Solutions to Problem sheet-1 IEC102

Q1) A 2-terminal device consumes energy as shown by the waveform in Fig. Q1. The current through the device is $i(t) = 2\cos(4000\pi t)$ A. Find the voltage across the device at $t = 0.5, 1.5, 4.75$, and 6.5 ms.

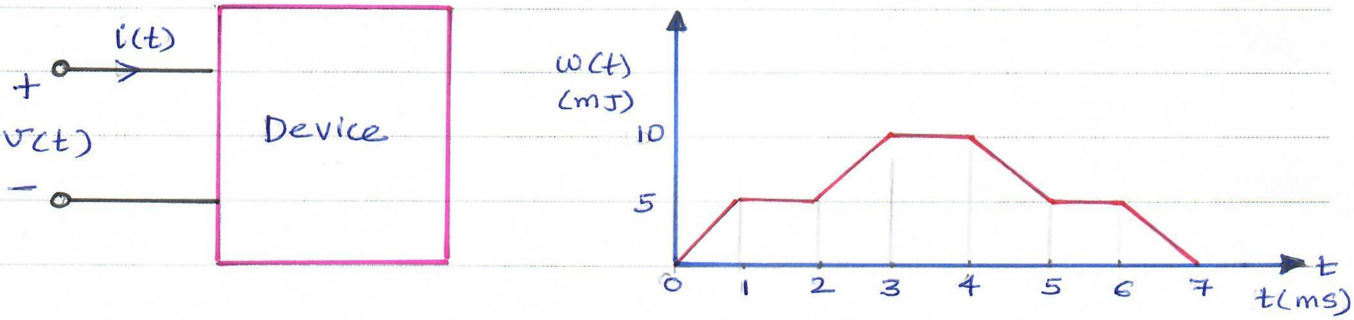

Fig. Q1

Sol. Power $= p(t) =$ Rate of change of energy $= \dfrac{dw}{dt}$

So, the power waveform can be plotted from the given energy waveform using the above equation.

The plot of $p(t)$ is shown in Fig. Q1a

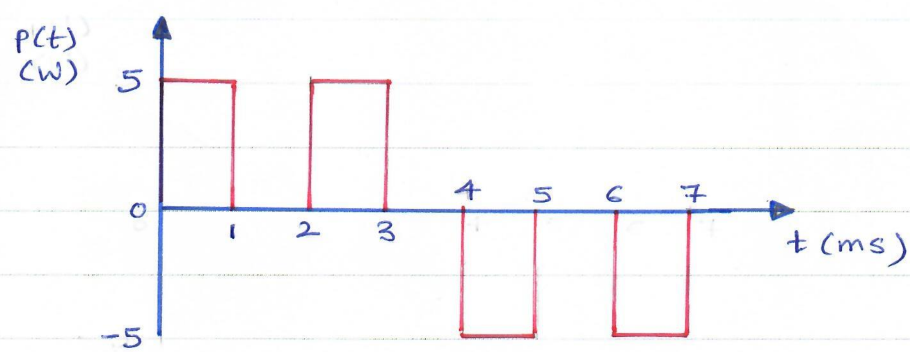

Fig. Q1a

At $t = 0.5$ ms, $p(t) = p(0.5\times10^{-3}) = 5\text{W}$ and

$i(t) = i(0.5\times10^{-3}) = 2\cos(4000\pi\times0.5\times10^{-3}) = 2\cos(2\pi) = 2\text{A}$

$p(t) = v(t)\,i(t) \Rightarrow v(t) = p(t)/i(t)$

$\therefore v(0.5\times10^{-3}) = p(0.5\times10^{-3})/i(0.5\times10^{-3}) = 5/2 = 2.5\text{V}$

At $t = 1.5\,ms$, $p(t) = 0$, $i(t) = 2\cos(4000\pi \times 1.5\times10^{-3}) = 2\cos(6\pi) = 2A$

$v(1.5\times10^{-3}) = p(1.5\times10^{-3}) / i(1.5\times10^{-3}) = 0/2 = 0$

At $t = 4.75\,ms$, $p(t) = -5$, $i(t) = 2\cos(4000\pi \times 4.75\times10^{-3}) = 2\cos(19\pi)$

$= -2A$

$v(4.75\times10^{-3}) = p(4.75\times10^{-3}) / i(4.75\times10^{-3}) = -5/-2 = 2.5V$

At $t = 6.5\,ms$, $p(t) = -5$; $i(t) = 2\cos(4000\pi \times 6.5\times10^{-3}) = 2\cos(26\pi)$

$= 2A$

$v(6.5\times10^{-3}) = p(6.5\times10^{-3}) / i(6.5\times10^{-3}) = -5/2 = -2.5V$

Q2 Find the power absorbed or supplied by the element 3 of the circuit shown in Fig. Q2

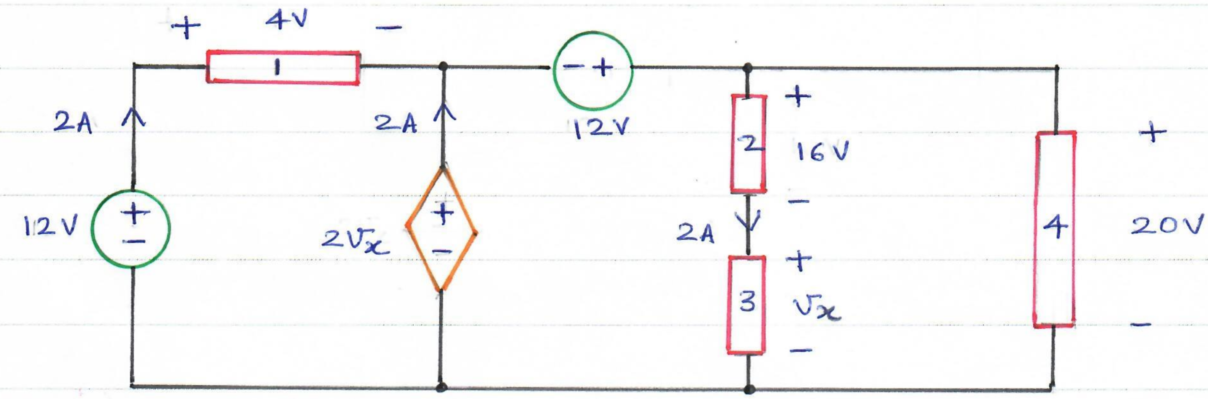

Fig. Q2

Sol. Labelling the nodes in the circuit as shown in Fig. Q2a

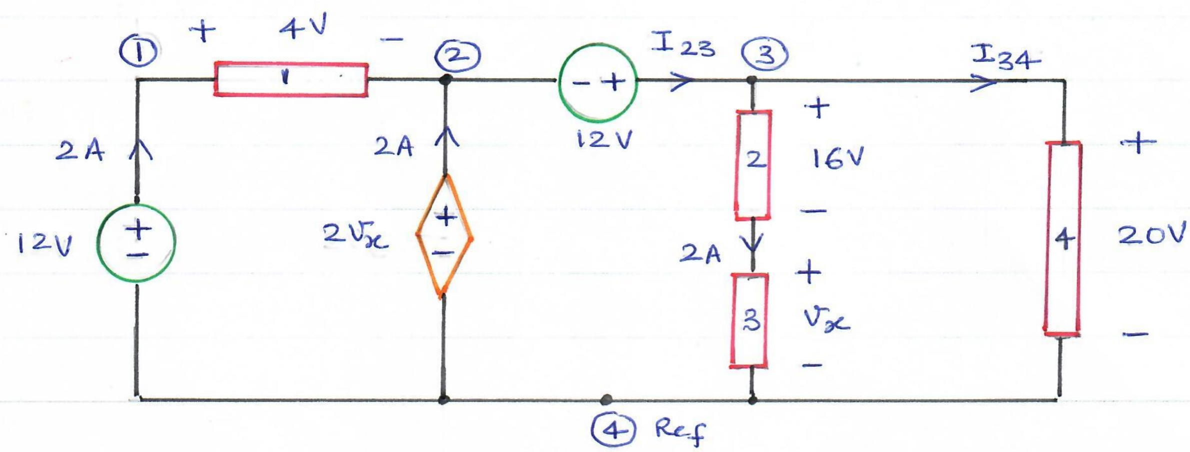

Fig. Q2a

Current through the Voltage source (12V) can be found by applying KCL at node 2.

$-2 - 2 + I_{23} = 0$ (currents moving away from node are considered positive)

$\Rightarrow I_{23} = 4A$ (current flowing from node 2 to 3)

current through element 4 from node 3 to 4 can be obtained by applying KCL at node 3.

$-I_{23} + 2 + I_{34} = 0$

$\Rightarrow I_{34} = I_{23} - 2 = 4 - 2 = 2A$

The voltage v_x across element-3 can be obtained by applying KVL around loop formed by elements 2, 3, and 4 in clockwise direction.

$$v_x + 16 - 20 = 0 \quad \text{(potential rises are considered +ve)}$$

$$\Rightarrow v_x = 4\,V$$

The polarity of the voltage and direction of current through element-3 is as shown in Fig. Q2b below.

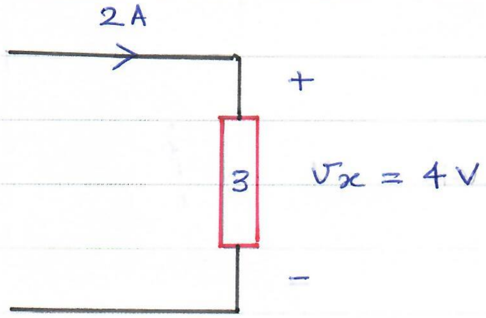

Fig. Q2b

Power absorbed by element-3 (using passive sign convention) is $2 \times 4 = 8\,W$.

Since the value is positive, the power is absorbed by element-3.

Q3 In the circuit shown in Fig. Q3, compute the power delivered & absorbed by the dependent source.

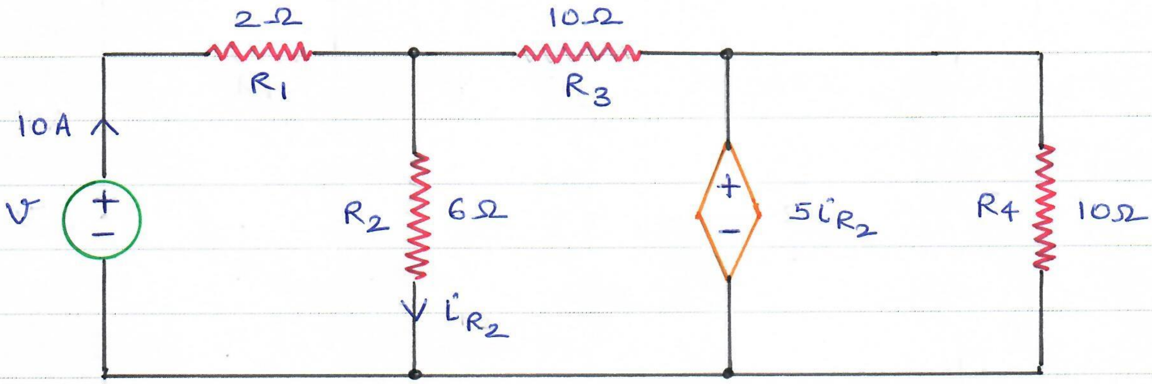

Fig. Q3

Sol. Labelling nodes and loops in the circuit shown in Fig. Q3a

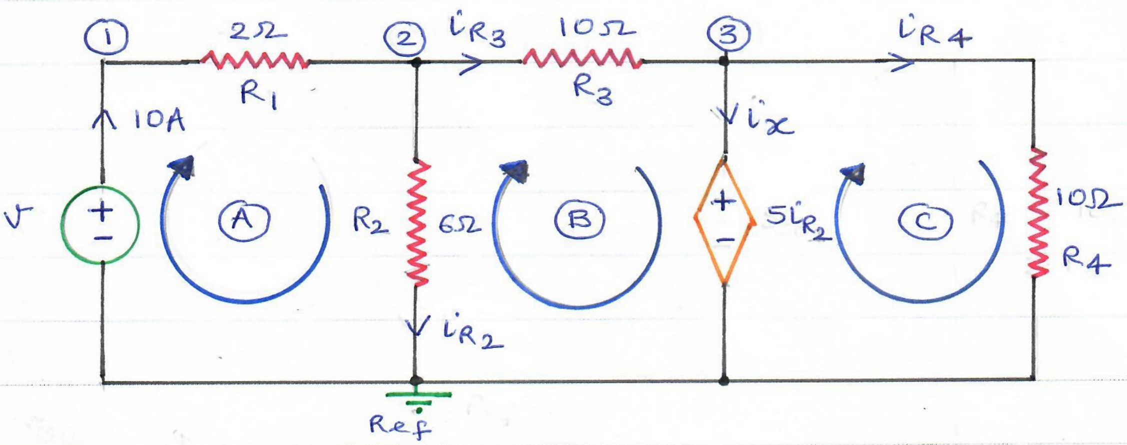

Fig. Q3 a

Applying KCL at node 2

$-10 + i_{R_2} + i_{R_3} = 0$ (currents leaving node are considered +ve)

$\Rightarrow \quad i_{R2} + i_{R_3} = 10 \quad \ldots \quad (I)$

Applying KVL around loop B in clockwise direction (potential rises are considered +ve)

$6 i_{R_2} - 10 i_{R_3} - 5 i_{R_2} = 0$

$\Rightarrow \quad i_{R_2} - 10 i_{R_3} = 0 \quad \ldots \quad (II)$

Solving equations I and II

$i_{R_2} = 100/11$ A and $i_{R_3} = 10/11$ A

Voltage across dependent source = voltage across R_4.

$\therefore \; 5 i_{R_2} = 10 i_{R_4}$

$\Rightarrow i_{R_4} = i_{R_2}/2 = 50/11$ A

current through the dependent source can be computed by applying KCL at node 3.

$-i_{R_3} + i_x + i_{R_4} = 0$

$\Rightarrow i_x = i_{R_3} - i_{R_4} = \frac{10}{11} - \frac{50}{11} = -\frac{40}{11}$ A

considering only the dependent source

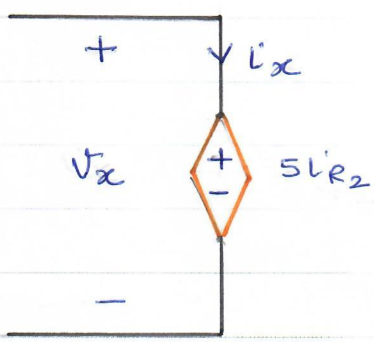

Fig. Q3b

$i_x = -40/11$ A and $v_x = 5 i_{R_2} = 500/11$ V

The power absorbed by the dependent source (using passive sign convention) = $v_x i_x$

$v_x i_x = \frac{500}{11} \times -\frac{40}{11} = \frac{-20000}{121}$ W

−ve sign indicates that actually power of 20000/121 W is supplied by the dependent source.

Q4 Find v_o in the circuit shown in Fig. Q4.

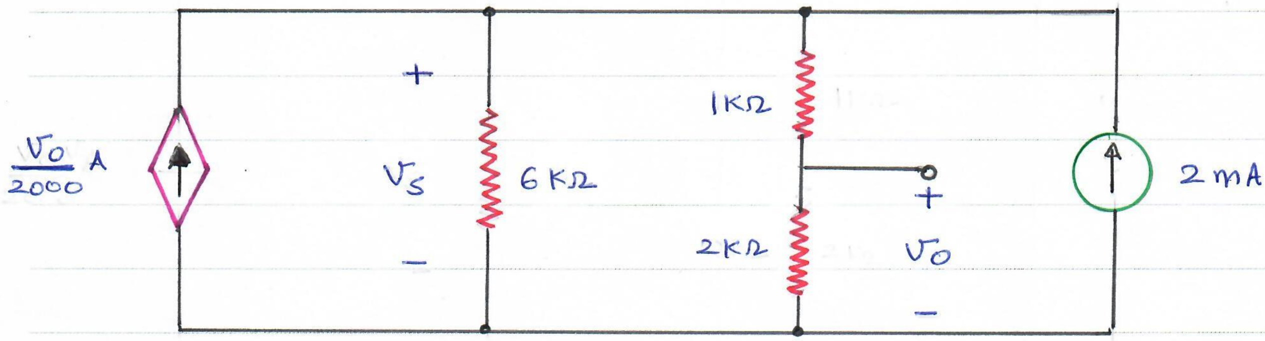

Fig. Q4

Sol. Labeling the nodes in the circuit as shown in Fig. Q4a

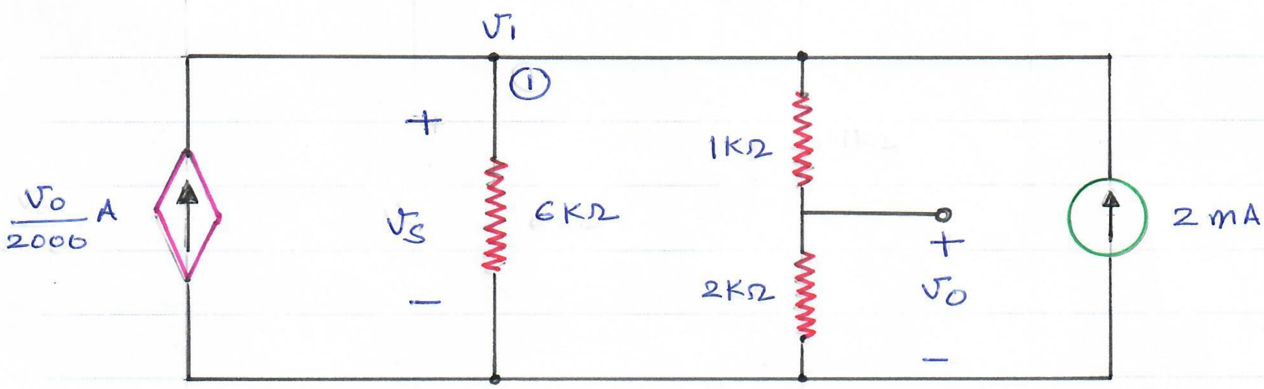

Fig. Q4a

Applying KCL at node 1

$$-\frac{v_o}{2000} + \frac{v_1}{6\times10^3} + \frac{v_1}{(2+1)\times10^3} - 2\times10^{-3} = 0 \quad \ldots (I)$$

and

$$v_o = \frac{2\times10^3}{(2+1)\times10^3} v_1 = \frac{2}{3} v_1$$

$$\Rightarrow v_1 = 3/2\, v_o \quad \ldots \quad (II)$$

Substituting the value of v_1 from eq. (II) in equation (I)

$$-\frac{v_o}{2000} + \frac{(3/2)v_o}{6000} + \frac{(3/2)v_o}{3000} = \frac{2}{1000}$$

$$\Rightarrow \frac{-v_o}{2000} + \frac{v_o}{4000} + \frac{v_o}{2000} = \frac{2}{1000}$$

$$\Rightarrow -\frac{v_o}{2} + \frac{v_o}{4} + \frac{v_o}{2} = 2$$

$$\Rightarrow v_o = 8\ V$$