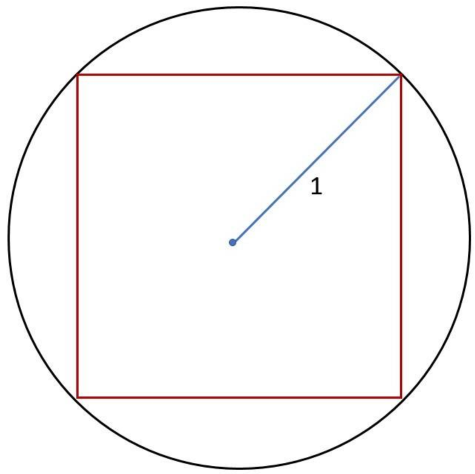
1. Consider the following figure where the value of the radius of the circle is 1. Write a Fortran code where you throw 1 lakh random points equally distributed inside the circle. Based on the fraction of points that fall inside the square calculate the value of π as a function of the number of points you have thrown. Plot the value of π versus the number of random points. [Hint: Consider polar coordinate].



2. Obtain a set of random numbers with the following distributions from 1 Lakh random numbers generated from in-built random number generator subroutine encoded in Fortran.
i. Distribution following exp(-5x), where x is random number. Plot the probability distribution of the random numbers as a function of the value of the random number.
ii. Distribution following $exp(-(x-0.5)^2/0.05)$. Plot the probability distribution of the random numbers as a function of the value of the random number.

3. Consider a 2D square lattice of size 10×10. Let us consider there is a magnetic spin at each lattice site with the spin value either +1 or -1. Starting with an initial configuration with all the spins up (+1), run Monte Carlo (MC) simulations at various temperatures (T = 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4). Plot average magnetic moment versus temperature. The length of the MC simulation at each temperature should be 1 Lakh MC steps where 1 MC step is equal to the 10 × 10 = 100 attempts to flip the spins. The total energy of the system is $E = \sum_{i,j} \sigma_i \sigma_j$ where i and j are nearest neighbors. σ_i and σ_j are the spin values of the magnetic spins at the lattice sites i and j, respectively.

4. Solve the harmonic oscillator problem, F=-kx, numerically and plot the position, velocity, kinetic energy, and potential energy as a function of time using:
i) Euler's method.
ii) Verlet Algorithm.
iii) Leapfrog Algorithm.
iv) Compare the three cases and draw the conclusions.