Final Assessment Test (FAT) – May 2017

VIT UNIVERSITY (Estd. u/s 3 of UGC Act 1956)

Course: **MAT1011 - Calculus for Engineers**

Class NBR(s): **4673** Slot: **E1+TE1**

Time: **Three Hours** Max. Marks: **100**

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. (a) (i) Find the second derivative of $\sin(x^2+3)$ **[10]**

   (ii) Sketch the graph of $f(x)=x^3-3x+8$. Find the intervals on which f is increasing, decreasing, concave up, concave down, critical points and points of inflection.

   (b) Find the area of the region enclosed by the curves $y=2-x^2$ and $y=-x$. **[10]**

2. (a) Let $f(x)=\begin{cases} x & if \quad 0\le x\le 1 \\ 2-x & if \quad 1\le x\le 2 \end{cases}$ **[10]**

   and $f(x)=f(x-2), x\ge 2$. Find Laplace transform of $f(x)$.

   (b) Find inverse Laplace transform of $\frac{1}{(s^2+a^2)^2}$ **[10]**

3. (a) Expand $f(x,y)=e^{x+y}$ in terms of x and y up to order 3. **[10]**

   (b) Find the shortest distance from the point $(1,0,-2)$ to the plane $x+2y+z=4$ **[10]**

4. (a) Sketch the region of integration for the integral $\int_0^2\int_{x^2}^{2x}(4x+2)dy\,dx$. Write and evaluate an equivalent integral by changing the order of integration . **[10]**

   (b) Evaluate $\iiint_B e^{(x^2+y^2+z^2)^{\frac{3}{2}}}dV$, where, $B=\{(x,y,z)|x^2+y^2+z^2\le 1\}$. **[10]**

5. (a) Find angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at $(2,-1,2)$. **[10]**

   (b) Let $\vec{F}(x,y)=(3+2xy)\vec{i}+(x^2-3y^2)\vec{j}$. Show that $\vec{F}$ is conservative and find f such that $\vec{F}=\nabla f$. **[10]**

6. (a) Let C be the curve of intersecting on of the plane $z=x$ and cylinder $x^2+y^2=1$ oriented counter clockwise, when viewed from above. If $\vec{F}=x\vec{i}+z\vec{j}+2y\vec{k}$, then evaluate $\oint_C F\,dr$. **[10]**

   (b) Verify Green's theorem for $P(x,y)=xy=Q(x,y)$, where (x,y) lies in the disc D with center $(0,0)$ and radius R. **[10]**

7. (a) Evaluate $\int_0^{\pi/2}\sqrt{tan\theta}\;d\theta$ using in terms of the gamma function. **[10]**

   (b) If $u=x^2-y^2, v=2xy$ and $x=arcos\theta, y=brsin\theta$ Find $\frac{\partial(u,v)}{\partial(r,\theta)}$. If $r\neq 0$, find $\delta\left(\frac{r,\theta}{u,v}\right)$. **[10]**

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