Final Assessment Test - November 2016

VIT UNIVERSITY (Estd. u/s 3 of UGC Act 1956) VELLORE ▪ CHENNAI www.vit.ac.in

Course: **MAT1011 - Calculus for Engineers**

Class NBR(s): **7312 / 7320 / 7328 / 7368 / 7483 / 7568 / 7576** Slot: **G1+TG1**

Time: **Three Hours** Max. Marks: **100**

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. (a) A dynamic blast blows a heavy rock straight up with a launch velocity of 160 ft/sec(about 109mph). It reaches a height of $S = 160t - 16t^2 ft$ after $'t' sec.$ **[10]**

   (i) How high does the rock go?

   (ii) What are the velocity and speed of the rock when it is 256 ft above the ground on the way up? on the way down?

   (iii) What is the acceleration of the rock at any time $'t'$ during its flight (after the blast)?

   (iv) When does the rock hit the ground again?

   (b) Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$ **[5]**

   (c) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$ **[5]**

2. (a) (i) Find $L\left\{ t\int_0^t \frac{e^{-t}\, sin(t)}{t} dt \right\}$ and **[5+5]**

   (ii) Using unit step function, find the Laplace transform of $f(t) = \begin{cases} sin(t), 0 \le t < \pi \\ sin(2t), 0 \le t < 2\pi \\ sin(3t), t \ge 2\pi \end{cases}$

   (b) (i) Find $L^{-1}\left\{ \frac{4s+15}{16s^2-25} \right\}$ and **[5+5]**

   (ii) Apply Convolution theorem to evaluate $L^{-1}\left\{ \frac{1}{(s^2+a^2)^2} \right\}$

3. (a) Use Taylor's formula for $f(x,y)$ at the origin to find cubic approximation to $f(x,y) = e^x log(1+y)$. **[10]**

   (b) Find the volume of the largest rectangular solid which can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ **[10]**

4. (a) Evaluate $\iint_R (x^2 + y^2) dxdy$ over the area bounded by the curves $y = 4x$, $x + y = 3, y = 0$ and $y = 2$. **[6]**

   (b) Change the order of integration in $\int_0^a \int_{a-\sqrt{(a^2-y^2)}}^{a+\sqrt{(a^2-y^2)}} xydxdy$ and hence, evaluate it. **[7]**

   (c) By transforming into cylindrical coordinates, evaluate the integral. **[7]**
   $\iiint (x^2 + y^2 + z^2) dxdydz$ taken over the region space defined by $x^2 + y^2 \le 1$ and $0 \le z \le 1$.

5. (a) Find the angle of intersection at the point (2,-1, 2) of the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-z-3$. **[6]**

(b) Find the constant 'a' if the divergence of the vector $\vec{F}=(x+z)\vec{i}+(3x+ay)\vec{j}+(x-5z)\vec{k}$. **[4]**

(c) Show that $\vec{F}=(y^2+2xz^2)\vec{i}+(2xy-z)\vec{j}+(2x^2z-y+2z)\vec{k}$ represents a conservative vector field and hence find its scalar potential. **[10]**

6. (a) Find the work done by $\vec{F}=(2x-y-z)\vec{i}+(x+y-z)\vec{j}+(3x-2y-5z)\vec{k}$ along a curve 'C' in the xy-plane given by $x^2+y^2=9, z=0$. **[10]**

(b) Verify divergence theorem for $\vec{F}=4x\vec{i}-2y^2\vec{j}+z^2\vec{k}$ taken over the region bounded by $x^2+y^2=4$, $z=0$ and $z=3$. **[10]**

7. (a) Prove that $\int_0^1 \frac{x^2dx}{\sqrt{(1-x^4)}}.\int_0^1 \frac{dx}{\sqrt{(1+x^4)}}=\frac{\pi}{4}$ using beta and gamma functions. **[10]**

(b) Show that the functions $u=x+y+z, v=x^2+y^2+z^2-2xy-2yz-2zx$ and $w=x^3+y^3+z^3-3xyz$ are functionally related. **[6]**

(c) If $U=log\left(x^2+y^2+z^2\right)$, prove that $\left(x^2+y^2+z^2\right)\left(\frac{\partial^2U}{\partial x^2}+\frac{\partial^2U}{\partial y^2}+\frac{\partial^2U}{\partial z^2}\right)=2$. **[4]**

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