

### **BOOSTING IN COX REGRESSION**

a comparison among the classical statistics-based, the likelihood-based and the model-based approaches

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### **OUTLINE**

- Introduction to the COX regression model
  - Background (Survival Analysis)
  - The COX model
  - FGD Boosting
- Application with the three approaches
  - Flexible boosting with the <u>AFT model</u>
  - mboost: model-based boosting in the COX model
  - CoxBoost: offset-based boosting in the COX model
- Comparison
- Allowing for mandatory variables
- Incorporating the pathway information
- Application with the mlr3 learner

# INTRODUCTION TO THE COX REGRESSION MODEL (10MIN)

WHAT IS SURVIVAL ANALYSIS

THE COX MODEL

FGD BOOSTING

#### WHAT IS SURVIVAL ANALYSIS

- statistics for analyzing the expected duration of time until one or more events of interest happen
  - e.g. death, failure in a mechanical system
- term
  - event
  - time: t
  - lacktriangle censoring observation:  $\delta$  (missing data)
    - <u>right</u>-censoring
    - <u>left</u>-censoring

#### THE COX MODEL

- <u>intuition</u>: for quantitive predictor variables, we use the COX PH hazard regression model
- hazard function  $\lambda$ 
  - def: event rate at time t conditional or survival until time t or later
  - consider the time-to-event data  $(t, x, \delta)$ 
    - the hazard function  $\lambda(t|X) = \lambda_0(t)exp(X^T\beta) = \lambda_0(t)exp(\beta_1X_{i1} + \ldots + \beta_pX_{ip})$
- estimator for  $\beta$ 
  - by maximizing the partial log-likelihood (MPLE)
  - $pl(\beta) = \sum_{i=1}^{n} \delta_i(X_i^T)\beta \log(\sum_{l \in R_i} exp\{(X_l^T\beta)\}$

#### **FGD BOOSTING**

- intuition: at each iteration, a weak learner is fitted on the modified version of data with the goal of minimizing the empirical loss function
- motivation: we will use boosting techniques when
  - the number of covariates is large or
  - it is hard to directly derive the partial log-likelihood

#### **FGD BOOSTING**

### semi-parametric boosting

• given: L(y, F(X)) is a generic loss function and F(x) is a statistical model.

• goal: to estimate F(X) by iteratively updating its value through a base learner h(y,X)

#### **FGD BOOSTING**

### semi-parametric boosting

- algorithm
  - 1. initialize the estimate, e.g.,  $\hat{F}(X) = constant$ ;
  - 2. compute the pseudo-residual vector,  $u = -\frac{\partial L(y,F(X))}{\partial F(X)}$ , where  $F(X) = \hat{F}(X)$ ;
  - 3. compute the update by:
    - 1. fit the base learner to the pseudo-residual vector,  $\hat{h}(u, X)$ ;
    - 2. penalize the value,  $\hat{f}(X) = v\hat{h}(u, X)$ ;
  - 4. update the estimate,  $\hat{F}(X) = \hat{F}(X) + \hat{f}(X)$

#### **FGD BOOSTING**

#### parametric boosting

- given: F(X) is a parameterized class of functions,  $F(X, \beta)$ .
- ullet the update process involves the estimate of the parameter, i.e. the regression coefficient eta

#### **FGD BOOSTING**

### parametric boosting

- algorithm
  - 1. initialize the estimate, e.g.,  $\hat{\beta} = (0, \dots, 0)$ ;
  - 2. compute the pseudo-residual vector,  $u = -\frac{\partial L(y, F(X, \beta))}{\partial F(X, \beta)}$ , where  $\beta = \hat{\beta}$ ;
  - 3. compute the possible updates by:
    - 1. fit the base learner to the pseudo-residual vector,  $\hat{h}(u, X_j)$ ;
    - 2. penalize the value,  $\hat{b}_j = v\hat{h}(u, X_j)$ ;
  - 4. select the best update  $j^*$
  - 5. update the estimate,  $\hat{\beta}_{j^*} = \hat{\beta}_{j^*} + \hat{b}_j$

#### **FGD BOOSTING**

boosting with the <u>regularized</u> empirical risk function

- ullet penalize over the empirical risk function by adding  $\lambda \cdot J(f)$ 
  - the complexity control parameter  $\lambda$
  - the complexity penalty J(f)
- algorithm

??

# APPLICATIONS WITH THE THREE APPROACHES (6MIN)

$$log(T) = f(X) + \sigma \cdot w$$

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- \* Otherwise, it is named a <u>semi-parametric</u> AFT model

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- 5. Update with the real-valued step length factor v and iterate  $m_{stop}$  times.

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# **COMPARISON (5MIN)**

**MBOOST** 

1.  $\sigma$  and w

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- 1.  $\sigma$  and w
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#### **MBOOST**

- 1. user-defined loss function
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#### **COXBOOST**

1. user-defined loss function

- 1.  $\sigma$  and w
- 2. semi-parametric and parametric

#### **MBOOST**

- 1. user-defined loss function
- 2. MPLE

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- 2. restricted MPLE

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#### **MBOOST**

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- 3. flexible penalty structure (variable selection, mandatory variables,

- 1.  $\sigma$  and w
- 2. semi-parametric and parametric

#### **MBOOST**

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- 2. restricted MPLE
- 3. flexible penalty structure (<u>variable selection</u>, <u>mandatory variables</u>, <u>correlated covariates</u>)

The three approaches are identical when

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- \* As for the AFT model: w is not specified
- \* As for the CoxBoost:  $\lambda$  is set to zero (no penalty is added)

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or

$$\star \lambda = \frac{X_j^T X_j + vpl_{\beta_j}(0|\hat{\beta})}{v}$$

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\* dimension is selected with the largest decrease of the penalized partial log-likelihood function

# ALLOWING FOR MANDATORY VARIABLES (3MIN)

#### FAVORING STRATEGY IN THE AFT MODEL

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- \* The covariates are divided into the mandatory and the non-mandatory groups
- $^*$  the penalization with v is applied only to the non-mandatory components

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### FAVORING STRATEGY IN CoxBoost

- 1. configure the penalty matrix P
- 2. mandatory variables can be introduced by updating their parameters before each step of componentwise CoxBoost

# INCORPORATING PATHWAY INFORMATION (1MIN)

\* correlated microarray features

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- \* trade-off between model complexity and the representation power

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# OBJECTIVE: DISCOURAGING THE SELECTION OF SINGLE MICROARRAY FEATURES

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# OBJECTIVE: DISCOURAGING THE SELECTION OF SINGLE MICROARRAY FEATURES

- 1. increasing the penalty for a selected covariate
- 2. decreasing the penalty for connected covariates

# APPLICATION WITH THE MLR3 LEARNER

 $a^2$