Solutions to Problem sheet-3 IEC102

Q1) Apply the superposition principle to find v_o in the circuit shown in Fig. Q1

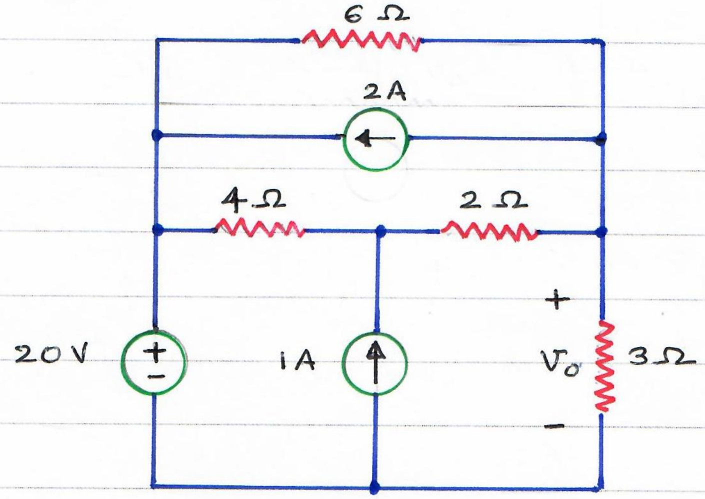


Fig. Q1

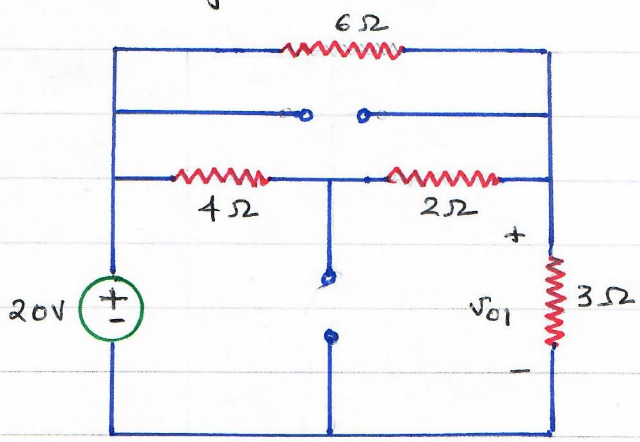
Sol.

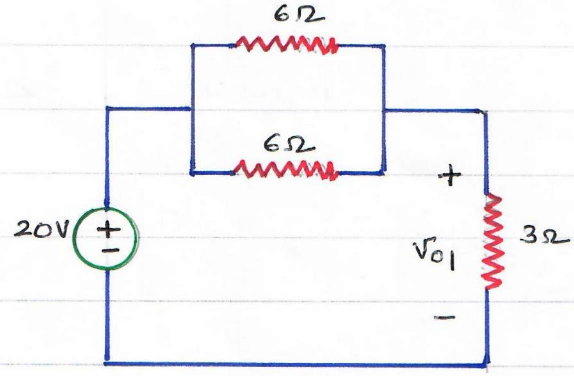
Let v_{o1} be the voltage across 3 Ω resistor due to 20 V source acting alone.
Similarly, let v_{o2} and v_{o3} be the voltage across 3 Ω resistor due to 2A and 1A current sources acting independently.

By superposition $v_o = v_{o1} + v_{o2} + v_{o3}$

v_{o1}

When only 20 V source is there and other sources are disabled.

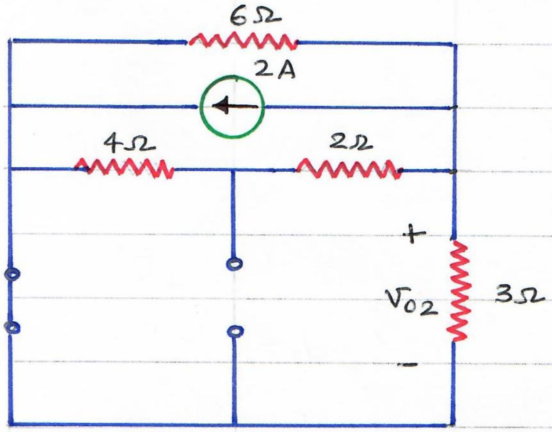


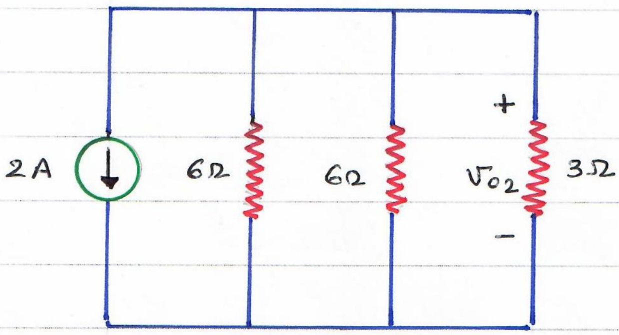


$$V_{o1} = \frac{20 \times 3}{6 \| 6 + 3} = \frac{20 \times 3}{6} = 10\text{ V}$$

V_{o2}

When only 2A current source is acting and other sources are disabled.



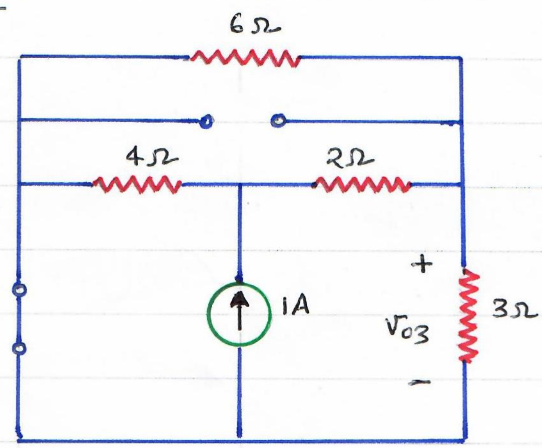


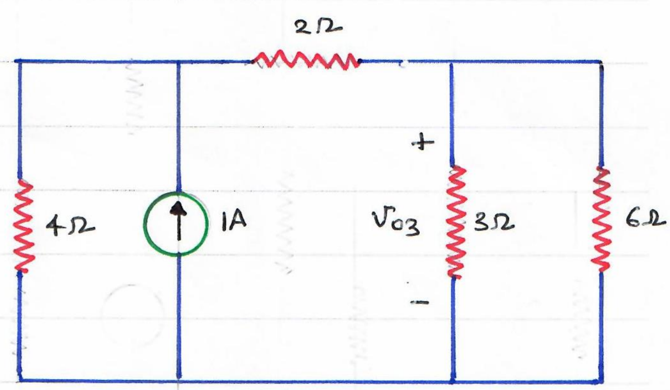
$$V_{o2} = -\frac{2 \times 6 \| 6}{6 \| 6 + 3} \times 3$$

$$= \frac{-2 \times 3}{3+3} \times 3$$

$$\Rightarrow V_{o2} = -3\,V$$

V_{o3}





$$V_{o3} = 1 \times \frac{4}{4 + 2 + 3\|6} \times (3\|6)$$

$$= \frac{4}{4+2+2} \times 2 = \frac{4}{8} \times 2 = 1\,V$$

When all the sources are acting the voltage across 3Ω resistor $= V_o = V_{o1} + V_{o2} + V_{o3} = 10 - 3 + 1 = 8\,V$

(Q2) Find the Thevenin & Norton equivalent circuit at terminals a-b of the circuit shown in Fig. Q2

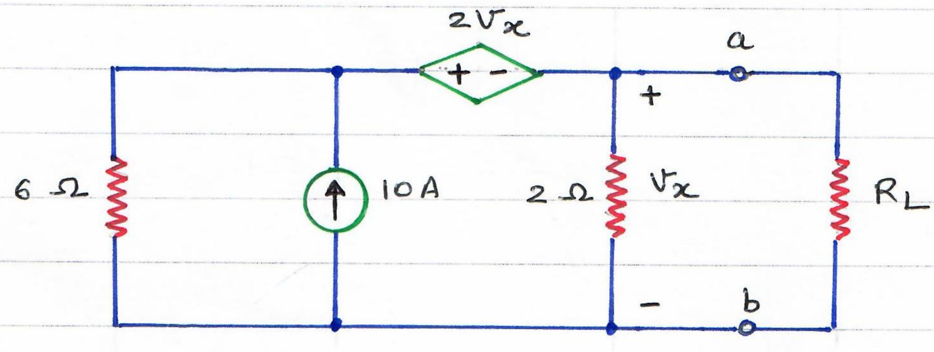
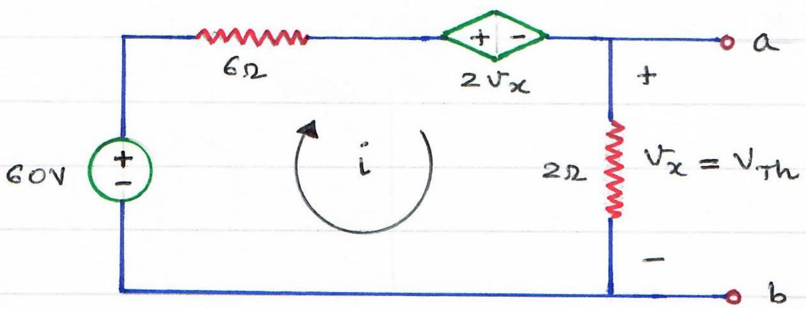


Fig. Q2

Sol.

Using source transformation on 10 A current source in parallel with 6Ω resistor.

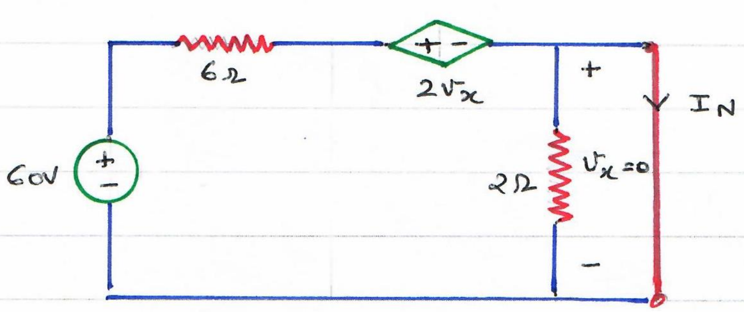


Applying KVL around loop

$$-60 + 6i + 2V_x + V_x = 0 \qquad V_x = 2i \Rightarrow i = V_x/2$$

$$-60 + 6\frac{V_x}{2} + 2V_x + V_x = 0$$

$$\Rightarrow 6V_x = 60 \Rightarrow V_x = V_{Th} = 10V$$



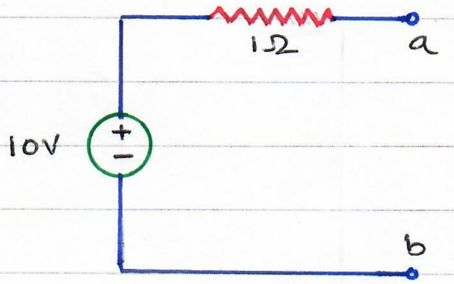
$$-60 + 6I_N + 2V_x = 0$$

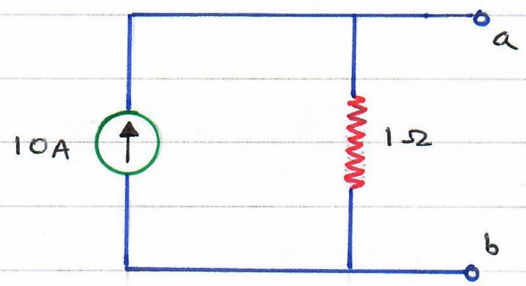
$$\Rightarrow -60 + 6I_N = 0 \qquad (\because V_x = 0)$$

$$\Rightarrow I_N = 10A$$

$$R_{eq} = R_{Th} = R_N = \frac{V_{Th}}{I_N} = \frac{10}{10} = 1\Omega$$

Thevenin and Norton equivalent circuits are





(Q3) Determine the Thevenin equivalent of the circuit shown in Fig. Q3

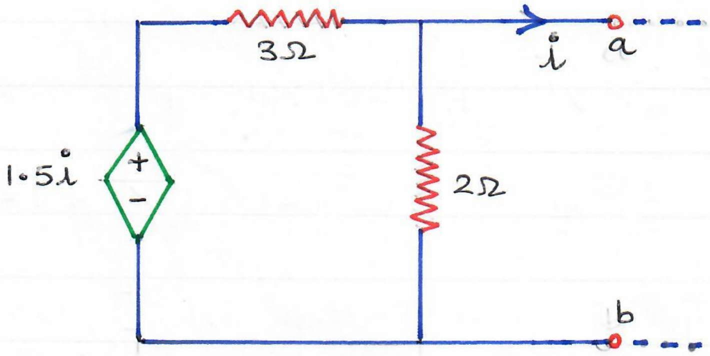


Fig. Q3

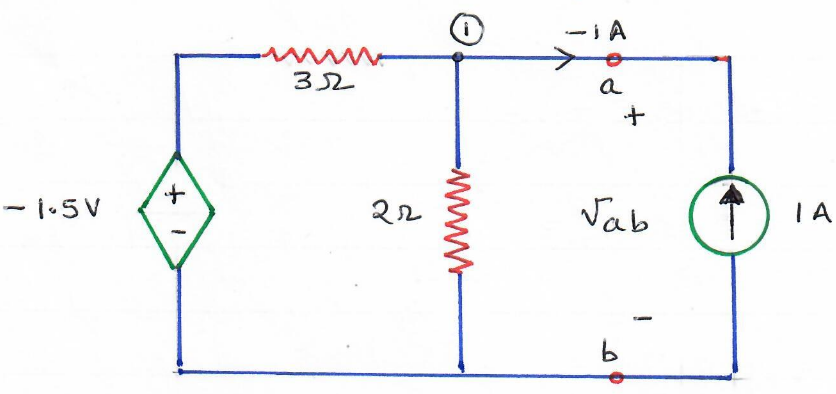
Sol.

$V_{Th} = I_N = 0$ (Since there are no independent sources)

<u>R_{Th}</u>

Insert a known test source at the terminals and find out the ratio of voltage to current to find R_{Th}.

Let us connect a test current source of 1A



$R_{Th} = \dfrac{V_{ab}}{1}$

$V_{ab} =$

Applying KCL at node (1)

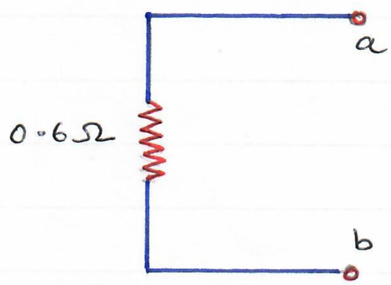
$$\frac{V_{ab} - (-1.5)}{3} + \frac{V_{ab}}{2} - 1 = 0$$

$$\Rightarrow V_{ab}\left(\frac{1}{3} + \frac{1}{2}\right) = 1 - \frac{1.5}{3} = 1 - 0.5 = 0.5$$

$$\Rightarrow V_{ab} \times \frac{5}{6} = 0.5 \quad ; \quad \Rightarrow V_{ab} = 0.6\,V$$

$\therefore\ R_{Th} = 0.6\,\Omega$

Thevenin equivalent circuit is



(Q4) Find the value of R_L for maximum power transfer in the circuit shown in Fig. Q4. Also, find the maximum power dissipated in R_L.

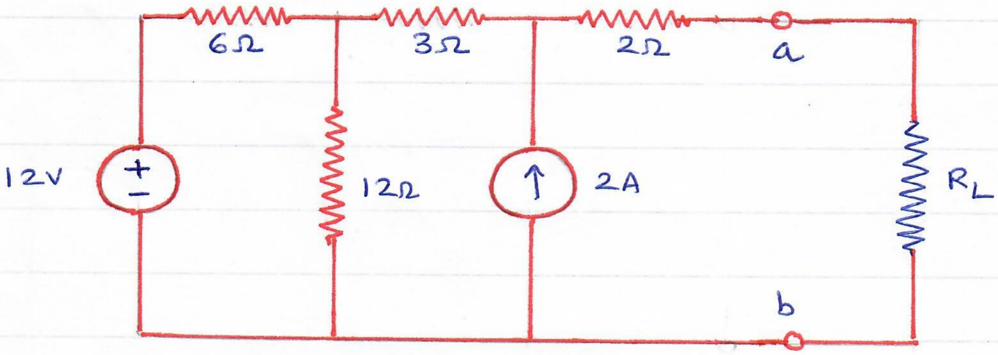
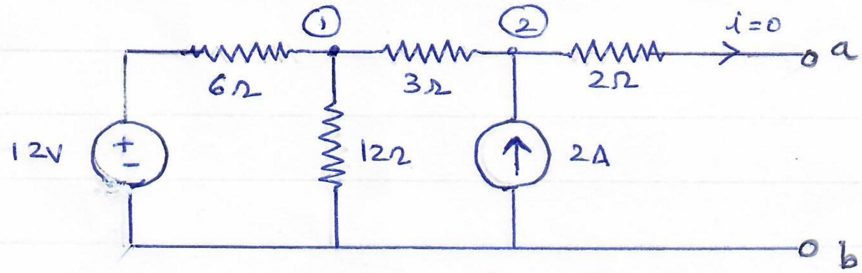


Fig. Q4

Sol.

Thevenize the circuit across terminals of R_L.



V_{th}

Applying KCL at node (1)

$$\frac{v_1 - 12}{6} + \frac{v_1}{12} + \frac{v_1 - v_2}{3} = 0$$

$$\Rightarrow \frac{2(v_1 - 12) + v_1 + 4(v_1 - v_2)}{12} = 0$$

$$\Rightarrow 7v_1 - 4v_2 = 24 \quad \cdots (A)$$

Applying KCL at node (2)

$$-2 + \frac{v_2 - v_1}{3} = 0$$

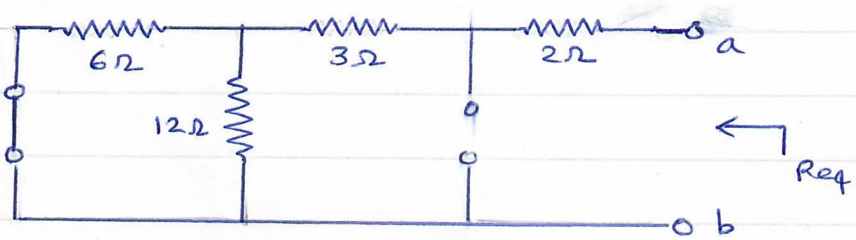
$$\Rightarrow v_2 - v_1 = 6 \quad \cdots (B)$$

Solving (A) and (B)

$$\boxed{v_2 = 22V = V_{Th}}$$

$\underline{R_{th} = R_{eq}}$

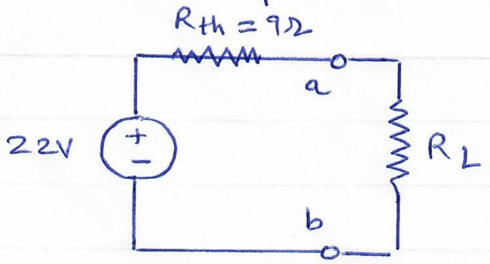
Turning off all the sources and calculating equivalent resistance across terminals a-b.



$$R_{eq} = R_{Th} = 2 + 3 + 6 \| 12$$

$$= 2 + 3 + 4 = 9\Omega$$

Thevenin equivalent circuit



Maximum power is transferred to R_L when $R_L = R_{Th} = 9\Omega$

Maximum power transferred to R_L when $R_L = R_{th} = 9\Omega$

$$P_{max} \text{ in } R_L = \left[\frac{22}{(9+9)}\right]^2 \times 9$$

$$= \left(\frac{\overset{11}{\cancel{22}}}{\underset{9}{\cancel{18}}}\right)^2 \times 9$$

$$\Rightarrow P_{max} = \left(\frac{11}{9}\right)^2 \times 9 = \frac{11 \times 11}{9} = \frac{121}{9} = 13.44\,W$$