HW4 Solutions

● Graded

Student

Chih-Hao Liao

Total Points

280 / 320 pts

Question 1

Problem 1

40 / 40 pts

✔ **– 0 pts** Correct

– 10 pts One minor mistake

– 10 pts Steps are a little unclear

– 20 pts One of 2 mentioned above:
1. One major mistake or two minor mistakes
2. Not provide the detail how to get optimal solution

– 30 pts Some reasonable effort but fundamentally wrong or wrong derivation

– 40 pts One of mentioned below:
1. No solution
2. Totally wrong solution

– 40 pts One of mentioned below:
1. Wrong answer
2. Choose wrong page
3. Can not see the content of solution (blur)

Question 2

Problem 2

40 / 40 pts

✔ **– 0 pts** Correct

– 2 pts Correct with some typos

– 10 pts 1. One minor mistake during the derivation steps
2. Minor mistakes in linear algebra operation

– 20 pts 1. One major mistake during the derivation steps
2. Major mistakes in linear algebra operation

– 30 pts Some reasonable effort but fundamentally wrong

– 40 pts 1. No page selected or wrong page selected
2. Little or no derivation steps
3. Wrong solution or no solution

Question 3

Problem 3

0 / 40 pts

- **– 0 pts** Correct
- **– 10 pts** Minor errors
- **– 20 pts** Major errors
- **– 30 pts** Reasonable effort
- ✔ **– 40 pts** Fundamentally wrong
- **– 40 pts** Blank

1. wrong difinition of kl divergence. should be summation over {y=+1, y=-1}
2. p_i undefined
3. wrong D_KL parameters.

Question 4

Problem 6

40 / 40 pts

- ✔ **– 0 pts** Correct
- **– 20 pts** Minor mistake
- **– 40 pts** No or wrong answer

Question 5

Problem 8

40 / 40 pts

- ✔ **– 0 pts** Correct
- **– 10 pts** One minor mistake
- **– 10 pts** Steps are a bit unclear
- **– 20 pts** One major mistake or two minor mistakes
- **– 30 pts** Some reasonable effort but fundamentally wrong
- **– 40 pts** Wrong answer
- **– 40 pts** Choose wrong page
- **– 40 pts** 1.No solution
  2. Totally wrong solution
  3. Almost missing solution

Question 6

Problem 9

40 / 40 pts

✔ **– 0 pts** Correct

– 10 pts Missing some computation details.

– 10 pts The answer is directly obtained by substituting the options, without explaining the optimality.

– 10 pts A minor mistake exists in the answer

– 20 pts A major mistake / several minor mistakes exist in the answer

– 30 pts Some reasonable effort

– 40 pts Wrong answer

– 40 pts Did not show the steps

– 20 pts Did not select the corresponding page

Question 7

Problem 10

40 / 40 pts

✔ **– 0 pts** Correct

– 10 pts One minor mistake or steps are a bit unclear for no explanation

– 20 pts One major mistake (such as wrong mathematical symbols in determine the bound) or no complete math derivation process.

– 30 pts Some reasonable effort but fundamentally wrong

– 40 pts No solution or almost missing solution

Question 8

Problem 11

40 / 40 pts

✔ **– 0 pts** Correct.

– 0 pts Correct. Have some minor typos but I can understand what you want to deliver, I will not deduct your points.

– 0 pts Correct. Have missed some trivial derivation steps but I can understand what you want to deliver, I will not deduct your points.

– 10 pts 1. One minor mistake during the derivation steps
2. One missing derivation step

– 20 pts 1. One major mistake
2. Missing several derivation steps
3. Several minor mistakes

– 30 pts 1. Some reasonable effort but fundamentally wrong

– 40 pts 1. Little or no derivation steps
2. Wrong solution or no solution
3. No page selected
4. wrong page selected

Question 9

Coding Part **0** / 0 pts

9.1 **Problem 12** **0** / 0 pts

✔ **– 0 pts** Correct

– 80 pts Unclear submission.

– 20 pts Unclear solutions

– 160 pts No submission

Question assigned to the following page: 1

HTML_2023_HW4

tags: `Personal`

```
Discuss with anonymous and TAs.
Ref: [TA Poy HTML 2023 HW4](https://hackmd.io/@Poy/SJVAvg0Qn)
```

More about Regularization

P1

In order to find the minimum w, we can set the derivative of the optimal solution to be zero, hence, we have

Questions assigned to the following page: 1 and 2

$$
\begin{aligned}
0 &= \frac{\partial}{\partial w}(\frac{1}{N}\sum_{n=1}^{N}(wx_n - y_n)^2 + \frac{\lambda}{N}w^2) \\
&= \frac{2}{N}\sum_{n=1}^{N}x_n(wx_n - y_n) + \frac{2w\lambda}{N} \\
&= \frac{2w}{N}\sum_{n=1}^{N}x_n^2 - \frac{2}{N}\sum_{n=1}^{N}x_n y_n + \frac{2w\lambda}{N} \\
&= \frac{2w}{N}(\sum_{n=1}^{N}x_n^2 + \lambda) - \frac{2}{N}\sum_{n=1}^{N}x_n y_n \\
\frac{2w}{N}(\sum_{n=1}^{N}x_n^2 + \lambda) &= \frac{2}{N}\sum_{n=1}^{N}x_n y_n \\
w(\sum_{n=1}^{N}x_n^2 + \lambda) &= \sum_{n=1}^{N}x_n y_n \\
w^* &= \frac{\sum_{n=1}^{N}x_n y_n}{\sum_{n=1}^{N}x_n^2 + \lambda} \\
\text{or we can use } w^* &= (x^T x + \lambda I)^{-1}x^T y \\
&= (\sum_{n=1}^{N}x_n^2 + \lambda)^{-1} \cdot \sum_{n=1}^{N}x_n y_n \\
&= \frac{\sum_{n=1}^{N}x_n y_n}{\sum_{n=1}^{N}x_n^2 + \lambda} \\
\because C &= (w^*)^2 \\
\therefore C &= (\frac{\sum_{n=1}^{N}x_n y_n}{\sum_{n=1}^{N}x_n^2 + \lambda})^2
\end{aligned}
$$

As a result, we should choose $[a]$ as our solution.

P2

Since the L2-regularized linear regressionin the $\mathcal{Z}$-space is equivalent to regularized linear regression in the $\mathcal{X}$-space, we have

$$
\begin{aligned}
&\min_{\tilde{w}\in\mathbb{R}^{d+1}}\frac{1}{N}\sum_{n=1}^{N}(\tilde{w}^T\Phi(x_n) - y_n)^2 + \frac{\lambda}{N}(\tilde{w}^T\tilde{w}) \\
=&\min_{\tilde{w}\in\mathbb{R}^{d+1}}\frac{1}{N}\sum_{n=1}^{N}(w^T x_n - y_n)^2 + \frac{\lambda}{N}\Omega(w) \\
\therefore &\ \tilde{w}^T\Phi(x_n) = \tilde{w}\Gamma^{-1}(x-u) = \tilde{w}\Gamma^{-1}x = w^T x_n, \tilde{w}^T\tilde{w} = \Omega(w)
\end{aligned}
$$

Since Γ is a diagonal matrix, we know that the inverse of diagonal matrix equals to itself, which means $\Gamma^T = \Gamma$ then we have

Questions assigned to the following page: 2 and 3

$$
\begin{aligned}
w^T &= \tilde{w}\Gamma^{-1} \\
\tilde{w}^T &= w^T\Gamma \\
\tilde{w} &= (w^T\Gamma)^T = \Gamma^T w \\
\therefore \Omega(w) &= \tilde{w}^T\tilde{w} = w^T\Gamma\Gamma w = w^T\Gamma^2 w
\end{aligned}
$$

As a result, we should choose $[b]$ as our solution.

P3

$$
\min_w \frac{1}{N}\sum_{n=1}^{N}\text{err}_{\text{smooth}}(w, x_n, y_n) = \min_w \frac{1}{N}\sum_{n=1}^{N}\text{err}(w, x_n, y_n) + \frac{\lambda}{N}\sum_{n=1}^{N}\Omega(w, x_n)
$$

$$
D_{KL}(P\|Q) = \sum_x P(x)\ln\frac{P(x)}{Q(x)} = \sum_x \frac{1}{2}\ln\frac{\frac{1}{2}}{\text{err}(w,x,y)}
$$

$$
\begin{aligned}
w_{lr} &= \underset{w}{\text{argmin}}\sum_{i=1}^{n} D_{KL}(P(Y_i)\|P(\hat{Y}_i) \\
&= \underset{w}{\text{argmin}}\sum_{i=1}^{n} y_i\ln\frac{y_i}{p_i} + (1-y_i)\ln\frac{(1-y_i)}{(1-p_i)} \\
&= \underset{w}{\text{argmin}}\sum_{i=1}^{n} y_i(\ln y_i - \ln p_i) + (1-y_i)(\ln(1-y_i) - \ln(1-p_i)) \\
&= \underset{w}{\text{argmin}}\sum_{i=1}^{n} -y_i\ln p_i - (1-y_i)\ln(1-p_i) + (y_i\ln y_i + (1-y_i)\ln(1-y_i)) \\
&= \underset{w}{\text{argmin}}\sum_{i=1}^{n} y_i\ln p_i + (1-y_i)\ln(1-p_i)) \\
&= \underset{w}{\text{argmin}}\sum_{i=1}^{n} H(P(Y_i)\|P(\hat{Y}_i) \\
\therefore \Omega(w,x) &= D_{KL}(P_u\|P_h)
\end{aligned}
$$

As a result, we should choose $[a]$ as our solution.

Validation

P4

Since constant hypothesis $h(x) = w_0$, we only need to take y into the consideration. By selecting two examples for predicting another one example, we can obtain the equation

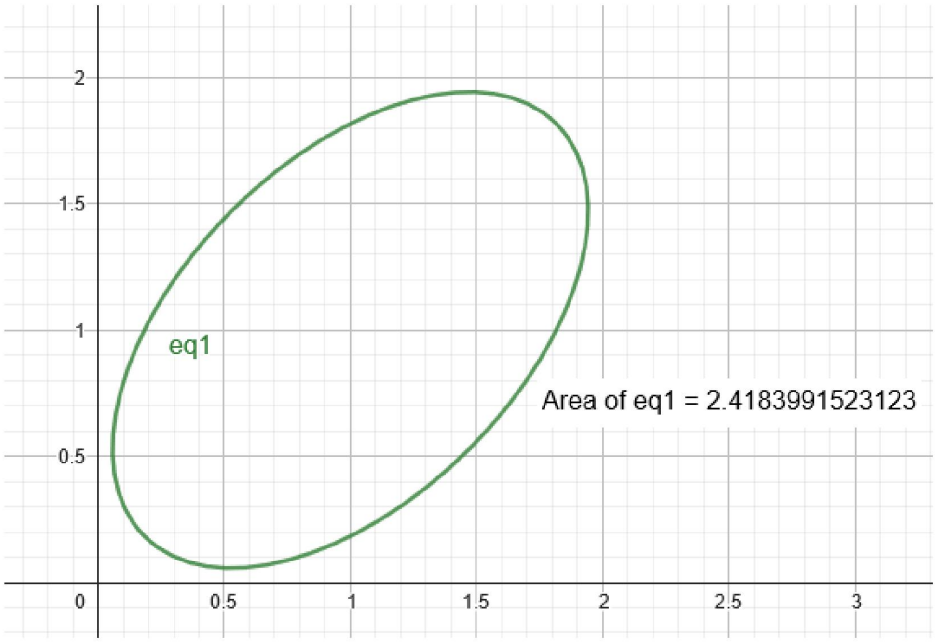
$$
\begin{aligned}
E_{loocv} &= \frac{1}{3}\left((y_1 - \frac{y_2+1}{2})^2 + (y_2 - \frac{y_1+1}{2})^2 + (1 - \frac{y_1+y_2}{2})^2\right) \\
&= \frac{1}{2}(y_1^2 + y_2^2 - y_1y_2 - y_1 - y_2 + 1)
\end{aligned}
$$

No questions assigned to the following page.

Since we are finding $E_{loocv} \leq \frac{1}{3}$, we can rewrite the equation to be

$$
\begin{aligned}
&(\frac{x+1}{2}-y)^2+(\frac{y+1}{2}-x)^2+(\frac{x+y}{2}-1)^2=1 \\
\Rightarrow &\frac{1}{2}(x^2+y^2-xy-x-y+1)=\frac{1}{3}
\end{aligned}
$$

Drawing the equation with `GeoBeBra`, we can simply get the area, which is $\approx 2.4183 \approx \frac{\pi}{3\sqrt{3}}$



As a result, we should choose $[b]$ as our solution.

Question assigned to the following page: 4

P5

$$
\begin{aligned}
&\because E_{out}(h) = \underset{x\sim P}{\mathcal{E}}[h(x) \neq f(x)] \\
&\therefore \underset{\mathcal{D}_{val}\sim\mathcal{P}^K}{\text{Variance}}\left[E_{val}(h)\right] \\
&= \underset{(x,y)\sim\mathcal{P}}{\text{Variance}}\left[\frac{1}{K}\sum_{n=1}^{K}\text{err}(h(x_n),y_n)\right] \\
&= \frac{1}{K^2}\underset{(x,y)\sim\mathcal{P}}{\text{Variance}}\left[\sum_{n=1}^{K}\text{err}(h(x_n),y_n)\right] \\
&= \frac{1}{K^2}\underset{(x,y)\sim\mathcal{P}}{\text{Variance}}\left[\text{err}(h(x_1),y_1)+\text{err}(h(x_2),y_2)+\cdots+\text{err}(h(x_K),y_K)\right] \\
&= \frac{1}{K^2}(K\cdot\underset{(x,y)\sim\mathcal{P}}{\text{Variance}}\left[\text{err}(h(x),y)\right]) \\
&= \frac{1}{K}\underset{(x,y)\sim\mathcal{P}}{\text{Variance}}\left[\text{err}(h(x),y)\right] \\
&\therefore \square = \frac{1}{K}
\end{aligned}
$$

Since the examples (x, y) are genetated from the i.i.d. distribution, the covariance the between examples should be zero.

$$
\begin{aligned}
\because \quad &\text{Covariance}\left[err(h(x_i),y_i), err(h(x_j),y_j)\right] = 0, \forall i,j \\
\therefore \text{Variance}\,(A+B) &= \text{Variance}\,(A) + \text{Variance}\,(B) + 2\cdot\text{Covariance}\,(A,B) \\
&= \text{Variance}\,(A) + \text{Variance}\,(B) + 0
\end{aligned}
$$

As a result, we should choose $[d]$ as our solution.

P6

Since single data set in a binary classification has N positive examples and N negative examples, we can perform leave-one-out validation by selecting one positive or one negative example into the validation data set.
Once we select a positive example as a validation data set, there are $N-1$ positive examples and N negative examples in the training data set, and the binary classification algorithm $A_{majority}$ will return negative to the training data set for the reason that it predicts the majority class. However, $A_{majority}$ will return positive to the validation data set since there is only one positive example, and vice versa.
As a result, we know that $A_{majority}$ will always return a different classification between the training data set and the validation data set, which means $E_{loocv}(A_{majority}) = 1$.

As a result, we should choose $[d]$ as our solution.

P7

The threshold of the decision stump model can be represented as

Questions assigned to the following page: 5 and 6

$$\theta \in \{-1\} \cup \{\frac{x_i + x_{i+1}}{2}\} : 1 \leq i \leq N-1 \text{ and } x_i \neq x_{i+1}$$

Since x is generated by a uniform distribution in $[-1, +1]$, and $y = \text{sign}(x)$, it's trivial that $E_{in} = 0$ when $\theta = 0$, and the threshold θ with the lowest E_{in} in the decision stump model will be trained in the middle of the smallest positive value and the largest negative value. Therefore, the trained $\theta = \frac{x_j + x_{j+1}}{2}$, where x_j is the largest negative value, and x_{j+1} is the smallest positive value.

If the x_j is selected as the validation data set, then $\theta = \frac{x_{j-1} + x_{j+1}}{2}$, where x_{j-1} is the second largest negative value. Both θ and the x_j are between x_{j-1} and x_{j+1}, then we have

- if $\theta \geq x_j \Rightarrow h(x_j) = -1$, which means the validation data set will be classified correctly.
- if $\theta < x_j \Rightarrow h(x_j) = +1$, which means the validation data set will be classified incorrectly.

And vice versa, if x_{j+1} is selected as the validation data set, then $\theta = \frac{xj + xj+2}{2}$, where x_{j+2} is the second smallest positive value. Both θ and the x_{j+1} are between x_j and x_{j+2}, then we have

- if $\theta \geq x_{j+1} \Rightarrow h(x_{j+1}) = -1$, which means the validation data set will be classified incorrectly.
- if $\theta < x_{j+1} \Rightarrow h(x_{j+1}) = +1$, which means the validation data set will be classified correctly.

Since only x_j and x_{j+1} will be classified incorrectly by the threshold θ, the tightest upper bound of the leave-one-out validation error to the decision stump model is equal to $\max(E_{loocv}) = 2/N$.

As a result, we should choose $[c]$ as our solution.

Support Vector Machine

P8

Since the examples are ordered, the hard-margin SVM without transformation can be considered as a decision stump model when the separation line is orthogonal to the coordinate of the examples. It's trivial that the perfect separation line must be the middle of x_M and x_{M+1}, therefore, the hypothesis can be represented as

$$g_{svm}(x) = \text{sign}(w^T x + b) = 0 = x - \frac{x_M + x_{M+1}}{2}$$

Therefore, the largest margin is equal to $\frac{1}{2}(x_{M+1} - x_M)$

As a result, we should choose $[e]$ as our solution.

P9

Based on the subjections of $\min(w, b) = \frac{1}{2} w^T w$, we have

Questions assigned to the following page: 6 and 7

$$
\begin{aligned}
(w^T x_n + b) &\geq 1 \qquad \text{for } y_n = +1 \\
-(w^T x_n + b) &\geq 1126 \quad \text{for } y_n = -1
\end{aligned}
$$

$$
y = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}^T \begin{bmatrix} 0 & 4 \\ 2 & 0 \\ -1 & 0 \\ 0 & 0 \end{bmatrix} + b = \begin{bmatrix} +1 \\ -1 \\ +1 \\ +1 \end{bmatrix} \Rightarrow \begin{bmatrix} (4w_2 + b) \geq 1 \\ -(2w_1 + b) \leq 1126 \\ (-w_1 + b) \geq 1 \\ (+b) \geq 1 \end{bmatrix}
$$

By drawing all data on the graph and from the subjections above, we know that the ratio of positive margin versus negative margin can represent as $r_m = \frac{y_n=+1}{y_n=-1} = \frac{1}{1126}$, hence, the hyperplane can be represented by the thinnest gap $-(w^T[2,0] + b = 1126)$ and $(w^T[0,0] + b = 1)$, which implies that $w^T x + b = 0 \Rightarrow -\frac{1127}{2}x + 1 = 0$, which means $w = (-\frac{1127}{2}, 0), b = 1$.

As a result, we should choose $[e]$ as our solution.

P10

Since $\alpha = 1, b = 0, h(x) = \text{sign}(\sum_{n=1}^{N} y_n K(x_n, x))$, then we have $E_{in}(h) = \frac{1}{N}\sum_{i=1}^{N}[h(x_i) \neq y_i]$. If $E_{in}(\hat{h}) = 0$, all predictions should be classified correctly, and there is no classification error, which means

$$
\left[\text{sign}\left(\sum_{n=1}^{N} y_n K(x_n, x_i)\right) = y_i\right], \forall i \overset{\text{rewrite}}{\Longrightarrow} \left[\left(\sum_{n=1}^{N} y_n K(x_n, x_i)\right) \cdot y_i > 0\right], \forall i
$$

To sum up the equation above, we have

$$
\begin{aligned}
\sum_{i=1}^{N}\left(y_i \sum_{n=1}^{N} y_n K(x_n, x_i)\right) &= \left[y_1^2 + y_1\left(y_2 K(x_2, x_1) + \cdots + y_N K(x_N, x_1)\right)\right] \\
&+ \left[y_2^2 + y_2\left(y_1 K(x_1, x_2) + \cdots + y_N K(x_N, x_2)\right)\right] \\
&+ \cdots \\
&+ \left[y_N^2 + y_N\left(y_1 K(x_1, x_N) + \cdots + y_{N-1} K(x_{N-1}, x_N)\right)\right] > 0
\end{aligned}
$$

Since $\gamma > 0, \|x_n \neq x_m\| \geq \epsilon, \forall n \neq m$, we have

$$
\begin{aligned}
K(x_n, x_m) = \exp(-\gamma\|x_n - x_m\|^2) &\leq \exp(-\gamma\epsilon^2) := \zeta \\
\text{if } n = i, K(x_n, x_i) = \exp(-\gamma\|x_n - x_i\|^2) &= \exp(-\gamma \cdot 0) = 0
\end{aligned}
$$

Taking ζ back to the equation as the upper bound, then we get

Questions assigned to the following page: 7 and 8

$$
\begin{aligned}
0 &< \sum_{i=1}^{N}\left(y_i\sum_{n=1}^{N}y_nK(x_n,x_i)\right) < \sum_{i=1}^{N}\left(y_i\sum_{n=1}^{N}y_n\exp(-\gamma\epsilon^2)\right) \\
&= \left[y_1^2+y_2^2+\cdots+y_N^2\right] \\
&+ \underbrace{\left[y_1\left(\underbrace{y_2\exp(-\gamma\epsilon^2)+\cdots+y_N\exp(-\gamma\epsilon^2)}_{N-1}\right)+\cdots+y_N\left(\underbrace{y_1\exp(-\gamma\epsilon^2)+\cdots+y_{N-1}\exp(-\gamma\epsilon^2)}_{N-1}\right)\right.}_{N}
\end{aligned}
$$

And we know that

$$
\begin{aligned}
&\forall n=m,\, y_m=(+1)\vee(-1)\Rightarrow y_i^2=1,\forall i \\
&\forall n\neq m,\, -1<y_ny_m<1 \Rightarrow \text{assumed that all } y_ny_m=-1
\end{aligned}
$$

Then, we have

$$
\begin{aligned}
0 &< \sum_{i=1}^{N}\left(y_i\sum_{n=1}^{N}y_n\exp(-\gamma\epsilon^2)\right) \\
&< \sum_{i=1}^{N}y_i^2+\sum_{i=1}^{N}(-1)\times(N-1)\exp(-\gamma\epsilon^2) \\
&= N+N\times[-(N-1)\exp(-\gamma\epsilon^2)] \\
&= N-N(N-1)\exp(-\gamma\epsilon^2) \\
1 &> (N-1)\exp(-\gamma\epsilon^2) \\
\frac{1}{N-1} &> \exp(-\gamma\epsilon^2) \\
\ln(N-1) &< \gamma\epsilon^2 \\
\gamma &> \frac{\ln(N-1)}{\epsilon^2}
\end{aligned}
$$

As a result, we should choose $[d]$ as our solution.

P11

Applying the feature transform ϕ to the Gaussian kernel, then we have

$$
\begin{aligned}
\|\phi(x)-\phi(x^{'})\|^2 &= <\phi(x)-\phi(x^{'}),\phi(x)-\phi(x^{'})> \\
&= <\phi(x),\phi(x)>-2<\phi(x),\phi(x^{'})>+<\phi(x^{'}),\phi(x^{'})> \\
&= \phi(x)^T\phi(x)-\phi(x)^T\phi(x^{'})-\phi(x^{'})^T\phi(x)+\phi(x^{'})^T\phi(x^{'}) \\
&= K(x,x)-2K(x,x^{'})+K(x^{'},x^{'}) \\
&= 2-2\exp(-\gamma\|x-x^{'}\|^2) \\
&\because \exp(-\gamma\|x-x^{'}\|^2)\in(0,1] \\
\therefore 2 &> \|\phi(x)-\phi(x^{'})\|^2 \\
\sqrt{2}\approx 1.5 &> \|\phi(x)-\phi(x^{'})\|
\end{aligned}
$$

Questions assigned to the following page: 9.1 and 8

Therefore, the tightest upper bound for the distane in the $\mathcal{Z}$-space is 1.5.

As a result, we should choose $[d]$ as our solution.

Experiments with Regularized Logistic Regression

P12

Since L2-regularized logistic regression `-s 0` in liblinear solves

$$w = \min_w \frac{1}{2} w^T w + C \sum_{i=1}^{N} \log(1 + \exp(-y_i w^T x_i))$$

and we solve

$$w_\lambda = \operatorname*{argmin}_w \frac{\lambda}{N} \|w\|^2 + \frac{1}{N} \sum_{n=1}^{N} \ln(1 + \exp(-y_n w^T \Phi_4(x_n))$$

Then we know that

$$\frac{1}{2} = \frac{\lambda}{N}, C = \frac{1}{N} \Rightarrow C = \frac{1}{2\lambda}$$

The parameter `-c cost : set the parameter C (default 1)` should be set to $C = \frac{1}{2\lambda}$.

```python
import numpy as np
from itertools import combinations_with_replacement
from liblinear.liblinearutil import *

def transformation(args, x):
    x_origin = x[:, 1:] # remove x_{n=0}=1
    transformed_x = x.copy()
    combination_tuples = list()
    iterable = list(i for i in range(1, args['dimension']))
    for r in range(1, args['Q']):
        combination_tuples.extend(list(combinations_with_replacement(iterable, r+1)))

    for combination in range(len(combination_tuples)):
        x_temp = 1
        for j in combination_tuples[combination]:
            x_temp = x_origin[:,j-1].reshape(-1, 1) * x_temp
        transformed_x = np.hstack((
            transformed_x, x_temp
            ))
    return transformed_x
```

Question assigned to the following page: 9.1

```
def read_file(args, filename):
    data = np.loadtxt(filename, dtype=float)
    x = data[:,:-1]
    x = np.c_[np.ones(len(x)), x] # x_{n=0}=1
    y = data[:,-1]
    args['dimension'] = x.shape[1]
    return x, y
```

```
def main(args):
    x_train, y_train = read_file(args, args['filename_train'])
    x_test, y_test = read_file(args, args['filename_test'])
    x_train = transformation(args, x_train)
    x_test = transformation(args, x_test)

    E_out = list()
    for lamb in args['lambda']:
        C = 1/(2*lamb)
        prob = problem(y_train, x_train)
        param = parameter('-s 0 -c {} -e 0.000001 -q'.format(C))
        m = train(prob, param)
        p_label, p_acc, p_val = predict(y_test, x_test, m)
        E_out.append(round(np.mean(y_test != p_label), 6))

    min_E_out = min(E_out)
    min_E_out_index = [i for i, v in enumerate(E_out) if v == min_E_out]
    print("E_out: ", E_out)
    print("min E_out: ", min(E_out))
    print("Choice: ", chr(97+max(min_E_out_index)))
```

```
if __name__ == '__main__':
    args = {
        'dimension': 0,
        'filename_test': "hw4_test.dat",
        'filename_train': "hw4_train.dat",
        'lambda': [10**(-6), 10**(-3), 10**(0), 10**3, 10**6],
        'Q': 4,
    }

    main(args)
```

Question assigned to the following page: 9.1

```
NumbaDeprecationWarning: The 'nopython' keyword argument was not supplied to the 'numl
  def csr_to_problem_jit(l, x_val, x_ind, x_rowptr, prob_val, prob_ind, prob_rowptr):
Accuracy = 77.4% (387/500) (classification)
Accuracy = 82.2% (411/500) (classification)
Accuracy = 84.6% (423/500) (classification)
Accuracy = 85.8% (429/500) (classification)
Accuracy = 81.2% (406/500) (classification)
E_out:  [0.226, 0.178, 0.154, 0.142, 0.188]
min E_out:  0.142
Choice:  d
```

Therefore, the best $\log_{10}(\lambda^*) = 3$

As a result, we should choose $[d]$ as our solution.

P13

We use the same code above, then update the `main` function. The updated functions show below.

```
def main(args):
    x_train, y_train = read_file(args, args['filename_train'])
    x_test, y_test = read_file(args, args['filename_test'])
    x_train = transformation(args, x_train)
    x_test = transformation(args, x_test)

    E_in = list()
    for lamb in args['lambda']:
        C = 1/(2*lamb)
        prob = problem(y_train, x_train)
        param = parameter('-s 0 -c {} -e 0.000001 -q'.format(C))
        m = train(prob, param)
        p_label, p_acc, p_val = predict(y_train, x_train, m)
        E_in.append(round(np.mean(y_train != p_label), 6))

    min_E_in = min(E_in)
    min_E_in_index = [i for i, v in enumerate(E_in) if v == min_E_in]
    print("E_in: ", E_in)
    print("min E_in: ", min(E_in))
    print("Choice: ", chr(97+max(min_E_in_index)))
```

No questions assigned to the following page.

```
NumbaDeprecationWarning: The 'nopython' keyword argument was not supplied to the 'numl
  def csr_to_problem_jit(l, x_val, x_ind, x_rowptr, prob_val, prob_ind, prob_rowptr):
Accuracy = 100% (200/200) (classification)
Accuracy = 100% (200/200) (classification)
Accuracy = 100% (200/200) (classification)
Accuracy = 96% (192/200) (classification)
Accuracy = 76% (152/200) (classification)
E_in:  [0.0, 0.0, 0.0, 0.04, 0.24]
min E_in:  0.0
Choice:  c
```

Since $E_{in}(w_{\log_{10}(\lambda^*)=-6}) = E_{in}(w_{\log_{10}(\lambda^*)=-3}) = E_{in}(w_{\log_{10}(\lambda^*)=0}) = 0$, by selecting the largest λ, therefore, the best $\log_{10}(\lambda^*) = 0$

As a result, we should choose $[c]$ as our solution.

P14

We use the same code above, then add a `train_test_split` function and update the `main` function and the parameter of args. The updated functions show below.

```
import numpy as np
import random
from itertools import combinations_with_replacement
from liblinear.liblinearutil import *

def train_test_split(args, x, y):
    rng = np.random.default_rng()
    idx = np.arange(len(x))
    rng.shuffle(idx)
    x_train = x[idx[:args['split']]]
    y_train = y[idx[:args['split']]]
    x_test = x[idx[args['split']:]]
    y_test = y[idx[args['split']:]]
    return x_train, y_train, x_test, y_test
```

No questions assigned to the following page.

```
def main(args):
    x_train, y_train = read_file(args, args['filename_train'])
    x_train = transformation(args, x_train)
    lamb_list = list()
    E_val_list = list()
    for i in range(args['repeat_time']):
        x_train_split, y_train_split, x_test_split, y_test_split = train_test_split(a
        args['size'] = x_train_split.shape[0]

        E_val = list()
        for lamb in args['lambda']:
            C = 1/(2*lamb)
            prob = problem(y_train_split, x_train_split)
            param = parameter('-s 0 -c {} -e 0.000001 -q'.format(C))
            m = train(prob, param)
            p_label, p_acc, p_val = predict(y_test_split, x_test_split, m)
            E_val.append(round(np.mean(y_test_split != p_label), 6))

        min_E_val = min(E_val)
        min_E_val_index = [i for i, v in enumerate(E_val) if v == min_E_val]
        lamb_list.append(max(min_E_val_index))
        E_val_list.append(min_E_val)

    best_lambda = args['lambda'][max(set(lamb_list), key = lamb_list.count)]
    print("best_lambda: ", best_lambda)
```

```
if __name__ == '__main__':
    args = {
        'dimension': 0,
        'filename_test': "hw4_test.dat",
        'filename_train': "hw4_train.dat",
        'lambda': [10**(-6), 10**(-3), 10**(0), 10**3, 10**6],
        'Q': 4,
        'repeat_time': 256,
        'seed': 1126,
        'size': 0,
        'split': 120
    }

    main(args)
```

```
best_lambda:  1000
```

As a result, we should choose $[d]$ as our solution.

No questions assigned to the following page.

P15

We use the same code above, then update the `main` function. The updated functions show below.

```
def main(args):
    x_train, y_train = read_file(args, args['filename_train'])
    x_train = transformation(args, x_train)
    x_test, y_test = read_file(args, args['filename_test'])
    x_test = transformation(args, x_test)
    lamb_list = list()
    E_val_list = list()
    E_out_list = list()
    for i in range(args['repeat_time']):
        x_train_split, y_train_split, x_test_split, y_test_split = train_test_split(a
        args['size'] = x_train_split.shape[0]

        model_list = list()
        E_val = list()
        for lamb in args['lambda']:
            C = 1/(2*lamb)
            prob = problem(y_train_split, x_train_split)
            param = parameter('-s 0 -c {} -e 0.000001 -q'.format(C))
            m = train(prob, param)
            p_label, p_acc, p_val = predict(y_test_split, x_test_split, m)
            E_val.append(round(np.mean(y_test_split != p_label), 6))
            model_list.append(m)

        min_E_val = min(E_val)
        min_E_val_index = [i for i, v in enumerate(E_val) if v == min_E_val]
        lamb_list.append(max(min_E_val_index))
        E_val_list.append(min_E_val)
        # predict test dataset
        p_label, p_acc, p_val = predict(y_test, x_test, model_list[max(min_E_val_inde
        E_out_list.append(round(np.mean(y_test != p_label), 6))

    best_lambda = args['lambda'][max(set(lamb_list), key = lamb_list.count)]
    print("best_lambda: ", best_lambda)
    print("E_out: ", np.mean(E_out_list))
```

```
best_lambda:  1000
E_out:  0.169109375
```

As a result, we should choose $[c]$ as our solution.

P16

We use the same code above, then update the `main` function. The updated functions show below.

No questions assigned to the following page.

```
def main(args):
    x_train, y_train = read_file(args, args['filename_train'])
    x_train = transformation(args, x_train)
    x_test, y_test = read_file(args, args['filename_test'])
    x_test = transformation(args, x_test)
    lamb_list = list()
    E_val_list = list()
    E_out_list = list()
    for i in range(args['repeat_time']):
        x_train_split, y_train_split, x_test_split, y_test_split = train_test_split(a
        args['size'] = x_train_split.shape[0]

        model_list = list()
        E_val = list()
        for lamb in args['lambda']:
            C = 1/(2*lamb)
            prob = problem(y_train_split, x_train_split)
            param = parameter('-s 0 -c {} -e 0.000001 -q'.format(C))
            m = train(prob, param)
            p_label, p_acc, p_val = predict(y_test_split, x_test_split, m)
            E_val.append(round(np.mean(y_test_split != p_label), 6))
            model_list.append(m)

        min_E_val = min(E_val)
        min_E_val_index = [i for i, v in enumerate(E_val) if v == min_E_val]
        lamb_list.append(max(min_E_val_index))
        E_val_list.append(min_E_val)

        # rebuild the model from raw training dataset
        prob = problem(y_train, x_train)
        param = parameter('-s 0 -c {} -e 0.000001 -q'.format(1/(2*(args['lambda'][max
        m = train(prob, param)
        # predict from raw testing dataset
        p_label, p_acc, p_val = predict(y_test, x_test, m)
        E_out_list.append(round(np.mean(y_test != p_label), 6))

    best_lambda = args['lambda'][max(set(lamb_list), key = lamb_list.count)]
    print("best_lambda: ", best_lambda)
    print("E_out: ", np.mean(E_out_list))
```

```
best_lambda:  1000
E_out:  0.15015624999999996
```

As a result, we should choose $[b]$ as our solution.

P17

We use the same code above, then add a `KFold` function and update the `main` function and the parameter of args. The updated functions show below.

No questions assigned to the following page.

```
def KFold(args, x, y):
    rng = np.random.default_rng()
    idx = np.arange(len(x))
    rng.shuffle(idx)
    x_split = np.array([x[idx[i*args['fold_size']:(i+1)*args['fold_size']]] for i in r
    y_split = np.array([y[idx[i*args['fold_size']:(i+1)*args['fold_size']]] for i in r
    return x_split, y_split
```

```
def main(args):
    x_train, y_train = read_file(args, args['filename_train'])
    x_train = transformation(args, x_train)
    x_test, y_test = read_file(args, args['filename_test'])
    x_test = transformation(args, x_test)
    args['size'] = x_train.shape[0]
    args['fold_size'] = int(args['size'] / args['fold'])

    E_cv_list = list()
    for i in range(args['repeat_time']):
        E_cv = list()
        x_split, y_split = KFold(args, x_train, y_train)

        for lamb in args['lambda']:
            C = 1/(2*lamb)
            E_val = list()
            for fold in range(args['fold']):
                x_valid_fold = x_split[fold]
                y_valid_fold = y_split[fold]
                x_train_fold = np.vstack(x_split[j] for j in range(args['fold']) if j
                y_train_fold = np.hstack(y_split[j] for j in range(args['fold']) if j

                prob = problem(y_train_fold, x_train_fold)
                param = parameter('-s 0 -c {} -e 0.000001 -q'.format(C))
                m = train(prob, param)
                p_label, p_acc, p_val = predict(y_valid_fold, x_valid_fold, m)
                E_val.append(round(np.mean(y_valid_fold != p_label), 6))
            E_cv.append(np.mean(E_val))
        E_cv_list.append(min(E_cv))

    print("E_cv: ", np.mean(E_cv_list))
```

No questions assigned to the following page.

```
if __name__ == '__main__':
    args = {
        'dimension': 0,
        'filename_test': "hw4_test.dat",
        'filename_train': "hw4_train.dat",
        'fold': 5,
        'fold_size': 0, # size of each fold
        'lambda': [10**(-6), 10**(-3), 10**(0), 10**3, 10**6],
        'Q': 4,
        'repeat_time': 256,
        'seed': 1126,
        'size': 0,
        'split': 120
    }

    main(args)
```

```
E_cv:  0.12927734375
```

As a result, we should choose $[a]$ as our solution.

P18

Since L1-regularized logistic regression `-s 6` in liblinear solves

$$w = \min_w \sum |w_j| + C \sum_{i=1}^{N} \log(1 + \exp(-y_i w^T x_i))$$

and we solve

$$w_\lambda = \underset{w}{\operatorname{argmin}} \frac{\lambda}{N} \|w\|_1 + \frac{1}{N} \sum_{n=1}^{N} \ln(1 + \exp(-y_n w^T \Phi_4(x_n))$$

Then we know that

$$1 = \frac{\lambda}{N}, C = \frac{1}{N} \Rightarrow C = \frac{1}{\lambda}$$

The parameter `-c cost : set the parameter C (default 1)` should be set to $C = \frac{1}{\lambda}$.

We use the same code from `P12`, then update parameter `C` and `param` in the `main` function. The updated functions show below.

```
C = 1/(lamb)
param = parameter('-s 6 -c {} -e 0.000001 -q'.format(C))
```

No questions assigned to the following page.

```
NumbaDeprecationWarning: The 'nopython' keyword argument was not supplied to the 'numl
  def csr_to_problem_jit(l, x_val, x_ind, x_rowptr, prob_val, prob_ind, prob_rowptr):
Accuracy = 76.8% (384/500) (classification)
Accuracy = 83.6% (418/500) (classification)
Accuracy = 84.6% (423/500) (classification)
Accuracy = 68% (340/500) (classification)
Accuracy = 49.2% (246/500) (classification)
E_out:  [0.232, 0.164, 0.154, 0.32, 0.508]
min E_out:  0.154
Choice:  c
```

As a result, we should choose $[c]$ as our solution.

P19

We use the same code above, then update the `main` function. The updated functions show below.

```
def main(args):
    x_train, y_train = read_file(args, args['filename_train'])
    x_test, y_test = read_file(args, args['filename_test'])
    x_train = transformation(args, x_train)
    x_test = transformation(args, x_test)
    args['dimension'] = x_train.shape[1]

    model_list = list()
    E_out = list()
    for lamb in args['lambda']:
        C = 1/(lamb)
        prob = problem(y_train, x_train)
        param = parameter('-s 6 -c {} -e 0.000001 -q'.format(C))
        m = train(prob, param)
        model_list.append(np.array([m.w[i] for i in range(args['dimension']) if np.abs
        p_label, p_acc, p_val = predict(y_test, x_test, m)
        E_out.append(round(np.mean(y_test != p_label), 6))

    min_E_out = min(E_out)
    min_E_out_index = [i for i, v in enumerate(E_out) if v == min_E_out]
    print("|m.w[i] <= 10**(-6)|: ", len(model_list[min_E_out_index[0]]))
```

No questions assigned to the following page.

```
NumbaDeprecationWarning: The 'nopython' keyword argument was not supplied to the 'numb
  def csr_to_problem_jit(l, x_val, x_ind, x_rowptr, prob_val, prob_ind, prob_rowptr):
Accuracy = 76.8% (384/500) (classification)
Accuracy = 83.6% (418/500) (classification)
Accuracy = 84.6% (423/500) (classification)
Accuracy = 68% (340/500) (classification)
Accuracy = 49.2% (246/500) (classification)
|m.w[i] <= 10**(-6)|:  960
```

As a result, we should choose $[e]$ as our solution.

P20

We use the same code above, then update parameter `C` and `param` in the `main` function. The updated functions show below.

```
C = 1/(2*lamb)
param = parameter('-s 0 -c {} -e 0.000001 -q'.format(C))
```

```
NumbaDeprecationWarning: The 'nopython' keyword argument was not supplied to the 'numb
  def csr_to_problem_jit(l, x_val, x_ind, x_rowptr, prob_val, prob_ind, prob_rowptr):
Accuracy = 77.4% (387/500) (classification)
Accuracy = 82.2% (411/500) (classification)
Accuracy = 84.6% (423/500) (classification)
Accuracy = 85.8% (429/500) (classification)
Accuracy = 81.2% (406/500) (classification)
|m.w[i] <= 10**(-6)|:  1
```

As a result, we should choose $[a]$ as our solution.