



Department of Mathematics
School of Advanced Sciences
Fall Semester 2018 – 19
Continuous Assessment Test 2

Course: Calculus for Engineers

Course Code: MAT1011

Slot:

Date: _____

Max. Marks: 50

Time: 90 minutes

Answer all the Questions.

1. If $U = x/z$ and $V = y/z$, where (x, y, z) is a point on the unit sphere with centre at the origin. Then compute the Jacobian of U and V with respect to x and y . Further, if $w = z$, compute the Jacobian of U , V and W with respect to x , y and z , and record your observations, with reasoning. (10 Marks)
2. (a) Steady-state temperature distributions in space, gravitational and electrostatic potentials in conservative fields are harmonic in the sense that they satisfy the Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$. Examine, if $f(x, y, z) = e^{3x+4y} \cos 5z$ is harmonic. (5 Marks)
- (b) Use Taylor's formula to find the second degree approximation for $f(x, y) = \cos x \cos y$ at the origin. (5 Marks)
3. Find the dimensions of a rectangular box of greatest volume that is bounded by the coordinate planes and the plane $x + 2y + z = 6$ in the positive octant. (10 Marks)
4. Reduce $\int_0^2 [\tan^{-1}(4-x) - \tan^{-1} x] dx$ into an appropriate double integral and sketch the region of integration, and compute the resulting double integral by changing the order of integration. (10 Marks)
5. A mechanical problem needs a wooden block \mathcal{D} , which is cut from the cylinder $x^2 + y^2 = 4$ by the xy -plane and the plane $x + z = 3$. Sketch the region enclosed by \mathcal{D} and compute its volume using a triple integral. (10 Marks)