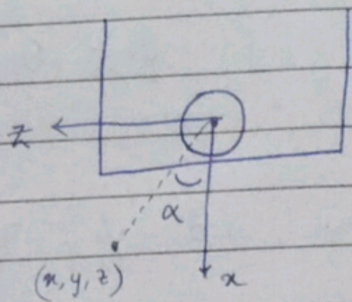
INVERSE KINEMATICS

\# Object at (x, y, z)



$\alpha = \text{Atan2}(z, x)$

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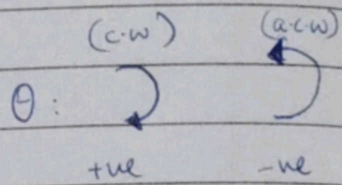
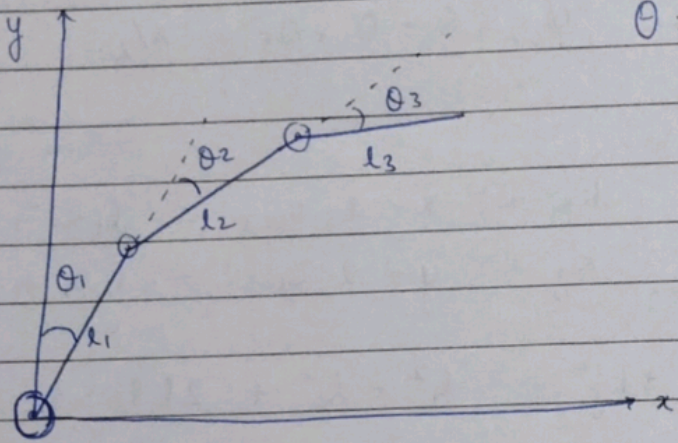
1^{st} motor rotation

↳ align object along x-axis





l_1 = joint to joint distance of 1^{st} arm

l_2 = " " " 2^{nd} arm

l_3 = joint to center of claw/gripper distance.

θ_1 = angle of 1^{st} arm from vertical

θ_2 = angle of 2^{nd} arm from arm 1

θ_3 = " 3^{rd} arm from arm 2

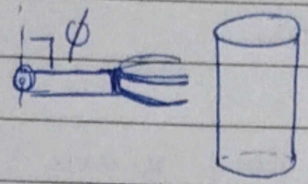
Now,

$$x = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3)$$

$$y = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)$$

Now if $\phi = \theta_1 + \theta_2 + \theta_3$: angle of arm 3 / gripper from vertical.

we need $\phi = \pi/2$ for parallel handelling of the cylinder



ie $\phi = \theta_1 + \theta_2 + \theta_3 = \pi/2$ } or, we can specify ϕ, later

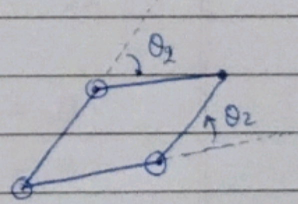
~~Note~~ $\Rightarrow$

$$k_x = x - l_3 s_{123} = l_1 s_1 + l_2 s_{12}$$
$$k_y = y - l_3 c_{123} = l_1 c_1 + l_2 c_{12}$$

$$k_x^2 + k_y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 s_1 s_{12} + 2 l_1 l_2 c_1 c_{12}$$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 c_2$$

$$\boxed{c_2 = \frac{k_x^2 + k_y^2 - l_1^2 - l_2^2}{2 l_1 l_2}}$$



and $s_2 = \pm \sqrt{1 - c_2^2}$

{ +ve when elbow is up
{ -ve when elbow is down

$$\therefore \boxed{\theta_2 = \mathrm{Atan2}(s_2, c_2)}$$

Again,

$$k_x = x - l_3 S_{123} = l_1 S_1 + l_2 S_1 C_2 + l_2 S_2 C_1$$
$$k_y = y - l_3 C_{123} = l_1 C_1 + l_2 C_1 C_2 - l_2 S_1 S_2$$

$$k_x = (l_1 + l_2 C_2) S_1 + (l_2 S_2) C_1$$
$$k_y = (-l_2 S_2) S_1 + (l_1 + l_2 C_2) C_1$$

$$\begin{bmatrix} k_x \\ k_y \end{bmatrix} = \underbrace{\begin{bmatrix} l_1 + l_2 C_2 & l_2 S_2 \\ -l_2 S_2 & l_1 + l_2 C_2 \end{bmatrix}}_{M} \begin{bmatrix} S_1 \\ C_1 \end{bmatrix}$$

premultiply by M^{-1}

$$M^{-1} = \frac{1}{(l_1 + l_2 C_2)^2 + (l_2 S_2)^2} \begin{bmatrix} l_1 + l_2 C_2 & -l_2 S_2 \\ l_2 S_2 & l_1 + l_2 C_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S_1 \\ C_1 \end{bmatrix} = \frac{1}{(l_1 + l_2 C_2)^2 + (l_2 S_2)^2} \begin{bmatrix} l_1 + l_2 C_2 & -l_2 S_2 \\ l_2 S_2 & l_1 + l_2 C_2 \end{bmatrix} \begin{bmatrix} k_x \\ k_y \end{bmatrix}$$

<u>Notice:</u> $k_x^2 + k_y^2 = (l_1 + l_2 C_2)^2 + (l_2 S_2)^2$

$$S_1 = \frac{(l_1 + l_2 C_2) k_x - (l_2 S_2) k_y}{k_x^2 + k_y^2}$$

$$C_1 = \frac{(l_1 + l_2 C_2) k_y + (l_2 S_2) k_x}{k_x^2 + k_y^2}$$

and $$\boxed{\theta_1 = A\tan 2(s_1, c_1)}$$

$$\theta_3 = \phi - \theta_1 - \theta_2$$

here

$$A\tan 2(y, x) = \begin{cases} \arctan(y/x) & x > 0 \\ \arctan(y/x) + \pi & x < 0,\ y \geq 0 \\ \arctan(y/x) - \pi & x < 0,\ y < 0 \\ +\pi/2 & x = 0 \quad y > 0 \\ -\pi/2 & x = 0 \quad y < 0 \\ \text{undefined} & x = 0 \quad y = 0 \end{cases}$$

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2-argument arctangent