# HTML_2023_HW1

***tags: `Personal`***

## Basics of Machine Learning

### P1

#### [A]

Machine learning is the process of training a model to learn patterns and relationships from data to make predictions or take action on new data. A voice assistant requires understanding natural language and context from various users, recognizing speech patterns in the language, and then generating an appropriate response based on that understanding. Therefore, this is an ideal use case for machine learning algorithms such as NLP and speech recognition.

#### [B]

Schrödinger's cat is a hypothetical cat, which may be considered simultaneously both alive and dead, while it is unobserved in a closed box, as a result of its fate is linked to a random subatomic event that may or may not occur. It's not a well-defined problem that has a deterministic solution, hence, it's not a suitable task for machine learning.

#### [C]

Searching for the shortest path to exit a maze is a well-defined algorithmic problem that can be solved using various graph search algorithms such as DFS, BFS, Dijkstra's algorithm, and A star algorithm, which doesn't require machine learning.

#### [D]

Machine learning is suitable for generating images, but it requires massive amounts of data and training while generating an image of Zeus that matches his actual facial look is not a suitable task for machine learning since Zeus doesn't have an actual facial look and it requires extensive knowledge of Greek mythology and art styles.
**As a result, we should choose $[a]$ as our solution.**

### P2

Let learning rate equals to $\eta_{n(t)}$. In order to find the $\eta_{n(t)}$ that $y_{n(t)}w_{t+1}^T x_{n(t)} > 0$, we can first find the $L_{n(t)} = \eta_{n(t)}$ that make the equation equal to zero.

$$
\begin{aligned}
y_{n(t)}w_{t+1}^T x_{n(t)} &= 0 = w_{t+1}^T \cdot x_{n(t)} \text{ since } y_{n(t)} \in \{-1, 1\} \\
(w_t + y_{n(t)} w_t^T x_{n(t)} L_{n(t)}) \cdot x_{n(t)} &= 0 \\
w_t \cdot x_{n(t)} &= -L_{n(t)} y_{n(t)} w_t^T x_{n(t)} \cdot x_{n(t)} \\
w_t^T x_{n(t)} &= -\frac{L_{n(t)} \|x_{n(t)}\|^2}{y_{n(t)}} \text{ since } y_{n(t)} \in \{-1, 1\} \\
L_{n(t)} &= \frac{-y_{n(t)} w_t^T x_{n(t)}}{\|x_{n(t)}\|^2}
\end{aligned}
$$

Since $L_{n(t)}$ exactly set $y_{n(t)}w_{t+1}^T x_{n(t)} = 0$, the real $\eta_{n(t)}$ must be bigger than $L_{n(t)}$ to strongly ensure that $w_{t+1}^T$ is correct on $(x_{n(t)}, y_{n(t)})$. It is straightforward to see that $\left\lfloor (\frac{-y_{n(t)} w_t^T x_{n(t)}}{\|x_{n(t)}\|^2} + 1) \right\rfloor = \lfloor L_{n(t)} + 1 \rfloor > L_{n(t)}$, therefore, this is the correct answer. Actually, the expression shows the smallest integer that can make $y_{n(t)} w_{t+1}^T x_{n(t)} > 0$.

**As a result, we should choose $[d]$ as our solution.**

## P3

Since the data is linear separable, there exists a perfect $w_f$ such that $y_n = \text{sign } (w_f^T z_n), \|w_f\| = 1$

$$
\begin{aligned}
w_f^T w_{t+1} &= w_f^T (w_t + y_{n(t)} z_{n(t)}) \\
&\geq w_f^T w_t + \underbrace{\min_n y_n w_f^T z_n}_{\rho > 0}
\end{aligned}
$$

PLA starts from $w_0 = 0$, it's trivial that $w_f^T w_T \geq \rho$

As for the length $\|w_T\|$

$$
\begin{aligned}
\|w_{t+1}\|^2 \;&=\; \|w_t + y_{n(t)} z_{n(t)}\|^2 \\
&= \|w_t\|^2 + 2y_{n(t)} w_t^T z_{n(t)} + \|y_{n(t)} z_{n(t)}\|^2 \\
&\Rightarrow \text{sign } (w_t^T z_{n(t)}) \neq y_{n(t)} \Leftrightarrow y_{n(t)} w_t^T z_{n(t)} \leq 0 \\
&\leq \|w_t\|^2 + 0 + \|y_{n(t)} z_{n(t)}\|^2 \\
&\leq \|w_t\|^2 + \max_n \|y_n z_n\|^2 \\
&\leq \|w_t\|^2 + \underbrace{\max_n \|z_n\|^2}_{R^2}
\end{aligned}
$$

PLA starts from $w_0 = 0$, it's trivial that $\|w_T\|^2 \leq R^2$

Combine the equations above, we have

$$
\begin{aligned}
1 \geq \frac{w_f^T}{\|w_f\|}\frac{w_T}{\|w_T\|} &\geq \frac{T\rho}{\|w_f\|\sqrt{T}R} \\
T &\leq \left(\frac{\|w_f\|R}{\rho}\right)^2 \\
\because z_n \leftarrow \frac{x_n}{\|x_n\|} &\therefore R^2 = 1 \text{ since } z_n \in \{0,1\} \\
T &\leq \left(\frac{\|w_f\|}{\rho}\right)^2 = \left(\min_n \frac{\|w_f\|}{y_n w_f^T z_n}\right)^2 = \frac{1}{\rho_z^2}
\end{aligned}
$$

**As a result, we should choose $[c]$ as our solution.**

### P4

From the previous question, we have

$$
\begin{aligned}
U = \frac{1}{\rho_z^2} &= \left(\frac{\sqrt{\max_n \|z_n\|^2}\|w_f\|}{\min_n y_n w_f^T z_n}\right)^2 \\
U_{\text{orig}} &= \left(\frac{\sqrt{\max_n \|x_n\|^2}\|w_f\|}{\min_n y_n w_f^T x_n}\right)^2
\end{aligned}
$$

Since $\min_n y_n w_f^T x_n > 0$ and $\min_n y_n w_f^T z_n > 0$, the denominator part can be ignored. Take the numerator part into consideration and take $\|w_f\| = 1$, then we have

$$
\begin{aligned}
U &\Rightarrow \max_n \|z_n\|^2 = \max_n \left\|\frac{x_n}{\|x_n\|}\right\|^2 \in \{0,1\} \\
U_{\text{orig}} &\Rightarrow \max_n \|x_n\|^2 \in \{0,\infty\} \\
\therefore U &\leq U_{\text{orig}}
\end{aligned}
$$

**As a result, we should choose $[b]$ as our solution.**

### P5

- Train examples by PLA
    1. Initialize $w_0 = [0,0,0]$
    2. $x_0 = [1,-2,2], y_0 = -1, \operatorname{sign}(w_0^T x_0) = \operatorname{sign}(0) = 0$
        - $w_1 = w_0 + y_0 x_0 = [0,0,0] + (-1)[1,-2,2] = [-1,2,-2]$
    3. $x_1 = [1,-1,2], y_1 = -1, \operatorname{sign}(w_1^T x_1) = \operatorname{sign}(-7) = -1$
        - $w_2 = w_1 = [-1,2,-2]$ since it is correct classified.
    4. $x_2 = [1,2,0], y_2 = 1, \operatorname{sign}(w_2^T x_2) = \operatorname{sign}(3) = 1$
        - $w_3 = w_2 = [-1,2,-2]$ since it is correct classified.

5. $x_3 = [1, -1, 0], y_3 = -1, \text{sign}(w_3^T x_3) = \text{sign}(-4) = -1$
    - $w_4 = w_3 = [-1, 2, -2]$ since it is correct classified.
6. $x_4 = [1, 1, 1], y_4 = 1, \text{sign}(w_4^T x_4) = \text{sign}(-1) = -1$
    - $w_5 = w_4 + y_4 x_4 = [-1, 2, -2] + (-1)[1, 1, 1] = [2, 1, -3]$

- Train examples by PAM
    1. Initialize $w_0 = [0, 0, 0], \tau = 5$
    2. $x_0 = [1, -2, 2], y_0 = -1, y_0 w_0^T x_0 = 0 < \tau$
        - $w_1 = w_0 + y_0 x_0 = [0, 0, 0] + (-1)[1, -2, 2] = [-1, 2, -2]$
    3. $x_1 = [1, -1, 2], y_1 = -1, y_1 w_1^T x_1 = 7 > \tau$
        - $w_2 = w_1 = [-1, 2, -2]$ since it is correct classified.
    4. $x_2 = [1, 2, 0], y_2 = 1, y_2 w_2^T x_2 = 3 < \tau$
        - $w_3 = w_2 + y_2 x_2 = [-1, 2, -2] + (1)[1, 2, 0] = [0, 4, -2]$
    5. $x_3 = [1, -1, 0], y_3 = -1, y_3 w_3^T x_3 = -5 < \tau$
        - $w_4 = w_3 + y_3 x_3 = [0, 4, -2] + (-1)[1, -1, 0] = [-1, 5, -2]$
    6. $x_4 = [1, 1, 1], y_4 = 1, y_4 w_4^T x_4 = 5 = \tau$
        - $w_5 = w_4 = [-1, 5, -2]$ since it is correct classified.
- Predict examples by PLA
    1. Set $w_{\text{PLA}} = [2, 1, -3]$
    2. $x_0 = [1, \frac{1}{2}, 2], y_0 = 1, \hat{y_0} = \text{sign}([2, 1, -3]^T [1, \frac{1}{2}, 2]) = \text{sign}(-\frac{7}{2}) = -1 \neq y_0$
    3. $x_1 = [1, \frac{1}{4}, 1], y_1 = 1, \hat{y_1} = \text{sign}([2, 1, -3]^T [1, \frac{1}{4}, 1]) = \text{sign}(-\frac{3}{4}) = -1 \neq y_1$
    4. $x_2 = [1, \frac{1}{2}, 0], y_2 = 1, \hat{y_2} = \text{sign}([2, 1, -3]^T [1, \frac{1}{2}, 0]) = \text{sign}(\frac{5}{2}) = 1 = y_2$
    5. $x_3 = [1, -\frac{1}{2}, 1], y_3 = -1, \hat{y_3} = \text{sign}([2, 1, -3]^T [1, -\frac{1}{2}, 1]) = \text{sign}(-\frac{3}{2}) = -1 = y_3$
- Predict examples by PAM
    1. Set $w_{\text{PAM}} = [-1, 5, -2]$
    2. $x_0 = [1, \frac{1}{2}, 2], y_0 = 1, \hat{y_0} = \text{sign}([-1, 5, -2]^T [1, \frac{1}{2}, 2]) = \text{sign}(-\frac{5}{2}) = -1 \neq y_0$
    3. $x_1 = [1, \frac{1}{4}, 1], y_1 = 1, \hat{y_1} = \text{sign}([-1, 5, -2]^T [1, \frac{1}{4}, 1]) = \text{sign}(-\frac{7}{4}) = -1 \neq y_1$
    4. $x_2 = [1, \frac{1}{2}, 0], y_2 = 1, \hat{y_2} = \text{sign}([-1, 5, -2]^T [1, \frac{1}{2}, 0]) = \text{sign}(\frac{3}{2}) = 1 = y_2$
    5. $x_3 = [1, -\frac{1}{2}, 1], y_3 = -1, \hat{y_3} = \text{sign}([-1, 5, -2]^T [1, -\frac{1}{2}, 1]) = \text{sign}(-\frac{11}{2}) = -1 = y$
- Test examples are wrongly predicted by PLA but correctly predicted by PAM
    - Wrong predicted by PLA: $x_0, x_1$
    - But correctly predicted by PAM: none

**As a result, we should choose $[e]$ as our solution.**

## The Learning Problems

### P6

It's trivial that the model learned the pattern based on a set of input features, which are all known ratings (supervised learning: all $y_n$), and the output value is the viewer's rating of the movie. The output space is a real number within the

range of $\{y|1 \leq y \leq 5\}$, which is a continuous range, making it a regression problem.
**As a result, we should choose $[a]$ as our solution.**

### P7

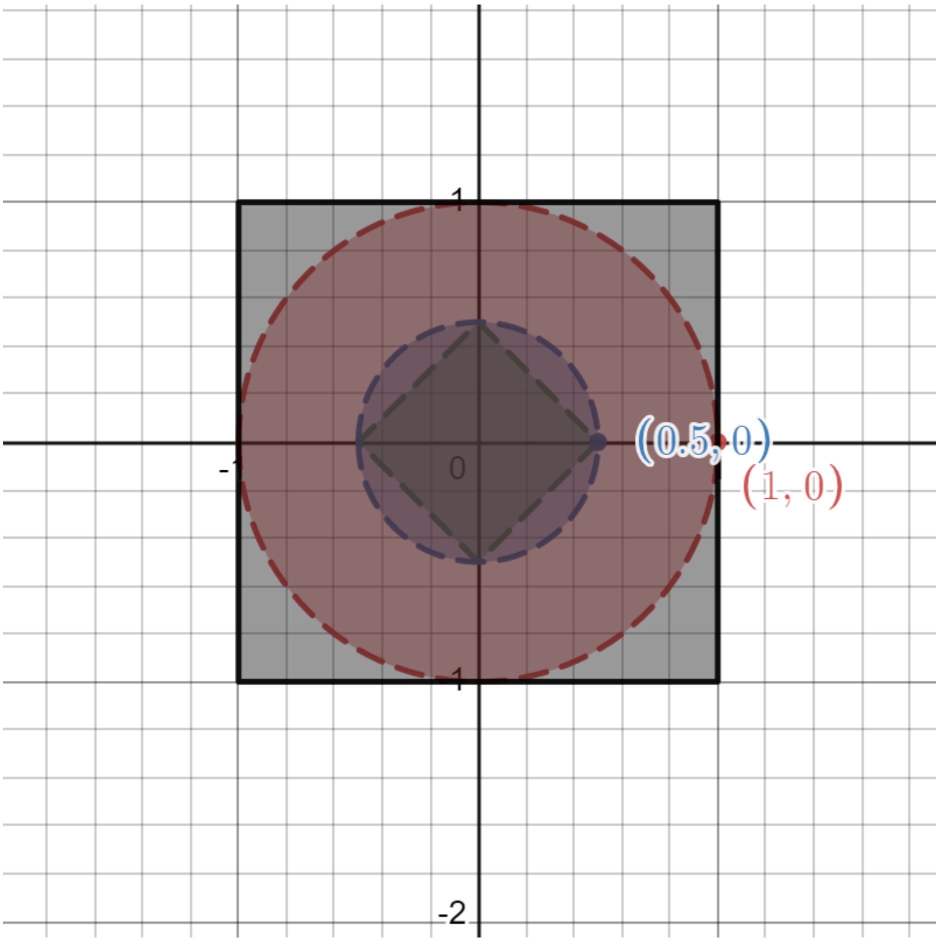
From the description, the labeler is always fed with two possible outputs and needs to select the better one, which means the task is to predict one of two possible outcomes given a set of input features that is model-generated output (binary classification: $y = \{-1, +1\}$). The labelers are effectively acting as a binary classifier, choosing between two options presented to them.
**As a result, we should choose $[b]$ as our solution.**

## Feasibility of Learning

### P8

Set input data $x$ to be $x_i(a_i, b_i), i = \{0, 1, \ldots, 5\}$.
Choose first three examples from $\mathcal{U}$ as $\mathcal{D}$, the perceptron hypothesis $h(x) = \text{sign}(2.5 - b)$ satisfied $y = +1$ for all $y \in \mathcal{D}$. It's trivial that $E_{in}(g) = 0$. and the hypothesis makes the evaluation of $g$ outside $\mathcal{D}$ all correct, which gives the best case $E_{ots} = 0$
Again, choose first three examples from $\mathcal{U}$ as $\mathcal{D}$, the perceptron hypothesis $h(x) = \text{sign}(a + b)$ satisfied $y = +1$ for all $y \in \mathcal{D}$. It's trivial that $E_{in}(g) = 0$. and the hypothesis makes the evaluation of $g$ outside $\mathcal{D}$ all wrong, which gives the worst case $E_{ots} = 1$.
Therefore, the smallest $E_{ots} = 0$, and the biggest $E_{ots} = 1$
**As a result, we should choose $[e]$ as our solution.**

### P9




Assumed that the area makes $y = +1$, we have

$$\begin{aligned} \text{Area} &= [+1, -1] \times [+1, -1] && \Rightarrow \text{black} \\ f(x) &= -\text{sign}(x_1^2 + x_2^2 - 0.25) = && \Rightarrow \text{blue} \\ h_1(x) &= -\text{sign}(x_1^2 + x_2^2 - 1) && \Rightarrow \text{red} \\ h_2(x) &= -\text{sign}(|x_1| + |x_2| - 0.5) && \Rightarrow \text{green} \end{aligned}$$

$$\begin{aligned} E_{out}(h_1) &= \mathbb{P}(\frac{h_1(x) \cap f(x)}{Area}) = \frac{\text{red-blue}}{Area} = \frac{\pi - 0.25\pi}{4} = \frac{3\pi}{16} \\ E_{out}(h_2) &= \mathbb{P}(\frac{f(x) \cap h_2(x)}{Area}) = \frac{\text{blue-green}}{Area} = \frac{0.25\pi - 0.5}{4} = \frac{\pi - 2}{16} \end{aligned}$$

$$\therefore (E_{out}(h_1), E_{out}(h_2)) = (\frac{3\pi}{16}, \frac{\pi - 2}{16})$$

**As a result, we should choose $[d]$ as our solution.**

## P10

$E_{in}(h_1) = E_{in}(h_2)$ means the examples that we draw has the same number of $f(x) \neq h_1(x)$ and $f(x) \neq h_1(x)$. In order to find $E_{in}(h_1) = E_{in}(h_2) = 0$, we have probability $\mathcal{P} = 1 - (\frac{3\pi}{16} + \frac{\pi-2}{16}) = \frac{18-4\pi}{16}$ of each drawing, hence, the probability of drawing 4 examples is $(\frac{18-4\pi}{16})^4 = 0.0133 \approx 0.01$.

Another easier way to solve this problem is to find the probability that both $h_1$ and $h_2$ are correct with respect to $f(x)$, hence, we have

$$E_{in}(h_1) = E_{in}(h_2) = 0$$
$$\Rightarrow \mathcal{P} = (\frac{\text{black-red+green}}{Area})^4 = (\frac{4-\pi+0.5}{4})^4 = 0.0133 \approx 0.01$$

**As a result, we should choose $[b]$ as our solution.**

### P11

We know that the value $\pi = 4 \cdot \frac{\text{number of darts landed in the circle}}{\text{total number of darts}}$, therefore we can estimate $\pi$ by $M_n = \frac{4}{n}\sum_{i=1}^{n} x_i$. Then we want $P(|M_n - \pi|) \geq 10^{-2} \leq 0.01$. $x_i$ is a bernolli random variable with $P(x_i = 1) = \frac{\pi}{4}$. The expectation of $M_n$ is

$$\mathbf{E}[M_n] = \mathbf{E}[\frac{4}{n}\sum_{i=1}^{n} x_i] = \frac{4}{n}\sum_{i=1}^{n}\mathbf{E}[x_i] = \frac{4}{n} \cdot n \cdot \frac{\pi}{4} = \pi$$

By Hoeffding’s inequality, we have

$$P(|M_n - \pi| \geq 10^{-2}) = P(|\frac{M_n}{4} - \frac{\pi}{4}| \geq 0.0025) \leq 2e^{-2 \cdot 0.0025^2 N}$$
$$2e^{-2 \cdot 0.0025^2 N} \leq 0.01 \Rightarrow N > \ln\frac{0.01}{2} \cdot -\frac{1}{2 \cdot 0.0025^2} \approx 4.23865 \times 10^5$$

**As a result, we should choose $[d]$ as our solution.**
Ref: **[Discrete Mathematics and Probability Theory](https://hkn.eecs.berkeley.edu/examfiles/cs70_fa14_f_sol.pdf)** (https://hkn.eecs.berkeley.edu/examfiles/cs70_fa14_f_sol.pdf)

### P12

With the probability at least $1 - \delta$, we can promise the data for all hypothesis in our hypothesis set is good

$$\delta \leq 2Mexp(-2(\frac{\epsilon}{2})^2 N)$$
$$\ln\frac{\delta}{2M} \leq -\frac{\epsilon^2 N}{2}$$
$$N \geq \frac{1}{2\epsilon^2}\ln\frac{M}{\delta}$$

**As a result, we should choose $[c]$ as our solution.**

## Experiments with Perceptron Learning Algorithm

### P13

```
1   import random
2   from tqdm import trange
3   
4   def average(lst):
5       return sum(lst) / len(lst)
6   
7   def cdot(args, w_t, x_list):
8       sum = 0
9       for i in range(args['feature']):
10          sum += w_t[i]*x_list[i]
11      return sum
12  
13  def E_in(args, w, x_train, y_train):
14      sum = 0
15      for i in range(args['size']):
16          wx = cdot(args, w, x_train[i])
17          if((wx <= 0 and y_train[i] > 0) or (wx > 0 and y_train[i] < 0)):
18              sum += 1
19      return sum/args['size']
20  
21  def PLA(args, seed, x_train, y_train):
22      random.seed(seed)
23      time = 0
24      M = args['size'] // 2
25      w_t = [0]*args['feature']
26  
27      while(time < M):
28          time += 1
29          pick = random.randint(0, args['size']-1)
30          wx = cdot(args, w_t, x_train[pick])
31          if((wx <= 0 and y_train[pick] > 0) or (wx > 0 and y_train[pick] < 0)):
32              time = 0
33              w_t = [w_t[i] + x_train[pick][i] * y_train[pick] for i in range(args['feature'])]
34      return w_t
35  
36  def read_file(args):
37      x_train = list()
38      y_train = list()
39      with open (args['filename'], 'r') as f:
40          lines = f.readlines()
41          for line in lines:
42              line_data = line.split()
43              line_data = [float(i) for i in line_data]
44              x_train.append([1]+line_data[:-1])  # set x_0 = 1 to every X_n
45              y_train.append(line_data[-1])
46      return x_train, y_train
47  
48  def main(args):
49      x_train, y_train = read_file(args)
50  
51      error_list = list()
52      for i in trange(args['repeat_time']):
53          w_pla = PLA(args, args['seed']+i, x_train, y_train)
54          error = E_in(args, w_pla, x_train, y_train)
55          error_list.append(error)
56  
57      error_avg = average(error_list)
58      print("average E_in(w_pla):", error_avg)
59  
60  if __name__ == '__main__':
61      args = {
62      'feature': 11,                 # include y
63      'filename': 'hw1_train.dat',
64      'seed': 1126,
65      'size': 256,                    # size = N
66      'repeat_time': 1000,
67      }
68  
69      main(args)
```

Output: $0.01959375 \approx 0.02$

**As a result, we should choose $[b]$ as our solution.**

## P14

We use the same code above, then substitute `M = 4 * args['size']` for `M = args['size'] // 2` in PLA procedure, which is line 24. The updated PLA function shows below.

```
def PLA(args, seed, x_train, y_train):
    random.seed(seed)
    time = 0
    M = 4 * args['size']
    w_t = [0]*args['feature']

    while(time < M):
        time += 1
        pick = random.randint(0, args['size']-1)
        wx = cdot(args, w_t, x_train[pick])
        if((wx <= 0 and y_train[pick] > 0) or (wx > 0 and y_train[pick] < 0)):
            time = 0
            w_t = [w_t[i] + x_train[pick][i] * y_train[pick] for i in range(args['feature'])]
    return w_t
```

Output: $0.000203125 \approx 0.00020$

**As a result, we should choose $[a]$ as our solution.**

## P15

```python
import random
from tqdm import trange

def average(lst):
    return sum(lst) / len(lst)

def cdot(args, w_t, x_list):
    sum = 0
    for i in range(args['feature']):
        sum += w_t[i]*x_list[i]
    return sum

def E_in(args, w, x_train, y_train):
    sum = 0
    for i in range(args['size']):
        wx = cdot(args, w, x_train[i])
        if((wx <= 0 and y_train[i] > 0) or (wx > 0 and y_train[i] < 0)):
            sum += 1
    return sum/args['size']

def median(lst):
    lst = sorted(lst)
    mid = len(lst) // 2
    res = (lst[mid] + lst[~mid]) / 2
    return res

def PLA(args, seed, x_train, y_train):
    random.seed(seed)
    time = 0
    M = 4 * args['size']
    w_t = [0]*args['feature']
    update = 0

    while(time < M):
        time += 1
        pick = random.randint(0, args['size']-1)
        wx = cdot(args, w_t, x_train[pick])
        if((wx <= 0 and y_train[pick] > 0) or (wx > 0 and y_train[pick] < 0)):
            time = 0
            update += 1
            w_t = [w_t[i] + x_train[pick][i] * y_train[pick] for i in range(args['feature'])]
    return w_t, update

def read_file(args):
    x_train = list()
    y_train = list()
    with open (args['filename'], 'r') as f:
        lines = f.readlines()
        for line in lines:
            line_data = line.split()
            line_data = [float(i) for i in line_data]
            x_train.append([1]+line_data[:-1])  # set x_0 = 1 to every X_n
            y_train.append(line_data[-1])
    return x_train, y_train

def main(args):
    x_train, y_train = read_file(args)

    error_list = list()
    update_list = list()
    for i in trange(args['repeat_time']):
        w_pla, update = PLA(args, args['seed']+i, x_train, y_train)
        error = E_in(args, w_pla, x_train, y_train)
        error_list.append(error)
        update_list.append(update)

    error_avg = average(error_list)
    update_median = median(update_list)
    print("average E_in(w_pla):", error_avg)
    print("median number of updates:", update_median)

if __name__ == '__main__':
    args = {
    'feature': 11,                 # include y
    'filename': 'hw1_train.dat',
    'seed': 1126,
    'size': 256,                    # size = N
    'repeat_time': 1000,
    }

    main(args)
```

Output: $454.0 \approx 400$

**As a result, we should choose $[d]$ as our solution.**

## P16

We use the same code above, then record all $w_0$ of each iteration into a list, and calculate the median value. The updated main function shows below.

```
def main(args):
    x_train, y_train = read_file(args)

    error_list = list()
    update_list = list()
    w_0_list = list()
    for i in trange(args['repeat_time']):
        w_pla, update = PLA(args, args['seed']+i, x_train, y_train)
        error = E_in(args, w_pla, x_train, y_train)
        error_list.append(error)
        update_list.append(update)
        w_0_list.append(w_pla[0])

    error_avg = average(error_list)
    update_median = median(update_list)
    w_0_median = median(w_0_list)
    print("average E_in(w_pla):", error_avg)
    print("median number of updates:", update_median)
    print("median of all w_0:", w_0_median)
```

Output: $35.0 \approx 40$

**As a result, we should choose $[e]$ as our solution.**

## P17

We use the same code above, then update the preprocessing of reading the files. The updated read_file function shows below.

```
def read_file(args):
    x_train = list()
    y_train = list()
    with open (args['filename'], 'r') as f:
        lines = f.readlines()
        for line in lines:
            line_data = line.split()
            line_data = [float(i) for i in line_data]
            x_train.append([0.5]+[line_data[:-1][i]*2 for i in range(args['feature']-1)])  # set x_0 = 1 to every X_
            y_train.append(line_data[-1])
    return x_train, y_train
```

Output: $452.0 \approx 400$

**As a result, we should choose $[d]$ as our solution.**

## P18

We use the same code above, then update the preprocessing of reading the files. The updated read_file function shows below.

```
def read_file(args):
    x_train = list()
    y_train = list()
    with open (args['filename'], 'r') as f:
        lines = f.readlines()
        for line in lines:
            line_data = line.split()
            line_data = [float(i) for i in line_data]
            x_train.append([0]+line_data[:-1])  # set x_0 = 1 to every X_n
            y_train.append(line_data[-1])
    return x_train, y_train
```

Output: $449.0 \approx 400$

**As a result, we should choose $[d]$ as our solution.**

## P19

We use the same code above, then record all $w_0x_0$ of each iteration into a list, and calculate the median value. The updated main function shows below.

```
def main(args):
    x_train, y_train = read_file(args)

    error_list = list()
    update_list = list()
    w_0_list = list()
    w_0x_0_list = list()
    for i in trange(args['repeat_time']):
        w_pla, update = PLA(args, args['seed']+i, x_train, y_train)
        error = E_in(args, w_pla, x_train, y_train)
        error_list.append(error)
        update_list.append(update)
        w_0_list.append(w_pla[0])
        w_0x_0_list.append(w_pla[0]*x_train[0][0])

    error_avg = average(error_list)
    update_median = median(update_list)
    w_0_median = median(w_0_list)
    w_0x_0_median = median(w_0x_0_list)
    print("average E_in(w_pla):", error_avg)
    print("median number of updates:", update_median)
    print("median of all w_0:", w_0_median)
    print("median of all w_0x_0:", w_0x_0_median)
```

Output: $35.0 \approx 40$
**As a result, we should choose $[e]$ as our solution.**

## P20

We use the same code above, then update the preprocessing of reading the files. The updated read_file function shows below.

```python
import random
from tqdm import trange

def average(lst):
    return sum(lst) / len(lst)

def cdot(args, w_t, x_list):
    sum = 0
    for i in range(args['feature']):
        sum += w_t[i]*x_list[i]
    return sum

def E_in(args, w, x_train, y_train):
    sum = 0
    for i in range(args['size']):
        wx = cdot(args, w, x_train[i])
        if((wx <= 0 and y_train[i] > 0) or (wx > 0 and y_train[i] < 0)):
            sum += 1
    return sum/args['size']

def median(lst):
    lst = sorted(lst)
    mid = len(lst) // 2
    res = (lst[mid] + lst[~mid]) / 2
    return res

def PLA(args, seed, x_train, y_train):
    random.seed(seed)
    time = 0
    M = 4 * args['size']
    w_t = [0]*args['feature']
    update = 0

    while(time < M):
        time += 1
        pick = random.randint(0, args['size']-1)
        wx = cdot(args, w_t, x_train[pick])
        if((wx <= 0 and y_train[pick] > 0) or (wx > 0 and y_train[pick] < 0)):
            time = 0
            update += 1
            w_t = [w_t[i] + x_train[pick][i] * y_train[pick] for i in range(args['feature'])]
    return w_t, update

def read_file(args):
    x_train = list()
    y_train = list()
    with open (args['filename'], 'r') as f:
        lines = f.readlines()
        for line in lines:
            line_data = line.split()
            line_data = [float(i) for i in line_data]
            x_train.append([0.1126]+line_data[:-1])  # set x_0 = 1 to every X_n
            y_train.append(line_data[-1])
    return x_train, y_train

def main(args):
    x_train, y_train = read_file(args)

    error_list = list()
    update_list = list()
    w_0_list = list()
    w_0x_0_list = list()
    for i in trange(args['repeat_time']):
        w_pla, update = PLA(args, args['seed']+i, x_train, y_train)
        error = E_in(args, w_pla, x_train, y_train)
        error_list.append(error)
        update_list.append(update)
        w_0_list.append(w_pla[0])
        w_0x_0_list.append(w_pla[0]*x_train[0][0])

    error_avg = average(error_list)
    update_median = median(update_list)
    w_0_median = median(w_0_list)
    w_0x_0_median = median(w_0x_0_list)
    print("average E_in(w_pla):", error_avg)
    print("median number of updates:", update_median)
    print("median of all w_0:", w_0_median)
    print("median of all w_0x_0:", w_0x_0_median)

if __name__ == '__main__':
    args = {
    'feature': 11,                  # include y
    'filename': 'hw1_train.dat',
    'seed': 1126,
    'size': 256,                    # size = N
    'repeat_time': 1000,
    }

    main(args)
```

```
    main(args)
```

Output: $0.4437 \approx 0.4$

**As a result, we should choose $[c]$ as our solution.**