

Index

Page numbers in *italics* are recommended to be consulted first. Page numbers in **bold** contain boxed algorithms.

- k*-armed bandits, 25–45
- absorbing state, 57
- access-control queuing example, 256
- action preferences, 322, 329, 336, 455
 - in bandit problems, 37, 42
- action-value function, *see* value function, action
- action-value methods, 321
 - for bandit problems, 27
- actor–critic, 21, 239, 321, 331–332, 338, 406
 - one-step (episodic), **332**
 - with eligibility traces (episodic), **332**
 - with eligibility traces (continuing), **333**
 - neural, 395–415
- addiction, 409–410
- advantage actor–critic methods, 338
- afterstates, 137, 140, 181, 182, 191, 424, 430
- agent–environment interface, 47–57, 467
- all-actions algorithm, 326
- AlphaGo, AlphaGo Zero, AlphaZero, 441–450
- Andreae, John, 17, 21, 69, 89
- ANN, *see* artificial neural networks
- applications and case studies, 421–457
- approximate dynamic programming, 15
- artificial intelligence, xvii, 1, 472, 475–478
- artificial neural networks, 223–228, 238–239, 395–398, 423, 430, 436–450, 472
- associative reinforcement learning, 45, 418
- associative search, 41
- asynchronous dynamic programming, 85, 88
- Atari video game play, 436–441
- auxiliary tasks, 460–461, 468, 474
- average reward setting, 249–255, 258, 464
- averagers, 264
- backgammon, 11, 21, 182, 184, 421–426
- backpropagation, 21, 225–227, 239, 407, 424, 436, 439
- backup diagram, 60, 139
 - for dynamic programming, 59, 61, 64, 172
 - for Monte Carlo methods, 94
 - for Q-learning, 134
 - for TD(0), 121
 - for Sarsa, 129
 - for Expected Sarsa, 134
 - for Sarsa(λ), 304
 - for TD(λ), 289
 - for Q(λ), 313
 - for Tree Backup(λ), 314
 - for truncated TD(λ), 296
 - for *n*-step Q(σ), 155
 - for *n*-step Expected Sarsa, 146
 - for *n*-step Sarsa, 146
 - for *n*-step TD, 142
 - for *n*-step Tree Backup, 152
 - for Samuel’s Checker Player, 428
 - compound, 288
 - half backups, 62
- backward view of eligibility traces, 288, 293
- Baird’s counterexample, 261–264, 280, 283, 285
- bandit algorithm, simple, **32**
- bandit problems, 25–45
- basal ganglia, 386
- baseline, 37–40, 329, 330, 331, 338
- behavior policy, 103, 110, *see* off-policy learning
- Bellman equation, 14
 - for v_π , 59
 - for q_π , 78
 - for optimal value functions: v_* and q_* , 63
 - differential, 250
 - for options, 463
- Bellman error, 268, 270, 272, 273
 - learnability of, 274–277
 - vector, 267–269
- Bellman operator, 267–269, 286
- Bellman residual, 286, *see* Bellman error
- Bellman, Richard, 14, 71, 89, 241
- binary features, 215, 222, 245, 304, 305
- bioreactor example, 51
- blackjack example, 93–94, 99, 106
- blocking maze example, 166
- bootstrapping, 89, 189, 308, 331
 - n*-step, 141–158, 255
 - and dynamic programming, 89
 - and function approximation, 208, 264–274

- and Monte Carlo methods, 95
 - and stability, 263–265
 - and TD learning, 120
 - assessment of, 124–128, 248, 264, 291, 318
 - in psychology, 345, 349, 354, 355
 - parameter (λ or n), 291, 307, 399
- BOXES, 18, 71, 237
- branching factor, 173–177, 422
- breakfast example, 5, 22
- bucket-brigade algorithm, 19, 21, 139
- catastrophic interference, 472
- certainty-equivalence estimate, 128
- chess, 4, 20, 54, 182, 450
- classical conditioning, 20, 343–357
 - blocking, 371
 - and higher-order conditioning, 345–355
 - delay and trace conditioning, 344
 - Rescorla-Wagner model, 346–349
 - TD model, 349–357
- classifier systems, 19, 21
- cliff walking example, 132, 133
- CMAC, *see* tile coding
- coarse coding, 215–220, 237
- cognitive maps, 363–364
- collective reinforcement learning, 404–407
- complex backups, *see* compound update
- compound stimulus, 345, 346–356, 371, 382
- compound update/backup, 288, 319
- conditioned/unconditioned stimulus, conditioned
 - response (CS/US, CR), 343
- constant- α MC, 120
- contextual bandits, 41
- continuing tasks, 54, 57, 70, 124, 249, 294
- continuous action, 73, 244, 335–336
- continuous state, 73, 223, 237
- continuous time, 11, 71
- control and prediction, 342
- control theory, 4, 70
- control variates, 150–152, 155, 281
 - and eligibility traces, 309–312
- credit assignment, 11, 17, 19, 47, 294, 401
 - in psychology, 346, 361
 - structural, 385, 405, 407
- critic, 18, 238, 346, 417, *see* actor-critic
- cumulant, 459
- curiosity, 474
- curse of dimensionality, 4, 14, 221, 231
- cybernetics, xvii, 477
- deadly triad, 264
- deep learning, 11, 223, 441, 472–474, 480
- deep reinforcement learning, 236
- deep residual learning, 226
- delayed reinforcement, 361–363
- delayed reward, 2, 47, 249
- dimensions of reinforcement learning methods, 189–191
- direct and indirect RL, 162, 164, 192
- discounting, 55, 199, 243, 249, 282, 324, 328, 427, 459
 - in pole balancing, 56
 - state dependent, 307
 - deprecated, 253, 256
- distribution models, 159, 185
- dopamine, 377, 381–387, 413–419
 - and addiction, 409–410
- double learning, 134–136, 140
- DP, *see* dynamic programming
- driving-home example, 122–123
- Dyna architecture, 164, 161–170
- dynamic programming, 13–15, 73–90, 174, 262
 - and artificial intelligence, 89
 - and function approximation, 241
 - and options, 463
 - and the deadly triad, 264
 - computational efficiency of, 87
- eligibility traces, 287–320, 350, 362, 398–403
 - accumulating, 301, 306, 310
 - replacing, 301, 306
 - dutch, 300–303
 - contingent/non-contingent, 399–403, 411
 - off-policy, 309–316
 - with state-dependent λ and γ , 309–316
- Emphatic-TD methods, 234–235, 315
 - off-policy, 281–282
- environment, 47–57
- episodes, episodic tasks, 11, 54–57, 91
- error reduction property, 144, 288
- evaluative feedback, 17, 25, 47
- evolution, 7, 359, 374, 471
- evolutionary methods, 7, 8–10, 19
- expected approximate value, 148, 155
- Expected Sarsa, 133, *see also* Sarsa, Expected
- expected update, 75, 172–181, 189
- experience replay, 440–441
- explore/exploit dilemma, 3, 103, 472
- exploring starts, 96, 98–100, 178

- feature construction, 210–223
- final time step (T), 54
- Fourier basis, 211–214
- function approximation, 195–200
 - n -step, 148–156
- incremental implementation
 - of averages, 30–33
 - of weighted averages, 109
- instrumental conditioning, 357–361, *see also*
 - Law of Effect
 - and motivation, 360–361
 - Thorndike’s puzzle boxes, 358
- interest and emphasis, 234–235, 282, 316
- inverse reinforcement learning, 470
- Jack’s car rental example, 81–82, 137, 210
- kernel-based function approximation, 232–233
- Klopf, A. Harry, *xv*, *xvii*, 19–21, 402–404, 411
- latent learning, 192, 363, 366
- Law of Effect, 15–16, 45, 342, 358–361, 417
- learning automata, 18
- Least Mean Square (LMS) algorithm, 279, 301
- Least-Squares TD (LSTD), 228–229
- linear function approx., 204–209, 266–269
- linear programming, 87, 90
- local and global optima, 200
- Markov decision process (MDP), 2, 14, 47–71
- Markov property, 49, 115, 465–468
- Markov reward process (MRP), 125
- maximization bias, 134–136
- maximum-likelihood estimate, 128
- MC, *see* Monte Carlo methods
- Mean Squared
 - Bellman Error, \overline{BE} , 268
 - Projected Bellman Error, \overline{PBE} , 269
 - Return Error, \overline{RE} , 275
 - TD Error, \overline{TDE} , 270
 - Value Error, \overline{VE} , 199–200
- memory-based function approx., 230–232
- Michie, Donald, 17, 71, 116
- Minsky, Marvin, 16, 17, 20, 89
- model of the environment, 7, 159
- model-based and model-free methods, 7, 159
 - in animal learning, 363–368
- model-based reinforcement learning, 159–193
 - in neuroscience, 407–409
- Monte Carlo methods, 91–117
 - first- and every-visit MC, 92
 - first-visit MC control, **101**
 - first-visit MC prediction, **92**
- gambler’s example, 84
- game theory, 19
- gazelle calf example, 5
- general value functions (GVFs), 459–463, 474
- generalized policy iteration (GPI), 86–87, 92, 97, 138, 189
- genetic algorithms, 19
- Gittins index, 43
- gliding/soaring case study, 453–457
- goal, *see* reward signal
- golf example, 61, 63, 66
- gradient, 201
- gradient descent, *see* stochastic gradient descent
- Gradient-TD methods, 278–281, 314–315
- greedy or ϵ -greedy
 - as exploiting, 26–28
 - as shortsighted, 64
 - ϵ -greedy policies, 100
- gridworld examples, 60, 65, 76, 147
 - cliff walking, 132
 - Dyna blocking maze, 166
 - Dyna maze, 164
 - Dyna shortcut maze, 167
 - windy, 130, 131
- habitual and goal-directed control, 364–368
- hedonistic neurons, 402–404
- heuristic search, 181–183, 190
 - as sequences of backups, 183
 - in Samuel’s checkers player, 426
 - in TD-Gammon, 425
- history of reinforcement learning, 13–21
- Holland, John, 19, 21, 44, 139, 241
- Hull, Clark, 16, 359, 360, 362–363
- importance sampling, 103–117, 151, 257
 - ratio, 104, 148, 258
 - weighted and ordinary, 105, 106
 - and eligibility traces, 309–312
 - and infinite variance, 106
 - discounting aware, 112–113
 - incremental implementation, 109
 - per-decision, 114–115

- gradient method for v_π , **202**
 - Monte Carlo ES (Exploring Starts), **99**
 - off-policy control, **111**, 110–112
 - off-policy prediction, 103–109, **110**
- Monte Carlo Tree Search (MCTS), 185–188
- motivation, 360–361
- mountain car example, 244–248, 305, 306
- multi-armed bandits, 25–45
- n*-step methods, 141–158
 - $Q(\sigma)$, **156**
 - Sarsa, **147**, **247**
 - differential, **255**
 - off-policy, **149**
 - TD, **144**
 - Tree Backup, **154**
 - truncated λ -return, 295
- naughts and crosses, *see* tic-tac-toe
- neural networks, *see* artificial neural networks
- neurodynamic programming, 15
- neuroeconomics, 413, 419
- neuroscience, 4, 21, 377–419
- nonstationarity, 30, 32–36, 41, 44, 255
 - inherent, 91, 198
- notation, xiii, *xix*
- observations, 464
- off-policy methods, 257–286
 - vs on-policy methods, 100, 103
 - Monte Carlo, 103–115
 - Q-learning, **131**
 - Expected Sarsa, 133–134
 - n*-step, 148–156
 - n*-step $Q(\sigma)$, **156**
 - n*-step Sarsa, **149**
 - n*-step Tree Backup, **154**
 - and eligibility traces, 309–316
 - Emphatic-TD(λ), 315
 - GQ(λ), 315
 - GTD(λ), 314
 - HTD(λ), 315
 - Q(λ), 312–314
 - Tree Backup(λ), 312–314
 - reducing variance, 283–284
- on-policy distribution, 175, 199, 208, 258, 262, 281, 282
 - vs uniform distribution, 176
- on-policy methods, 100
 - actor-critic, **332**, **333**
 - approximate
 - control, **244**, **247**, **251**, **255**
 - prediction, **202**, **203**, **209**
 - Monte Carlo, **101**, 100–103, **328**, **330**
 - n*-step, **144**, **147**
 - Sarsa, **130**, 129–131
 - TD(0), **120**, 119–128
 - with eligibility traces, **293**, **300**, **305**, **307**
- operant conditioning, *see* instrumental learning
- optimal control, 2, 14–15, 21
- optimistic initial values, 34–35, 192
- optimizing memory control, 432–436
- options, 461–464
 - models of, 462
- pain and pleasure, 6, 16, 413
- Partially Observable MDPs (POMDPs), 466
- Pavlov, Ivan, 16, 343–345, 362
- Pavlovian
 - conditioning, *see* classical conditioning
 - control, 343, 371, 373, 479
- personalizing web services, 450–453
- planning, 3, 5, 7, 11, 138, 159–193
 - in psychology, 363, 364, 366
 - with learned models, 161–168, 473
 - with options, 461, 463
- policy, 6, 41, 58
 - hierarchical, 462
 - soft and ε -soft, 100–103, 110
- policy approximation, 321–324
- policy evaluation, 74–76, *see also* prediction
 - iterative, **75**
- policy gradient methods, 321–338
 - REINFORCE, **328**, **330**
 - actor-critic, **332**, **333**
- policy gradient theorem, 324–326
 - proof, episodic case, 325
 - proof, continuing case, 334
- policy improvement, 76–80
 - theorem, 78, 101
- policy iteration, 14, **80**, 80–82
- polynomial basis, 210–211
- prediction, 74–76, *see also* policy evaluation
 - and control, 342
 - Monte Carlo, 92–97
 - off-policy, 103–108
 - TD, 119–126
 - with approximation, 197–241
- prior knowledge, 11, 34, 54, 137, 236, 324, 470

- prioritized sweeping, **170**, 168–171
- projected Bellman error, 285
 - vector, 267, 269
- proximal TD methods, 286
- pseudo termination, 282, 308
- psychology, 4, 13, 18, 20, 341–376
- $Q(\lambda)$, Watkins’s, 312–314
- Q-function, *see* action-value function
- Q-learning, 21, **131**, 131–135
 - double, **136**
- Q-planning, **161**
- $Q(\sigma)$, **156**, 154–156
- queuing example, 251
- R-learning, 256
- racetrack exercise, 111
- radial basis functions (RBFs), 221–222
- random walk, 95
 - 5-state, 125, 126, 127
 - 19-state, 144, 291
 - TD(λ) results on, 294, 295, 299
 - 1000-state, 203–209, 217, 218
 - Fourier and polynomial bases, 214
- real-time dynamic programming, 177–180
- recycling robot example, 52
- REINFORCE, **328**, 326–331
 - with baseline, **330**
- reinforcement learning, 1–21
- reinforcement signal, 380
- representation learning, 473
- residual-gradient algorithm, 272–274, 277
 - naïve, 270, 271
- return, 54–57
 - n -step, 143
 - for $Q(\sigma)$, 155
 - for action values, 146
 - for Expected Sarsa, 148
 - for Tree Backup, 153
 - with control variates, 150, 151
 - with function approximation, 209
 - differential, 250, 255, 334
 - flat partial, 113
 - with state-dependent termination, 308
 - λ -return, 288–291
 - truncated, 296
- reward prediction error hypothesis, 381–383, 387–395
- reward signal, 1, 6, 48, 53, 361, 380, 383, 397
 - and reinforcement, 373–375, 380–381
 - design of, 469–472, 476
 - intrinsic, 474
 - sparse, 469–470
- rod maneuvering example, 171
- rollout algorithms, 183–185
- root mean-squared (RMS) error, 125
- safety, 434, 475–478
- sample and expected updates, 121, 170–174
- sample or simulation model, 115
- sample-average method, 27
- Samuel’s checkers player, 20, 241, 426–429
- Sarsa, **130**, 129–131, **244**
 - vs Q-learning, 132
 - differential, one-step, **251**
 - Expected, 133–134, 140
 - n -step, 148
 - n -step off-policy, 150
 - double, 136
 - n -step, **147**, 145–148, **247**
 - differential, **255**
 - off-policy, **149**
- Sarsa(λ), **305**, 303–307
 - true online, **307**
- Schultz, Wolfram, 387–395, 410
- search control, 163
- secondary reinforcement, 20, 346, 354, 369
- selective bootstrap adaptation, 239
- semi-gradient methods, 202, 258–259
- SGD, *see* stochastic gradient descent
- Shannon, Claude, 16, 20, 70, 71, 426
- shaping, 360, 470
- Skinner, B. F., 359–360, 375, 470, 480
- soap bubble example, 95
- soft and ε -soft policies, 100–103, 110
- soft-max, 322–323, 329, 336, 400, 445, 455
 - for bandits, 37, 45
- spike-timing-dependent plasticity (STDP), 401
- state, 7, 48, 49
 - k th-order history approach, 468
 - and observations, 464–468
 - Markov property, 465–468
 - belief, 466
 - latent, 466
 - observable operator models (OOMs), 467
 - partially observable MDPs, 14, 466
 - predictive state representations, 466
 - state-update function, 466

- state aggregation, 203–204
- state-update function, 466
- step-size parameter, 10, 31–33, 120, 125, 126
 - automatic adaptation, 238
 - in DQN, 439, 440
 - in psychological models, 347, 348
 - selecting manually, 222–223
 - with coarse coding, 216
 - with Fourier features, 213
 - with tile coding, 217, 223
- stochastic approx. convergence conditions, 33
- stochastic gradient descent (SGD), 200–204
 - in the Bellman error, 269–277
- strong and weak methods, 4
- supervised learning, xvii, 2, 16–19, 198
- sweeps, 75, 160, *see also* prioritized sweeping
- synaptic plasticity, 379
 - Hebbian, 400
 - two-factor and three factor, 400
- system identification, 364
- tabular solution methods, 23
- target
 - policy, 103, 110
 - of update, 31, 143, 198
- TD, *see* temporal-difference learning
- TD error, 121
 - n -step, 255
 - differential, 250
 - with function approximation, 270
- TD(λ), **293**, 292–295
 - truncated, 295–297
 - true online, **300**, 299–301
- TD-Gammon, 21, 421–426
- temporal abstraction, 461–464
- temporal-difference learning, 10, 119–140
 - history of, 20–21
 - advantages of, 124–126
 - optimality of, 126–128
 - TD(0), **120**, **203**
 - TD(1), 294
 - TD(λ), **293**, 292–295
 - true online, **300**, 299–301
 - λ -return methods
 - offline, 290
 - online, 297–299
 - n -step, **144**, 141–158, **209**
- termination function, 307, 459
- Thompson sampling, 43, 45
- Thorndike, Edward, *see* Law of Effect
- tic-tac-toe, 8–13, 17, 137
- tile coding, 217–221, 223, 238, 246, 434, 435
- Tolman, Edward, 364, 408
- trace-decay parameter (λ), 287, 289, 290, 292
 - state dependent, 307
- trajectory sampling, 174–177
- transition probabilities, 49
- Tree Backup
 - n -step, 152–153, **154**
 - Tree-Backup(λ), 312–314
- trial-and-error, 2, 7, 15–21, 403, 404, *see also*
 - instrumental conditioning
- true online TD(λ), **300**, 299–301
- Tsitsiklis and Van Roy’s Counterexample, 263
- undiscounted continuing tasks, *see* average re-ward setting
- unsupervised learning, 2, 226
- value, 6, 26, 47
- value function, 6, 58–67
 - for a given policy: v_π and q_π , 58
 - for an optimal policy: v_* and q_* , 62
 - action, 58, 63, 64, 71, 129, 131
 - approximate action values: $\hat{q}(s, a, \mathbf{w})$, 243
 - approximate state values: $\hat{v}(s, \mathbf{w})$, 197
 - differential, 243
 - vs evolutionary methods, 10
- value iteration, **83**, 82–85
- value-function approximation, 198
- Watkins, Chris, 14, 21, 89, 320
- Watson (*Jeopardy!* player), 429–432
- Werbos, Paul, 14, 21, 69, 89, 139, 238
- Witten, Ian, 21, 69

Adaptive Computation and Machine Learning

Francis Bach, Editor

Bioinformatics: The Machine Learning Approach, Pierre Baldi and Søren Brunak

Reinforcement Learning: An Introduction, Richard S. Sutton and Andrew G. Barto

Graphical Models for Machine Learning and Digital Communication, Brendan J. Frey

Learning in Graphical Models, Michael I. Jordan

Causation, Prediction, and Search, second edition, Peter Spirtes, Clark Glymour, and Richard Scheines

Principles of Data Mining, David Hand, Heikki Mannila, and Padhraic Smyth

Bioinformatics: The Machine Learning Approach, second edition, Pierre Baldi and Søren Brunak

Learning Kernel Classifiers: Theory and Algorithms, Ralf Herbrich

Learning with Kernels: Support Vector Machines, Regularization, Optimization, and Beyond, Bernhard Schölkopf and Alexander J. Smola

Introduction to Machine Learning, Ethem Alpaydin

Gaussian Processes for Machine Learning, Carl Edward Rasmussen and Christopher K.I. Williams

Semi-Supervised Learning, Olivier Chapelle, Bernhard Schölkopf, and Alexander Zien, Eds.

The Minimum Description Length Principle, Peter D. Grünwald

Introduction to Statistical Relational Learning, Lise Getoor and Ben Taskar, Eds.

Probabilistic Graphical Models: Principles and Techniques, Daphne Koller and Nir Friedman

Introduction to Machine Learning, second edition, Ethem Alpaydin

Machine Learning in Non-Stationary Environments: Introduction to Covariate Shift Adaptation, Masashi Sugiyama and Motoaki Kawanabe

Boosting: Foundations and Algorithms, Robert E. Schapire and Yoav Freund

Machine Learning: A Probabilistic Perspective, Kevin P. Murphy

Foundations of Machine Learning, Mehryar Mohri, Afshin Rostami, and Ameet Talwalker

Introduction to Machine Learning, third edition, Ethem Alpaydin

Deep Learning, Ian Goodfellow, Yoshua Bengio, and Aaron Courville

Elements of Causal Inference, Jonas Peters, Dominik Janzing, and Bernhard Schölkopf

Machine Learning for Data Streams, with Practical Examples in MOA, Albert Bifet, Ricard Gavaldà, Geoffrey Holmes, Bernhard Pfahringer