Solutions to Problem Sheet - 2 IEC102

Q1. Use nodal analysis to find I_o in the circuit shown in Fig. Q1

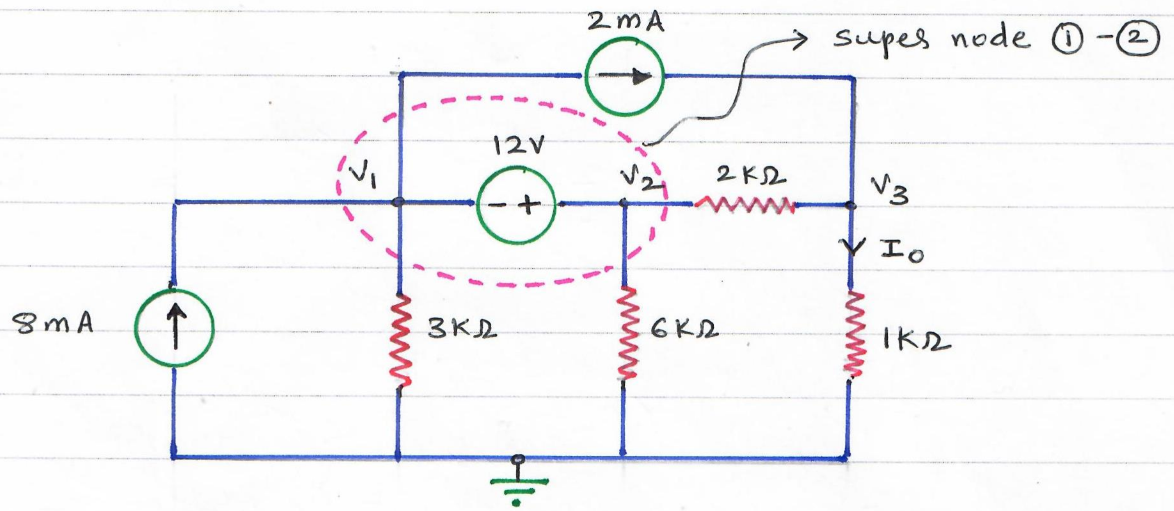

Fig. Q1

Nodal equation at super node ① - ②

$$-8\times10^{-3} + 2\times10^{-3} + \frac{V_1}{3K} + \frac{V_2}{6K} + \frac{V_2 - V_3}{2K} = 0$$

$$\Rightarrow \frac{V_1}{3K} + V_2\left(\frac{1}{6K} + \frac{1}{2K}\right) - \frac{V_3}{2K} = 6\times10^{-3}$$

$$\Rightarrow \frac{V_1}{3K} + \frac{2}{3K}V_2 - \frac{V_3}{2K} = 6\times10^{-3} \quad \ldots (A)$$

Also $V_2 - V_1 = 12V$ & $V_1 = -12 + V_2$ ... (B)

Substiting the value of V_2 from (B) in (A)

$$\frac{V_2 - 12}{3K} + \frac{2}{3K}V_2 - \frac{V_3}{2K} = 6\times10^{-3}$$

$$\Rightarrow V_2\left(\frac{1}{3K} + \frac{2}{3K}\right) - \frac{V_3}{2K} = 10\times10^{-3}$$

$$\Rightarrow \frac{V_2}{1K} - \frac{V_3}{2K} = 10\times10^{-3} \Rightarrow \boxed{2V_2 - V_3 = 20} \quad \ldots (C)$$

Applying KCL at node -3

$$\frac{V_3 - V_2}{2K} + \frac{V_3}{1K} - 2\times10^{-3} = 0$$

$$\Rightarrow -\frac{V_2}{2K} + V_3\left(\frac{1}{2K} + \frac{1}{1K}\right) = 2\times10^{-3}$$

$$\Rightarrow \boxed{-V_2 + 3V_3 = 4} \quad \ldots (D)$$

$I_o = V_3/1K$

From eqn (C) and (D)

$$V_3 = 28/5\,V$$

$$\therefore I_o = \frac{28/5}{10^3} = \frac{28}{5}\,mA$$

$$\Rightarrow \boxed{I_o = 5.6\,mA}$$

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Q2 Using mesh analysis, calculate I_o in the circuit shown in Fig. Q2

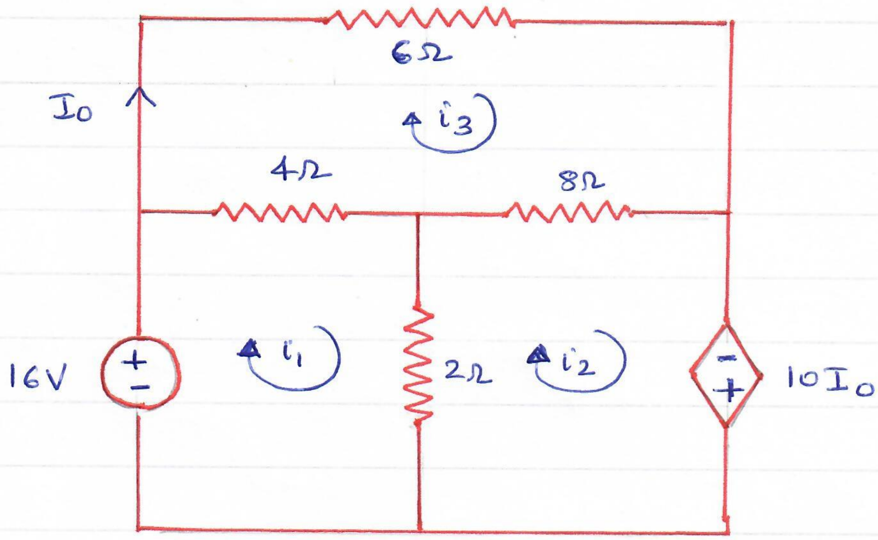


Fig. Q2

Sol.

$$I_o = i_3$$

Applying KVL around loop 1

$$-16 + 4(i_1 - i_3) + 2(i_1 - i_2) = 0$$

$$\Rightarrow \boxed{6 i_1 - 2 i_2 - 4 i_3 = 16} \quad \ldots (A)$$

Applying KVL around loop 2

$$2(i_2 - i_1) + 8(i_2 - i_3) - 10 I_o = 0$$

but $I_o = i_3$

$$\therefore \quad 2(i_2 - i_1) + 8(i_2 - i_3) - 10 i_3 = 0$$

$$\Rightarrow \boxed{-2 i_1 + 10 i_2 - 18 i_3 = 0} \quad \ldots (B)$$

Applying KVL around loop 3

$$6(i_3) + 8(i_3 - i_2) + 4(i_3 - i_1) = 0$$

$$\Rightarrow \boxed{-4 i_1 - 8 i_2 + 18 i_3 = 0} \quad \ldots (C)$$

Writing ~~e~~eqns (A), (B), and (C) in matrix form

$$\begin{bmatrix} 6 & -2 & -4 \\ -2 & 10 & -18 \\ -4 & -8 & 18 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \begin{bmatrix} 16 \\ 0 \\ 0 \end{bmatrix}$$

Solving the above simultaneous eqns.

$$\boxed{i_3 = I_o = -4A}$$

(Q3) Use superposition to find v_x in the circuit shown in Fig. Q3

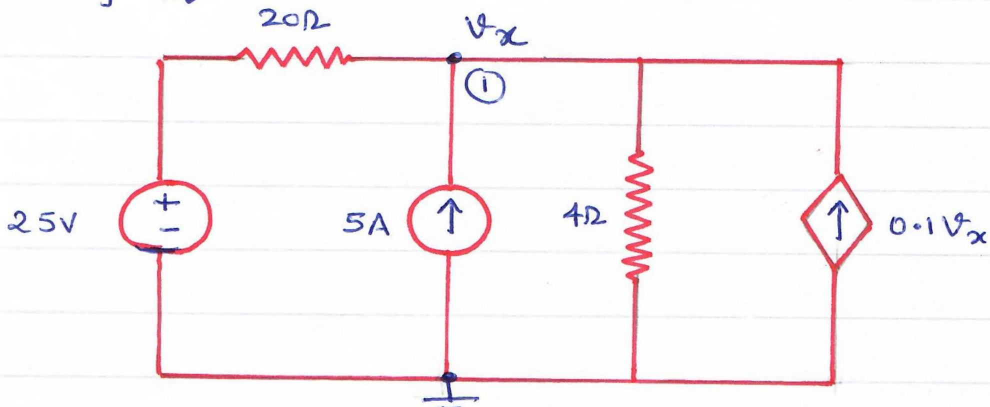
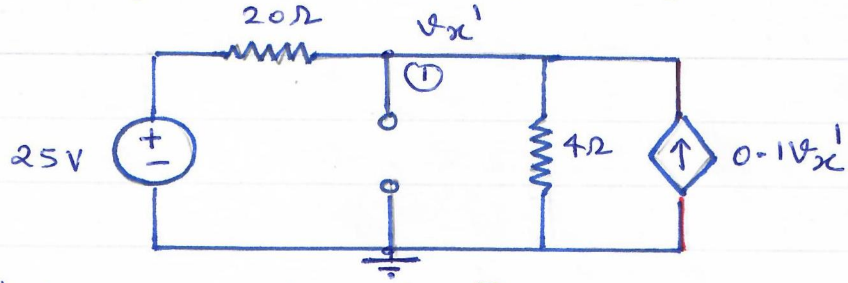


Fig. Q3

Sol.

Let node 1 voltage be v_x' due to voltage source alone. (open circuit the current source)



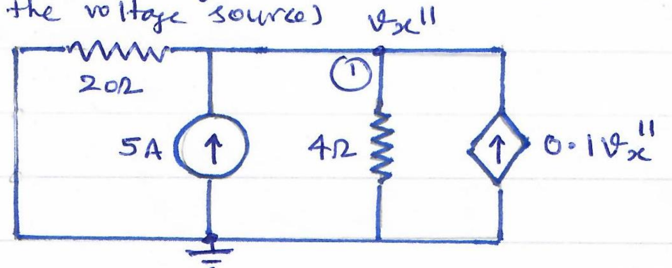
Applying KCL at node 1

$$\frac{v_x' - 25}{20} + \frac{v_x'}{4} - 0.1v_x' = 0$$

$$\Rightarrow \quad \frac{v_x'}{20} + \frac{v_x'}{4} - \frac{v_x'}{10} = \frac{25}{20}$$

$$\Rightarrow \quad \boxed{v_x' = \frac{25}{4} = 6.25\,V}$$

Let node 1 Voltage be v_x'' due to the current source alone (short circuit the voltage source)



Applying KCL at node 1

$$\frac{v_x''}{20} - 5 + \frac{v_x''}{4} - 0.1v_x'' = 0$$

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$$\Rightarrow v_x''\left(\frac{1}{20}+\frac{1}{4}-\frac{1}{10}\right)=5$$

$$\Rightarrow \boxed{v_x''=25}$$

The voltage at node ① when both sources are present is

$$v_x = v_x' + v_x'' = 6.25 + 25 = 31.25\,V$$

Q4 Calculate v_x using source transformation (Fig. Q4)

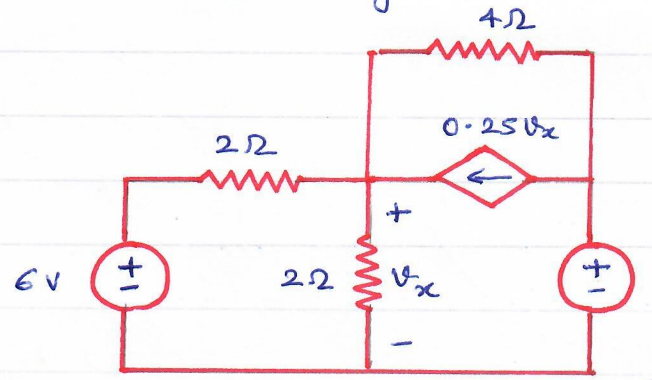
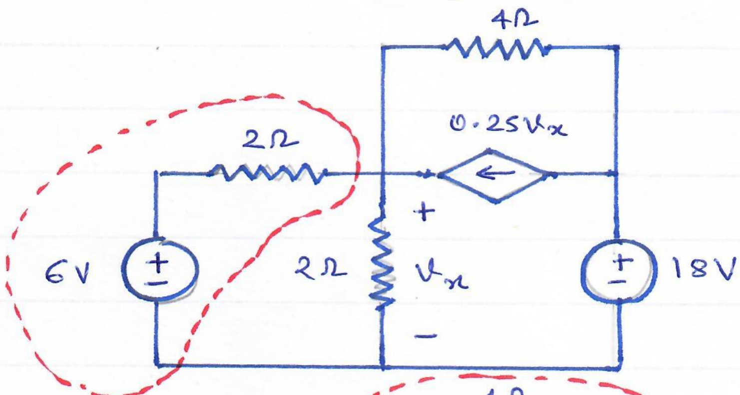
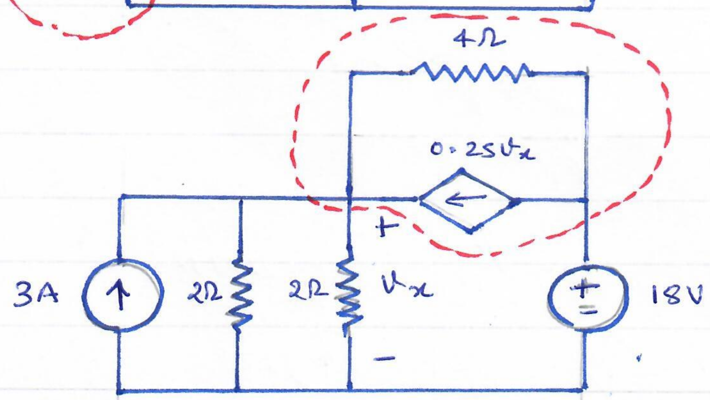


Fig. Q4

Sol.





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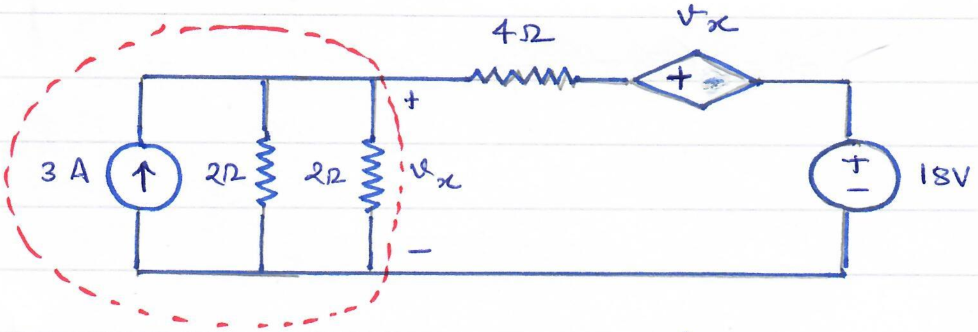
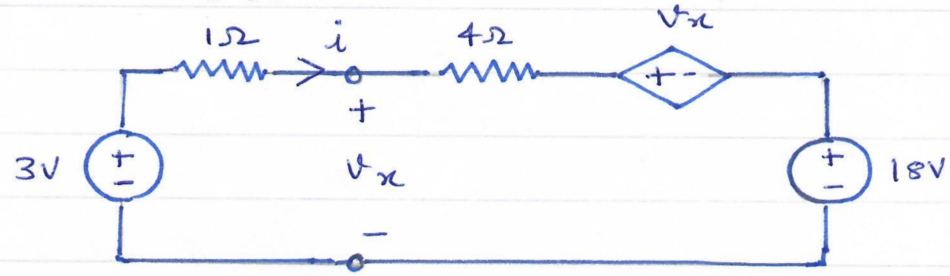


Transformed voltage source $= 0.25v_x \times 4 = v_x$



Apply KVL around the loop

$$-3 + 1\times i + 4i + v_x + 18 = 0$$

$$\Rightarrow -3 + 5i + v_x + 18 = 0 \cdots (A)$$

but $v_x = 3 - 1\times i$

$$\Rightarrow i = -v_x + 3$$

substituting $i = 3 - v_x$ in (A)

$$-3 + 5(3 - v_x) + v_x + 18 = 0$$

$$\Rightarrow \boxed{v_x = \frac{30}{4} = 7.5\,V}$$

Q5 Using Thevenin's theorem, find the equivalent circuit to the left of the terminals (a-b) in the circuit of Fig. Q5. Then find I.

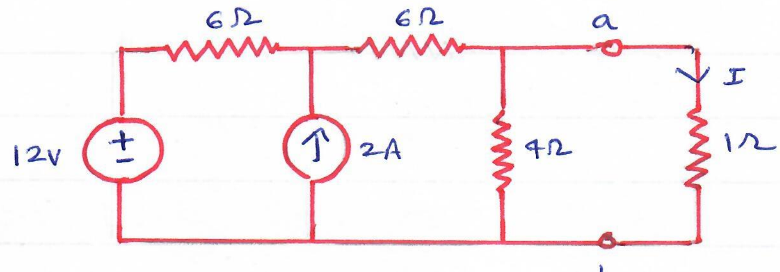


Fig. Q5

Sol, To find the V_{th}, open ~~the~~ circuit the terminals a-b (remove 1Ω resistor), and find the open circuit voltage across a-b.

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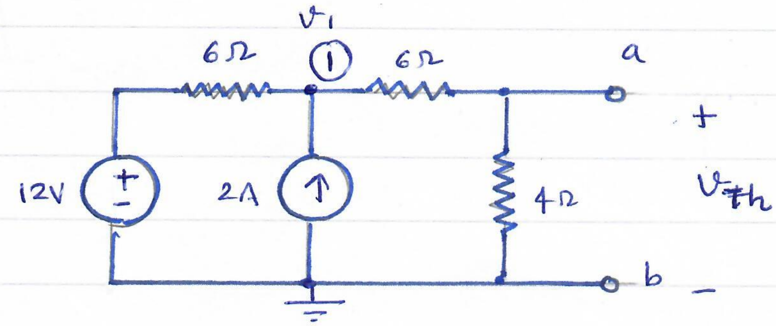
Sol.



Applying KCL at node ①

$$\frac{v_1 - 12}{6} - 2 + \frac{v_1}{10} = 0$$

$$\Rightarrow \quad \frac{v_1}{6} + \frac{v_1}{10} = 4$$

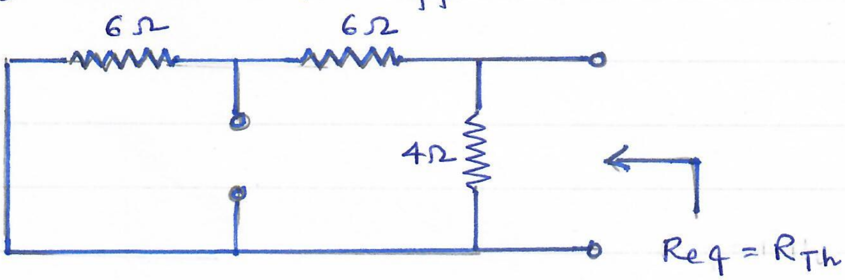
$$\Rightarrow \quad v_1 = 15\,V$$

$$V_{Th} = \frac{v_1 \times 4}{4+6} \quad \text{(voltage across } 4\Omega \text{ resistor)}$$

$$= \frac{4}{10} \times 15 = 6\,V$$

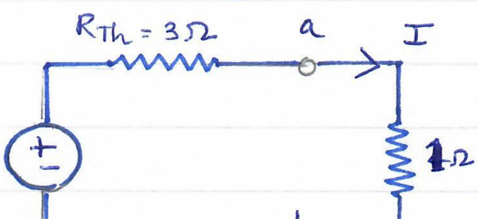
$$\boxed{V_{Th} = 6V}$$

R_{Th} is the equivalent resistance across a-b with all the independent sources turned off.



$$\boxed{R_{Th} = 12 \| 4 = \frac{12 \times 4}{16} = 3\Omega}$$

The Thevenin equivalent circuit is



$I = 6/(3+1) = 6/4 = 1.5\,A$