HW0

Graded

Student

Chih-Hao Liao

Total Points

40 / 40 pts

Question 1

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 2

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 3

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 4

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 5

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 6

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 7

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 8

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 9

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 10

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 11

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

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Question 12

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

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Question 13

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

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Question 14

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

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Question 15

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

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Question 16

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

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Question 17

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

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Question 18

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 19

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 20

(no title) **2** / 2 pts

✔ **+ 2 pts** Correct

+ 0 pts Incorrect

Question 21

(no title) **0** / 0 pts

✔ **+ 0 pts** Correct

+ 0 pts Incorrect

Q1

2 Points

Let $C(N,K)=1$ for $K=0$ or $K=N$, and $C(N,K)=C(N-1,K)+C(N-1,K-1)$ for $N\geq 1$. What is the closed-form equation of $C(N,K)$ for $N\geq 1$ and $0\leq K\leq N$?

- (●) $C(N,K)=\frac{N!}{K!(N-K)!}$
- ( ) $C(N,K)=\sum_{k=0}^{K}\frac{N!}{k!(N-k)!}$
- ( ) $C(N,K)=\frac{K!(N-K)!}{K!}$
- ( ) $C(N,K)=\sum_{k=0}^{K}\frac{k!(N-k)!}{N!}$
- ( ) none of the other choices

Q2

2 Points

What is the probability of getting exactly 4 heads when flipping 10 fair coins? Choose the closest number.

- ( ) 0.0
- ( ) 0.1
- (●) 0.2
- ( ) 0.3
- ( ) 0.4

Q3

2 Points

If your friend flipped a fair coin three times, and then tells you that one of the tosses resulted in head, what is the probability that all three tosses resulted in heads?

- ○ $1/8$
- ○ $3/8$
- ○ $7/8$
- ◉ $1/7$
- ○ $1/3$

Q4

2 Points

A program selects a random integer x like this: a random bit is first generated uniformly. If the bit is 0, x is drawn uniformly from $\{0, 1, \ldots, 7\}$; otherwise, x is drawn uniformly from $\{0, -1, -2, -3\}$. If we get an x from the program with $|x| = 1$, what is the probability that x is negative?

- ○ $1/3$
- ○ $1/4$
- ○ $1/2$
- ○ $1/12$
- ◉ $2/3$

Q5

2 Points

For N random variables $x_1, x_2, \ldots, x_N$, let their mean be $\bar{x} = \frac{1}{N}\sum_{n=1}^{N} x_n$ and variance be $\sigma_x^2 = \frac{1}{N-1}\sum_{n=1}^{N}(x_n - \bar{x})^2$. Which of the following is provably the same as σ_x^2?

- ○ $\frac{1}{N}\sum_{n=1}^{N}(x_n^2 - \bar{x}^2)$
- ◉ $\frac{1}{N-1}\sum_{n=1}^{N}(x_n^2 - \bar{x}^2)$
- ○ $\frac{1}{N-1}\sum_{n=1}^{N}(\bar{x}^2 - x_n^2)$
- ○ $\frac{N}{N-1}(\bar{x}^2)$
- ○ none of the other choices

Q6

2 Points

For two events A and B, if their probability $P(A) = 0.3$ and $P(B) = 0.4$, what is the tightest possible range of $P(A \cup B)$?

- ○ $[0.3, 0.4]$
- ○ $[0, 0.4]$
- ○ $[0, 0.7]$
- ○ $[0.3, 1]$
- ◉ $[0.4, 0.7]$

Q7
2 Points

What is the rank of $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 1 & 1 & 2 \end{pmatrix}$?

- ○ 0
- ○ 1
- ◉ 2
- ○ 3
- ○ none of the other choices

Q8
2 Points

What is the diagonal on the inverse of $\begin{pmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{pmatrix}$?

- ○ $[3/4, 1/4, 1/8]$
- ○ $[1/4, 1/8, 3/4]$
- ○ $[1/4, 3/4, 1/8]$
- ◉ $[1/8, 3/4, 1/4]$
- ○ none of the other choices

Q9

2 Points

What is the largest eigenvalue of $\begin{pmatrix} 2023 & 1 & 1 \\ 2 & 2024 & 2 \\ -1 & -1 & 2021 \end{pmatrix}$?

- ○ 2020
- ○ 2021
- ○ 2022
- ○ 2023
- ◉ 2024

Q10

2 Points

For a real matrix M, let $\mathrm{M} = \mathrm{U}\Sigma\mathrm{V}^T$ be its singular value decomposition, with U and V being unitary matrices. Define $\mathrm{M}^\dagger = \mathrm{V}\Sigma^\dagger\mathrm{U}^T$, where $\Sigma^\dagger[j][i] = \frac{1}{\Sigma[i][j]}$ when $\Sigma[i][j]$ is nonzero, and 0 otherwise. Which of the following is always the same as $\mathrm{M}\mathrm{M}^\dagger\mathrm{M}$?

- ○ $\mathrm{M}\mathrm{M}^T\mathrm{M}$
- ○ $\mathrm{M}\mathrm{V}^T$
- ○ $\mathrm{U}^T\mathrm{M}$
- ○ $\mathrm{U}^T\mathrm{M}\mathrm{V}^T$
- ◉ M

Q11

2 Points

Which of the following matrix is not guaranteed to be positive semi-definite?

- ○ $\mathrm{Z}^T\mathrm{Z}$ for any real matrix Z
- ○ a real symmetric matrix S whose eigenvalues are all non-negative
- ○ an all-zero square matrix
- ◉ a real symmetric matrix whose entries are all positive
- ○ none of the other choices

Q12

2 Points

Consider a fixed $\mathbf{x} \in \mathbb{R}^d$ and some varying $\mathbf{u} \in \mathbb{R}^d$ with $||\mathbf{u}|| = 1$. Which of the following is the smallest value of $\mathbf{u}^T\mathbf{x}$?

- ○ 0
- ○ $-\infty$
- ◉ $-||\mathbf{x}||$
- ○ $-||\mathbf{u}||$
- ○ none of the other choices

Q13

2 Points

Consider two parallel hyperplanes in R^d:
$H_1 : \mathbf{w}^T\mathbf{x} = +3$,
$H_2 : \mathbf{w}^T\mathbf{x} = -2$
What is the distance between H_1 and H_2?

- ○ 5
- ◉ $5/||\mathbf{w}||$
- ○ $5/||\mathbf{w}||^2$
- ○ $5 \cdot ||\mathbf{w}||$
- ○ none of the other choices

Q14

2 Points

Let $g(x,y) = e^x + e^{2y} + e^{3xy^2}$. What is $\dfrac{\partial g(x,y)}{\partial y}$?

- ○ $e^x + 2e^{2y} + 6xye^{3xy^2}$
- ◉ $2e^{2y} + 6xye^{3xy^2}$
- ○ $2e^{2y} + 3xye^{3xy^2}$
- ○ $2e^y + 6xye^y$
- ○ none of the other choices

Q15

2 Points

Let $f(x,y) = xy$, $x(u,v) = \cos(u+v)$, $y(u,v) = \sin(u-v)$. What is $\dfrac{\partial f}{\partial v}$?

- ◉ $-\sin(u+v)\sin(u-v) - \cos(u+v)\cos(u-v)$
- ○ $+\sin(u+v)\sin(u-v) - \cos(u+v)\cos(u-v)$
- ○ $-\sin(u+v)\sin(u-v) + \cos(u+v)\cos(u-v)$
- ○ $+\sin(u+v)\sin(u-v) + \cos(u+v)\cos(u-v)$
- ○ none of the other choices

Q16

2 Points

Let $E(u,v) = (ue^v - 2ve^{-u})^2$. Calculate the gradient $\nabla E(u,v) = \begin{pmatrix} \frac{\partial E}{\partial u} \\ \frac{\partial E}{\partial v} \end{pmatrix}$ at $[u,v] = [1,1]$.

- ○ $[-13.70, -7.86]$
- ○ $[-13.70, +7.86]$
- ○ $[+13.70, -7.86]$
- ◉ $[+13.70, +7.86]$
- ○ $[1, 1]$

Q17
2 Points

For some given $A > 0, B > 0$, what is the optimal α that solves

$\min_\alpha Ae^{\alpha} + Be^{-2\alpha}$?

- (selected) $\frac{1}{3}\ln(\frac{2B}{A})$
- $\frac{1}{3}\ln(\frac{A}{2B})$
- $\ln(\frac{2B}{A})$
- $\ln(\frac{A}{2B})$
- none of the other choices

Q18
2 Points

Let $\mathbf{w}$ be a vector in $\mathbb{R}^d$ and $E(\mathbf{w}) = \frac{1}{2}\mathbf{w}^T \mathrm{A}\mathbf{w} + \mathbf{b}^T\mathbf{w}$ for some symmetric matrix A and vector $\mathbf{b}$. What is the gradient $\nabla E(\mathbf{w})$?

- $\mathbf{w}^T\mathrm{A}\mathbf{w} + \mathbf{w}^T\mathbf{b}$
- $\mathbf{w}^T\mathrm{A}\mathbf{w} - \mathbf{w}^T\mathbf{b}$
- (selected) $\mathrm{A}\mathbf{w} + \mathbf{b}$
- $\mathrm{A}\mathbf{w} - \mathbf{b}$
- none of the other choices

Q19

2 Points

Let $\mathbf{w}$ be a vector in $\mathbb{R}^d$ and $E(\mathbf{w}) = \frac{1}{2}\mathbf{w}^T \mathrm{A}\mathbf{w} + \mathbf{b}^T\mathbf{w}$ for some symmetric ***and strictly positive definite*** matrix A and vector $\mathbf{b}$. What is the optimal $\mathbf{w}$ that minimizes $E(\mathbf{w})$?

- ○ $+\mathrm{A}^{-1}\mathbf{b}$
- ◉ $-\mathrm{A}^{-1}\mathbf{b}$
- ○ $-\mathrm{A}^{-1}\mathbf{1} + \mathbf{b}$, where $\mathbf{1}$ is a vector of all 1's
- ○ $+\mathrm{A}^{-1}\mathbf{1} - \mathbf{b}$
- ○ none of the other choices

Q20

2 Points

Solve

$$\min_{w_1, w_2, w_3} \frac{1}{2}(w_1^2 + 2w_2^2 + 3w_3^2)$$

$$\text{subject to } w_1 + w_2 + w_3 = 11.$$

What is the optimal w_1? (Hint: refresh your memory on ''Lagrange multipliers'')

- ○ 0
- ○ 1
- ○ 2
- ○ 3
- ◉ 6

Q21
0 Points

How many gold medals do you want to use for this homework (every gold medal extends the deadline of this homework by 12 hours, and you have four gold medals in total this semester)

- (●) 0
- ( ) 1
- ( ) 2
- ( ) 3
- ( ) 4