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Final Assessment Test - November 2016

Course: **MAT1011 - Calculus for Engineers**

Class NBR(s):**3827** Slot: **C1+TC1**

Time: **Three Hours** Max. Marks: **100**

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. (a) Determine the regions of increasing and decreasing for $f(x) = x^4 - 4x^3 + 10$ and also, find its minima, maxima and point of inflection. **[10]**

   (b) Find the area of the region R bounded by the parabola $y = x^2$ and the line $y = x$ in the first quadrant of the XY plane. If R is revolved about the Y-axis to generate a solid, find the volume of the solid. **[10]**

2. (a) Find the Laplace transform of $\frac{\cos t - \cos 2t}{t}$ and hence, evaluate $L\left(\frac{e^{-2t}(\cos t - \cos 2t)}{t}\right)$. **[10]**

   (b) Using Convolution Theorem, evaluate $L^{-1}\left(\frac{s^2}{(s^2+1)(s^2+4)}\right)$. **[10]**

3. (a) Expand $f(x, y) = e^x \log(1+y)$ in powers of x and y upto the terms of third degree. **[10]**

   (b) Find the volume of the greatest rectangular box that can be inscribed in the sphere $x^2 + y^2 + z^2 = a^2$. **[10]**

4. (a) Change the order of the integral $\int_0^1 \int_{x^2}^{2-x} xy \, dxdy$ and hence, evaluate it. **[10]**

   (b) Using spherical co-ordinates, evaluate the following triple integral **[10]**

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$$

5. (a) If $r = \sqrt{x^2+y^2+z^2}$, show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ and hence, find $\nabla^2 \log(r)$. **[10]**

   (b) Show that $\vec{F} = 2xyz^2\vec{i} + (x^2z^2 + z\cos(yz))\vec{j} + (2x^2yz + y\cos(yz))\vec{k}$ is irrotaional and, find its scalar potential. **[10]**

6. (a) Compute the work done in moving a particle in the force field $\vec{F} = (2y+3)\vec{i} + xz\vec{j} + (yz - x)\vec{k}$ along $x = 2t^2, y = t, z = t^3$ from the point (0,0,0) to the point (2,1,1). **[10]**

   (b) Verify Gauss divergence theorem for the function $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ over the cylindrical region bounded by $x^2 + y^2 = 9$, $z = 0$ and $z = 2$. **[10]**

7. (a) Using Gamma and Beta functions, evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{xy} \, dxdy$. **[10]**

   (b) Show that $u = \frac{x^2 - y^2}{x^2 + y^2}$ and $v = \frac{2xy}{x^2 + y^2}$ are functionally dependent and find the relation between u and v. **[10]**

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