Presentation Template

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Problem Statement

A circle C passes through

$$\mathbf{P} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \tag{1}$$

and touches the y-axis at

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}. \tag{2}$$

Which one of the following equations can represent a diameter of this circle?

(i) $\begin{pmatrix} 4 & 5 \end{pmatrix} \mathbf{x} = 6$

(iii) $\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} = 3$

(ii) $\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} + 10 = 0$

(iv) $\begin{pmatrix} 5 & 2 \end{pmatrix} \mathbf{x} + 4 = 0$

Finding the Centre

Let $\mathbf{O}$ be the centre of C. Then the equation of the normal, OQ is

$$(0 \quad 1) (\mathbf{O} - \mathbf{Q}) = 0$$
$$\implies (0 \quad 1) \mathbf{O} = 2 \tag{3}$$

Also,

$$\|\mathbf{O} - \mathbf{P}\|^2 = \|\mathbf{O} - \mathbf{Q}\|^2$$
$$\implies 2 (\mathbf{P} - \mathbf{Q})^T \mathbf{O} = \|\mathbf{P}\|^2 - \|\mathbf{Q}\|^2$$
$$\text{or,} \quad (1 \quad -1) \mathbf{O} = -4 \tag{4}$$

(3) and (4) result in the matrix equation

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \mathbf{O} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \tag{5}$$

yielding the augmented matrix

$$\begin{pmatrix} 1 & -1 & -4 \\ 0 & 1 & 2 \end{pmatrix} \leftrightarrow \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \end{pmatrix} \implies \mathbf{O} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \tag{6}$$

Finding the Diameter

(i) $\begin{pmatrix} 4 & 5 \end{pmatrix} \mathbf{O} = 2 \neq 6$. Incorrect.

(ii) $\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{O} + 10 = 0$. Correct.

(iii) $\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{O} = 2 \neq 3$. Incorrect.

(iv) $\begin{pmatrix} 5 & 2 \end{pmatrix} \mathbf{O} + 4 = -2 \neq 0$. Incorrect

Finding the Radius

The radius of C is obtained as

$$r = \|O - P\| = 2 \tag{7}$$
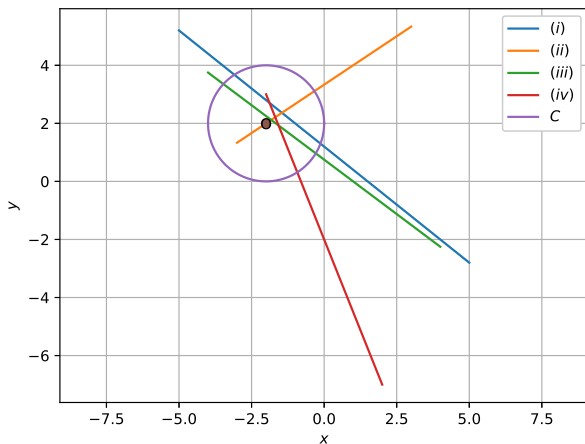
Plot

The code in plots Fig. 1.

Figure: Circle C and all lines (i)-(iv). (ii) is a diameter.