Lambda Cases (lcases)

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1 Introduction

Haskell is a delightful language. Yet, for some reason, it doesn't seem to have it's rightful place in terms of popularity in industry. Why is it so? Is it inherently hard to learn and therefore only the brave dare to use it, or could it be that the syntax is perplexing to the amateur eye? It is my belief that with some syntax changes that give a greater familiarity to the new user, there would be no language more compelling than (the new) Haskell. In an attempt to achieve that familiarity, I present some (hopefully useful) new syntax, of which some is closer to the imperative/OOP style (to attract more already experienced programmers from these languages), some is closer to mathematics (in which most programmers should be experienced) and some is closer to natural language (in which we are all already experienced).

2 Language Description: General

2.1 Program Structure

An leases program consists of a set of definitions and theorems. Definitions are split into value definitions, type definitions and type proposition definitions. Theorems are proven type propositions. The definition of the "main" value determines the program's behaviour. Functions as well as "Environment Actions" (see section 3.2.5) are also considered values.

Program example: extended euclidean alogirthm

```
// type definitions

tuple_type Coeffs
value (previous, current) : Int x Int

tuple_type GcdAndCoeffs
value (gcd, a, b) : Int x Int x Int

// algorithm

ext_euc
: (Int, Int) => GcdAndCoeffs
= ext_euc_rec(init_a_coeffs, init_b_coeffs)
    where
    init_a_coeffs, init_b_coeffs
```

```
: all Coeffs
      = (1, 0), (0, 1)
    ext_euc_rec
      : (Coeffs, Coeffs, Int, Int) => GcdAndCoeffs
      = (a_coeffs, b_coeffs, x, cases) =>
        0 => (x, a_coeffs.previous, b_coeffs.previous)
        y => ext_euc_rec(next <- a_coeffs, next <- b_coeffs, y, x -> mod <- y)
          where
          next
            : Coeffs => Coeffs
            = cs => (cs.current, cs.previous - x -> div <- y * cs.current)
// reading, printing and main
read_two_ints
  : (Int x Int) IOAction
  = print <- "Please give me 2 ints";</pre>
    get_line ;> split_words o> apply(from_string)to_all o> cases =>
      [x, y] \Rightarrow (x, y) \rightarrow io_action
      ... => io_error <- "You didn't give me 2 ints"
print_gcd_equation
  : (Int, Int, GcdAndCoeffs) => (EmptyVal)IOAction
  = (x, y, gcd_cs) => print("gcd = " + gcd_cs.gcd + " = " + linear_comb_string)
    linear_comb_string
      : String
      = gcd_cs.a + " * " + x + " + " + gcd_cs.b + " * " + y
main
  : (EmptyVal)IOAction
  = read_two_ints ;> ints =>
    print_gcd_equation(ints.1st, ints.2nd, ext_euc(ints.1st, ints.2nd))
Program grammar
```

```
\langle program \rangle ::= (\langle value-def \rangle | \langle type-def \rangle | \langle type-prop-def \rangle) +
```

2.2 Keywords

The leases keywords are the following:

```
cases where all tuple_type value or_type values type_predicate function type_proposition equivalent type_theorem proof
```

Each keyword's functionality is described in the respective section shown in the table below:

Keyword	Section
cases	3.3 Function Expressions
where all	3.4 Value Definitions
tuple_type value or_type values	4.1 Types
type_predicate function type_proposition equivalent	4.2 Type Logic
type_theorem proof	

The "cases" and "where" keywords are also reserved words. Therefore, even though they can be generated by the "identifiers" grammar, they cannot be used as identifiers (see "Literals and Identifiers" section 3.1.1).

3 Language Description: Values

3.1 Basic Expressions

3.1.1 Literals and Identifiers

Literals

 \bullet Examples

```
1 2 17 42 -100

1.61 2.71 3.14 -1234.567

'a' 'b' 'c' 'x' 'y' 'z' '.' ',' '\n'

"Hello World!" "What's up, doc?" "Alrighty then!"
```

• Description

There are literals for the four basic types: Int, Real, Char, String. These are the usual integers, real numbes, characters and strings. The exact specification of literals is the same as in the Haskell report.

• Grammar

```
\langle literal \rangle ::= \langle literal \rangle
```

TODO add the grammar from the haskell report

Identifiers

• Examples

```
x y z
a1 a2 a3
funny_identifier
unnecessarily_long_identifier
apply()to_all
```

• Description

An identifier is the name of a value or a parameter. It is used in the definition of a value (see "value definition" section 3.4) and in expressions that use that value, or in the parameters of a function and in the body of that function.

An identifier starts with a lower case letter and is followed by lower case letters or underscores. It is also possible to have pairs of parentheses in the middle of an identifier (see "Parenthesis Function Application" section 3.1.3 for why this can be useful). Finally, an identifier can be ended with a digit.

 \bullet Grammar

```
\langle identifier \rangle ::= [a-z] [a-z_]^* ( '() ' [a-z_] + )^* [ [0-9] ]
```

Even though the "cases" and "where" keywords can be generated by this grammar, they cannot be used as identifiers.

3.1.2 Parenthesis, Tuples and Lists

Parenthesis

• Examples

```
(1 + 2)
(((1 + 2) * 3) ^ 4)
(x => f(x) + 1)
(s => "f(val) + 1 is: " + s)
(val -> (x => f(x) + 1) -> to_string -> (s => "f(val) + 1 is: " + s))
("Line is: " + line)
(get_line ;> line => print("Line is: " + line))
(do(3)times <- (get_line ;> line => print("Line is: " + line)))
```

• Description

An expression is put in parenthesis to prioritize it or isolate it in a bigger (operator) expression. The expressions inside parethesis are operator or function expressions.

• Grammar

```
\langle paren-expr \rangle ::= '(' \langle op-or-func-expr \rangle ')'
\langle op-or-func-expr \rangle ::= \langle simple-op-expr \rangle \mid \langle op-expr-func-end \rangle \mid \langle simple-func-expr \rangle
```

Tuples

• Examples

```
(1, "What's up, doc?")
(2, "Alrighty then!", 3.14)
(x, y, z, w)
(1, my_function, (x, y, z) => (x ^ 2 + y ^ 2 + z ^ 2) ^ (1/2))
```

ullet Description

Tuples are used to group many values (of possibly different types) into one. The type of the tuple can be either the product of the types of the fields or a defined tuple_type which is equivalent to the afformentioned product type (see "tuple_type" section 4.1.2). For example, the type of the second tuple above could be:

```
Int x String x Real
or:
MyType
assuming "MyType" has been defined in a similar way to the following:
tuple_type MyType
value (my_int, my_string, my_real) : Int x String x Real
```

• Big Tuples

Example

```
my_big_tuple
  : String x Int x Real x String x String x (String x Real x Real)
  = ( "Hey, I'm the first field and I'm also a relatively big string."
    , 42, 3.14, "Hey, I'm the first small string", "Hey, I'm the second small string"
    , ("Hey, I'm a string inside the nested tuple", 2.71, 1.61)
    )
```

Description

It is possible to stretch a (big) tuple expression over multiple lines (only) in a seperate value definition (see "Value Definitions" section 3.4.1). In that case:

- The character '(' is after the "=" part of the value definition and the first field must be in the same line.
- The tuple can split in a new line only at a ',' character. Every such line must be indented so that the ',' is in same column where the '(' character was in the first line.
- The tuple must be ended by a line that only contains the ')' character and is also indented so that the ')' is in same column where the '(' character was in the first line.
- The precise indentation rules are described in the section "Indentation System" 3.5.1.
- Grammar

```
 \langle tuple \rangle ::= `(` \langle line-expr \rangle `, \_' \langle comma-sep-line-exprs \rangle `)` \\ \langle comma-sep-line-exprs \rangle ::= \langle line-expr \rangle ( `, \_' \langle line-expr \rangle )^* \\ \langle line-expr \rangle ::= \langle no-paren-op-arg \rangle \mid \langle op-or-func-expr \rangle \\ \langle big-tuple \rangle ::= \\ `(` \langle line-expr \rangle [ ``n' \langle indent \rangle ] `, \_' \langle comma-sep-line-exprs \rangle \\ `(``n' \langle indent \rangle `, ` \langle comma-sep-line-exprs \rangle )^* \\ ``n' \langle indent \rangle `)`
```

Lists

• Examples

```
[1, 2, 17, 42, -100]
[1.61, 2.71, 3.14, -1234.567]
["Hello World!", "What's up, doc?", "Alrighty then!"]
[x => x + 1, x => x + 2, x => x + 3]
[x, y, z, w]
```

• Description

Lists are used to group many values of the same type into one. The type of the list is ListOf(A)s where A is the type of every value inside. Therefore, the types of the first four examples are:

```
ListOf(Int)s
ListOf(Real)s
ListOf(String)s
(A)And(Int)AddTo(B) --> ListOf(A => B)s
```

And the last list is only legal if x, y, z and w all have the same type. Assuming they do and it's the type T, the type of the list is:

ListOf(T)s

• Big Lists

It is possible to stretch a (big) list expression over multiple lines (only) in a seperate value definition (see "Value Definitions" section 3.4.1). In that case:

- The character '[' is after the "=" part of the value definition and the first element must be in the same line.
- The list can split in a new line only at a ',' character. Every such line must be indented so that the ',' is in same column where the '[' character was in the first line.
- The tuple must be ended by a line that only contains the ']' character and is also indented so that the ']' is in same column where the '[' character was in the first line.
- The precise indentation rules are described in the section "Indentation System" 3.5.1.

Example:

```
my_big_list
  : ListOf(Int => (EmptyVal)IOAction)s
  = [ x => print("I'm the first function and x + 1 is: " + (x + 1))
    , x => print("I'm the second function and x + 2 is: " + (x + 2))
    , x => print("I'm the third function and x + 3 is: " + (x + 3))
  ]
```

• Grammar

```
\langle list \rangle ::= `[' [ \langle comma-sep-line-exprs \rangle ] `]'
\langle big-list \rangle ::= `[' \langle comma-sep-line-exprs \rangle ( `\n' \langle indent \rangle `, ' \langle comma-sep-line-exprs \rangle ) * `\n' \langle indent \rangle `]'
```

3.1.3 Parenthesis Function Application

• Examples

```
f(x)
f(x, y, z)
(x)to_string
apply(f)to_all
apply(f)to_all(1)
```

• Description

Function application in leases can be done in many different ways in an attempt to maximize readability. In this section, we discuss the ways function application can be done with parenthesis.

In the first two examples, we have the usual mathematical function application which is also used in most programming languages and should be familiar to the reader. That is, function application is done with the arguments of the function in parenthesis separated by commas and **appended** to the function identifier.

We extend this idea by allowing the arguments to be **prepended** to the function identifier (third example). Finally, it is also possible to to have the arguments **inside** the function identifier provided the function has been **defined with parentheses inside the identifier**. For example, below is the definition of "apply()to_all":

```
apply()to_all
  : (A => B, ListOf(A)s) => ListOf(B)s
  = (f, cases) =>
    empty => empty
    non_empty:1 => non_empty:(f <- l.head, apply(f)to_all <- l.tail)</pre>
```

The actual definition doesn't matter at this point, what matters is that the identifier is "apply()to_all" with the parentheses **included**. This is very useful for defining functions where the argument in the middle makes the function application look and sound more like natural language.

It is possible to have many parentheses pairs in a single function application (last example). The arguments are always inserted to the function from **left to right**. Therefore, when multiple parentheses pairs are present the arguments of the leftmost parentheses are inserted first then the next ones to the right and so on.

• Grammar

3.1.4 Prefix and Postfix Functions

Prefix Functions

• Examples

```
the_value:1
non_empty:1
error:e
result:r
apply(the_value:)to_all
```

• Description

Prefix functions are automatically generated from or_type definitions (see "Or Types" section 4.1.3). They are functions that convert a value of a particular type to a value that is a case of an or_type and has values of this type inside. For example in the first example above we have:

```
1
  : Int
the_value:1
  : Possibly(Int)
```

Where the function thevalue: is automatically generated from the definition of the Possibly type:

```
or_type Possibly(A)
values the_value:A | no_value
```

And it has the type $A \Rightarrow Possibly(A)$.

These functions are called prefix functions because they are prepended to their argument. However, they can also be used as any other function. An illustration of the aforementioned is the last example, where the function the_value: is an argument of the function apply()to_all. Prefix functions always end with a colon.

• Grammar

```
\langle pre\text{-}func \rangle ::= \langle identifier \rangle \text{ ':'}
\langle pre\text{-}func\text{-}app \rangle ::= \langle pre\text{-}func \rangle \text{ (} \langle basic\text{-}expr \rangle \text{ | } \langle paren\text{-}expr \rangle \text{ | } \langle pre\text{-}func\text{-}app \rangle \text{ )}}
\langle basic\text{-}expr \rangle ::= \langle literal \rangle \text{ | } \langle identifier \rangle \text{ | } \langle tuple \rangle \text{ | } \langle list \rangle \text{ | } \langle paren\text{-}func\text{-}app \rangle \text{ | } \langle post\text{-}func\text{-}app \rangle \text{ }}
```

Postfix Functions

• Examples

```
name.first_name
list.head
date.year
tuple.1st
apply(.1st)to_all
```

• Description

Postfix functions are automatically generated from tuple_type definitions (see "Tuple Types" section 4.1.2). They are functions that take a tuple_type value and return a particular field (i.e. projection functions). For example in the first example above we have:

```
name
    : Name
name.first_name
    : String
```

Where the function .first name is automatically generated from the definition of the Name type:

```
tuple_type Name
value (first_name, last_name) : String x String
```

And it has the type Name => String.

There are also the following special projection functions for **product types**: .1st .2nd .3rd .4th .5th. For the 4th example above, assuming:

```
tuple
  : Int x String
We have:
tuple.1st
  : Int
```

The general types of these functions are:

```
.1st
   : (A)Is(B)sFirst --> B => A
.2nd
   : (A)Is(B)sSecond --> B => A
```

These functions are called postfix functions because they are appended to their argument. However, they can also be used as any other function. An illustration of the aforementioned is the last example, where the function .1st is an argument of the function apply()to_all. Postfix functions always begin with a dot.

• Grammar

```
\langle post\text{-}func \rangle ::= `.` \langle identifier \rangle
\langle post\text{-}func\text{-}app \rangle ::= (\langle paren\text{-}expr \rangle \mid \langle basic\text{-}expr \rangle) \langle post\text{-}func \rangle
```

3.2 Operators

3.2.1 Function Application Operators

Operator	Type	
->	(A, A => B) => B	
<-	(A => B, A) => B	

The function application operators "->" and "<-" are a different way to apply functions to arguments than the usual parenthesis function application. They are meant to look like arrows that point from the argument to the function. These operators are very useful for chaining many function applications without the clutter of having to open and close parentheses for each one of the functions. For example, assuming we have the following functions with the behaviour suggested by their names and types:

```
apply()to_all
  : (A => B, ListOf(A)s) => ListOf(B)s
string_length
  : String => Int
filter_with
  : (A => Bool, ListOf(A)s) => ListOf(A)s
is_odd
  : Int => Bool
sum_ints
  : ListOf(Int)s => Int
And a list of strings:
strings
  : ListOf(String)s
```

Here is a simple way to get the total number of characters in all the strings that have odd length:

These operators can also be used together to put a function between two arguments if that function is commonly used that way in math (or if it looks better for a certain function). For example the "mod" function can be used like so:

$$x \rightarrow mod \leftarrow y$$

Which is equivalent to:

3.2.2 Function Composition Operators

Operator	Type
0>	(A => B, B => C) => (A => C)
<o< th=""><td>(B => C, A => B) => (A => C)</td></o<>	(B => C, A => B) => (A => C)

The function composition operators "o>" and "<o" are used to compose functions, each one in the corresponding direction. The use of the letter 'o' is meant to be similar to the mathematical function composition symbol 'o' and the symbols '>', '<' are used so that the operator points from the function which is applied first to the function which is applied second. A neat example using function composition is the following. Assuming we have the following functions with the behaviour suggested by their names and types:

split_words
 : String => ListOf(String)s
apply()to_all
 : (A => B, ListOf(A)s) => ListOf(B)s
reverse_string
 : String => String
merge_words
 : ListOf(String)s => String

We can reverse the all the words in a string like so:

reverse_words

- : String => String
- = split_words o> apply(reverse_string)to_all o> merge_words

Ofcourse this can be done equivalently using the other operator:

reverse_words

- : String => String
- = merge_words <o apply(reverse_string)to_all <o split_words

3.2.3 Arithmetic Operators

Operator	Type	
^	(A) ToThe (B) Gives $(C) \longrightarrow (A, B) \Longrightarrow C$	
*	$(A)And(B)MultiplyTo(C) \longrightarrow (A, B) \Longrightarrow C$	
/	(A) Divides (B) To $(C) \longrightarrow (B, A) \Longrightarrow C$	
+	$(A)And(B)AddTo(C) \longrightarrow (A, B) \Longrightarrow C$	
-	(A)SubtractsFrom(B)To(C) \longrightarrow (B, A) \Longrightarrow C	

The usual arithmetic operators work as they are expected, similarly to mathematics and other programming languages for the usual types. However, they are generalized. The examples below show their generality:

```
>> 1 + 1
2
>> 1 + 3.14
4.14
>> 'a' + 'b'
"ab"
>> 'w' + "ord"
"word"
>> "Hello " + "World!"
"Hello World!"
>> 5 * 'a'
"aaaaa"
>> 5 * "hi"
"hihihihihi"
>> "1,2,3" - ','
"123"
```

Let's analyze further the example of addition. The type can be read as such: the '+' operator has the type (A, B) => C, provided that the type proposition (A)And(B)AddTo(C) holds. This proposition being true, means that addition has been defined for these three types (see section "Type Logic" 4.2 for more on type propositions). For example, by the examples above we can deduce that the following propositions are true (in the order of the examples):

```
(Int)And(Int)AddTo(Int)
(Int)And(Real)AddTo(Real)
(Char)And(Char)AddTo(String)
(Char)And(String)AddTo(String)
(Int)And(Char)MultiplyTo(String)
(Int)And(String)MultiplyTo(String)
(Char)SubtractsFrom(String)To(String)
```

This allows us to use the familiar arithmetic operators in types that are not necessarily numbers but it is somewhat intuitively obvious what the should do in those other types. Furthermore, their behaviour can be defined by the user for new user defined types!

3.2.4 Comparison and Boolean Operators

Operator	Type	
= /=	(A)HasEquality> (A, A) => Bool	
> < >= <=	(A)HasOrder $>$ (A, A) $=>$ Bool	
&	(Bool, Bool) => Bool	

The comparison and boolean operators behave the same as in Haskell and very similarly to most programming languages. The main difference is that in leases the "equals", "and" and "or" operators have the symbol once (= & |) rather than twice (== && | |).

3.2.5 Environment Action Operators

Operator	Type
;>	(E)IsAnEnvAction \longrightarrow (E(A), A \Longrightarrow E(B)) \Longrightarrow E(B)
;	(E)IsAnEnvAction $>$ (E(A), E(B)) $=>$ E(B)

Simple Example

```
print_string("I'll repeat the line.") ; get_line ;> print_string
```

The example above demonstrates the use of the environment action operators with the IOAction type, which is how IO is done in leases. Some light can be shed on how this is done, if we take a look at the types (as always!):

```
print_string
  : String => (EmptyVal)IOAction
print_string("I'll repeat the line.")
  : (EmptyVal)IOAction
get_line
  : (String) IOAction
  : (E) IsAnEnvAction --> (E(A), E(B)) \Rightarrow E(B)
print_string("I'll repeat the line.") ; get_line
  : (String) IOAction
  where (IOAction)IsAnEnvAction is true, E = IOAction, A = EmptyVal, B = String
  : (E)IsAnEnvAction --> (E(A), A => E(B)) => E(B)
print_string("I'll repeat the line.") ; get_line ;> print_string
  : (EmptyVal)IOAction
  where (IOAction) IsAnEnvAction is true, E = IOAction, A = String, B = EmptyVal
Example program
main
  : (EmptyVal)IOAction
  = print_string <- "Hello! What's your name?" ; get_line ;> name =>
    print_string("Oh hi " + name + "! What's your age?") ; get_line ;> age =>
    print_string("Oh that's crazy " + name + "! I didn't expect you to be " + age + "!");
In this bigger but similar example the types are:
print_string
  : String => (EmptyVal)IOAction
get_line
  : (String) IOAction
print_string <- "Hello! ... "</pre>
  : (EmptyVal)IOAction
print_string("Oh hi...)
  : (EmptyVal)IOAction
print_string("Oh that's crazy...)
  : (EmptyVal)IOAction
  : (E)IsAnEnvAction --> (E(A), E(B)) => E(B)
```

```
print_string("Oh hi...) ; get_line
  : (String) IOAction
  where (IOAction)IsAnEnvAction is true, E = IOAction, A = EmptyVal, B = String
age => print_string("Oh that's crazy...)
  : String => (EmptyVal)IOAction
;>
  : (E)IsAnEnvAction --> (E(A), A => E(B)) => E(B)
print_string("Oh hi...) ; get_line ;> age =>
print_string("Oh that's crazy...)
  : (EmptyVal)IOAction
  where (IOAction)IsAnEnvAction is true, E = IOAction, A = String, B = EmptyVal
print_string <- "Hello..." ; get_line</pre>
  : (String) IOAction
name => print_string("Oh hi ... (till the end)
  : String => (EmptyVal)IOAction
print_string <- "Hello..." ; get_line ;> name =>
print_string("Oh hi ... (till the end)
  : (EmptyVal)IOAction
```

Therefore, "main: (EmptyVal)IOAction" checks out. The key here is to remember that function expressions extend to the end of the whole expression. Therefore, we have "name => ... (till the end)" and "age => ... (till the end)" as the second arguments of the two occurrences of the ";>" operator.

Description

The environment action operators are used to combine values that do environment actions into values that do more complicated environment actions. Environment actions are also represented by types. More acurately, type constructors that take a type as an argument and produce a new type (just like ListOf()s). A value of the type E(A) where (E)IsAnEnvAction does an environment action of type E that produces value of type A.

The effect of the ";" operator described in words is the following: given a value of type E(A) and a value of type E(B) (which do environment actions that produce values of type A and B respectively), create a new value the does both actions (provided the first did not result in an error). The overall effect is a value that does an environment action of type E (the combination of the "smaller" actions) which produces a value of type B (the one produced by the second action) and therefore it is of type E(B).

Note that the value of type A produced by the first action is not used anywhere. This happens mostly when A = EmptyVal and it is because values of type E(EmptyVal) are used for their environment action only (e.g. print_string(...) : (EmptyVal)IOAction).

How the two environment actions of the E(A) and E(B) values are combined to produce the new environment action is specific to the environment action type E.

The effect of the ";>" operator described in words is the following: given a value of type E(A) (which does an environment action of type E(A) that produces a value of type E(A) and a value of type E(B) (which is a function that takes a value of type E(A) and returns an environment action of type E(A) that produces a value of type E(A), combine those two values by creating a value that does the following:

- Performs the first action that produces a value of type A
- Takes the value of type A produced (provided there was no error) and passes it to the function of type A => E(B) that then returns an action
- Perfoms the resulting action

The overall effect is an environment action of type E that at the end produces a value is of type B and therefore the new value is of type E(B).

3.2.6 Operator Expressions

• Examples

```
1 + 2
1 + x * 3 ^ y
"Hello " + "World!"
x -> f -> g
f o> g o> h
x = y
x >= y - z & x < 2 * y
get_line; get_line; > line => print("Second line: " + line)
```

• Description

Operator expressions are expressions that use operators. Operators act like two-argument-functions that are placed in between their arguments. Therefore, they have function types and they act as it is described in their respective sections above this one.

An operator expression might have multiple operators. The order of operations is explained in the next section ("Complete Table, Precedence and Associativity") in Table 2.

Just like functions, the sub-expressions that act as arguments to an operator, must have types that match the types expected by the operator.

It is possible to end an operator expression with a function. This is mostly useful with the ";>" operator (see previous section: "Environment Operators"), but it is also possible with the following operators: "->", "o>", "<o".

• Big Operator Expressions

It is possible to stretch a (big) operator expression over multiple lines. In that case:

- The operator expression must split in a new line after an operator (not an argument).
- Every line after the first must be indented so that in begins at the column where the first character of the operator expression was in the first line.
- The precise indentation rules are described in the section "Indentation System" 3.5.1.
- Grammar

```
\langle op\text{-}expr\rangle ::= \langle simple\text{-}op\text{-}expr\rangle \mid \langle op\text{-}expr\text{-}func\text{-}end\rangle \mid \langle big\text{-}op\text{-}expr\rangle \mid \langle cases\text{-}op\text{-}expr\rangle
\langle simple\text{-}op\text{-}expr\rangle ::= \langle op\text{-}arg\rangle \ ( \ `\sqcup' \ \langle op\rangle \ `\sqcup' \ \langle op\text{-}arg\rangle \ ) +
\langle op\text{-}expr\text{-}func\text{-}end\rangle ::= \langle simple\text{-}op\text{-}expr\rangle \ `\sqcup' \ \langle op\rangle \ `\sqcup' \ \langle simple\text{-}func\text{-}expr\rangle
```

3.2.7 Complete Table, Precedence and Associativity

Table 1: The complete table of leases operators along with their types and their short descriptions.

Operator	Type	Description
->	(A, A => B) => B	Right function application
<-	(A => B, A) => B	Left function application
0>	(A => B, B => C) => (A => C)	Right function composition
<0	(B => C, A => B) => (A => C)	Left function composition
^	(A)ToThe(B)Gives(C) \longrightarrow (A, B) \Longrightarrow C	General exponentiation
*	(A)And(B)MultiplyTo(C) \longrightarrow (A, B) \Longrightarrow C	General multiplication
/	(A)Divides (B) To (C) > (B, A) => C	General division
+	$(A)And(B)AddTo(C) \longrightarrow (A, B) \Longrightarrow C$	General addition
-	(A)SubtractsFrom(B)To(C)> (B, A) => C	General subtraction
= /=	(A)HasEquality $>$ (A, A) $=>$ Bool	Equality operators
> < >= <=	(A) HasOrder> $(A, A) \Rightarrow$ Bool	Order operators
&	(Bool, Bool) => Bool	Boolean operators
;>	(E)IsAnEnvAction> (E(A), A => E(B)) => E(B)	Monad bind
;	(E)IsAnEnvAction> (E(A), E(B)) => E(B)	Monad then

The order of operations is done from highest to lowest precedence. In the same level of precedence the order is done from left to right if the associativity is "Left" and from right to left if the associativity is "Right". For the operators that have associativity "None" it is not allowed to place them in the same operator expression. The precedence and associativity of the operators is shown in the table below.

Table 2:	The table of	precedence and	associativity of	the leases operators.

Operator	Precedence	Associativity
->	10 (highest)	Left
<-	9	Right
0> <0	8	Left
^	7	Right
* /	6	Left
+ -	5	Left
= /= > < >= <=	4	None
&	3	Left
I	2	Left
;> ;	1	Left

3.3 Function Expressions

 $\langle func\text{-}expr \rangle ::= \langle simple\text{-}func\text{-}expr \rangle \mid \langle big\text{-}func\text{-}expr \rangle \mid \langle cases\text{-}func\text{-}expr \rangle$

3.3.1 Regular Function Expressions

• Examples

$$a \Rightarrow 17 * a + 42$$

 $(x, y, z) \Rightarrow (x^2 + y^2 + z^2)^(1/2)$

• Description

Regular function expressions are used to define functions or be part of bigger expressions as anonymous functions. They are comprised by their parameters and their body. Parameter are specified by identifiers. The parameters are either only one, in which case there is no parenthesis, or they are many, in which case they are in parenthesis, seperated by commas. The parameters and the body are seperated by an arrow (" => "). The body is an operator expression.

• Big Function Expressions

It is possible to stretch a (big) function expression over multiple lines. In that case:

- The function expression must split in a new line after the arrow ("⇒") following the parameters.
- Every line after the first must be indented so that in begins at the column where the first character of the parameters was in the first line.
- The precise indentation rules are described in the section "Indentation System" 3.5.1.

• Grammar

 $\langle simple-func-expr \rangle ::= \langle parameters \rangle ` => ' \langle simple-func-body \rangle$

```
\langle big\text{-}func\text{-}expr\rangle ::= \langle parameters\rangle \text{`}_{\square} => \text{`}_{\square} \text{`}_{\square} \text{'}_{\square} \text{
```

3.3.2 "cases" Function Expressions

• Examples

```
print_sentimental_bool
  : Bool => (EmptyVal)IOAction
  = cases =>
    true => print <- "It's true!! :)"</pre>
    false => print <- "It's false... :("</pre>
or_type TrafficLight
values green | amber | red
print_sentimental_traffic_light
  : TrafficLight => (EmptyVal)IOAction
  = cases =>
    green => print <- "It's green! Let's go!!! :)"</pre>
    amber => print <- "Go go go, fast!"</pre>
    red => print <- "Stop right now! You're going to kill us!!"</pre>
is_not_red
  : TrafficLight => Bool
  = cases =>
    green => true
    amber => true
    red => false
is_seventeen_or_forty_two
  : Int => Bool
  = cases =>
    17 => true
    42 => true
    ... => false
traffic_lights_match
  : (TrafficLight, TrafficLight) => Bool
  = (cases, cases) =>
    (green, green) => true
    (amber, amber) => true
    (red, red) => true
    ... => false
gcd
  : (Int, Int) => Int
  = (x, cases) =>
   0 => x
    y \Rightarrow gcd(y, x \rightarrow mod \leftarrow y)
```

```
is_empty
  : ListOf(A)s => Bool
  = cases =>
    empty => true
    non_empty:anything => false

apply()to_all
  : (A => B, ListOf(A)s) => ListOf(B)s
  = (f, cases) =>
    empty => empty
    non_empty:list => non_empty:(f <- list.head, apply(f)to_all <- list.tail)</pre>
```

• Description

"cases" is a keyword that works as a special parameter. The difference is that instead of giving the name "cases" to that parameter, it allows the programmer to pattern match on the possible values of that parameter and return a different result for each particular case.

The "cases" keyword can only be used on parameters that have either one of the basic types (Int, Real, Char, String) or an or_type (e.g. Bool, ListOf(A)s).

The last case can be "... => (body of default case)" to capture all remaining cases while dismissing the value (e.g. is_seventeen_or_forty_two example), or it can be "some_id => (body of default case)" to capture all remaining cases while being able to use the value with the name "some_id" (e.g. "y" in gcd example).

It is possible to use the "cases" keyword in multiple parameters to match on all of them. By doing that, each case represents a particular combination of values for the parameters involved (e.g. traffic_lights_match example).

It is also possible to use a "where" expression below a particular case. The "where" expression must be indented two spaces more than than the line where that particular case begins.

A function expression that uses the "cases" syntax must contain the "cases" keyword in at least one parameter. The number of matching expressions in all cases must be the same as the number of parameters with the "cases" keyword.

• Grammar

```
 \langle cases\text{-}func\text{-}expr\rangle ::= \langle cases\text{-}parameters\rangle \text{ `$\sqcup$=>'} \langle case\rangle + \langle end\text{-}case\rangle 
 \langle cases\text{-}parameters\rangle ::= \langle cases\text{-}parameter\rangle \mid \text{`$('$} \langle cases\text{-}parameter\rangle \text{ (`,}_{\sqcup}', \langle cases\text{-}parameter\rangle \text{ )+ '})' 
 \langle cases\text{-}parameter\rangle ::= \langle parameter\rangle \mid \text{`}cases' 
 \langle case\rangle ::= \text{`$\backslash n'$} \langle indent\rangle \langle matching\rangle \text{`}_{\sqcup}=>' \langle case\text{-}body\rangle 
 \langle end\text{-}case\rangle ::= \text{`$\backslash n'$} \langle indent\rangle \text{ (`...'} \mid \langle matching\rangle \text{ )'}_{\sqcup}=>' \langle case\text{-}body\rangle 
 \langle matching\rangle ::= \langle literal\rangle \mid \langle identifier\rangle \mid \langle pre\text{-}func\rangle \langle matching\rangle \mid \langle tuple\text{-}matching\rangle \mid \langle list\text{-}matching\rangle
```

```
\langle tuple-matching \rangle ::= '(' \langle matching \rangle (',' \langle matching \rangle) + ')'
\langle list-matching \rangle ::= '[' [ \langle matching \rangle (',' \langle matching \rangle)^* [', ...']]']'
\langle case-body \rangle ::= ( \langle simple-func-body \rangle | \langle big-op-expr \rangle ) [ \langle where-expr \rangle ]
```

3.4 Value Definitions and "where" Expressions

3.4.1 Value Definitions

• Examples

```
foo
  : Int
  = 42

val1, val2, val3
  : Int, Bool, Char
  = 42, true, 'a'

int1, int2, int3
  : all Int
  = 1, 2, 3

f
  : (Int, Int, Int) => Int
  = (a, b, c) => a + b * c
```

• Description

Value definitions are the main building block of leases programs. To define a new value you give it a name, a type and an expression. The name is in the first line. The second line is indented two spaces more and begins by ": " and continues with the type expression. The third line is indented as the second, begins by "=" and continues with the value expression (which extends to as many lines as needed).

A value definition is either in the first column, where it can be "seen" by all other value definitions, or it is in a "where" expression (see section below), where it can be "seen" by the expression above the "where" and all the other definitions in the same "where" expression.

A value definition can be followed by a "where" expression where intermediate values used in the value expression are defined. In that case, the "where" expression must be indented two spaces more than the "=" line of the value definition.

It is possible to group value definitions together by seperating the names, the types and the expressions with commas. This is very useful for not cluttering the program with many definitions for values with small expressions (e.g. constants). When grouping definitions together it is also possible to use the keyword "all" to give the same type to all the values.

• Grammar

```
\langle value\text{-}def \rangle ::= \langle indent \rangle \ \langle identifier \rangle \ \text{``n'} \ \langle indent \rangle \ \text{`:} \ \text{``l'} \ \langle type \rangle \ \text{``n'} \ \langle indent \rangle \ \text{`=} \ \text{`} \ \langle value\text{-}expr \rangle \ [ \ \langle where\text{-}expr \rangle \ ]
\langle value\text{-}expr \rangle ::= \langle no\text{-}paren\text{-}op\text{-}arg \rangle \ | \ \langle op\text{-}expr \rangle \ | \ \langle func\text{-}expr \rangle \ | \ \langle big\text{-}tuple \rangle \ | \ \langle big\text{-}list \rangle
```

```
\langle grouped\text{-}value\text{-}defs \rangle ::=
          \langle indent \rangle \langle identifier \rangle (`, \sqcup', \langle identifier \rangle) +
          '\n' \langle indent \rangle ': \Box' (\langle type \rangle (', \Box' \langle type \rangle) + | 'all' \langle type \rangle)
          \n' (indent) = \n' (comma-sep-line-exprs) ( \n' (indent) , \comma-sep-line-exprs) 
3.4.2 "where" Expressions
   • Examples
     sort :
        (A)HasOrder --> ListOf(A)s => ListOf(A)s
        = cases =>
          empty => empty
          non_empty:l => sort(less_l) + l.head + sort(greater_l)
            where
            less_1, greater_1
               : all ListOf(A)s
               = filter_with(x => x < l.head, l.tail)</pre>
               , filter_with(x \Rightarrow x \Rightarrow 1.head, 1.tail)
     tuple_type Coeffs
     value (previous, current) : Int x Int
     tuple_type GcdAndCoeffs
     value (gcd, a, b) : Int x Int x Int
     ext_euc
        : (Int, Int) => GcdAndCoeffs
        = ext_euc_rec((1, 0), (0, 1))
     ext_euc_rec
        : (Coeffs, Coeffs, Int, Int) => GcdAndCoeffs
        = (a_coeffs, b_coeffs, x, cases) =>
          0 => (x, a_coeffs.previous, b_coeffs.previous)
          y => ext_euc_rec(next <- a_coeffs, next <- b_coeffs, y, x -> mod <- y)
            where
            next: Coeffs => Coeffs
               = cs => (cs.current, cs.previous - x / y * cs.current)
     big_string
        : String
        = s1 + s2 + s3 + s4
          s1, s2, s3, s4 : all String
            = "Hello, my name is Struggling Programmer."
             , "I have tried way too many times to fit a big chunk of text"
              "inside my program, without it hitting the half-screen mark!"
```

• Description

"where" expressions allow the programmer to use values inside an expression and define them below it. They are very useful for reusing or abbreviating expressions that are specific to a particular definition or case.

, "I am so glad I finally discovered lcases!!!"

A "where" expression begins by a line that only has word "where" in it. It is indented as described in the "Value Definitions" (3.4.1) or "cases' Function Expressions" (3.3.2) section according to which of the two is above it. The definitions are placed below the "where" line and must have the same indentation.

• Grammar

```
\langle where\text{-}expr \rangle ::= \text{`\n'} \langle indent \rangle \text{ `where\n'} (\langle value\text{-}def \rangle | \langle grouped\text{-}value\text{-}defs \rangle) +
```

3.5 Indentation and Complete Grammar for Values

3.5.1 Indentation System

The $\langle indent \rangle$ nonterminal in not a normal BNF nonterminal. It is a context sensitive construct that enforces the indentation rules of leases. It depends on a integer value that shall be named "indentation level" (il). The $\langle indent \rangle$ nonterminal corresponds to 2*il space characters. The indentation level follows the rules below:

Indentation Rules

- At the beginning: il = 0.
- At the end of the first line of a value definition: $il \leftarrow il + 1$. This also applies to grouped value definitions.
- At the end of the third line ("=" line) of a (single) value definition: $il \leftarrow il + 1$.
- At the end of a (single) value definition: $il \leftarrow il 2$.
- At the end of grouped value definitions: $il \leftarrow il 1$.
- After the "=>" arrow in a case: $il \leftarrow il + 1$.
- At the end of a case body: $il \leftarrow il 1$.
- In a cases function expression which does not begin in the "=" line of a value definition:
 - After the arrow "=>" at the end of the paremeters: $il \leftarrow il + 1$.
 - At the end of the whole cases function expression: $il \leftarrow il 1$.

3.5.2 Complete Grammar for Values

```
\langle literal \rangle ::= \langle literal \rangle
\langle identifier \rangle ::= [a-z] [a-z_{-}]^* ( '()' [a-z_{-}]^+ )^* [ [0-9] ]
\langle paren-expr \rangle ::= '(' \langle op-or-func-expr \rangle ')'
\langle op-or-func-expr \rangle ::= \langle simple-op-expr \rangle | \langle op-expr-func-end \rangle | \langle simple-func-expr \rangle
\langle tuple \rangle ::= '(' \langle line-expr \rangle ', _{\square}' \langle comma-sep-line-exprs \rangle ')'
\langle comma-sep-line-exprs \rangle ::= \langle line-expr \rangle ( ',_{\square}' \langle line-expr \rangle )^*
\langle line-expr \rangle ::= \langle no-paren-op-arg \rangle | \langle op-or-func-expr \rangle
```

```
\langle biq\text{-}tuple \rangle ::=
         '(' \langle line-expr \rangle ['\n' \langle indent \rangle ]', \( \)' \langle comma-sep-line-exprs \rangle
         ( '\n' \langle indent \rangle ', ' \langle comma-sep-line-exprs \rangle )^*
         '\n' \(\langle indent\)')'
\langle list \rangle ::= '[' [\langle comma-sep-line-exprs \rangle]']'
\langle biq-list \rangle := '[' \langle comma-sep-line-exprs \rangle (' n' \langle indent \rangle ', ' \langle comma-sep-line-exprs \rangle) * ' n' \langle indent \rangle ']'
\langle paren-func-app \rangle ::=
           [\langle arguments \rangle] \langle identifier\text{-}with\text{-}arguments \rangle [\langle arguments \rangle]
           \langle arguments \rangle \langle identifier \rangle [\langle arguments \rangle]
           \langle identifier \rangle \langle arguments \rangle
\langle arguments \rangle ::= '(' \langle comma-sep-line-exprs \rangle ')'
\langle identifier\text{-}with\text{-}arguments \rangle ::=
           [a-z] [a-z]* ( '() '[a-z_]+ )* (arguments) [a-z_]+ ( ( '() ' | (arguments) ) [a-z_]+ )* [ [0-9] ]
\langle pre\text{-}func \rangle ::= \langle identifier \rangle ':'
\langle pre\text{-}func\text{-}app \rangle ::= \langle pre\text{-}func \rangle \ (\langle basic\text{-}expr \rangle \mid \langle paren\text{-}expr \rangle \mid \langle pre\text{-}func\text{-}app \rangle \ )
\langle basic\text{-}expr \rangle ::= \langle literal \rangle \mid \langle identifier \rangle \mid \langle tuple \rangle \mid \langle list \rangle \mid \langle paren\text{-}func\text{-}app \rangle \mid \langle post\text{-}func\text{-}app \rangle
\langle post\text{-}func \rangle ::= `.` \langle identifier \rangle
\langle post\text{-}func\text{-}app \rangle ::= (\langle paren\text{-}expr \rangle \mid \langle basic\text{-}expr \rangle) \langle post\text{-}func \rangle
\langle op\text{-}expr \rangle ::= \langle simple\text{-}op\text{-}expr \rangle \mid \langle op\text{-}expr\text{-}func\text{-}end \rangle \mid \langle big\text{-}op\text{-}expr \rangle \mid \langle cases\text{-}op\text{-}expr \rangle
\langle simple-op-expr \rangle ::= \langle op-arg \rangle \ (`` \sqcup ` \langle op \rangle ` \sqcup ` \langle op-arg \rangle \ )+
\langle op\text{-}expr\text{-}func\text{-}end \rangle ::= \langle simple\text{-}op\text{-}expr \rangle `` \Box' \langle op \rangle `` \Box' \langle simple\text{-}func\text{-}expr \rangle
\langle big\text{-}op\text{-}expr\rangle ::=
          \langle op\text{-}expr\text{-}line \rangle ( '\n' \langle indent \rangle \langle op\text{-}expr\text{-}line \rangle )*
         `\n' \langle indent \rangle \ (\ \langle op\text{-}arg \rangle \ | \ \langle simple\text{-}op\text{-}expr \rangle \ | \ [\ \langle op\text{-}expr\text{-}line \rangle \ `\ \lrcorner' \ ] \ (\ \langle simple\text{-}func\text{-}expr \rangle \ | \ \langle big\text{-}func\text{-}expr \rangle) \ )
\langle cases-op-expr \rangle ::= \langle op-expr-line \rangle \ ( `\n' \langle indent \rangle \ \langle op-expr-line \rangle \ )^* \ ( `\n' \langle indent \rangle \ | `_{\Box}' \ ) \ \langle cases-func-expr \rangle
\langle op\text{-}expr\text{-}line \rangle ::= (\langle op\text{-}arg \rangle \mid \langle simple\text{-}op\text{-}expr \rangle) ' ' ' \langle op \rangle
\langle op\text{-}arq \rangle ::= \langle no\text{-}paren\text{-}op\text{-}arq \rangle \mid \langle paren\text{-}expr \rangle
\langle no\text{-}paren\text{-}op\text{-}arg \rangle ::= \langle basic\text{-}expr \rangle \mid \langle pre\text{-}func \rangle \mid \langle post\text{-}func \rangle \mid \langle pre\text{-}func\text{-}app \rangle
⟨op⟩ ::= '->' | '<-' | 'o>' | '<o' | '^' | '*' | '/' | '+' | '-' | '=' | '/=' | '>' | '<' | '>=' | '<=' | '&' | '|' | ';'
```

```
\langle func\text{-}expr \rangle ::= \langle simple\text{-}func\text{-}expr \rangle \mid \langle big\text{-}func\text{-}expr \rangle \mid \langle cases\text{-}func\text{-}expr \rangle
\langle simple-func-expr \rangle ::= \langle parameters \rangle ` \Box = > \Box ` \langle simple-func-body \rangle
\langle parameters \rangle ::= \langle identifier \rangle \mid (\langle identifier \rangle (\langle identifier \rangle ) + (\langle identifie
\langle simple-func-body \rangle ::= \langle no-paren-op-arg \rangle \mid \langle simple-op-expr \rangle \mid \langle op-expr-func-end \rangle
\langle cases\text{-}func\text{-}expr \rangle ::= \langle cases\text{-}parameters \rangle `=>` \langle case \rangle + \langle end\text{-}case \rangle
\langle cases\text{-}parameter \rangle ::= \langle cases\text{-}parameter \rangle \mid \text{`('} \langle cases\text{-}parameter \rangle \mid \text{`,}_{\square} \langle cases\text{-}parameter \rangle \mid \text{')'}
\langle cases-parameter \rangle ::= \langle parameter \rangle \mid 'cases'
\langle case \rangle ::= '\n' \langle indent \rangle \langle matching \rangle ' =>' \langle case-body \rangle
\langle end\text{-}case \rangle ::= \text{`\n'} \langle indent \rangle \text{ (`...'} | \langle matching \rangle \text{ )'} =>' \langle case\text{-}body \rangle
\langle matching \rangle ::= \langle literal \rangle \mid \langle identifier \rangle \mid \langle pre-func \rangle \langle matching \rangle \mid \langle tuple-matching \rangle \mid \langle list-matching \rangle
\langle tuple\text{-}matching \rangle ::= '(' \langle matching \rangle (',' \langle matching \rangle )+ ')'
\langle list\text{-}matching \rangle ::= `[' [ \langle matching \rangle ( `,' \langle matching \rangle )* [ `, ...' ] ] `]'
\langle case-body \rangle ::= (``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``" | ``"
\langle value\text{-}def \rangle ::= \langle indent \rangle \langle identifier \rangle \text{ '\n'} \langle indent \rangle \text{ ':} \text{ ':} \text{ '} \langle type \rangle \text{ '\n'} \langle indent \rangle \text{ '=} \text{ '} \langle value\text{-}expr \rangle \text{ } [ \langle where\text{-}expr \rangle \text{ } ]
\langle value-expr \rangle ::= \langle no-paren-op-arg \rangle \mid \langle op-expr \rangle \mid \langle func-expr \rangle \mid \langle big-tuple \rangle \mid \langle big-list \rangle
\langle grouped\text{-}value\text{-}defs \rangle ::=
                        \langle indent \rangle \langle identifier \rangle ( ', \Box' \langle identifier \rangle )+
                        \verb|`\n'| \langle indent \rangle `: \bot' ( \langle type \rangle ( `, \bot' \langle type \rangle ) + | `all' \langle type \rangle )|
                       '\n' \langle indent\rangle '=_1' \langle comma-sep-line-exprs\rangle ('\n' \langle indent\rangle ', '\comma-sep-line-exprs\rangle )*
\langle where\text{-}expr \rangle ::= \text{`\n'} \langle indent \rangle \text{ `where\n'} (\langle value\text{-}def \rangle | \langle grouped\text{-}value\text{-}defs \rangle) +
```

4 Language Description: Types and Type Logic

4.1 Types

4.1.1 Type Expressions

Type Identifiers

• Examples

Int
Real
Char
String
FunnyType
MyDefinedType

 \bullet Description

A type identifier is either the name of a basic type (Int, Real, Char, String) or the name of some defined type that has no type parameters. It begins with a capital letter and is followed by one or more capital or lowercase letters.

• Grammar

```
\langle \, type\text{-}id \, \rangle \, ::= \, \, [\text{A-Z}] \, \, [\text{A-Za-z}] +
```

Type Variables

• Examples

Α

В

С

X

Y

Z

• Examples of type variables inside bigger type expressions

```
A \Rightarrow A

(A \Rightarrow B, B \Rightarrow C) \Rightarrow (A \Rightarrow C)

((A, A) \Rightarrow A, A, ListOf(A)s) \Rightarrow A
```

 \bullet Description

Type Variables are used inside larger type expressions of polymorphic types. A polymorphic type is a type where any function of that type can be used as a function of any type that corresponds to substituting every type variable of the polymorphic type with a particular type. The easiest example of a polymorphic type is the type of the identity function where we have:

```
id
  : A => A
  = x => x

id(1)
  : Int
  where A is substituted by Int and id gets the type Int => Int

id("Hello")
  : String
  where A is substituted by String and id gets the type String => String
```

A type variable is a single capital letter.

 \bullet Grammar

```
\langle type\text{-}var \rangle ::= [A-Z]
```

Function Types

- Examples
- $\bullet \ \ Description$
- Grammar

```
\langle literal \rangle ::= \langle literal \rangle
```

Product Types

- \bullet Examples
- Description
- Grammar

```
\langle literal \rangle ::= \langle literal \rangle
```

Type Application Types

- \bullet Examples
- ullet Description
- Grammar

```
\langle \mathit{literal} \rangle ::= \langle \mathit{literal} \rangle
```

Conditional Types

- \bullet Examples
- ullet Description
- Grammar

```
\langle literal \rangle ::= \langle literal \rangle
```

Examples

Int

String => String

Int x Int

Int x Int => Real

A => A

 $(A \Rightarrow B, B \Rightarrow C) \Rightarrow (A \Rightarrow C)$

 $((A, A) \Rightarrow A, A, ListOf(A)s) \Rightarrow A$

 $((B, A) \Rightarrow B, B, ListOf(A)s) \Rightarrow B$

(T)HasStringRepresantion --> T => String

Description

Every value definition has a type annotation which is a type expression. Type expressions are divided into 5 categories:

• Function Types

Function type expressions are comprised of the following:

- The parameter type expression(s). If the parameter type expressions are more than one they are in parenthesis seperated by commas.
- The function type operator " ⇒ "
- The result type expression
- Product Types

Product type expressions are comprised by the subtype type expressions separated by "x".

• Type Application Types

Type application types are comprised of the following:

- The name of a tuple_type or or_type that is defined expecting type parameters.
- The argument type expressions in parenthesis and comma seperated. Just like Parenthesis Function Application (section 3.1.3) the arguments can be before, in the middle or after the name of the type tuple_type or or_type.

Grammar

$$\langle type \rangle ::= \langle type\text{-}id \rangle \mid \langle type\text{-}app \rangle \mid \langle prod\text{-}type \rangle \mid \langle func\text{-}type \rangle$$

$$\langle type-id \rangle ::= [A-Z] [A-Za-z]^*$$

```
\langle type-id-with-args \rangle ::= \langle type-id \rangle (\langle types-in-paren \rangle [A-Za-z]+)+
\langle type-app \rangle ::=
        [\ \langle types\text{-}in\text{-}paren\rangle\ ]\ \langle type\text{-}id\text{-}with\text{-}args\rangle\ [\ \langle types\text{-}in\text{-}paren\rangle\ ]
        \langle types-in-paren \rangle \langle type-id \rangle [\langle types-in-paren \rangle]
        \langle type-id \rangle \langle types-in-paren \rangle
\langle types-in-paren \rangle ::= '(' \langle type \rangle (', ' \langle type \rangle) * ')'
\langle prod-type \rangle ::= \langle product-subtype \rangle \ (` ` \sqcup x \sqcup ` \langle product-subtype \rangle \ ) +
\langle product\text{-}subtype \rangle ::= \langle type\text{-}id \rangle \mid \langle type\text{-}app \rangle \mid \text{`('} (\langle func\text{-}type \rangle \mid \langle prod\text{-}type \rangle ) ')'
\langle func\text{-}type \rangle ::= \langle input\text{-}types\text{-}expression \rangle ` \Box = > \Box ` \langle one\text{-}type \rangle
\langle input-types-expression \rangle ::= \langle one-type \rangle \mid \langle two-or-more-types-in-paren \rangle
\langle one\text{-}type \rangle ::= \langle type\text{-}id \rangle \mid \langle type\text{-}app \rangle \mid \langle prod\text{-}type \rangle \mid \text{`('} \langle func\text{-}type \rangle ')'
\langle two\text{-}or\text{-}more\text{-}types\text{-}in\text{-}paren \rangle ::= '(' \langle type \rangle (', ' \langle type \rangle) + ')'
4.1.2 Type Definitions: Tuple Types
Definition Examples
tuple_type Name
value (first_name, last_name) : String x String
tuple_type ClientInfo
value (name, age, nationality) : Name x Int x String
tuple_type Date
value (day, month, year) : Int x Int x Int
tuple_type (A)And(B)
value (a_value, b_value) : A x B
tuple_type (ExprT)WithPosition
value (expr, line, column) : ExprT x Int x Int
Usage Examples
giorgos_info
   : ClientInfo
   = (("Giorgos", "Papadopoulos"), 42, "Greek")
john_info
   : ClientInfo
   = (("John", "Doe"), 42, "American")
name_to_string
   : Name => String
   = n => "First Name: " + n.first_name + "\nLast Name: " + n.last_name
```

Description

Tuple types group many values into a single value. They are specified by their name, the names of their fields and the types of their fields. They generate projection functions for all of their fields by using a '.' before the name of the field. For example the ClientInfo type above generates the following functions:

```
.name
  : ClientInfo => String
.age
  : ClientInfo => Int
.nationality
  : ClientInfo => String
```

These functions shall be named "postfix functions" as the can just be appended to their argument.

Definition Grammar

4.1.3 Type Definitions: Or Types

Examples

```
or_type Bool
values true | false

or_type Possibly(A)
values the_value:A | no_value

or_type ListOf(A)s
values non_empty:NonEmptyListOf(A)s | empty

or_type Error(A)OrResult(B)
values error:A | result:B

Usage

tuple_type NonEmptyListOf(A)s
value (head, tail) : A x ListOf(A)s
```

```
is_empty
  : ListOf(A)s => Bool
  = cases =>
    empty => true
    non_empty:anything => false

get_head
  : ListOf(A)s => Possibly(A)
  = cases =>
    empty => no_value
    non_empty:list => the_value:list.head

print_err_or_res
  : Error(A)OrResult(B) => (EmptyVal)IOAction
  = cases =>
    error:e => print("Error occured: " + e -> to_string)
    result:r => print("All good! The result is: " + r -> to_string)
```

Description

Values of an or_type are one of many cases that possibly have other values inside. The cases which have other values inside are followed by a semicolon and the type of the internal value. The same syntax can be used for matching that particular case in a function using the "cases" syntax, with the difference that after the colon, we write the name given to the value inside. Or_types and basic types are the only types on which the "cases" syntax can be used. The cases of an or_type which have a value inside create functions. For example, the case "non_empty" of a list creates the function "non_empty:" for which we can say:

```
non_empty:
    : NonEmptyListOf(A)s => ListOf(A)s
Similarly:
the_value:
    : A => Possibly(A)
```

These functions shall be named "prefix functions" as they are prepended to their argument. For example:

```
head_and_tail
  : NonEmptyListOf(Int)s
  = (1, [2, 3, 4])

list
  : ListOf(Int)s
  = non_empty:head_and_tail
```

These functions can be used like any other function as arguments to other functions. For example:

Definition Grammar

```
\langle or\text{-}type\text{-}definition \rangle ::=
\text{`or\_}type_{\sqcup'} \langle type\text{-}app \rangle
\text{``nvalues}_{\sqcup'} \langle identifier \rangle [ `:' \langle type \rangle ] ( `_{\sqcup}|_{\sqcup'} \langle identifier \rangle [ `:' \langle type \rangle ])^*
```

- 4.2 Type Logic
- 4.2.1 Type Proposition
- 4.2.2 Type Theorem
- 5 Predefined
- 5.1 Constants
- 5.2 Functions
- 5.3 Types
- 5.4 Type Propositions
- 6 Parser implimentation

The parser was implemented using the parsec library.

- 6.1 AST Types
- 6.2 Parsers
- 7 Translation to Haskell
- 8 Running examples
- 9 Conclusion