Early Internet Advertising

- Advertisers place ads (banners) on a website.
- The owner of the website charges the advertisers on a per-impression basis, typically a flat fee to show their ads a fixed number of times (one thousand impressions).
- Major problems:
  1. Contracts were negotiated on a case-by-case basis, mostly are quite small (few thousand dollars per month).
  2. The capacity of a website is limited. A banner ad is shown to every one visiting the website, no matter whether it is relevant or not. Obviously the web resource is largely wasted.

Sponsored search and first-price ad auction

- An advertiser chooses a set of keywords that are related to the product it wishes to sell.
- Each advertiser submits a bid for each keyword, i.e., the amount to be paid per click.
- When a user's search query matches a keyword, a set of ads is displayed. These ads are ranked by bids. The ad with the highest bid receives the best position; i.e., the position that is mostly likely to be clicked on by the user.
- Advertisers pay by clicks. Every time a consumer clicked on a sponsored link, the associated advertisers account is automatically billed the amount of the advertisers most recent bid.
- This approach was introduced by Overture Services (now part of Yahoo) in 1997.

Weakness of first-price ad auctions

Suppose there are two slots on a page and three bidders. An ad in the first slot receives 200 clicks per hour while the second slot gets 100. The bidders valuations on each click are

	Bidder 1	Bidder 2	Bidder 3
valuation	\$10	\$4	\$2

After a period of time when bidder 2 knew that bidder 3's bid was \$2, he reduced his bid to \$2.01 to guarantee that he got a slot. Then bidder 1 would not want to bid more than \$2.02. But then bidder 2 would want to revise his bid to \$2.03 to get the top spot, bidder 1 would in turn raise his bid to \$2.04, and so on.

Weakness of first-price ad auctions

- There is no pure strategic equilibrium in the one-shot auction therefore the game is unstable.
- The bidder who could react to its competitors moves fastest had a substantial advantage. The mechanism therefore encouraged inefficient investments in gaming the system.
- It also created volatile prices that in turn caused allocative inefficiencies.

Ad auction: second-price

- Advertisers are invited to submit bids.
- Advertisers' links are arranged on the page in descending order of their bids.
- The advertiser in the first position pays a price per click that equals to the bid of the second advertiser plus an increment (say 1 cent); the second advertiser pays the price offered by the third advertiser plus an increment and so forth
- This auction mechanism is known as Ad auction, name by Google, or Generalized Second-Price Auction (GSP auction) by economists.
- Ad auction was introduced by Google in 2002. In 2004, Over 98% of Googles total revenue was generated from Ad auctions ($3.189 billion).

Ad auctions: second-price

	Bidder 1	Bidder 2	Bidder 3
value	\$10	\$4	\$2

If all advertisers bid truthfully, then bids are \$10, \$4, \$2. Under second-price ad auction, payments will be \$4.01 and \$2.01. Note that total payments of the bidders are around \$800 and \$200, respectively while the first-price auction gives about \$400 and \$200 at its worst case, respectively. Google says their "unique auction model uses Nobel Prize-winning economic theory to eliminate ... that feeling that you've paid too much".

The model of GSP auctions

- K bidders: the advertisers $k = 1, 2, \cdots, K$.
- N items: the positions on the screen, where ads related to a keyword can be displayed, $i = 1, 2, \cdots, N$.
- Click Through Rates (CTR) α_i: the expected clicked if an ad placed in position i during a period. Without loss of generality, we assume that $\alpha_1 > \alpha_2 > \alpha_3 > \cdots > \alpha_N$.
- Value per click s_k: the value to bidder k if it receives a click.
- Payoff $u_k(i)$: bidder k's payoff from an ad placed at position i.

$$u_k(i) = \alpha_i s_k$$

Properties of GSP auctions

- Truth-telling is not a dominant strategy under GSP
- This means that a bidder does not have to bid using its true value to maximize its profit.
- For a market mechanism, if it is not a truth-telling mechanism, bidders would be able to manipulate the mechanism (play tricks) to get better outcomes.

Why GSP is not a truth-telling mechanism?

Lower your position:
Assume that three bidders bid for two ad slots with private values \$10, \$4 and \$2, respectively. The two slots receive a similar clicks per hour, say 200 and 199, respectively. The first bidder bids with its true value receiving a payoff: $(\$10 - \$4) * 200 = \$1200$. If it bids \$3, it receives a payoff: $(\$10 - \$2) * 199 = \$1592 > \$1200$.

Why GSP is stable?

Envy Free Equilibrium:

- Suppose that you won a slot at position l and your competitive opponent won the slot immediate above yours and pays your bidding price. To give your opponent a lesson, you raise your bid a bit, which does not affect your payoff but will reduce the payoff of your opponent.
- However, your opponent can easily retaliate you with the following strategy. Because she knows your truth valuation, she can simply reduce her bid to your valuation, which will force you move up one position (therefore you will have to pay exactly your valuation plus an increment). This means that you will receive a negative utility.
- Such a property is called envy free. A GSP procedure guarantees to reach an envy free equilibrium.

In the reality, Ad auction has been combined with many other factors, such as link quality, to make it work more smoothly.