1 Distribution Overview

1.1 Discrete Distributions

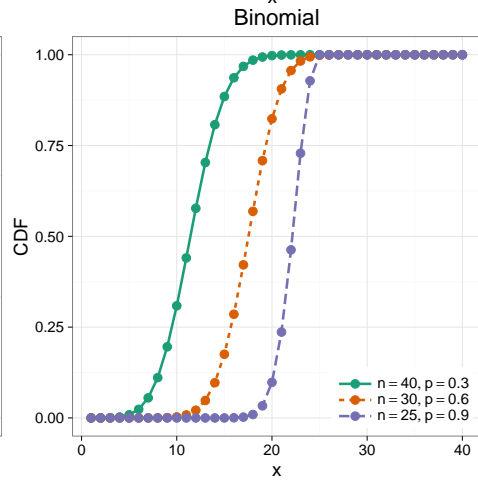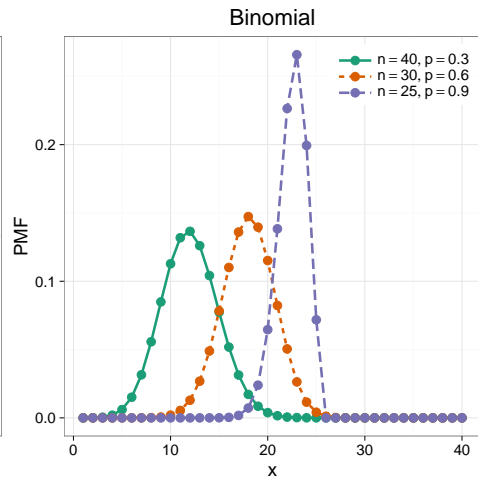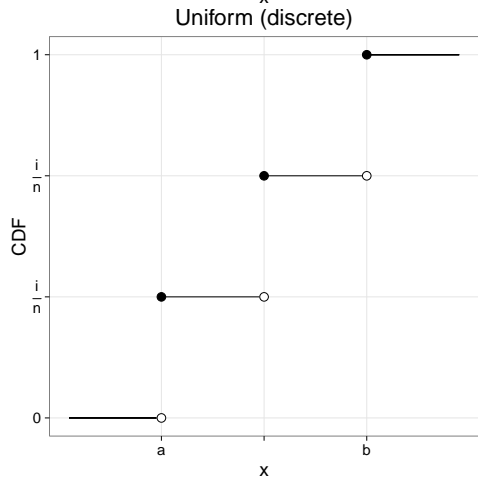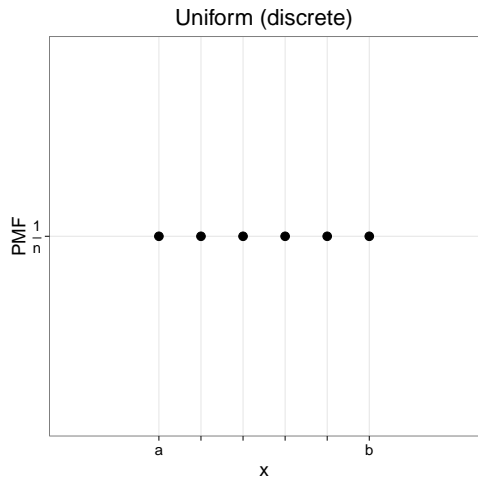|	Notation[1]	$F_X(x)$	$f_X(x)$	$\mathbb{E}[X]$	$\mathbb{V}[X]$	$M_X(s)$
Uniform	$\mathrm{Unif}\{a,\dots,b\}$	$\begin{cases} 0 & x < a \\ \frac{\lfloor x \rfloor - a + 1}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}$	$\frac{I(a \le x \le b)}{b-a+1}$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$	$\frac{e^{as}-e^{-(b+1)s}}{s(b-a)}$
Bernoulli	$\mathrm{Bern}(p)$	$(1-p)^{1-x}$	$p^x(1-p)^{1-x}$	p	$p(1-p)$	$1-p+pe^s$
Binomial	$\mathrm{Bin}(n,p)$	$I_{1-p}(n-x, x+1)$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pe^s)^n$
Multinomial	$\mathrm{Mult}(n,p)$		$\frac{n!}{x_1!\dots x_k!}p_1^{x_1}\cdots p_k^{x_k} \quad \sum_{i=1}^{k} x_i = n$	$\begin{pmatrix} np_1 \\ \vdots \\ np_k \end{pmatrix}$	$\begin{pmatrix} np_1(1-p_1) & -np_1p_2 \\ -np_2p_1 & \ddots \end{pmatrix}$	$\left(\sum_{i=0}^{k} p_i e^{s_i}\right)^n$
Hypergeometric	$\mathrm{Hyp}(N,m,n)$	$\approx \Phi\left(\frac{x-np}{\sqrt{np(1-p)}}\right)$	$\frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}$	$\frac{nm}{N}$	$\frac{nm(N-n)(N-m)}{N^2(N-1)}$	
Negative Binomial	$\mathrm{NBin}(r,p)$	$I_p(r, x+1)$	$\binom{x+r-1}{r-1}p^r(1-p)^x$	$r\frac{1-p}{p}$	$r\frac{1-p}{p^2}$	$\left(\frac{p}{1-(1-p)e^s}\right)^r$
Geometric	$\mathrm{Geo}(p)$	$1-(1-p)^x \quad x \in \mathbb{N}^+$	$p(1-p)^{x-1} \quad x \in \mathbb{N}^+$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^s}{1-(1-p)e^s}$
Poisson	$\mathrm{Po}(\lambda)$	$e^{-\lambda}\sum_{i=0}^{x}\frac{\lambda^i}{i!}$	$\frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ	$e^{\lambda(e^s-1)}$

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[1]We use the notation $\gamma(s,x)$ and $\Gamma(x)$ to refer to the Gamma functions (see §22.1), and use $\mathrm{B}(x,y)$ and I_x to refer to the Beta functions (see §22.2).
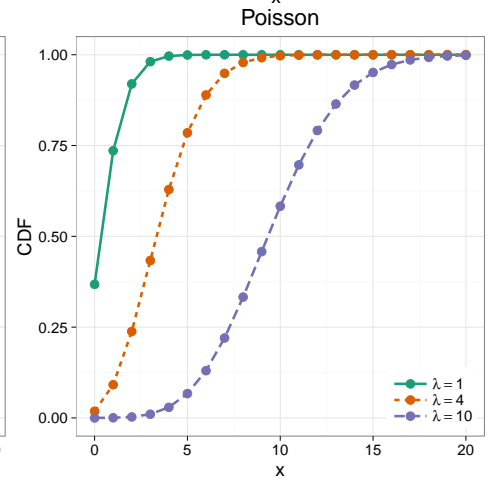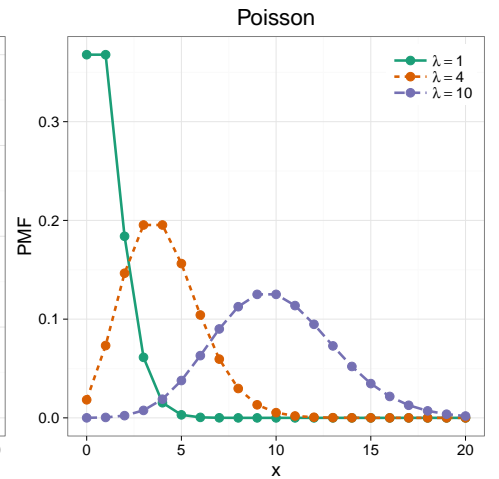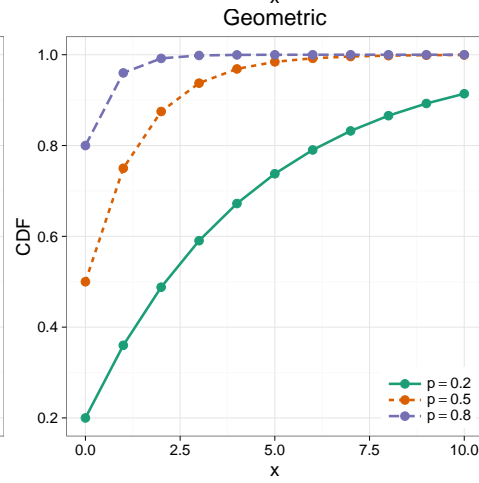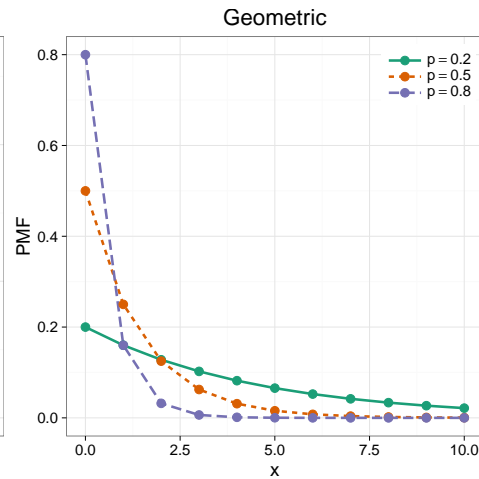
1.2 Continuous Distributions

	Notation	$F_X(x)$	$f_X(x)$	$\mathbb{E}[X]$	$\mathbb{V}[X]$	$M_X(s)$
Uniform	$\text{Unif}(a,b)$	$\begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x > b \end{cases}$	$\frac{I(a < x < b)}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{sb} - e^{sa}}{s(b-a)}$
Normal	$\mathcal{N}(\mu, \sigma^2)$	$\Phi(x) = \int_{-\infty}^{x} \phi(t)\,dt$	$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{ -\frac{(x-\mu)^2}{2\sigma^2} \right\}$	μ	σ^2	$\exp\left\{ \mu s + \frac{\sigma^2 s^2}{2} \right\}$
Log-Normal	$\ln\mathcal{N}(\mu, \sigma^2)$	$\frac{1}{2} + \frac{1}{2}\,\text{erf}\left[\frac{\ln x - \mu}{\sqrt{2\sigma^2}} \right]$	$\frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(\ln x - \mu)^2}{2\sigma^2} \right\}$	$e^{\mu+\sigma^2/2}$	$(e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$	
Multivariate Normal	$\text{MVN}(\mu, \Sigma)$		$(2\pi)^{-k/2}|\Sigma|^{-1/2}e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}$	μ	Σ	$\exp\left\{ \mu^T s + \frac{1}{2}s^T\Sigma s \right\}$
Student's t	$\text{Student}(\nu)$	$I_x\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$	$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}$	$0 \quad \nu > 1$	$\begin{cases} \frac{\nu}{\nu-2} & \nu > 2 \\ \infty & 1 < \nu \le 2 \end{cases}$	
Chi-square	χ_k^2	$\frac{1}{\Gamma(k/2)}\gamma\left(\frac{k}{2}, \frac{x}{2}\right)$	$\frac{1}{2^{k/2}\Gamma(k/2)}x^{k/2-1}e^{-x/2}$	k	$2k$	$(1 - 2s)^{-k/2}\ s < 1/2$
F	$\text{F}(d_1, d_2)$	$I_{\frac{d_1 x}{d_1 x + d_2}}\left(\frac{d_1}{2}, \frac{d_1}{2}\right)$	$\frac{\sqrt{\frac{(d_1 x)^{d_1} d_2^{d_2}}{(d_1 x + d_2)^{d_1 + d_2}}}}{x\text{B}\left(\frac{d_1}{2}, \frac{d_1}{2}\right)}$	$\frac{d_2}{d_2 - 2}$	$\frac{2d_2^2(d_1 + d_2 - 2)}{d_1(d_2 - 2)^2(d_2 - 4)}$	
Exponential*	$\text{Exp}(\beta)$	$1 - e^{-x/\beta}$	$\frac{1}{\beta}e^{-x/\beta}$	β	β^2	$\frac{1}{1 - \frac{s}{\beta}}\ (s < \beta)$
Gamma*	$\text{Gamma}(\alpha, \beta)$	$\frac{\gamma(\alpha, \beta x)}{\Gamma(\alpha)}$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	$\left(\frac{1}{1 - \frac{s}{\beta}}\right)^\alpha\ (s < \beta)$
Inverse Gamma	$\text{InvGamma}(\alpha, \beta)$	$\frac{\Gamma\left(\alpha, \frac{\beta}{x}\right)}{\Gamma(\alpha)}$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{-\alpha-1}e^{-\beta/x}$	$\frac{\beta}{\alpha-1}\ \alpha > 1$	$\frac{\beta^2}{(\alpha-1)^2(\alpha-2)}\ \alpha > 2$	$\frac{2(-\beta s)^{\alpha/2}}{\Gamma(\alpha)}K_\alpha\left(\sqrt{-4\beta s}\right)$
Dirichlet	$\text{Dir}(\alpha)$		$\frac{\Gamma\left(\sum_{i=1}^k \alpha_i\right)}{\prod_{i=1}^k \Gamma(\alpha_i)}\prod_{i=1}^k x_i^{\alpha_i - 1}$	$\frac{\alpha_i}{\sum_{i=1}^k \alpha_i}$	$\frac{\mathbb{E}[X_i](1 - \mathbb{E}[X_i])}{\sum_{i=1}^k \alpha_i + 1}$	
Beta	$\text{Beta}(\alpha, \beta)$	$I_x(\alpha, \beta)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$1 + \sum_{k=1}^\infty \left(\prod_{r=0}^{k-1}\frac{\alpha+r}{\alpha+\beta+r}\right)\frac{s^k}{k!}$
Weibull	$\text{Weibull}(\lambda, k)$	$1 - e^{-(x/\lambda)^k}$	$\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}e^{-(x/\lambda)^k}$	$\lambda\Gamma\left(1 + \frac{1}{k}\right)$	$\lambda^2\Gamma\left(1 + \frac{2}{k}\right) - \mu^2$	$\sum_{n=0}^\infty \frac{s^n\lambda^n}{n!}\Gamma\left(1 + \frac{n}{k}\right)$
Pareto	$\text{Pareto}(x_m, \alpha)$	$1 - \left(\frac{x_m}{x}\right)^\alpha\ x \ge x_m$	$\alpha\frac{x_m^\alpha}{x^{\alpha+1}}\quad x \ge x_m$	$\frac{\alpha x_m}{\alpha-1}\ \alpha > 1$	$\frac{x_m^2\alpha}{(\alpha-1)^2(\alpha-2)}\ \alpha > 2$	$\alpha(-x_m s)^\alpha\Gamma(-\alpha, -x_m s)\ s < 0$

* We use the *rate* parameterization where $\beta = \frac{1}{\lambda}$. Some textbooks use β as *scale* parameter instead [6].