rest mass squared is $\mu^2 = (1.04 \pm 1.2) \times 10^{-19}$ cm^{-2}. Expressed as a deviation from Coulomb's law of the form $1/r^{2+q}$, our experiment gives $q = (2.7 \pm 3.1) \times 10^{-16}$. This result extends the validity of Coulomb's law by two orders of magnitude.

- 24 decades validity in distance proved by Pioneer 10 data on the magnetic field mapping of Jupiter
- Charge is additive

If there were only two charges to experiment we could never measure them separately. Only we could verify the nature of the force-- central

If we have three charge $q_{1,}$ $q_{2\ \text{and}}$ q_3
We can measure the force in following cases





We find by measurement that force on q_1 is $\sum forces\ in(a)and(b)$

⇒Force of interaction between two charges is not affected by the presence of third ----basis of

principle of superposition: electrical forces add according to the law of vector addition.

$$\overrightarrow{F_{2+3,1}} = \overrightarrow{F_{2,1}} + \overrightarrow{F_{3,1}}$$

Generalizing:

Force due to some arrangement of charges

q_1 , q_2, q_3,----q_n on some charge q_0

$$\vec{F}_0 = \frac{1}{4\pi\epsilon_0} \sum_{j=1}^{n} \frac{q_0 q_j}{{r_{0j}}^2} \widehat{r_{0j}}$$

Electric Field

$$\vec{F}_0 = \frac{1}{4\pi\epsilon_0}\sum_{j=1}^{n}\frac{q_0 q_j}{{r_{0j}}^2}\widehat{r_{0j}}$$

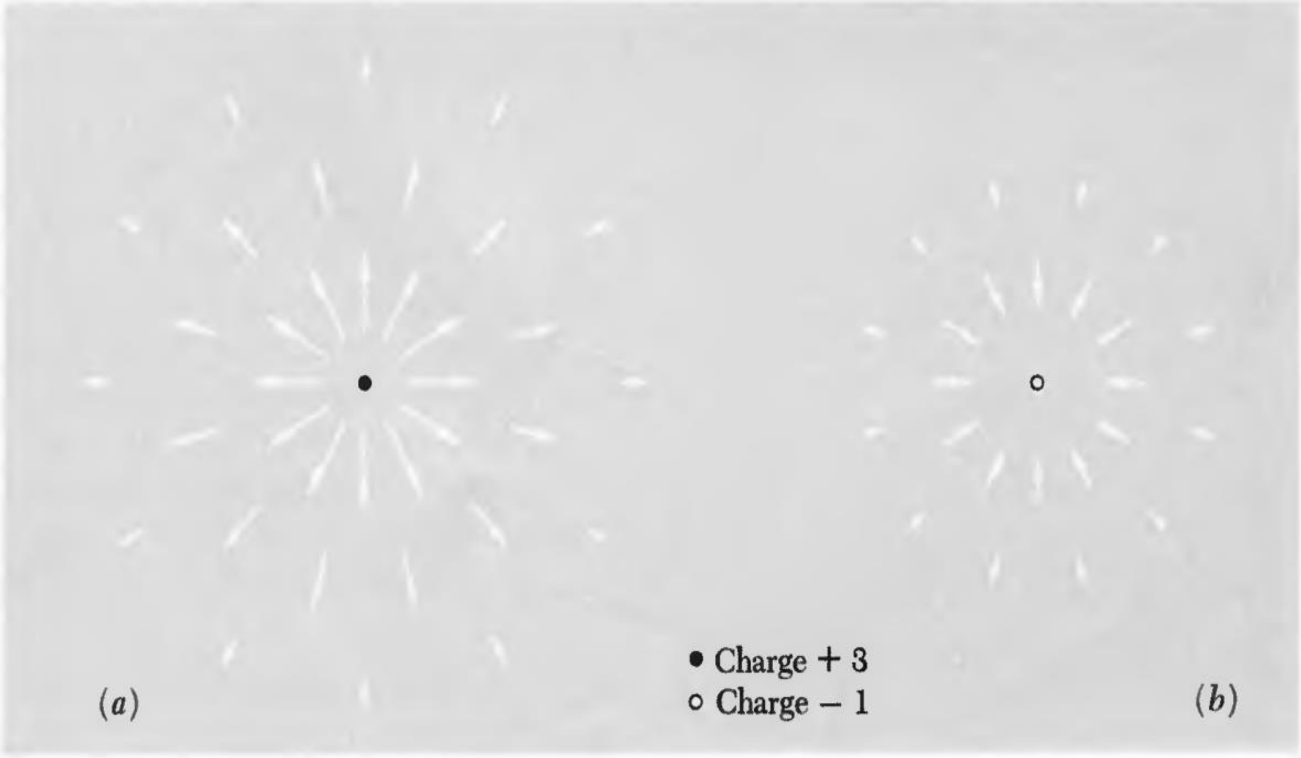
$$\overrightarrow{E(x,y,z)} = \frac{1}{4\pi\epsilon_0}\sum_{j=1}^{n}\frac{q_j}{{r_{0j}}^2}\widehat{r_{0j}}$$

--Vector quantity--Electric field -force per unit charge(N/C) depends only on structure of the our system of charges q_1 , q_2, q_3,----q_n (source charges) and on the r_{0j}.

The Field Concept

- A field is any physical quantity which takes on different values at different points in space (and time), i.e, it is a continuous function of the coordinates (x, y, z, t)
- Eg. Temperature $T(x, y, z, t)$—scalar field
- Velocity of flowing liquid $\boldsymbol{v}(x, y, z, t)$—eg. of a vector field
- Strength of a vector field can be represented at every point of space by drawing a vector—specifying magnitude and direction
- $\mathbf{E}(x, y, z, t)$ and $\mathbf{B}(x, y, z, t)$ are vector fields

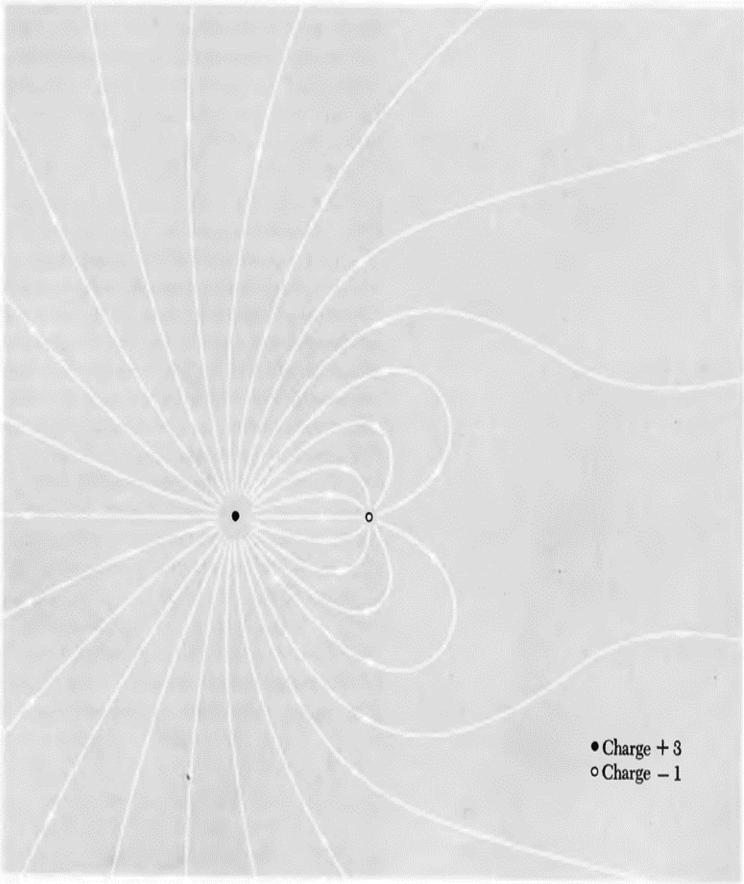
Strength of the vector field can be represented at every point in its influence zone by a vector--giving direction and magnitude



Another representation

Draw field line--tangent to the curve at any point gives the direction of the field.--smooth and continuous except at singularity.
---Field lines converge towards region of strong field and diverge in regions of weak fields.

---crude way of describing the field as
- These do not contain the principle of superposition.
- Direct interaction picture offered may give vivid picture in electrostatics but not for moving charges.



Next we look at the usefulness of this concept

Why do you want the field ?

Objection: I was happy to calculate vector sum of forces

Why this vector field $= \dfrac{F}{q_0}$ with loopy diagrams?

Resolution: We are interested in the effect one set of charges produces on another set

Split this in two pieces

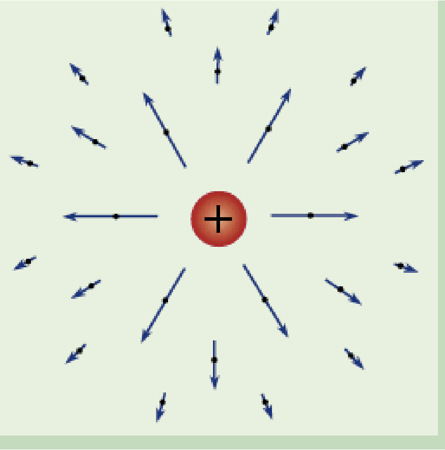
1) Calculate the field due to a set of charges, without worrying about *other* charges nearby

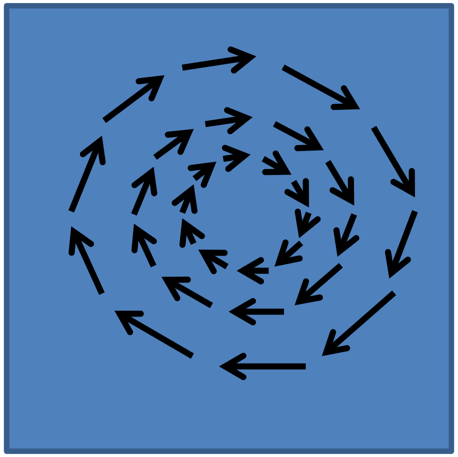
2) Calculate the effect of a field on a set of charges, without worrying about what charges produced the field

Here are some examples of Vector Fields

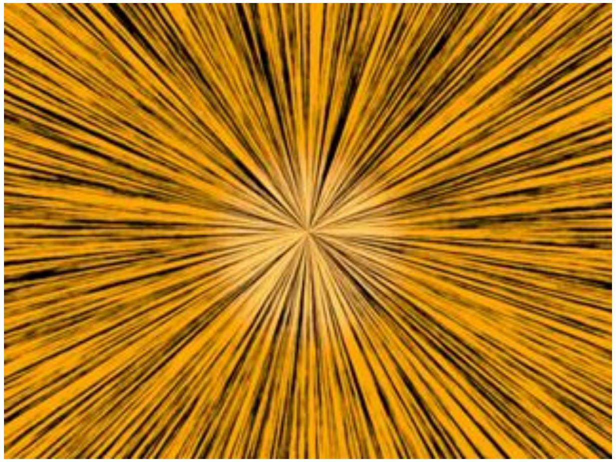
We can use multiple representations

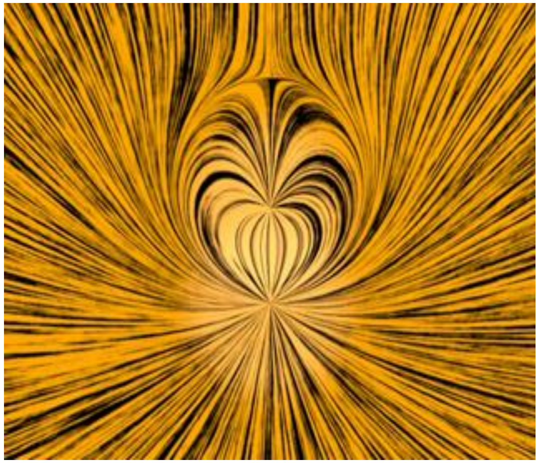
Arrows





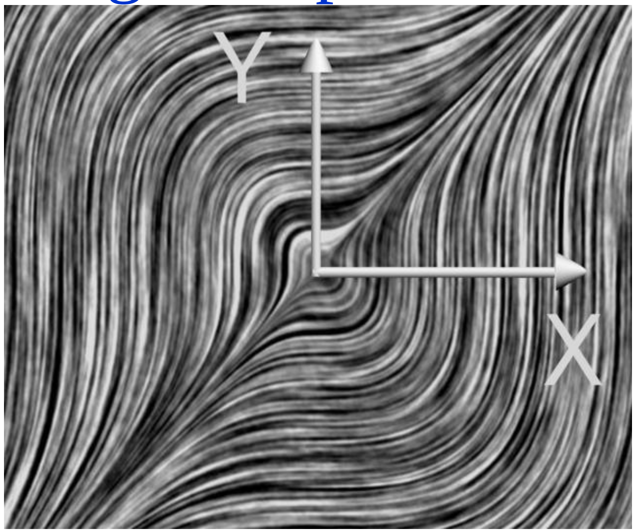
Field Lines





What are vector fields useful for, generally?

In *(x,y)* plane, this figure represents a vector field



The Vector field shown is….:

1. $\vec{E}(x,y) = x^2\hat{\imath} + y^2\hat{\jmath}$
2. $\vec{E}(x,y) = y^2\hat{\imath} + x^2\hat{\jmath}$
3. $\vec{E}(x,y) = \sin(x)\hat{\imath} + \cos(y)\hat{\jmath}$
4. $\vec{E}(x,y) = \cos(x)\hat{\imath} + \sin(y)\hat{\jmath}$
5. NOT SURE

Practice visualizing fields

Sketch the following vector function:

Use arrows of proper direction and length

$$\vec{E}(x,y) = \frac{1}{\sqrt{x^2+y^2}}(-y\hat{\imath} + x\hat{\jmath})$$

Advanced Definition:
Vector field is defined in terms of behavior of its components under rotation of coordinate axes.

Consider displacement vector $\vec{r}$
In two different coordinate systems. One rotated w.r.t. other

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Since a vector can be represented by coordinates of a point, components of a vector must transform as coordinates of a point

$$\begin{pmatrix} A_x{}' \\ A_y{}' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

a_{ij} ---direction cosines between primed and unprimed system w.r.t. positive axes.

$$\Rightarrow \qquad x_i' = \sum_{j=1}^{2} a_{ij} \cdot x_j$$

$$V_i' = \sum a_{ij} \cdot V_j$$

Definition of a Vector

- Consider a vector $\mathbf{r} = x\,\hat{\imath} + y\,\hat{\jmath}$ in two dimensions in the XY plane . The magnitude (length) of the vector is $r = \sqrt{x^2 + y^2}$
- By elementary geometry: $x = r\cos\theta$ and $y = r\sin\theta$
- If the co-ordinate system is now rotated by an angle ϕ w.r.t the XY system, then:

$$x' = x\cos\phi + y\sin\phi$$
$$y' = -x\ \sin\phi + y\cos\phi$$

Inverting the above relations:

$$x = x' \cos\phi - y' \sin\phi$$
$$y = x' \sin\phi + y' \cos\phi$$

Thus $$\frac{\partial x}{\partial x'} = \cos\phi \quad \text{and} \quad \frac{\partial x}{\partial y'} = -\sin\phi$$
$$\frac{\partial y}{\partial x'} = \sin\ \phi \quad \text{and} \quad \frac{\partial y}{\partial y'} = \cos\phi$$

By chain rule $$\frac{\partial}{\partial x'} = \frac{\partial x}{\partial x'}\frac{\partial}{\partial x} + \frac{\partial y}{\partial x'}\frac{\partial}{\partial y} = \cos\phi\frac{\partial}{\partial x} + \sin\phi\frac{\partial}{\partial y}$$
$$\frac{\partial}{\partial y'} = \frac{\partial x}{\partial y'}\frac{\partial}{\partial x} + \frac{\partial y}{\partial y'}\frac{\partial}{\partial y} = -\sin\ \phi\frac{\partial}{\partial x} + \cos\ \phi\frac{\partial}{\partial y}$$

Thus $\boldsymbol{\nabla} = \boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y}$ *transforms* like the position vector and hence is a vector (operator) under rotations