Mathematics III Assignment 1

Zubair Abid, 20171076

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1. X is a random variable with PDF given by

$$f(n) = \begin{cases} cx^2 & \text{if } x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Constant c

   Summation of PDF over domain adds up to 1.
   Or,

$$\int_{-\infty}^{\infty} cx^2 = 1$$

   Since the function returns 0 everywhere except at $[-1, 1]$, we just calculate

$$\begin{aligned} \int_{-1}^{1} cx^2 &= 1 \\ c \times \left.\frac{x^3}{3}\right|_{-1}^{1} &= 1 \\ c \times \frac{2}{3} &= 1 \\ c &= 1.5 \text{ (Answer)} \end{aligned}$$

(b) $E\left[X\right]$ and $Var\left(X\right)$ $E[X]$ is

$$\begin{aligned}\int\limits_{-1}^{1} xcx^2 &= 1.5 \times \int\limits_{-1}^{1} x^3 \\ &= 1.5t \times \left.\frac{x^4}{4}\right|_{-1}^{1} \\ &= 0\end{aligned}$$

Now, $Var(X) = E[x^2] - (E[X])^2$, or

$$\begin{aligned}Var(X) &= 1.5 \times \int\limits_{-1}^{1} x^4 - 0 \\ &= 1.5 \times \left.\frac{x^5}{5}\right|_{-1}^{1} \\ &= 1.5 \times 0.4 \\ &= 0.6\end{aligned}$$

(c) $P\left(X \geq \frac{1}{2}\right)$

Since the function given is a PDF, to get the $P(X \geq \frac{1}{2})$, all we need to do is integrate $f(x)$ from $\frac{1}{2}$ to 1
Or,

$$\begin{aligned}P\left(X \geq \frac{1}{2}\right) &= \int\limits_{\frac{1}{2}}^{1} cx^2 \\ &= 1.5 \times \left.\frac{x^3}{3}\right|_{\frac{1}{2}}^{1} \\ &= 1.5 \times \frac{7}{24} \\ &= 0.4375\end{aligned}$$

2. Given, the CDF is:

$$F(x) = \frac{x^3 + k}{40} \quad x = 1, 2, 3$$

(a) Value of k
Since $F(x)$ is a CDF, value of $F(3) = 1$
or

$$\frac{27+k}{40} = 1$$
$$k = 13 \quad \text{(Q.E.D)}$$

(b) Find the probability distribution of X
This can be obtained by simple subtraction, answer is

$$\begin{aligned}
P(X=1) &= \frac{1+13}{40} \\
&= \frac{7}{20} \\
P(X=2) &= \frac{21-14}{40} \\
&= \frac{7}{40} \\
P(X=3) &= \frac{40-21}{40} \\
&= \frac{19}{40}
\end{aligned}$$