## Contents

1	Introduction	11
	Games with Ordinal Payoffs	
2	Ordinal Games in Strategic Form	17
2.1	Game frames and games	17
2.2	Strict and weak dominance	24
2.3	Second-price auction	29
2.4	The pivotal mechanism	32
2.5	Iterated deletion procedures	35
2.5.1	IDSDS	35
2.5.2	IDWDS	37
2.6	Nash equilibrium	39
2.7	Games with infinite strategy sets	44
2.8	Proofs of theorems	46
2.9	Exercises	50
2.9.1	Exercises for Section 2.1: Game frames and games	50
2.9.2	Exercises for Section 2.2: Strict/weak dominance	51
2.9.3	Exercises for Section 2.3: Second price auction	52
2.9.4	Exercises for Section 2.4: The pivotal mechanism	54
2.9.5	Exercises for Section 2.5: Iterated deletion procedures	55
2.9.6	Exercises for Section 2.6: Nash equilibrium	57

2.9.7	Exercises for Section 2.7: Games with infinite strategy sets	. 57
2.10	Solutions to exercises	59
3	Perfect-information Games	. 75
3.1	Trees, frames and games	75
3.2	Backward induction	80
3.3	Strategies in perfect-information games	83
3.4	Relationship between backward induction and other solutions	86
3.5	Perfect-information games with two players	90
3.6	Exercises	94
3.6.1 3.6.2 3.6.3 3.6.4 <b>3.7</b>	Exercises for Section 3.1: Trees, frames and games	. 95 . 96
4	General Dynamic Games	117
4.1	Imperfect Information	117
4.2	Strategies	124
4.3	Subgames	126
4.4	Subgame-perfect equilibrium	128
4.5	Games with chance moves	134
4.6	Exercises	140
4.6.1	Exercises for Section 4.1: Imperfect information	
4.6.2	Exercises for Section 4.2: Strategies	
4.6.3 4.6.4	Exercises for Section 4.3: Subgames	141
4.6.5	Exercises for Section 4.5: Games with chance moves	144
4.7	Solutions to exercises	148
	Games with Cardinal Payoffs	
5	Expected Utility Theory	169
5.1	Money lotteries and attitudes to risk	169
5.2	Expected utility: theorems	171
5.3	Expected utility: the axioms	177
5.4	Exercises	184
5.4.1	Exercises for Section 5.1: Money lotteries and attitudes to risk	184
5.4.2 5.4.3	Exercises for Section 5.2: Expected utility theory Exercises for Section 5.3: Expected utility axioms	185 186

5.5	Solutions to Exercises	187
6	Strategic-form Games	193
6.1	Strategic-form games with cardinal payoffs	193
6.2	Mixed strategies	196
6.3	Computing the mixed-strategy Nash equilibria	201
6.4	Strict dominance and rationalizability	206
6.5	Exercises	210
6.5.1	Exercises for Section 6.1: Strategic-form games with cardinal payoffs .	210
6.5.2	Exercises for Section 6.2: Mixed strategies	
6.5.3	Exercises for Section 6.3: Computing the mixed-strategy Nash equilibria	
6.5.4	Exercises for Section 6.4: Strict dominance and rationalizability	
6.6	Solutions to exercises	216
7	Extensive-form Games	227
7.1	Behavioral strategies in dynamic games	227
7.2	Subgame-perfect equilibrium revisited	237
7.3	Problems with the notion of subgame-perfect equilibrium	240
7.4	Exercises	242
7.4.1	Exercises for section 7.1: Behavioral strategies in dynamic games	242
7.4.2	Exercises for section 7.2: Subgame-perfect equilibrium revisited	244
7.5	Solutions to exercises	249
II	Knowledge, Common Knowledge, Belie	fs
8	Common Knowledge	261
8.1	Individual knowledge	261
8.2	Interactive knowledge	264
8.3	Common knowledge	271
8.4	•	276
8.4.1	Exercises for Section 8.1: Individual knowledge	_
8.4.2	Exercises for Section 8.2: Interactive knowledge	
8.4.3	Exercises for Section 8.3: Common knowledge	281
8.5	Solutions to Exercises	284
9	Adding Beliefs to Knowledge	295
9.1	Sets and probability: brief review	295
9.1.1	Sets	295
9.1.2	Probability	297

9.2	Probabilistic beliefs	298
9.3	Conditional probability and Bayes' rule	300
9.3.1	Conditional probability	300
9.4	Changing beliefs in response to new information	303
9.4.1	Belief updating	303
9.4.2	Belief revision	305
9.5	Harsanyi consistency of beliefs or like-mindedness	308
9.6	Agreeing to disagree	314
9.7	Proof of the Agreement Theorem	322
9.8	Exercises	323
9.8.1	Exercises for Section 9.1: Sets and probability	323
9.8.2	Exercises for Section 9.2: Probabilistic beliefs	324
9.8.3	Exercises for Section 9.3: Conditional probability and Bayes' rule	324
9.8.4	Exercises for Section 9.4: Changing beliefs in response to new informa 327	tion
9.8.5	Exercises for Section 9.5: Harsanyi consistency of beliefs or like-mindedr 330	ness
9.8.6	Exercises for Section 9.6: Agreeing to disagree	332
9.9	Solutions to Exercises	333
10	•	347
10.1	Models of strategic-form games	347
10.2	Common knowledge of rationality in strategic-form games	352
10.3	Common knowledge of rationality in extensive-form games	354
10.4	Proofs of Theorems	356
10.5	Exercises	358
10.5.1	Exercises for Section 10.1: Model of strategic-form games	358
10.5.2	Exercises for Section 10.2:  Common knowledge of rationality in strategic-form games	360
10.5.3	Exercises for Section 10.3:	
	Common knowledge of rationality in extensive-form games	361
10.6	Solutions to Exercises	362
I۱	Refinements of Subgame Perfection	
11	Weak Sequential Equilibrium	369
11.1	Assessments and sequential rationality	369
11.2	Bayesian updating at reached information sets	376
11.3		378
11.4	·	387
	Exercises for Section 11.1: Assessments and sequential rationality	

11.4.2	Exercises for Section 11.2:  Payorian undating at reached information sets	300
11.4.3	Bayesian updating at reached information sets	
	Solutions to Exercises	393
12	Sequential Equilibrium	403
12.1	Consistent assessments	403
12.2	Sequential equilibrium	408
12.3	Is 'consistency' a good notion?	411
12.4	Exercises	413
	Exercises for Section 12.1: Consistent assessments	_
	Exercises for Section 12.2: Sequential equilibrium	
12.5		418
13	Perfect Bayesian Equilibrium	429
13.1	Belief revision and AGM consistency	429
13.2	Bayesian consistency	433
13.3	Perfect Bayesian equilibrium	436
13.4	Adding independence	440
13.4.1	Weak independence	440
13.4.2	Strong independence	444
13.5	Characterization of SE in terms of PBE	446
13.6	History-based definition of extensive-form game	449
13.7	Proofs	452
13.8	Exercises	454
	Exercises for Section 13.1: Belief revision and AGM consistency	
	Exercises for Section 13.2: Bayesian consistency	
	Exercises for Section 13.3: Perfect Bayesian equilibrium	
	Exercises for Section 13.5:	400
	Characterization of sequential equilibrium in terms of PBE	459
13.9	Solutions to Exercises	463
V	Incomplete Information	
14	Static Games	483
14.1	Interactive situations with incomplete information	483
14.2	One-sided complete information	485
14.3	Two-sided incomplete information	492
14.4	Multi-sided incomplete information	496

14.5	Exercises	499
14.5.1	Exercises for Section 14.2: One-sided incomplete information	499
14.5.2	Exercises for Section 14.3: Two-sided incomplete information	501
14.5.3	Exercises for Section 14.4: Multi-sided incomplete information	504
14.6	Solutions to Exercises	508
15	Dynamic Games	<b>521</b>
15.1	One-sided incomplete information	521
15.2	Multi-sided incomplete information	538
15.3	Exercises	544
15.3.1	Exercises for Section 15.1: One-sided incomplete information	544
15.3.2	Exercises for Section 15.2: Multi-sided incomplete information	548
15.4	Solutions to Exercises	551
16	The Type-Space Approach	567
16.1	Types of players	567
16.2	Types that know their payoffs	568
16.3	The general case	571
16.4	Exercises	574
16.4.1	Exercises for Section 16.2: Types that know their own payoffs	574
16.4.2	Exercises for Section 16.3: The general case	576
16.5	Solutions to Exercises	578
	Index	589