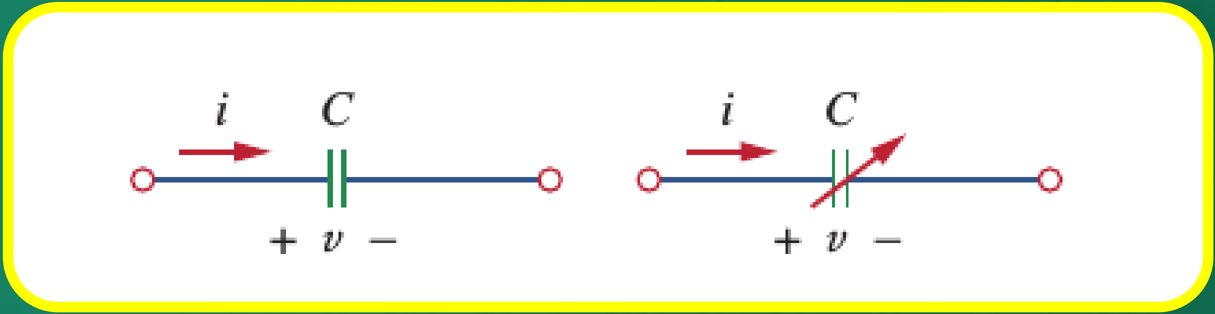
Electrical Science - I
(IEC-102)

Lecture-06

Capacitors and Inductors

The Capacitor

- The ideal capacitor is a passive element with circuit symbol



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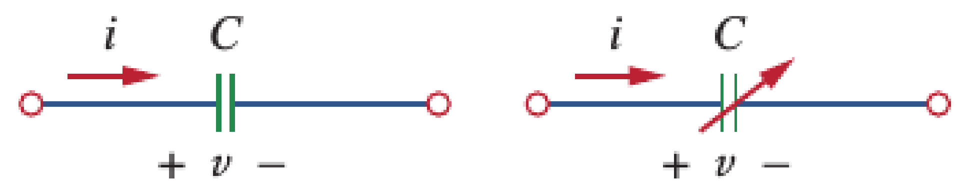

- The current-voltage relation is

$$i = C\frac{dv}{dt}$$

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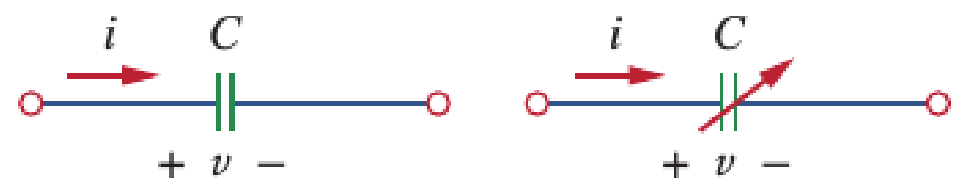

- The current-voltage relation is

$$i = C\frac{dv}{dt}$$

- The capacitance C is measured in farads (F)

The Capacitor

- The voltage-current relation is

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v(t_0)}$$

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- Instantaneous power

$$p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$$

The Capacitor

- Energy stored in a capacitor

$$w = \int_{-\infty}^{t} p\,dt = C\int_{-\infty}^{t} v\frac{dv}{dt}dt = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

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$$w = \frac{1}{2}Cv^2 \quad \text{if} \quad v(-\infty) = 0$$

Some Capacitors

Typical values range from pF to μF

Key Capacitor Behaviors

- Acts as an open circuit to dc (in steady state).

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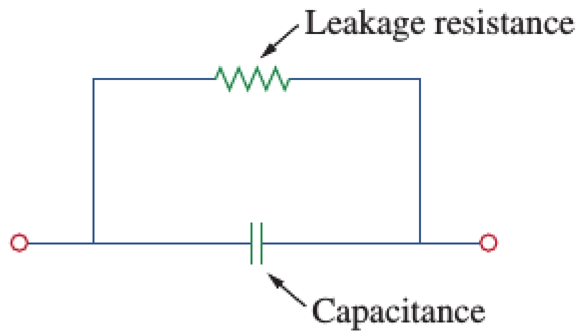
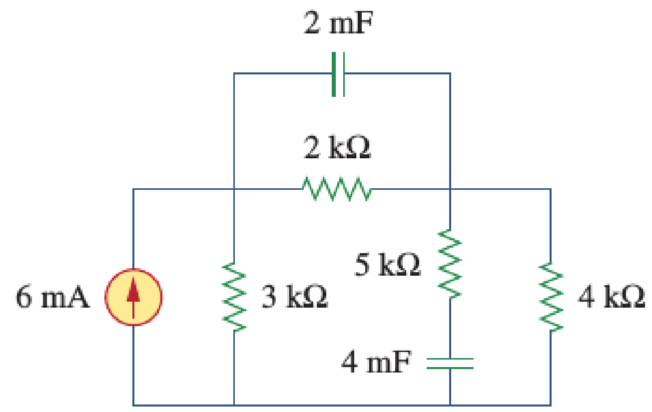
- ❑ Voltage across a capacitor cannot change abruptly.
- ❑ Ideal capacitors does not dissipate energy.
- ❑ Non-ideal capacitor has a leakage resistance.



Example: Capacitor Energy

Calculate energy stored in each capacitor under DC conditions (or in steady state).



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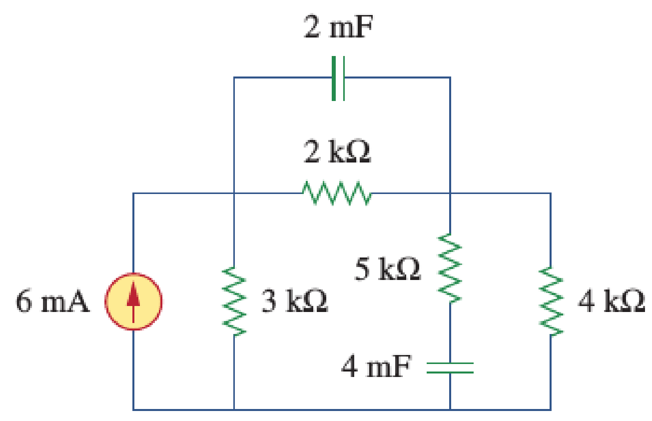

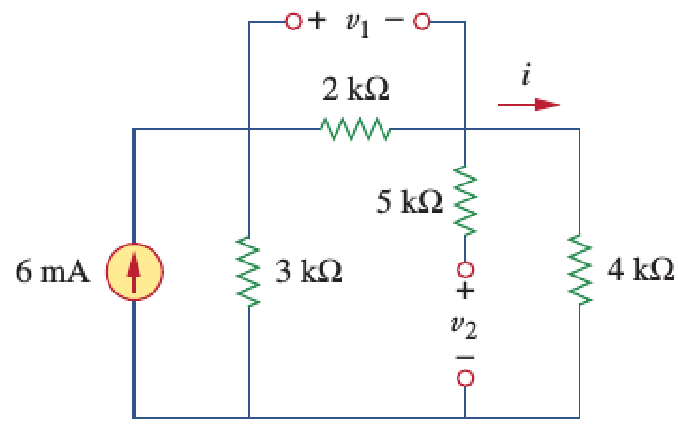

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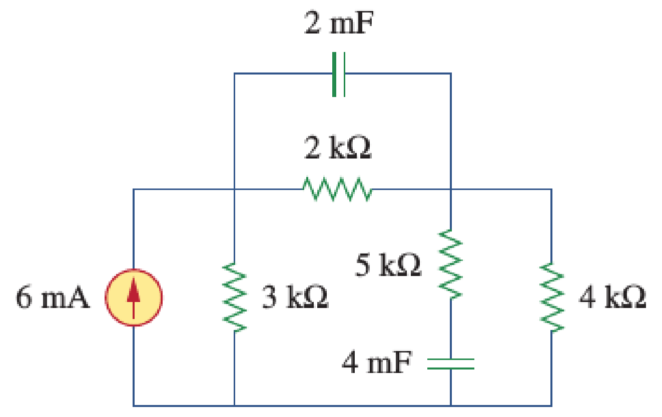
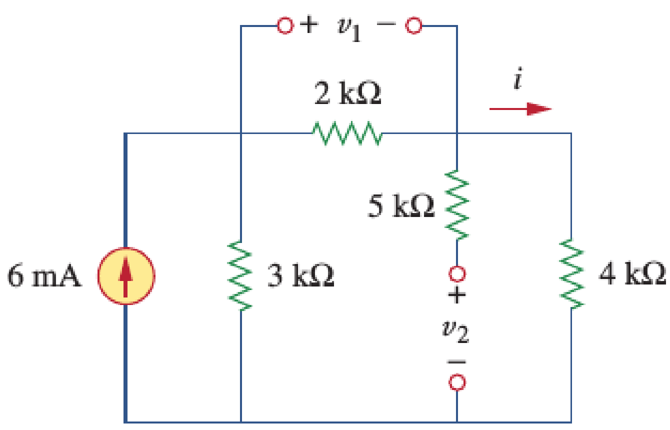
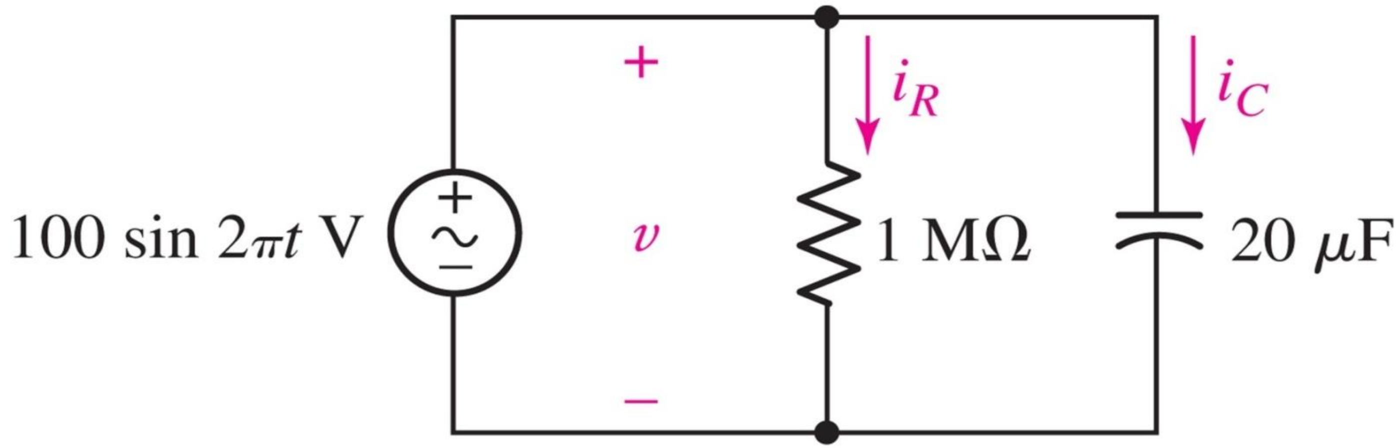




Answer: $W_1 = 16$ mJ ; $W_2 = 128$ mJ

Example: Capacitor Energy

Determine the maximum energy stored in the capacitor, and plot i_R and i_C.



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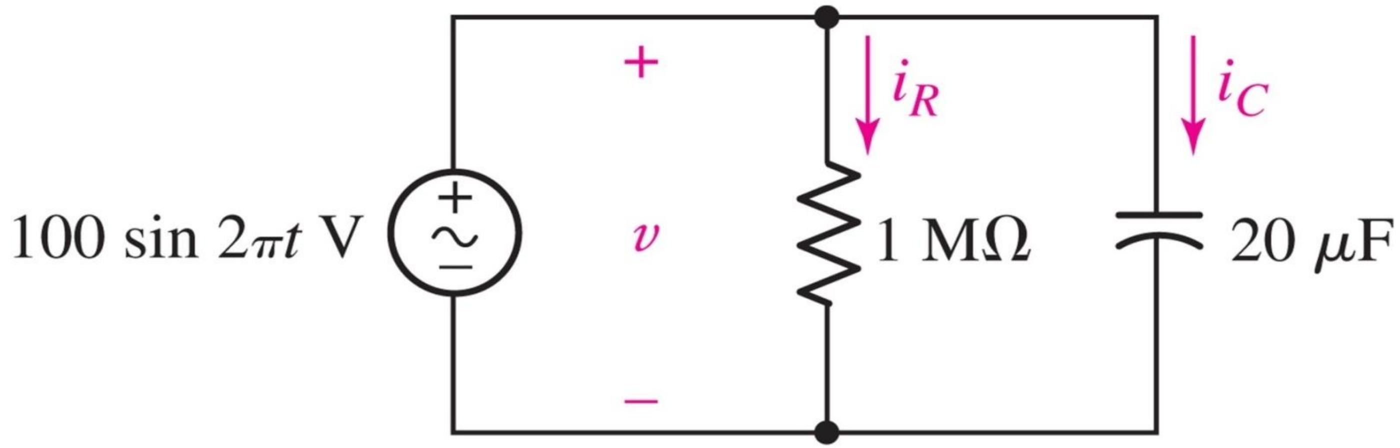
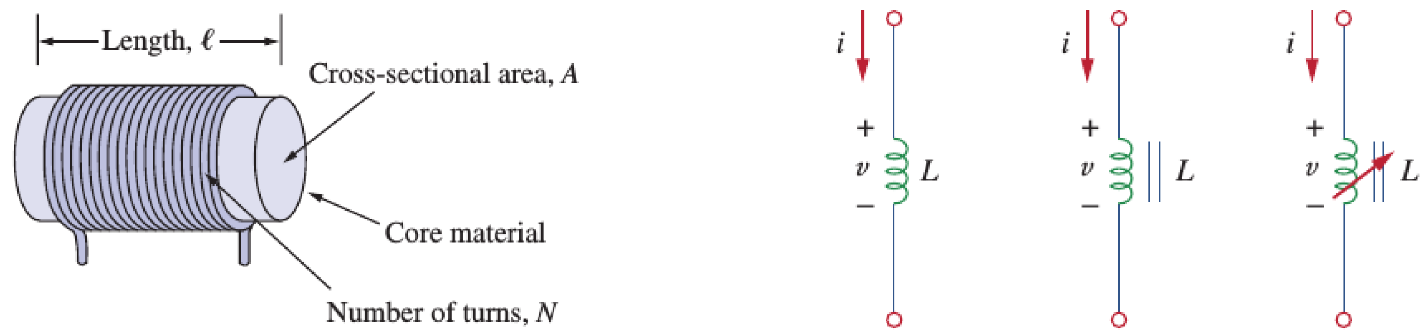


Answer: Maximum energy = 0.1 J

The Inductor

- The ideal inductor is a passive element with circuit symbol



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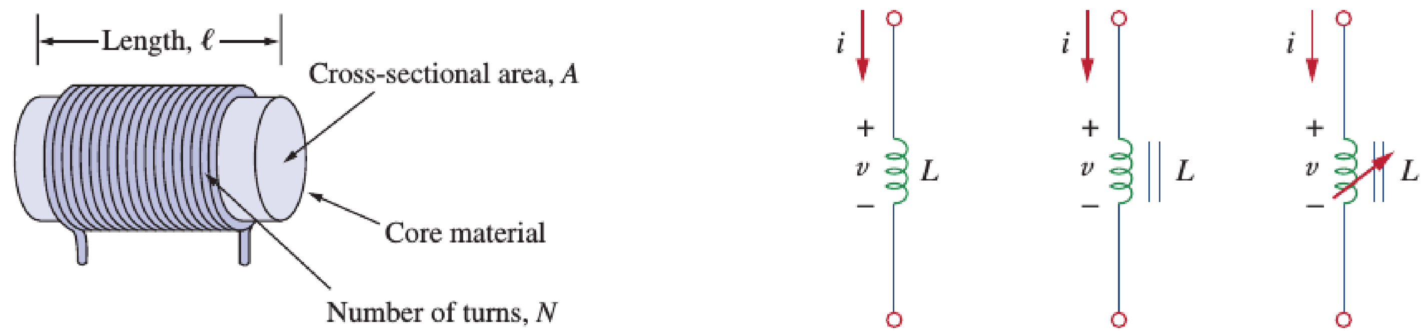

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$$v = L\frac{di}{dt}$$

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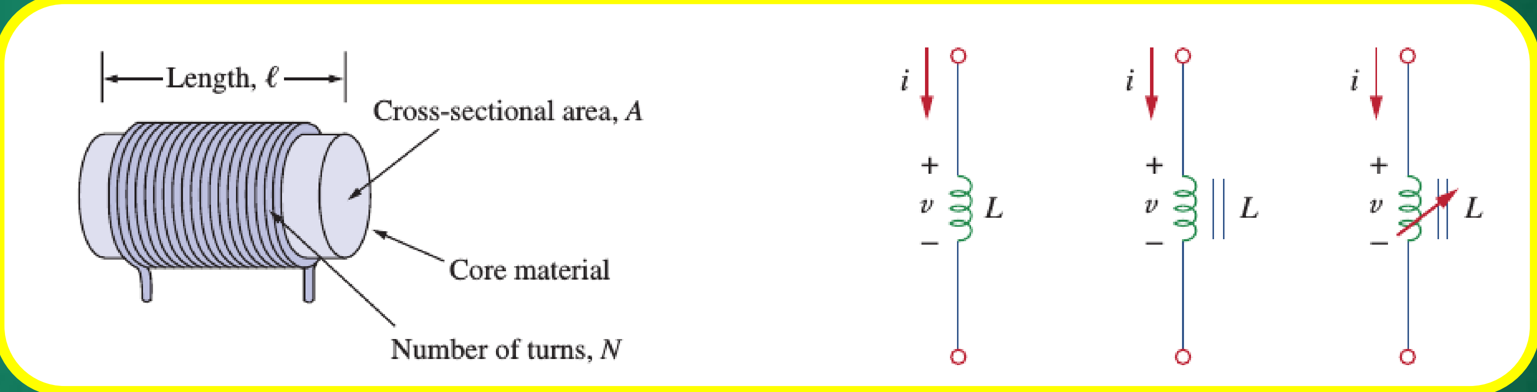

- The current-voltage relation is

$$v = L\frac{di}{dt}$$

- The inductance L is measured in Henry (H)

The Inductor

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$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt$$

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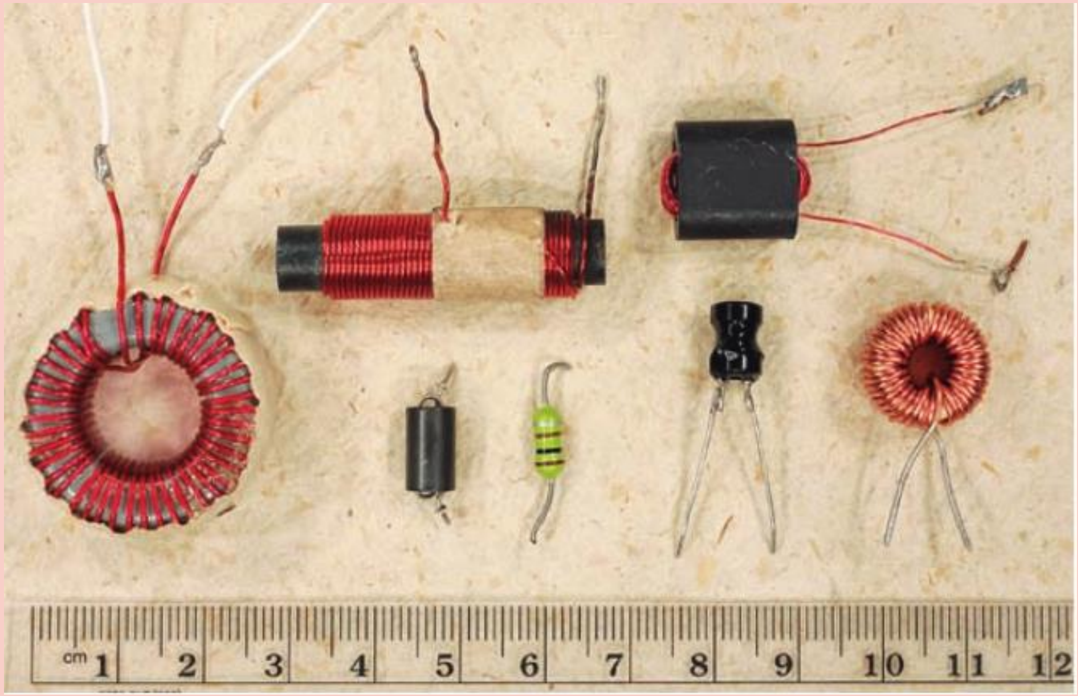
$$\boxed{i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt}$$

- Energy stored in an inductor

$$\boxed{w = \frac{1}{2}Li^2} \quad \text{if} \quad \boxed{i(-\infty) = 0}$$

Some Inductors

Typical values range from μH to H



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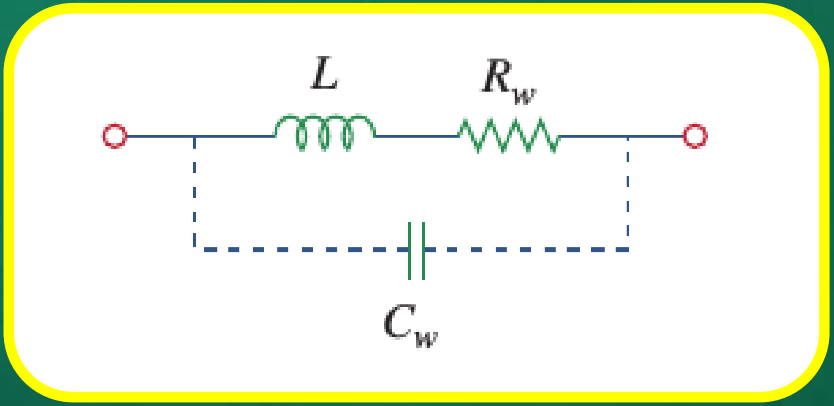
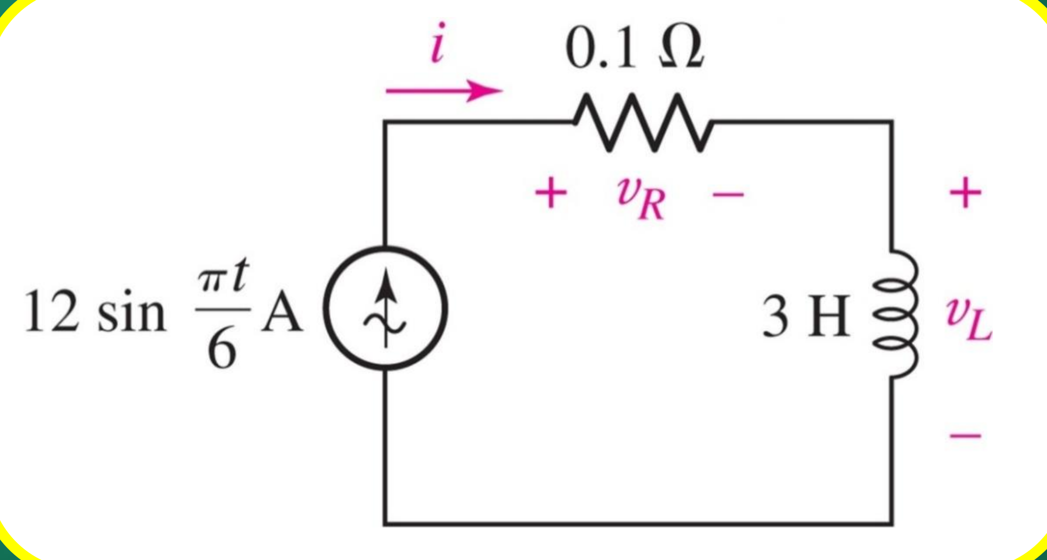
- Current through inductor cannot change abruptly.
- Ideal inductors does not dissipate energy.
- Non-ideal inductor can be modeled as shown.



Example: Energy in L

Determine the maximum energy stored in the inductor, and find the energy lost by resistor from $t = 0$ to $t = 6$ s.



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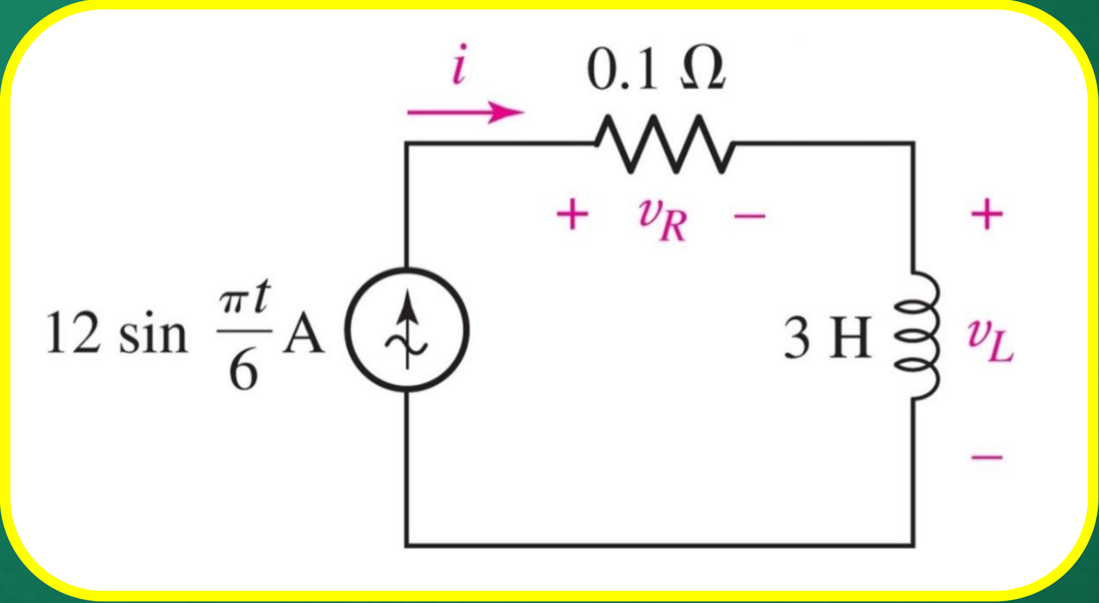
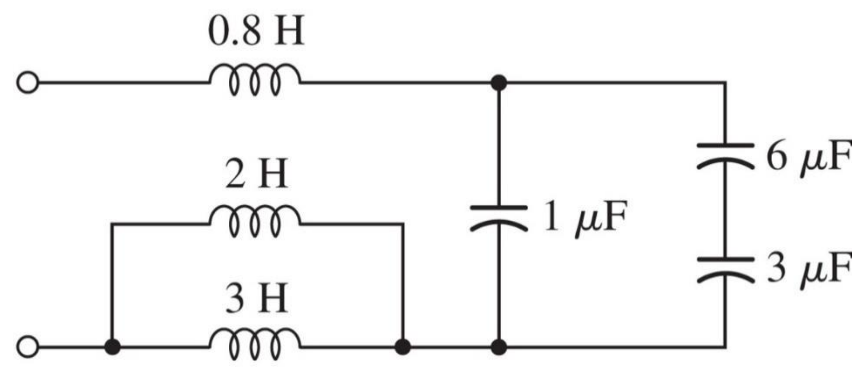


Answer: 216 J, 43.2 J

Example: Simplifying LC Circuit

Simplify the circuit



Example: Simplifying LC Circuit

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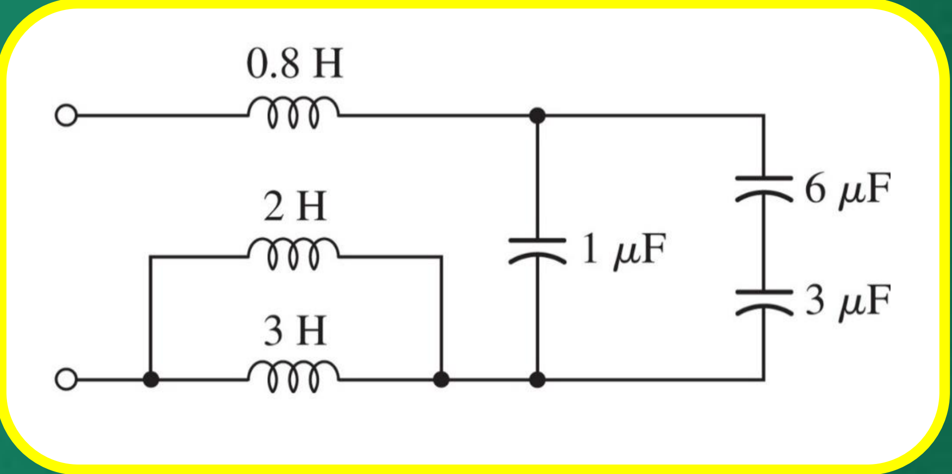
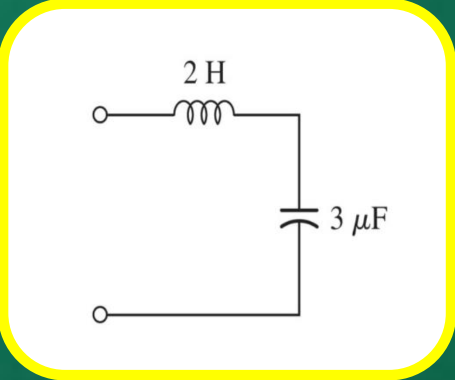
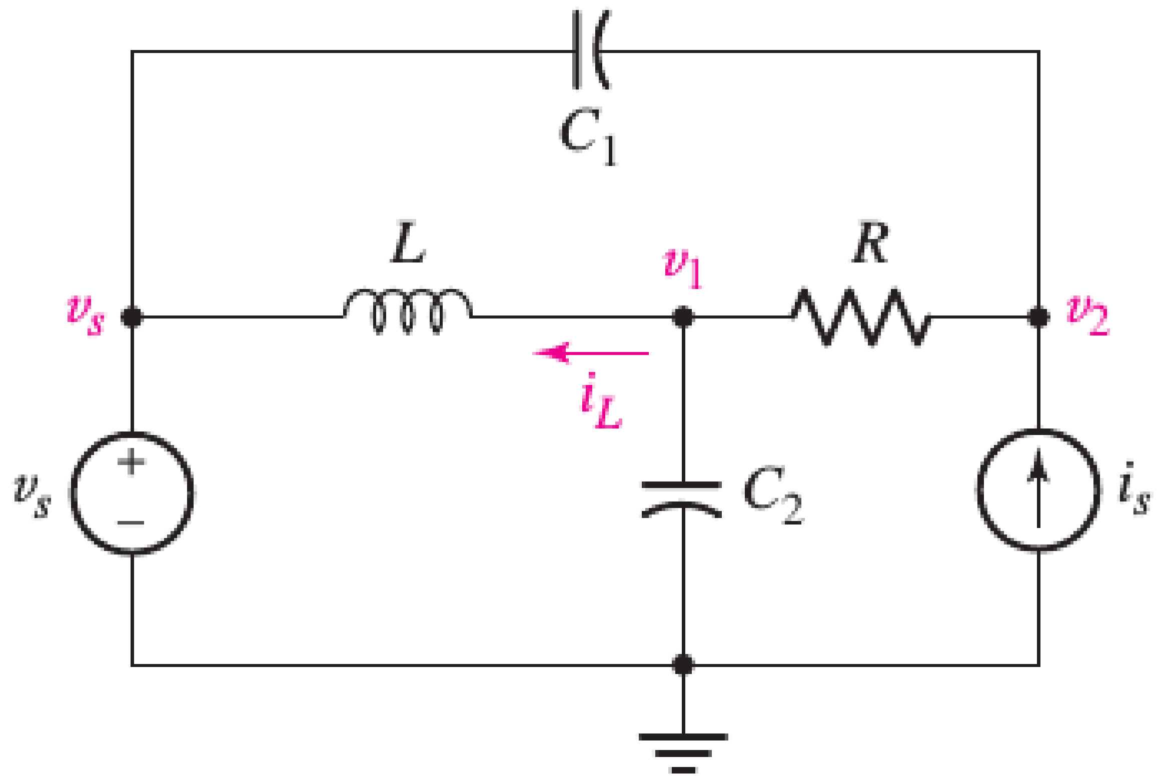




Nodal Equation for an RLC Circuit



Nodal Equation for an RLC Circuit

$$\frac{1}{L}\int_{t_0}^{t}(v_1 - v_s)dt' + i_L(t_0) + \frac{v_1 - v_2}{R} + C_2\frac{dv_1}{dt} = 0$$

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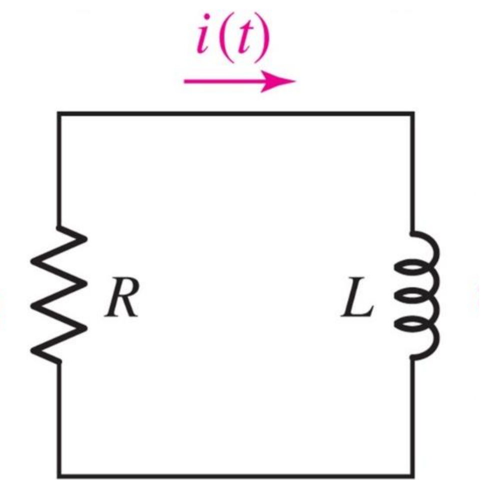
$$\frac{v_1}{R} + C_2\frac{dv_1}{dt} + \frac{1}{L}\int_{t_0}^{t} v_1 dt' - \frac{v_2}{R} = \frac{1}{L}\int_{t_0}^{t} v_s dt' - i_L(t_0)$$

$$-\frac{v_1}{R} + \frac{v_2}{R} + C_1\frac{dv_2}{dt} = C_1\frac{dv_s}{dt} + i_s$$

Source Free RL and RC Circuits

The Source-Free RL Circuit



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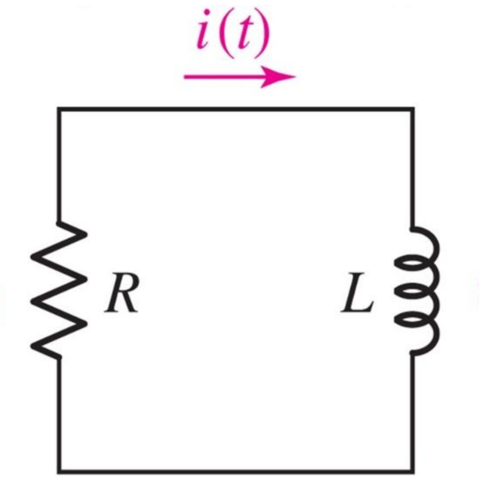

Applying KVL

$$\frac{di}{dt}+\frac{R}{L}i=0$$

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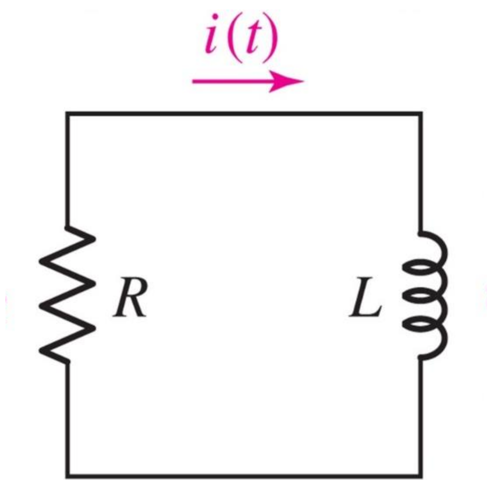

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We can solve for the natural response if we know the initial condition $i(0)=I_0$

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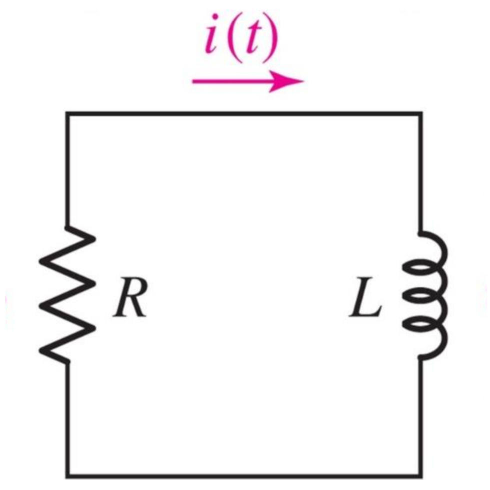

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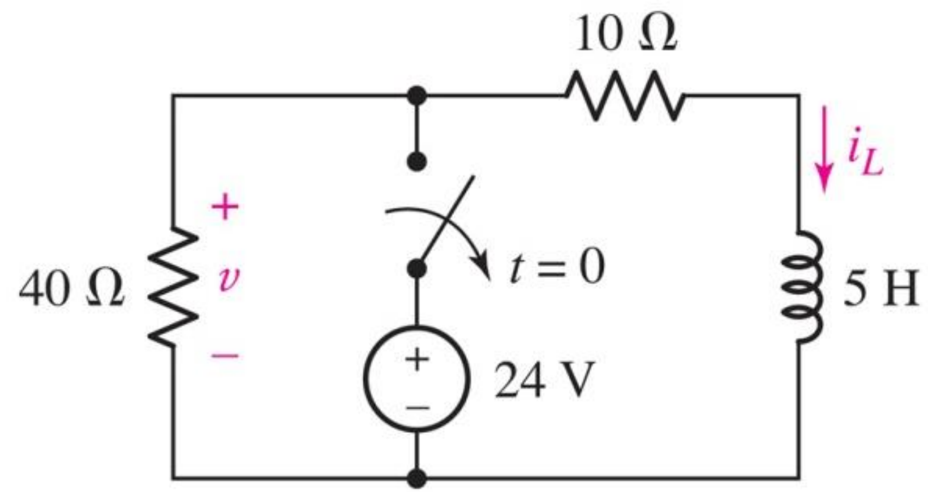
$$i(t)=I_0e^{-Rt/L} \quad \text{for} \quad t\geq 0$$

Example: RL with a Switch



Show that the voltage $v(t)$ **will be** -12.99 V **at** t = 200 ms **(Assume that switch is in closed position since very long time before it is open)**

The Exponential Response

$$i(t)=I_0e^{-t/\tau} \text{ for } t \geq 0$$

$$\tau = L/R$$

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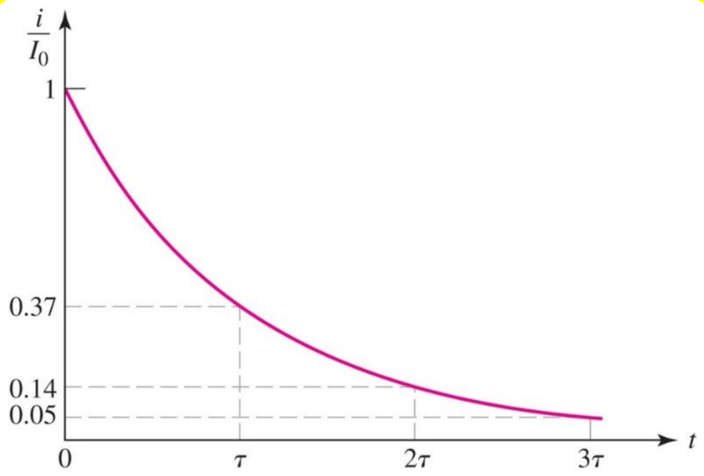

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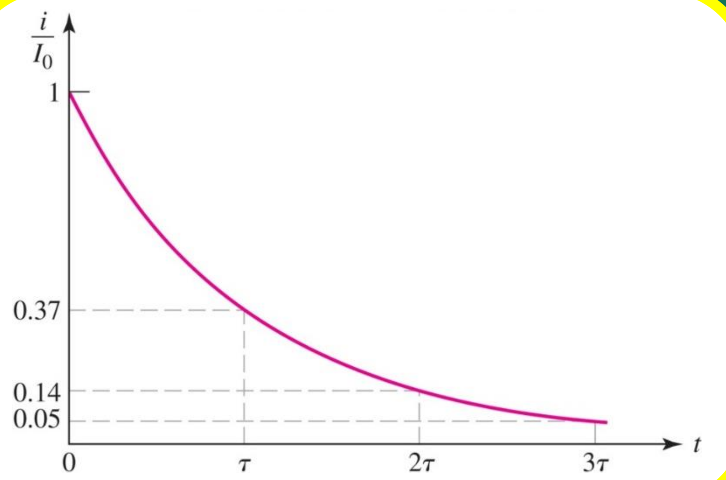
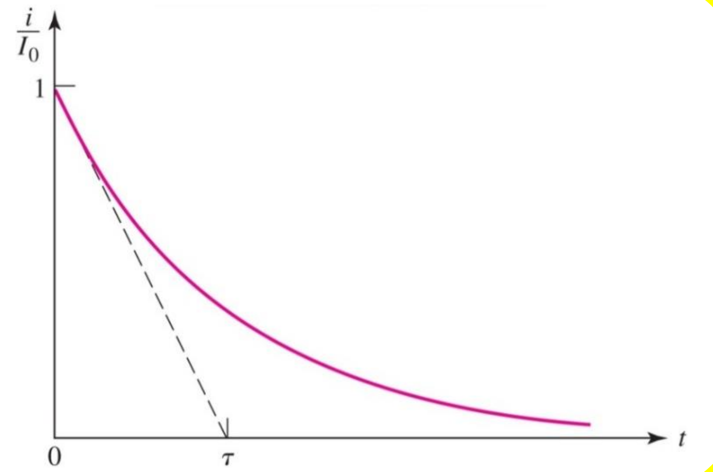
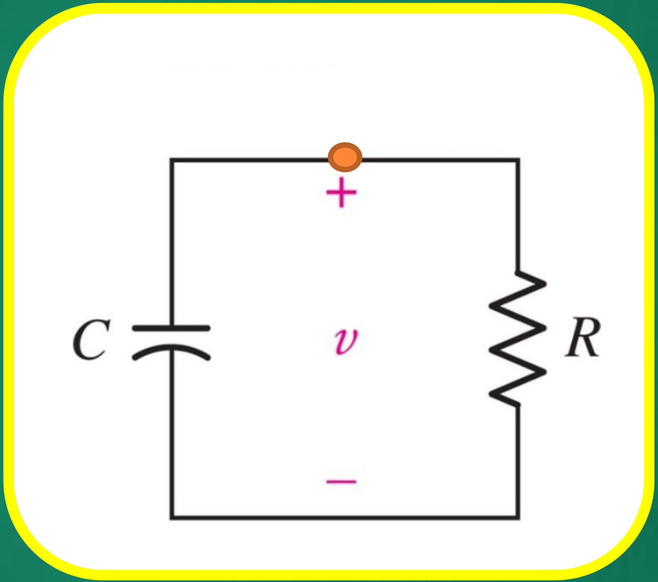




The Source-Free RC Circuit



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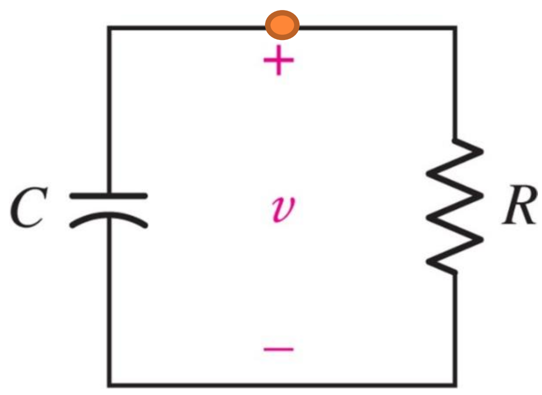

Applying KCL

$$\frac{dv}{dt}+\frac{1}{RC}v=0$$

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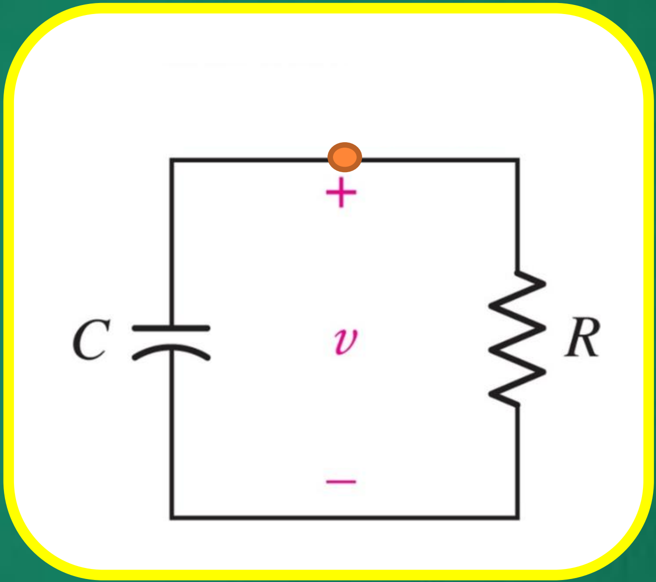

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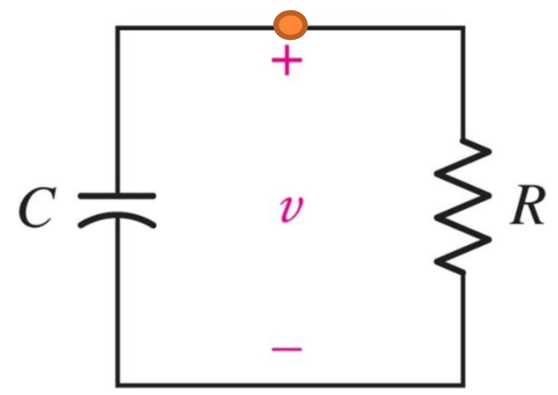

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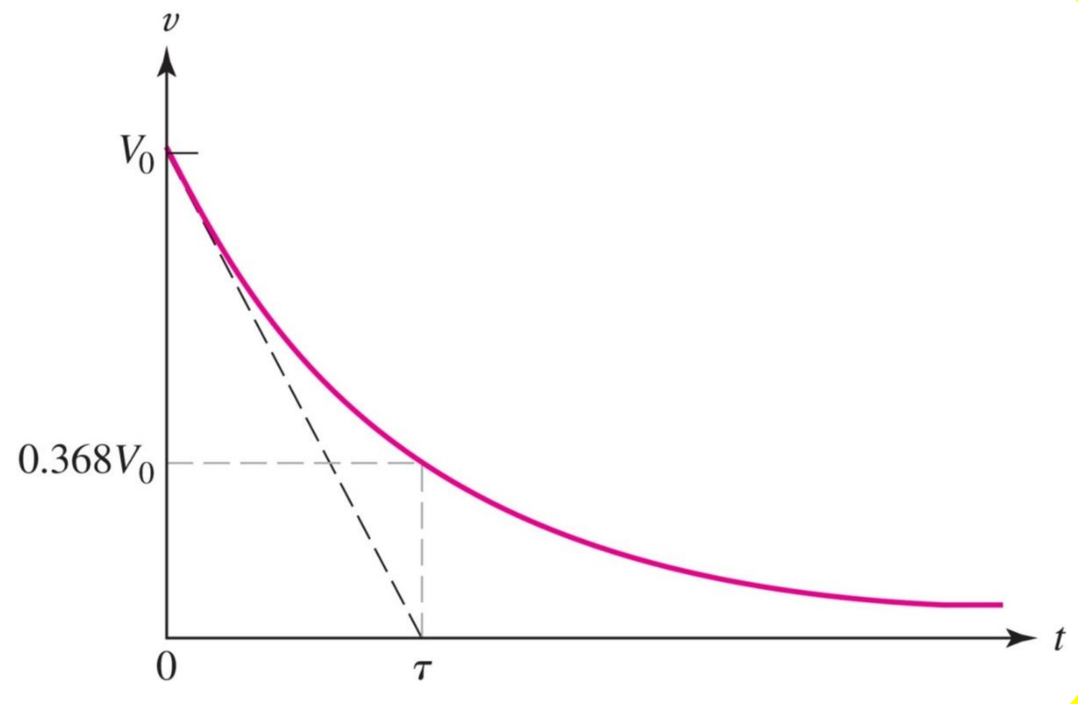
$$v(t)=V_0 e^{-t/RC} \quad \text{for} \quad t \geq 0$$

RC Natural Response



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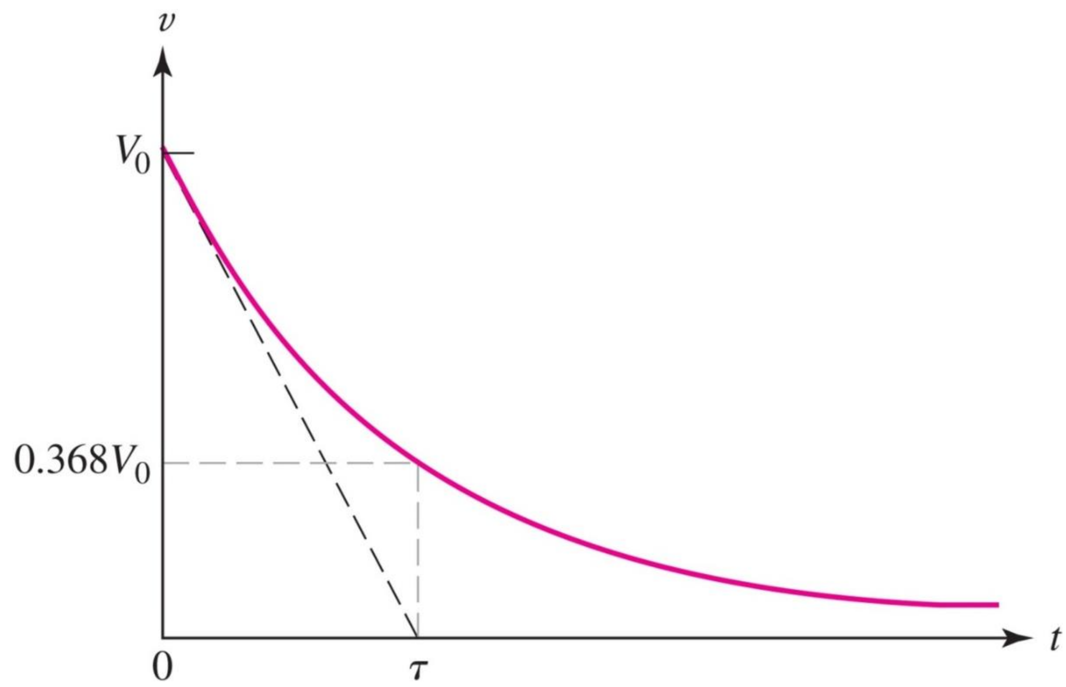
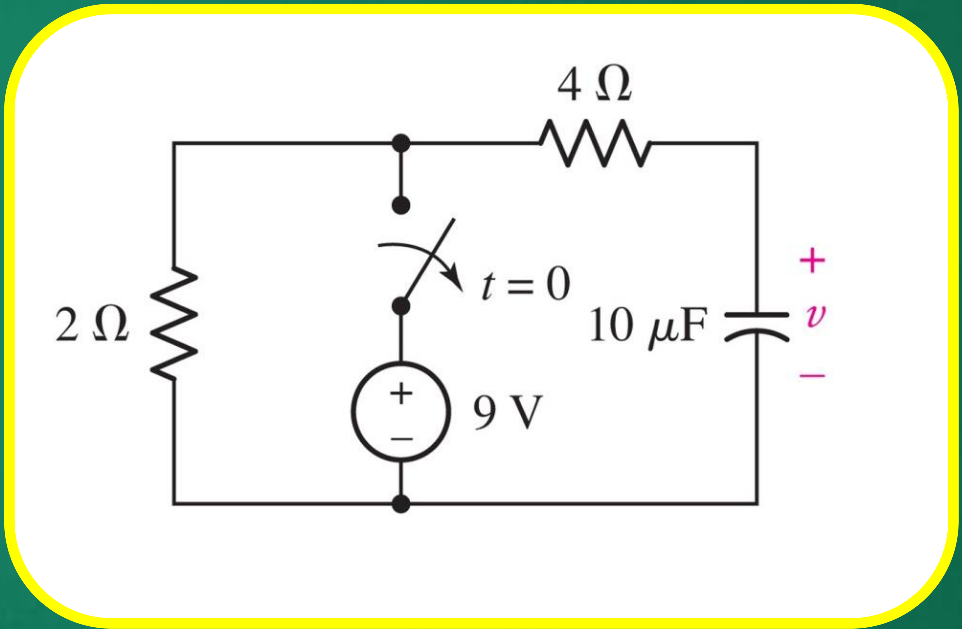


The time constant is $\tau = RC$

Example: Source Free RC Circuit



Show that the voltage $v(t)$ is 321 mV at $t = 200$ μs.

(Assume that switch is in closed position since very long time before it is open)