
Chapter

8

Electrical Noise: An Overview

The word “noise,” in everyday use, is any wanted or unwanted sound that can be heard. Electrical noise has a significantly different meaning. Electrical noise is a current or voltage signal that is *unwanted* in an electrical circuit. Real signals are the sum of this unwanted noise and the desired signals. Types of noise include 1) noise resulting from the discrete and random movement of charge in a wire or device (which we’ll call inherent circuit noise, e.g., thermal noise, shot noise, or flicker noise), 2) quantization noise (resulting from the finite digital word size when changing an analog signal into a digital signal), and 3) coupled noise (resulting from the signals of adjacent circuits feeding into each other and interfering). We focus the discussion in this chapter on inherent noise. Quantization and coupled noise are discussed in other parts of the book, e.g., Ch. 28.

Electrical noise is introduced early in our study of circuits in this book because it is often the limiting factor in a system’s performance. For example, the distance a radio receiver can pick up a transmitted signal increases as the noise performance of the receiver improves. When studying noise, the units, terminology, and procedures used in a noise analysis can present a barrier to understanding what circuit element limits the noise performance (that is, how the circuit’s noise performance can be improved). To obviate this barrier, we’ll take an unusual approach: we’ll use a few simple signals and measurements to illustrate some of the noise analysis procedures.

8.1 Signals

In this section we briefly review some fundamentals related to electrical signals.

8.1.1 Power and Energy

Figure 8.1 shows a simple sinewave. At the risk of stating the obvious, the average value of this signal is zero. Does this mean that if this signal is applied across a resistor zero average power is dissipated? Of course not (or else many people wouldn’t be able to cook food or heat their homes). In order to characterize the *average* amount of power dissipated by a resistor when this sinusoidal signal is applied across it, Fig. 8.2, we can begin by writing the instantaneous power dissipated by the resistor (the power dissipated by the resistor at a specific time t) as

$$P_{inst}(t) = \frac{v^2(t)}{R} = \frac{V_P^2 \sin^2(2\pi \cdot \frac{t}{T})}{R} \quad (8.1)$$

Summing this power over time results in

$$Energy = \int_0^{\infty} \frac{V_P^2 \sin^2(2\pi \cdot \frac{t}{T})}{R} \cdot dt \quad (8.2)$$

with units of $W \cdot s$ or Joules (energy). Clearly, summing the instantaneous power doesn't result in the average power dissipated by the resistor but rather the energy the sinewave source supplies to the resistor. Because we didn't place an upper bound on the integration in Eq. (8.2), the energy supplied tends to infinity. (A resistor dissipating power over an infinite period of time is being driven by a source with an infinite amount of energy.)

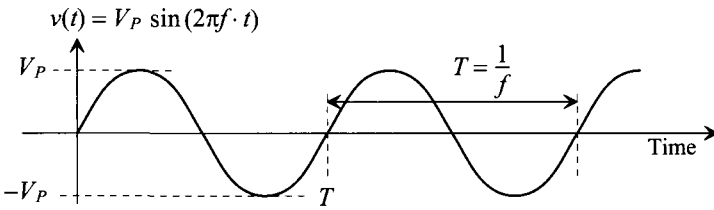


Figure 8.1 A sinewave.

Examining Fig. 8.1 for a moment, we realize that because the sinewave is periodic, we need only estimate the average power over one cycle (say from time zero to T) to estimate the average power supplied to a resistor by the entire waveform. We can do this by summing the instantaneous power for one cycle (which results in the energy supplied by the sinewave source during this cycle) and then dividing by the cycle time T . This can be written as

$$\text{Average power dissipated} = \frac{1}{T} \int_0^T \frac{V_P^2 \sin^2(2\pi \cdot \frac{t}{T})}{R} \cdot dt \quad (8.3)$$

noting that the resistor's value can be moved outside of the integration. This result, with units of watts, is the mean (average) squared voltage of the sine wave signal dropped across the resistor divided by the resistor's value. For the sine wave in Figs. 8.1 and 8.2, we can write

$$\text{Mean-squared voltage} = \overline{v^2} = \frac{1}{T} \int_0^T V_P^2 \sin^2(2\pi \cdot \frac{t}{T}) \cdot dt = \frac{V_P^2}{2} \quad (8.4)$$

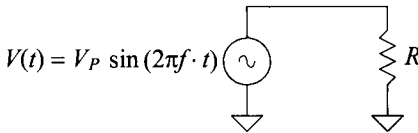


Figure 8.2 A sinewave driving a resistor.

with units of volts squared. Equation (8.3) simplifies to the mean-squared voltage divided by R , i.e., $\overline{v^2}/R$, or, for a sinusoid $V_P^2/2R$. This should be compared to the power dissipated by a resistor with a DC voltage across it, i.e., V_{DC}^2/R . When $V_P^2/2 = V_{DC}^2$, the power dissipation is the same. If we take the square root of the mean-squared voltage (the result is called the root-mean-squared, RMS, voltage) or

$$\text{RMS voltage of a sinewave, } \sqrt{\overline{v^2}} = V_{RMS} = \sqrt{\frac{1}{T} \int_0^T V_P^2 \sin^2(2\pi f \cdot t) \cdot dt} = \frac{V_P}{\sqrt{2}} \quad (8.5)$$

then we can say that when $V_{RMS} = V_{DC}$, the power dissipated by the resistor with either a sinusoidal or DC source is the same.

Comments

Power is heat. If a circuit is battery-powered, then the less heat it generates the longer the battery life. Batteries are often characterized by their capacity to supply power (the potential energy they hold with units of Joules, watthours, or if the battery voltage is known, mA-hours). A battery can also be characterized by its ability to supply a given amount of power in a period of time (kinetic energy).

For example, two 1.5 V batteries may be rated with the same capacity, i.e., potential energies (say 1 W·hr, 3600 J, or 667 mA·hr) but one may only be capable of supplying a maximum current of 1 mA, while the other may supply a maximum of 10 mA. In a one-hour period, and at maximum current draw, the first battery can supply a power of 1.5 mW to a circuit, while the second battery can supply up to 15 mW. The energy (kinetic) supplied at these maximums in one hour is then 5.40 J (the battery supplying 1.5 mW for one hour) and 54 J (the battery supplying 15 mW for one hour). Question: Which battery will discharge first? Answer: The one supplying 10 mA of current will discharge first because both started with the same capacity. It will discharge in 66.7 hours.

When a voltage signal containing many sinusoids (and perhaps DC) is applied across a resistor, we add the power from each sinusoid (and DC) to obtain the total power dissipated by the resistor. In other words, we sum the mean-squared voltages from each sinewave (the mean-squared voltage of a DC source, V_{DC} , is V_{DC}^2) and divide by the resistor's value to obtain the total power dissipated. Taking the square-root of the sum of the mean-squared voltages, we obtain the RMS (root OF THE mean OF THE square) value of the signal applied to the resistor. This is *important* because we will regularly sum mean-squared voltages when performing noise analysis. (We regularly add the power [mean-squared value] contributed from each noise source to a circuit.) We *never* sum RMS voltages.

8.1.2 Power Spectral Density

Figure 8.3a shows spectral plots of the sinewave in Fig. 8.1. Notice how we use either a single line or a dot to indicate spectral content at a frequency of f . Figure 8.3b shows the spectrum of a signal containing two sinewaves and a DC component (with arbitrary amplitudes) using lines and dots. In Fig. 8.3c we show a signal where the spectrum is occupied with sinusoids from DC to some maximum f_{\max} . Even though we show a constant amplitude (a continuous spectral density from DC to f_{\max}), it is understood that we are representing a spectrum with spectral components that are, or may be, changing.

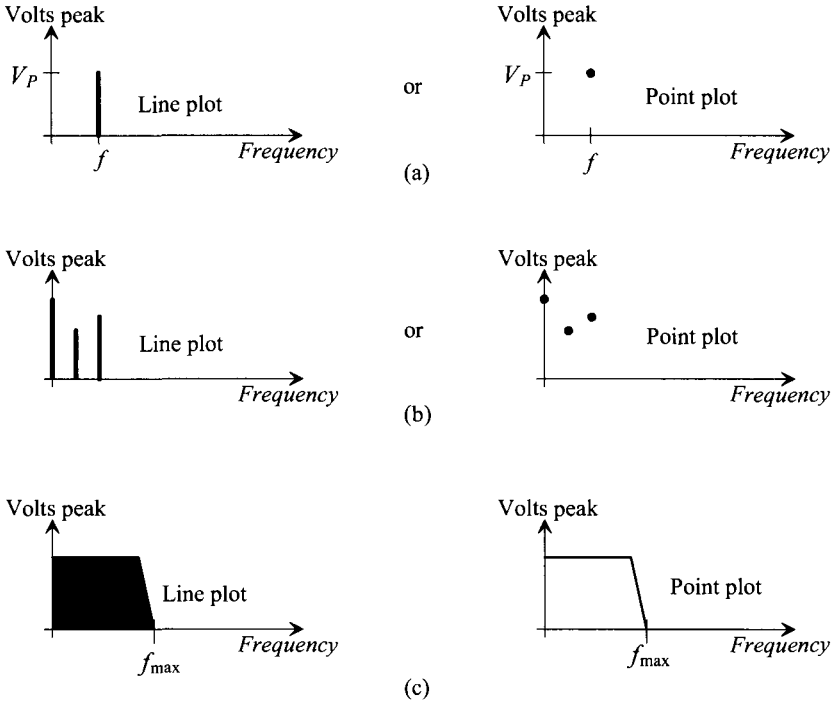


Figure 8.3 Spectrums of various signals (see text).

Spectrum Analyzers

A spectrum analyzer (SA) is an instrument for evaluating the spectral content of a signal. The SA outputs plots similar to those seen in Fig. 8.3 (point-by-point). A block diagram of an SA is seen in Fig. 8.4. The input of the SA is multiplied by a sinusoid to frequency shift the input signal. After the multiplication, the bandpass filter limits the range of frequencies to a bandwidth of f_{res} (the resolution bandwidth of the measurement). The power in the output signal of the bandpass filter is then measured. The counting index, n , varies the measurement from a start frequency, f_{start} , to a stop frequency, f_{stop} . This range of frequencies makes up the x-axis in our spectral plots (as seen in Fig. 8.3). The power measured at each corresponding point sets the y-axis values.

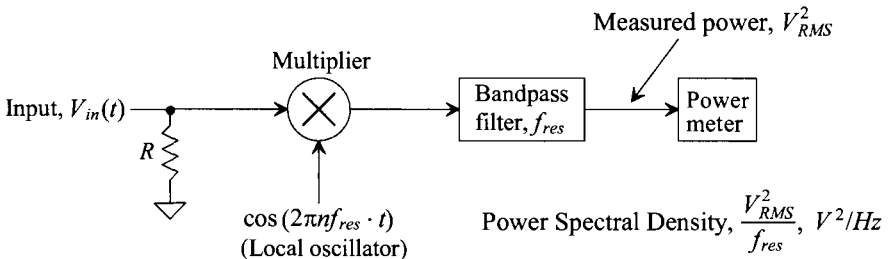


Figure 8.4 Block diagram of a spectrum analyzer.

Let's make some assumptions and give an example to illustrate how the SA operates. Let's assume that we want to look at the spectrum of a signal from DC ($f = 0 = f_{start}$) to 10 kHz ($= f_{stop}$) with a resolution of 100 Hz ($= f_{res}$). Further let's assume that the signal we want to look at is a 1 V peak amplitude sinewave at 4.05 kHz in series with 1 V DC, that is, $V_{in}(t) = 1 + \sin 2\pi \cdot 4.05 \text{ kHz} \cdot t$ V. To begin, we calculate the range of the counting index, n ,

$$f_{stop} = 10 \text{ kHz} = n_{max} \cdot f_{res} \rightarrow n_{max} = 100 \quad (8.6)$$

The counting index will vary from 0 to 100. In other words, the frequency of the cosine signal in Fig. 8.4 will vary from 0 to 10 kHz in 100 Hz steps. Our plot will have 101 points.

With our input signal applied, the power at the first point, $n = 0$ (or for the x-axis $f = 0$), is measured. The cosine term applied to the multiplier is 1. The entire input signal is applied to the bandpass filter. In a general SA, a bandpass filter is used. In this example, we use a lowpass filter that passes DC to < 100 Hz. The signal applied to the power meter is $(1 \text{ V})^2$. We are ready to plot the first point in the spectral analysis. For this first point at $f = 0$ (and $n = 0$), the y-axis units can be peak voltages ($= 1 \text{ V}$), mean-squared ($= 1/2 \text{ V}^2$), RMS ($= 1/\sqrt{2} \text{ V} = V_{RMS}$), power ($= 1/2R = V_{RMS}^2/R$ watts where the R is the input resistance of the SA seen in Fig. 8.4), power spectral density (PSD) ($= 1/[2Rf_{res}]$ with units of W/Hz or Joules¹), or voltage spectral density ($= V_{RMS} / \sqrt{f_{res}} = 1/\sqrt{2 \cdot 100}$ with units of $V/\sqrt{\text{Hz}}$).

Often we'll use a PSD with units of V^2/Hz . This is the result of eliminating the resistance from the calculation **so that the PSD is V_{RMS}^2/f_{res}** ($= 1/2f_{res} = 5 \times 10^{-3} \text{ V}^2/\text{Hz}$ for this first point). We use the PSD in noise analysis (and elsewhere) because, for a continuous spectrum (something that we don't have in this example but that is common for noise signals), increasing f_{res} increases the power we measure (V_{RMS}^2), resulting in a constant number when we take the ratio V_{RMS}^2/f_{res} . (The PSD of a continuous spectrum doesn't change with measurement resolution.) Unfortunately, when a single sinewave is measured, increasing f_{res} decreases the sinewave's amplitude PSD amplitude, as seen in Fig. 8.5.

Continuing on with our measurement when $n = 1$ and the multiplying (local oscillator) frequency is 100 Hz, the output of the multiplier, in Fig. 8.4, is

$$\cos(2\pi \cdot 100 \cdot t) \cdot (1 + \sin[2\pi \cdot 4.05 \text{ kHz} \cdot t]) \text{ volts} \quad (8.7)$$

or knowing $\cos A \cdot \sin B = \frac{1}{2}(\sin[B - A] + \sin[A + B])$, we get

$$\cos(2\pi \cdot 100t) + \frac{1}{2}(\sin[2\pi(3.95k)t] + \sin[2\pi(4.15k)t]) \text{ volts} \quad (8.8)$$

This multiplier output signal (remembering $n = 1$) contains three sinusoids with frequencies of 100 Hz, 3.95 kHz, and 4.15 kHz. Passing this signal through the filter, which allows frequencies from DC to < 100 Hz through, results in zero measured power. This assumes an ideal filter that doesn't pass 100 Hz (see the comment at the end of this

¹ From the units, $\text{W/Hz} = \text{W} \cdot \text{s} = \text{J}$, we can tell that this is the amount of power dissipated by R (for the frequency range of sine waves that pass through the bandpass filter) in one second (the energy used). This shouldn't be confused with energy spectral density (ESD) which has units of $\text{V}^2/\text{Hz} \cdot \text{s}$ (or V^2), for transient signals (for a finite amount of time, i.e., one second).

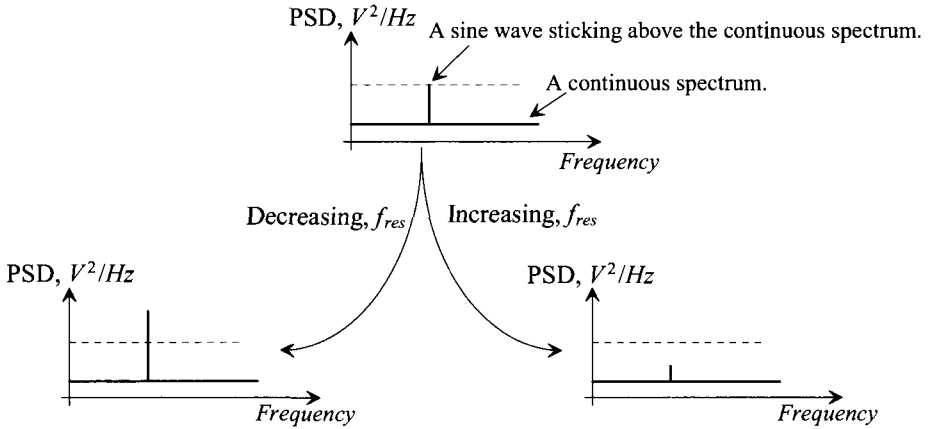


Figure 8.5 Changing the resolution bandwidth doesn't affect the PSD amplitude of the continuous spectrum but does affect the amplitude of the sine wave sticking up above the continuous spectrum.

section). In fact, it's easy to show that we get zero measured signal until $nf_{res} = 4 \text{ kHz}$. When the local oscillator frequency is 4 kHz, the output of the multiplier is

$$\cos(2\pi \cdot 4kt) + \frac{1}{2}(\sin[2\pi(50)t] + \sin[2\pi(8.05k)t]) \text{ volts} \quad (8.9a)$$

that is, a signal containing power at frequencies of 50 Hz, 4 kHz, and 8.05 kHz. Again, the filter passes the signal content in the range of $0 \leq f < 100 \text{ Hz}$ to the power meter. What comes out of the filter is a sinewave at 50 Hz with a peak amplitude of 0.5 V. The next measured point occurs when $nf_{res} = 4.1 \text{ kHz}$. The signal applied to the filter (the output of the multiplier) is

$$\cos(2\pi \cdot 4.1kt) + \frac{1}{2}(\sin[2\pi(-50)t] + \sin[2\pi(8.15k)t]) \text{ volts} \quad (8.9b)$$

The *negative frequency* signal present here can be thought of as a phase-shifted positive frequency signal. Because the sine function is an odd function, $\sin(-2\pi f \cdot t) = -\sin(2\pi f \cdot t) = \sin(2\pi f \cdot t \pm \pi)$, there is still spectral content at 50 Hz. The signal applied to the meter is, again, a 50 Hz sinewave with an amplitude of 0.5 V. The total (measured) signal amplitude from the 4.05 kHz component of the input is the sum of the measured signal amplitudes when nf_{res} is 4 kHz and 4.1 kHz (that is, a sum of 1 V). When plotting points, the amplitudes at measured adjacent points are summed. For example, we can plot a point at 4.05 kHz with an amplitude of 1 V by summing the measured signals when nf_{res} is 4 kHz and 4.1 kHz. We can plot points at 3.95 and 4.15 kHz with amplitudes of 0.5 V because at 3.9 kHz and 4.2 kHz ($= nf_{res}$), we have zero measured signal.

Looking at the spectrums of signals using a discrete Fourier transform (DFT) of analog waveforms is analogous to using an SA, to look at spectrums. For more detailed information on using both a DFT and an SA the interested reader is referred to the book *CMOS Mixed-Signal Circuit Design*. In this book additional information and discussions are given on these topics.

8.2 Circuit Noise

The electrical noise coming out of a circuit can be measured using a SA, as seen in Fig. 8.6. The circuit under test (CUT) is connected to a spectrum analyzer with no source signal applied. If the noise coming out of a CUT is lower than the noise floor of the SA, a low-noise amplifier (LNA) is inserted between the CUT and SA to help with the noise measurement (the measured noise is then increased by the LNA's gain). As mentioned in the previous section, we generally plot PSD versus frequency to ultimately characterize the power in a signal or the signal's RMS value.

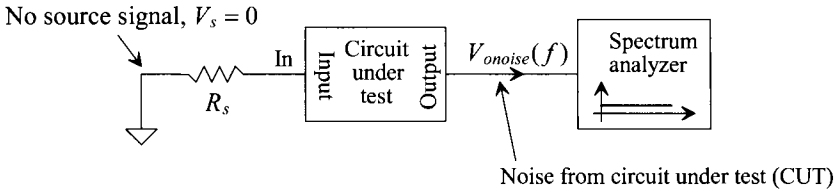


Figure 8.6 Circuit used for noise measurement.

8.2.1 Calculating and Modeling Circuit Noise

Figure 8.7 shows a time domain representation of a noise signal. If this noise signal's PSD is called $V_{noise}^2(f)$ (units, V^2/Hz), we can determine its RMS value using

$$V_{RMS} = \sqrt{\int_{f_L}^{f_H} V_{noise}^2(f) \cdot df} \quad \text{Volts} \quad (8.10)$$

where f_L and f_H are the lowest and highest frequencies of interest. If the noise PSD is flat (often called a “white noise spectrum” after white light that contains spectral content at all visible wavelengths), then this equation reduces to

$$V_{RMS} = \sqrt{(f_H - f_L) \cdot V_{noise}^2(f)} = \sqrt{B} \cdot \sqrt{V_{noise}^2(f)} \quad (8.11)$$

where the bandwidth of the measurement, B , is $f_H - f_L$.

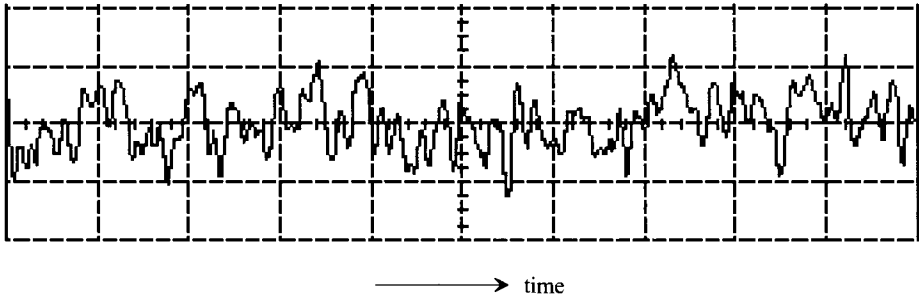


Figure 8.7 Time domain view of circuit noise.

Input-Referred Noise I

Noise is measured on the output of a circuit. It can, however, be referred back to the input of the circuit for comparison with an input signal. This input-referred noise isn't really present on the input of the CUT. This is especially true for CMOS circuits where the input to a CUT may be the polysilicon gate of a MOSFET. While a direct tunneling gate current noise may be present it's generally much less important than the average (DC) tunneling gate current and usually a very small contributor to the measured output noise.

Consider the amplifier models seen in Fig. 8.8. Figure 8.8a shows the measured output noise PSD. We can calculate the input-referred PSD, Fig. 8.8b, by simply dividing the output PSD by the gain, A , of the amplifier squared (or divide the voltage spectral density, $= \sqrt{PSD}$, by the gain).

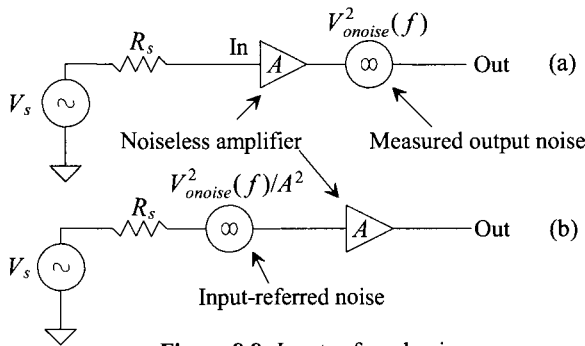


Figure 8.8 Input-referred noise.

Example 8.1

Suppose the measured noise voltage spectral density on the output of an amplifier is a white spectrum from DC to 100 MHz ($= B = f_H - f_L$) and has a value of $10 \text{ nV}/\sqrt{\text{Hz}}$. Estimate the amplifier's input-referred noise if its gain is 100.

The amplifier's output noise PSD is

$$V_{onoise}^2(f) = \left(10 \text{ nV}/\sqrt{\text{Hz}}\right)^2 = 100 \times 10^{-18} \text{ V}^2/\text{Hz}$$

The RMS value of this spectral density is

$$V_{onoise,RMS} = \sqrt{\int_0^{100\text{MHz}} 100 \times 10^{-18} df} = 100 \text{ }\mu\text{V}$$

Referring this noise back to the input of the amplifier results in

$$V_{inoise,RMS} = \frac{V_{onoise,RMS}}{A} = 1 \text{ }\mu\text{V} \quad (8.12)$$

■

Noise Equivalent Bandwidth

In the previous example, the bandwidth of the noise calculation was given. However, we may wonder, in a real circuit, how we determine f_L and f_H . In the ideal situation, we would

use an infinite bandwidth when calculating the noise. Figure 8.9a shows a white noise spectrum. If we were to calculate the RMS value of the noise voltage from this spectrum, over an infinite bandwidth, we would end up with an infinite RMS noise voltage. In real circuits the signals, and noise, are bandlimited (their spectral content goes to zero at some frequencies). This bandlimiting can be the result of intentionally added or parasitic capacitances present in the circuit. Figure 8.9b shows a noise spectrum if the CUT shows a simple single-pole roll-off. In this case, our noise is no longer white at high frequencies but rolls off above the 3-dB frequency of the circuit, f_{3dB} .

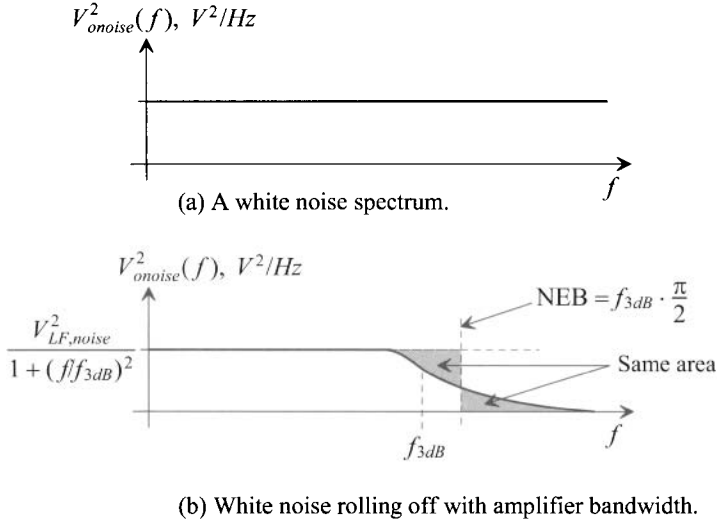


Figure 8.9 Noise PSDs with (a) a white spectrum and (b) a spectrum that rolls-off with circuit bandwidth (assuming a dominant pole circuit).

If we assume that a CUT has a single-pole roll-off, as seen in Fig. 8.9b, and the low-frequency noise of the CUT is white (again as seen in Fig. 8.9b), then we can calculate the RMS output noise using

$$V_{noise,RMS}^2 = \int_0^{\infty} \frac{\overbrace{V_{noise}^2(f)}^{V_{LF,noise}^2}}{\left(\sqrt{1 + (ff_{3dB})^2}\right)^2} df \quad (8.13)$$

If we use

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C \quad (8.14)$$

then

$$\begin{aligned} V_{noise,RMS}^2 &= V_{LF,noise}^2 \cdot f_{3dB} \cdot [\tan^{-1} 2\pi f f_{3dB}]_0^{\infty} \\ &= V_{LF,noise}^2 \cdot \overbrace{f_{3dB} \cdot \frac{\pi}{2}}^{NEB} \end{aligned} \quad (8.15)$$

where NEB is the noise-equivalent bandwidth. Rewriting Eq. (8.11), we get

$$V_{noise,RMS} = \sqrt{NEB} \cdot \sqrt{V_{LF,noise}^2} \quad (8.16)$$

Of course this assumes $V_{LF,noise}^2$ is constant until the frequency approaches f_{3dB} .

Example 8.2

Repeat Ex. 8.1 if the amplifier's output noise power spectral density starts to roll off at 1 MHz as seen in Fig. 8.10.

We begin by using Eq. (8.16) directly, that is,

$$V_{noise,RMS} = \sqrt{f_{3dB} \cdot \frac{\pi}{2}} \cdot \sqrt{V_{LF,noise}^2} \quad (8.17)$$

which evaluates to 12.5 μV . The gain of the amplifier is 100 so $V_{noise,RMS} = 125$ nV. ■

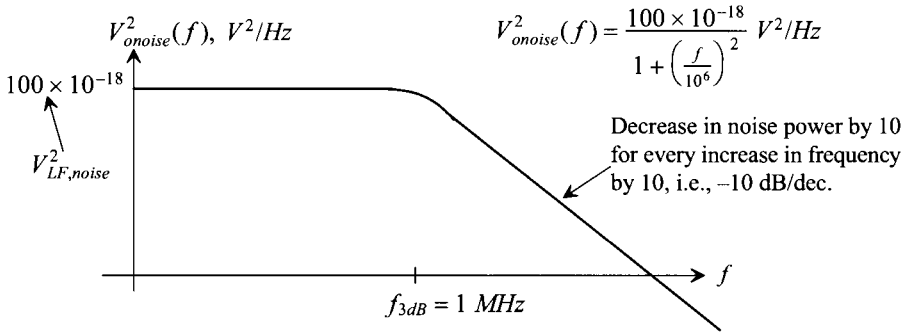


Figure 8.10 Measured output noise spectrum for the CUT discussed in Ex. 8.2.

In this example, we assumed the voltage gain of the amplifier was 100 independent of frequency when we calculated the input-referred RMS noise voltage. In practice, the frequency response of the noise may follow (be correlated with) the amplifier's frequency response. If this is the case, we can write the frequency response of the amplifier in Ex. 8.2 as

$$A(f) = \frac{V_{out}}{V_{in}} = \frac{A_{DC}}{1 + j\frac{f}{f_{3dB}}} = \frac{100}{1 + j\frac{f}{f_{3dB}}} \quad (8.18)$$

where $f_{3dB} = 1$ MHz. We can determine the input-referred noise PSD using

$$V_{in,noise}^2(f) = \frac{V_{onoise}^2(f)}{|A(f)|^2} = \frac{V_{LF,noise}^2}{1 + (f/f_{3dB})^2} \cdot \frac{1 + (f/f_{3dB})^2}{A_{DC}^2} = \frac{V_{LF,noise}^2}{A_{DC}^2} \quad (8.19)$$

showing the input-referred PSD of the output noise is a flat spectrum from DC to infinity! The problem with this is that if, to determine the RMS input-referred noise, we integrate this PSD over an infinite bandwidth, we get an infinite RMS voltage. We must remember that the input-referred noise is used to model (with obvious limitations) the circuit's output noise (again, you can't measure input-referred noise because it isn't really there!).

Input-referred noise is only used to get an idea of how a circuit will corrupt an input signal. This latter point is important because two amplifiers with identical bandwidths may have the same output noise PSD but different gains. The amplifier with the larger gain will have a smaller input-referred noise and thus result in an output signal with a better signal-to-noise ratio. Knowing that the output noise is bandlimited and that the input-referred RMS voltage should reflect this, we can write

$$V_{\text{inoise,RMS}} = \frac{\sqrt{NEB \cdot V_{LF,\text{noise}}^2}}{A_{DC}} = \frac{V_{\text{onoise,RMS}}}{A_{DC}} \quad (8.20)$$

which is, of course, what we used in Ex. 8.2.

Input-Referred Noise in Cascaded Amplifiers

Figure 8.11a shows a cascade of noisy amplifiers with corresponding input-referred noise sources. The output noise power in (a) can be written as

$$V_{\text{onoise,RMS}}^2 = (A_1 A_2 A_3)^2 V_{\text{inoise,RMS1}}^2 + (A_2 A_3)^2 V_{\text{inoise,RMS2}}^2 + A_3^2 V_{\text{inoise,RMS3}}^2 \quad (8.21)$$

again remembering that we add the noise powers from each amplifier. The input-referred noise in (b) is given by

$$V_{\text{inoise,RMS}}^2 = V_{\text{inoise,RMS1}}^2 + V_{\text{inoise,RMS2}}^2 / A_1^2 + V_{\text{inoise,RMS3}}^2 / (A_1 A_2)^2 \quad (8.22)$$

The key point to notice here is that the noise of the first amplifier has the largest effect on the noise performance of the amplifier chain. For good noise performance, the design of the first stage is critical.

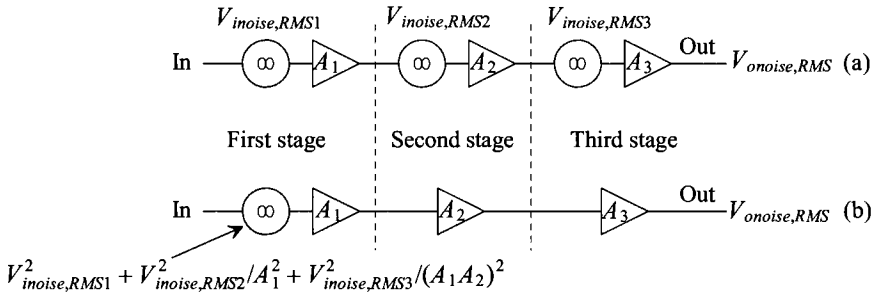


Figure 8.11 Noise performance of a cascade of amplifiers.

Example 8.3

Comment on the limitations of Eqs. (8.21) and (8.22) for calculating noise.

Measured output noise usually includes the thermal noise of the source resistance. If the effective source resistance changes when we cascade the amplifiers, the value calculated for input-referred noise, $V_{\text{inoise,RMS}}$, will also change.

A perhaps more important concern is the change in the bandwidth of the noise. Cascading amplifiers results in a reduction in the circuit's bandwidth. The point of Eqs. (8.21) and (8.22) is still valid, that is, that the first stage's output noise and

gain (equivalent to saying “input-referred noise”) are critical for overall low-noise performance. However, to accurately determine the input-referred noise, it is best to measure the noise on the output of the cascade and then refer it back to the cascade’s input. At the risk of stating the obvious, the gain of the cascaded amplifiers is determined by applying a small sinewave signal to the cascade’s input at a frequency that falls within the amplifier’s passband (not too high or too low). Taking the ratio of the cascade’s output sinewave amplitude to the input sinewave amplitude is the gain. The input-referred noise is then the output RMS noise divided by the gain of the overall cascade. ■

Calculating $V_{noise,RMS}$ from a Spectrum: A Summary

Before leaving this section, let’s show, Fig. 8.12, a summary of how the output RMS noise voltage is calculated from a noise spectrum.

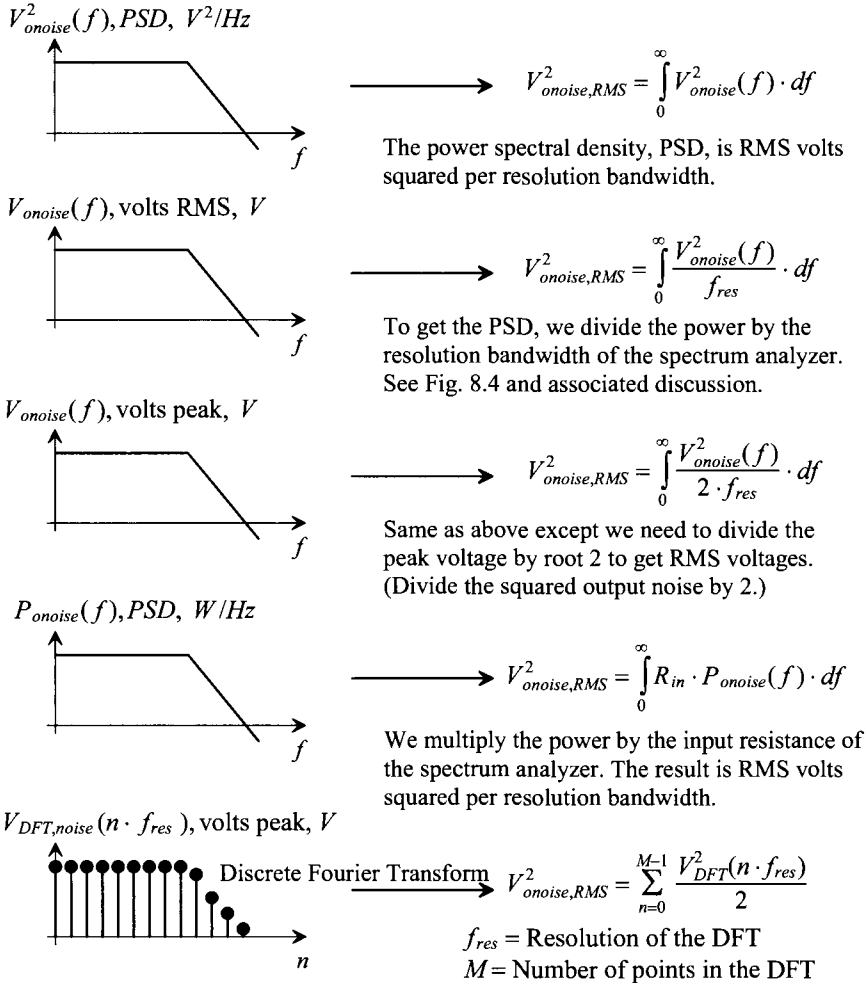


Figure 8.12 Calculating RMS output noise from a noise spectrum.

8.2.2 Thermal Noise

Noise in a resistor is primarily the result of random motion of electrons due to thermal effects. This type of noise is termed *thermal noise*, or Johnson noise, after J. B. Johnson who is credited for first observing the effect. Thermal noise in a resistor can be characterized by a PSD of

$$V_R^2(f) = 4kTR \text{ with units of } V^2/\text{Hz} \quad (8.23)$$

(noting thermal noise is white and independent of frequency) where

$$k = \text{Boltzmann's constant} = 13.8 \times 10^{-24} \text{ Watt} \cdot \text{sec}/^\circ \text{K (or J}/^\circ \text{K)}$$

$$T = \text{temperature in } ^\circ \text{K}$$

$$R = \text{resistance in } \Omega$$

Example 8.4

Verify that the thermal noise PSD given by Eq. (8.23) does indeed have units of V^2/Hz . The term kT has units of Joules (or Watt·sec) and can be thought of as thermal energy. The units for Joules can be written as

$$kT, \text{ units} = \text{Joules} = \text{Watt} \cdot \text{sec} = \text{volts} \cdot \text{amps} \cdot \text{sec} = \text{volts} \cdot \text{columbs}$$

knowing amps = columbs/sec. The units of resistance, R , can be written as

$$R, \text{ units of } \Omega = \frac{\text{volts}}{\text{amps}} = \frac{\text{volts}}{\text{columbs/sec}} = \frac{\text{volts}}{\text{columbs} \cdot \text{Hz}}$$

The units for thermal noise PSD, $4kTR$, are then V^2/Hz . ■

Figure 8.13 shows how the thermal noise from a resistor is modeled and added to a circuit. The output noise PSD in (a) is $(4kT/R) \cdot R^2$ (we multiply by R^2). Heat (power) is absorbed by the resistor when it is not at 0°K . The heat causes lattice vibrations in the resistive material and thus randomizes the motion of electrons moving in or traveling through the resistor. The net current that flows out of the resistor due to heat is zero but,

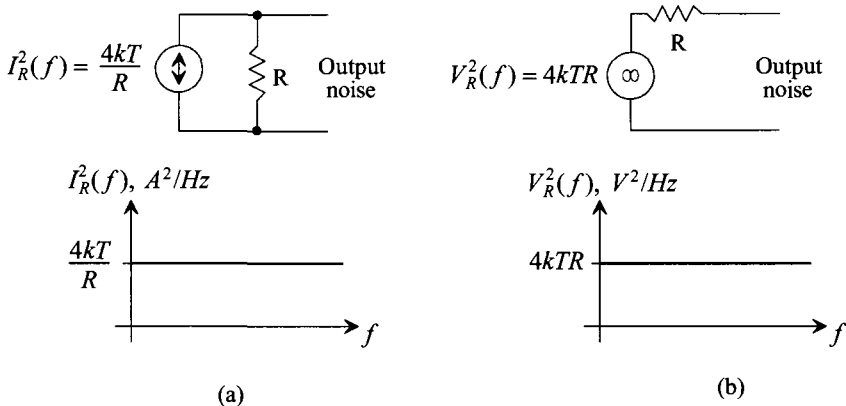


Figure 8.13 Circuit representations and their corresponding PSDs for thermal noise.

over a finite period of time, the current crossing the terminal of a resistor may have a net nonzero value (either into or out of the resistor). In other words, electrons move back and forth from the resistor to the metal wire with an overall net zero charge transfer but, over a short finite period of time, a net nonzero current flow is possible.

The term kT , at 300 °K, is 4.14×10^{-21} Joules. For a one-second time frame, again at 300 °K, the resistor dissipates 4.14×10^{-21} Watts (absorbs this much heat from a heat source). For one hour, the energy supplied to keep the resistor heated to 300 °K is $(3600 \text{ s}) \cdot (4.14 \times 10^{-21} \text{ watts}) = 14.9 \times 10^{-18}$ Joules.

Example 8.5

For the circuit shown in Fig. 8.14, determine the RMS output and input-referred noise over a bandwidth from DC to 1 kHz. Verify your answer with SPICE.

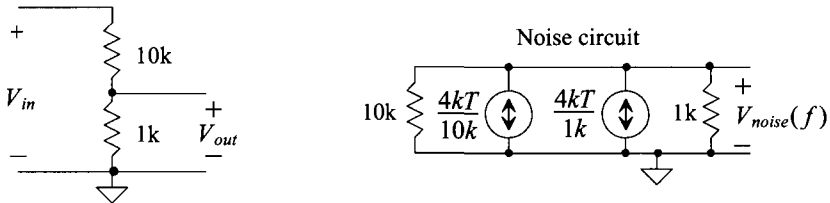


Figure 8.14 Circuit used in Ex. 8.5.

The PSD of the noise currents are given by (again noting, as seen in Fig. 8.6, that we apply zero volts to the input when calculating output noise PSD, that is, we ground the input)

$$I_{10k}^2(f) = \frac{4 \cdot (13.8 \times 10^{-24}) \cdot (300)}{10,000} = 1.66 \times 10^{-24} \frac{A^2}{Hz} \rightarrow I_{10k}(f) = 1.29 \times 10^{-12} \frac{A}{\sqrt{Hz}}$$

$$I_{1k}^2(f) = 16.66 \times 10^{-24} \frac{A^2}{Hz} \rightarrow I_{1k}(f) = 4.1 \times 10^{-12} \frac{A}{\sqrt{Hz}}$$

The output thermal noise PSD due to the 10k resistor is

$$I_{10k}^2(f) \cdot \left[\frac{1k \cdot 10k}{1k + 10k} \right]^2 \text{ units } V^2/Hz$$

In order to avoid analyzing circuits in a different way, (for example, $V^2 = I^2 R^2$, $i_d^2 = g_m^2 v_{gs}^2$, $V^2 = I^2 (R_1 + R_2)^2$, etc.), we can use the voltage or current spectral densities (square-root of PSD) of the noise when doing circuit noise calculations

$$V_{10k}(f) = I_{10k}(f) \cdot \frac{1k \cdot 10k}{1k + 10k} \text{ units } V/\sqrt{Hz}$$

which evaluates to $1.2 \text{ nV}/\sqrt{Hz}$. The output noise voltage-spectral density $V_{1k}(f)$ from the 1k resistor is $3.7 \text{ nV}/\sqrt{Hz}$. The total output noise PSD, $V_{noise}^2(f)$, is the sum of $V_{10k}^2(f)$ and $V_{1k}^2(f)$. Again, we sum the PSDs (or power) but **not** the voltage-spectral densities (or the RMS voltages).

The mean-squared output noise voltage over a bandwidth of 1 kHz is

$$V_{noise,RMS}^2 = \int_{f_L}^{f_H} V_{noise}^2(f) \cdot df = \int_0^{1kHz} \left(V_{10k}^2(f) + V_{1k}^2(f) \right) \cdot df = 15.1 \times 10^{-15} V^2$$

The RMS output noise voltage is

$$V_{noise,RMS} = 123 \text{ nV}$$

The input-referred noise voltage is

$$V_{inoise,RMS} = 123 \text{ nV} \cdot \frac{10k + 1k}{1k} = 1.35 \text{ } \mu\text{V}$$

SPICE simulation gives an output mean squared noise of $1.5053 \times 10^{-14} V^2$ (= 123 nV RMS) and an input-referred noise referenced to an input voltage of 1 V of 1.8215×10^{-12} (= 1.35 μV RMS). The SPICE netlist is shown below.

*** Example 8.5 CMOS: Circuit Design, Layout, and Simulation ***

```
.control
destroy all
run
print all
.endc

.noise v(2,0) Vin dec 100 1 1k
R1 1 2 10k
R2 2 0 1k
Vin 1 0 dc 0 ac 1
.print noise all
.end
```

The SPICE output is seen below

```
TEMP=27 deg C
Noise analysis ... 100%
inoise_total = 1.821510e-12
onoise_total = 1.505380e-14
```

A bandwidth of 1 to 1,000 Hz was used in this simulation rather than DC to 1 kHz. Also note that the reference supply, *Vin* in the netlist, was a voltage, so the units of the SPICE output are V^2 . The input AC source has a magnitude of 1 V, so the input noise SPICE gives is divided by 1 V squared. If we had used a 1 mV AC supply for *Vin*, then the *inoise_total* above would be $1.8215 \times 10^{-6} V^2$.

Here, in SPICE, we used a single input-referred voltage source to model the input-referred noise, see Eq. (8.22) and the associated discussion. In SPICE we always refer the input noise to a source (voltage or current) and not to a node (like we do in our analysis). This means that the input-referred noise in SPICE is always a single source in series (voltage input) or parallel (current input) with the input of the circuit. ■

Example 8.6

Estimate $V_{noise,RMS}$ for the circuit seen in Fig. 8.15. Verify the answer with SPICE.

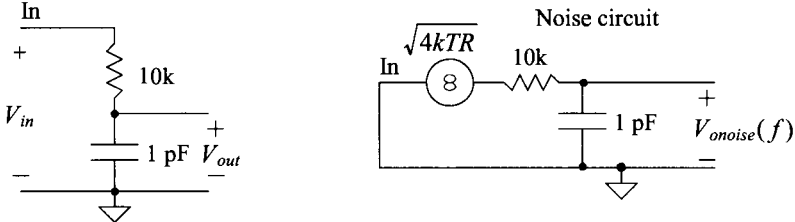


Figure 8.15 Circuit used in Ex. 8.6.

The only element in this circuit that generates noise is the resistor. It generates thermal noise. The resistor's noise voltage spectral density is $\sqrt{4kTR}$. The output noise spectral density is then

$$V_{noise}(f) = \sqrt{4kTR} \frac{1/j\omega C}{1/j\omega C + R} = \frac{\sqrt{4kTR}}{1 + j\frac{f}{f_{3dB}}} \text{ units, } V/\sqrt{\text{Hz}}$$

or

$$V_{noise}^2(f) = \frac{4kTR}{|1 + j\frac{f}{f_{3dB}}|^2} = \frac{4kTR}{\left(\sqrt{1 + (ff_{3dB})^2}\right)^2} = \frac{4kTR}{1 + (ff_{3dB})^2} \text{ units, } V^2/\text{Hz}$$

where $f_{3dB} = 1/2\pi RC$. This single-pole roll-off was the reason we discussed noise-equivalent bandwidth (NEB) earlier, Ex. 8.2. Using Eq. (8.17), the output RMS noise voltage is

$$V_{noise,RMS} = \sqrt{\frac{1}{2\pi RC} \cdot \frac{\pi}{2} \cdot 4kTR} = \sqrt{\frac{kT}{C}} \quad (8.24)$$

The RMS value of the thermal noise in this circuit is limited by the size of the capacitor and independent of the size of the resistor. This result is very useful. This “Kay Tee over Cee” noise is frequently used to determine the size of the capacitors used in filtering or sampling circuits.

The SPICE netlist and output are seen below.

*** Example 8.6 CMOS: Circuit Design, Layout, and Simulation ***

```
.noise v(Vout,0) Vin dec 100 1 1G
R1 Vin Vout 10k
C1 Vout 0 1p
Vin Vin 0 dc 0 ac 1
.print noise all
.end
```

TEMP=27 deg C

Noise analysis ... 100%


```
inoise_total = 1.695223e-07
onoise_total = 4.101864e-09
```

Using Eq. (8.24) at 300 °K, we get $V_{onoise,RMS}^2 = 4.14 \times 10^{-9} V^2$. This is close to what SPICE gives above for `onoise_total` (close but not exact; we stopped the simulation at 1 GHz not infinity). The output RMS noise is 64 μV (keeping in mind that the peak-to-peak value of the thermal noise will be larger than this).

The input-referred noise, `inoise_total`, in this simulation example is somewhat meaningless. Changing the stop frequency in the simulation from 1 GHz to 10 GHz has little effect on the output RMS noise but does cause the input-referred noise to increase. As indicated in Eq. (8.19) and the associated discussion, the input-referred noise spectral density increases indefinitely. Integrating this spectral density from DC to infinity results in an infinite RMS input-referred voltage. Since the low-frequency gain here is one, we would specify $V_{inoise,RMS} = V_{onoise,RMS}$. ■

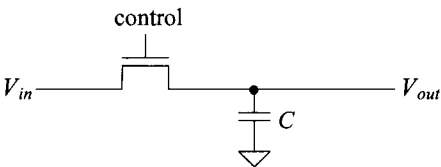


Figure 8.16 A sample-and-hold circuit.

For an example of the usefulness of Eq. (8.24), consider the sample-and-hold circuit seen in Fig. 8.16. When the MOSFET gate is driven to V_{DD} , the MOSFET behaves like a resistor and permits V_{in} to charge the capacitor. When the MOSFET is off, the capacitor remains charged and the MOSFET behaves like an open. The voltage on the capacitor doesn't change again until the MOSFET turns back on. We can think of the sampling operation as sampling both the input signal and the kT/C noise onto the capacitor. Table 8.1 provides a comparison between capacitor sizes and kT/C noise.

Table 8.1 Capacitor size and corresponding kT/C noise at 300 °K.

Capacitor size, pF	$\sqrt{kT/C}$, μV
0.01	640
0.1	200
1	64
10	20
100	6.4

8.2.3 Signal-to-Noise Ratio

Signal-to-noise ratio, SNR , can be defined in general terms by

$$SNR = \frac{\text{desired signal power, } P_s}{\text{undesired signal power (noise), } P_{noise}} \quad (8.25)$$

SNR can be specified using dB as

$$SNR = 10 \log \frac{P_s}{P_{noise}} \quad (8.26)$$

If the power is normalized to a $1\text{-}\Omega$ load (e.g., $P_s = V_{s,RMS}^2/(1\text{ }\Omega)$), we can write

$$SNR = 10 \log \frac{P_s}{P_{noise}} = 20 \log \frac{\sqrt{P_s}}{\sqrt{P_{noise}}} = 20 \log \frac{V_{s,RMS}}{V_{noise,RMS}} \quad (8.27)$$

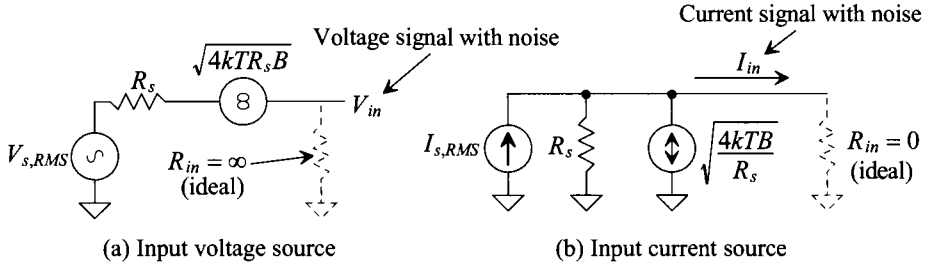


Figure 8.17 Calculating input SNR.

Figure 8.17 shows two equivalent input sources with associated thermal noise models. With the ideal R_{in} the SNR associated with these circuits is (noting $V_{s,RMS}^2 = I_{s,RMS}^2 \cdot R_s^2$) an open (voltage input) or a short (current input)

$$SNR_{in} = \frac{V_{s,RMS}^2}{4kTR_sB} = \frac{I_{s,RMS}^2}{4kTB/R_s} \quad (8.28)$$

In Fig. 8.17a, V_{in} is dropped across an infinite R_{in} (an open). In Fig. 8.17b, I_{in} drives zero R_{in} (a short). In the practical case, R_{in} is finite and nonzero. For Fig. 8.17a, the input signal and noise are attenuated by the voltage divider formed between R_s and R_{in}

$$SNR_{in} = \frac{V_{s,RMS}^2 \cdot \left[\frac{R_{in}}{R_{in} + R_s} \right]^2}{4kTR_sB \cdot \left[\frac{R_{in}}{R_{in} + R_s} \right]^2} = \frac{V_{s,RMS}^2}{4kTR_sB} \quad (8.29)$$

which has no effect on the SNR_{in} . We can also show that there is no change in SNR_{in} if R_{in} is nonzero in Fig. 8.17b.

Examine the amplifier model with noise seen in Fig. 8.18. The output noise, $V_{noise,RMS}$, includes the thermal noise contributions from R_s . We measured the output noise with the source connected to the amplifier (see Figs. 8.6 and 8.8). The SNR associated with the output of the amplifier (where the noise is due to the amplifier and the thermal noise from R_s) is then

$$SNR_{out} = \frac{V_{s,RMS}^2 \cdot \left[\frac{R_{in}}{R_{in} + R_s} \right]^2 \cdot A^2}{V_{noise,RMS}^2} \quad (8.30)$$

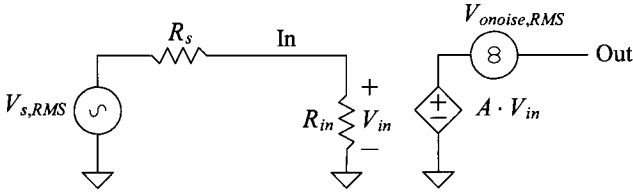


Figure 8.18 Calculating output SNR.

Input-Referred Noise II

When we discussed input-referred noise in Fig. 8.8, we tacitly assumed that the input resistance of the amplifier R_{in} was infinite. The input-referred noise could then be modeled with a single voltage source in series with the input signal source V_s . Keeping in mind that we are measuring $V_{noise,RMS}$ at a fixed value of R_s , Fig. 8.19a, we can show that if the amplifier's input resistance R_{in} is not infinite, this single input-referred voltage source can still manage to model the amplifier's output noise. Looking at Fig. 8.19b, we can write

$$A \cdot V_{noise,RMS} \cdot \frac{R_{in}}{R_{in} + R_s} = V_{noise,RMS} \quad (8.31)$$

If the input resistance R_{in} goes to infinity (common for low-frequency CMOS amplifiers) or R_s is zero (a voltage source is connected to the amplifier's input), a single input-referred noise voltage is ideal for modeling the output noise. (And we'll use this simpler model in most of our noise discussions in this book.)

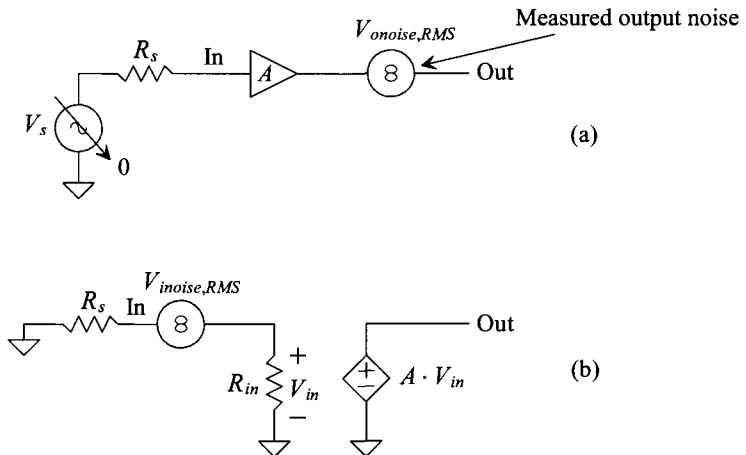


Figure 8.19 Modeling measured output noise with a single input-referred source.

Using a single voltage source to model input-referred noise does have some limitations though. For example, what happens if our input signal is a current (meaning R_s is infinite), as seen in Fig. 8.20a? In this case, the input-referred noise voltage is irrelevant. The input voltage is independent of $V_{noise,RMS}$ (the output of the circuit is free of noise). To avoid this situation and to make the input-referred noise sources *independent* of R_s , we can use the noise model seen in Fig. 8.20b. The output noise using this model is

$$V_{onoise,RMS}^2 = 4kTR_sB \cdot \left(\frac{AR_{in}}{R_s + R_{in}} \right)^2 + I_{inoise,RMS}^2 \cdot \left(\frac{AR_sR_{in}}{R_s + R_{in}} \right)^2 + V_{inoise,RMS}^2 \cdot \left(\frac{AR_{in}}{R_s + R_{in}} \right)^2 \quad (8.32)$$

noting that we sum the power from each source to get the total output noise power. It's important to note that our input-referred sources no longer depend on the thermal noise contributions from the source resistance (we've added it separately). Looking at Eq. (8.32), we see that if R_s is zero or infinite, the thermal noise contributions from R_s to the output noise are zero. We should also see that if R_{in} is large (or R_s is small), $V_{inoise,RMS}$ alone is sufficient to model the input-referred noise. Similarly, if R_s is large (or R_{in} is small), $I_{inoise,RMS}$ alone can be used to model the input-referred noise.

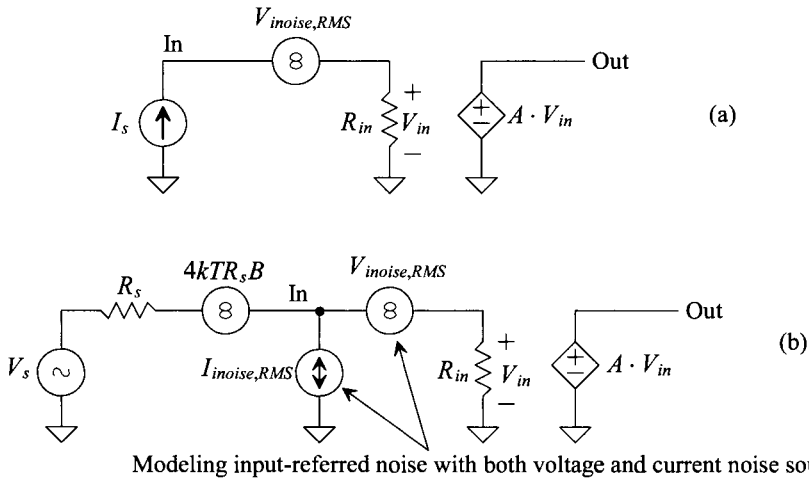


Figure 8.20 Using two input-referred noise sources to model an amplifier's output noise.

Example 8.7

Discuss how to determine $V_{inoise,RMS}$ and $I_{inoise,RMS}$.

Looking at Eq. (8.32), we see that if R_s is zero, then $V_{inoise,RMS}$ is sufficient to model the output noise. We can therefore write

$$V_{inoise,RMS} = \frac{V_{onoise,RMS,R_s=0}}{A} \quad (V_{onoise,RMS} \text{ measured with } R_s \text{ shorted}) \quad (8.33)$$

If R_s is infinite (think of the input as seen in Fig. 8.17b), then $I_{noise,RMS}$ is sufficient to model the amplifier's output noise

$$I_{noise,RMS} = \frac{V_{noise,RMS,R_s=\infty}}{AR_{in}} (V_{noise,RMS} \text{ measured with } R_s \text{ opened}) \quad (8.34)$$

■

Noise Figure

The noise figure, NF , of an amplifier is given by

$$NF = 10 \log \frac{SNR_{in}}{SNR_{out}} = 10 \log [\text{noise factor}, F] \quad (8.35)$$

If the amplifier doesn't degrade the SNR , then the input and output $SNRs$ are the same, the noise factor, F , is 1, and the NF is 0 dB. Using Eqs. (8.29) and (8.30), we can write the NF as

$$NF = 10 \log \frac{V_{noise,RMS}^2}{4kTR_s B \cdot \left[\frac{R_{in}}{R_{in} + R_s} \right]^2 \cdot A^2} \quad (8.36)$$

When the measuring bandwidth B is small (common for narrowband amplifiers used in communication circuits), we are measuring the spot NF (noting, from Eq. [8.10], the bandwidth used to calculate $V_{noise,RMS}$ is reduced). If the measuring bandwidth is large (common in general analog CMOS design), we are measuring the average NF .

The numerator in Eq. (8.36) is the total output noise power and the denominator is the output noise power due to the source resistance, that is,

$$F = \frac{\text{total output noise power}}{\text{output noise power due to source resistance}} \quad (8.37)$$

If the amplifier is noise-free ($V_{noise,RMS} = \sqrt{4kTR_s B} \cdot A \cdot R_{in}/[R_{in} + R_s]$, that is, the output contains only the thermal noise from the source resistance), the NF is, again, 0 dB. For a low-noise amplifier (LNA), the NF may typically range from 0.5 to 5 dB.

Note that if SNR_{in} is infinite, the NF is meaningless. Infinite SNR_{in} could occur if we use the impractical case, see Fig. 8.17, of $R_s = 0$ (an ideal voltage source input where, also, ideally $R_{in} = \infty$) or $R_s = \infty$ (an ideal current source input where, also, ideally $R_{in} = 0$).

An Important Limitation of the Noise Figure

Looking at Eq. (8.36), we see that if R_{in} is large compared to R_s ($R_{in} \gg R_s$), F approaches

$$F = \frac{V_{noise,RMS}^2}{4kTR_s B \cdot A^2} \quad (8.38)$$

This equation shows why CMOS amplifiers work so well for low-noise amplification with large source resistances. At low frequencies, the gate of the MOSFET is an open. If we take $R_s \rightarrow \infty$, then F goes to 1 which is a perfect noiseless amplifier right? After reviewing how R_s affects SNR_{in} in Eq. (8.29), we see that the cost for this good amplifier NF is an SNR_{in} that approaches zero. The input voltage signal is so noisy that it makes the amplifier noise irrelevant and thus the NF is 0 dB. Note that if R_{in} is infinite, we must use the model seen in Fig. 8.17a. In Fig. 8.17b I_{in} would be zero if $R_{in} = \infty$.

Example 8.8

Suppose an input signal source has an SNR of 60 dB and a source resistance R_s . Further, suppose this signal is amplified with an amplifier having an NF of 1 dB when driven from a source with a resistance R_s . Estimate the SNR on the output of the amplifier.

We can begin by rewriting Eq. (8.31)

$$NF = 10 \log SNR_{in} - 10 \log SNR_{out} \quad (8.39)$$

illustrating the beauty of using the NF . That is, SNR_{out} is calculated by subtracting the NF from SNR_{in} . In this case, the SNR_{out} is 59 dB. ■

Example 8.9

Calculate the input-referred noise, F , and $SNRs$ for the circuit seen in Fig. 8.21.

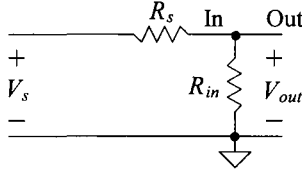


Figure 8.21 Circuit used in Ex. 8.9.

Let's begin by adding the noise voltage spectral density to the circuit, Fig. 8.22a. The output noise PSD is

$$V_{noise}^2(f) = 4kTR_s \left[\frac{R_{in}}{R_{in} + R_s} \right]^2 + 4kTR_{in} \left[\frac{R_s}{R_{in} + R_s} \right]^2$$

To determine $V_{noise,RMS}$, we integrate this PSD over the bandwidth of interest B or

$$V_{noise,RMS}^2 = \int_{f_L}^{f_H} V_{noise}^2(f) \cdot df = 4kTBR_s \left[\frac{R_{in}}{R_{in} + R_s} \right]^2 + 4kTBR_{in} \left[\frac{R_s}{R_{in} + R_s} \right]^2$$

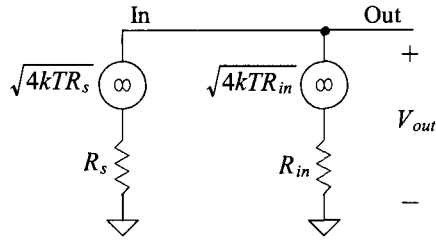
Noting our gain A ($= V_{out}/V_{in}$ not V_{out}/V_s) is one, we can use the model shown in Fig. 8.22b. To determine the input-referred noise sources, we can use Eq. (8.32) and the results in Ex. 8.7. To determine $V_{noise,RMS}$, we short the input to ground ($R_s = 0$ in Fig. 8.21 and the equation above), Fig. 8.22c, and equate the circuit output to $V_{noise,RMS}$. This gives

$$V_{noise,RMS,R_s=0} = V_{noise,RMS} = 0$$

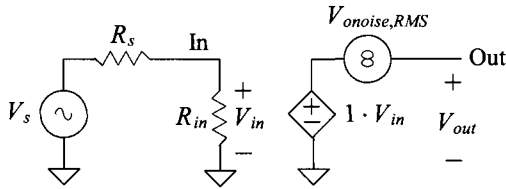
To determine $I_{noise,RMS}$, we open the input ($R_s = \infty$), Fig. 8.22d, and equate the circuit's output to $V_{noise,RMS}$ (from the equation above). This gives

$$R_{in}^2 \cdot I_{noise,RMS}^2 = V_{noise,RMS,R_s=\infty}^2 = 4kTBR_{in} \rightarrow I_{noise,RMS} = \sqrt{\frac{4kTB}{R_{in}}}$$

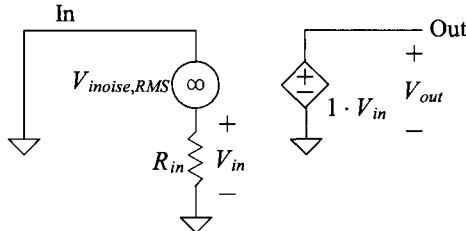
The input SNR is given in Eq. (8.29). The output SNR , Fig. 8.22e, is



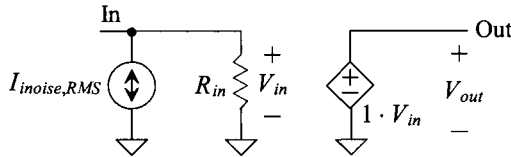
(a) Adding noise sources to the circuit.



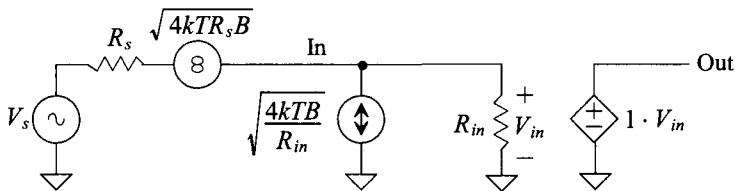
(b) Model showing output noise.



(c) Shorting the input.



(d) Opening the input.



(e) Input-referred noise model.

Figure 8.22 Noise analysis procedure for Ex. 8.9.

$$SNR_{out} = \frac{V_{s,RMS}^2 \cdot \left[\frac{R_{in}}{R_s + R_{in}} \right]^2}{V_{noise,RMS}^2} = \frac{V_{s,RMS}^2}{4kTB \cdot R_s (1 + R_s/R_{in})} \quad (8.40)$$

The noise factor is then

$$F = 1 + \frac{R_s}{R_{in}} \quad (8.41)$$

To minimize the NF, we can decrease R_s or increase R_{in} . Decreasing R_s causes SNR_{in} and SNR_{out} to increase, as seen in Eqs. (8.29) and (8.40). At the same time, increasing R_{in} causes SNR_{out} to move towards SNR_{in} , Eq. (8.40), resulting in F moving towards 1. ■

Optimum Source Resistance

Looking at Eq. (8.36), notice that if R_s approaches a short or an open, the NF gets really large. To determine the optimum source resistance (to minimize NF), let's write F using Eqs. (8.32) and (8.36) as

$$F = \frac{4kTR_s B + I_{noise,RMS}^2 \cdot R_s^2 + V_{noise,RMS}^2}{4kTR_s B} \quad (8.42)$$

Taking the derivative with respect to R_s and setting the result equal to zero (to determine the minimum), results in

$$I_{noise,RMS}^2 + 2 \cdot I_{noise,RMS} \cdot R_s \cdot \frac{\partial I_{noise,RMS}}{\partial R_s} - \frac{V_{noise,RMS}^2}{R_s^2} + \frac{2V_{noise,RMS}}{R_s} \cdot \frac{\partial V_{noise,RMS}}{\partial R_s} = 0 \quad (8.43)$$

Since the input-referred noise sources do not vary with R_s (the derivatives are zero), we can write the optimum source $R_{s,opt}$ as

$$R_{s,opt} = \frac{V_{noise,RMS}}{I_{noise,RMS}} \quad (8.44)$$

From Ex. 8.9 we can use this equation to see that $R_{s,opt} = 0$.

To determine the optimum noise factor F_{opt} , we can substitute Eq. (8.44) into Eq. (8.42) to get

$$F_{opt} = 1 + \frac{(V_{noise,RMS} \cdot I_{noise,RMS})/2}{kTB} \quad (8.45)$$

The best noise performance is characterized by a ratio of the input-referred noise power of the amplifier, $(V_{noise,RMS} \cdot I_{noise,RMS})/2$, to the thermal power (the thermal energy kT in a bandwidth of B). This result isn't too useful because it indicates that for good noise performance we need to use an amplifier with low input-referred noise power (which is as profound as saying "to get good noise performance use a low-noise amplifier").

Simulating Noiseless Resistors

To verify the NF of a circuit using SPICE, it may be useful to replace one or more resistors with elements that don't generate noise. To simulate a noiseless resistor, a voltage-controlled current source (VCCS) can be used, Fig. 8.23. Current flows from

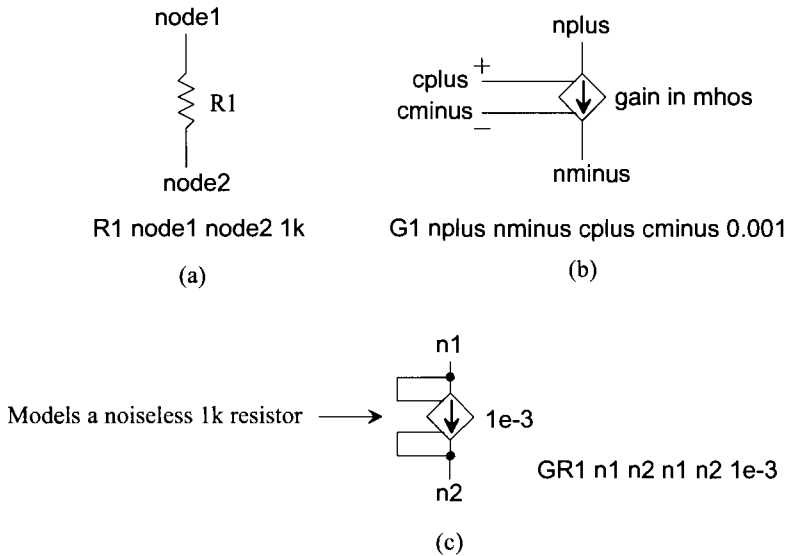


Figure 8.23 Modeling a noiseless resistor in SPICE.

node1 to node2 in the resistor of (a) while it flows from nplus to nminus in the VCCS in (b). The current in (b) is controlled by the voltage applied to the controlling nodes cplus and cminus. The input to the VCCS is a voltage, and the output is a current (meaning the gain is current/volts or transconductance). To implement a noiseless resistor, we can use the VCCS where the controlling nodes are tied across the VCCS, as seen in (c). The value of the resistor is one over the transconductance of the VCCS.

Example 8.10

Suppose, in Fig. 8.21, R_s is 10k and R_{in} is 1k. Use SPICE to verify the value of noise factor derived in Ex. 8.9 (that is, Eq. [8.41]).

Using Eq. (8.41), the noise factor is 11 (NF = 10.4 dB). Note that Eq. (8.41) is not dependent on bandwidth. We simulated this circuit already in Ex. 8.5 over a bandwidth of 1 to 1kHz. The result was a $V_{noise,RMS}^2$ of $1.5053 \times 10^{-14} V^2$. Referring to Eq. (8.37), this is the “total output noise power.” To determine the “output noise power due to source resistance,” we can replace R_{in} with a noiseless resistor, as seen in Fig. 8.24. The SPICE netlist is given below.

*** Example 8.10 CMOS: Circuit Design, Layout, and Simulation ***

```
.control
destroy all
run
print all
.endc
```

```
.noise V(Vout,0) Vs dec 100 1 1k

Rs Vs Vout 10k
Gin Vout 0 Vout 0 1e-3
```

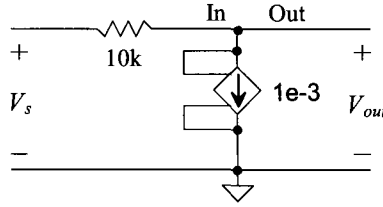


Figure 8.24 Replacing R_{in} with a noiseless resistor.

```
Vs      Vs      0      dc      0      ac      1

.print noise all
.end

TEMP=27 deg C
Noise analysis ... 100%
inoise_total = 1.655918e-13
onoise_total = 1.368527e-15
```

The noise factor is calculated from the simulation results as

$$F = \frac{1.5053 \times 10^{-14} V^2}{1.3685 \times 10^{-15} V^2} = 11$$

which is what we calculated using Eq. (8.41). ■

In a general SPICE simulation, we can determine F by eliminating the noise contributed by R_s . To do so, we first need to rewrite Eq. (8.37) as

$$F = \frac{\text{total output noise power}}{\text{total output noise power} - \text{total output noise power with } R_s \text{ noiseless}} \quad (8.46)$$

Example 8.11

Using SPICE and Eq. (8.46), calculate the noise factor of the circuit in Ex. 8.10 (verify, once again, that it is 11).

Again, as determined in Ex. 8.5, the total output noise power for this circuit is $1.5053 \times 10^{-14} V^2$. If we simulate the noise performance of the circuit in Fig. 8.21 with R_s being noiseless, we get

```
TEMP=27 deg C
Noise analysis ... 100%
inoise_total = 1.655918e-12
onoise_total = 1.368527e-14
```

Using Eq. (8.46), we get

$$F = \frac{1.5053 \times 10^{-14}}{1.5053 \times 10^{-14} - 1.3685 \times 10^{-14}} = 11$$

Simulating with a noiseless source resistor is the preferred way to simulate NF with SPICE. ■

Noise Temperature

Sometimes the term *noise temperature* is used to characterize the noise performance of an amplifier. In Fig. 8.25a we measure an amplifier's output RMS noise, $V_{noise,RMS}$, with the input source connected but with $V_s = 0$ (the source contributes thermal noise to the output noise PSD). In Fig. 8.25b we refer this noise back to the input of the amplifier (this input-referred voltage includes both the amplifier and source resistance noise). After referring the noise back to the input, we think of the amplifier as being noiseless. In Fig. 8.23c we remove the input-referred source but, to have the same output noise as in (a) or (b), we change the effective temperature of the source's resistor and include its corresponding thermal noise model. By adjusting the temperature of the source resistor in our calculations (called the *noise temperature*, T_n), we can get the same total output noise power as in (a) or (b). We can relate the noise factor to the noise temperature by re-writing Eq. (8.36) as

$$F = \frac{\overbrace{4kTR_s B \cdot A^2 \cdot \left(\frac{R_{in}}{R_{in}+R_s}\right)^2}^{\text{Output noise from } R_s} + \overbrace{4kT_n R_s B \cdot A^2 \cdot \left(\frac{R_{in}}{R_{in}+R_s}\right)^2}^{\text{Amplifier's output noise alone}}}{\underbrace{4kTR_s B \cdot A^2 \cdot \left(\frac{R_{in}}{R_{in}+R_s}\right)^2}_{\text{Output noise from } R_s}} = 1 + \frac{T_n}{T} \quad \text{where } T_n \geq 0 \quad (8.47)$$

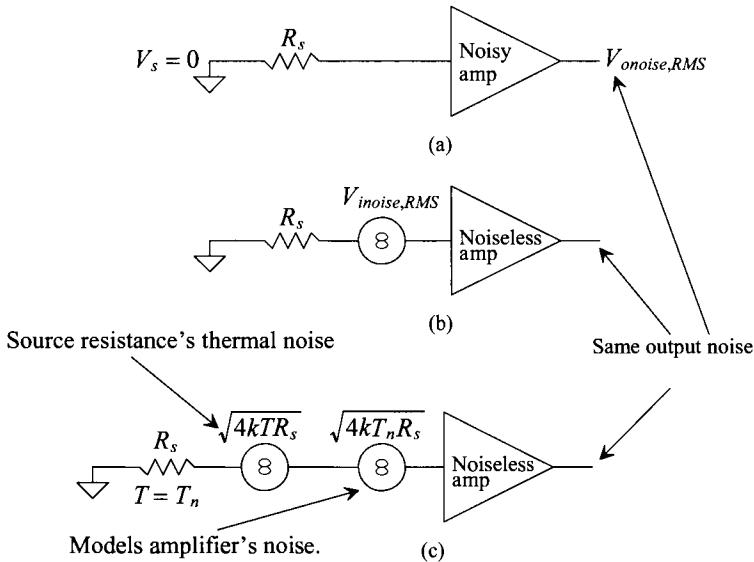


Figure 8.25 Determining the noise temperature of an amplifier.

where T is the temperature, in Kelvin, at which the total output noise was measured in (a). We know that if the amplifier is noiseless, $F = 1$. A noiseless amplifier has $T_n = 0$. Note that we could now relate Eq. (8.47) to Eq. (8.45) to write the optimum noise temperature of an amplifier in terms of $R_{s,opt}$. Again this wouldn't be too useful because it would state that we should use an amplifier with a low input-referred noise power to get a low value of T_n (good amplifier noise performance).

Table 8.2 provides a listing of NFs and the corresponding values of F and T_n . Note that an NF , less than 3 dB indicates that more than half of the output noise is due to the thermal noise of the source resistance.

Table 8.2 NFs and corresponding values of F and T_n (with $T = 300^\circ \text{K}$).

Noise figure, NF , dB	Noise factor, F	Noise temp, T_n , °K
0	1	0
0.5	1.12	36.61
0.75	1.19	56.55
1	1.26	77.68
1.5	1.41	123.8
2	1.59	175.5
3	2	298.6
4	2.51	453.6
5	3.16	648.7

Averaging White Noise

Before moving on, let's comment on what happens if we average white noise (like thermal noise or shot noise discussed next). If the number of samples averaged is K , then the spectral density, and RMS value, of the noise is reduced by K . For thermal noise, we would rewrite Eq. (8.23) as

$$V_{RMS}^2 = \frac{4kTRB}{K} \text{ with units of } V^2 \quad (8.48)$$

Averaging can be thought of as lowpass filtering the random signal. Looking at Eq. (8.48), we see that the RMS value of the noise is reduced by the square root of the number of points averaged. Reviewing Ex. 8.6 (kT/C noise), we see the same result. Increasing C reduces the bandwidth of the thermal noise in the output of the RC circuit. This then causes the RMS value of the output noise to decrease by the root of C .

Figure 8.26a shows a noisy voltage signal with no averaging. The peak-to-peak amplitude of this waveform is roughly 5 divisions. In (b) we are averaging 4 of the waveforms in (a). We expect the amplitude to go to $5/\sqrt{4}$ or 2.5 divisions. If 16 averages are used, we should get an amplitude of 1.25 divisions, Fig. 8.26c. Finally, Fig. 8.26d shows how the noise is reduced if 64 waveforms are averaged. We would expect an amplitude of $5/\sqrt{64}$ or 0.625 divisions.

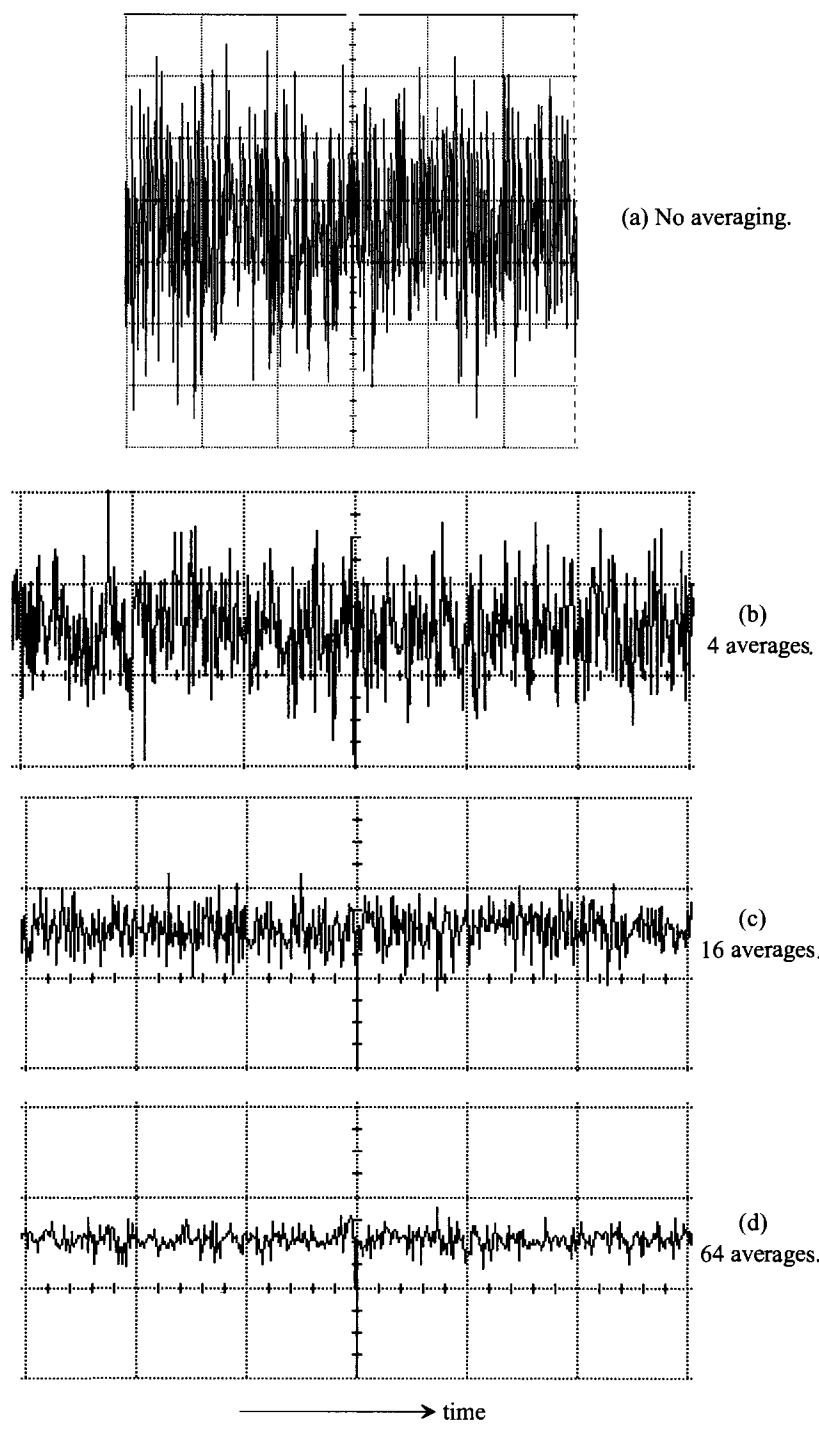


Figure 8.26 Averaging reduces the amplitude of white noise.

8.2.4 Shot Noise

Shot noise results from the discrete movement of charge across a potential barrier (e.g., a diode). To understand the origin of shot noise, imagine dumping marbles onto a table. As the marbles move around on the table, some will fall to the floor. The rate the marbles drop to the floor is random even though the rate we dump them onto the table may be constant. When current flows in a diode (when we dump the marbles onto the table), the movement of charge across the depletion region is random (the rate the marbles fall to the floor is random). For example, when a majority carrier, say an electron (a marble on top of the table), moves into the space-charge region (falls off the table), the electric field sweeps it across the junction (gravity causes the marble to fall to the floor). On the p-type side (the floor), where it is now a minority carrier, it diffuses out until it recombines (the marble moves until it stops). Note the name “shot” has to do with using shot from a shotgun shell (little balls of lead or, nowadays for waterfowl, steel used in shotgun shells) in the example above instead of marbles.

The power spectral density of shot noise can be determined empirically and is given by

$$I_{shot}^2(f) = 2qI_{DC} \text{ with units of } A^2/Hz \quad (8.49)$$

Shot noise is different from thermal noise (although both are modeled with a white noise PSD). Recall that the carriers can randomly move either into or out of the resistive material (with net zero charge transfer) due to thermal noise. Thermal noise is present even without a current flowing in the resistor. For shot noise to be present, we must have both a potential barrier and a current flowing. (There is net charge transfer with an average equal to the current flowing across the barrier.) The movement across the barrier is random and in one direction (the marbles don’t hop back up on the table).

Shot noise is not present in long-channel MOSFETs (however, see the discussion in [4]). Shot noise is present in short-channel MOSFETs ($t_{ox} < 20$ nm) because of the gate tunneling current. When a long-channel MOSFET is operating in the saturation region, the entrance of charge into the depletion region, between the channel and the drain, has a discrete nature. However, this discrete movement is the result of thermal variations in the MOSFET’s channel resistance and not carriers crossing a potential barrier and being swept up by an electric field.

Example 8.12

Estimate the output noise in the circuit seen in Fig. 8.27a. Verify your answer with SPICE. Assume that the diode’s minority carrier lifetime (see Ch. 2) is 10 ns.

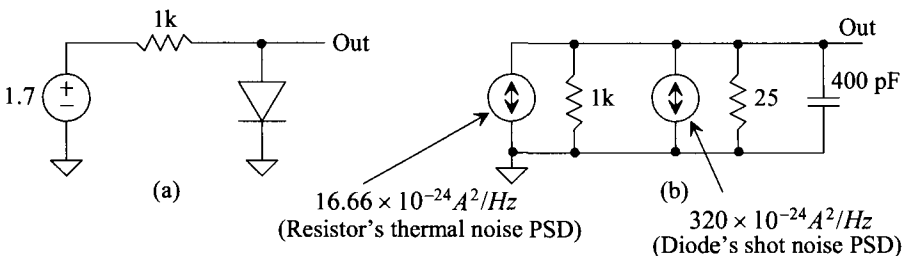


Figure 8.27 Calculating output noise for Ex. 8.12.

Figure 8.27b shows the circuit used for noise analysis. The diode is forward-biased at roughly 0.7 V. One volt is dropped across the 1k resistor and so 1 mA of current flows in the circuit. The diode's small-signal resistance is

$$r_d = \frac{V_T}{I_{DC}} = \frac{kT}{qI_{DC}} \approx 25 \Omega$$

noting that the small-signal resistance of the diode does not generate thermal noise (it's a model) but that any diode series resistance will. (We set the diode's series resistance to zero in the SPICE model statement for this example.) The diode's storage capacitance is

$$C_S = \frac{I_{DC}}{V_T} \cdot \tau_T = \frac{\tau_T}{r_d} = 400 \text{ pF}$$

The diode's shot noise PSD is

$$I_{shot}^2(f) = 2qI_{DC} = 2 \cdot (1.6 \times 10^{-19}) \cdot 1 \text{ mA} = 320 \times 10^{-24} \text{ A}^2/\text{Hz}$$

The resistor's thermal noise PSD was calculated in Ex. 8.5 as $16.66 \times 10^{-24} \text{ A}^2/\text{Hz}$. The circuit output PSD is then the sum of the thermal and the shot noise contributions times the parallel connection of the 1k and 25 ohm resistors or

$$V_{noise}^2(f) = 336.6 \times 10^{-24} \cdot (25 || 1k)^2 = 200 \times 10^{-21} \text{ V}^2/\text{Hz}$$

To calculate the output RMS noise voltage, we need to integrate this PSD from DC to infinity as indicated in Fig. 8.12. However, notice that this circuit has a single time constant of $(1k || 25) \cdot 400 \text{ pF}$ or approximately 10 ns (the diodes minority carrier lifetime). The noise equivalent bandwidth, NEB from Eq. (8.15), is roughly

$$NEB = \frac{1}{2\pi \cdot 10 \text{ ns}} \cdot \frac{\pi}{2} = 25 \text{ MHz}$$

The RMS output noise is then

$$V_{noise,RMS} = \sqrt{25 \times 10^6 \times 200 \times 10^{-21}} = 2.23 \mu\text{V}$$

The SPICE netlist and simulation output are seen below.

*** Example 8.12 CMOS: Circuit Design, Layout, and Simulation ***

```
.control
destroy all
run
print all
.endc

.noise V(Vout,0) Vs dec 100 1 100G

Vs Vs 0 dc 1.7 ac 1
Rs Vs Vout 1k
D1 Vout 0 Diode

.model Diode D TT=10n Rs=0
.print noise all
.end
```

```
TEMP=27 deg C
Noise analysis ... 100%
inoise_total = 3.592225e-05
onoise_total = 5.257612e-12
```

The RMS output noise calculated using SPICE is $\sqrt{5.26 \times 10^{-12}} = 2.28 \mu V$. Again note that the input-referred noise is meaningless in this example, as seen in Eq. (8.19) and the associated discussion. ■

Example 8.13

Estimate the output noise in the circuit seen in Fig. 8.27a if the diode's anode and cathode are swapped. Assume that the diode's zero bias depletion capacitance C_{J0} is 25 fF, its built-in potential V_J is 1 V, and its grading coefficient m is 0.5.

The diode is now reverse-biased. The small reverse leakage current that flows in the diode will result in shot noise but it will be tiny compared to the thermal noise generated by the resistor. To calculate the shot noise PSD, we use the reverse leakage current for I_{DC} in Eq. (8.49). Because the diode is reverse-biased, it can be thought of as a capacitor. The value of the diode's depletion capacitance is calculated as

$$C_J = \frac{C_{J0}}{\left(1 + \frac{V_d}{V_J}\right)^m} = \frac{25 \text{ fF}}{\sqrt{1 + \frac{1.7}{1}}} = 15.2 \text{ fF}$$

We can use the circuit seen in Fig. 8.28 for the noise analysis. After examining this figure for a moment, we see that the output RMS noise is simply, from Ex. 8.6 and Eq. (8.24), kT/C noise. For this example then, $V_{noise,RMS} = 521 \mu V$ (verified with SPICE). The fundamental way to reduce the noise is to decrease the bandwidth of the circuit by increasing the capacitance shunting the output. Changing the resistor size has practically no effect on the RMS output noise. ■

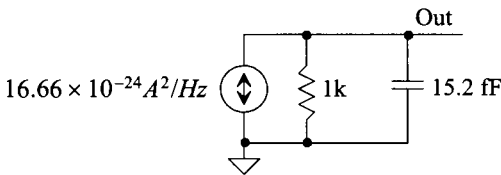


Figure 8.28 Calculating output noise for Ex. 8.12.

8.2.5 Flicker Noise

Flicker noise is a low-frequency noise and is probably the most important, and most misunderstood, noise source in CMOS circuit design. Flicker noise is also known as pink noise (a reddish color present in the lower range of the visible spectrum) or $1/f$ noise (pronounced “one over f” because its PSD, as we shall see, is inversely proportional to frequency). Before going too much further, let’s do an example.

Example 8.14

Estimate the output noise PSD and the RMS value for the integrator circuit seen in Fig. 8.29 (which includes the only noise source, that is, the thermal noise from the resistor). Assume that the op-amp is ideal (noiseless, infinite gain, and infinite bandwidth).

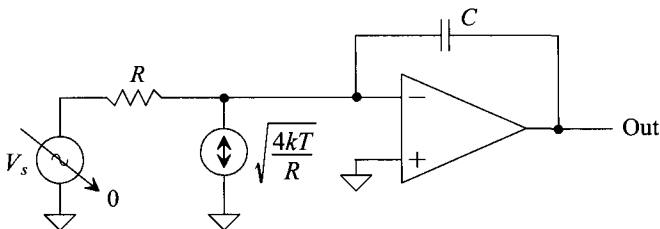


Figure 8.29 Integrating thermal noise.

The op-amp will keep the inverting input at the same potential as the noninverting input, that is, ground. This means that all of the thermal noise from the resistor will flow through the feedback capacitor (both sides of R are at ground). The integrator's output noise PSD is then

$$V_{noise}^2(f) = \left| \frac{1}{j\omega C} \right|^2 \cdot \frac{4kT}{R} = \frac{4kT}{R} \cdot \frac{1}{(2\pi C)^2} \cdot \frac{1}{f^2} \text{ units of } V^2/\text{Hz}$$

Noise with a $1/f^2$ PSD shape is often called red noise, Fig. 8.30. Looking at this spectrum, we see that the noise PSD becomes infinite as we approach DC. Remember, from our discussion at the beginning of the chapter concerning how a spectrum analyzer operates, that a point representing the PSD at a particular frequency is the measured power, V_{RMS}^2 , divided by the resolution bandwidth of the spectrum analyzer, f_{res} (that is, V_{RMS}^2/f_{res}). To make the low-frequency measurements, f_{res} must decrease, e.g., go from 1 Hz to 0.01 Hz to 0.00001 Hz, etc. A DC signal (or a sinewave for that matter, see Fig. 8.5) results in an infinite PSD point if we take $f_{res} \rightarrow 0$.

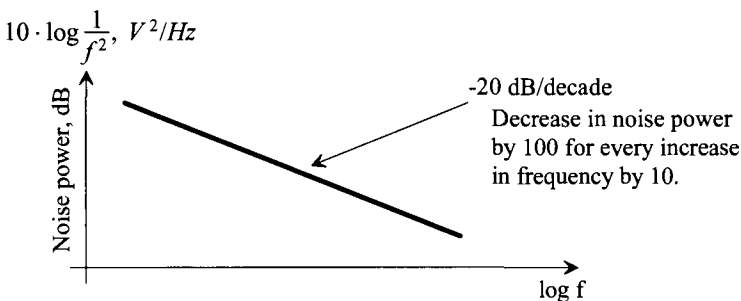


Figure 8.30 PSD of integrated thermal noise (red noise).

Let's use Eq. (8.10) to determine the RMS output noise voltage for this circuit

$$V_{noise,RMS}^2 = \int_{f_L}^{f_H} V_{noise}^2(f) \cdot df = \frac{4kT}{R} \cdot \frac{1}{(2\pi C)^2} \cdot \left(\frac{1}{f_L} - \frac{1}{f_H} \right) \quad (8.50)$$

If we take $f_H \rightarrow \infty$, this equation becomes

$$V_{noise,RMS} = \sqrt{\frac{4kT}{R}} \cdot \frac{1}{2\pi C} \cdot \frac{1}{\sqrt{f_L}} \quad (8.51)$$

If we call the length of time we integrate (or measure an input signal since integrators are often used for sensing) T_{meas} , then we can get an approximation for lower integration frequency as

$$T_{meas} \approx \frac{1}{f_L} \quad (8.52)$$

or, rewriting Eq. (8.51),

$$V_{noise,RMS} \approx \sqrt{\frac{4kT}{R}} \cdot \frac{1}{2\pi C} \cdot \sqrt{T_{meas}} \quad (8.53)$$

This result is practically very important. If we average (the same as integrating, which is just summing a variable) a signal with a $1/f^2$ noise spectrum (integrated thermal noise), the RMS output noise voltage actually gets bigger the longer we average (unlike averaging thermal noise, see Eq. [8.48]). A signal containing $1/f^3$ noise (e.g., integrating flicker noise) shows a linear increase in its RMS noise voltage with measurement time. The result is an *SNR* that doesn't get better, and may get worse, by increasing the measurement time. These results pose a limiting factor when imaging in astronomy. The night sky contains this type of noise (flicker, red, etc.). We can make images brighter by exposing our imaging system for longer periods of time but the images don't get clearer. ■

Flicker noise (a name used because when viewed in a light source flickering is observed) is modeled using

$$I_{1/f}^2(f) \text{ or } V_{1/f}^2(f) = \frac{FNN}{f} \text{ units of } A^2/Hz \text{ or } V^2/Hz \quad (8.54)$$

where *FNN* is the flicker noise numerator. The RMS value of a $1/f$ noise signal is

$$V_{noise,RMS}^2 = \int_{f_L}^{f_H} \frac{FNN}{f} \cdot df = FNN \cdot \ln \frac{f_H}{f_L} \text{ units of } V^2 \quad (8.55)$$

If f_H is 100 GHz and f_L is once every 10^{-10} Hz (roughly once every 320 years), this equation becomes

$$V_{noise,RMS} = 7 \cdot \sqrt{FNN} \text{ Volts} \quad (8.56)$$

where *FNN* has units of V^2 . After reviewing Ex. 8.14, we can write

$$V_{noise,RMS} \propto \sqrt{\ln T_{meas}} \quad (8.57)$$

For all intents and purposes, $V_{noise,RMS}$ stabilizes as measurement time increases.

Example 8.15

The input-referred noise voltage spectral density of the TLC220x (a low-noise CMOS op-amp) is seen in Fig. 8.31. The input-referred noise current is not shown but listed on the data sheet as typically $0.6 \text{ fA}/\sqrt{\text{Hz}}$ (a tiny number and so it will be neglected in the following example). For the unity follower op-amp seen in Fig. 8.32, estimate the RMS output noise voltage.

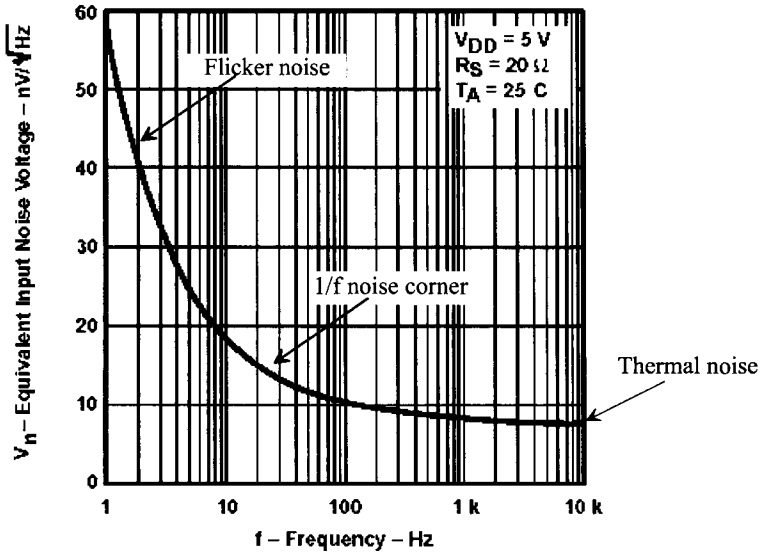


Figure 8.31 Input-referred noise for the TLC220x low-noise op-amp.

The input-referred thermal noise PSD is roughly $\left(8 \text{ nV}/\sqrt{\text{Hz}}\right)^2$. The square root of the flicker noise input-referred PSD is

$$V_{inoise,1/f}(f) = \sqrt{\frac{FNN}{f}} = \frac{56 \text{ nV}}{\sqrt{\text{Hz}}} \text{ by looking at } f = 1 \text{ Hz}$$

then

$$FNN = (56 \text{ nV})^2 = 3.14 \times 10^{-15} \text{ V}^2$$

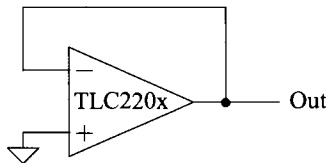


Figure 8.32 Estimating RMS output noise in a voltage follower.

As a quick check we see, in Fig. 8.31, that at 4 Hz the spectral density is roughly $28 \text{ nV}/\sqrt{\text{Hz}}$ or

$$V_{\text{noise},1/f}(f) = \sqrt{\frac{FNN}{f}} = \sqrt{\frac{3.14 \times 10^{-15} \text{ V}^2}{4 \text{ Hz}}} = \frac{56 \text{ nV}}{\sqrt{4 \text{ Hz}}} = 28 \text{ nV}/\sqrt{\text{Hz}}$$

Note, in Fig. 8.31, that the point where the thermal noise PSD gets comparable to the $1/f$ noise PSD is sometimes called the $1/f$ noise corner. Because the gain of the op-amp is one, we can write the input or output noise PSD as

$$V_{\text{noise}}^2(f) = V_{\text{noise}}^2(f) = \underbrace{\frac{3.14 \times 10^{-15}}{f}}_{V_{\text{noise},1/f}(f), \text{ flicker noise}} + \underbrace{64 \times 10^{-18}}_{\text{thermal noise}} \text{ with units of } \text{V}^2/\text{Hz}$$

To determine the output or input-referred RMS noise voltage, we integrate this noise spectrum. The gain-bandwidth product of the op-amp is roughly 2 MHz. This means, because the closed-loop gain of the op-amp is one, its $f_{3\text{dB}}$ frequency is 2 MHz. Knowing the $f_{3\text{dB}}$ of the circuit, at least for the thermal noise, we can use Eq. (8.15). With the help of Eq. (8.56) for the flicker noise contributions, we can then write

$$V_{\text{noise,RMS}}^2 = V_{\text{noise,RMS}}^2 = \overbrace{49 \cdot (3.14 \times 10^{-15})}^{\text{using Eq. (8.56)}} + \overbrace{(64 \times 10^{-18}) \cdot (2 \times 10^6) \cdot \frac{\pi}{2}}^{\text{using Eq. (8.15)}}$$

or noting the main contributor here is thermal noise. (The contributions from flicker noise, relative to the thermal noise present, fall off quickly above 10 Hz. Therefore, not using the amplifier's bandwidth when calculating the flicker noise contributions doesn't result in any significant error.) Solving this equation gives

$$V_{\text{noise,RMS}} = V_{\text{noise,RMS}} = 15 \mu\text{V}$$

or roughly 100 μV peak-to-peak if the noise has a Gaussian probability distribution function, Fig. 8.33, (where the standard deviation 1σ is the RMS value of the noise voltage and the peak-to-peak value is roughly 6σ).

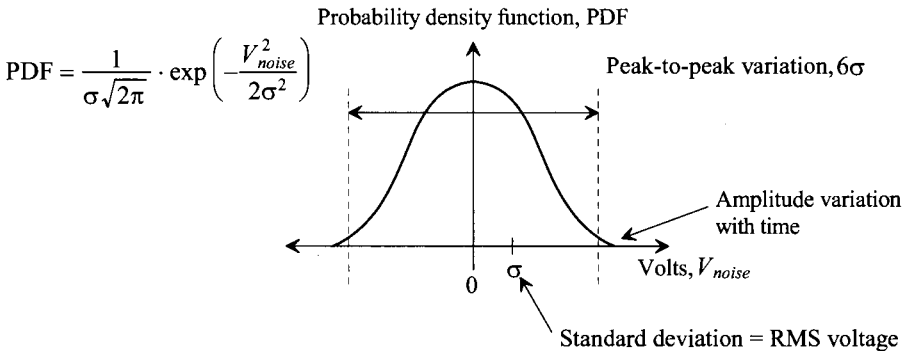


Figure 8.33 Gaussian probability distribution.

Note that this last result is useful for any of the RMS values we've calculated. To get an estimate for the peak-to-peak amplitude of the noise, we simply multiply by 6. Going the other way, we can estimate the RMS value of a noise waveform by looking at its peak-to-peak amplitude in the time domain and dividing by 6. ■

Example 8.16

On the TLC220x data sheet, the equivalent RMS input noise voltage for frequencies between 0.1 Hz and 1 Hz is 0.5 μV peak-to-peak. Verify this result with the data given in Fig. 8.31.

Using the numbers from the previous example, we see, in this 0.9 Hz bandwidth, that the contributions from thermal noise are negligible. The contributions from Flicker noise, using Eq. (8.55), are

$$V_{\text{noise,RMS}}^2 = \int_{f_L}^{f_H} \frac{3.14 \times 10^{-15}}{f} \cdot df = 3.14 \times 10^{-15} \cdot \ln \frac{1}{0.1}$$

or

$$V_{\text{noise,RMS}} = 85 \text{ nV}$$

The peak-to-peak input-referred noise voltage is roughly six times this or 510 nV (verifying the specification matches the data we get with the plot). ■

Example 8.17

Estimate the input-referred noise for the circuit seen in Fig. 8.34.

This amplifier is a transimpedance amplifier, that is, current input and voltage output. The amplifier takes the current generated by the photodiode and converts it into an output voltage. The low-frequency gain of the amplifier is R_F or 100k. The 1,000 pF feedback capacitor limits the bandwidth of the circuit to reduce noise. The photodiode is reverse-biased. Its noise contributions are related to the conversion of light to electrons (which we won't add to our noise analysis). The diode's reverse leakage current will be small and so the shot noise contributions to the circuit noise will be insignificant. The op-amp holds the diode's anode at a fixed potential (ground). The diode's junction capacitance won't affect the noise in the circuit (both sides of the capacitance are at AC ground).

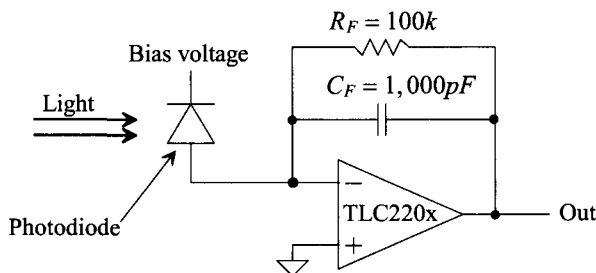


Figure 8.34 Estimating RMS output noise in a transimpedance amplifier.

The basic noise circuit for Fig. 8.34 is seen in Fig. 8.35. All of the output is fed back to the input so that the op-amp's thermal and flicker noise sources appear directly in the output signal (to keep the inverting and noninverting terminals at the same potential). The resistor's thermal noise (keeping in mind that we only look at one noise source at a time so that the inverting and noninverting inputs of the op-amp are at 0V), will create an output voltage that is the product of the noise current and the parallel combination of the 100k and 1,000 pF.

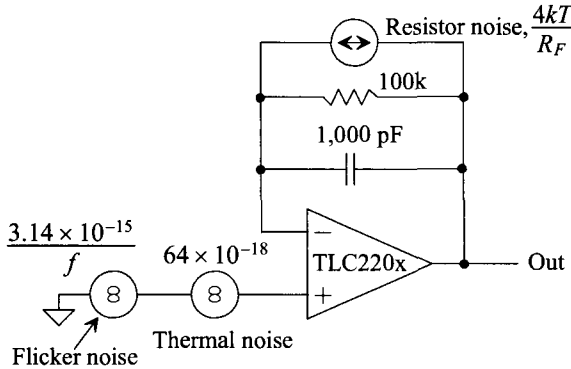


Figure 8.35 Noise model for the transimpedance amplifier in Fig. 8.34.

The output noise PSD can then be written as

$$V_{noise}^2(f) = \frac{3.14 \times 10^{-15}}{f} + 64 \times 10^{-18} + \frac{4kTR_F}{R_F} \cdot \frac{1}{1 + (2\pi f R_F C_F)^2} \cdot \left| \frac{R_F \cdot 1/j\omega C_F}{R_F + 1/j\omega C_F} \right|^2$$

We should recognize the last term in this PSD as simply kT/C noise (Ex. 8.6). The bandwidth of the amplifier can be determined using Fig. 8.36 as

$$A(f) = R_m = \frac{v_{out}}{i_s} = R_F \parallel \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega R_F C_F}$$

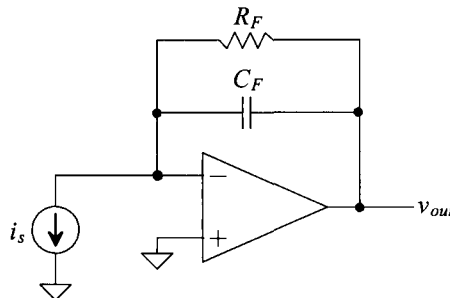


Figure 8.36 Determining the transfer function of a transimpedance amplifier.

or using

$$f_{3dB} = \frac{1}{2\pi R_F C_F}$$

which, for the present example, is 1.59 kHz. We can estimate the output RMS noise remembering the gain-bandwidth product, f_{un} , of the op-amp is 2 MHz and noting that the transfer function of the input-referred noise to the output is one as

$$V_{noise,RMS}^2 = \overbrace{49 \cdot 3.14 \times 10^{-15}}^{=154 \times 10^{-15}} + \overbrace{64 \times 10^{-18} \cdot \frac{\pi}{2} \cdot 2 \text{ MHz}}^{=200 \times 10^{-12}} + \overbrace{\frac{kT}{C_F}}^{=4 \times 10^{-12}}$$

$$V_{noise,RMS} \approx \sqrt{(0.392)^2 + (14.14)^2 + (2.07)^2} = 14.3 \mu V$$

noting that the op-amp's thermal noise dominates the output RMS noise. Our estimate for the flicker noise is high. However, it's easy to throw some numbers in for high and low frequencies in Eq. (8.55) and convince ourselves that the result isn't impacted too much. For example, we used $\ln \frac{10^{11}}{10^{-10}} \approx 49$ (yes, it's closer to 48 but we use 49 because the square-root is a whole number, i.e., 7) when we derived Eq. (8.55). If we were to use instead $\ln \frac{10^5}{10^{-10}}$ (a million times smaller upper frequency), we get 34.5 (a square root of roughly 6). The impact on the result is too small to worry about (unless, of course, the bandwidth is very narrow as in Ex. 8.16).

Dropping the feedback capacitance to 1 pF increases the bandwidth to 1.59 MHz and increases the RMS output noise due to R_F to 64 μV .

Our output signal is determined using $i_s \cdot R_F$. An SNR_{out} of 0 dB, in this example with $C_F = 1,000$ pF, would correspond to an i_s of roughly 143 pA. If we wanted an input signal to produce an output that is larger than the peak-to-peak value of the output noise (here, from Fig. 8.33, $> 6 \times 14.3 \mu V$), then $i_s > 858$ pA.

■

Example 8.18

Verify Ex. 8.17 using SPICE. Use a voltage-controlled voltage source for the op-amp (neglect the op-amp noise).

The schematic used for simulations is seen in Fig. 8.37. Instead of connecting the op-amp's noninverting input (the + input) to ground directly, we connect it to

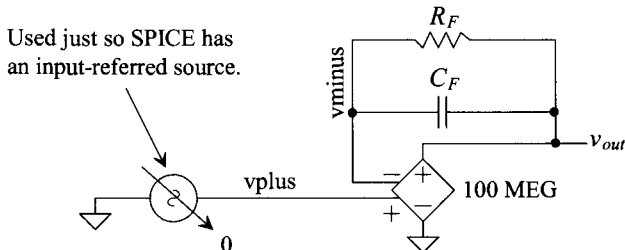


Figure 8.37 Using a voltage-controlled voltage source for an op-amp in SPICE.

ground via a zero volt DC supply so that, in the simulation, we have an “input.” (Recall comments about how the AC magnitude of this input source affects the input-referred noise calculation on the bottom of page 227.) The SPICE netlist is

```

Comment line; the first line of a netlist is ignored by SPICE
.noise V(Vout,0) Vplus dec 100 1 1000G
Vplus Vplus 0 dc 0 ac 1
Rf Vout Vminus 100k
Cf Vout Vminus 1000pf
Eopamp Vout 0 Vplus Vminus 100MEG
.print noise all
.end

```

The SPICE output is

```

TEMP=27 deg C
inoise_total = 4.142190e-12
onoise_total = 4.142190e-12

```

For the 1,000 pF capacitor, this gives 2.03 μV . Resimulating with the 1 pF capacitor, gives 64.3 μV . ■

8.2.6 Other Noise Sources

Thermal and flicker noise are the dominant noise sources in CMOS circuit design; however, there are other noise mechanisms. In this section we briefly discuss other types of noise, including random telegraph signal (aka burst or popcorn) and avalanche noise.

Random Telegraph Signal Noise

Random telegraph signal (RTS) noise is seen, in the time-domain, in Fig. 8.38. RTS noise is often called popcorn or burst noise. When a signal containing RTS noise is played on a speaker, the RTS noise component sounds like popping corn or the sound a telegraph makes. The origin of RTS noise is thought to be related to how generation-recombination, GR, centers affect the diffusion of carriers in a pn junction. If gold, for example, is introduced into a semiconductor, its inclusion results in nonuniformity in the silicon crystal lattice and thus additional GR sites. With a large number of these sites present, the variation in the diffusion rate, that is, the rate that the carriers diffuse away from a depletion region, can be significant. Since an RTS noise burst, Fig. 8.38, can last hundreds of microseconds, it is likely that at times more sites are filled and thus more carriers can diffuse further into the diode with a smaller number landing in a recombination site. This results in a build up of stored charge in the diode. The popping in the signal occurs when this additional stored charge exits the diode.

Although RTS noise doesn't have a Gaussian noise distribution, its PSD can be modeled using

$$I_{RTS}^2(f) = \frac{K_{RTS} \cdot I_{DC}}{1 + \left(\frac{f}{f_{3dB}}\right)^2} \text{ units of } A^2/\text{Hz} \quad (8.58)$$

where K_{RTS} has units of A/Hz and f_{3dB} (e.g., 100 Hz) is the point where the noise starts to roll off and is related to the number of pops per second, Fig. 8.39. Note that at very low frequencies, RTS noise is white (like thermal and shot noises). However, at higher frequencies, the noise rolls off as $1/f^2$.

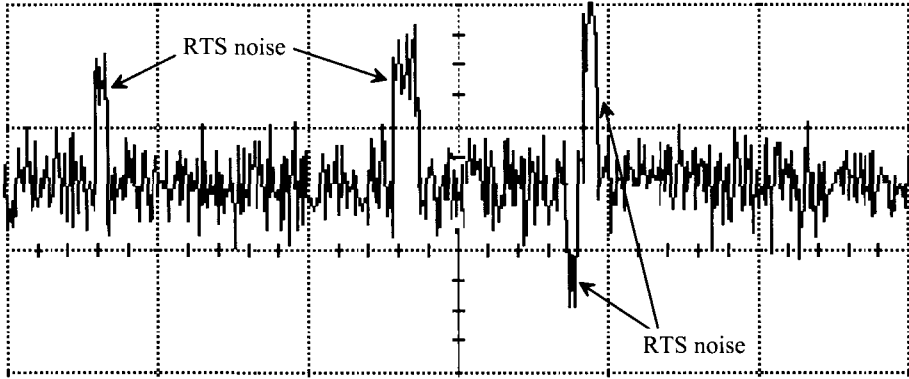


Figure 8.38 RTS noise viewed in the time domain.

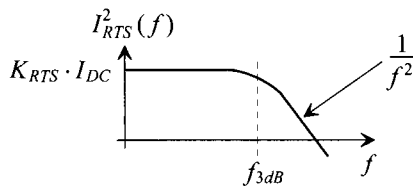


Figure 8.39 PSD of RTS noise.

Excess Noise (Flicker Noise)

Thermal noise in a resistor is present independent of the amount of current flowing in the resistor, as seen in Eq. (8.23). However, there is also flicker noise present in the resistor, dubbed *excess* noise, see Eq. (8.54). Flicker noise is present whenever a direct current flows in a discontinuous material (e.g., a material's surface or the interface between a MOSFET's gate oxide and the silicon used as the channel of the MOSFET). The electrons “jump” from one location to the next while sometimes being randomly trapped and released. Excess (flicker) noise is not dependent on temperature, unlike thermal noise, and so it can be especially troublesome when making low noise, temperature, and frequency measurements.

Avalanche Noise

When the electric field in a reverse-biased pn junction gets really large, carriers are accelerated to a point where they can strike the lattice and dislodge additional electron-hole pairs. The result is an increase in diode current. This additional generation of current is termed avalanche multiplication and is used in many Zener diodes to break the diode down. Tunneling is used in the lower voltage, < 6 V, Zener diodes which may have a shot noise spectrum because carriers tunnel across a potential barrier. Avalanche breakdown is a very noisy process. A typical PSD (noting a dependence on the current flowing in the reverse-biased junction) is $10^{-16} V^2/\text{Hz}$. The PSD is generally white, which has led to the use of Zener diodes as white noise generators, Fig. 8.40. In low-noise design it can be challenging to design a low-noise voltage reference using a Zener diode.

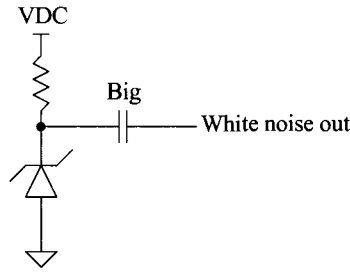


Figure 8.40 A Zener diode-based white noise generator.

Avalanche noise can also be troublesome in small geometry CMOS processes where a large drain-source voltage can lead to avalanche multiplication (and thus make MOSFET thermal noise appear to be very large).

8.3 Discussion

In this section we'll provide additional discussions and examples to help explain electrical noise.

8.3.1 Correlation

Consider the two signals seen in Fig. 8.41. If the RMS values of $v_1(t)$ and $v_2(t)$ are called V_{1RMS} and V_{2RMS} , respectively, then the power dissipated by the resistor is

$$P_{AVG} = \frac{V_{1RMS}^2 + V_{2RMS}^2}{R} \quad (8.59)$$

as discussed in Sec. 8.1.1 and used throughout the chapter. Let's look at this in more detail because it's easy to show that under certain circumstances this equation is misleading (and wrong!). In particular, if the two voltages sources are correlated, we can get a totally different value for the power the resistor dissipates. For example, we know that the voltage dropped across the resistor is $v_1(t) + v_2(t)$. What happens if $v_1(t) = V_P \cdot \sin(2\pi f \cdot t)$ and $v_2(t) = V_P \cdot \sin(2\pi f \cdot t + \pi)$ (which, of course, is the same as $v_2(t) = -V_P \cdot \sin[2\pi f \cdot t]$). The two sinewaves sum to zero. The voltage dropped across the resistor is zero and so is the power dissipated by the resistor. In a noise analysis, if our

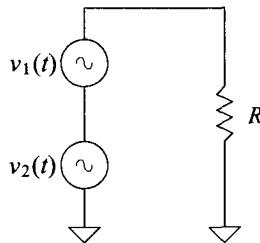


Figure 8.41 Series combination of voltage signals.

noise sources are correlated, then we can't simply add their noise contributions on the output of a network as we've done throughout the chapter. If the noise sources are correlated, the analysis can be much more challenging.

To describe how signals can be correlated in a little more detail, we can write, from Eq. (8.1),

$$P_{inst}(t) = \frac{V^2(t)}{R} = \frac{[v_1(t) + v_2(t)]^2}{R} \quad (8.60)$$

$$= \frac{v_1^2(t)}{R} + \frac{v_2^2(t)}{R} + \frac{2v_1(t)v_2(t)}{R} \quad (8.61)$$

The first two terms in the equation are independent. We can calculate the RMS value of these terms, and thus the average power supplied to R from each source, using

$$P_{AVG} = \frac{V_{RMS}^2}{R} = \frac{1}{T_{meas}} \cdot \int_0^{T_{meas}} \frac{v^2(t)}{R} \cdot dt \quad (8.62)$$

where T_{meas} is the amount of time used to measure the average power. Of course, we use this equation twice (once for each source) and then sum the average powers to get the average power supplied to R (which is Eq. [8.59]). If no correlation exists between $v_1(t)$ and $v_2(t)$, then the third term in Eq. (8.61) is zero. A simple example of two signals with no correlation are sine waves with different frequencies. Because each sinewave will go positive and negative (the signals are no longer squared and thus always positive), the summation of the signal (the integration) results in zero power added or subtracted from the total power. An example of 100% correlation is when

$$v_1(t) = v_2(t) = V_P \cdot \sin 2\pi f \cdot t \quad (8.63)$$

The signal applied to R in Fig. 8.41 has a peak amplitude of $2V_P$, an RMS value of $2V_P/\sqrt{2}$, and supplies a power to R of $2V_P^2/R$. Using Eqs. (8.61) and (8.62), we can write

$$P_{AVG} = \frac{V_P^2}{2R} + \frac{V_P^2}{2R} + \overbrace{\frac{2V_P^2}{2R}}^{100\% \text{ correlation}} = \frac{2V_P^2}{R} \quad (8.64)$$

We showed an example of -100% correlation a moment ago when

$$v_1(t) = V_P \cdot \sin 2\pi f \cdot t \text{ and } v_2(t) = -V_P \sin 2\pi f \cdot t \quad (8.65)$$

or

$$P_{AVG} = \frac{V_P^2}{2R} + \frac{V_P^2}{2R} + \overbrace{\frac{2(V_P)(-V_P)}{2R}}^{-100\% \text{ correlation}} = 0 \quad (8.66)$$

In general, the correlation between two waveforms is specified using C as

$$V_{1RMS}^2 + V_{2RMS}^2 + 2CV_{1RMS}V_{2RMS} \quad (8.67)$$

where $-1 \leq C \leq 1$. If C is zero, no correlation exists (again what we've used throughout the chapter). An example of two signals partially correlated are two sine waves at the same frequency but different amplitudes or phases. Note that this discussion used RMS values for the signals. However, it is also possible to discuss correlation in PSDs.

Correlation of Input-Referred Noise Sources

When we discussed input-referred noise and derived Eq. (8.32), we assumed zero correlation between $I_{noise,RMS}$ and $V_{noise,RMS}$. We can rewrite Eq. (8.32) without the thermal noise from the source resistance and with the help of Fig. 8.42 as

$$V_{noise,RMS}^2 = \left(\frac{AR_{in}}{R_s + R_{in}} \right)^2 [V_{noise,RMS} + I_{noise,RMS} \cdot R_s]^2 \quad (8.68)$$

If the correlation is zero, Eq. (8.68) is equivalent to Eq. (8.32) (without thermal noise contributions from R_s). With correlation, we can write

$$V_{noise,RMS}^2 = \left(\frac{AR_{in}}{R_s + R_{in}} \right)^2 \left[V_{noise,RMS}^2 + I_{noise,RMS}^2 \cdot R_s^2 + 2C \cdot V_{noise,RMS} \cdot I_{noise,RMS} \right] \quad (8.69)$$

To use the input-referred noise models, we generally assume no correlation ($C = 0$) between $I_{noise,RMS}$ and $V_{noise,RMS}$. In a practical circuit, the correlation can be significant because both are derived from the same noise mechanisms.

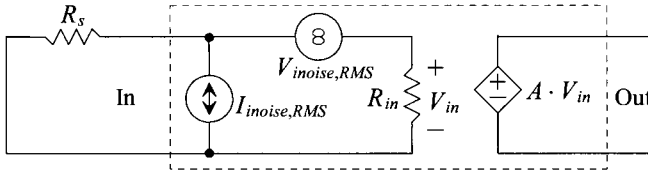


Figure 8.42 Correlation calculation of input-referred noise sources.

Complex Input Impedance

We've used a real input resistance in all of our calculations in this chapter. It's straightforward to extend our discussions and derivation to a general input impedance, Z_{in} . Our input voltage divider changes to

$$\frac{R_{in}}{R_s + R_{in}} \rightarrow \frac{Z_{in}}{R_s + Z_{in}} \quad (8.70)$$

When we multiply this divider by, say, V_s , to get V_{in} , we use

$$V_{in} = V_s \cdot \frac{|Z_{in}|}{|R_s + Z_{in}|} \text{ or } V_{in}^2 = V_s^2 \cdot \frac{|Z_{in}|^2}{|R_s + Z_{in}|^2} \quad (8.71)$$

It's important to take the magnitude before squaring. For example, if $Z_{in} = 1/j\omega C_{in} = -j/\omega C_{in}$ (the input impedance is a capacitor), then the correct way of calculating the denominator in Eq. (8.71) is

$$|R_s + Z_{in}|^2 = \left(\sqrt{R_s^2 + (1/\omega C_{in})^2} \right)^2 = R_s^2 + (1/\omega C_{in})^2 \quad (8.72)$$

The incorrect way is

$$\begin{aligned}
 |R_s + Z_{in}|^2 &\neq |(R_s + 1/j\omega C_{in})(R_s + 1/j\omega C_{in})| = \overbrace{|R_s^2 + 1/\omega^2 C_{in}^2|}^{\text{real term}} + \overbrace{|j \cdot (-2/\omega C_{in})|}^{\text{imaginary term}} \\
 &= \sqrt{(R_s^2 + 1/\omega^2 C_{in}^2)^2 + (2/\omega C_{in})^2} \quad (\text{again wrong!})
 \end{aligned}
 \tag{8.73}$$

It appears that there may be correlation in the equation above but it is really just bad complex algebra. We used the magnitude squared earlier in Eq. (8.13), that is, $|1 + j(f/f_{3dB})|^2$ or $1 + (f/f_{3dB})^2$. The point is to always take the magnitude of the expressions prior to squaring.

Let's use a complex input impedance, Z_{in} , to derive the optimum source resistance, Eq. (8.44). As we saw in Ex. 8.7 and 8.9, we measure the output noise with R_s shorted (or a big capacitor placed across the input of the circuit, which is essentially a short for all frequencies of interest) or opened to determine the input-referred noise sources. If the measured output noise (excluding thermal noise contributions from R_s) doesn't change with variations in R_s (quite common in amplifiers whose bias is independent of R_s like AC-coupled input amplifiers, but not the case for the circuit in Ex. 8.9), that is,

$$V_{\text{noise,RMS},R_s=\infty} = V_{\text{noise,RMS},R_s=0} = V_{\text{noise,RMS}} \tag{8.74}$$

and the input-referred noise sources are uncorrelated then, for a real input resistance,

$$R_{s,opt} = R_{in} \tag{8.75}$$

We may think that we can get both maximum power transfer and the best noise performance by matching the input resistance to the source resistance. However, if we use a complex input impedance, Eq. (8.75) becomes

$$R_{s,opt} = |Z_{in}| = |R_{in} + jX_{in}| = \sqrt{R_{in}^2 + X_{in}^2} \tag{8.76}$$

which can be valid without regard for maximum power transfer. Knowing that the requirement for maximum power transfer is making the source impedance the complex conjugate of the load impedance, we see that if the source impedance is purely real, it will be impossible to achieve maximum power transfer when the input resistance has an imaginary component. In radio-frequency circuits (narrow bandwidths where the impedances are relatively constant), impedance transformation circuits are used to improve both power transfer and noise performance. From the source side, the input impedance is transformed to maximize the transfer of power. From the input side, the source impedance is transformed to provide the best noise performance.

Example 8.19

Suppose an amplifier has an output noise PSD, $V_{\text{noise}}(f)$, of $1 \mu\text{V}/\sqrt{\text{Hz}}$ (we get this PSD if R_s is a short or an open), a gain of 100, and an input capacitance of 1 pF (infinite input resistance). Determine the input-referred noise sources for the amplifier.

With the input shorted, Fig. 8.43a, we get an input-referred noise voltage spectral density of

$$A \cdot V_{innoise}(f) = V_{onnoise}(f) = 1 \mu V / \sqrt{Hz}$$

or since $A = 100$

$$V_{innoise}(f) = 10 nV / \sqrt{Hz}$$

With the input opened, Fig. 8.43b, we get

$$I_{innoise}(f) \cdot |Z_{in}| \cdot A = \frac{I_{innoise}(f)}{2\pi f \cdot 1pF} \cdot 100 = 1 \mu V / \sqrt{Hz}$$

or

$$I_{innoise}(f) = 62.8 \times 10^{-21} \cdot f A / \sqrt{Hz}$$

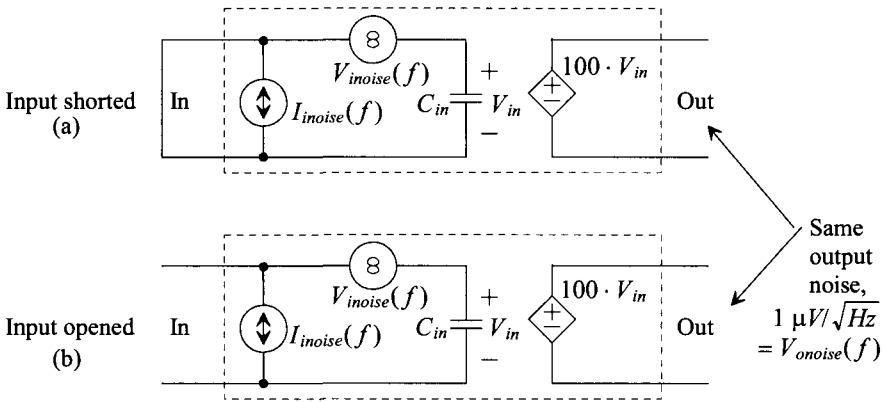


Figure 8.43 Determining input-referred noise in Ex. 8.19.

Note that the spectral density increases with f (Fig. 8.44). The noise current contributions to the output noise is insignificant until the frequency gets comparable to $1/(2\pi R_s C_{in})$ (noting that squaring $I_{innoise}(f)$ and integrating to find the RMS value results in an f^3 term). Unless the source resistance, frequencies of interest, or input capacitance are relatively large, the single input-referred noise voltage source is all that is needed to model the amplifier's output noise. ■

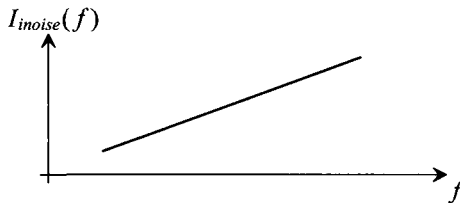


Figure 8.44 Input-referred noise current increasing with frequency when the input impedance is capacitive.

8.3.2 Noise and Feedback

Figure 8.45a shows the basic block diagram of a feedback circuit. In Fig. 8.45b we show the input-referred noise source. The output of the summer in (a) or (b) can be written as $V_{in} - \beta \cdot V_{out}$. The output of the circuit in (b) is

$$V_{out} = [(V_{in} - \beta V_{out}) + V_{inoise,RMS}] \cdot A \quad (8.77)$$

or

$$V_{out} = \frac{A}{1 + \beta A} \cdot (V_{in} + V_{inoise,RMS}) \quad (8.78)$$

In other words, feedback doesn't affect the circuit's noise performance. The input-referred noise adds directly to the input signal independent of the feedback. In practical circuits, the addition of resistors to provide feedback degrades the noise performance of the amplifier.

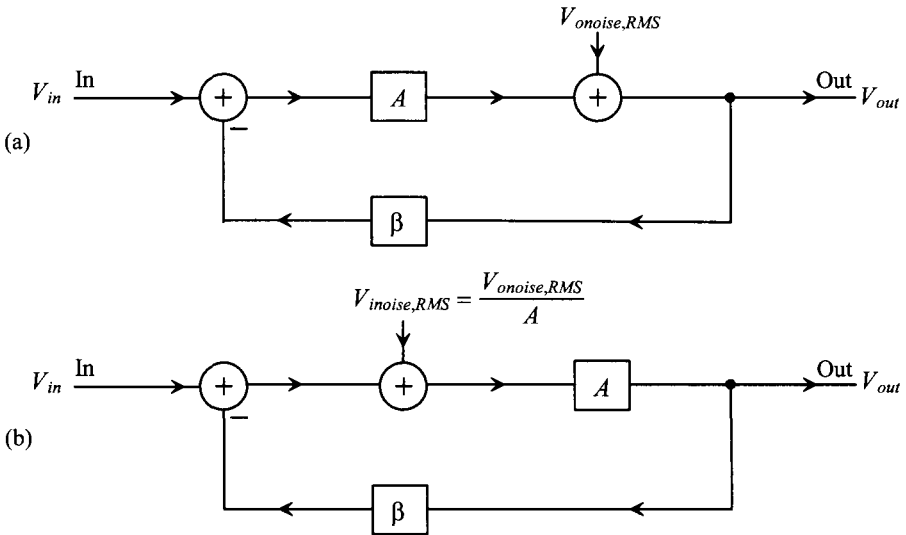


Figure 8.45 Noise in a feedback circuit.

Op-Amp Noise Modeling

Figure 8.46a shows how input-referred noise sources are added to an op-amp to model output noise. The input-referred noise current is connected to the inverting terminal of the op-amp, while the input-referred noise voltage is connected in series with the noninverting terminal. Figure 8.46b shows the general implementation of a feedback amplifier using an op-amp, including noise sources modeling the resistor's thermal noise. If the input to the op-amp circuit is on the left side of R_1 , then the gain to the output is inverting, $(-R_2/R_1)$. If the input is connected directly to the noninverting terminal of the op-amp, then the gain is positive, $(1 + R_2/R_1)$. Remember, we look at only one noise source at a time (superposition) when we calculate the total output noise. Also, remember that the op-amp, through the feedback action, holds the inverting terminal at ground.

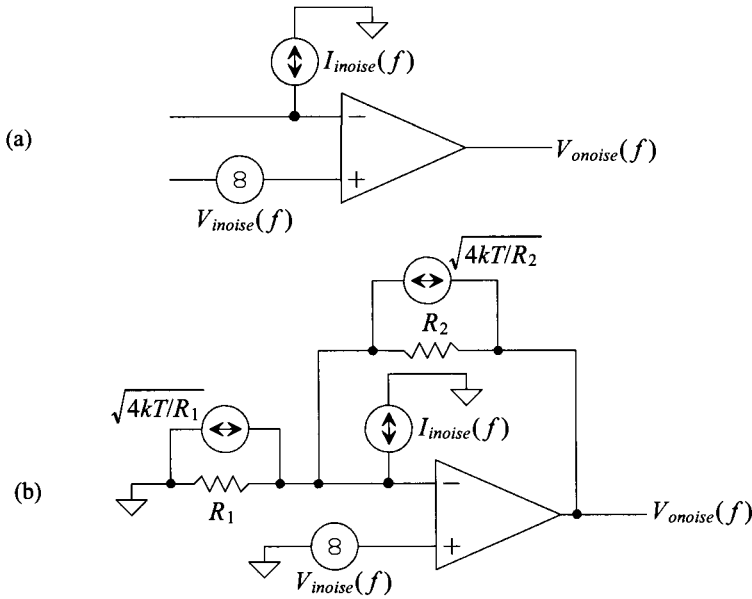


Figure 8.46 Modeling op-amp noise.

Let's write the output noise, for Fig. 8.46b, in terms of the input-referred noise sources and the thermal noise from the resistors. Because both sides of R_1 are at ground (one side is physically tied to ground while the other side is held at ground by the op-amp), we can write

$$V_{onoise}^2(f) = V_{inoise}^2(f) \cdot \left(1 + \frac{R_2}{R_1}\right)^2 + \left[I_{inoise}^2(f) + \frac{4kT}{R_1} + \frac{4kT}{R_2}\right] \cdot R_2^2 \quad (8.79)$$

The gain of this circuit for the sake of calculating bandwidth is $(1 + R_2/R_1)$. The closed-loop value of bandwidth is written in terms of the op-amp's gain-bandwidth product, f_{un} (= the frequency when the open-loop gain of the op-amp is unity, see pages 792-793) as

$$\text{op-amp's gain bandwidth} = f_{un} = \left(1 + \frac{R_2}{R_1}\right) \cdot f_{3dB} \quad (8.80)$$

The NEB, Eq. (8.15), can be written as

$$NEB = \frac{\pi}{2} \cdot f_{un} \cdot \frac{R_1}{R_1 + R_2} \quad (8.81)$$

Assuming that the input-referred sources are not correlated and the NEB of the output noise limits the RMS values, see Eq. (8.15) and associated discussion, we can write

$$V_{onoise,RMS} = \sqrt{V_{onoise}^2(f) \cdot \frac{\pi}{2} \cdot f_{un} \cdot \frac{R_1}{R_1 + R_2}} \quad (8.82)$$

where $V_{onoise}^2(f)$ is assumed to be white until it roll-offs with the amplifier's bandwidth, Fig. 8.9b.

As an example, Fig. 8.47 shows the input-referred noise sources for the LT1364 op-amp. Neglecting the low-frequency (flicker) noise, we see that $V_{innoise}(f) = 9 \text{ nV}/\sqrt{\text{Hz}}$ ($= e_n$ in Fig. 8.47) and $I_{innoise}(f) = 0.85 \text{ pA}/\sqrt{\text{Hz}}$ ($= i_n$ in Fig. 8.47). From the datasheet, the gain-bandwidth product (f_{un}) of this op-amp is nominally 70 MHz (and so an op-amp in a gain of 10 ($= 1 + R_2/R_1$) configuration has $f_{3dB} = 7 \text{ MHz}$).

Equations (8.79) and (8.82) can be interpreted in many ways. If our op-amp is used as a transimpedance amplifier as in Fig. 8.34 (Ex. 8.17), then we want R_1 to be infinite to minimize the output noise. We think of the input source, i_s , as a current connected to the inverting op-amp input, the amplifier as having a gain of $|R_2|$, and the output voltage as $i_s \cdot R_2$. (The inverting input to the op-amp is a virtual ground, which is the ideal input resistance for an input current signal.) Looking at the SNR_{out} or $(i_s \cdot R_2)/V_{onnoise,RMS}$, we see that reducing R_2 lowers both the gain and the noise while at the same time causing the SNR_{out} to drop.

If the op-amp is used as an inverting voltage amplifier with a gain of $-R_2/R_1$ (input signal connected to the left side of R_1), or as a noninverting voltage amplifier with a gain of $1 + R_2/R_1$ (input signal connected to the noninverting op-amp input terminal) then increasing R_1 , as seen in Eq. (8.79), reduces the output noise. If R_2 is held constant then increasing R_1 also reduces the amplifier gain causing the SNR_{out} to decrease (where $SNR_{out} = (v_s \cdot R_2/R_1)/V_{onnoise,RMS}$ or $(v_s \cdot [1 + R_2/R_1])/V_{onnoise,RMS}$. In this situation, for large SNR_{out} , we want both R_1 (for large gain) and R_2 (to minimize the output noise as seen in Eq. [8.79]) to be as small as possible. For example, if we want a gain of 10, it is preferable to use an R_1 and R_2 of 10k and 1k over using 100k and 10k. Of course, using lower values results in more power dissipation in these feedback resistors.

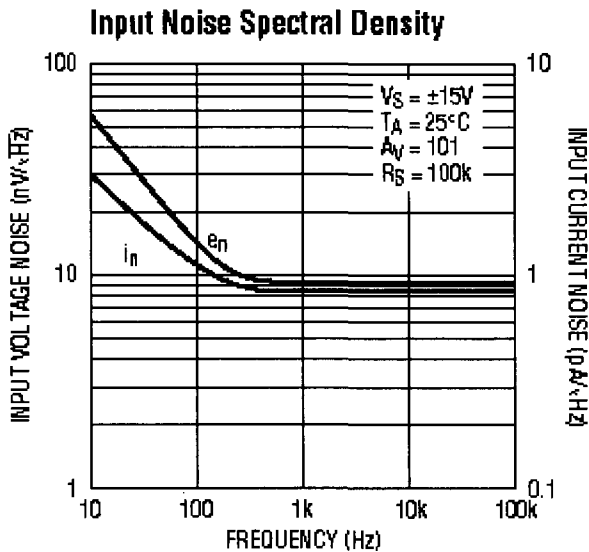


Figure 8.47 Input-referred noise sources for the LT1364 high-speed op-amp.

8.3.3 Some Final Notes Concerning Notation

When we calculate the mean-squared value of a noise voltage signal (which means that the desired signal components are not present in the signal), we used

$$\overline{v^2} = V_{RMS}^2 = \frac{1}{T_{meas}} \int_0^{T_{meas}} v^2(t) \cdot dt \quad (8.83)$$

The square root of the mean-squared voltage is the RMS value of a noise waveform. In Ex. 8.14 we showed that the RMS value of the integrated thermal noise increases with measuring time. Does this mean that if we measure, or look, at the value of this noise signal at a specific time it can't be zero? As seen in Fig. 8.26, a noise voltage can be positive, zero, or negative. What we are saying when we say the RMS value is increasing is that the voltage excursions away from zero are growing (but $v(t)$ is still zero at some times).

The next thing we should note is that the mean-squared value is a number not a spectral density. Strictly speaking, it is incorrect to write, for example, the thermal noise of a resistor as

$$\overline{v^2} = 4kTR \text{ or } \overline{i^2} = \frac{4kT}{R} \quad (8.84)$$

A number (the left side of the equation) is not equal to a spectrum (the right side of the equation is a PSD). When we talked about measuring PSD with a spectrum analyzer in Sec. 8.1.2, we said that we plot a point by dividing the measured power by the resolution bandwidth, that is, V_{RMS}^2/f_{res} , at a particular frequency. Sometimes the notation, Δf , is used for the resolution bandwidth f_{res} . It is correct to write the PSD of thermal noise as

$$V_R^2(f) = \frac{V_{RMS}^2}{f_{res}} = \frac{\overline{v^2}}{\Delta f} = 4kTR \text{ units of } \frac{V^2}{Hz} \quad (8.85)$$

It is also correct to write

$$\overline{v^2} = 4kTR \cdot \Delta f \text{ volts} \quad (8.86)$$

where the resolution bandwidth is assumed narrow (for a general noise signal that may or may not be white). What makes this more confusing is that the mean-squared values are used for both narrow band signals, as seen in Eqs. (8.85) and (8.86), and for entire noise spectrums. For example, to calculate the RMS value of a wideband noise signal, we use

$$V_{RMS}^2 = \overline{v^2} = \int_{f_L}^{f_H} V_R^2(f) \cdot df = 4kTR \cdot B \quad (8.87)$$

where $B = f_H - f_L \neq \Delta f = f_{res}$. Note that Δf has nothing to do with NEB or B .

As another example, if we were to measure the PSD of flicker noise, we would get a value of

$$V_{1/f}^2(f) = \frac{\overline{v^2}}{\Delta f} = \frac{FNN}{f} \quad (8.88)$$

at each frequency f . The measured power in the narrow bandwidth $\Delta f (= f_{res})$, $\overline{v^2} (= V_{RMS}^2)$, decreases as f goes up resulting in the $1/f$ PSD.

ADDITIONAL READING AND/OR INFORMATION

- [1] R. J. Baker, *CMOS: Mixed-Signal Circuit Design, Second Edition*, Wiley-IEEE Press, 2009. Covers noise/averaging and the affects on signal-to-noise ratio. Good complement to the material in this chapter.
- [2] W. T. (Tim) Holman, Private communication, Feb. 2002. Information on how the RMS value of a signal containing $1/f$ noise (and $1/f^2$, $1/f^3$, etc.) grows with measuring time.
- [3] C. D. Motchenbacher and J. A. Connelly, *Low-Noise Electronic System Design*, John Wiley and Sons, 1993. ISBN 0-471-57742-1. Excellent reference for low-noise design.
- [4] R. Sarpeshkar, T. Delbrück, and C. A. Mead, "White Noise in MOS Transistors and Resistors," *IEEE Circuits and Devices Magazine*, Nov. 1993. Shows that both shot and thermal noise mechanisms in MOSFETs have the same origins.
- [5] D. M. Binkley, J. M. Rochelle, M. J. Paulus, and M. E. Casey, "A Low-Noise, Wideband, Integrated CMOS Transimpedance Preamplifier for Photodiode Applications," in *IEEE Transactions on Nuclear Science*, vol. NS-39, no. 4, August 1992, pp. 747–752. Covers the design of low-noise transimpedance amplifiers.
- [6] H. L. Krauss, C. W. Bostian, and F. H. Raab, *Solid State Radio Engineering*, John Wiley and Sons, 1980. ISBN 0-471-03018-X. Reference for the design of radio circuits.
- [7] W. R. Bennett, *Electrical Noise*, McGraw-Hill, 1960. Fundamental reference on electrical noise. Written by a pioneer in noise theory. No longer in print but available used through various book sellers.
- [8] A. Van der Ziel, *Noise; Sources, Characterization, Measurement*, Prentice-Hall. Also written by a pioneer in noise theory.
- [9] H. F. Friis, "Noise Figures of Radio Receivers," in *Proceedings of the IRE* (Institute of Radio Engineers), vol. 32, no. 7, 1944, pp. 419–422. Fundamental paper covering noise figure.

LIST OF SYMBOLS/ACRONYMS

A - Voltage gain of a circuit.

A_{DC} - DC gain of a circuit.

B - Bandwidth of a noise measurement, i.e., $f_H - f_L$.

C - Correlation term, see Eq. (8.67).

C_{in} - Input capacitance.

C_j - Diodes junction capacitance (also called depletion capacitance).

C_{j0} - Depletion capacitance measured with zero volts across the diode.

CUT - Circuit under test.

Δf - Resolution bandwidth, same as f_{res} .

DFT - Discrete Fourier transform.

ESD - Energy spectral density.

F - Noise factor.

FNN - Flicker noise numerator.

F_{opt} - Optimum noise factor.

f - Frequency of a sinewave.

f_{3dB} - 3 dB frequency of a circuit (the power is half the low frequency value at f_{3dB} or the voltage is 0.707 its low-frequency value at a frequency of f_{3dB}).

f_H - Higher frequency in an RMS noise calculation, see Eq. (8.10).

f_L - Lower frequency in an RMS noise calculation, see Eq. (8.10).

f_{res} - Resolution bandwidth of a spectrum analyzer or a DFT (same as Δf).

f_{start} - Starting frequency in a DFT or a spectrum analyzer measurement.

f_{stop} - Final frequency in a DFT or a spectrum analyzer measurement.

f_{un} - Unity gain frequency of an op-amp (the open-loop gain is 1).

GR - Generation-recombination.

$\overline{i^2}$ - Mean-squared current ($= I_{RMS}^2$).

$I(f)$ - Square root of a current PSD, units of A/\sqrt{Hz} .

$I^2(f)$ - PSD of a noise current, units of A^2/Hz .

$I_{1/f}^2(f)$ - PSD of flicker noise current, units of A^2/Hz .

$I_{noise}^2(f)$ - PSD of the input-referred noise of a circuit, units of A^2/Hz .

$I_{noise,RMS}$ - Input-referred RMS noise current, unit of A .

I_{RMS} - Root mean-squared value of a current waveform, units of A , also written in this chapter as $\sqrt{\overline{i^2}}$.

$I_R^2(f)$ - PSD of thermal noise, units of A^2/Hz .

I_{RMS}^2 - Mean-squared value of a current waveform, units of A^2 , also written as $\overline{i^2}$.

$I_{RTS}^2(f)$ - PSD of random telegraph signal noise current, units of A^2/Hz .

$I_{shot}^2(f)$ - Shot noise current PSD, units of A^2/Hz .

j - $\sqrt{-1}$.

K - Number of averages, see Eq. (8.48).

k - Boltzmann's constant, $13.8 \times 10^{-24} J/K$.

LNA - Low-noise amplifier.

n - A counting index.

NEB - Noise-equivalent bandwidth, see Eq. (8.15).

NF - Noise figure, see Eq. (8.35).

$P_{noise}(f)$ - PSD of a signal with units of W/Hz .

P_{AVG} - Average power dissipated.

PDF - Probability density function, see Fig. 8.33.

$P_{inst}(t)$ - Instantaneous power dissipation (the power dissipated at a specific time). Units of watts (Joules/s).

PSD - Power spectral density, units of V^2/Hz .

q - Electron charge of 1.6×10^{-19} coulombs.

R - Resistance in ohms.

R_s - Resistance of an input source signal.

RMS - Root mean square.

SA - Spectrum analyzer.

SNR_{in} - Input signal-to-noise ratio, see Eq. (8.28).

SNR_{out} - Output signal-to-noise ratio, see Eq. (8.40).

t - Time in seconds.

T - Temperature in Kelvin.

T_{meas} - Time a measurement is taken, units of seconds.

$\overline{v^2}$ - Mean-squared voltage $\left(= V_{RMS}^2\right)$.

$V(f)$ - Square root of a PSD, units of V/\sqrt{Hz} (also called a voltage spectral density).

$v(t)$ - Time domain voltage waveform.

V_{in} - Input voltage.

$V_{inoise}^2(f)$ - PSD of the input-referred noise of a circuit.

$V_{inoise,RMS}$ - Input-referred RMS noise voltage, units of V .

$V_{LF,noise}^2(f)$ - Low-frequency white noise, see Fig. 8.9.

$V_{noise}^2(f)$ - PSD of a noise signal.

$V_{onoise}^2(f)$ - PSD of the output noise of a circuit.

$V_{onoise,RMS}$ - Output RMS noise voltage.

V_p - Peak voltage of a sine waves wave.

$V_R^2(f)$ - PSD of thermal noise, units of V^2/Hz .

$V_{1/f}^2(f)$ - PSD of flicker noise voltage, units of V^2/Hz .

V_{RMS} - Root mean-squared value of a voltage waveform, units of V , also written in this chapter as $\sqrt{v^2}$.

V_{RMS}^2 - Mean-squared value of a voltage waveform, units of V^2 , also written in this chapter as $\overline{v^2}$.

V_s - Source input voltage.

V_T - Thermal voltage, kT/q or 26 mV at room temperature.

ω - $2\pi f$.

Z_{in} - A complex input impedance.

PROBLEMS

- 8.1 Suppose that the power company is charging \$ 0.25 for a kilowatthour (1,000 W of power supplied for one hour) of energy. How much are you paying for a Joule of energy? How much energy, in Joules, is supplied to a 100 W lightbulb in one hour? Is it potential energy or kinetic energy the power company sells?
- 8.2 The power company attempts to hold, in the United States, the RMS value of the voltage supplied to households to 120 V RMS with $\pm 3V$ at a frequency of 60 Hz. What is the peak-to-peak value of this voltage? If we were to look at the PSD of this signal in a 10 Hz bandwidth, what amplitude would we see?
- 8.3 Suppose a SA measures a noise voltage spectral density of $1 \mu V/\sqrt{\text{Hz}}$. What is the RMS value of this noisy signal over a bandwidth of DC to 1 MHz?
- 8.4 Suppose the noise signal in problem 3 shows a 3-dB frequency of 5 MHz. What is its RMS value over an infinite bandwidth?
- 8.5 Estimate the RMS output noise in the following circuit over a bandwidth of 1 to 1 kHz. Verify your answer with SPICE. (Hint: simplify the circuit by combining resistors.)

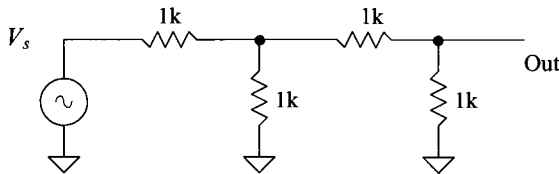


Figure 8.48 Circuit for Problem 8.5.

- 8.6 Estimate the RMS output noise over an infinite bandwidth for the circuit in Fig. 8.48 if the output is shunted with a 1 pF capacitor.
- 8.7 Show, Fig. 8.49, that the calculation of SNR_{in} results in the same value independent of treating the input as a voltage or a current.

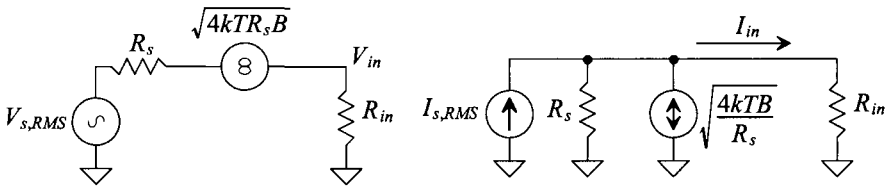


Figure 8.49 Calculating input SNR in Problem 8.7.

- 8.8** Using the input-referred noise model seen in Fig. 8.20b, verify that if the input resistance becomes infinite, the output noise is adequately modeled using a single input-referred noise voltage.
- 8.9** If an amplifier has a 0 dB NF, does that indicate the amplifier's output is free of noise? Why or why not?
- 8.10** Verify, as seen in Eq. (8.45), that the input-referred noise power of an amplifier is $(V_{innoise,RMS} \cdot I_{innoise,RMS})/2$.
- 8.11** Verify that the units for the shot noise PSD seen in Eq. (8.49) are indeed A^2/Hz .
- 8.12** Repeat Ex. 8.12 if the input voltage is reduced from 1.7 V to 1 V.
- 8.13** Verify the comment made in Sec. 8.2.5 that the RMS value of integrated flicker noise signal increases linearly with measurement time.
- 8.14** If the maximum allowable RMS output noise of a transimpedance amplifier built using the TLC220x is 100 μV in a bandwidth of 1 MHz is needed, what are the maximum values of C_F and R_F ? What is the peak-to-peak value of the noise in the time domain?
- 8.15** Estimate the RMS output noise for the circuit seen in Fig. 8.50. Compare the hand-calculated result to SPICE simulation results using an ideal op-amp (as used in Ex. 8.18).

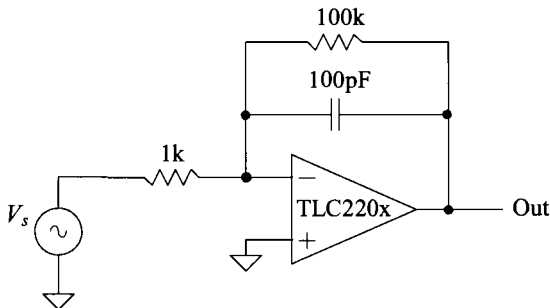


Figure 8.50 Circuit for Problem 8.15.

- 8.16** Suppose a 4.7 V Zener diode is used in a noise generator, as seen in Fig. 8.40. Estimate the output noise voltage PSD in terms of the diode's noise current and the biasing resistor (use 1k) if the power supply is 9 V. (Note: the noise mechanism is shot noise not avalanche noise.)
- 8.17** Rewrite Eq. (8.72) if $Z_{in} = R_{in} + 1/j\omega C_{in}$.
- 8.18** Estimate the RMS output noise for the amplifier seen in Fig. 8.51. What would placing a capacitor across the feedback resistor do to the RMS output noise and to the speed (bandwidth) of the amplifier? Name two ways to lower the RMS output noise for this amplifier. What is the cost for the lower output noise?

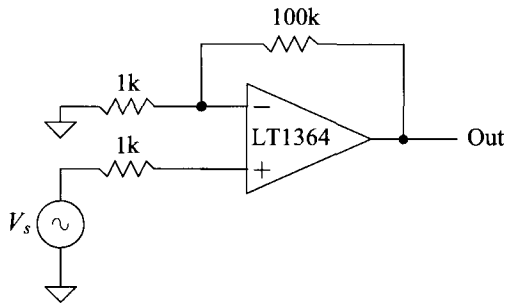


Figure 8.51 Circuit for Problem 8.18.