COMP 353: Databases
Assignment 2
Section: F
Group Name: The Unique Keys

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Q1.

Exercise #1

Here are the two sets of FDs for R = {A, B, C, D, E}.

S = {A→B ; AB→C ; D→AC ; D→E}
T = {A→BC ; D→AE}

Are they equivalent?

S Implies T:

If S covers T, then ($T \subseteq S$).
If T covers S, then ($S \subseteq T$).

If both are held true, then we can say that $S = T$ and S is equivalent to T and vice versa.

$S = \{A \to B,\ AB \to C,\ D \to AC,\ D \to E\}$

We need to show that S implies T, which means we need to prove that every FD in T can be derived from S.

$T = \{A \to BC,\ D \to AE\}$

$A \to BC$ in T:

To derive $A \to BC$ from S, we can use the transitive rule on $A \to B$ from S and $AB \to C$ from S:

- $A \to B$ (from S)
- $AB \to C$ (from S)
- Therefore, $A \to BC$ can be derived from S

$D \to AE$ in T:

To derive $D \to AE$ from S, we can use the transitive rule on $D \to AC$ from S and $A \to B$ from S:

- $D \to AC$ (from S)
- $A \to B$ (from S)
- Therefore, $D \to AE$ can be derived from S

T Implies S:

Now, let's check if T implies S:

$T = \{A \to BC,\ D \to AE\}$

We need to show that T implies S, which means we need to prove that every FD in S can be derived from T.

$\underline{A \rightarrow B \text{ in } S}$:

To derive $A \rightarrow B$ from T, we can use the augmentation rule. We have $A \rightarrow BC$ in T.

- $A \rightarrow BC$ (from T)
- Therefore, $A \rightarrow B$ can be derived from T

$\underline{AB \rightarrow C \text{ in } S}$:

This FD is already in T as $A \rightarrow BC$. Therefore, $AB \rightarrow C$ can be derived from T.

$\underline{D \rightarrow AC \text{ in } S}$:

To derive $D \rightarrow AC$ from T, we can use the transitive rule on $D \rightarrow AE$ from T and $A \rightarrow BC$ from T:

- $D \rightarrow AE$ (from T)
- $A \rightarrow BC$ (from T)
- Therefore, $D \rightarrow AC$ can be derived from T

$\underline{D \rightarrow E \text{ in } S}$:

This FD is already in T as $D \rightarrow AE$. Therefore, $D \rightarrow E$ can be derived from T.

So, T implies S.

Since we have shown that S implies T and T implies S, the two sets of FDs are equivalent.

$\underline{\text{Set } S \text{ Covers } T}$:
$S = \{A \rightarrow B,\ AB \rightarrow C,\ D \rightarrow AC,\ D \rightarrow E\}$
$A^+ = \{A, B, C\}$, so $A \rightarrow BC$
$D^+ = \{D, A, C, E\}$, so $D \rightarrow AE$

$\underline{\text{Set } T \text{ Covers } S}$:
$T = \{A \rightarrow BC,\ D \rightarrow AE\}$
$A^+ = \{A, B, C\}$, so $A \rightarrow B$
$AB^+ = \{A, B, C\}$, so $AB \rightarrow C$
$D^+ = \{D, A, E, B, C\}$, so $D \rightarrow AC$ and $D \rightarrow E$

Q2.

Exercise #2

a) Compute the closure of the following set F of functional dependencies for relation schema R = {A, B, C, D, E}.

$$F = \{A \to BC ; CD \to E ; B \to D ; E \to A\}$$

b) List the candidate keys for R.

a) Closure for A:

Using $A \to BC$, we get $A^+ = \{A\} \cup \{A, B, C\} = \{A, B, C\}$.

Using $B \to D$, we get $A^+ = \{A, B, C\} \cup \{B, D\} = \{A, B, C, D\}$.

Using $CD \to E$, we get $A^+ = \{A, B, C, D\} \cup \{C, D, E\} = \{A, B, C, D, E\}$.

Therefore, $A^+ = \{A, B, C, D, E\}$.

Closure for BC:

Using $BC \to BC$, we get $BC^+ = \{B, C\}$.

Using $B \to D$, we get $BC^+ = \{B, C\} \cup \{B, D\} = \{B, C, D\}$.

Using $CD \to E$, we get $BC^+ = \{B, C, D\} \cup \{C, D, E\} = \{B, C, D, E\}$.

Using $E \to A$, we get $BC^+ = \{B, C, D, E\} \cup \{E, A\} = \{A, B, C, D, E\}$.

Therefore, $BC^+ = \{A, B, C, D, E\}$.

Closure for C:

Using $C \to C$, we get $C^+ = \{C\}$.

Therefore, $C^+ = \{C\}$.

Closure for CD:

Using $CD \to E$, we get $CD^+ = \{C, D\} \cup \{C, D, E\} = \{C, D, E\}$.

Using $E \to A$, we get $CD^+ = \{C, D, E\} \cup \{E, A\} = \{A, C, D, E\}$.

Using $A \to BC$, we get $CD^+ = \{A, C, D, E\} \cup \{A, B, C\} = \{A, B, C, D, E\}$.

Therefore, $CD^+ = \{A, B, C, D, E\}$.

Closure for B:

Using $B \to D$, we get $B^+ = \{B\} \cup \{B, D\} = \{B, D\}$.

Therefore, $B^+ = \{B, D\}$.

Closure for D:

Using $D \rightarrow D$, we get $D^+ = \{D\}$.

Therefore, $D^+ = \{D\}$.

Closure for E:

Using $E \rightarrow A$, we get $E^+ = \{E\} \cup \{E, A\} = \{A, E\}$.

Using $A \rightarrow BC$, we get $E^+ = \{A, E\} \cup \{A, B, C\} = \{A, B, C, E\}$.

Using $B \rightarrow D$, we get $E^+ = \{A, B, C, E\} \cup \{B, D\} = \{A, B, C, D, E\}$.

Therefore, $E^+ = \{A, B, C, D, E\}$.

b) Since candidate keys list all attributes of R and must be minimal with no proper key subsets, then A, CD, BC and E are candidate keys for R because all their closures equal $\{A, B, C, D, E\}$. On the other hand, B, C and D are not candidate keys because each of their closures equal $\{B, D\}$, $\{C\}$ and $\{D\}$ instead.

Q3.

Exercise #3

Consider the following decomposition of the table ENROLLMENT in two tables Student and Course.

Table ENROLLMENT

StudentID	StudentName	CourseName	Credits
1111111	William Smith	COMP218	4
2222222	Michel Cyr	COMP353	4
3333333	Charles Fisher	COMP348	4
4444444	Patricia Roubaix	COMP353	4
2222222	Paul Paul	COMP352	3
5555555	Lucie Trembaly	COMP354	3

Table Student

StudentID	StudentName	Credits
1111111	William Smith	4
2222222	Michel Cyr	4
3333333	Charles Latan	4
4444444	Patricia Roubaix	4
2222222	Paul Paul	3
5555555	Lucie Trembaly	3

Table Course

Credits	CourseName
4	COMP218
4	COMP353
4	COMP348
4	COMP353
3	COMP352
3	COMP354

Question: Is this decomposition lossless? Justify.

The decomposition of the relation Enrollment into the relation Student and Course will be a lossless join decomposition in DBMS when Table Student ∩ Table Course = Table Enrollment.

In our case, that is not the case that the decomposition is lossless. When looking at Table Course, we see the new table connects with the Credits column. In our cases, each Credits column from Table Course matches four different types of ID in the Table Student, which increases the original Table Enrollment from a 6-row data table to a 20-row data table. This redundancy creates losses in the data. Thus, the decomposition of the Table Enrollment into Table Student and Table Course is not a lossless joint decomposition.

Result of the Newly-Created Table with Table Student Intersection Table Course:

StudentID	**StudentName**	**CourseName**	**Credits**
1111111	William Smith	COMP218	4
1111111	William Smith	COMP 353	4
1111111	William Smith	COMP348	4
1111111	William Smith	COMP353	4
2222222	Michel Cyr	COMP218	4
2222222	Michel Cyr	COMP 353	4
2222222	Michel Cyr	COMP348	4
2222222	Michel Cyr	COMP353	4
3333333	Charles Fisher	COMP218	4
3333333	Charles Fisher	COMP 353	4
3333333	Charles Fisher	COMP348	4
3333333	Charles Fisher	COMP353	4
4444444	Patricia Roubaix	COMP218	4
4444444	Patricia Roubaix	COMP 353	4
4444444	Patricia Roubaix	COMP348	4
4444444	Patricia Roubaix	COMP353	4
2222222	Paul Paul	COMP352	3
2222222	Paul Paul	COMP354	3

5555555	Lucie Tremblay	COMP352	3
5555555	Lucie Tremblay	COMP354	3

Q4.

Exercise #4

Using the Functional Dependencies,

$F = \{A \to BC ; CD \to E ; B \to D ; E \to A\}$

a) Compute the closure of F (F^+).

b) Is true / false : $F \models E \to BC$?

c) Provide the minimal cover F^c (*min(F)*).

d) List of the candidate keys for R

a) When $F = \{A \to BC,\ CD \to E,\ B \to D,\ E \to A\}$, determine F^+:

<u>$A \to BC$:</u>

Using $A \to BC$, we get $A \to BC$ which is already in F^+.
Using $A \to BC$, we get $A \to B$.
Using $A \to BC$, we get $A \to C$.

<u>$CD \to E$:</u>

Using $CD \to E$, we get $CD \to E$ which is already in F^+.
Using $CD \to E$, we get $C \to E$.
Using $CD \to E$, we get $D \to E$.

<u>$B \to D$:</u>

Using $B \to D$, we get $B \to D$ which is already in F^+.

<u>$E \to A$:</u>

Using $E \to A$, we get $E \to A$ which is already in F^+.

$A^+ = \{A, B, C, D, E\}$
$B^+ = \{B, D\}$
$C^+ = \{C\}$
$D^+ = \{D\}$

$E^{+} = \{A, B, C, D, E\}$

$BC^{+} = \{A, B, C, D, E\}$

$CD^{+} = \{A, B, C, D, E\}$

$F^{+} = \{A, B, C, D, E\}$

b) $F \vDash E \rightarrow BC$:

We have the following functional dependencies:
$E \rightarrow A$
$A \rightarrow BC$

According to the transitive rule, if A determines B and B determines C, then A must also determine C. Hence, the application here would result using the given $E \rightarrow A$, then we can derive $E \rightarrow BC$, so $E^{+} = \{A, B, C\}$. Therefore, $E \rightarrow BC$ is true.

c) Find the minimal cover (F^{c}) for the given set of functional dependencies.
$F = \{A \rightarrow BC,\ CD \rightarrow E,\ B \rightarrow D,\ E \rightarrow A\}$:

Step 1: Decomposition

Using $A \rightarrow BC$, we get $A \rightarrow B$ according to composition properties.
Using $A \rightarrow BC$, we get $A \rightarrow C$ according to composition properties.
$CD \rightarrow E$
$B \rightarrow D$
$E \rightarrow A$

Step 2: Redundancy Removal

When $A^{+} = \{A, C\}$, then we don't get $A \rightarrow B$ from it, so we keep it.
When $A^{+} = \{A, B, D\}$, we don't get $A \rightarrow C$ from it, so we keep it.
When $B^{+} = \{B\}$ (Reflexive), we don't get $B \rightarrow D$ from it, so we keep it.
When $E^{+} = \{E\}$ (Reflexive) we don't get $E \rightarrow A$ from it, so we keep it.
When $CD^{+} = \{C, D\}$ (Reflexive), we don't get $CD \rightarrow E$ from it, so we keep it.

After obtaining the closure of the given set, we see that there is no redundancy. So, we are left with $\{A \rightarrow B,\ A \rightarrow C,\ B \rightarrow D,\ E \rightarrow A,\ CD \rightarrow E\}$. Hence, F^{c} is equal to F.

d) Candidate keys derive all attributes found in relation R. From the following list, A, E, BC and CD are candidate keys for relation R as the closure for each of them is $\{A, B, C, D, E\}$.

$$A^+ = \{A, B, C, D, E\}$$
$$B^+ = \{B, D\}$$
$$C^+ = \{C\}$$
$$D^+ = \{D\}$$
$$E^+ = \{A, B, C, D, E\}$$
$$BC^+ = \{A, B, C, D, E\}$$
$$CD^+ = \{A, B, C, D, E\}$$