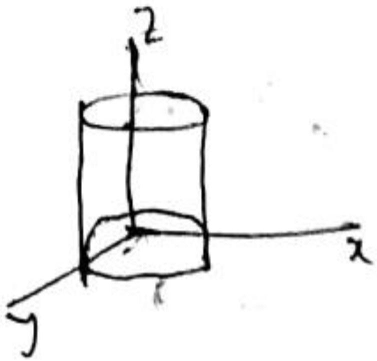
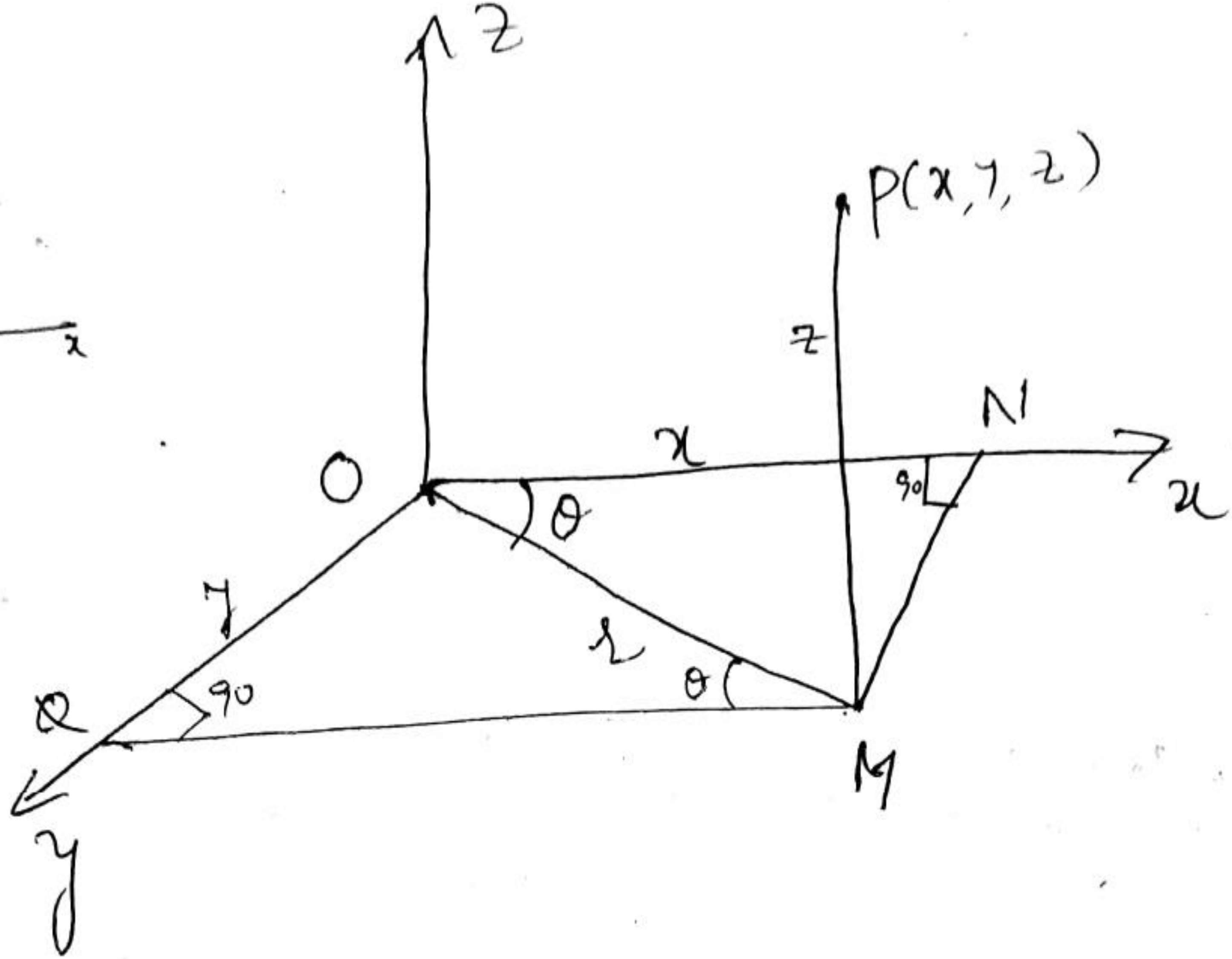
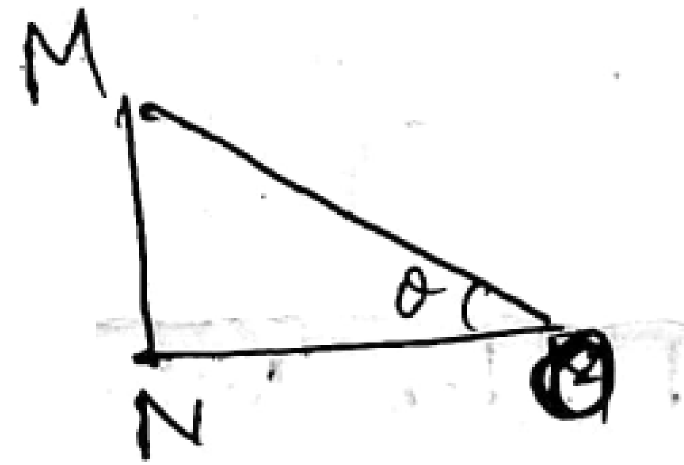
Cylindrical Coordinates

Change to cylindrical polar coordinates.





From $\triangle^{le}$ MON :



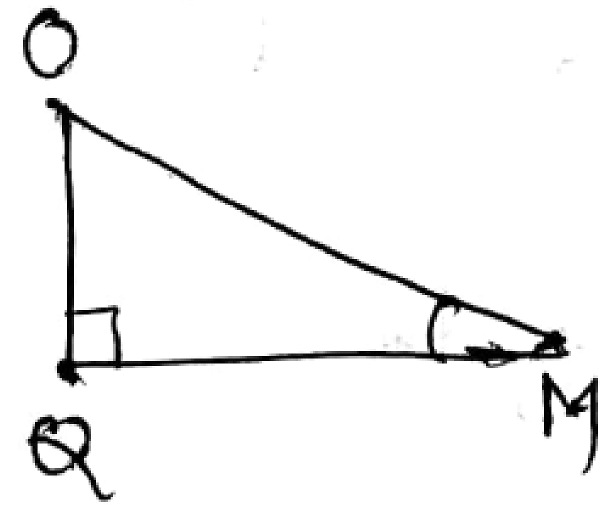
$$\cos\theta = \frac{NO}{MO}$$

$$\Rightarrow NO = MO\cos\theta$$

$$NO = r\cos\theta$$

$$\boxed{x = r\cos\theta}$$

From $\triangle^{le}$ QMO :



$$\sin\theta = \frac{OQ}{OM}$$

$$OQ = OM \sin\theta$$

$$\boxed{y = r \sin\theta}$$

$$dx\,dy\,dz = |J|\,dr\,d\theta\,dz$$

$$= \frac{\partial(x, y, z)}{\partial(r, \theta, z)}\,dr\,d\theta\,dz$$

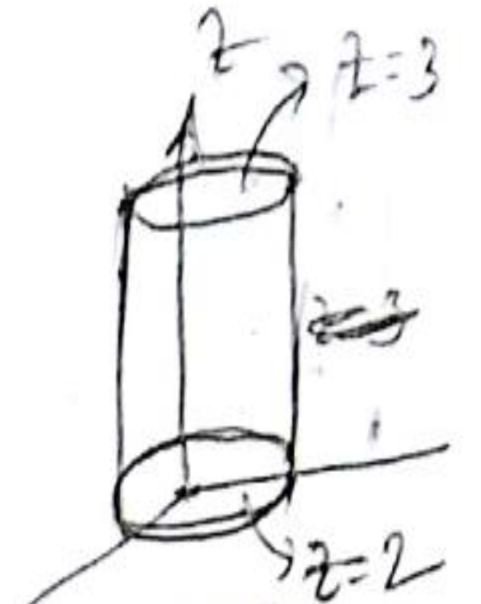
$$\boxed{dx\,dy\,dz = r\,dr\,d\theta\,dz}$$

★ Evaluate $\iiint\limits_{\substack{x^2+y^2\le 1 \\ 2\le z\le 3}} z(x^2+y^2)\,dx\,dy\,dz$ by changing into cylindrical polar coordi

Sol

put $x = r\cos\theta$; , $y = r\sin\theta$; $z = z$

$dx\,dy\,dz = r\,dr\,d\theta\,dz$



$$= \int_{z=2}^{3}\int_{\theta=0}^{2\pi}\int_{r=0}^{1} z\, r^2 \cdot r\, dr\, d\theta\, dz$$

$$= \int_{z=2}^{3}\int_{\theta=0}^{2\pi}\left(\frac{r^4}{4}\right)_0^1 z\, d\theta\, dz$$

$$= \frac{1}{4}\int_{z=2}^{3} z\,(\theta)_0^{2\pi}\, dz$$

$$= \frac{2\pi}{4}\left[\frac{z^2}{2}\right]_2^3 = \frac{\pi}{4}\left[9-4\right] = 5\pi/4$$

☆ Using Cylindrical Coordinates, find the Vol. of the Cylinder with base radius a and height 'h'.

Sol: The region of integration is bounded by
$x^2+y^2 \le a^2, \quad 0 \le z \le h.$

The same region in cylindrical coordinates will be as follows.

$r : 0$ to a ; $\theta : 0$ to 2π ; $z : 0$ to h

$$\text{Req. Vol.} = \iiint dx\,dy\,dz$$

$$= \int_{z=0}^{h} \int_{\theta=0}^{2\pi} \int_{r=0}^{a} r\,dr\,d\theta\,dz$$

$$= \int_0^h \int_0^{2\pi} \left(\frac{r^2}{2}\right)_0^a d\theta\, dz = \frac{a^2}{2} \int_0^h (\theta)_0^{2\pi}\, dz$$

$$= \frac{a^2}{2}(2\pi) \int_0^h dz = \pi a^2 h$$