Solutions to Problem Sheet-4 IEC102

Q1. The Thevenin equivalent at terminals a-b of the linear network shown in Fig. Q1 is to be determined by measurement. When a 10 KΩ resistor is connected to terminals a-b, the voltage Vab is measured as 6 V. When a 30 KΩ resistor is connected to the terminals, Vab is measured as 12 V.

Determine

a) The Thevenin equivalent at terminals a-b.

b) Vab when a 20 KΩ resistor is connected to the terminals a-b.

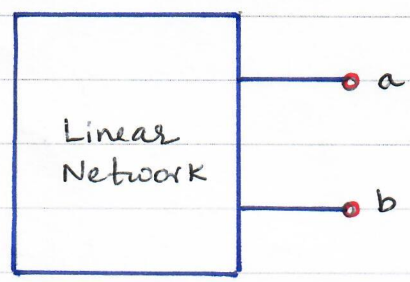
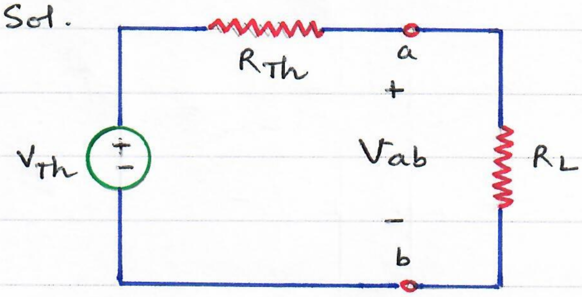


Fig. Q1

Sol.



When $R_L = 10\,K\Omega$; $V_{ab} = 6V$

$R_L = 30\,K\Omega$ $V_{ab} = 12V$

V_{Th} $V_{ab} = \dfrac{V_{Th}}{R_{Th} + R_L} \times R_L$

$\Rightarrow V_{Th} R_L = V_{ab}(R_{Th} + R_L)$

$6(R_{Th} + 10K) = V_{Th} \times 10K \quad \cdots (1)$

$12(R_{Th} + 30K) = V_{Th} \times 30K \quad \cdots (2)$

$$\frac{30}{10} = \frac{12}{6}\,\frac{(R_{Th} + 30K)}{(R_{Th} + 10K)}$$

$$\Rightarrow \quad 3 = \frac{2\,(R_{Th}+30K)}{(R_{Th}+10K)}$$

$$\Rightarrow \quad 3\,(R_{Th}+10K) = 2\,(R_{Th}+30K)$$

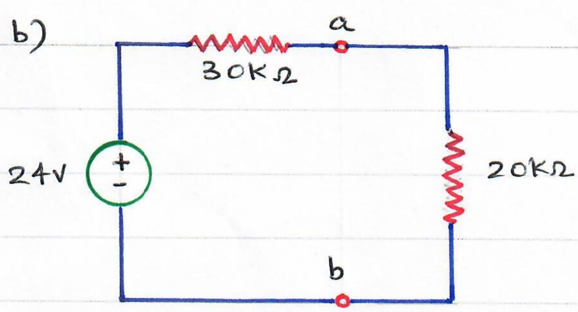
$$\boxed{R_{Th} = 60K - 30K = 30K\,\Omega}$$

$$V_{Th} \times 10K = 6\,(R_{Th}+10K) = 6\,(30K+10K)$$

$$V_{Th} \times 10K = 6 \times 40K$$

$$\Rightarrow \boxed{V_{Th} = 24\,V}$$

b)



$$V_{ab} = \frac{24 \times 20K}{30K+20K}$$

$$= 9.6\,V$$

Q2 For the bridge network shown in Fig. Q2, find R_{ab} and i.

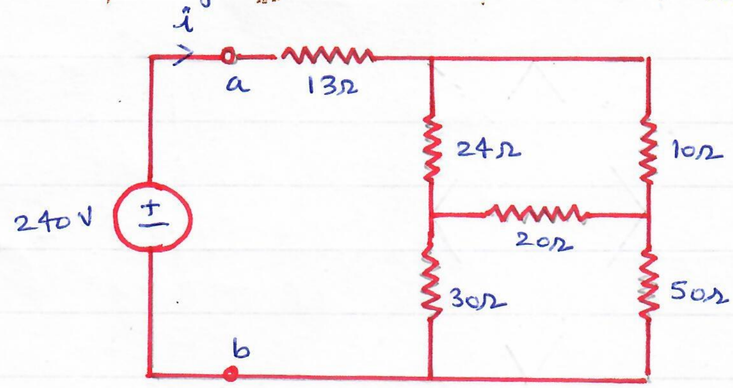
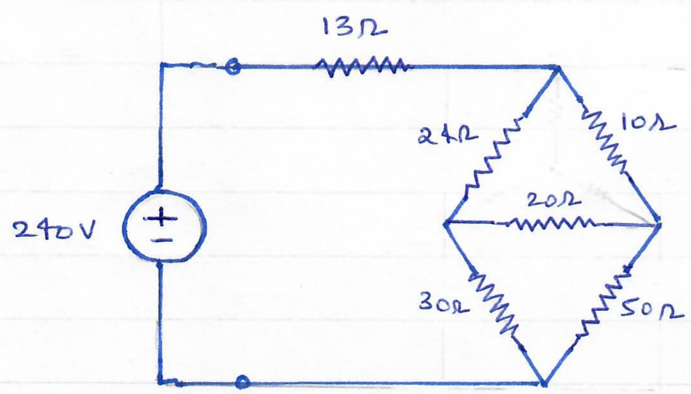


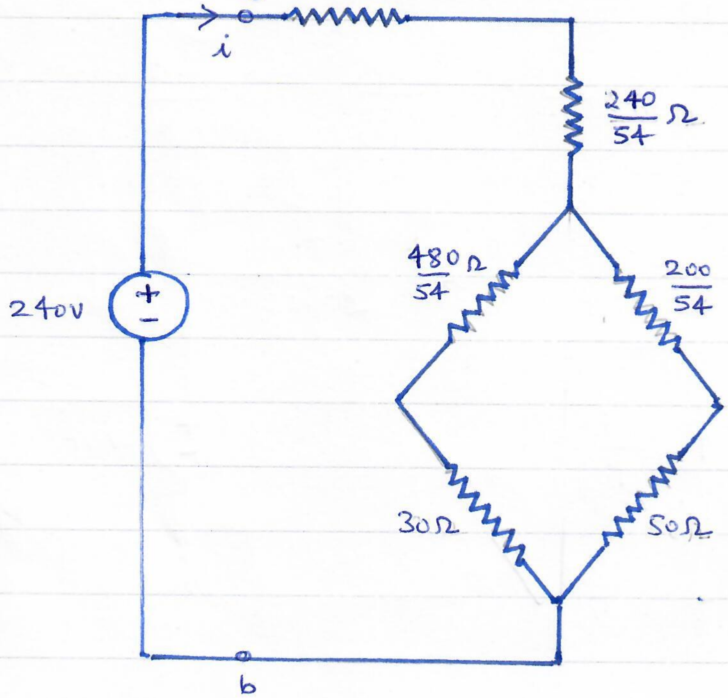
Fig. Q2

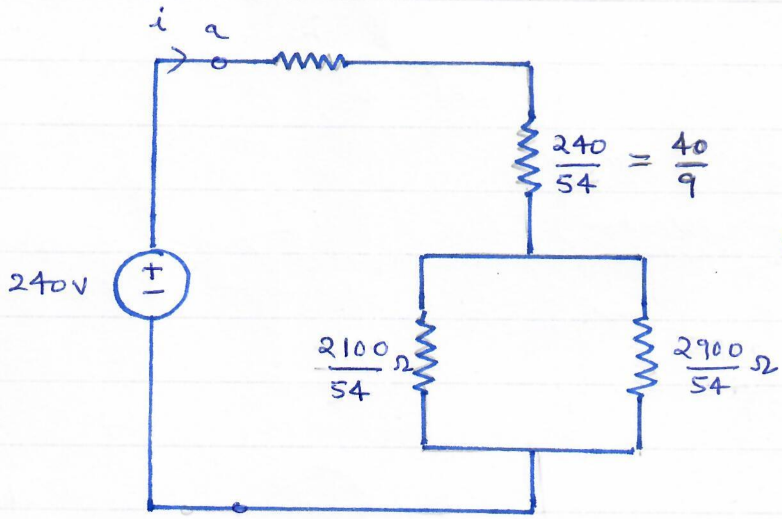
Sol. circuit can be redrawn as

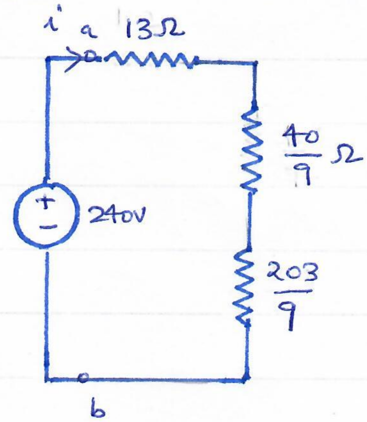


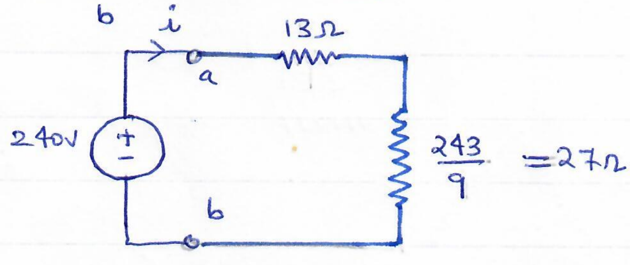
PTO

Transforming upper delta network to Y.









$$R_{ab} = 13 + 27 = 40\,\Omega$$

$$i = \frac{240}{40} = 6\,A$$

Q3 The voltage source $v_s(t)$ in the circuit shown in Fig. Q3a) has the source waveform as shown in Fig. Q3b)

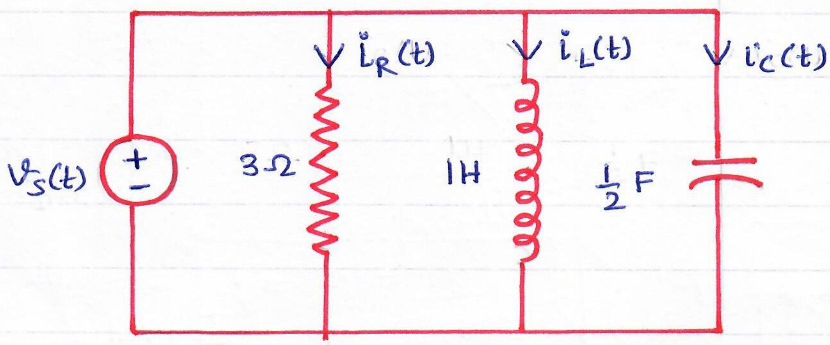


Fig. Q3a)

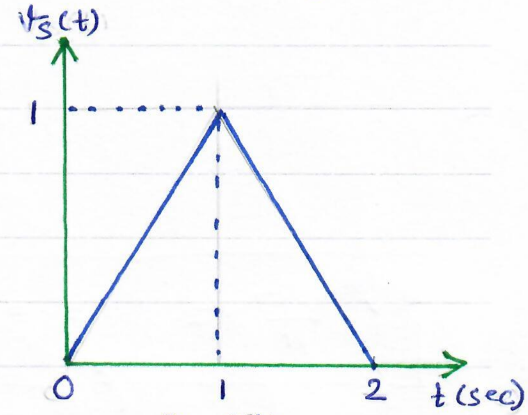


Fig Q3b)

a) Sketch $i_R(t)$

b) Sketch $i_C(t)$

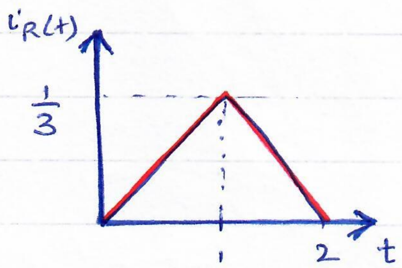
c) Sketch $i_L(t)$, assume $i_L(0) = 0$.

Sol. $i_R(t) = \dfrac{v_s(t)}{3}$; $i_C(t) = C\dfrac{dv_s(t)}{dt}$; $i_L(t) = \dfrac{1}{L}\displaystyle\int_0^t v_s(t)\,dt$

The voltage across all the elements is $v_s(t)$ as they are connected in parallel.

$$v_s(t) = \begin{cases} t & 0 \le t < 1 \\ -t+2 & 1 \le t \le 2 \\ 0 & t > 2 \end{cases}$$

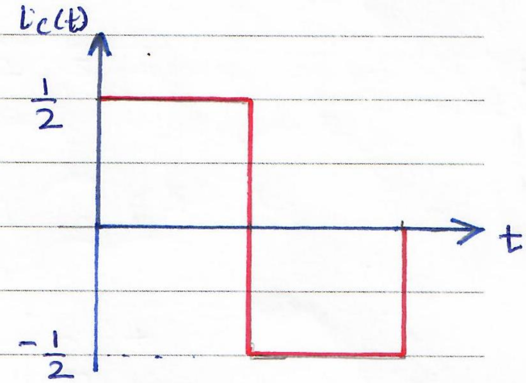
a) $$i_R(t) = \frac{v_s(t)}{3} = \begin{cases} \dfrac{t}{3} & 0 \le t < 1 \\ -\dfrac{t+2}{3} & 1 \le t \le 2 \\ 0 & t > 2 \end{cases}$$



b) $i_C(t) = \dfrac{C\,dv_s(t)}{dt} = \dfrac{1}{2}\dfrac{dv_s(t)}{dt}$ $\quad \because C = \frac{1}{2}F$

$$i_C = \begin{cases} \frac{1}{2}\frac{d}{dt}(t) & 0 \le t < 1 \\ \frac{1}{2}\frac{d}{dt}(-t+2) & 1 \le t \le 2 \\ 0 & t > 2 \end{cases}$$

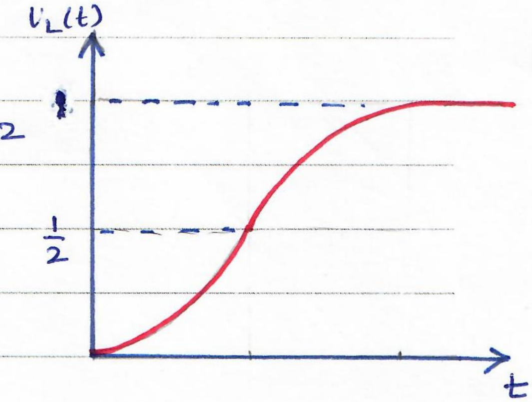
$$= \begin{cases} \frac{1}{2} & 0 \le t < 1 \\ -\frac{1}{2} & 1 \le t \le 2 \\ 0 & t > 2 \end{cases}$$



c) $i_L(t) = \dfrac{1}{L}\displaystyle\int_{-\infty}^{t} v_s(t) = \dfrac{1}{L}\int_0^t v_s(t)\,dt = \int_0^t v_s(t)\,dt$ $\quad \because L = 1H$

$$i_L(t) = \begin{cases} \int_0^t v_s(t)\,dt + i_L(0) & 0 \le t < 1 \\ \int_1^t (-t+2)\,dt + i_L(1) & 1 \le t \le 2 \\ 0 + i_L(2) & t > 2 \end{cases}$$

$$= \begin{cases} \frac{t^2}{2} & 0 \le t < 1 \\ -\frac{t^2}{2} + 2t - 1 & 1 \le t \le 2 \\ 1 & t > 2 \end{cases}$$



Free Lined Graph Paper from http://incompetech.com/graphpaper/lined/

Q4 For the circuit shown in Fig. Q4, calculate the value of 'R' that will make energy stored in the capacitor the same as that stored in the inductor under dc conditions.

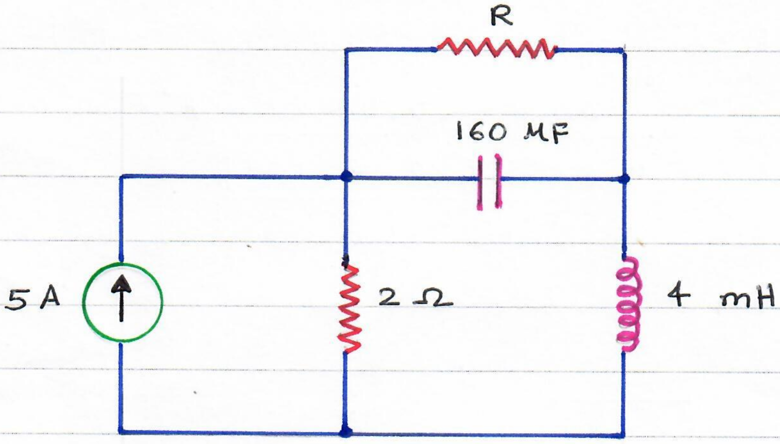
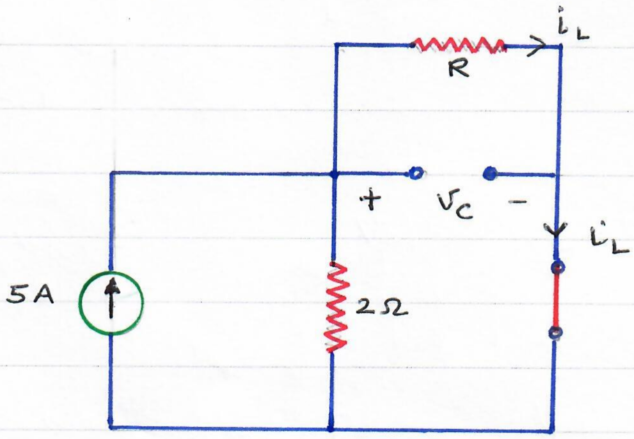


Fig. Q4

Sol. Under dc conditions capacitor acts as open circuit and inductor acts as short circuit.

The equivalent circuit under dc conditions is



$$i_L = 5 \times \frac{2}{2+R} = \frac{10}{R+2}$$

$$V_c = i_L R$$

$$= \frac{10R}{R+2}$$

Energy stored in capacitor $= \frac{1}{2} C V_c^2 = W_c$

$$\Rightarrow W_c = \frac{1}{2} \times 160 \times 10^{-6} \times \left(\frac{10R}{(R+2)}\right)^2$$

$$= \frac{1}{2} \times 160 \times 10^{-6} \times \frac{100R^2}{(R+2)^2}$$

$$= \frac{80 \times 10^{-4} R^2}{(R+2)^2}$$

Energy stored in the inductor $= \frac{1}{2} L i_L^2 = W_L$

$$= \frac{1}{2} \times 4 \times 10^{-3} \times \left(\frac{10}{R+2}\right)^2$$

$$= \frac{0.2}{(R+2)^2}$$

If $W_C = W_L$

$$\therefore \quad \frac{80 \times 10^{-4} R^2}{(R+2)^2} = \frac{0.2}{(R+2)^2}$$

$$R^2 = \frac{0.2}{80 \times 10^{-4}} = 25$$

$$\therefore \quad R = 5\,\Omega$$