

# Thermodynamics

## 1. Heat transfer between SPH particles

$$\frac{\Delta T_i}{\Delta t} = \alpha \sum_{j \in N_i} m_j \frac{(T_j - T_i)}{\rho_j} \nabla^2 W(x_{ij}, H_h)$$

$\alpha$  : thermal diffusion coefficient, 열확산도 =  $\frac{K_c}{p C_p}$

Scaled up by a factor  $S_f$

(status of particle i, i.e. ice, water or rigid)

$C_p$  : specific heat capacity value, 비열용량

$C_{p_{water}}$	Specific heat capacity water	$4186 \text{ J kg}^{-1} \text{ K}^{-1}$
$C_{p_{snow}}$	Specific heat capacity snow	$2090 \text{ J kg}^{-1} \text{ K}^{-1}$
$C_{p_{ice}}$	Specific heat capacity ice	$2050 \text{ J kg}^{-1} \text{ K}^{-1}$

$K_c$  : thermal conductivity, 열 전도율

$p$ : 밀도

인용한 논문에서 사용하는 thermal diffusion constant 값

ice thermal diffusion constant	0.5
water thermal diffusion constant	0.1

@ Fast particle-based visual simulation of ice melting

$m_j$  : particle j의 질량

$T_j - T_i$  : particle j와 i의 온도 차

$x_{ij}$  : distance between  $x_i$  and  $x_j$

$N_i$  : a set of neighboring particles whose distances are smaller than  $H_h$  or support radius from particle i

$\Delta T_i$  : obtained change in temperature for particle i  
= particle i의 온도변화

$H_h$  : the effective radius

$\rho_j$  : ????????

$W$  : smoothing kernel

is the same as that used for computing viscosity

$$W_{\text{viscosity}}(\mathbf{r}, h) = \frac{15}{2\pi h^3} \begin{cases} -\frac{r^3}{2h^3} + \frac{r^2}{h^2} + \frac{h}{2r} - 1 & 0 \leq r \leq h \\ 0 & \text{otherwise.} \end{cases}$$

$$\nabla^2 W(\mathbf{r}, h) = \frac{45}{\pi h^6} (h - r)$$

$$W(|\mathbf{r}| = h, h) = 0$$

$$\nabla W(|\mathbf{r}| = h, h) = \mathbf{0}$$

@ Müller et al / Particle-Based Fluid Simulation for Interactive Applications

## 2. Heat loss/gain with the neighboring particles

$$Q_{i_{\text{neighbors}}} = C_p m \Delta T_i$$

$\Delta T_i$  : 앞에서 계산한 값 사용, 온도 변화량

$C_p$  : 비열

$C_{p_{\text{water}}}$	Specific heat capacity water	$4186 \text{ Jkg}^{-1} \text{K}^{-1}$
$C_{p_{\text{snow}}}$	Specific heat capacity snow	$2090 \text{ Jkg}^{-1} \text{K}^{-1}$
$C_{p_{\text{ice}}}$	Specific heat capacity ice	$2050 \text{ Jkg}^{-1} \text{K}^{-1}$

$m$  : 질량

## 3. Heat exchange between the snow particle i and the air and the ground

Newton's law of cooling

$$Q_{i_{\text{air}}} = h_T (T_{\text{air}} - T_i) \delta A_{\text{air}}$$

$Q_{i_{\text{ground}}}$  : Air와 같은 방식으로 계산

$h_T$  : heat transfer coefficient(thermal conductivity)

$h_{T_{\text{water}}}$	Thermal conductivity of water	$0.602 \text{ Wm}^{-2} \text{K}^{-1}$
$h_{T_{\text{snow}}}$	Thermal conductivity of snow	$0.1 \text{ Wm}^{-2} \text{K}^{-1}$
$h_{T_{\text{ice}}}$	Thermal conductivity of snow	$0.7 \text{ Wm}^{-2} \text{K}^{-1}$

$h_T$  for a snow particle is interpolated using its snow content  $\eta$  and  $h_{T_{\text{snow}}}$  and  $h_{T_{\text{ice}}}$

$T_{\text{air}}, T_i$ : Temperature for air and particle i

// air의 온도는 상수 값 사용

$\delta A_{\text{air}}$  : the area of particle i exposed to the air

- How to calculate  $\delta A$ ?

Particles are generated at the centers of the voxels of the voxelized ice object

Surface area  $A$

= the sum of the areas of the six surfaces of the voxel

Therefore, the number of faces touching the air among the six faces is calculated by subtracting the number of neighboring particles from 6.

$$\delta A = \frac{6-n}{6} A$$

$n$ : the number of neighboring snow, ice and particles

// if  $n$  exceeds 6,  $\delta A$  is clamped to 0

$A$ : total surface area for the particle in consideration

#### 4. Latent heat

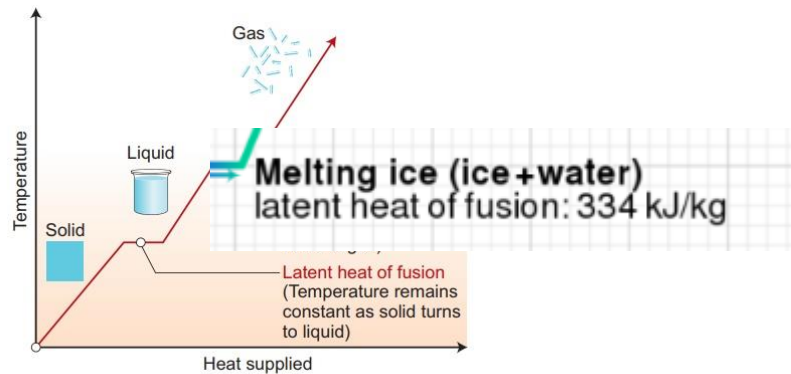
$$Q_f = mL\beta$$

$\beta$ : amount of water melted within the particle  
( variable, float )

잠열이 파티클 내의 녹은 물의 양으로 저장됨  
만약  $\beta$ 값이 1이 되면 파티클이 물이 된다.

범위가 0~1인 듯함

L: the specific latent heat of fusion for the material.



$$Q = m r$$

Q : 열량 [Kcal/Kg] // 기존의 잠열 계산 식

m : 질량 [Kg] (기호 G(중량)으로도 표시 합니다.)

r : 잠열 [Kcal]

#### 5. Total heat value received for a particle

$$Q_i = Q_{i_{air}} + Q_{i_{ground}} + Q_{i_{neighbors}} + Q_f$$

계산한 값 사용 -> Temperature(변수)값 변화  
이 값은 눈의 녹는점 계산에만 사용한다

#### 6. Phase transition of each particle

온도가 273[K]초과 +  $\beta$ 가 1이 됨 -> 녹음