1) <u>Filter Design</u> :-

<u>Filter properties</u>:-

*) 5×5 filter with 1 at center and zeros at rest so that it can act only on center pixel.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

*) To make above filter shift image up by 1 pixel.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

*) To make above filter shift image to right by 2 pixels

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This is how the filter should look after flipping.

*) So we have to flip it again to look how it is before flipping

Note:- For 2D convolution flipping occurs on both x & y axis.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[\text{y-axis}]{\text{flip over}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[\text{x-axis}]{\text{flip over}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

∴ Resultant filter :-

Assump tion if the filter is already flipped $\longrightarrow$ $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Assumption if the filter is not flipped $\longrightarrow$ $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

2) Gaussian Filter Separbility

Given Gaussian filter : $\frac{1}{100}\begin{bmatrix} 1 & 2 & 4 & 2 & 1 \\ 2 & 4 & 8 & 4 & 2 \\ 4 & 8 & 16 & 8 & 4 \\ 2 & 4 & 8 & 4 & 2 \\ 1 & 2 & 4 & 2 & 1 \end{bmatrix}$

→ The above gaussian distribution filter is symmetric across both x-axis and y-axis.

→ As we know a 2D gaussian filter can be represented as a product of 2 1-D gaussian filters. In this case as the filter is symmetric across both axies both 1-D gaussian filters should look similar.

→ Looking at the matrix each row and each column is the scaled version of the 1D gaussian distribution.

$\therefore$ The corresponding 1-D gaussian filter is

$$= \frac{1}{10}\begin{bmatrix} 1 & 2 & 4 & 2 & 1 \end{bmatrix}$$

$$\text{1-D Gaussian}_{row} = \frac{1}{10}\begin{bmatrix} 1 & 2 & 4 & 2 & 1 \end{bmatrix}$$

$$\text{1-D Gaussian}_{column} = \left[\frac{1}{10}\begin{bmatrix} 1 & 2 & 4 & 2 & 1 \end{bmatrix}\right]^T = \frac{1}{10}\begin{bmatrix} 1 \\ 2 \\ 4 \\ 2 \\ 1 \end{bmatrix}$$

Verification :-

$$\text{2D-gaussian} = \text{1-D gaussian}_{row} \times \text{1-D gaussian}_{col}$$

$$= \frac{1}{10}\begin{bmatrix} 1 & 2 & 4 & 2 & 1 \end{bmatrix} \times \frac{1}{10}\begin{bmatrix} 1 \\ 2 \\ 4 \\ 2 \\ 1 \end{bmatrix}$$

$$= \frac{1}{100}\begin{bmatrix} 1 & 2 & 4 & 2 & 1 \\ 2 & 4 & 8 & 4 & 2 \\ 4 & 8 & 16 & 8 & 4 \\ 2 & 4 & 8 & 4 & 2 \\ 1 & 2 & 4 & 2 & 1 \end{bmatrix}$$

3) Harris Corner Detector :-

Given :- $E(u,v) = [u, v]\, M \begin{bmatrix} u \\ v \end{bmatrix}$

$$M = \sum_{(x,y)} w(x,y) \begin{bmatrix} I_x^2(x,y) & I_x(x,y)\, I_y(x,y) \\ I_x(x,y)\, I_y(x,y) & I_y^2(x,y) \end{bmatrix}$$

window matrix $W = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$I_x = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix} \;;\; I_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

3.1) Find M ?

$$M = \Sigma_{xy} \, W(x,y) \begin{bmatrix} I_x^2(x,y) & I_x(x,y)\, I_y(x,y) \\ I_x(x,y)\, I_y(x,y) & I_y^2(x,y) \end{bmatrix}$$

$$= W(0,0) \begin{bmatrix} I_x^2(0,0) & I_x(0,0)\, I_y(0,0) \\ I_x(0,0)\, I_y(0,0) & I_y^2(0,0) \end{bmatrix} +$$

$$W(0,1) \begin{bmatrix} I_x^2(0,1) & I_x(0,1)\, I_y(0,1) \\ I_x(0,1)\, I_y(0,1) & I_y^2(0,1) \end{bmatrix} +$$

$$W(1,0) \begin{bmatrix} I_x^2(1,0) & I_x(1,0)\, I_y(1,0) \\ I_x(1,0)\, I_y(1,0) & I_y^2(1,0) \end{bmatrix} +$$

$$W(1,1) \begin{bmatrix} I_x^2(1,1) & I_x(1,1)\, I_y(1,1) \\ I_x(1,1)\, I_y(1,1) & I_y^2(1,1) \end{bmatrix}$$

$$= 1 \begin{bmatrix} 0^2 & 0\times1 \\ 0\times1 & 1^2 \end{bmatrix} + 1 \begin{bmatrix} 0^2 & 0\times0 \\ 0\times0 & 0^2 \end{bmatrix} + 1 \begin{bmatrix} 1^2 & 1\times0 \\ 1\times0 & 0^2 \end{bmatrix} + 1 \begin{bmatrix} 2^2 & (-1)\times2 \\ 2(-1) & (-1)^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+1+4 & 0+0+0-2 \\ 0+0+0-2 & 1+0+0+1 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\boxed{\therefore \; M = \begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}}$$

3.2) calculate λ_1, λ_2.

characterstic Equation :- $|M - \lambda I| = 0$

$$\cancel{[M - \lambda I] = 0}$$

$$\left| \begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 5-\lambda & -2 \\ -2 & 2-\lambda \end{bmatrix} \right| = 0$$

$$(5-\lambda)(2-\lambda) - (-2)(-2) = 0$$

$$10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda^2 - \lambda - 6\lambda + 6 = 0$$

$$\lambda(\lambda - 1) - 6(\lambda - 1) = 0$$

$$(\lambda - 1)(\lambda - 6) = 0$$

$$\therefore \lambda_1 = 1 \text{ and } \lambda_2 = 6$$

3.3) calculate f? $\quad f = \dfrac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$

$$f = \frac{1 \times 6}{6+1} = \frac{6}{7} = 0.857$$

3.4) As $f = 0.857 < 1$

$\therefore$ The window does not contain a corner.

4) <u>Convolution</u>:-

Given input image $= \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

Given convolution kernel $= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Given stride $= 1$

padding $= 0$

As the padding is zero, means there is only one position that kernel can convolve with input image that is when they overlay on top of each other.

We came to the conclusion because ~~the~~ input image size is equal to convolution kernel size.

Hence the output should result in a single scalar value.

<u>Assuming that convolution kernel is not flipped</u>:-

Flip the kernel on both x-axis and y-axis.

$\rightarrow$ Flip the kernel on both x-axis and y-axis.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow[\text{y-axis}]{\text{flip on}} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow[\text{x-axis}]{\text{flip on}} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

→ Applying the operation on the image

$$3\times1 + 1\times1 + 0\times2 + 1\times1 + 2\times0 + 1\times1 + 0\times2 + 1\times1 + 1\times0$$

$$= 3+1+0+1+0+1+0+1+0$$

$$= 7$$

Assuming convolution kernel is ~~not~~ flipped already :-

→ As the filter is already flipped then we only have to perform operation.

→ Applying the operation on the image

$$3\times0 + 1\times1 + 2\times0 + 1\times1 + 2\times0 + 1\times1 + 2\times0 + 1\times1 + 1\times1$$

$$= 0+1+0+1+0+1+0+1+1$$

$$= 5$$

∴ Output of the convolution operation :-

→ If the kernel is not flipped = 7

→ If the given kernel is already flipped = 5

5) <u>Template</u> <u>Matching</u> :-

Given kernel = $\begin{bmatrix} 3 & 10 & 20 \\ 18 & 1 & 5 \\ 2 & 30 & 3 \end{bmatrix}$

Each locations After necessary zero-padding :-

Location 1 : $\begin{bmatrix} 0 & 0 & 0 \\ 72 & 0 & 84 \\ 170 & 26 & 54 \end{bmatrix}$

Location 2 : $\begin{bmatrix} 75 & 127 & 52 \\ 87 & 86 & 0 \\ 12 & 188 & 176 \end{bmatrix}$

Location 3 : $\begin{bmatrix} 3 & 9 & 208 \\ 1 & 2 & 6 \\ 22 & 40 & 9 \end{bmatrix}$

5.1) we know

$$SSD = \sum_{(x,y)} \left[I(x+i, y+j) - T(i,j) \right]^2$$

For location 1 : $(3-0)^2 + (10-0)^2 + (20-0)^2 + (18-72)^2 + (1-0)^2 + (84-5)^2 +$
$(170-2)^2 + (26-30)^2 + (54-3)^2$

$= 9 + 100 + 400 + 2916 + 1 + 6241 + 28224 + 16 + 2601$

$= 40508$

For location 2 :

$$(3-75)^2+(10-127)^2+(20-52)^2+(18-87)^2+(1-86)^2+(5-0)^2+$$
$$(2-120)^2+(30-188)^2+(3-176)^2$$

$$= 5184+13689+1024+4761+7225+25+100+24964+29929$$

$$= 86901$$

For location 3 :

$$(3-3)^2+(10-9)^2+(20-208)^2+(18-1)^2+(1-2)^2+(5-6)^2+(2-22)^2+$$
$$(30-40)^2+(3-9)^2$$

$$= 0+1+35344+289+1+1+400+100+36$$

$$= 36172$$

The best match for Sum of Squared difference will be less.

∴ The best match occurs at location 3 i.e., location @ 2 with value 36172.

5.2) We know

$$N_{tf}[i,j] = \frac{\sum_m \sum_n f[m,n]\ t[m-i,n-j]}{\sqrt{\sum_m \sum_n f^2[m,n]}\ \sqrt{\sum_m \sum_n t^2[m-i,n-j]}}$$

$$\sqrt{\sum_m \sum_n t^2[m-i,n-j]} = \sqrt{3^2+10^2+20^2+18^2+1^2+5^2+2^2+30^2+3^2}$$
$$= \sqrt{9+100+400+324+1+25+4+900+9}$$
$$= \sqrt{1772}$$
$$= 42.095$$

@ Location 1 NCC :-

$$\sqrt{\sum_m \sum_n f_1^2[m,n]} = \sqrt{0^2 + 0^2 + 0^2 + 72^2 + 0^2 + 84^2 + 170^2 + 26^2 + 54^2}$$

$$= \sqrt{0 + 0 + 0 + 5184 + 0 + 7056 + 28900 + 676 + 2916}$$

$$= \sqrt{44732}$$

$$= 211.4994$$

$$NCC = \frac{3 \times 0 + 10 \times 0 + 20 \times 0 + 72 \times 18 + 1 \times 0 + 5 \times 84 + 170 \times 2 + 30 \times 26 + 3 \times 54}{\sqrt{44732} \times \sqrt{1772}}$$

$$= \frac{0 + 0 + 0 + 1296 + 0 + 420 + 340 + 780 + 162}{\sqrt{44732} \times \sqrt{1772}}$$

$$= 0.3367$$

@ Location 2 NCC :-

$$\sqrt{\sum_m \sum_n f_2^2[m,n]} = \sqrt{75^2 + 127^2 + 52^2 + 87^2 + 86^2 + 0^2 + 12^2 + 188^2 + 176^2}$$

$$= \sqrt{5625 + 16129 + 2704 + 7569 + 0 + 7396 + 144 + 35344 + 30976}$$

$$= \sqrt{105887}$$

$$= 325.40$$

$$NCC = \frac{3 \times 75 + 10 \times 127 + 20 \times 52 + 18 \times 87 + 1 \times 86 + 5 \times 0 + 2 \times 12 + 30 \times 188 + 3 \times 176}{\sqrt{105887} \times \sqrt{1772}}$$

$$= \frac{225 + 1270 + 1040 + 1566 + 86 + 0 + 24 + 5640 + 528}{\sqrt{105817} \times \sqrt{1772}}$$

$$= 0.7577$$

@ Location 3 NCC :-

$$\sqrt{\sum_m \sum_n f_3^2[m,n]} = \sqrt{3^2 + 9^2 + 208^2 + 1^2 + 2^2 + 6^2 + 22^2 + 40^2 + 9^2}$$

$$= \sqrt{9 + 81 + 43264 + 1 + 4 + 36 + 484 + 1600 + 81}$$

$$= \sqrt{45560}$$

$$= 213.447$$

$$NCC = \frac{3\times3 + 9\times10 + 20\times208 + 18\times1 + 1\times2 + 5\times6 + 2\times22 + 30\times40 + 3\times9}{\sqrt{45560} \times \sqrt{1772}}$$

$$= \frac{9 + 90 + 4160 + 18 + 2 + 30 + 44 + 1200 + 27}{\sqrt{45560} \times \sqrt{1772}}$$

$$= 0.6210$$

For the best match Normalized cross correlation will be higher.

$\therefore$ The best match occurs at location 2 i.e., location @ 86 with value 0.7577