Digital Signal Processing Laboratory (EE39203)

P Manoj Kumar (21IE10027)

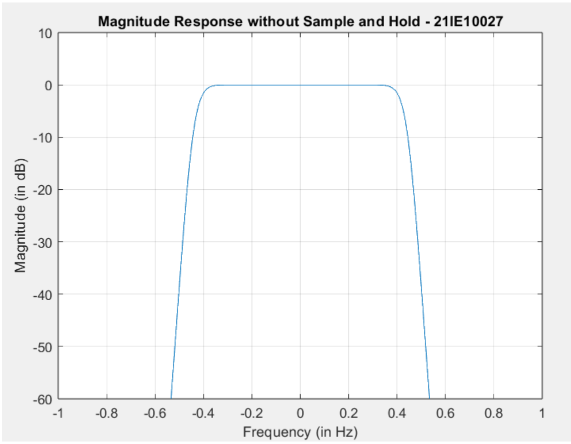
Experiment 4 - Sampling and Reconstruction of Continuous-Time Signals, Interpolation and Decimation

1 Sampling and Reconstruction Using Sample-and-Hold

1.1 Without Sample and Hold (only Two cascaded Butterworth Filters)

The MATLAB code the System and Magnitude response of the System is given below.

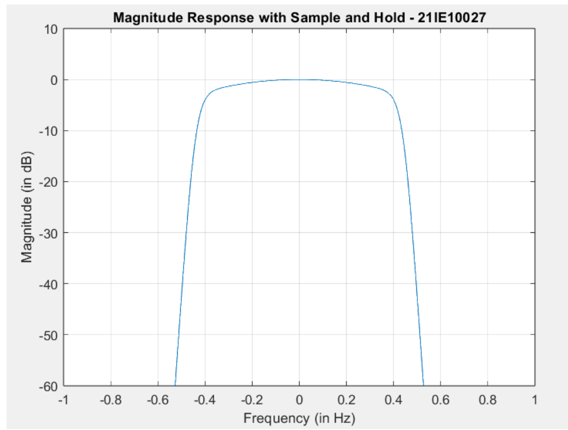
```
fs=1;
fc=0.45; % Cut-Off Frequency
N=20;    % Filter Order
f=-1:0.001:1;
% Frequency Response
% of One Butterworth Filter
H1=1./(1+(f/fc).^(N));
% Overall Frequency Response
% of Cascaded System
H_cascade= H1.^2;
plot(f,20*log10(abs(H_cascade)));
title(['Magnitude Response without' ...
    ' Sample and Hold - 21IE10027']);
xlabel('Frequency (in Hz)');
ylabel('Magnitude (in dB)');
grid on;
axis([-1 1 -60 10]);
```



1.2 With Sample and Hold and Two cascaded Butterworth Filters

The MATLAB code the System and Magnitude response of the System is given below.

```
fs=1;
fc=0.45; % Cut-Off Frequency
N=20;    % Filter Order
f=-1:0.001:1;
% Frequency Response
% of One Butterworth Filter
H1=1./(1+(f/fc).^(N));
% Sinc Function
H2= sinc(f/fs);
% Overall Frequency Response
% of Cascaded System
H_cascade= H1.*H2.*H1;
plot(f,20*log10(abs(H_cascade)));
title(['Magnitude Response with' ...
    ' Sample and Hold - 21IE10027']);
xlabel('Frequency (in Hz)');
ylabel('Magnitude (in dB)');
grid on;
axis([-1 1 -60 10]);
```



1.3 High Quality Audio CD player

1. **Magnitude Response Without Sample-and-Hold Device:** The magnitude response without the sample-and-hold device exhibits the typical characteristics of the filtering elements in the system, which consist of two 20th order Butterworth filters connected in cascade. It demonstrates a smooth, roll-off behavior with a cutoff frequency at 0.45 Hz. The response is marked by its capability to attenuate higher frequencies beyond the cutoff frequency. This reflects the analog filtering traits of the system.

2. **Magnitude Response of the Complete System with Sample-and-Hold Device:** The complete system's magnitude response, incorporating the sample-and-hold device, is shaped not only by the analog filters but also by the characteristics of the sample-and-hold operation. This includes effects like discrete-time sampling and quantization. Depending on the particulars of the sample-and-hold device, it can introduce frequency domain artifacts like aliasing (due to insufficient sampling rates) or quantization noise (due to limited bit-depth).

1.4 How the magnitude response of the sample-and-hold device might affect the design considerations of a high-quality audio CD player:

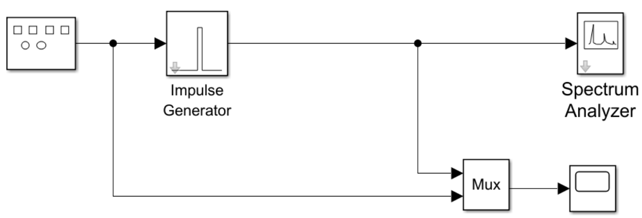
1. Aliasing Concerns: In cases where the sample-and-hold device fails to sufficiently suppress higher frequency components prior to sampling, aliasing may occur. Aliasing is an undesirable

phenomenon in which higher frequency components wrap back into the audible range, leading to distortion. When developing a top-tier CD player, meticulous anti-aliasing filtering and high sample rates become imperative in order to guarantee that the magnitude response of the sample-and-hold device doesn't introduce discernible audio artifacts.

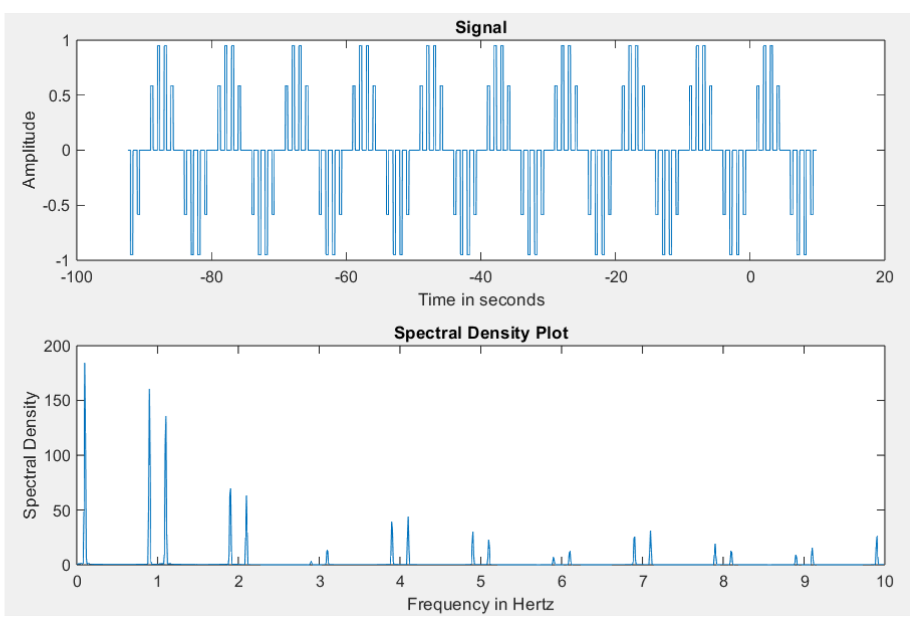
2. Quantization Noise: The bit-depth of the sample-and-hold device defines the precision of the digital representation of the analog signal. A lower bit-depth can lead to quantization noise, which may manifest as an audible hiss. Premium CD players typically employ converters with high bit-depth to reduce quantization noise to imperceptible levels.

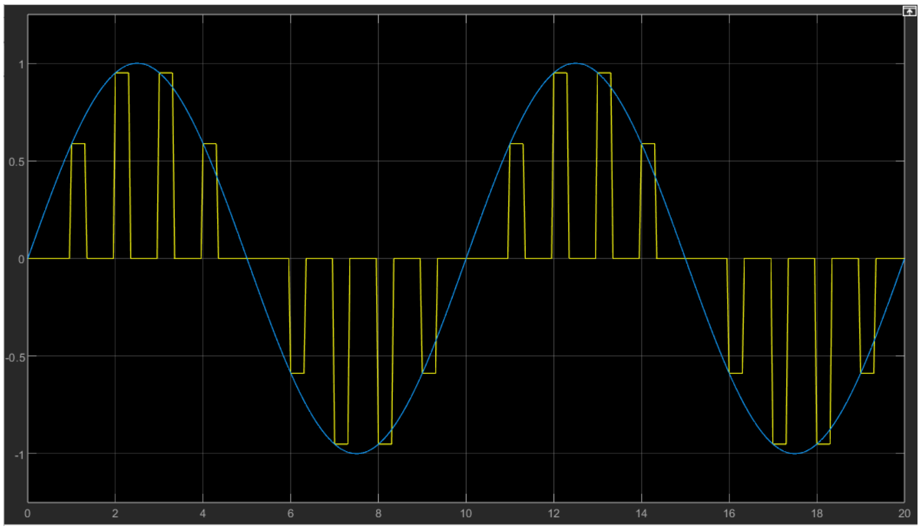
3. Reconstruction Filtering: Following the sample-and-hold stage, the signal usually undergoes reconstruction filtering to eliminate the "staircase" artifacts resulting from discrete sampling. The design and quality of this filter are pivotal in achieving a high-fidelity audio output.

2 Sampling and Reconstruction with an Impulse Generator

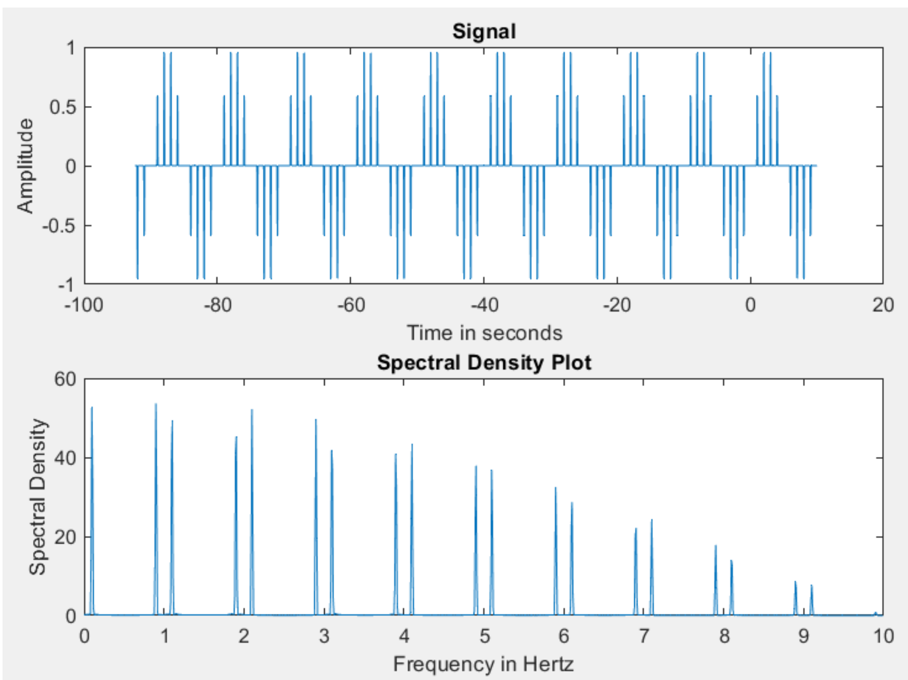


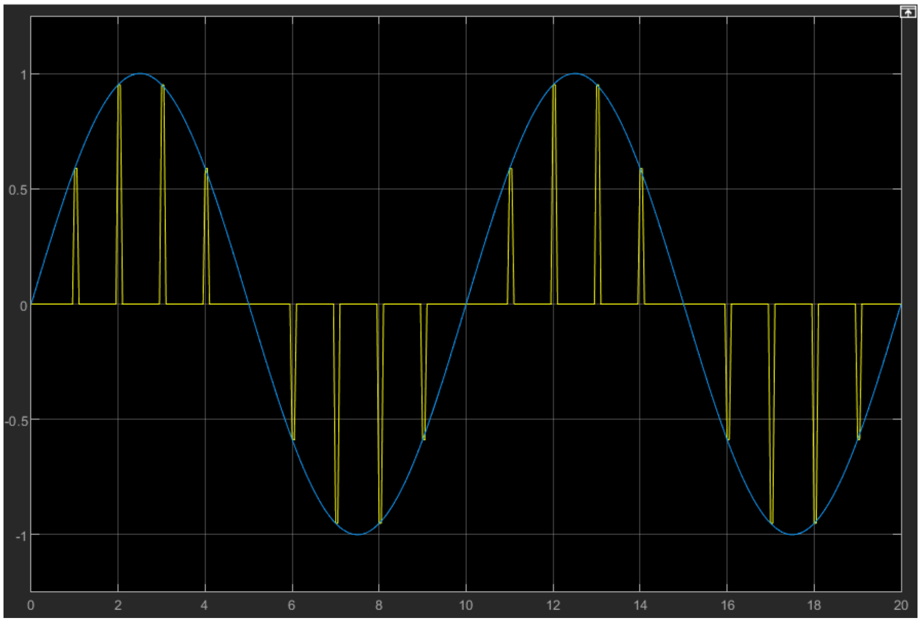
a) frequency of Input Sine wave = 1Hz, pulse width = 0.3s
The Output of Spectrum Analyzer and Scope is given below.



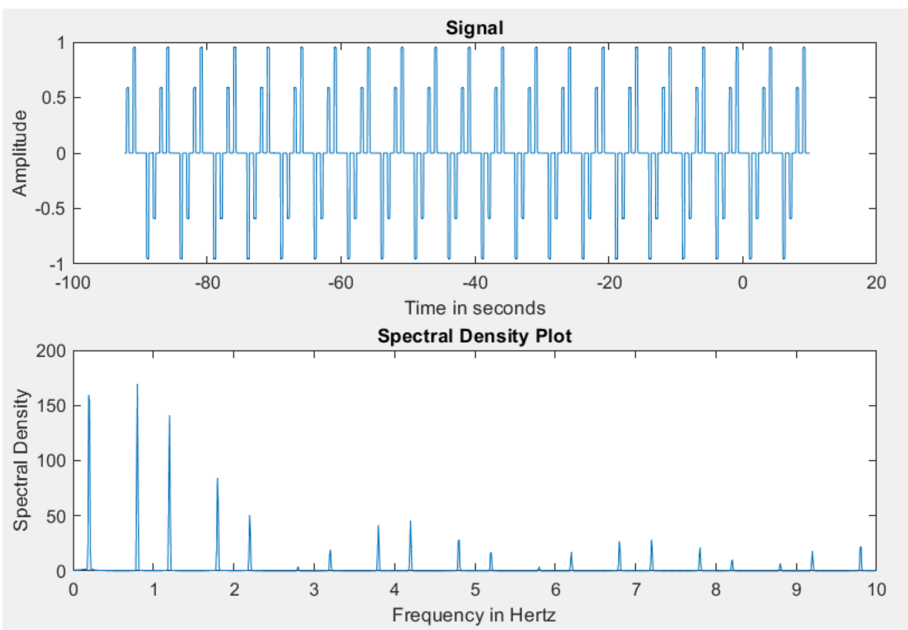


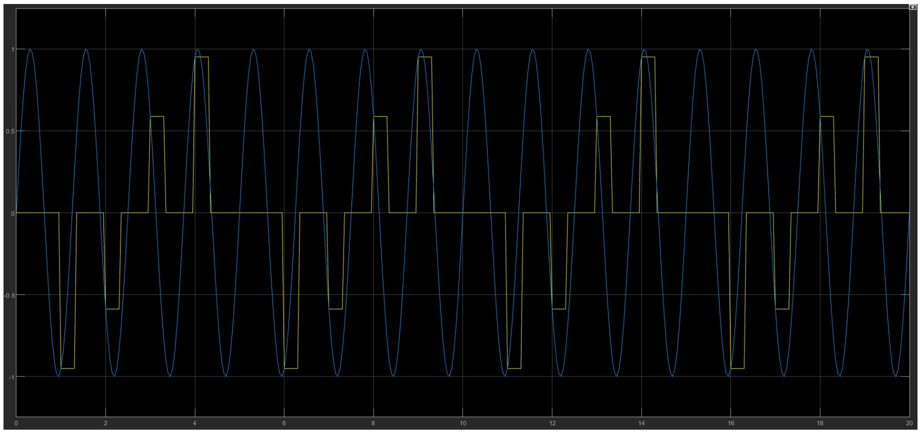
b) frequency of Input Sine wave = 1Hz, pulse width = 0.1s
The Output of Spectrum Analyzer and Scope is given below.



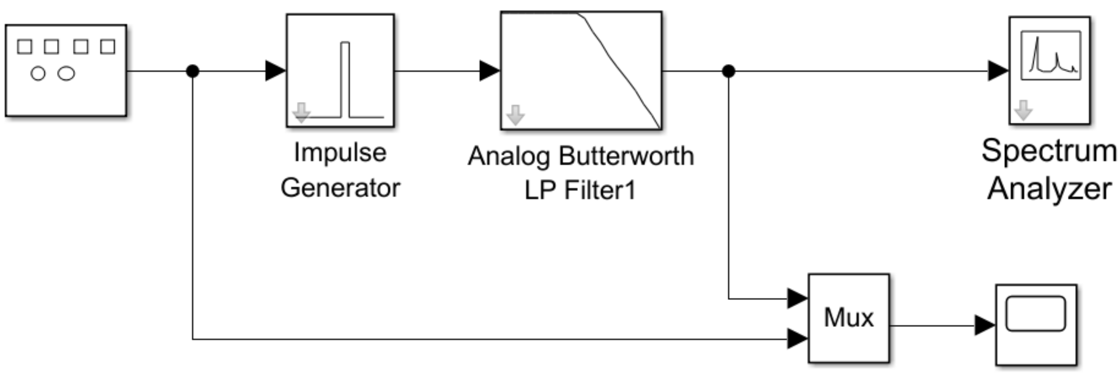


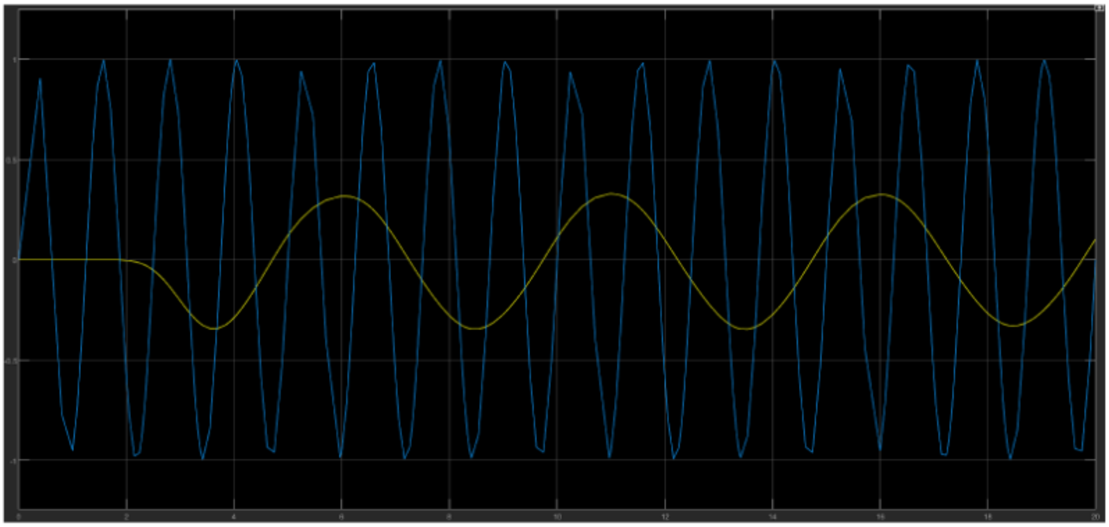
c) frequency of Input Sine wave = 0.8Hz, pulse width = 0.3s
The Output of Spectrum Analyzer and Scope is given below.

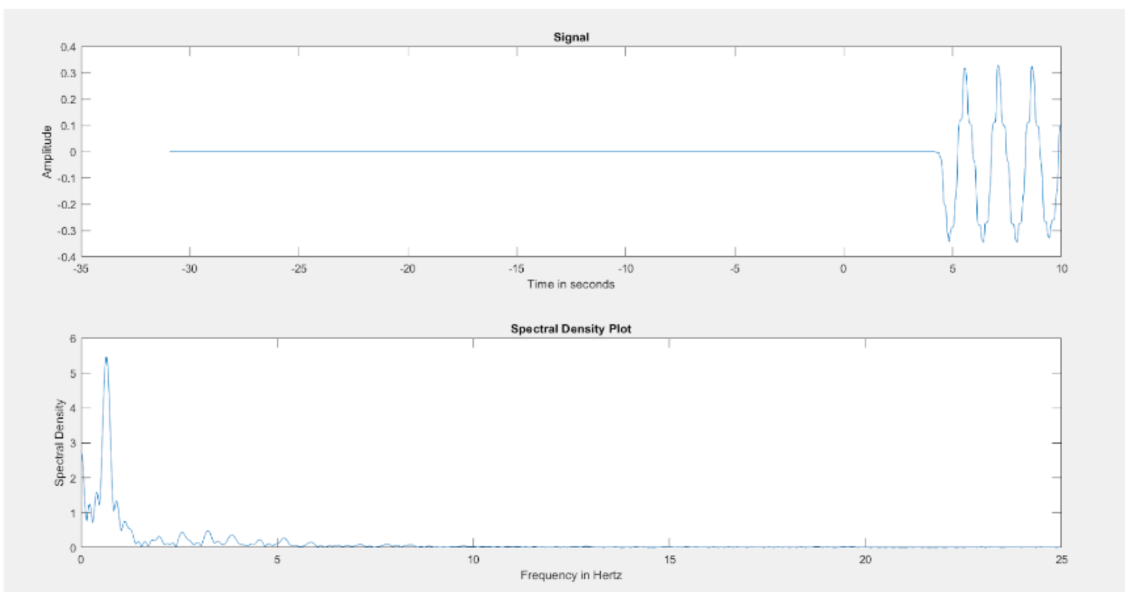




d) With Butterworth Filter, Cutoff frequency = 0.5Hz, f = 0.8Hz, pulse width = 0.3s
The Block Diagram, Output of Spectrum Analyzer and Scope is given below.

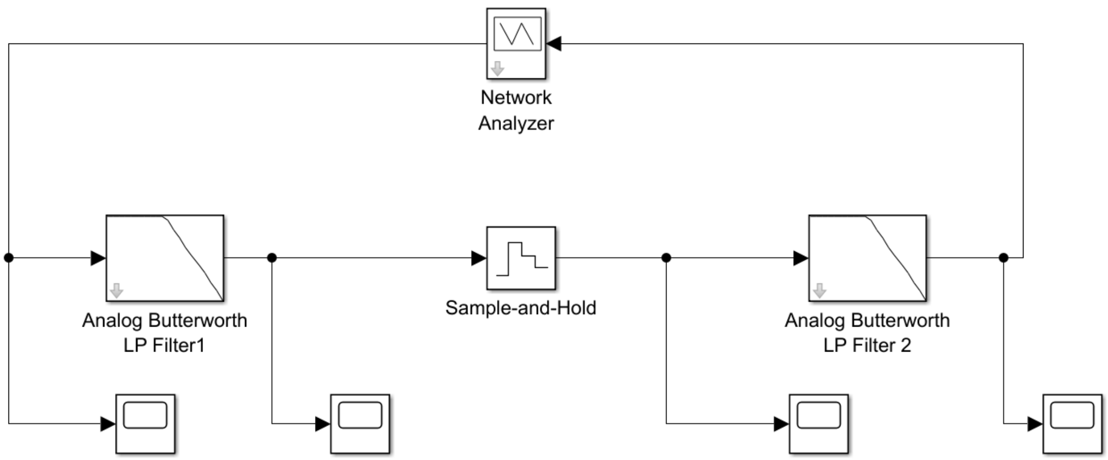




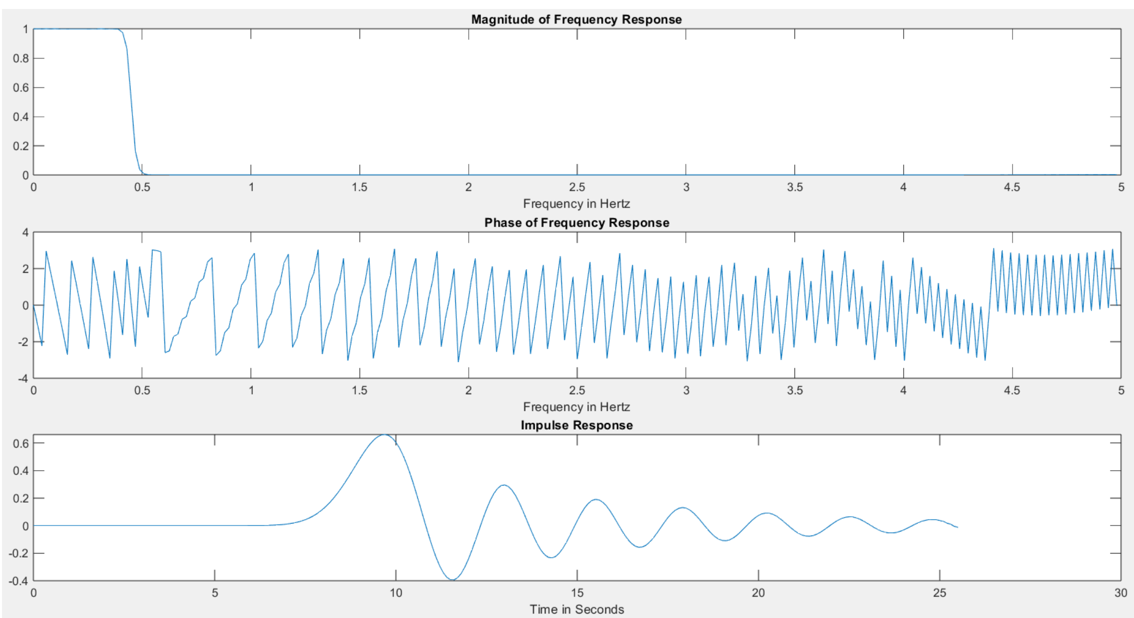

Signal
Amplitude
Time in seconds
Spectral Density Plot
Spectral Density
Frequency in Hertz

3 Sampling and Reconstruction with Sample and Hold

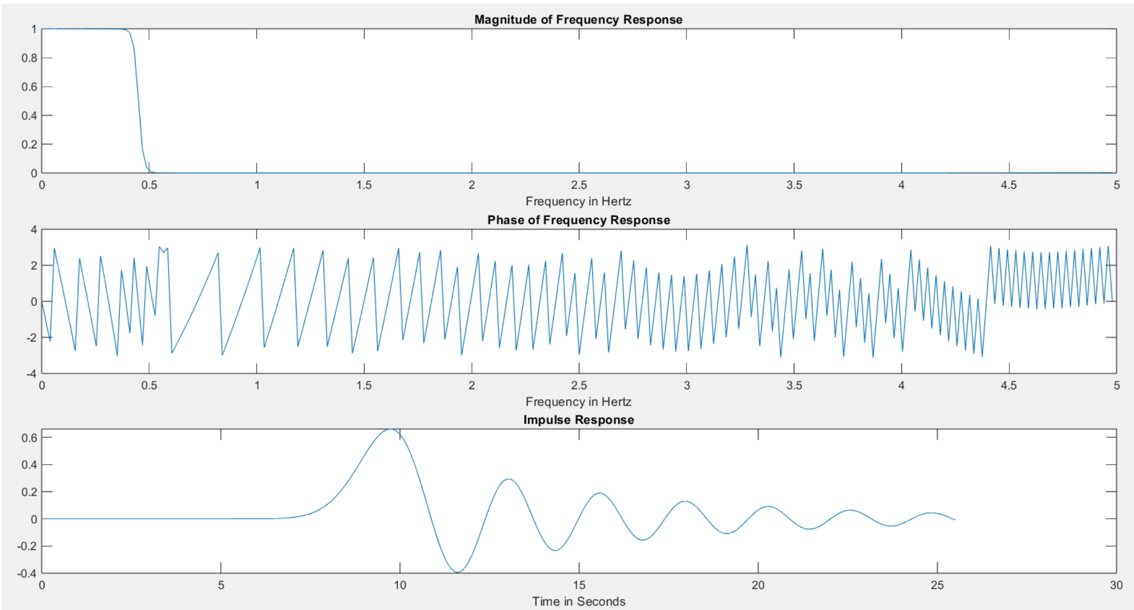
3.1 Simulink Diagram for the System



3.2 Without Sample and Hold Block



3.3 With Sample and Hold Block



The variation in the magnitude response observed in the second experiment arises from the inclusion of the sample-and-hold circuit. This circuit effectively functions as a pulse generator, characterized by a pulse duration equivalent to one sampling period. The introduction of this pulse shape incorporates a sinc function into the frequency response of the system, ultimately resulting in the sharp roll-off observed at higher frequencies.

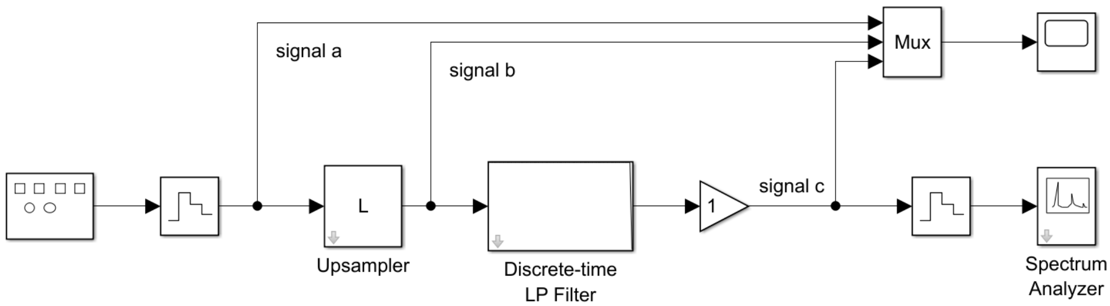
The analytical expression for the behavior of the magnitude plot for frequencies below 0.45 Hz is:

$$|H(f)| = 1 - 0.5\mathrm{sinc}(0.45f)$$

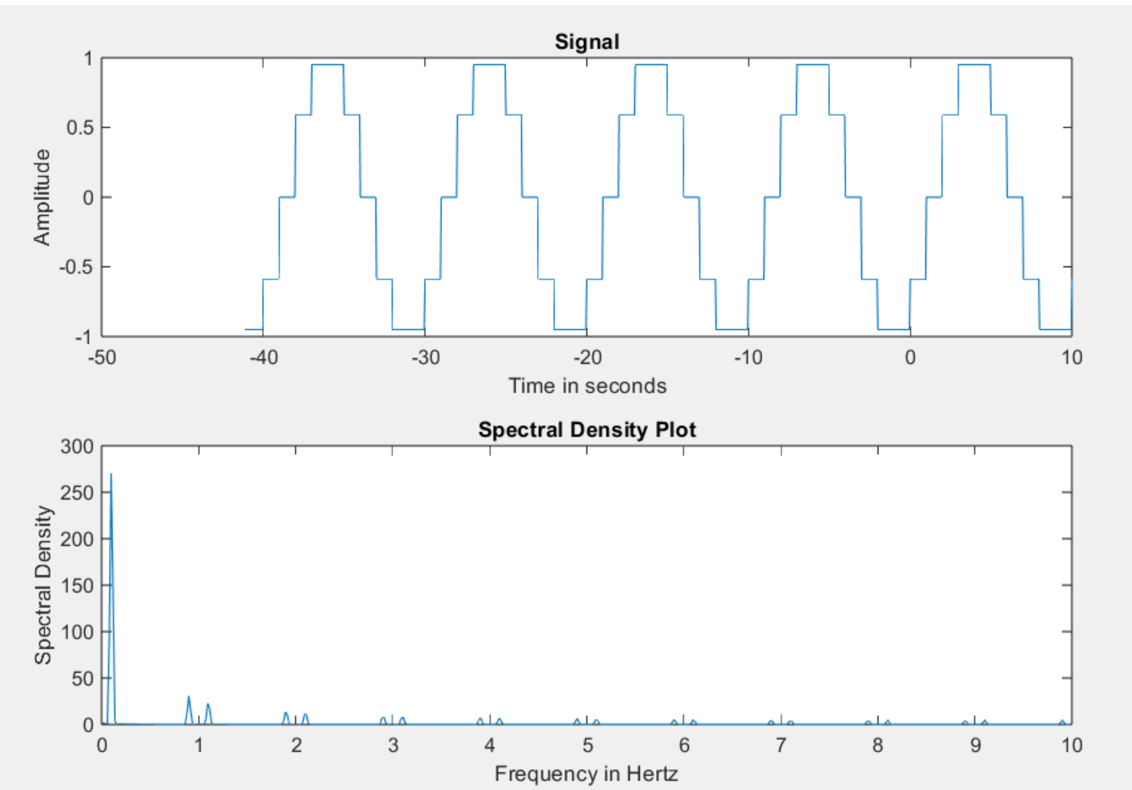
$$\mathrm{sinc}(x) = \frac{\sin(x)}{x}$$

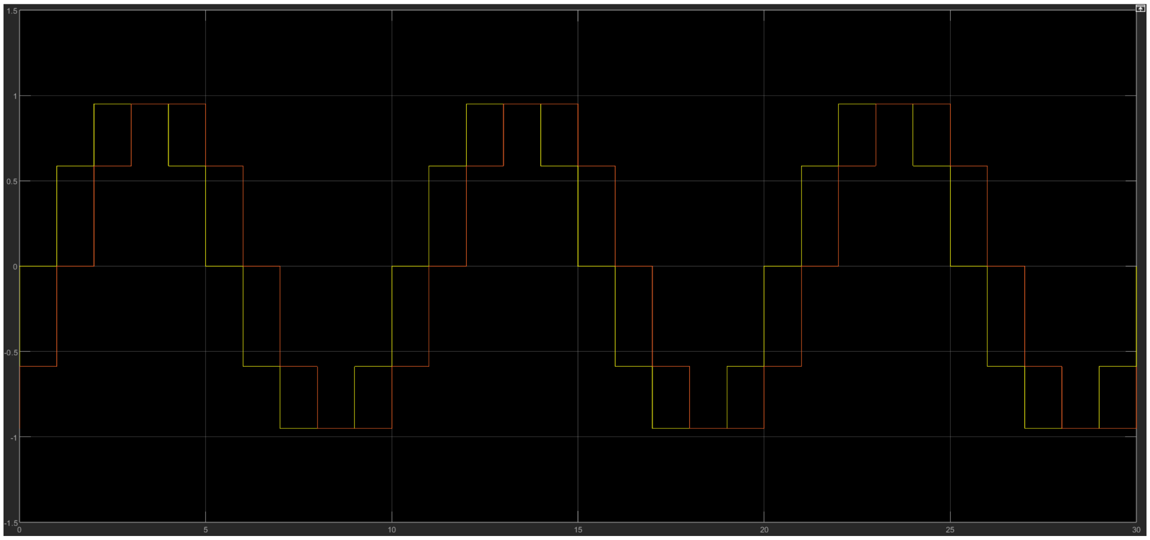
The sinc function, with a sharp peak at x = 0 and a rapid decay to zero, combines with the low-pass filters to shape the magnitude response of the sample-and-hold circuit. This introduces a sharp roll-off at high frequencies. Meanwhile, the low-pass filters work to suppress frequencies beyond the sampling frequency. This difference results in a distinct frequency response when compared to the system without the sample-and-hold circuit, which exhibits a more gradual roll-off.

4 Discrete Time Interpolation

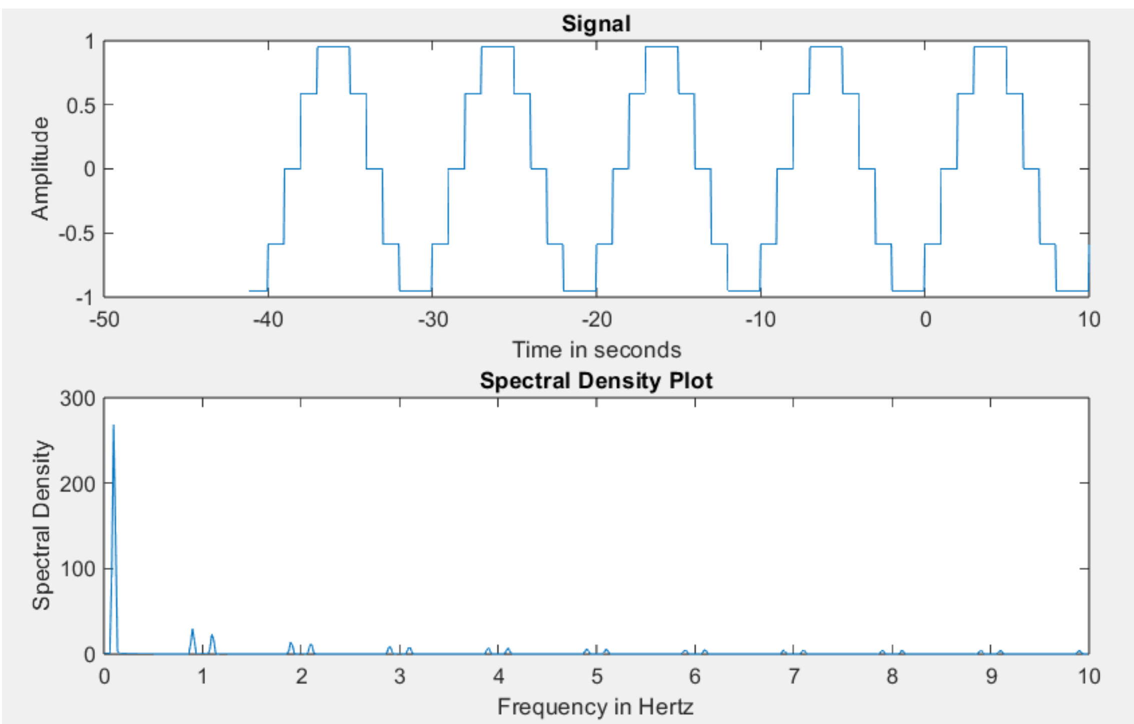


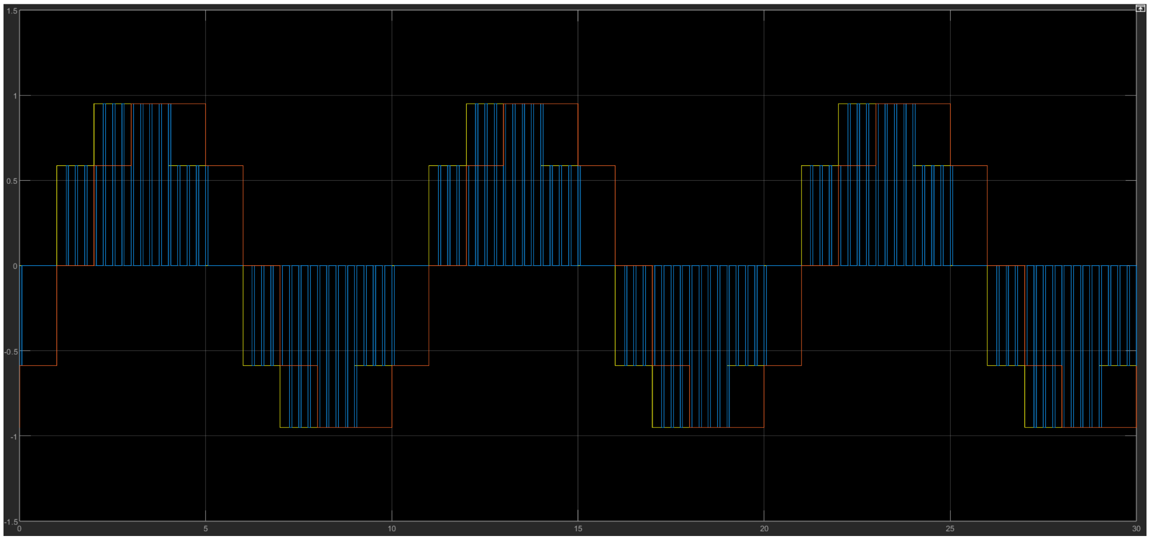
4.1 Upsampling Factor: 1, Sampling Frequency: 1 Hz





4.2 Upsampling Factor: 4, Sampling Frequency: 4 Hz





5 Discrete Time Decimation

5.1 MATLAB Code for Generating Signal sig1

```
% Specify the file path to your AU format sound file
file_path = 'music.au';
% Set the desired sampling rate (16 kHz)
desired_fs = 16000;
% Use audioread to load the sound file and
% resample it to the desired sampling rate
[x, fs] = audioread(file_path, [desired_fs, Inf]);
% Play the original signal
sound(x, fs);
% Create sig1 by subsampling every other sample
sig1 = x(1:2:end);
% Play sig1
sound(sig1, desired_fs);
```

5.2 MATLAB Code for Generating Signal sig2 by Low-Pass Filtering

```
% Specify the file path to your AU format sound file
file_path = 'music.au';
% Set the desired sampling rate (16 kHz)
desired_fs = 16000;
% Use audioread to load the sound file and
% resample it to the desired sampling rate
[x, fs] = audioread(file_path, [desired_fs, Inf]);
% Design a low-pass filter with a cutoff frequency of pi/2
M = 20; % Filter length
Wc = pi/2; % Cutoff frequency
h = fir1(M, Wc/pi);
% Apply the filter to the original signal
filtered_signal = conv(x, h);
% Decimate the filtered signal by 2
sig2 = decimate(filtered_signal, 2);
% Play sig2
sound(sig2, desired_fs);
```

5.3 Quality Assessment of Decimated Audio Signals

Signal: `sig1` (Subsampling)

Subsampling (decimating by a factor of 2) is a direct approach. However, it effectively halves the signal's sampling rate. The challenge arises if the original signal contains frequency components surpassing the new Nyquist frequency (half of the new sampling rate), potentially leading to aliasing and audio distortion.

Signal: `sig2` (Low-Pass Filtering + Decimation)

Employing a pre-decimation low-pass filter to eliminate high-frequency components, `sig2` is designed with a cutoff frequency of $\pi/2$ radians/sample. This strategic filtering effectively prevents aliasing. Consequently, `sig2` is expected to provide superior audio quality compared to `sig1`.

In practice, `sig1` may exhibit noticeably inferior audio quality compared to `sig2`, particularly if the original signal contains high-frequency components. `sig1` is more susceptible to experiencing aliasing artifacts. Conversely, `sig2` is anticipated to offer superior audio quality owing to the implemented low-pass filtering.

To evaluate quality and potential distortion, you can listen to both `sig1` and `sig2` and provide subjective observations. Pay attention to:

- Clipping or distortion artifacts: Noted in `sig1`.

- Loss of high-frequency content: Begins with a high frequency and transitions to a lower pitched sound in `sig1`.

Ultimately, perceived audio quality and the presence of distortion will hinge on the original audio signal's characteristics, the filter design, and your subjective perception as a listener.