Department of Electrical Engineering

Indian Institute of Technology Kharagpur

Digital Signal Processing Laboratory (EE39203)

Autumn, 2022-23

Experiment 3 **Frequency Analysis**

Slot: Date:

Student Name: Roll No.:

Grading Rubric

	Tick the best applicable per row			Points
	Below Expectation	Lacking in Some	Meets all Expectation	
Completeness of the report				
Organization of the report (5 pts) *With cover sheet, answers are in the same order as questions in the lab, copies of the questions are included in report, prepared in LaTeX*				
Quality of figures (5 pts) *Correctly labelled with title, x-axis, y-axis, and name(s)*				
Ability to compute Fourier series expansion and synthesize periodic signals using the expansion (15 pts) *Derivation and sketch, plots of synthesized signals, questions*				
Implementation of DTFT (25 pts) *Matlab function*				
Magnitude and Phase Response of DTFT (25 pts) *DTFT's magnitude and phase plots*				
Discrete time system analysis (25 pts) *Exercises in 3.3, completed block diagram, table of measurements, impulse and frequency response*				
			TOTAL (100 pts)	

Total Points (100): TA Name: TA Initials:

Digital Signal Processing Laboratory (EE39203)

P Manoj Kumar (21IE10027)

Experiment 3 - Frequency Analysis

1 Background Exercises - Synthesis of Periodic Signals

1. For a period $T_0 = 2$ with $t \in [0, 2]$:

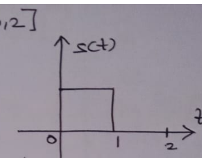
$$s(t) = \begin{cases} \text{rect}(t - 0.5), & \text{if } 0 \le t < 2 \\ 0, & \text{otherwise} \end{cases}$$

2. For a period $T_0 = 1$ with $t \in \left[-\frac{1}{2}, \frac{1}{2}\right]$:

$$s(t) = \begin{cases} \text{rect}\left(2t + \frac{1}{2}\right), & \text{if } -\frac{1}{2} \le t < 0 \\ \text{rect}\left(2t - \frac{1}{2}\right), & \text{if } 0 \le t \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

1. Period $T_0 = 2$ For $t \in [0, 2]$

$s(t) = \text{rect}(t - \frac{1}{2})$



$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt$$

$$= \frac{1}{2}\int_0^2 \text{rect}(t - \tfrac{1}{2})\,dt = \frac{1}{2}\int_0^1 1\,dt = \frac{1}{2}$$

$$a_k = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos(k\omega_0 t)\,dt \qquad \omega_0 = \frac{2\pi}{T} = \pi$$

$$= \int_0^1 \cos k\pi t\,dt = \frac{\sin k\pi}{k\pi} = 0 \quad (\text{as } k \in \mathbb{Z})$$

$$b_k = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin(k\omega_0 t)\,dt \qquad \omega_0 = \frac{2\pi}{T} = \pi$$

$$= \int_0^1 \sin(k\omega_0 t)\,dt = \frac{1 - \cos k\pi}{k\pi} = \begin{cases} 0 & \text{if } k \text{ is even.} \\ \frac{2}{k\pi} & \text{if } k \text{ is odd.} \end{cases}$$

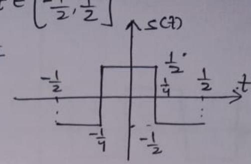
In given expression,

$$s(t) = a_0 + \sum_{k=1}^{\infty} A_k \sin(2\pi k f_0 t + \phi_k)$$

$$\Rightarrow \boxed{s(t) = \frac{1}{2} + \sum_{k=1,3,5\ldots}^{\infty} \frac{2}{k\pi}\sin(2\pi k f_0 t)}$$

2. Period $T_0 = 1$ For $t \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$s(t) = rect(2t) - \frac{1}{2}$



$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} s(t)\,dt = 0$$

$$a_k = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos(k\cdot 2\pi t)\,dt$$

$$= 2\left[\frac{1}{2}\int_{-\frac{1}{4}}^{\frac{1}{4}}\cos(2k\pi t)\,dt - \frac{1}{2}\left\{\int_{-\frac{1}{2}}^{-\frac{1}{4}}\cos(2\pi k t)\,dt + \int_{\frac{1}{4}}^{\frac{1}{2}}\cos(2\pi k t)\,dt\right\}\right]$$

$$= 2\left[\frac{1}{4\pi k}\left[\sin 2\pi k t\right]_{-\frac{1}{4}}^{\frac{1}{4}} - \frac{1}{4\pi k}\left(\sin 2\pi k t\Big|_{-\frac{1}{2}}^{-\frac{1}{4}} + \sin 2\pi k t\Big|_{\frac{1}{4}}^{\frac{1}{2}}\right)\right]$$

$$= \frac{1}{2\pi k}\left\{2\sin\left(\frac{k\pi}{2}\right) + 2\sin\left(\frac{k\pi}{2}\right)\right\} = \frac{\sin\left(\frac{k\pi}{2}\right)}{\left(\frac{k\pi}{2}\right)}$$

$$b_k = \frac{1}{2\pi k}\left\{\cos 2\pi k t\Big|_{\frac{1}{4}}^{-\frac{1}{4}} - \left(\cos 2\pi k t\Big|_{-\frac{1}{2}}^{-\frac{1}{4}} + \cos 2\pi k t\Big|_{\frac{1}{2}}^{\frac{1}{4}}\right)\right\}$$

$$= 0$$

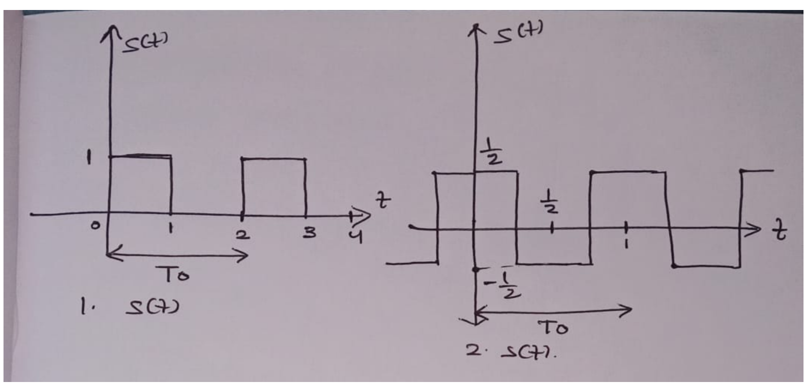
In given Expression,

$$s(t) = a_0 + \sum_{k=1}^{\infty} A_k \sin(2\pi k f_0 t + \theta_k)$$

$$\boxed{s(t) = \sum_{k=1}^{\infty} \frac{2}{k\pi}\sin\frac{k\pi}{2}\sin\left(2\pi k f_0 t + \frac{\pi}{2}\right)}$$

$$\boxed{s(t) = \sum_{k=1,3,5}^{\infty} \frac{2}{\pi k}\cos(2\pi k t)}$$

The sketch of both the signals are given below on the interval $[0, T_0]$.



2 Discrete-Time Fourier Transform

The MATLAB code for DTFT(x,n0,dw) is given below. The Magnitude and Phase responses of the below three functions are given below.

1. $x[n] = \delta[n]$
2. $x[n] = \delta[n-5]$
3. $x[n] = (0.5)^n u[n]$

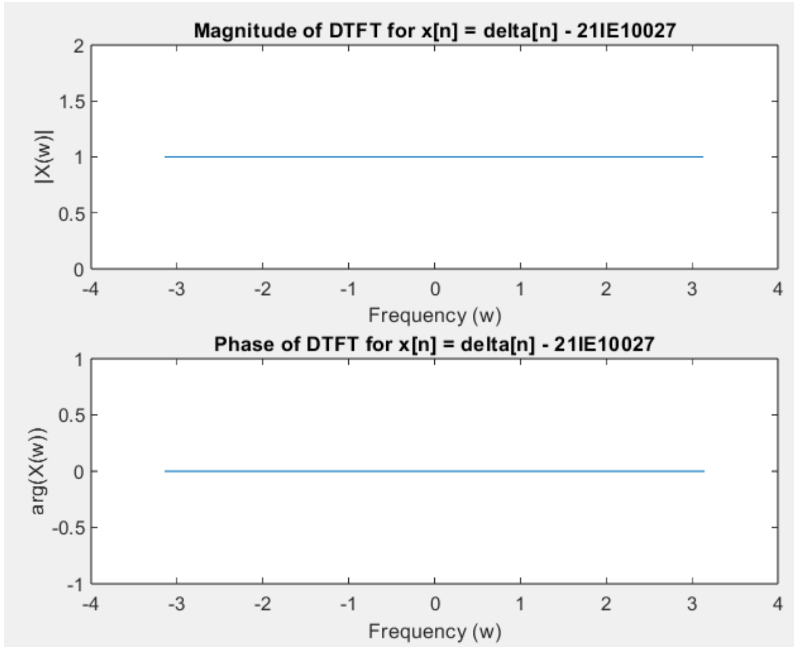
```
% Define the time index n0, spacing dw, and
% compute the DTFT for each signal
n0 = 0;
dw = 0.01; % Adjust this value for the desired frequency resolution
w = -pi:dw:pi;
% Signal 1: x[n] = delta[n]
x1 = 1;
X1 = DTFT(x1, n0, dw);
% Signal 2: x[n] = delta[n-5]
x2 = [0, 0, 0, 0, 0, 1]; % Shifted delta function
X2 = DTFT(x2, n0, dw);
% Signal 3: x[n] = (0.5)^n * u[n]
n = 0:30; % Define the time index for this signal
x3 = (0.5).^n .* (n >= 0); % Generate the signal
X3 = DTFT(x3, n0, dw);
```

```
figure;
subplot(2,1,1);
plot(w, abs(X2));
title('Magnitude of DTFT for x[n] = delta[n-5] - 21IE10027');
xlabel('Frequency (w)');
ylabel('|X(w)|');
subplot(2,1,2);
plot(w, angle(X2));
title('Phase of DTFT for x[n] = delta[n-5] - 21IE10027');
xlabel('Frequency (w)');
ylabel('arg(X(w))');
```

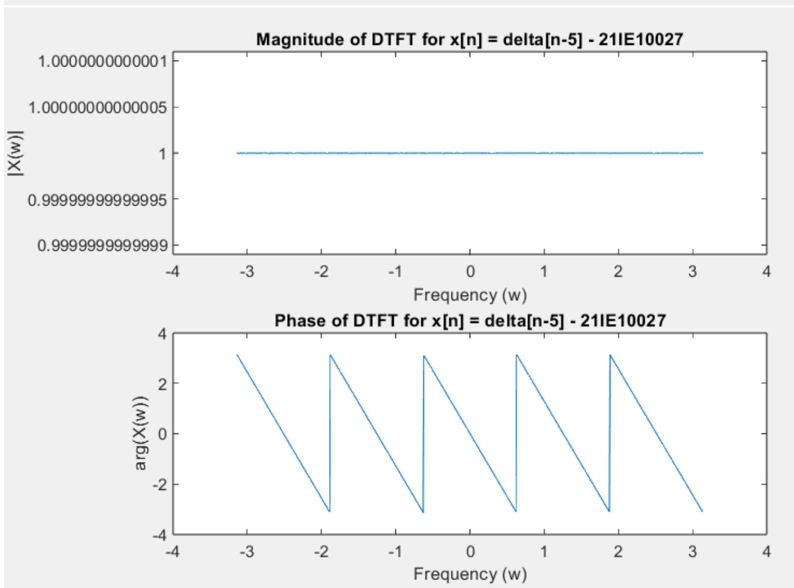
```
figure;
subplot(2,1, 1);
plot(w, abs(X1));
title('Magnitude of DTFT for x[n] = delta[n] - 21IE10027');
xlabel('Frequency (w)');
ylabel('|X(w)|');
subplot(2,1,2);
plot(w, angle(X1));
title('Phase of DTFT for x[n] = delta[n] - 21IE10027');
xlabel('Frequency (w)');
ylabel('arg(X(w))');
```

```
figure;
subplot(2,1,1);
plot(w, abs(X3));
title('Magnitude of DTFT for x[n] = (0.5)^n * u[n] - 21IE10027');
xlabel('Frequency (w)');
ylabel('|X(w)|');
subplot(2,1,2);
plot(w, angle(X3));
title('Phase of DTFT for x[n] = (0.5)^n * u[n] - 21IE10027');
xlabel('Frequency (w)');
ylabel('arg(X(w))');
```

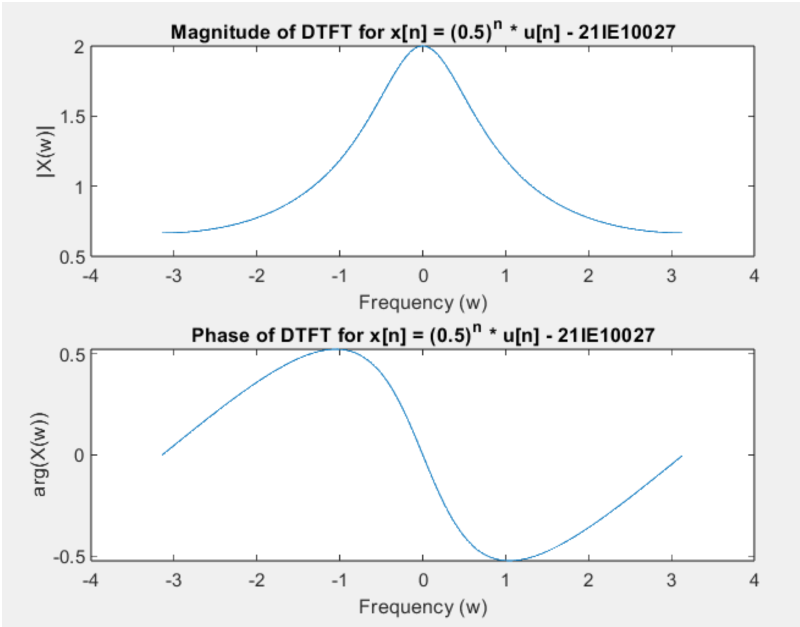
```
function X = DTFT(x, n0, dw)
    % Compute the DTFT of the discrete-time signal x
    % Create the frequency vector
    w = -pi:dw:pi;
    N=length(x);
    % Initialize X as a vector of zeros
    X = zeros(size(w));
    % Compute the DTFT using the formula
    for k = 1:length(w)
        X(k) = sum(x .* exp(-1j * w(k) * (0:N-1 + n0)));
    end
end
```



Magnitude of DTFT for x[n] = delta[n] - 21IE10027
|X(w)|
2
1.5
1
0.5
0
-4
-3
-2
-1
0
1
2
3
4
Frequency (w)
Phase of DTFT for x[n] = delta[n] - 21IE10027
arg(X(w))
1
0.5
0
-0.5
-1
-4
-3
-2
-1
0
1
2
3
4
Frequency (w)



Magnitude of DTFT for x[n] = delta[n-5] - 21IE10027
|X(w)|
1.0000000000001
1.00000000000005
1
0.99999999999995
0.9999999999999
-4
-3
-2
-1
0
1
2
3
4
Frequency (w)
Phase of DTFT for x[n] = delta[n-5] - 21IE10027
arg(X(w))
4
2
0
-2
-4
-4
-3
-2
-1
0
1
2
3
4
Frequency (w)

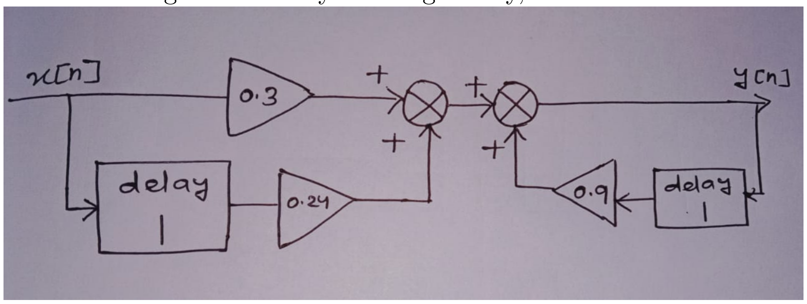


3 Magnitude and Phase of the Frequency Response of a DiscreteTime Systems

The given difference equation is:

$$y[n] = 0.9y[n-1] + 0.3x[n] + 0.24x[n-1]$$

The block diagram of the system is given by,



To obtain the impulse response, we replace $x[n]$ with $\delta[n]$ (the discrete-time impulse function) and set up the initial conditions for causality.
When $x[n] = \delta[n]$, the difference equation becomes:

$$h[n] = 0.9h[n-1] + 0.3\delta[n] + 0.24\delta[n-1]$$

The initial condition is $h[0] = 0.3$ (due to the impulse $\delta[n]$ at $n = 0$).

Using the recursion relation $h[n] = 0.9h[n-1]$, we can find the impulse response for positive values of n:

$$h[1] = 0.9 * h[0] + 0.24 = 0.9 \times 0.3 + 0.24 = 0.51$$
$$h[2] = 0.9 * h[1] = 0.9 \times 0.51 = 0.459$$
$$h[3] = 0.9 * h[2] = 0.9 \times 0.459 = 0.4131$$

So, the causal impulse response $h[n]$ is given by:

$$h[n] = 0.3\delta[n] + 0.51\delta[n-1] + 0.459\delta[n-2] + 0.4131\delta[n-3] + ...$$

The Frequency response of the system can be derived from the above impulse response of system ,

$$H(\omega) = 0.3 + 0.51e^{-j\omega} + 0.459e^{-2j\omega} + 0.4131e^{-3j\omega} + ...$$

To find the frequency response of the system using the DTFT method, we'll take the DTFT of both sides of the given difference equation:

$$y[n] = 0.9y[n-1] + 0.3x[n] + 0.24x[n-1]$$

Applying the DTFT to both sides:

$$\mathcal{Y}(\omega) = 0.9\mathcal{Y}(\omega)e^{-j\omega} + 0.3\mathcal{X}(\omega) + 0.24e^{-j\omega}\mathcal{X}(\omega)$$

Where $\mathcal{Y}(\omega)$ and $\mathcal{X}(\omega)$ are the DTFTs of $y[n]$ and $x[n]$ respectively.
Simplifying, we get:

$$\mathcal{Y}(\omega) = \frac{0.3 + 0.24e^{-j\omega}}{1 - 0.9e^{-j\omega}}\mathcal{X}(\omega)$$

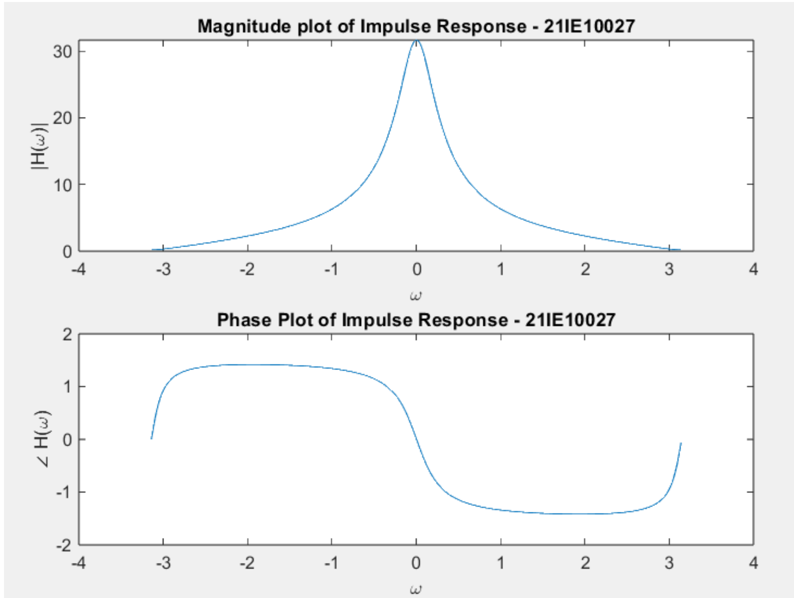
The transfer function $H(\omega)$ is defined as the ratio of the DTFT of the output $y[n]$ to the input $x[n]$:

$$H(\omega) = \frac{\mathcal{Y}(\omega)}{\mathcal{X}(\omega)} = \frac{0.3 + 0.24e^{-j\omega}}{1 - 0.9e^{-j\omega}}$$

This is the frequency response of the system.
Here is the MATLAB code for plotting the Magnitude and Phase response of the system.

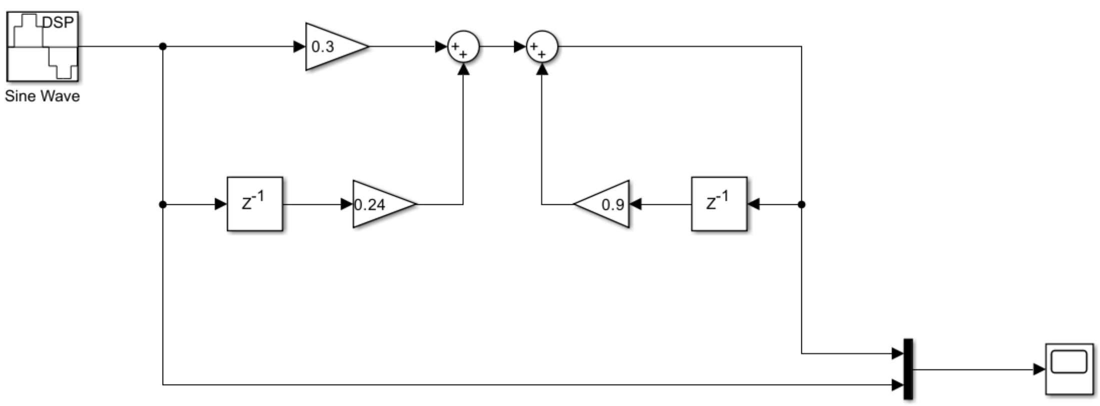
```
% Define the system coefficients
a = [1, -0.9];
b = [0.3, 0.24];
omega = linspace(-pi, pi, 1000);
% Compute the DTFT of the left-hand side (LHS) and
% right-hand side (RHS) separately
LHS = fft(a, N); % DTFT of y[n]
RHS = fft(b, N); % DTFT of the input x[n]
% Compute the frequency response H(omega)
H = LHS ./ RHS;
% Compute magnitude and phase
magnitude_H = abs(H);
phase_H = angle(H);
```

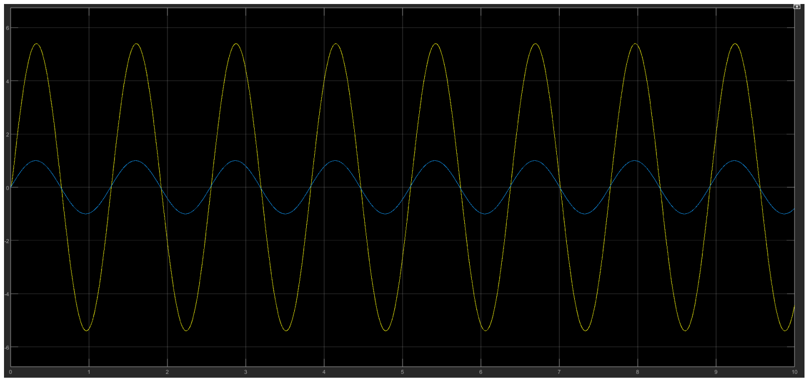
```
% Plot the magnitude response
subplot(2, 1, 1);
plot(omega, magnitude_H);
title('Magnitude plot of Impulse Response - 21IE10027');
xlabel('\omega');
ylabel('|H(\omega)|');
% Plot the phase response
subplot(2, 1, 2);
plot(omega, phase_H);
title('Phase Plot of Impulse Response - 21IE10027');
xlabel('\omega');
ylabel('\angle H(\omega)');
```



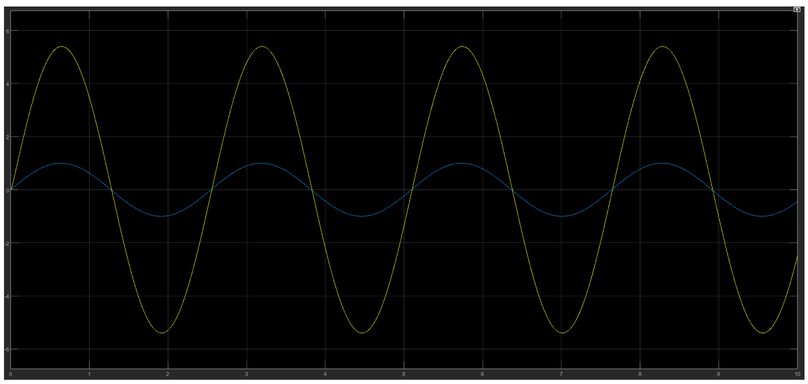
4 System Analysis

A sinusoidal input of variable frequency was given as an excitation to the given system modelled by a Simulink-block diagram. As expected, the output signal was also sinusoidal as the system under consideration is an LTI system.
Here is the SIMULINK diagram of the system.

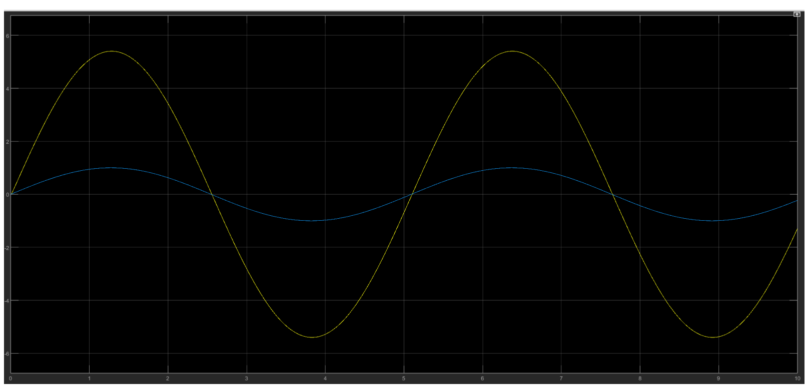




$\omega = \pi/4$



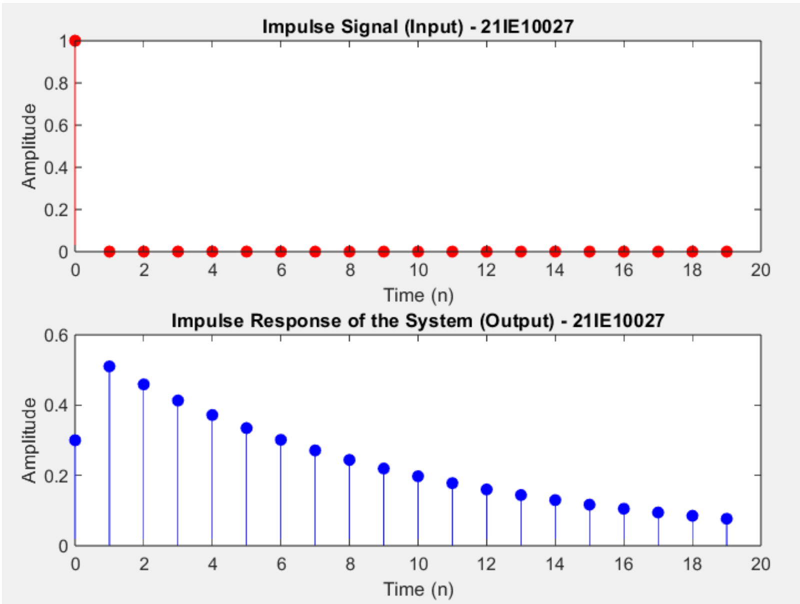
$\omega = \pi/8$



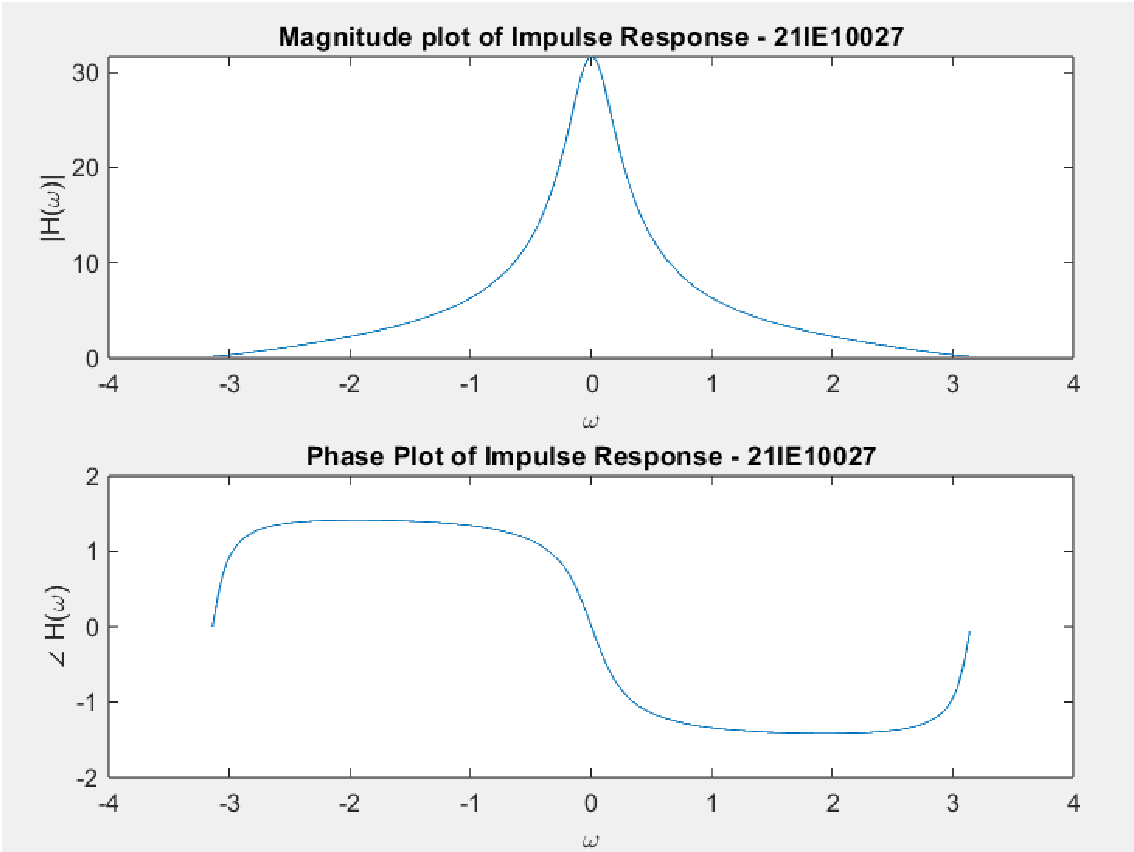
$\omega = \pi/16$

The MATLAB code for the impulse response of the system is given below.

```
% Define the system coefficients
a = [1, -0.9];
b = [0.3, 0.24];
% Compute the impulse response assuming a length of 20
impulse_response = filter(b, a, [1, zeros(1,19)]);
% Define the time index
n = 0:length(impulse_response) - 1;
% Impulse Signal
subplot(2, 1, 1);
stem(n, [1, zeros(1, 19)], 'r', 'filled');
title('Impulse Signal (Input) - 21IE10027');
xlabel('Time (n)');
ylabel('Amplitude');
% Impulse Response
subplot(2, 1, 2);
stem(n, impulse_response, 'b', 'filled');
title('Impulse Response of the System (Output) - 21IE10027');
xlabel('Time (n)');
ylabel('Amplitude');
```



The Magnitude and Phase response of the System is plotted below.



The table contains both the practical values of amplitude measurements and their theoretical values. We can infer from Table 1 that the theoretical and observed values are almost equal.

Frequency	Theoretical	Observed
$\pi/16$	2.545	2.52
$\pi/8$	1.3816	1.36
$\pi/4$	0.6815	0.688

Table 1:Comparison of Theoretical and Observed values