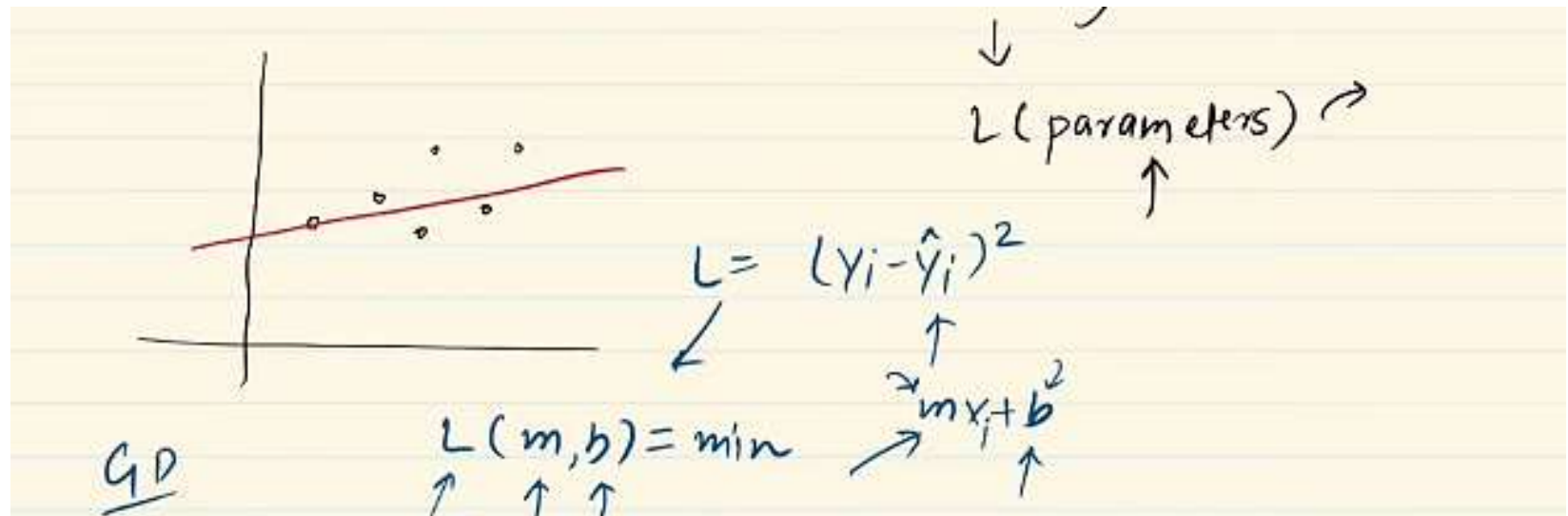


Loss function is a method of evaluating how well your algorithm is modelling your dataset.

- High means Poor Model
- Low means great Model



Why is Loss function important?

[you can't improve what you can't measure.]

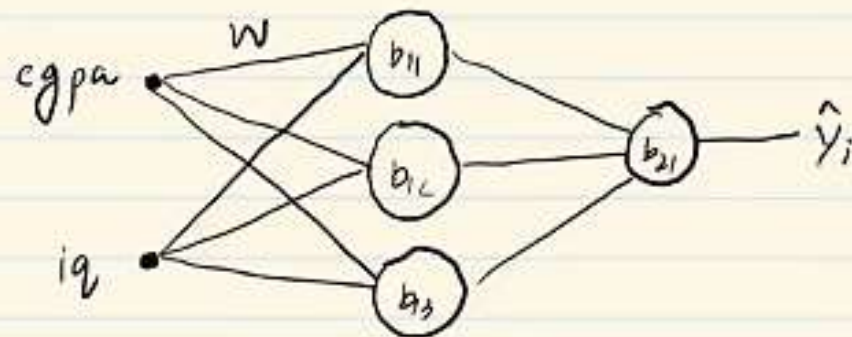
Peter Drucker

Loss Function in Deep Learning?

Backprop

cgpa | iq | package (epa)

7.1	83	3.2
8.5	91	4.5
6.3	102	6.1
5.1	87	2.7
...	...	...
...	...	...



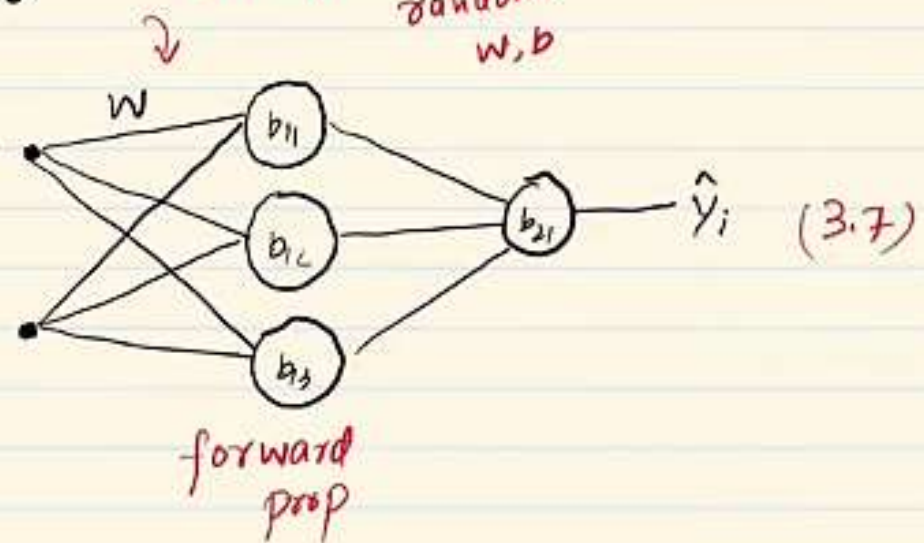
## Loss Function in Deep Learning?

✓ cgpa | iq | package (lpa)

7.1	83	3.2
8.5	91	4.5
6.3	102	6.1
5.1	87	2.7
...	...	...
...	...	...
...	...	...

7.1 cgpa

83 iq



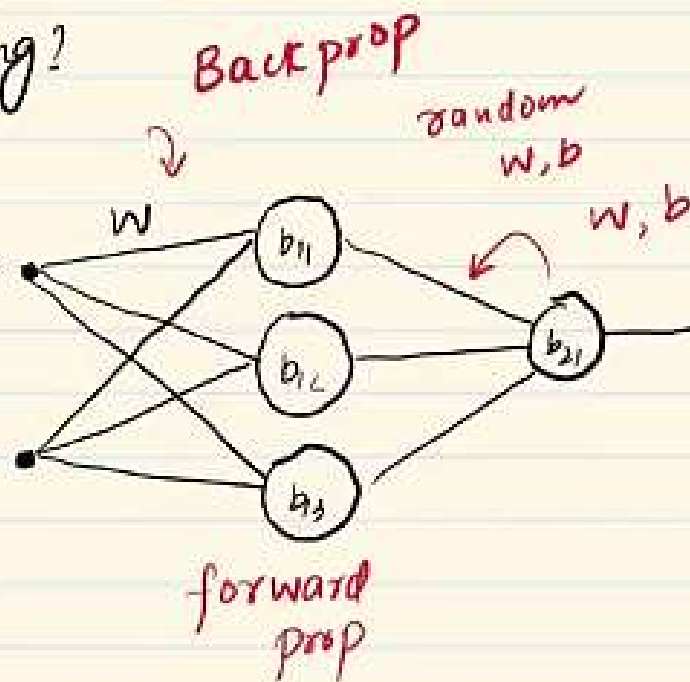
# Loss Function in Deep Learning?

✓ cgpa | iq | package(epa)

7.1	83	3.2
8.5	91	4.5
6.3	102	6.1
5.1	87	2.7
...	...	...
...	...	...
...	...	...

7.1 cgpa

83 iq



$$\hat{y}_i \quad (3.7)$$

$$\mathcal{L} = (y_i - \hat{y}_i)^2$$

$$(3.2 - 3.7)^2$$

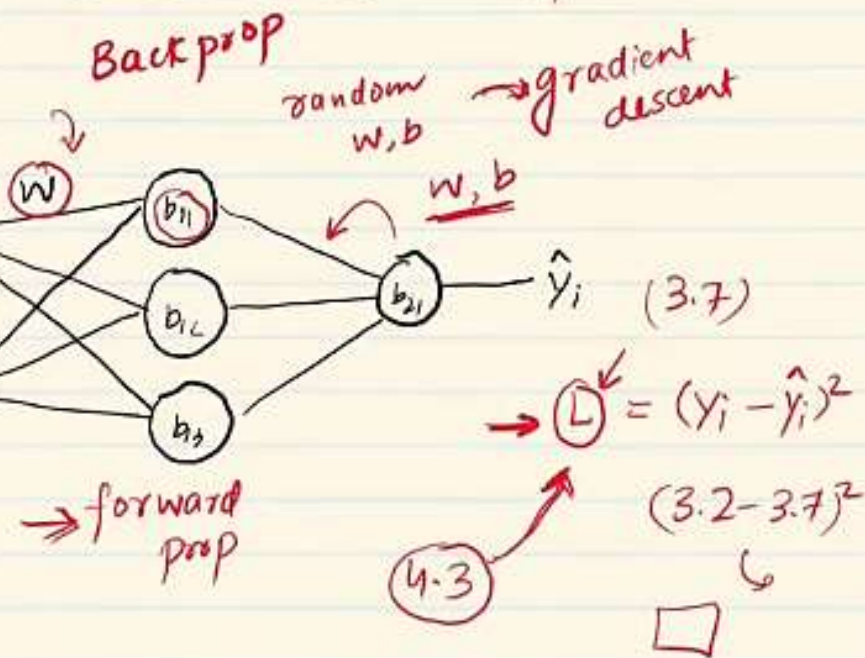
# Loss Function in Deep Learning?

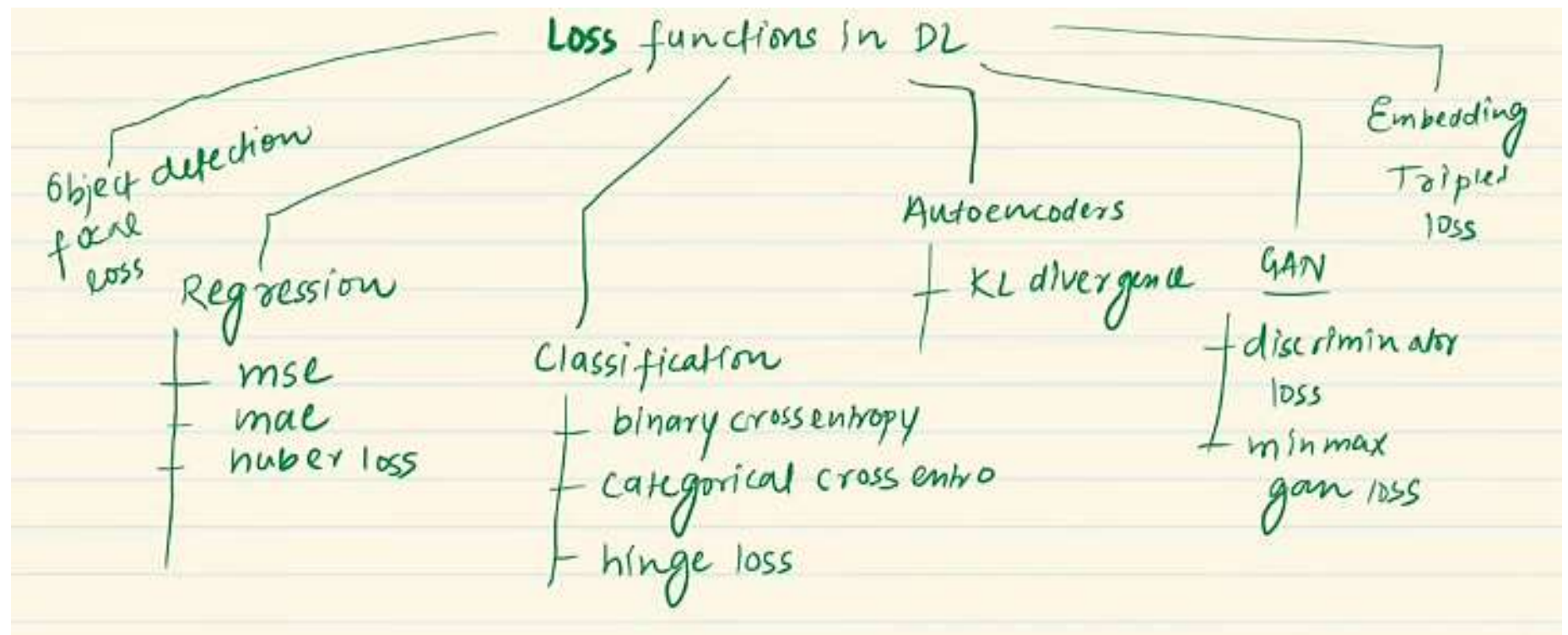
✓ cgpa | iq | package(epa)

7.1	83	3.2
8.5	91	4.5
6.3	102	6.1
5.1	87	2.7
...	...	...
...	...	...
...	...	...

7.1 cgpa

83 iq



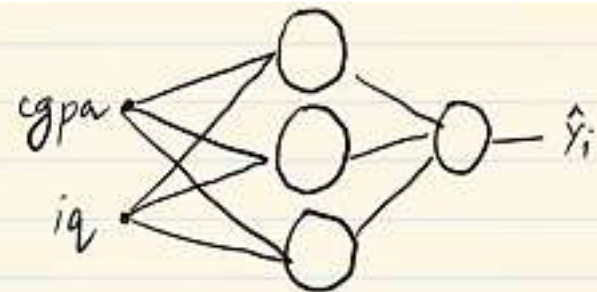




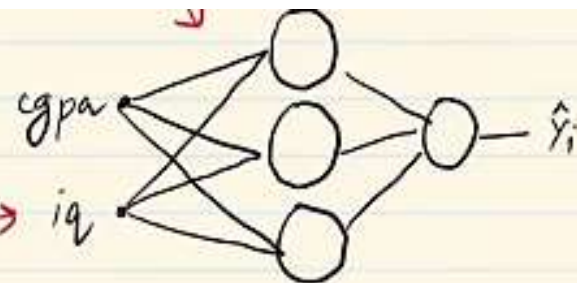
# Loss Function vs Cost Function

cgpa | iq |  $(y_i)$  package |  $\hat{y}_i$  Prediction

6.3	100	6.3	6.1
7.1	91	4.1	4
8.5	83	3.5	3.7
9.2	102	7.2	7



# Loss Function vs Cost Function



cgpa	iq	(y <sub>i</sub> ) package	y <sub>i</sub> Prediction
6.3	100	6.3	6.1
7.1	91	4.1	4
8.5	83	3.5	3.7
9.2	102	7.2	7

Loss function → single training eg -

$$y_i = 6.3 \quad \hat{y}_i = 6.1$$

$$(y_i - \hat{y}_i)^2$$

$$(6.3 - 6.1)^2 =$$

# Loss Function Vs Cost Function

cgpa | iq | package |  $\hat{y}_i$   
Prediction

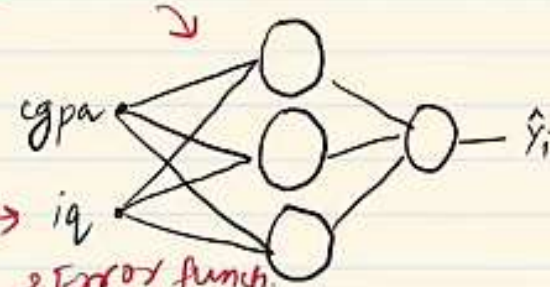
6.3	100	6.3
7.1	91	4.1
8.5	83	3.5
9.2	102	7.2

6.1
4
3.7
7

batch

Cost function

$$\frac{1}{4} [(6.1 - 6.3)^2 + (4.1 - 4)^2 + (3.5 - 3.7)^2 + (7.2 - 7)^2] = CF$$



Error function

Loss function → single training eg

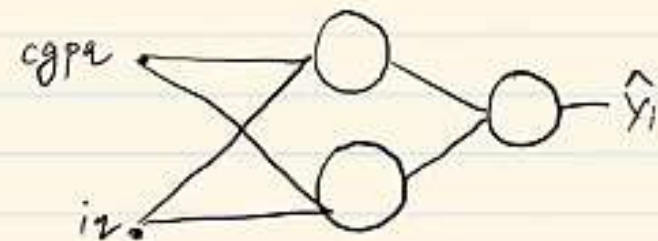
$$y_i = 6.3 \quad \hat{y}_i = 6.1$$

$$(y_i - \hat{y}_i)^2$$

$$(6.3 - 6.1)^2 =$$

1. Mean Squared Error (MSE)  
 squared loss L2 loss

$y$ cgpa	$x$ iq	$y_i$ package	$\hat{y}_i$ Prediction
6.3	100	6.3	6.1
7.1	91	4.1	4
8.5	83	3.5	3.7
9.2	102	7.2	7



# 4. Mean Squared Error (MSE)

Squared loss L2 loss

$$(y_i - \hat{y}_i)^2$$

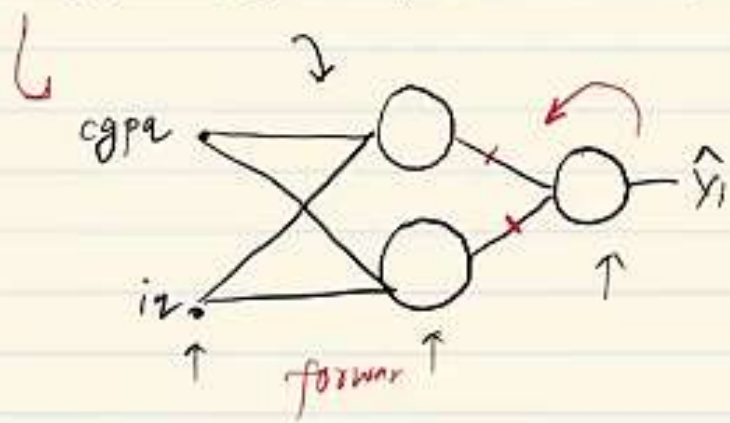
Advan  $\rightarrow$  DBADP

$(\text{true} - \text{predict})^2$

$$(6.3 - 6.1)^2 = -$$

$$(y_i - \hat{y}_i)^2$$

cgpa	iq	$y_i$ package	$\hat{y}_i$ Prediction
6.3	100	6.3	6.1
7.1	91	4.1	4
8.5	83	3.5	3.7
9.2	102	7.2	7





# 1. Mean Squared Error (MSE)

Squared loss L2 Loss

$$(y_i - \hat{y}_i)^2$$

$$(true - predict)^2$$

$$(6.3 - 6.1)^2 = -$$

$$(y_i - \hat{y}_i)^2$$

$$(y_i - \hat{y}_i)^2$$

Advan DBAAs

$y_i$	$\hat{y}_i$	
cgpa	iq	package
6.3	100	6.3
7.1	91	4.1
8.5	83	3.5
9.2	102	7.2
		Prediction
		6.1
		4
		3.7
		7

$y_i - \hat{y}_i$

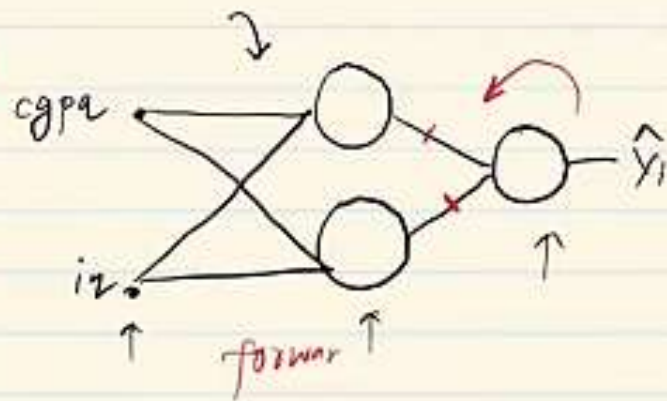
0.2

0.1

-0.2

0.2

Overall error



## 1. Mean Squared Error (MSE)

Squared loss L2 loss

$$(y_i - \hat{y}_i)^2$$

$$(true - predict)^2$$

$$(6.3 - 6.1)^2 = -$$

$$(y_i - \hat{y}_i)^2$$

Advant  
mae  
DRAAs

$$\frac{(y_i - \hat{y}_i)^2}{2}$$

punish

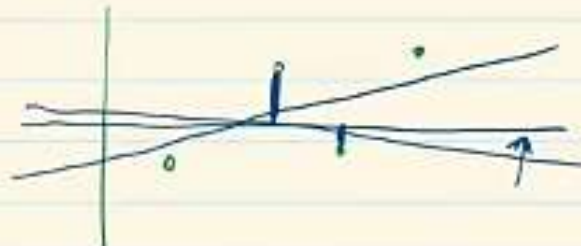
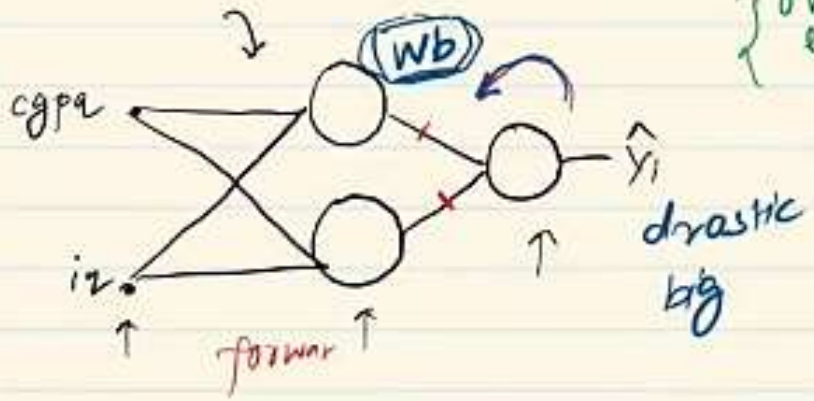
true - predic

magnify

1 unit → 1 unit  
2 unit → 4 unit  
4 unit → 16 unit

cgpa	iq	package	Prediction	$y_i - \hat{y}_i$
6.3	100	6.3	6.1	0.2
7.1	91	4.1	4	0.1
8.5	83	3.5	3.7	-0.2
9.2	102	7.2	7	0.2

overall error



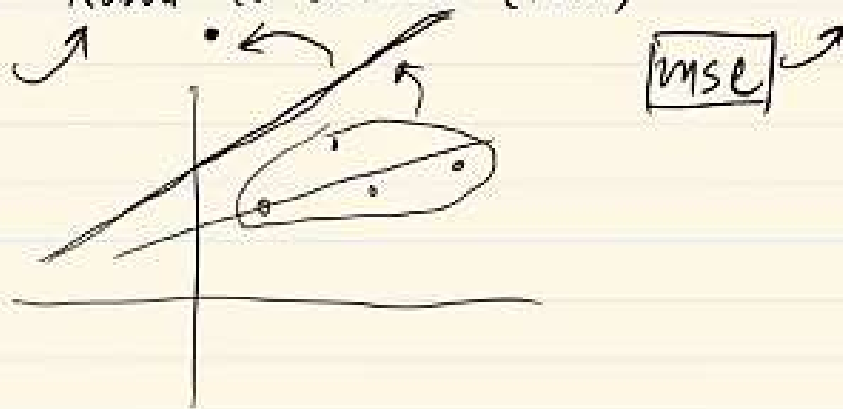


### Advantages

- 1) Easy to interpret
- 2) Differentiable (GD)
- 3) 1 local minima

### Disadvantage

- 1) Error unit (squared)  $\rightarrow$  diff
- 2) Robust to Outliers (Not)





## 1. Mean Squared Error (MSE)

Squared Loss L2 Loss

$$(y_i - \hat{y}_i)^2$$

$$(true - predict)^2$$

$$(6.3 - 6.1)^2 = -$$

$$(y_i - \hat{y}_i)^2$$

Advan DBAs

mae

$$(y_i - \hat{y}_i)^2$$

quadratic  $\wedge^2$

punish

cgpa | iq | package | Prediction  $(y_i - \hat{y}_i)$

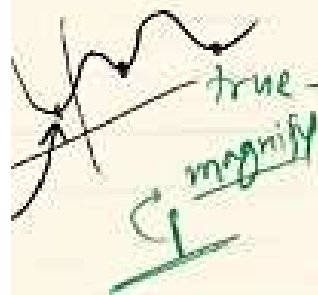
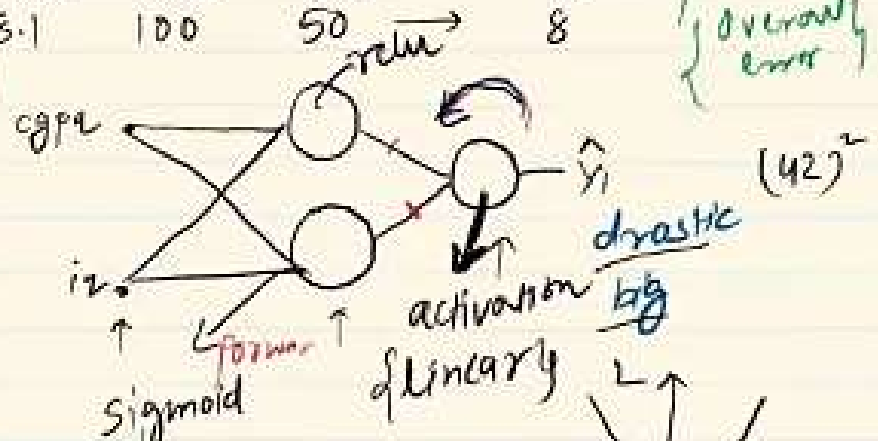
6.3	100
7.1	91

8.5	83
9.2	102
8.1	100

6.3	6.1
4.1	4
3.5	3.7
7.2	7
50	8

0.2
0.1
-0.2
0.2

overall error

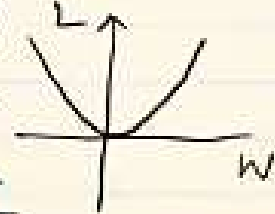


true - predic

$$(y_i - \hat{y}_i)^2$$

1 unit → 1 unit  
2 unit → 4 unit  
4 unit → 16 unit

$$(w, b) \rightarrow \text{min}$$



2. Mean Absolute Error (MAE)  $\rightarrow$   $L_1$  loss

$$L = |y_i - \hat{y}_i|$$

$$C = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$$

$\xrightarrow{\text{punish}}$   $\xrightarrow{\text{2/abs}}$   $\xrightarrow{\text{true-predict}}$

$2 \text{ lpa}$   
 $CSP | y | \text{ pace lpa}$   
 $|y_i - \hat{y}_i|$   
 $\hookrightarrow \text{lpa}$

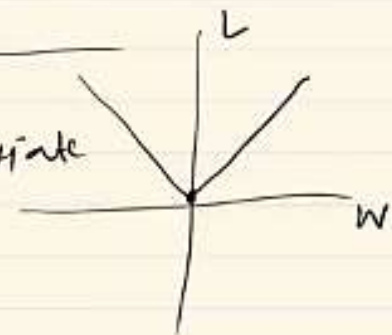
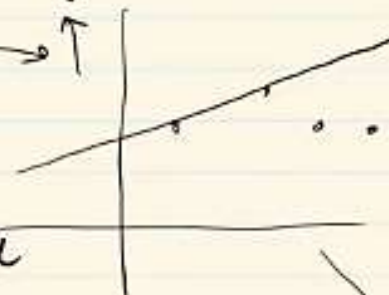
Advantages

- 1) Intuitive and easy
- 2) Unit  $\rightarrow$  same - y
- 3) Robust to outliers

Disadvantage

Not differentiable

$\hookrightarrow$  GD  $\rightarrow$  differentiate  
 $\hookrightarrow$  Subgradient

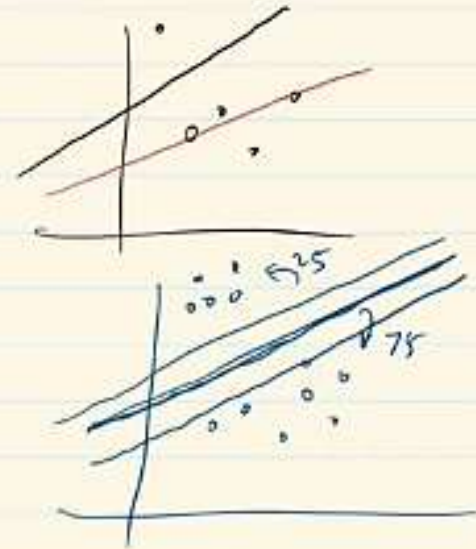


3. Huber Loss ✓

$$\rightarrow L = \begin{cases} \frac{1}{2} (y - \hat{y})^2 & \text{for } |y - \hat{y}| \leq \delta \\ \delta |y - \hat{y}| - \frac{1}{2} \delta^2 & \text{otherwise} \end{cases}$$

mse - outliers ✓

mae - normal points ✓



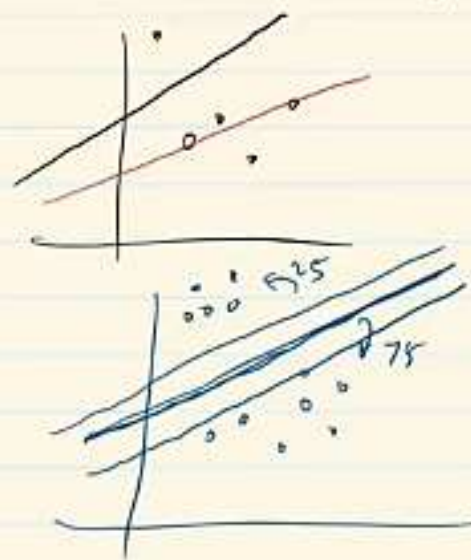
3. Huber Loss ✓

$$L = \begin{cases} \frac{1}{2} (y - \hat{y})^2 & \text{for } |y - \hat{y}| \leq \delta \\ \delta |y - \hat{y}| - \frac{1}{2} \delta^2 & \text{otherwise} \end{cases}$$

Annotations:   
 -  $\frac{1}{2} (y - \hat{y})^2$  is labeled **mse** (Mean Squared Error).   
 -  $\delta |y - \hat{y}| - \frac{1}{2} \delta^2$  is labeled **mae** (Mean Absolute Error).   
 - The entire expression is labeled **Huber**.   
 - The condition  $|y - \hat{y}| \leq \delta$  is labeled **Subgradient**.

hyper

mse - outliers ✓   
 mae - normal point ✓



#### 4. Binary Cross Entropy

- classification
- Two classes

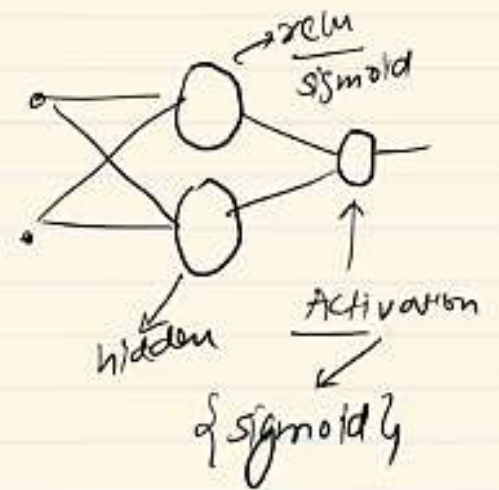
cgpa / iq / placement → 1 0

8	80	1
7	70	0
6	60	0

$$\text{Loss function} = -y \log(\hat{y}) - (1-y) \log(1-\hat{y})$$

$y$  → actual value / target

$\hat{y}$  → NN prediction



#### 4. Binary Cross Entropy

- classification
- Two classes

cgpa | iq | placement

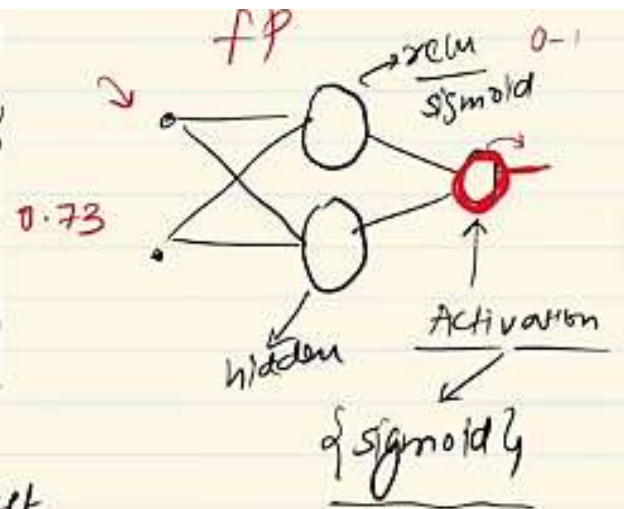
8	80	1
7	70	0
6	60	0

Loss function =  $-y \log(\hat{y}) - (1-y) \log(1-\hat{y})$

$y \rightarrow$  actual value / target

$\hat{y} \rightarrow$  NN prediction

Cost function =  $-\frac{1}{n} \left[ \sum_{i=1}^n x_i \log \hat{y}_i + (1-x_i) \log (1-\hat{y}_i) \right]$





#### 4. Binary Cross Entropy

- classification ✓
- Two classes ✓

100 days

cgpa / iq / placement

8	80	1
7	70	0
6	60	0

0.12

$$\text{Loss function} = -y \log(\hat{y}) - (1-y) \log(1-\hat{y})$$

Keras

$$- (1-0) \log(1-0.25) - 1 \log(0.75)$$

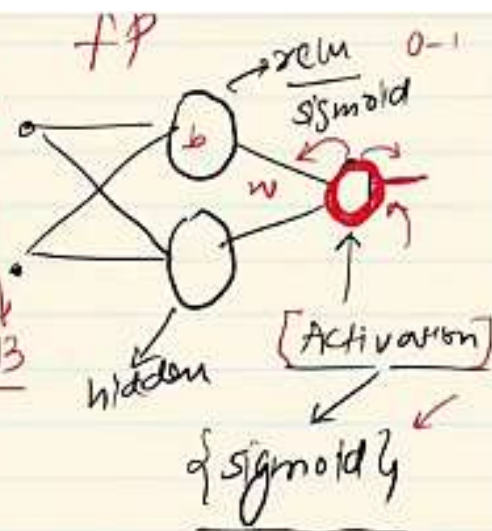
$$\text{cost function} = -\frac{1}{n} \left[ \sum_{i=1}^n x_i \log \hat{y}_i + (1-x_i) \log(1-\hat{y}_i) \right]$$

maximum likelihood

$$-1 \log(0.75)$$

$$-1 \times -0.13 = 0.13$$

Logistic Reg



## 5. Categorical Cross Entropy [used in Softmax Regression] ↓

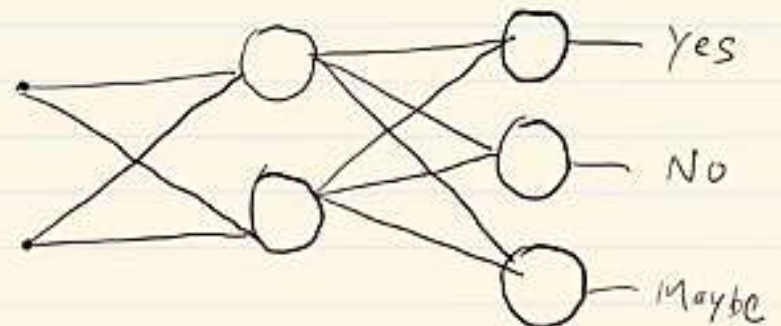
→ [Multi-class] classification

$$L = - \sum_{j=1}^K x_j \log(\hat{y}_j)$$

where K is #classes in the data

cgpa	iq	placed?
8	80	Yes 1
6	60	No 2
7	70	Maybe 3

Yes	No	maybe
1	0	0
0	1	0
0	0	1





## 5. Categorical Cross Entropy [used in Softmax Regression] ↓

→ [Multi-class] classification

1 point

$$L = - \sum_{j=1}^K y_j \log(\hat{y}_j)$$

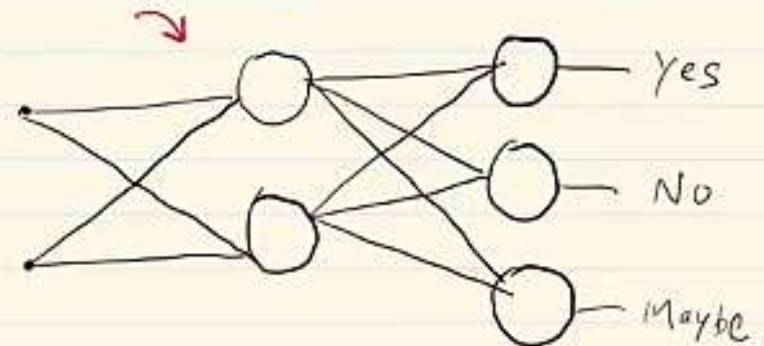
where  $K$  is # classes in the data  
↘ 3

1 point

$$L = - y_1 \log(\hat{y}_1) - y_2 \log(\hat{y}_2) - y_3 \log(\hat{y}_3)$$

cgpa	iq	placed?
8	80	Yes 1
6	60	No 2
7	70	Maybe 3

Yes	No	maybe
1	0	0
0	1	0
0	0	1



## 5. Categorical Cross Entropy [used in Softmax Regression] ↓

→ [Multi-class] classification

1 point  
→

$$L = - \sum_{j=1}^K y_j \log(\hat{y}_j)$$

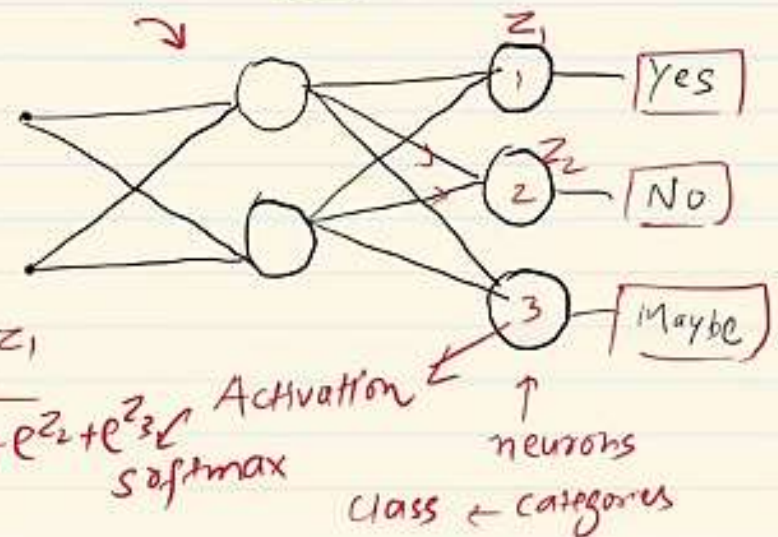
where  $K$  is # classes in the data  
↳ 3

1 point

$$L = - y_1 \log(\hat{y}_1) - y_2 \log(\hat{y}_2) - y_3 \log(\hat{y}_3)$$

$$\frac{e^{z_3}}{e^{z_1} + e^{z_2} + e^{z_3}}$$

$$f(z) = \frac{e^{z_1}}{e^{z_1} + e^{z_2} + e^{z_3}}$$



cgpa	iq	placed?
8	80	<u>Yes</u> 1
6	60	<u>No</u> 2
7	70	<u>Maybe</u> 3

Yes	No	Maybe
1	0	0
0	1	0
0	0	1

## 5. Categorical Cross Entropy [used in Softmax Regression]

→ [Multi-class] classification

$$L = - \sum_{j=1}^K y_j \log(\hat{y}_j)$$

where  $K$  is # classes in the data

cgpa	iq
8	80
6	60
7	70

placed?
Yes 1
No 2
Maybe 3

OHE

Yes	No	Maybe
1	0	0
0	1	0
0	0	1

Yes  
No  
Maybe

$$L = - y_1 \log(\hat{y}_1) - y_2 \log(\hat{y}_2) - y_3 \log(\hat{y}_3)$$

$$\begin{bmatrix} 0.2 & 0.3 & 0.5 \end{bmatrix} \frac{e^{z_3}}{e^{z_1} + e^{z_2} + e^{z_3}}$$

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \frac{e^{z_1}}{e^{z_1} + e^{z_2} + e^{z_3}}$$

$$f(z) = \frac{e^{z_1}}{e^{z_1} + e^{z_2} + e^{z_3}}$$

