Electrical Science - I
(IEC-102)

Lecture-03

Nodal & Mesh Analysis

Nodal and Mesh Analysis

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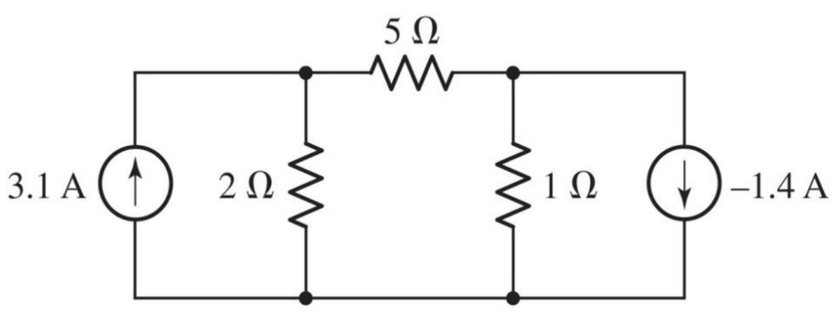
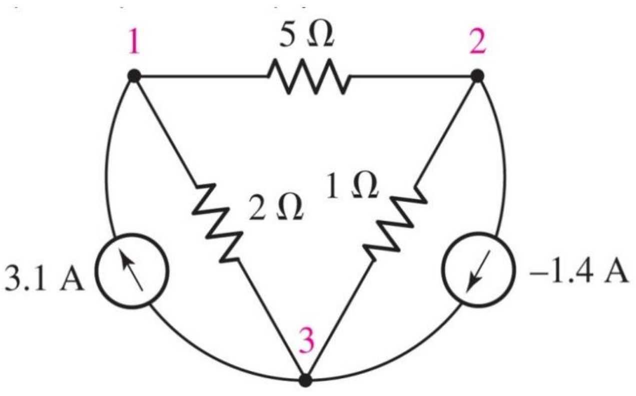
- Nodal analysis assigns voltages to each node, and then we apply KCL.

- Mesh analysis assigns currents to each mesh, and then we apply KVL.

The Nodal Analysis Method

- ❑ Identify the nodes in the given circuit.
- ❑ Choose one node among them as a reference.
- ❑ Assign voltages to every other node relative to a reference node.



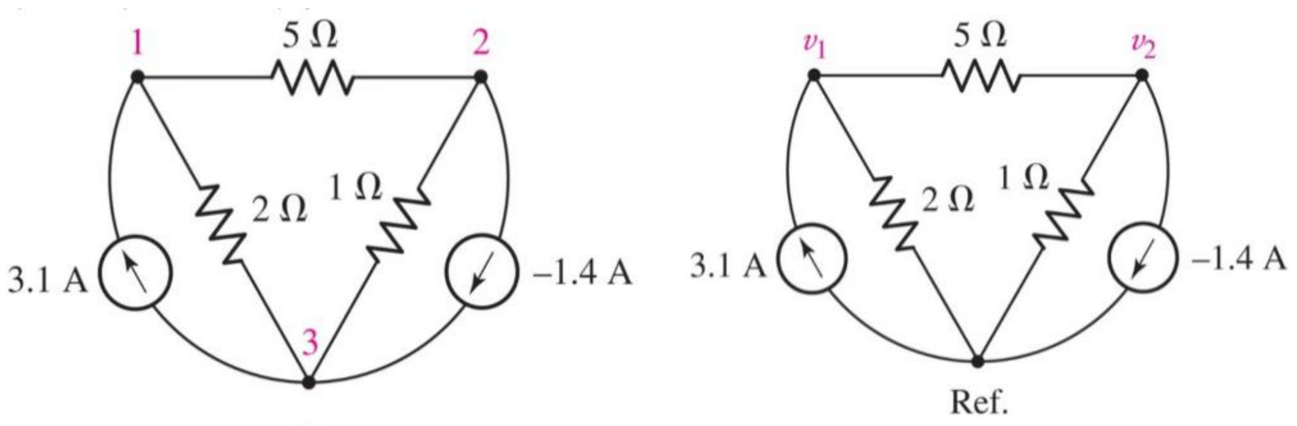


In the circuit above, there are three nodes.

Choosing the Reference Node

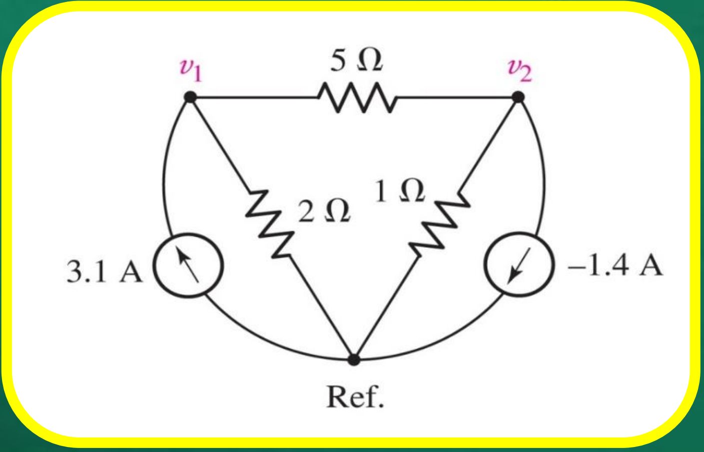
- The bottom node, or
- the ground connection, if there is one, or
- a node with many connections

Assign voltages to other nodes relative to reference.



Apply KCL to Find Voltages

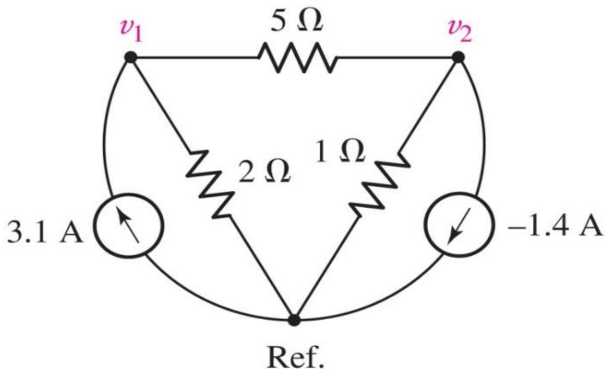
Apply KCL to node 1 (Σout = 0) and Ohm's law to each resistor.



Apply KCL to Find Voltages

Apply KCL to node 1 (Σout = 0) and Ohm's law to each resistor.

$$-3.1+\frac{v_1}{2}+\frac{v_1-v_2}{5}=0$$



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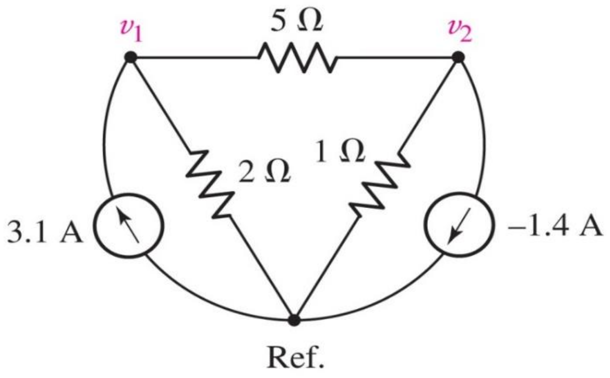
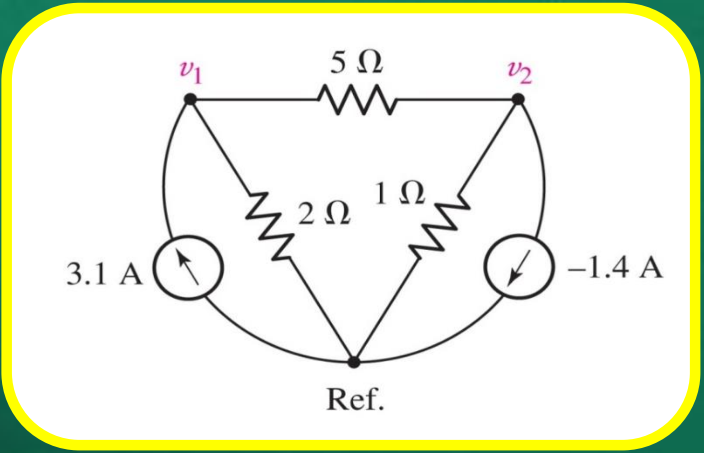


Note: **the current flowing out of node 1 through the 5 Ω resistor is** $(v_1\text{-}v_2)/5$

Apply KCL to Find Voltages

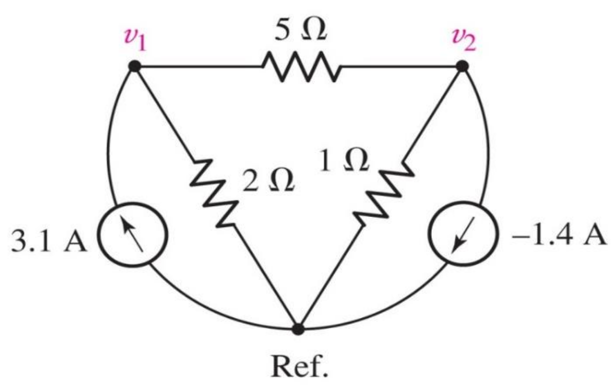
Apply KCL to node 2 (Σout $= 0$) and Ohm's law to each resistor.



Apply KCL to Find Voltages

Apply KCL to node 2 (Σout = 0) and Ohm's law to each resistor.

$$\frac{v_2}{1}+\frac{v_2-v_1}{5}-1.4=0$$



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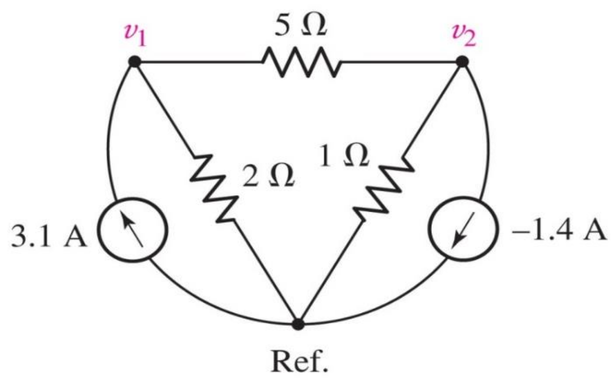


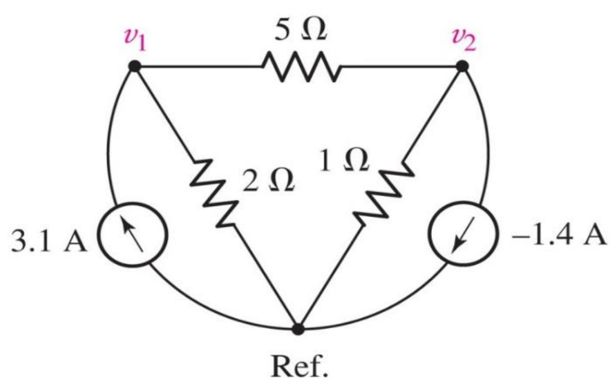
Note: the current flowing out of node 2 through the 5 Ω resistor is $(v_2-v_1)/5$

Solve for Node Voltages

$$-3.1+\frac{v_1}{2}+\frac{v_1-v_2}{5}=0$$

$$\frac{v_2}{1}+\frac{v_2-v_1}{5}-1.4=0$$

We now have two equations for the two unknowns v_1 and v_2 and can be solved.



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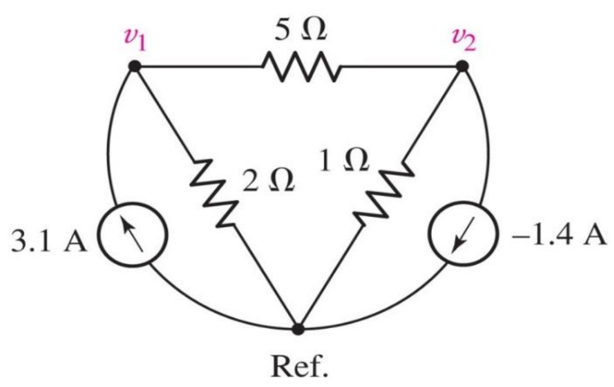
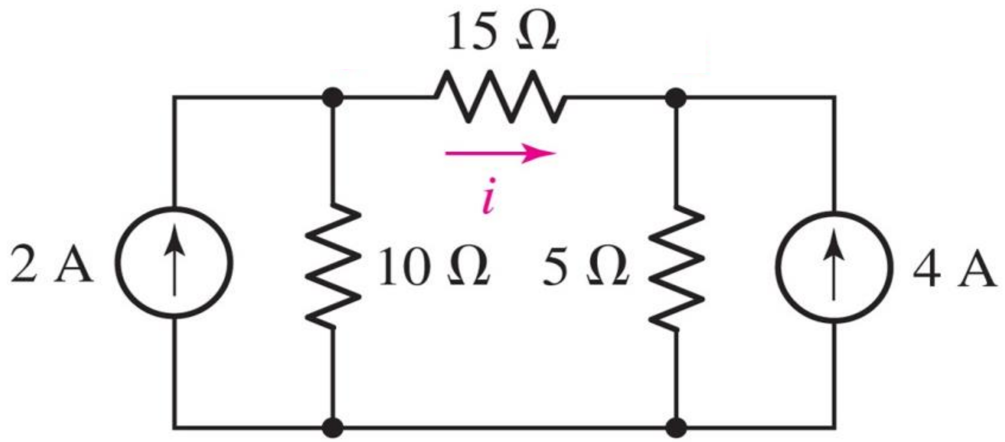


$v_1 = 5$ V and $v_2 = 2$ V

Example: Nodal Analysis

Find the current i in the circuit.



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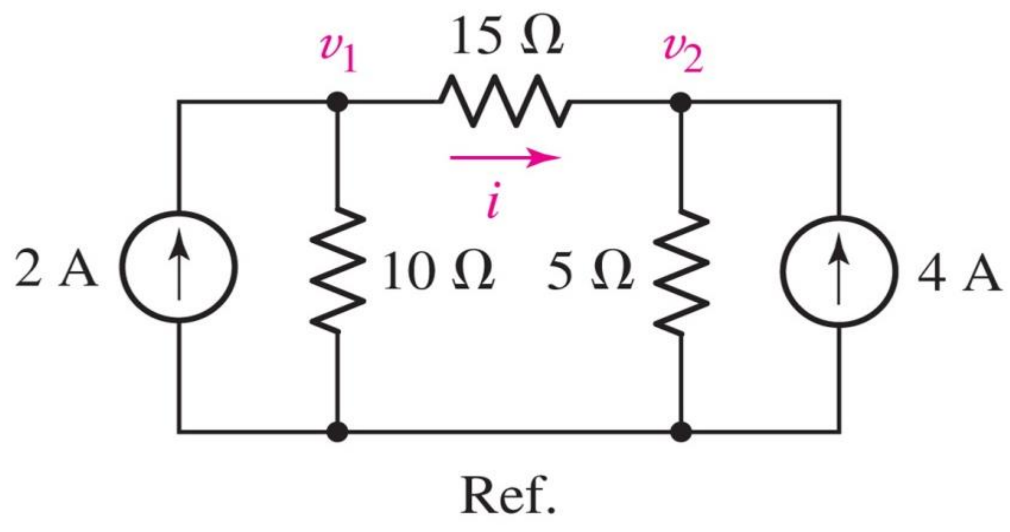

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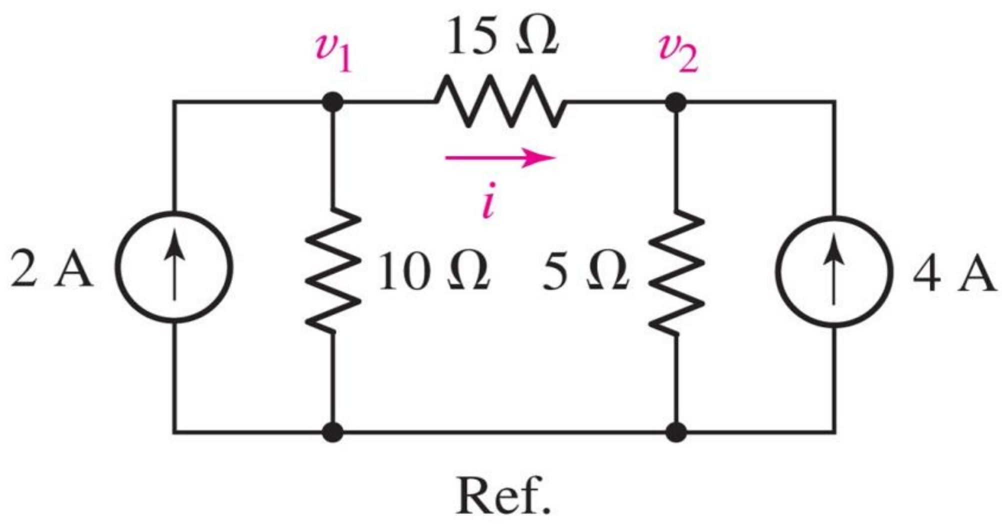
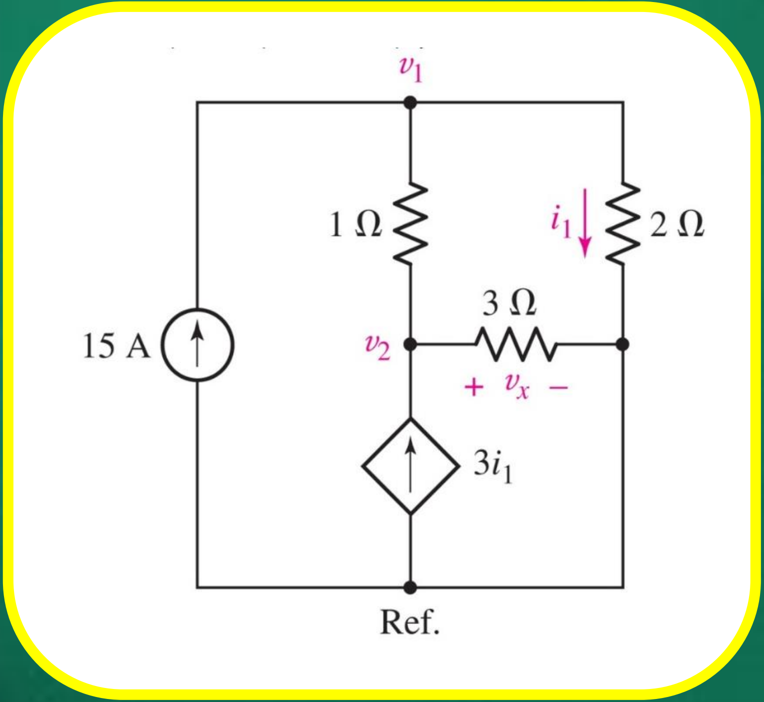


Answer: $i = 0$ (since $v_1 = v_2 = 20$ V)

Nodal Analysis: Dependent Source Example

Determine the power supplied by the dependent source.



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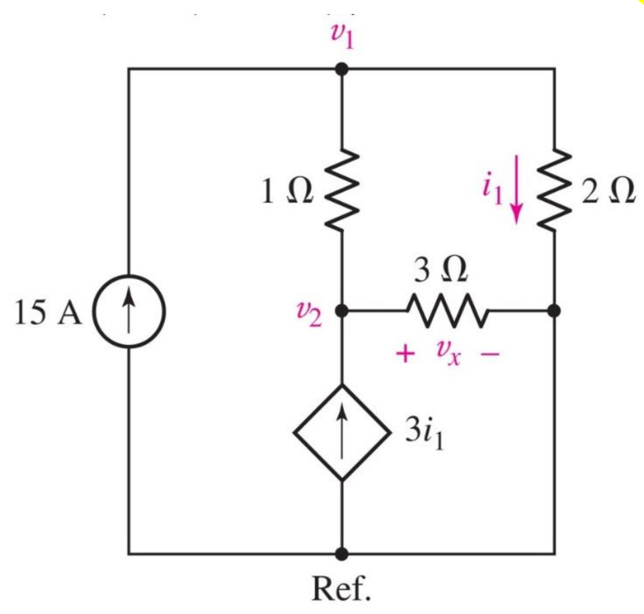

Key step: **eliminate i_1 from the equations using $v_1 = 2i_1$**

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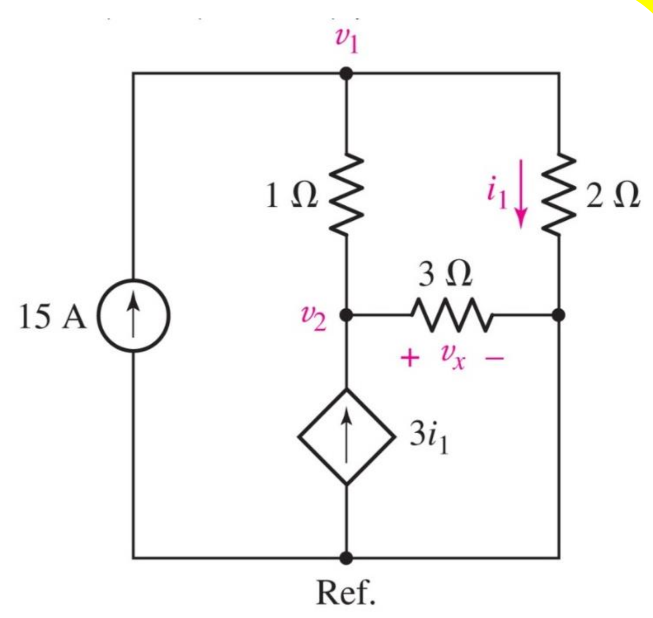
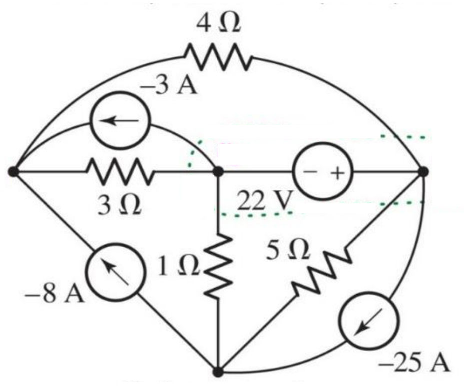


Key step: eliminate i_1 from the equations using $v_1 = 2i_1$

Answer: 4.5 kW

Voltage Sources and the Super node

What is the current through a voltage source connected between nodes?



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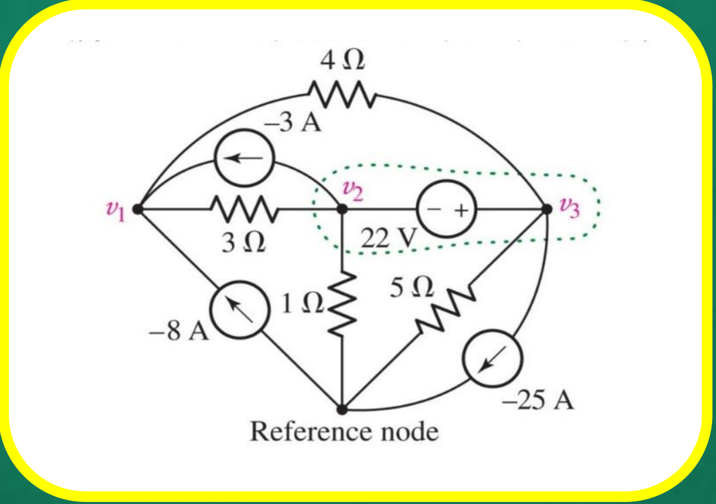

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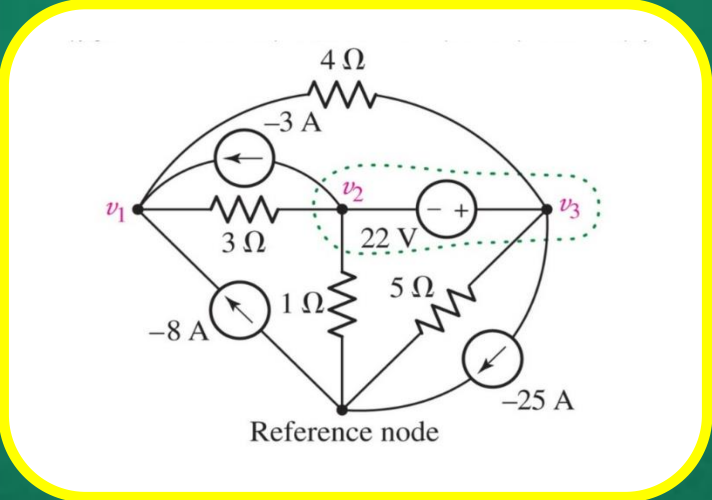

We can eliminate the need for introducing a current variable by applying KCL to the super node.

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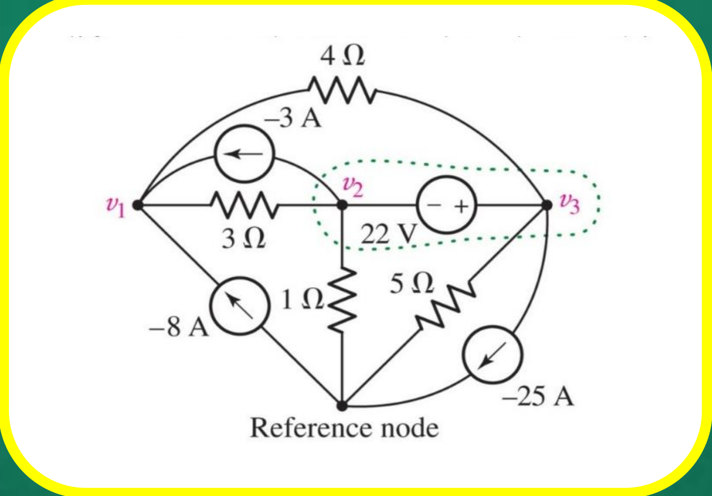

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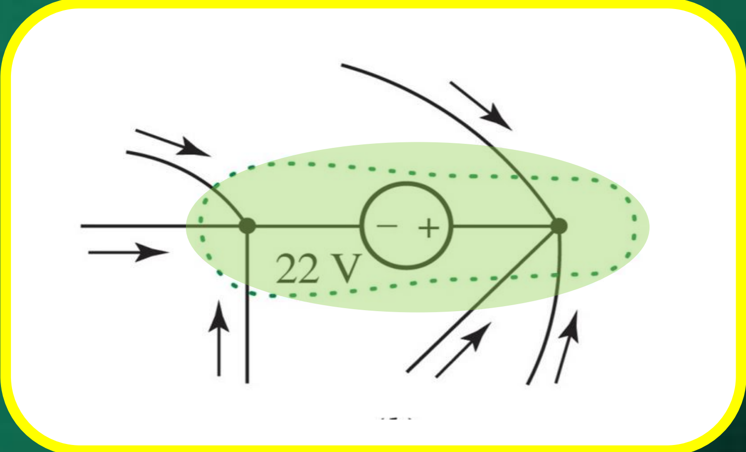
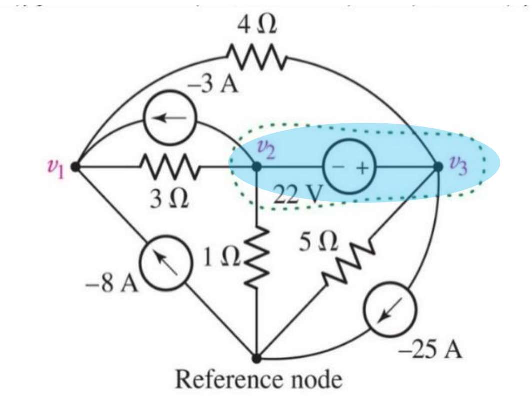


Super node



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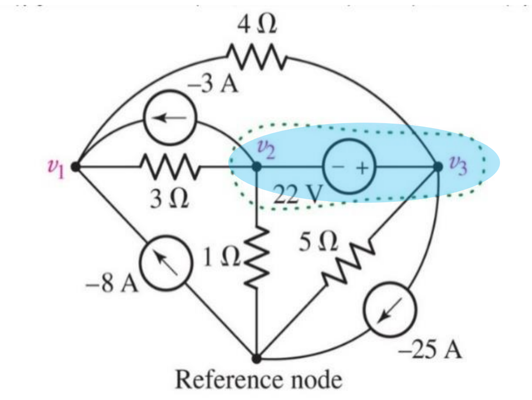


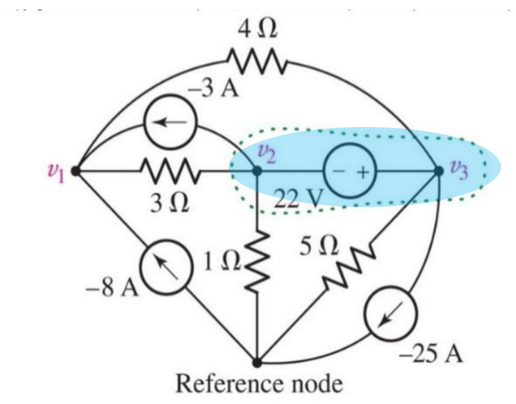
- Apply KCL at Node 1.

$$-(-8)-(-3)+\frac{v_1-v_3}{4}+\frac{v_1-v_2}{3}=0$$

$$-3+\frac{v_2}{1}+\frac{v_2-v_1}{3}+\frac{v_3}{5}+\frac{v_3-v_1}{4}-25=0$$

$$v_3-v_2=22$$

Super node



- ❑ Apply KCL at Node 1.
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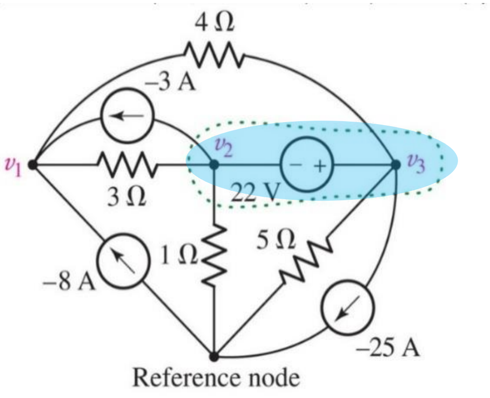

- Apply KCL at Node 1.
- Apply KCL at the super node.
- Add the equation for the voltage source inside the super node.

$$-(-8)-(-3)+\frac{v_1-v_3}{4}+\frac{v_1-v_2}{3}=0$$

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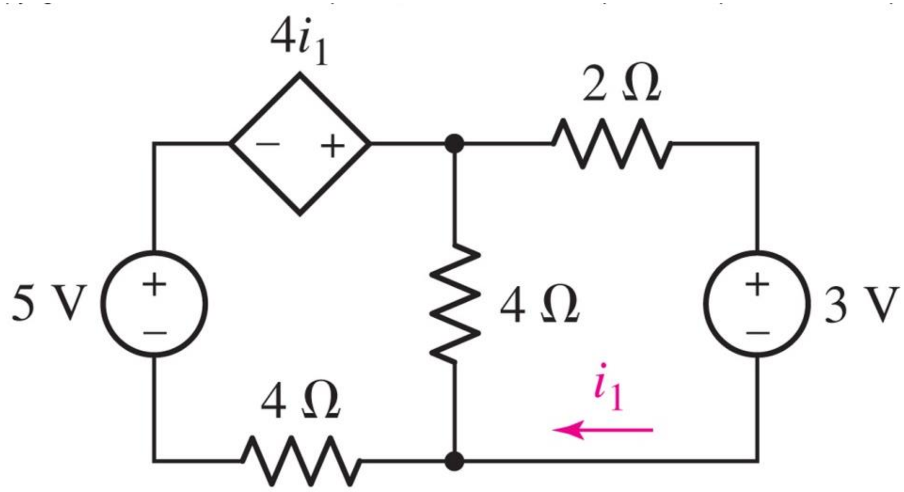
Example (With Dependent Source)

Find i_1 using Nodal Analysis



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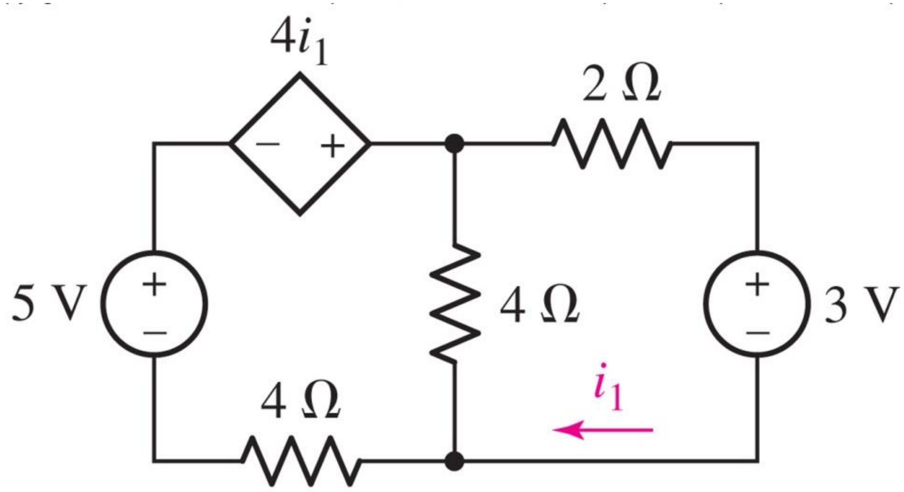

Answer: $i_1 = -250$ mA

Mesh Analysis (Alternative)

- A mesh is a loop which does not contain any other loops within it.
- In mesh analysis, we assign currents to each mess and solve them using KVL.
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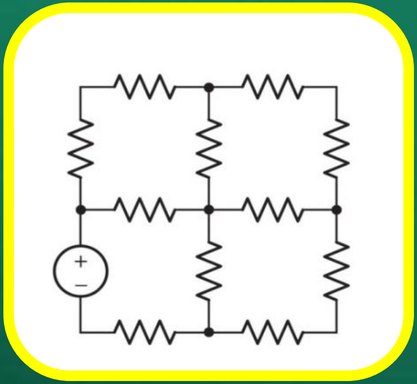

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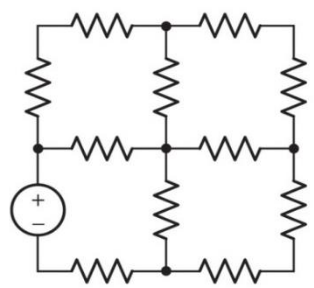
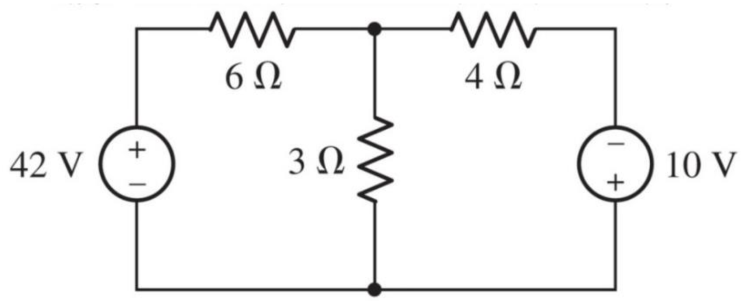


This circuit has 4 meshes.

The Mesh Analysis Method



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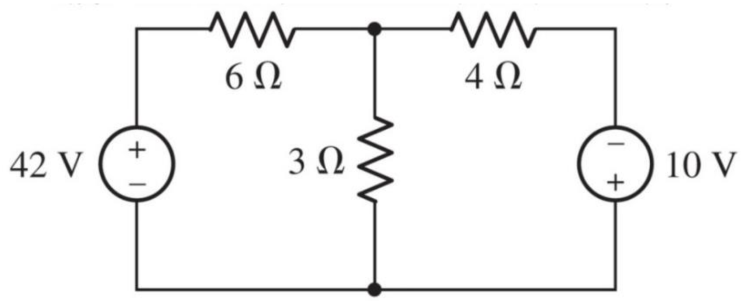
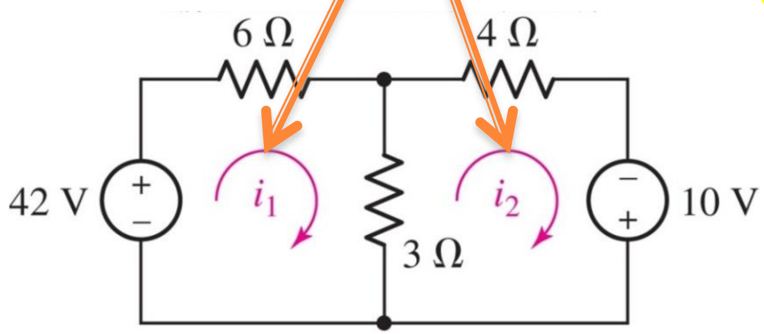


Mesh currents



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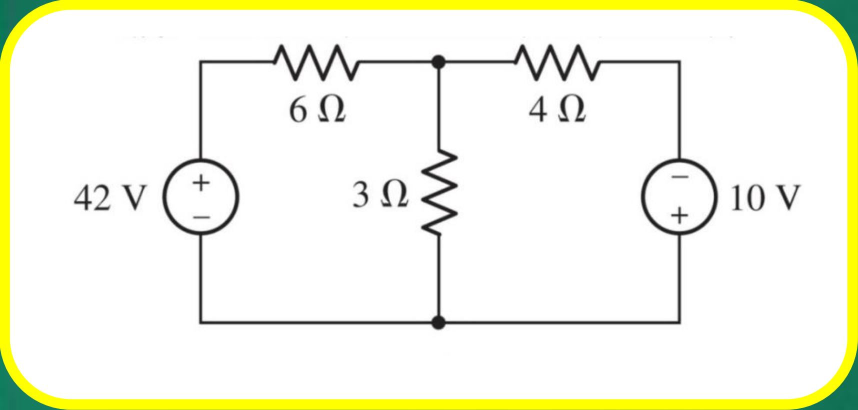

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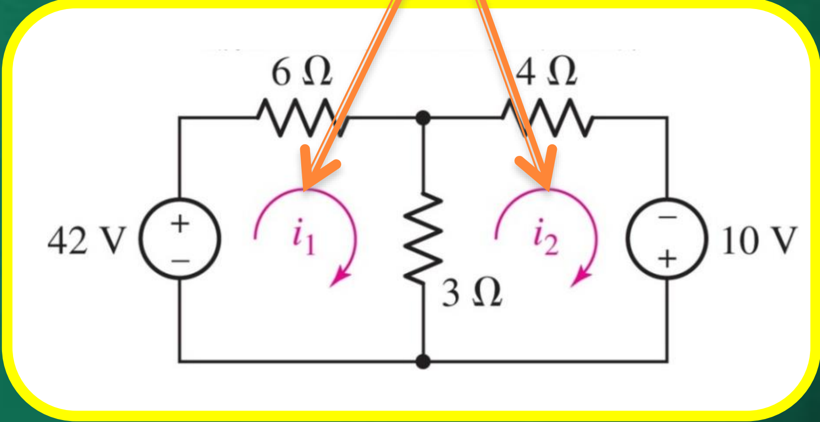
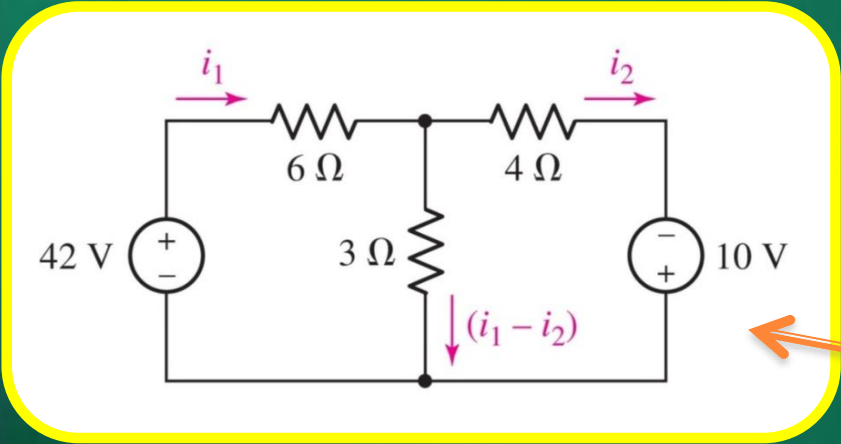
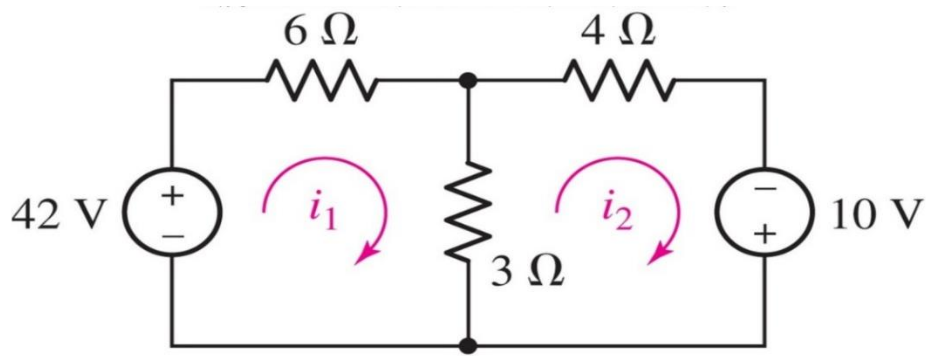




Branch currents

Mesh: Apply KVL



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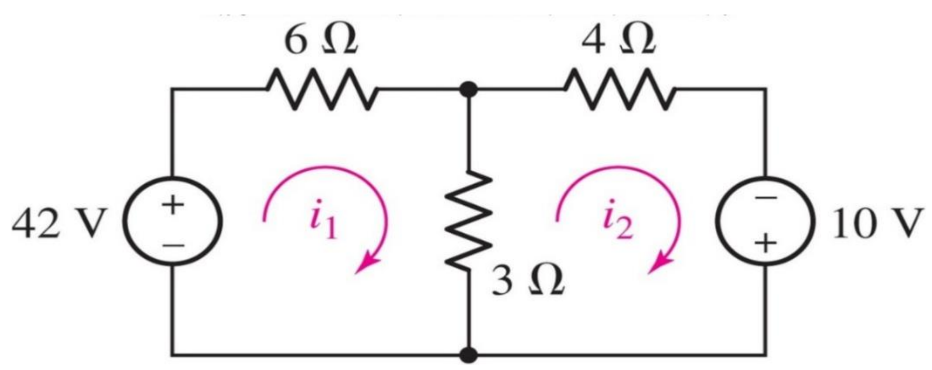
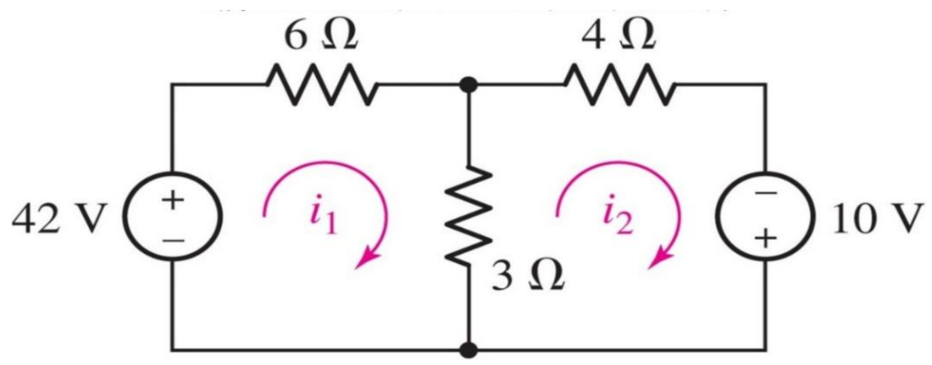


Apply KVL to mesh 1 (Σ drops = 0)

$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

Mesh: Apply KVL



Apply KVL to mesh 1 (Σ drops = 0)

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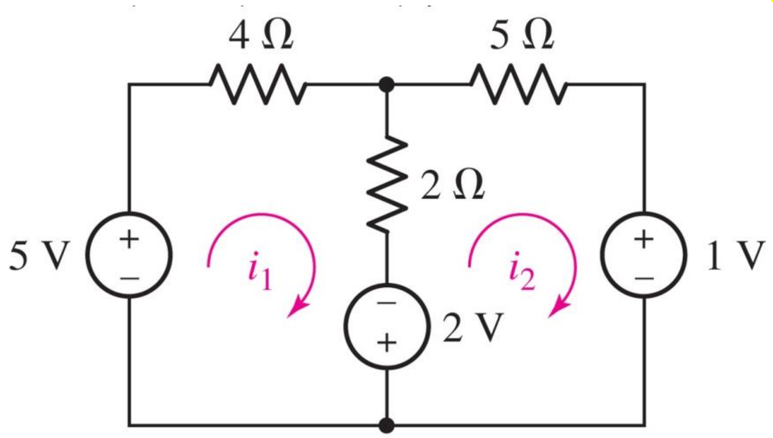
Apply KVL to mesh 2 (Σ drops = 0)

$$3(i_2 - i_1) + 4i_2 - 10 = 0$$

Example: Mesh Analysis

Determine the power supplied by the 2 V source.



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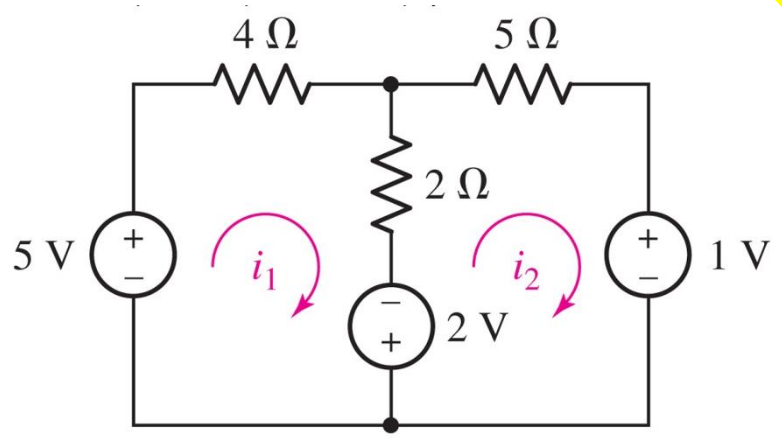

Applying KVL to the meshes

$$-5+4i_1+2(i_1-i_2)-2=0$$

$$+2+2(i_2-i_1)+5i_2+1=0$$

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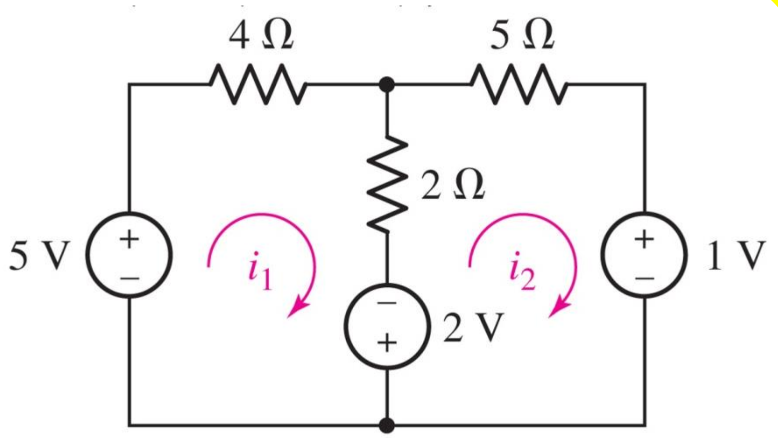

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$i_1 = 1.1316$ A, $i_2 = -0.1053$ A

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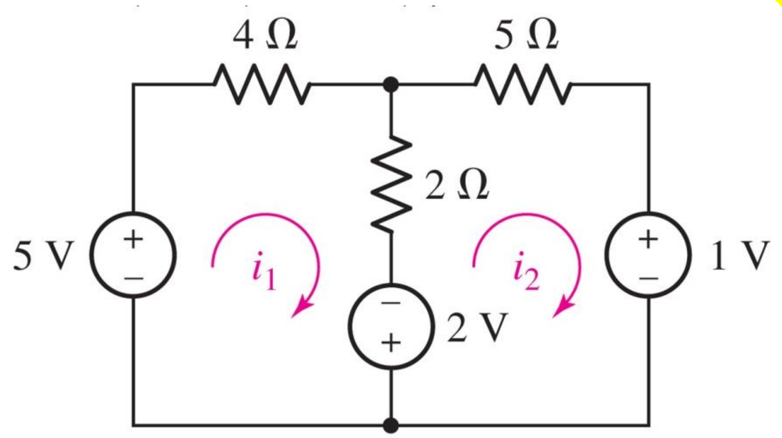

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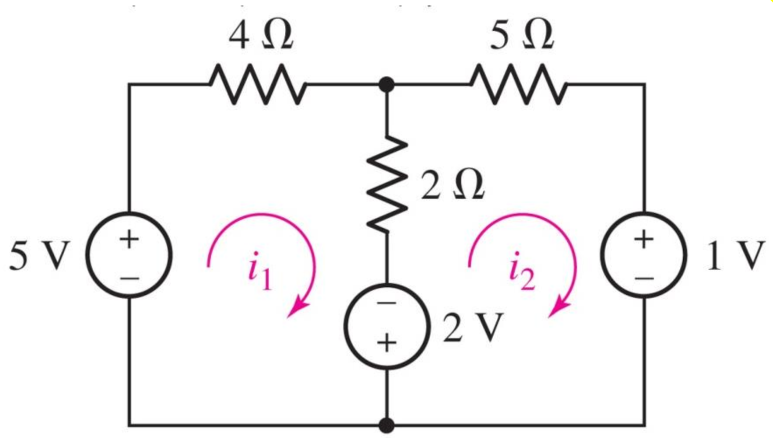

Applying KVL to the meshes

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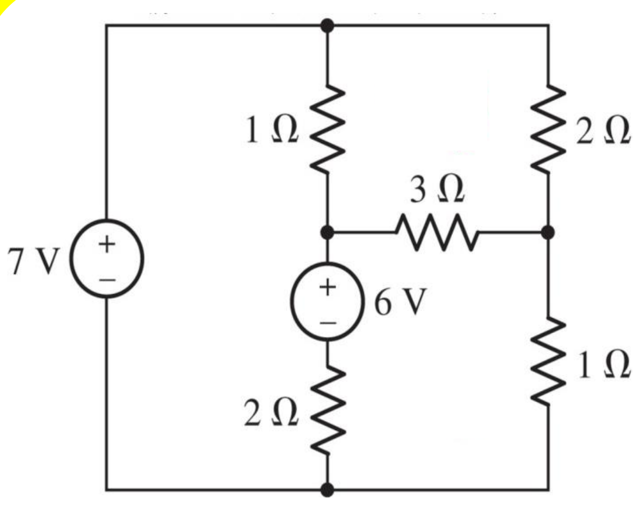
$i_1 = 1.1316$ A, $i_2 = -0.1053$ A $\quad P_{2V} = (i_2 - i_1) \times 2 = -2.4746$ W

Answer: Power supplied by 2V source is 2.4738 W

A Three Mesh Example



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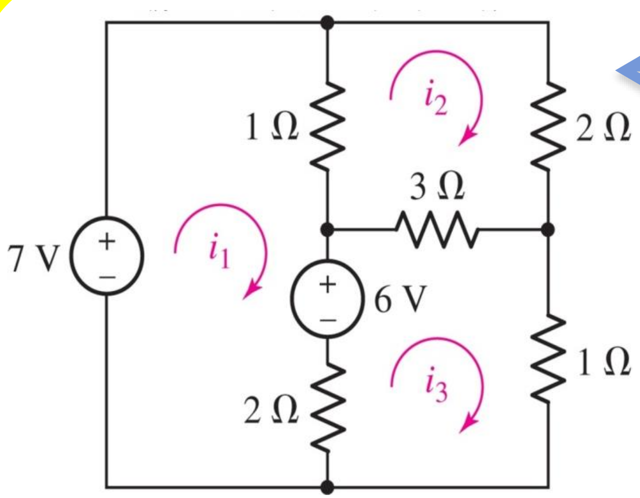
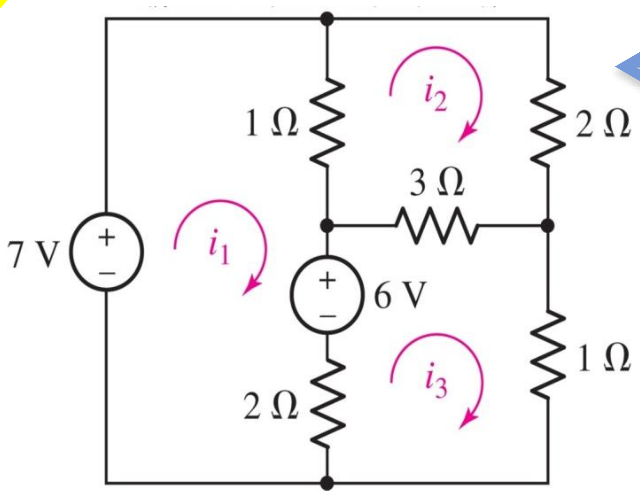


Follow each mesh clockwise

A Three Mesh Example



Follow each mesh clockwise

$$
\begin{aligned}
-7+1(i_1-i_2)+6+2(i_1-i_3) &= 0\\
1(i_2-i_1)+2i_2+3(i_2-i_3) &= 0\\
2(i_3-i_1)-6+3(i_3-i_2)+1i_3 &= 0
\end{aligned}
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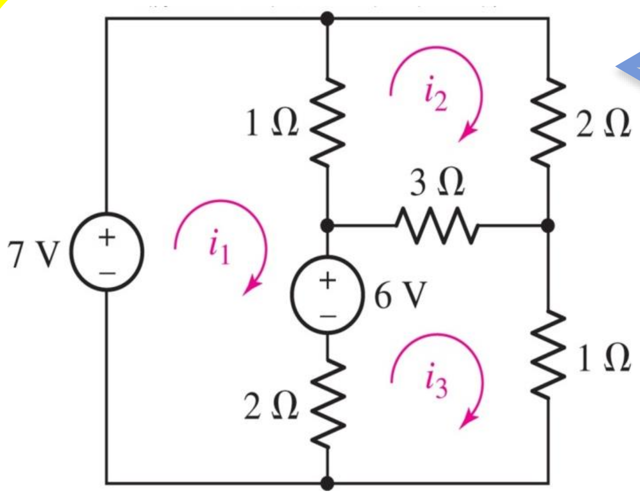

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\end{aligned}
$$

On simplification

$$
\begin{aligned}
+3i_1-i_2-2i_3 &= 1\\
-i_1+6i_2-3i_3 &= 0\\
-2i_1-3i_2+6i_3 &= 6
\end{aligned}
$$

A Three Mesh Example

Equations in matrix form

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$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

A Three Mesh Example

Equations in matrix form

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A Three Mesh Example

Equations in matrix form

$$\underbrace{\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}}_{I} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

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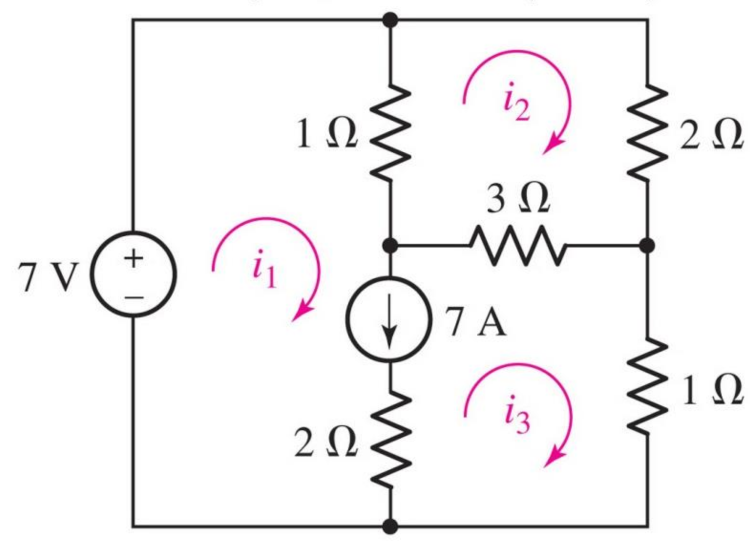
$A \qquad I \qquad B$

$$\boxed{I = A^{-1} \times B}$$

Answer: $i_1 = 3$, $i_2 = 3$, $i_3 = 3$

Current Sources and Supermesh

What is the voltage across a current source in between two meshes?



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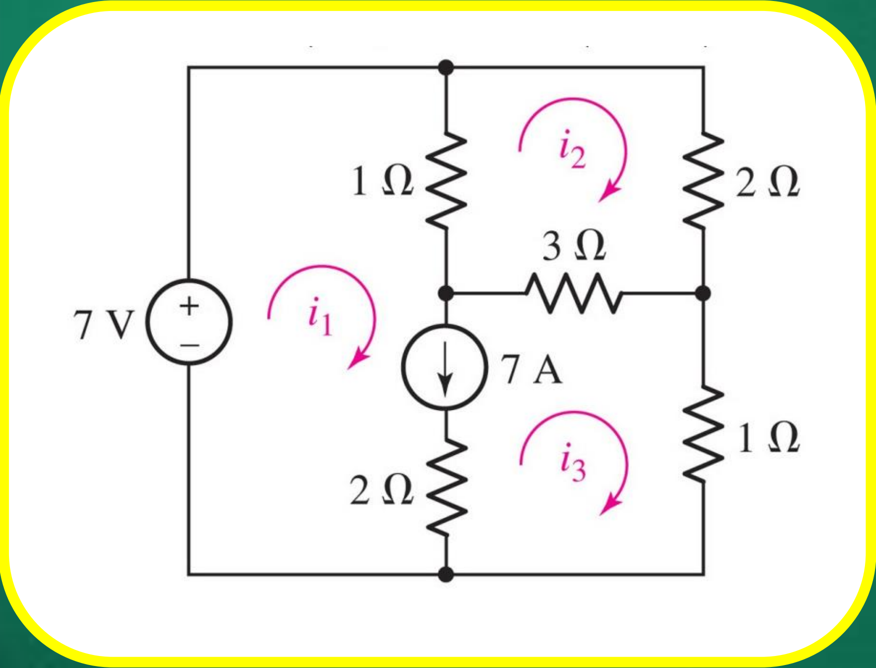

We can eliminate the need for introducing a voltage variable by applying KVL to the super mesh formed by joining mesh 1 and mesh 3.

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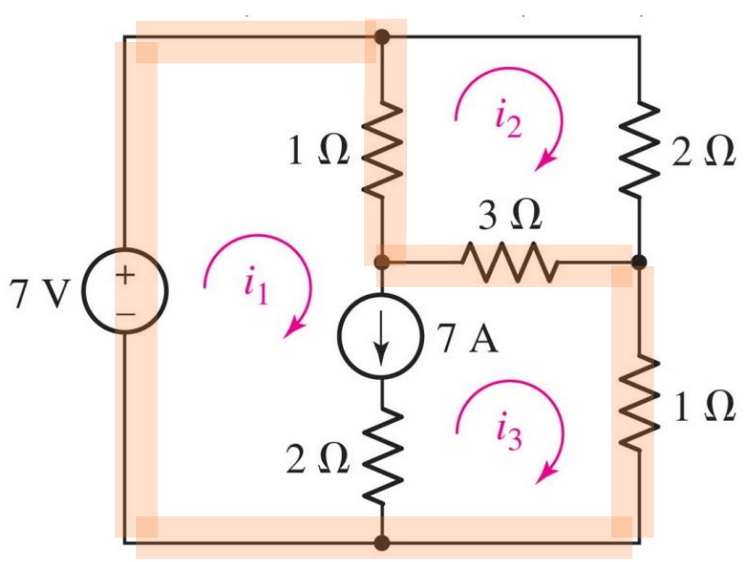

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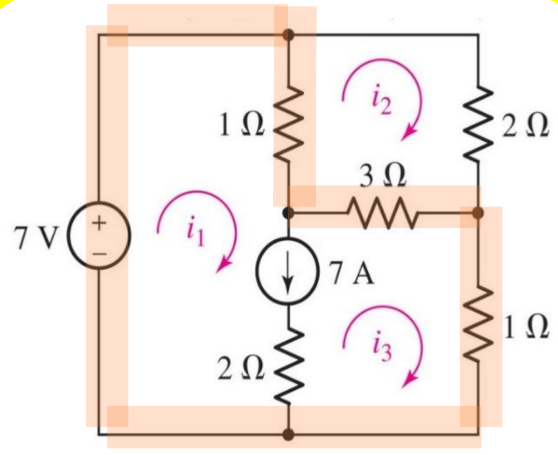
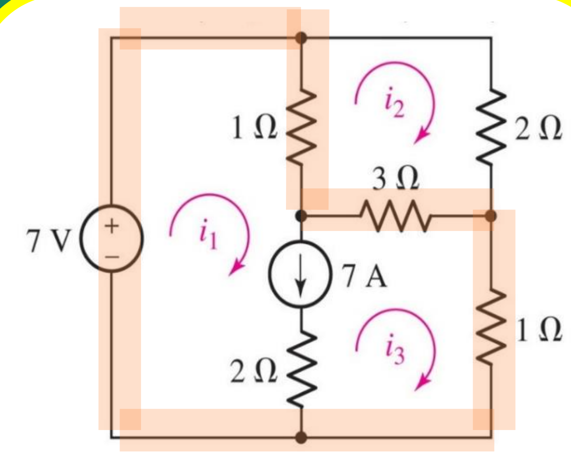
Super mesh

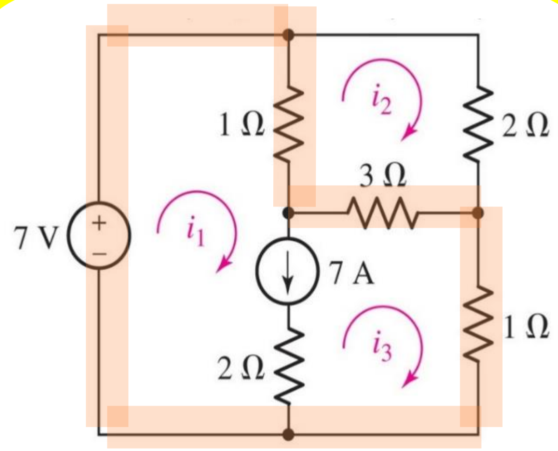


Super mesh



Apply KVL to mesh 2: $1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3)$

Super mesh

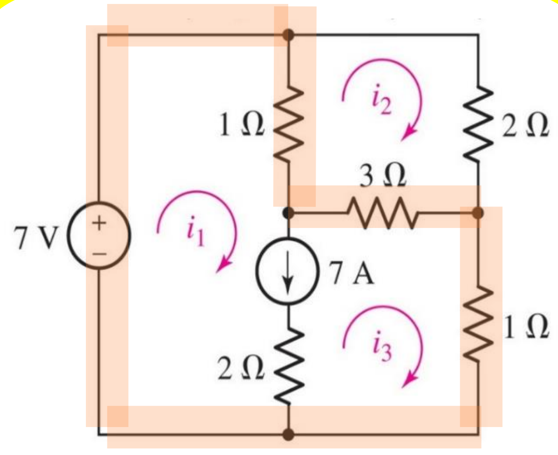


Apply KVL to mesh 2: $1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3)$

Apply KVL supermesh 1/3:

$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3$$

Super mesh



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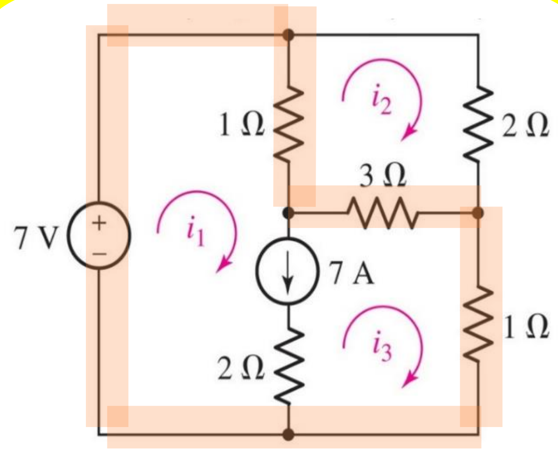
Apply KVL supermesh 1/3:

$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3$$

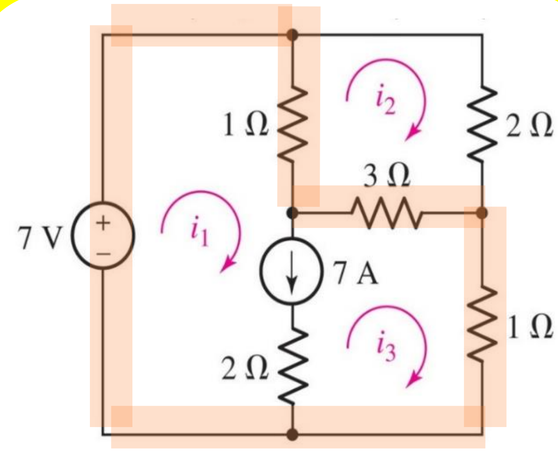
Add the current source: $i_1 - i_3 = 7$

Super mesh



$$\begin{aligned} -i_1 + 6i_2 - 3i_3 &= 0 \\ i_1 - 4i_2 + 4i_3 &= 7 \\ i_1 - i_3 &= 7 \end{aligned}$$

Super mesh

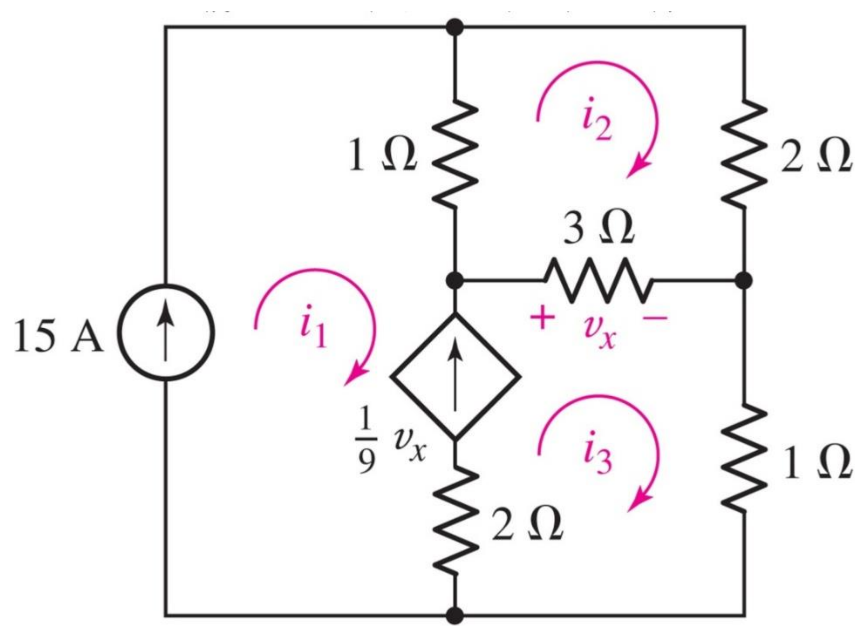


$$
\begin{aligned}
-i_1 + 6i_2 - 3i_3 &= 0 \\
i_1 - 4i_2 + 4i_3 &= 7 \\
i_1 - i_3 &= 7
\end{aligned}
$$

Answer: $i_1 = 9$, $i_2 = 2.5$, $i_3 = 2$

Dependent Source Example

Compute all the mesh currents.



Dependent Source Example

In mesh 1: $$\boxed{i_1 = 15}$$

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In mesh 2:

$$\boxed{i_3 - i_1 = \frac{V_x}{9} \implies i_3 - 15 = \frac{V_x}{9}}$$

Dependent Source Example

In mesh 1: $$i_1 = 15$$

In mesh 2: $$i_3 - i_1 = \frac{V_x}{9} \implies i_3 - 15 = \frac{V_x}{9}$$

Apply KVL to mesh 2 $$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

Dependent Source Example

In mesh 1: $\boxed{i_1 = 15}$

In mesh 2: $$\boxed{i_3 - i_1 = \frac{V_x}{9} \implies i_3 - 15 = \frac{V_x}{9}}$$

Apply KVL to mesh 2 $$\boxed{1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0}$$

$$\boxed{\implies i_2 - 15 + 2i_2 + 3(i_2 - i_3) = 0}$$

Dependent Source Example

In mesh 1: $$i_1 = 15$$

In mesh 2: $$i_3 - i_1 = \frac{V_x}{9} \implies i_3 - 15 = \frac{V_x}{9}$$

Apply KVL to mesh 2 $$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

$$\implies i_2 - 15 + 2i_2 + 3(i_2 - i_3) = 0$$

$$v_x = 3(i_3 - i_2)$$

Dependent Source Example

$$\begin{aligned} i_1 &= 15 \\ 6i_2 - 3i_3 &= 15 \\ 3i_2 + 6i_3 &= 135 \end{aligned}$$

Answer: $i_1 = 15$, $i_2 = 11$, $i_3 = 17$

Node or Mesh: How to Choose?

- Use the one with fewer equations, or
- Use the method you like best, or
- Use both (as a check)

Example

Use both Nodal and Mesh Analysis to find v_1