Remainder Theorem

1.Find x such that 3x ≡ 7 (mod 10)

- x ≡19 (mod 10).
- x ≡ 9 (mod 11).
- x ≡ 7 (mod 10).
- **x ≡ 9 (mod 10).**

Ans: x ≡ 9 (mod 10).

> **Solution.** Based on our previous work, we know that 3 has a multiplicative inverse modulo 10, namely $3^{\varphi(10)-1}$. Moreover, $\varphi(10) = 4$, so the inverse of 3 modulo 10 is $3^3 \equiv 27 \equiv 7 \pmod{10}$. Hence, multiplying both sides of the above equation by 7, we obtain
>
> $$\begin{aligned} & 3x \equiv 7 \pmod{10} \\ \Leftrightarrow\ & 7 \cdot 3x \equiv 7 \cdot 7 \pmod{10} \\ \Leftrightarrow\ & x \equiv 49 \equiv 9 \pmod{10} \end{aligned}$$
>
> Hence, the solution is $x \equiv 9 \pmod{10}$.

2. Which of the following statements is true regarding the Chinese Remainder Theorem?

- It can only be applied to solve systems of congruences with prime moduli.
- **It can be applied to solve systems of congruences with coprime moduli.**
- It can be applied to solve systems of congruences with any moduli.
- It can only be applied to solve systems of congruences with integer moduli.

Ans:

- It can be applied to solve systems of congruences with coprime moduli.

Explanation: The Chinese Remainder Theorem (CRT) is applicable when the moduli are pairwise coprime (i.e., their greatest common divisors are 1). This property allows for a unique solution to the system of congruences.

3.What will be the output of the following code?

```
int num = 8;
int divisor = 3;
int quotient = num / divisor;
int remainder = num % divisor;
System.out.println(quotient + " " + remainder);
```

Ans:**2 2**

Explanation:

- **quotient** is calculated as the result of integer division (8 / 3), which is 2.
- **remainder** is calculated as the remainder of the division (8 % 3), which is 2.
- The **System.out.println** statement then prints the values of **quotient** and **remainder** separated by a space.

4.Consider a number that leaves a remainder of 2 when divided by 3, a remainder of 4 when divided by 5, and a remainder of 6 when divided by 7. What is the number according to the Chinese Remainder Theorem?

- **114**
- 119
- 123
- 456

Ans: 114

5.What is the remainder theorem?

- It is a theorem in calculus that relates to finding the remainder of a polynomial function after dividing it by another polynomial function.
- It is a theorem in number theory that states that for any integer n and any integer a, there exists a unique integer q and r such that n = aq + r, where r is the remainder.
- **It is a theorem in algebra that states that if a polynomial f(x) is divided by x-a, then the remainder is equal to f.**
- It is a theorem in statistics that relates to finding the remainder of a set of data after dividing it by another set of data.

Ans:
The correct statement describing the Remainder Theorem is:

It is a theorem in algebra that states that if a polynomial $f(x)$ is divided by $x-a$, then the remainder is equal to $f(a)$.

6.The Chinese Remainder Theorem is often used in which field?

- Number theory
- Geometry
- Algebra
- Calculus

Ans: **Number theory**

7.Which of the following is a disadvantage of the Chinese Remainder Theorem?

- It requires advanced knowledge of complex numbers.
- It can only be applied to linear equations.
- It has limited applicability to certain types of problems.
- It is computationally intensive and time-consuming.

Ans: **It has limited applicability to certain types of problems.**

8.Consider a number that leaves a remainder of 2 when divided by 5, a remainder of 3 when divided by 7, and a remainder of 4 when divided by 9. What is the number according to the Chinese Remainder Theorem?

- 76
- **158**
- 156
- 67

Ans:158

9.$x \equiv 1 \pmod 5$
$x \equiv 3 \pmod 7$
What is the smallest positive integer that satisfies these congruences using the Chinese Remainder Theorem?

- **17**
- 11
- 10
- 13

Ans: 17

10.In cryptography, the Chinese Remainder Theorem is used for:

- Generating random numbers.
- Encrypting messages.
- Decrypting messages.
- Hashing algorithms.

Ans: **Decrypting messages.**