EULER'S PHI FUNCTION

1. What is Euler's phi function?

- A function that counts the number of divisors of an integer.
- A function that counts the number of prime factors of an integer.
- **A function that counts the number of integers less than a given integer n that are relatively prime to n.**
- A function that counts the number of integers less than a given integer n that are
  divisible by n.

Ans: **A function that counts the number of integers less than a given integer n that are relatively prime to n.**

2.What is the relationship between the values of $\phi(n)$ and $\phi(p^k)$ for prime p and positive integer k?

- $\phi(n) = p^k - 1$
- B)$\phi(n) = p^{(k-1)}$
- C)$\phi(n) = p^k$
- **D)$\phi(n) = (p-1)*p^{(k-1)}$**

Ans: D)$\phi(n) = (p-1)*p^{(k-1)}$

3. What is the relationship between the values of $\phi(n)$ and $\phi(m)$ for coprime positive integers n and m?

- $\phi(nm) = \phi(n) + \phi(m)$
- **B)$\phi(nm) = \phi(n)\phi(m)$**
- C)$\phi(nm) = \phi(n) - \phi(m)$
- D)None of the above

Ans: B. $\phi(nm) = \phi(n)\ \phi(m)$

This property of Euler's phi function is known as multiplicativity, and it holds true for any two coprime positive integers.

4. What is the output of phi(324)?

- 98
- 90
- **108**
- 120

Ans:108

324=2power2 × 3power4 means 2^2 * 3^4.

1. Euler's phi function formula: For each prime factor pi raised to power ki, the contribution to phi(n) is (pi - 1) * pi^(ki-1).

2. Applying the formula:

- For 2^2: (2 - 1) * 2^(2-1) = 1 * 2 = 2
- For 3^4: (3 - 1) * 3^(4-1) = 2 * 3^3 = 54

3. Multiplicativity property: Since 2 and 3 are coprime, we multiply the individual contributions: phi(324) = 2 * 54 = 108

5.What is the φ function of the number 3?

- 0
- 1
- **2**
- 3

Ans: 2

The Euler's ϕ (phi) function of a prime number p is given by:

$\phi(p) = p - 1$

For $p = 3$:

$\phi(3) = 3 - 1 = 2$

So, the correct option is:

C) 2

6. What is the φ function of the number 8?

- 0
- 1
- 2
- **4**

Ans: 4

7.What is the value of phi(n) for n = 12 using Euler's phi function?

- 1
- 2
- 6
- **4**

Ans: 4

To find $\phi(12)$, we need to use Euler's totient function formula:

$$\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\left(1 - \frac{1}{p_k}\right)$$

where $p_1, p_2, \ldots, p_k$ are the distinct prime factors of n.

For $n = 12$:

$$\phi(12) = 12\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)$$

$$\phi(12) = 12 \cdot \frac{1}{2} \cdot \frac{2}{3}$$

$$\phi(12) = 4$$

So, the correct option is:

D) 4

8.What is the φ function of the number 12?

- 0
- 1
- 2
- 4

Ans: 4

The ϕ (phi) function of the number 12 is calculated using Euler's totient function formula:

$$\phi(12) = 12\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)$$

$$\phi(12) = 12 \cdot \frac{1}{2} \cdot \frac{2}{3}$$

$$\phi(12) = 4$$

So, the correct option is:

D) 4

9.Which of the following is a property of Euler's phi function?

- φ(n) is always even for any positive integer n.
- φ(n) is equal to the number of divisors of n.
- **φ(p) = p-1 for any prime number p.**
- φ(n) is always greater than n for any positive integer n.

Ans: φ(p) = p-1 for any prime number p.

This is a property of Euler's phi function.

For a prime number p, ϕ(p) is equal to p−1.

10.What is the value of Euler's Totient Function for the number 2000?

- A)789
- B)880
- C)800
- D)670

Ans: 800

$$\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\left(1 - \frac{1}{p_k}\right)$$

where $p_1, p_2, \ldots, p_k$ are the distinct prime factors of n.

For $n = 2000$, we can factorize it as follows:

$$2000 = 2^4 \times 5^3$$

Now, use the formula:

$$\phi(2000) = 2000\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{5}\right)$$

$$\phi(2000) = 2000 \cdot \frac{1}{2} \cdot \frac{4}{5}$$

$$\phi(2000) = 800$$

So, the correct option is:

C) 800