Department of Electrical Engineering
Indian Institute of Technology Kharagpur

Digital Signal Processing Laboratory (EE39203)

Autumn, 2022-23

Experiment 2 **Discrete Time Systems**

Slot: **X** Date: **16/08/2023**

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Grading Rubric

	Tick the best applicable per row			Points
	Below Expectation	Lacking in Some	Meets all Expectation	
Completeness of the report				
Organization of the report (5 pts) *With cover sheet, answers are in the same order as questions in the lab, copies of the questions are included in report, prepared in LaTeX*				
Quality of figures (5 pts) *Correctly labelled with title, x-axis, y-axis, and name(s)*				
Ability to process given signals (15 pts) *Matlab figures, questions*				
Understanding and ability to implement difference equations (50 pts) *Manual computation, Matlab figures, Matlab codes, questions*				
Ability to test for linearity and time-invariance of systems (25 pts) *Matlab figures, Matlab codes, questions*				
			TOTAL (100 pts)	

Total Points (100): TA Name: TA Initials:

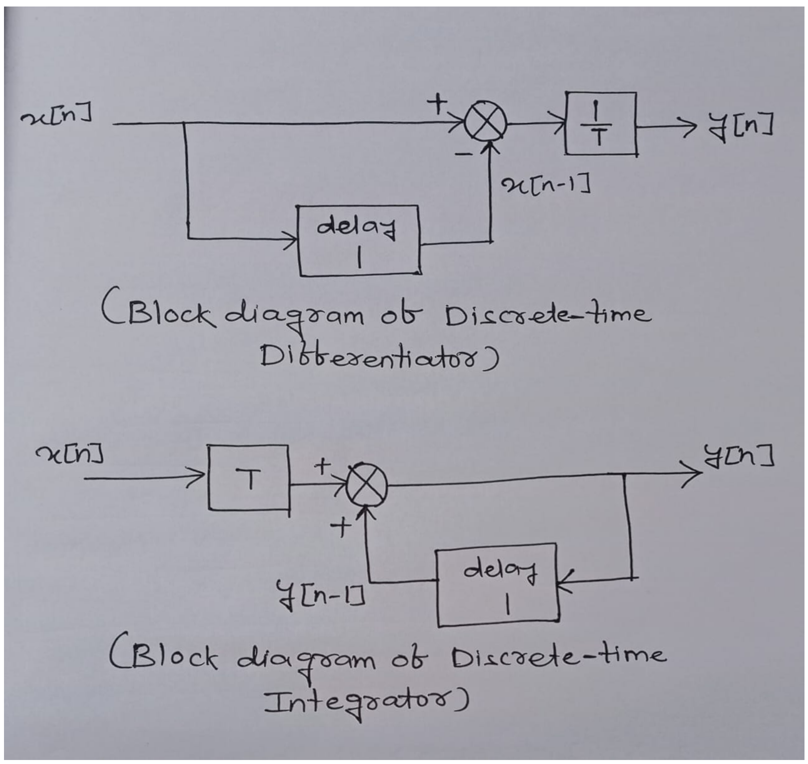
Digital Signal Processing Laboratory (EE39203)

P Manoj Kumar (21IE10027)

Experiment 2 - Discrete Time Systems

1 Example of Discrete-time Systems

1.1 Block Diagram of Discrete-Time Differentiator and Integrator



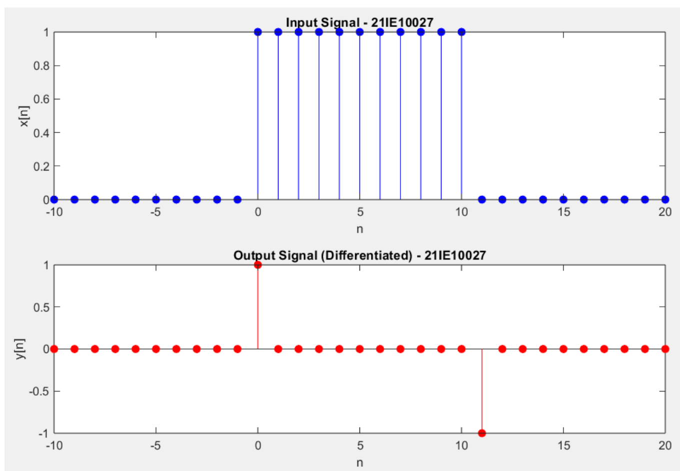
1.2 Discrete-Time Differentiation and Integration of a Pulse Signal

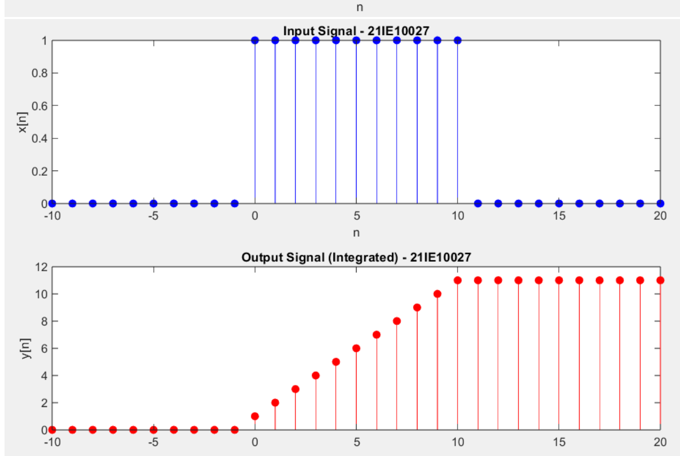
x[n] = $(u[n] - u[n-11]), \quad n \in [-10, 20]$

The discrete-time differentiation and integration of discrete time function x[n] is plotted below for assuming a time stamp of T=1 The MATLAB code also given for respective plots.

```
n = -10:20; % n values
x = (n>=0)-(n>=11); % Input Signal
T=1;
y = zeros(size(n));
% Implement the difference equation for the differentiator
for k = 2:length(n)
    y(k) = (x(k) - x(k - 1)) / T;
end
% Plot the input and output signals
subplot(2, 1, 1);
stem(n, x, 'b', 'filled');
xlabel('n');
ylabel('x[n]');
title('Input Signal - 21IE10027');
subplot(2, 1, 2);
stem(n, y, 'r', 'filled');
xlabel('n');
ylabel('y[n]');
title('Output Signal (Differentiated) - 21IE10027');
```

```
n = -10:20; % n values
x = (n>=0)-(n>=11); % Input Signal
T=1;
y = zeros(size(n));
% Implement the difference equation for the integrator
for k = 2:length(n)
    y(k) = y(k - 1) + x(k) * T;
end
% Plot the input and output signals
subplot(2, 1, 1);
stem(n, x, 'b','filled');
xlabel('n');
ylabel('x[n]');
title('Input Signal - 21IE10027');
subplot(2, 1, 2);
stem(n, y, 'r','filled');
xlabel('n');
ylabel('y[n]');
title('Output Signal (Integrated) - 21IE10027');
```



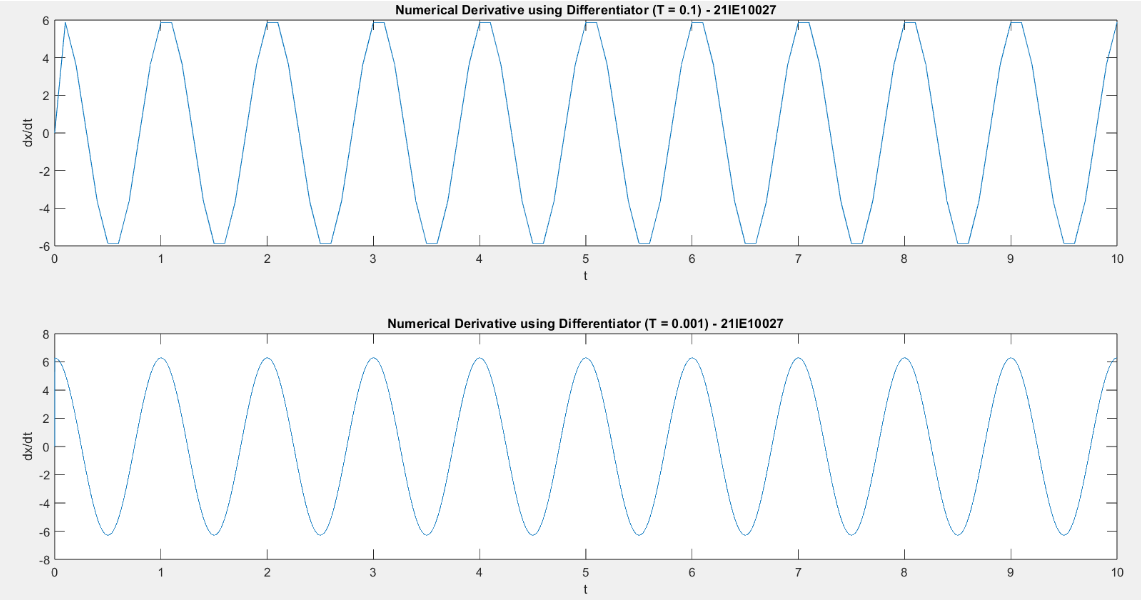


1.3 Discrete-Time Differentiation of a Sinusoidal Signal

The below MATLAB code is for evaluating numerically the discrete-time differentiation of $x(t) = sin(2\pi t), \quad n \in [0, 10]$ assuming a time stamp of T= 0.1 and 0.001.

```
T=0.1; % Sampling Period
t = 0:T:10; % Time values
x = sin(2*pi*t); % Input Signal
y = zeros(size(t));
% Implement the difference equation for the differentiator
for k = 2:length(t)
    y(k) = (x(k) - x(k - 1)) / T;
end
% Display results
subplot(2,1,1);
plot(t,y);
title(['Numerical Derivative ' ...
    'using Differentiator (T = 0.1) - 21IE10027']);
xlabel('t');
ylabel('dx/dt');
```

```
T=0.001; % Sampling Period
t = 0:T:10; % Time values
x = sin(2*pi*t); % Input Signal
y = zeros(size(t));
% Implement the difference equation for the differentiator
for k = 2:length(t)
    y(k) = (x(k) - x(k - 1)) / T;
end
% Display results
subplot(2,1,2);
plot(t,y);
title(['Numerical Derivative ' ...
    'using Differentiator (T = 0.001) - 21IE10027']);
xlabel('t');
ylabel('dx/dt');
```



2 Difference Equations

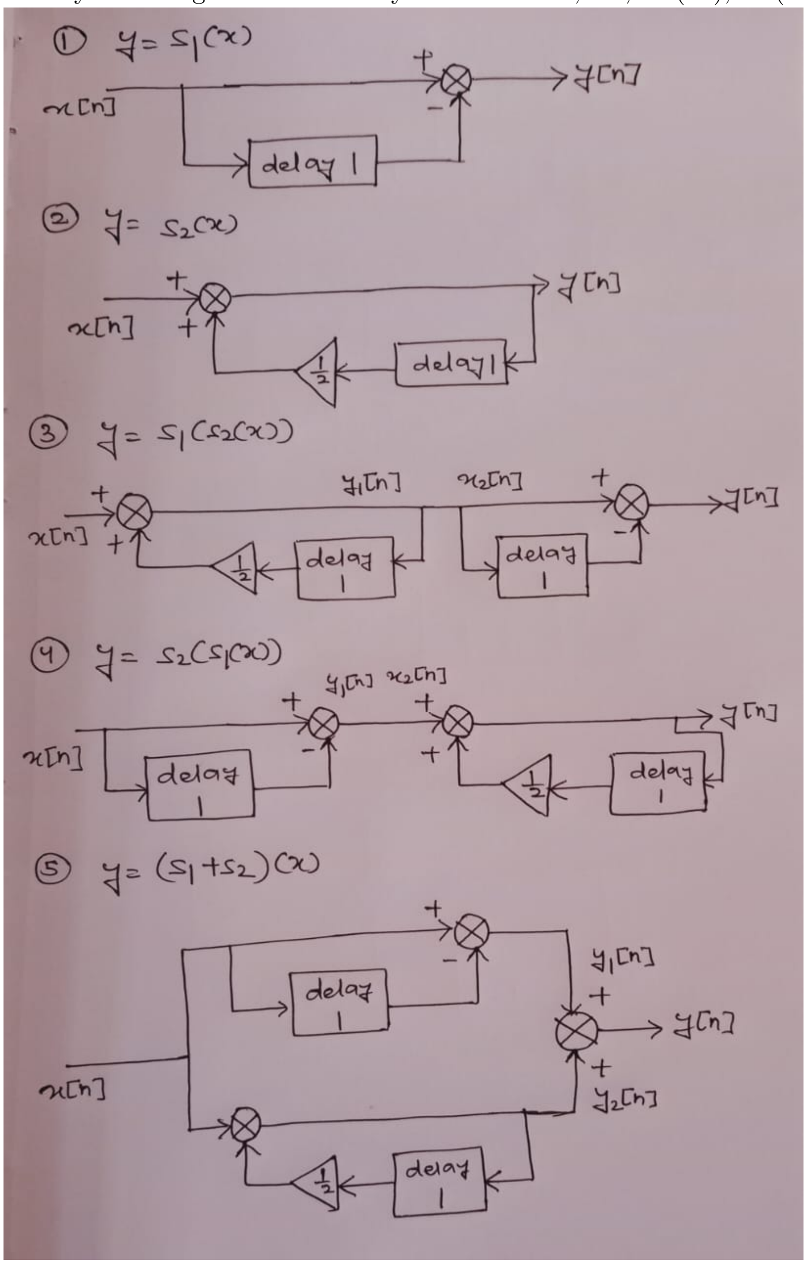
The first filter is y = S1(x) which obeys the difference equation

$$y[n] = x[n] - x[n-1]$$

The second filter is y=S2(x) which obeys the difference equation

$$y[n] = \frac{1}{2}y[n-1] + x[n]$$

The system diagrams of all 5 systems i.e. S1, S2, S1(S2), S2(S1), S1+S2 are drawn below.



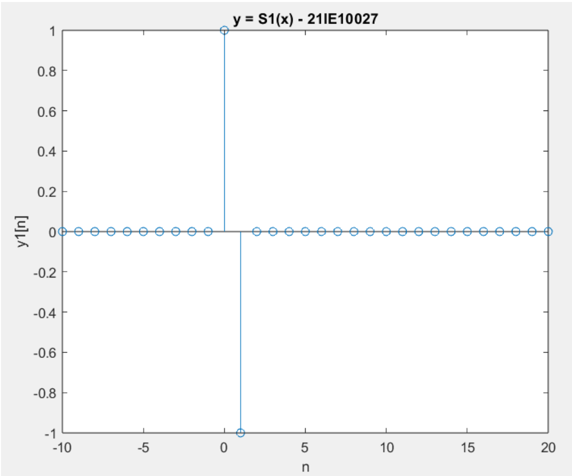
The MATLAB code for the filter and impulse response of all 5 systems are given below. The individual impulse response of systems are plotted below.

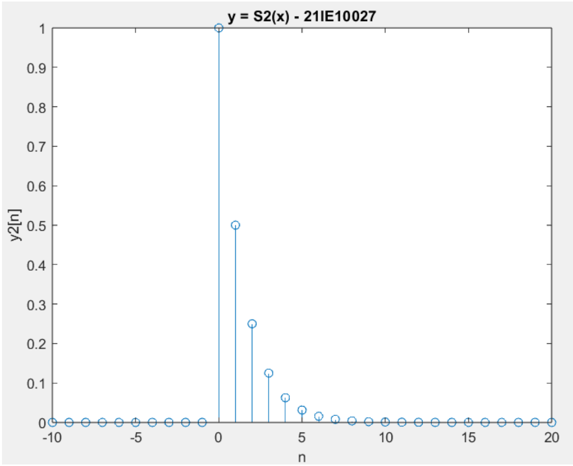
```
function y=S1(x)
    y=zeros(size(x));
    for i=2:length(x)
        y(i)=x(i)-x(i-1);
    end
end
```

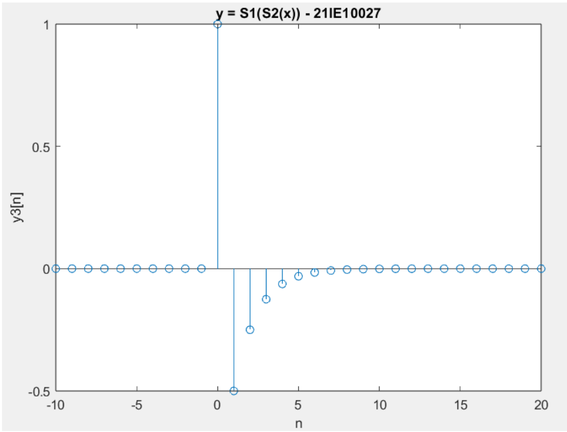
```
function y=S2(x)
    y=zeros(size(x));
    for i=2:length(x)
        y(i)=0.5*y(i-1)+x(i);
    end
end
```

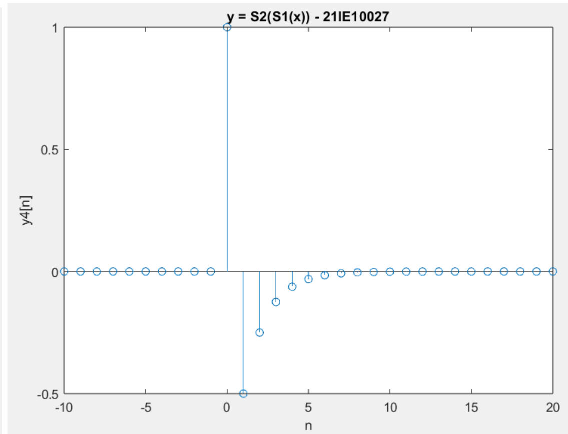
```
n=-10:20; % n values
x=n==0; % Impulse Signal
figure;
y1=S1(x); % S1(x)
stem(n,y1);
xlabel('n');
ylabel('y1[n]');
title('y = S1(x) - 21IE10027')
figure;
y2=S2(x); % S2(x)
stem(n,y2);
xlabel('n');
ylabel('y2[n]');
title('y = S2(x) - 21IE10027')
figure;
```

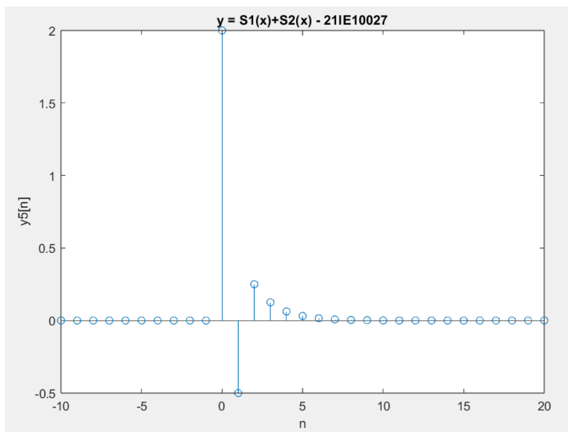
```
y3=S1(S2(x)); % S1(S2(x))
stem(n,y3);
xlabel('n');
ylabel('y3[n]');
title('y = S1(S2(x)) - 21IE10027')
figure;
y4=S2(S1(x)); % S2(S1(x))
stem(n,y4);
xlabel('n');
ylabel('y4[n]');
title('y = S2(S1(x)) - 21IE10027')
figure;
y5=S1(x)+S2(x); % S1(x)+S2(x)
stem(n,y5);
xlabel('n');
ylabel('y5[n]');
title('y = S1(x)+S2(x) - 21IE10027')
```











3 Inverse Systems

Given the system $S2 : y[n] = 0.5 \cdot y[n-1] + x[n]$, to find its inverse (S3), we need to express $x[n]$ in terms of $y[n]$:

$$y[n] = 0.5 \cdot y[n-1] + x[n]$$

$$x[n] = y[n] - 0.5 \cdot y[n-1]$$

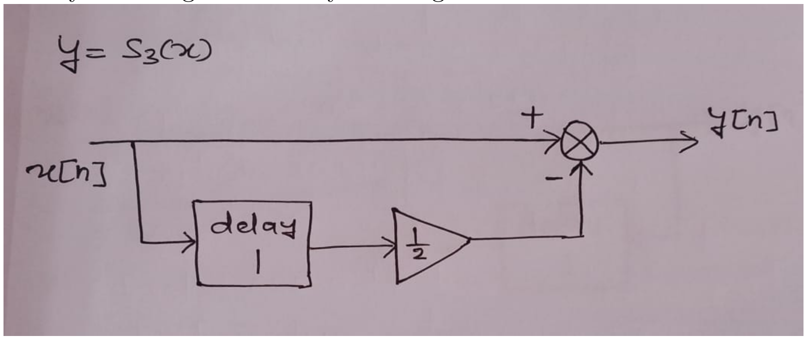
Thus, the inverse system $S2^{-1}$ can be expressed as:

$$S2^{-1} : x[n] = y[n] - 0.5 \cdot y[n-1]$$

The Inverse filter of S2 is given by,

$$S3 : y[n] = x[n] - 0.5 \cdot x[n-1]$$

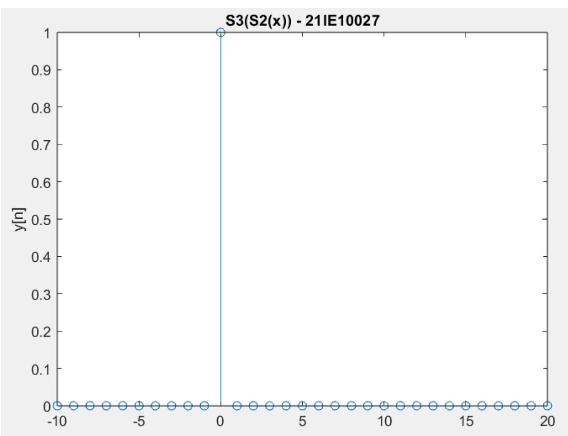
The system diagram of S3 system is given below.

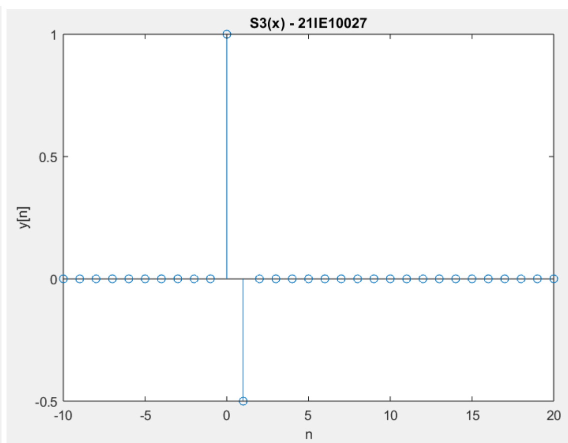


The MATLAB code for the inverse filter S3 and it's impulse response is given below. The impulse response of S3 and S3(S2) is plotted below.

```
% Inverse Function of S2
function y = S3(x)
    y = zeros(size(x));
    for i = 2:length(x)
        y(i) = x(i) - 0.5 * x(i - 1);
    end
end
```

```
n = -10:20; % n values
x = n == 0; % input signal
figure;
y = S3(x); % Impulse Response of S3(x)
stem(n, y);
ylabel('y[n]');
xlabel('n');
title('S3(x) - 21IE10027');
figure;
y = S3(S2(x)); % S3(S2(x))
stem(n, y);
ylabel('y[n]');
xlabel('n');
title('S3(S2(x)) - 21IE10027');
```



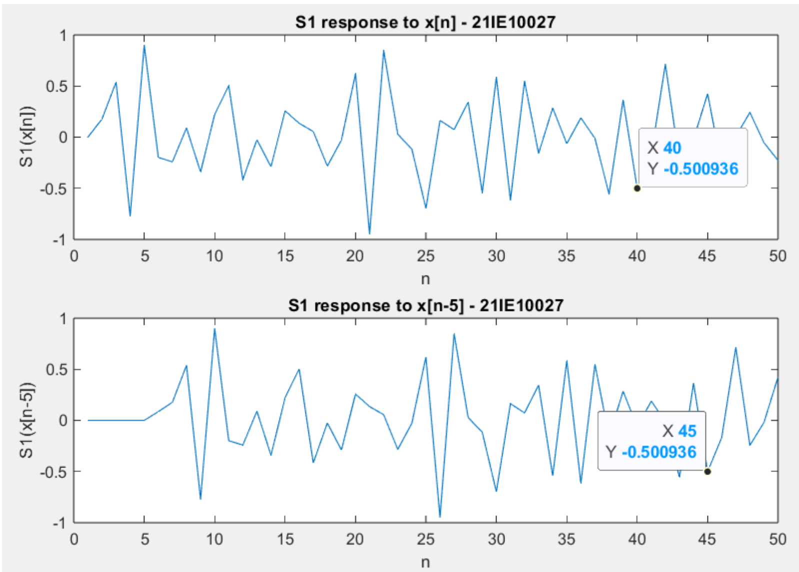


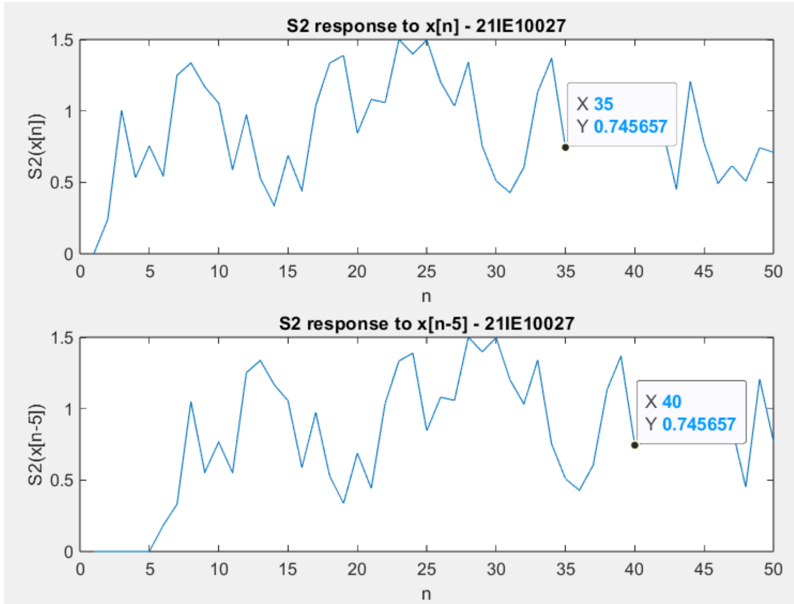
4 System Tests

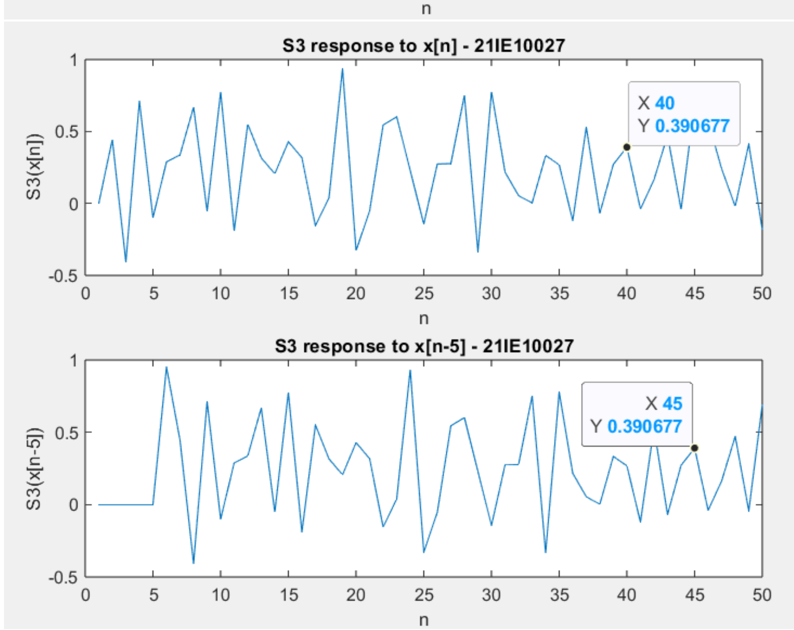
4.1 Time Invariance of Systems

The below MATLAB code is for testing whether the three systems S1, S2, S3 are time variant or time invariant.

```
n=50;% no. of samples
n1=1:1:n;
x_in1=rand(1,n);% Random Input Signal
x_in2=zeros(1,n);
% Shifting the signal in time
% x_in2[n] = x_in1[n-5]
for i=6:n
    x_in2(i)=x_in1(i-5);
end
y1=S1(x_in1);
y2=S1(x_in2);
subplot(2,1,1);
plot(n1,y1);
xlabel('n');ylabel('S1(x[n])');
title('S1 response to x[n] - 21IE10027');
subplot(2,1,2);
plot(n1,y2);
xlabel('n');ylabel('S1(x[n-5])');
title('S1 response to x[n-5] - 21IE10027');
figure;
y1=S2(x_in1);
y2=S2(x_in2);
subplot(2,1,1);
plot(n1,y1);
xlabel('n');ylabel('S2(x[n])');
title('S2 response to x[n] - 21IE10027');
subplot(2,1,2);
plot(n1,y2);
xlabel('n');ylabel('S2(x[n-5])');
title('S2 response to x[n-5] - 21IE10027');
figure;
y1=S3(x_in1);
y2=S3(x_in2);
subplot(2,1,1);
plot(n1,y1);
xlabel('n');ylabel('S3(x[n])');
title('S3 response to x[n] - 21IE10027');
subplot(2,1,2);
plot(n1,y2);
xlabel('n');ylabel('S3(x[n-5])');
title('S3 response to x[n-5] - 21IE10027');
```





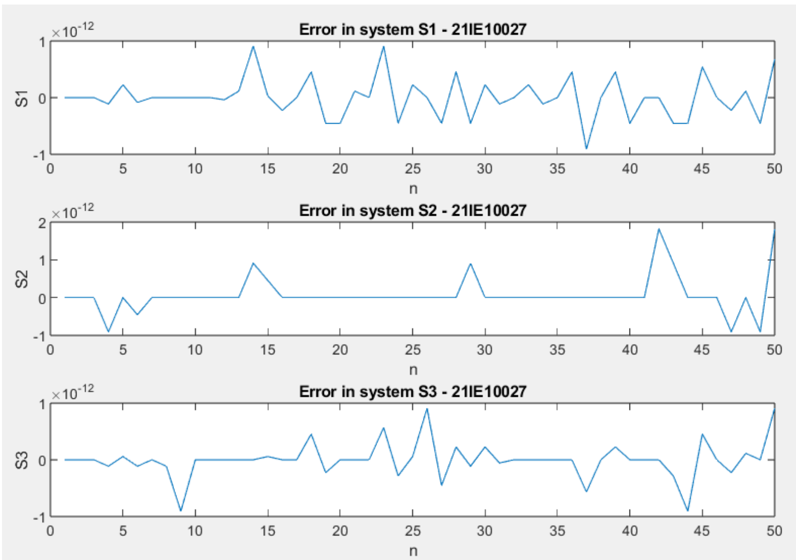


As the values of the response to the delayed input is same as the value off response to the original input, The three systems are time-invariant.

4.2 Linearity of Systems

The below MATLAB code is for testing whether the three systems S1, S2, S3 is linear or non-linear.

```
a=randi([1,100],1);
b=randi([1,100],1);
n=50;% No of samples
n1=1:1:n;
x1=randi(100,[1,n]);
x2=sin(n1);
x3=a*x1+b*x2;
y1=S1(x1);y2=S1(x2);y3=S1(x3);
subplot(3,1,1);
plot(n1,y3-(a*y1+b*y2));
xlabel('n');ylabel('S1');
title('Error in system S1 - 21IE10027');
y1=S2(x1);y2=S2(x2);y3=S2(x3);
subplot(3,1,2);
plot(n1,y3-(a*y1+b*y2));
xlabel('n');ylabel('S2');
title('Error in system S2 - 21IE10027');
y1=S3(x1);y2=S3(x2);y3=S3(x3);
subplot(3,1,3);
plot(n1,y3-(a*y1+b*y2));
xlabel('n');ylabel('S3');
title('Error in system S3 - 21IE10027');
```



The differences are of the order of 10^{-12} and arise due to approximations by MATLAB while processing and are hence negligible. Thus, all the systems are linear.

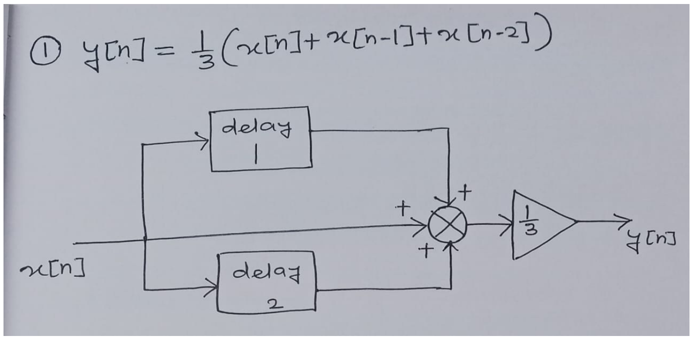
5 Stock Market Example

5.1 Block Diagram Impulse Response Calculation - 1

The equation can be written as,

$$y[n] = \frac{1}{3} \cdot (x[n] + x[n-1] + x[n-2])$$

The block diagram of the system is given below.



Given $x[n] = \delta[n]$, the input signal is an impulse function.
Substituting $x[n] = \delta[n]$ into the equation:

$$y[n] = \frac{1}{3} \cdot (\delta[n] + \delta[n-1] + \delta[n-2])$$

Therefore, the impulse response of the given system is:

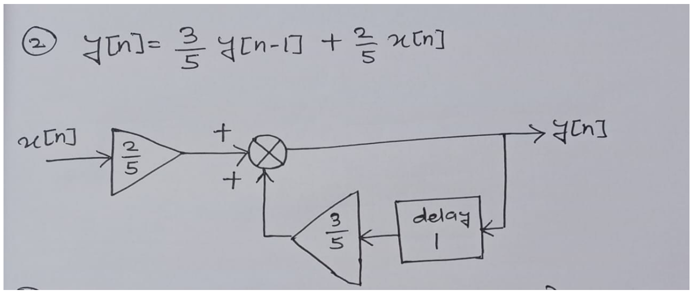
$$y[n] = h[n] = \frac{1}{3} \cdot (\delta[n] + \delta[n-1] + \delta[n-2])$$

5.2 Block Diagram Impulse Response Calculation - 2

The equation can be written as,

$$y[n] = 0.6y[n-1] + 0.4x[n]$$

The block diagram of the system is given below.



Given $x[n] = \delta[n]$, the input signal is an impulse function.
The system equation with the impulse input is:

$$y[n] = 0.6y[n-1] + 0.4\delta[n]$$

Taking the Z-transform of both sides of the equation:

$$Y(z) = 0.6z^{-1}Y(z) + 0.4$$

Solving for $Y(z)$:

$$Y(z)(1 - 0.6z^{-1}) = 0.4$$

$$Y(z) = \frac{0.4}{1 - 0.6z^{-1}}$$

Applying the inverse Z-transform to $Y(z)$:

$$y[n] = \mathcal{Z}^{-1}\left\{\frac{0.4}{1 - 0.6z^{-1}}\right\}$$

Using the property of the Z-transform: $\mathcal{Z}\{a^n u[n]\} = \frac{z}{z-a}$, where $u[n]$ is the unit step function, we can write:

$$y[n] = 0.4 \cdot (0.6)^n \cdot u[n]$$

Therefore, the impulse response of the given system is:

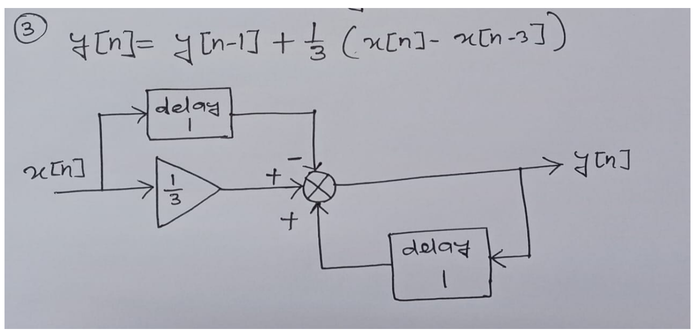
$$y[n] = h[n] = 0.4 \cdot (0.6)^n \cdot u[n]$$

5.3 Block Diagram Impulse Response Calculation - 3

The equation can be written as,

$$y[n] = y[n-1] + \frac{1}{3} \cdot (x[n] - x[n-3])$$

The block diagram of the system is given below.



Given $x[n] = \delta[n]$, the input signal is an impulse function. The system equation with the impulse input is:

$$y[n] = y[n-1] + \frac{1}{3} \cdot (\delta[n] - \delta[n-3])$$

Substituting $x[n] = \delta[n]$ into the equation:

$$y[n] = y[n-1] + \frac{1}{3} \cdot (1 - \delta[n-3])$$

Therefore, the impulse response of the given system is:

$$y[n] = h[n] = y[n-1] + \frac{1}{3} \cdot (1 - \delta[n-3])$$

5.4 Moving Average

1.

$$y[n] = \frac{1}{3} \cdot (x[n] + x[n-1] + x[n-2])$$

This system calculates the output $y[n]$ as the average of the current input $x[n]$ and the two previous input samples $x[n-1]$ and $x[n-2]$, each multiplied by $\frac{1}{3}$. Thus, the output at any time index is given by the weighted sum of these input samples, effectively performing a moving average operation over a window of three samples. This system acts as a moving average filter with a window size of three, which helps to smooth out fluctuations and noise present in the input signal.

2.

$$y[n] = y[n-1] + \frac{1}{3} \cdot (x[n] - x[n-3])$$

In this system, the output $y[n]$ is computed by adding the previous output $y[n-1]$ to a fraction ($\frac{1}{3}$) of the difference between the current input $x[n]$ and the input three time steps ago $x[n-3]$. This mechanism effectively calculates a moving average of the input differences over a window of three samples. The output is updated using both the previous output value and the weighted difference between the current and past inputs.