

## Week 3 – Problem Set

9/10 分 (90%)

测验, 10 个问题

✓ 恭喜! 您通过了!

下一项



1 / 1 分

1.

Suppose a MAC system  $(S, V)$  is used to protect files in a file system

by appending a MAC tag to each file. The MAC signing algorithm  $S$

is applied to the file contents and nothing else. What tampering attacks

are not prevented by this system?



Changing the last modification time of a file.



正确

The MAC signing algorithm is only applied to the file contents and

does not protect the file meta data.



Replacing the contents of a file with the concatenation of two files  
on the file system.



Changing the first byte of the file contents.



Replacing the tag and contents of one file with the tag and contents of a file  
from another computer protected by the same MAC system, but a different key.



1 / 1 分

2.

Let  $(S, V)$  be a secure MAC defined over  $(K, M, T)$  where  $M = \{0, 1\}^n$  and  $T = \{0, 1\}^{128}$ . That is, the key space is  $K$ , message space is  $\{0, 1\}^n$ , and tag space is  $\{0, 1\}^{128}$ .

Which of the following is a secure MAC: (as usual, we use  $\parallel$  to denote string concatenation)



$$S'(k, m) = \begin{cases} S(k, 1^n) & \text{if } m = 0^n \\ S(k, m) & \text{otherwise} \end{cases} \quad \text{and}$$

$$V'(k, m) = \begin{cases} V(k, 1^n, t) & \text{if } m = 0^n \\ V(k, m, t) & \text{otherwise} \end{cases}$$


未选择的是正确的



$$S'((k_1, k_2), m) = (S(k_1, m), S(k_2, m)) \quad \text{and}$$

$$V'((k_1, k_2), m, (t_1, t_2)) = [V(k_1, m, t_1) \text{ and } V(k_2, m, t_2)]$$

## Week 3 – Problem Set

(i.e.,  $V'((k_1, k_2), m, (t_1, t_2))$  outputs "1" if both  $t_1$  and  $t_2$  are valid tags)

9/10 分 (90%)

测验, 10 个问题

正确

a forger for  $(S', V')$  gives a forger for  $(S, V)$ .

☐  $S'(k, m) = [t \leftarrow S(k, m), \text{ output } (t, t)]$  and

$$V'(k, m, (t_1, t_2)) = \begin{cases} V(k, m, t_1) & \text{if } t_1 = t_2 \\ "0" & \text{otherwise} \end{cases}$$

(i.e.,  $V'(k, m, (t_1, t_2))$  only outputs "1"

if  $t_1$  and  $t_2$  are equal and valid)

正确

a forger for  $(S', V')$  gives a forger for  $(S, V)$ .

☐  $S'(k, m) = S(k, m)$  and

$$V'(k, m, t) = [V(k, m, t) \text{ or } V(k, m \oplus 1^n, t)]$$

(i.e.,  $V'(k, m, t)$  outputs "1" if  $t$  is a valid

tag for either  $m$  or  $m \oplus 1^n$ )

未选择的是正确的

☐  $S'(k, m) = S(k, m \oplus m)$  and

$$V'(k, m, t) = V(k, m \oplus m, t)$$

未选择的是正确的

☐  $S'(k, m) = S(k, m \oplus 1^n)$  and

$$V'(k, m, t) = V(k, m \oplus 1^n, t).$$

正确

a forger for  $(S', V')$  gives a forger for  $(S, V)$ .



1 / 1 分

3.

Recall that the ECBC-MAC uses a fixed IV (in the lecture we simply set the IV to 0). Suppose instead we chose a random IV for every message being signed and include the IV in the tag.

## Week 3 – Problem Set

9/10 分 (90%)

测验, 10 个问题

In other words,  $S(k, m) := (r, \text{ECBC}_r(k, m))$

where  $\text{ECBC}_r(k, m)$  refers to the ECBC function using  $r$  as the IV. The verification algorithm  $V$  given key  $k$ , message  $m$ , and tag  $(r, t)$  outputs "1" if  $t = \text{ECBC}_r(k, m)$  and outputs "0" otherwise.

The resulting MAC system is insecure.

An attacker can query for the tag of the 1-block message  $m$  and obtain the tag  $(r, t)$ . He can then generate the following existential forgery: (we assume that the underlying block cipher operates on  $n$ -bit blocks)

☒ The tag  $(r \oplus m, t)$  is a valid tag for the 1-block message  $0^n$ .



正确

The CBC chain initiated with the IV  $r \oplus m$  and applied to the message  $0^n$  will produce exactly the same output as the CBC chain initiated with the IV  $r$  and applied to the message  $m$ . Therefore, the tag  $(r \oplus m, t)$  is a valid existential forgery for the message 0.

☐ The tag  $(r, t \oplus r)$  is a valid tag for the 1-block message  $0^n$ .

☐ The tag  $(m \oplus t, r)$  is a valid tag for the 1-block message  $0^n$ .

☐ The tag  $(m \oplus t, t)$  is a valid tag for the 1-block message  $0^n$ .



1 / 1 分

4.

Suppose Alice is broadcasting packets to 6 recipients

## Week 3 - Problem Set

9/10 分 (90%)

测验, 10 个问题

$B_1, \dots, B_6$  may not be important but integrity is.

In other words, each of  $B_1, \dots, B_6$  should be assured that the

packets he is receiving were sent by Alice.

Alice decides to use a MAC. Suppose Alice and  $B_1, \dots, B_6$  all

share a secret key  $k$ . Alice computes a tag for every packet she

sends using key  $k$ . Each user  $B_i$  verifies the tag when

receiving the packet and drops the packet if the tag is invalid.

Alice notices that this scheme is insecure because user  $B_1$  can

use the key  $k$  to send packets with a valid tag to

users  $B_2, \dots, B_6$  and they will all be fooled into thinking

that these packets are from Alice.

Instead, Alice sets up a set of 4 secret keys  $S = \{k_1, \dots, k_4\}$ .

She gives each user  $B_i$  some subset  $S_i \subseteq S$

of the keys. When Alice transmits a packet she appends 4 tags to it

by computing the tag with each of her 4 keys. When user  $B_i$  receives

a packet he accepts it as valid only if all tags corresponding

to his keys in  $S_i$  are valid. For example, if user  $B_1$  is given keys  $\{k_1, k_2\}$  he will accept an incoming packet only if the first and second tags are valid. Note that  $B_1$  cannot validate the 3rd and 4th tags because he does not have  $k_3$  or  $k_4$ .

How should Alice assign keys to the 6 users so that no single user

can forge packets on behalf of Alice and fool some other user?

☐  $S_1 = \{k_1, k_2\}, S_2 = \{k_2, k_3\}, S_3 = \{k_3, k_4\}, S_4 = \{k_1, k_3\}, S_5 = \{k_1, k_2\}, S_6 = \{k_1, k_4\}$

未选择的是正确的

☐  $S_1 = \{k_1, k_2\}, S_2 = \{k_1, k_3\}, S_3 = \{k_1, k_4\}, S_4 = \{k_2, k_3, k_4\}, S_5 = \{k_2, k_3\}, S_6 = \{k_3, k_4\}$

未选择的是正确的

☐  $S_1 = \{k_1, k_2\}, S_2 = \{k_1, k_3, k_4\}, S_3 = \{k_1, k_4\}, S_4 = \{k_2, k_3\}, S_5 = \{k_2, k_3, k_4\}, S_6 = \{k_3, k_4\}$

未选择的是正确的

☐  $S_1 = \{k_1, k_2\}, S_2 = \{k_1, k_3\}, S_3 = \{k_1, k_4\}, S_4 = \{k_2, k_3\}, S_5 = \{k_2, k_4\}, S_6 = \{k_3, k_4\}$

正确

Every user can only generate tags with the two keys he has.

Since no set  $S_i$  is contained in another set  $S_j$ , no user  $i$

## Week 3 – Problem Set

cannot fool a user  $j$  into accepting a message sent by  $i$ .

9/10 分 (90%)

测验, 10 个问题



1 / 1 分

5.

Consider the encrypted CBC MAC built from AES. Suppose we

compute the tag for a long message  $m$  comprising of  $n$  AES blocks.

Let  $m'$  be the  $n$ -block message obtained from  $m$  by flipping the

last bit of  $m$  (i.e. if the last bit of  $m$  is  $b$  then the last bit

of  $m'$  is  $b \oplus 1$ ). How many calls to AES would it take

to compute the tag for  $m'$  from the tag for  $m$  and the MAC key? (in this question please ignore message padding and simply assume that the message length is always a multiple of the AES block size)

☐  $n$

☐ 5

☒ 4



正确

You would decrypt the final CBC MAC encryption step done using  $k_2$ ,

the decrypt the last CBC MAC encryption step done using  $k_1$ ,

flip the last bit of the result, and re-apply the two encryptions.

☐ 6



0 / 1 分

6.

Let  $H : M \rightarrow T$  be a collision resistant hash function.

Which of the following is collision resistant:

(as usual, we use  $\parallel$  to denote string concatenation)

☒  $H'(m) = H(m) \parallel H(0)$



正确

a collision finder for  $H'$  gives a collision finder for  $H$ .

☐  $H'(m) = H(0)$



未选择的是正确的

☐  $H'(m) = H(m)[0, \dots, 31]$

## Week 3 – Problem Set

(We compute the first 32 bits of the hash)

9/10 分 (90%)

测验, 10 个问题

这个选项的答案不正确

This construction is not collision resistant

because an attacker can find a collision in time  $2^{16}$  using the birthday paradox.

☐  $H'(m) = H(m) \oplus H(m \oplus 1^{|m|})$

(where  $m \oplus 1^{|m|}$  is the complement of  $m$ )

未选择的是正确的

☐  $H'(m) = H(m \| m)$

正确

a collision finder for  $H'$  gives a collision finder for  $H$ .

☐  $H'(m) = H(m) \oplus H(m)$

未选择的是正确的

☐  $H'(m) = H(H(m))$

正确

a collision finder for  $H'$  gives a collision finder for  $H$ .



1 / 1 分

7.

Suppose  $H_1$  and  $H_2$  are collision resistant

hash functions mapping inputs in a set  $M$  to  $\{0, 1\}^{256}$ .

Our goal is to show that the function  $H_2(H_1(m))$  is also

collision resistant. We prove the contra-positive:

suppose  $H_2(H_1(\cdot))$  is not collision resistant, that is, we are

given  $x \neq y$  such that  $H_2(H_1(x)) = H_2(H_1(y))$ .

We build a collision for either  $H_1$  or for  $H_2$ .

This will prove that if  $H_1$  and  $H_2$  are collision resistant

then so is  $H_2(H_1(\cdot))$ . Which of the following must be true:



Either  $H_2(x), H_2(y)$  are a collision for  $H_1$  or

$x, y$  are a collision for  $H_2$ .

## Week 3 - Problem Set

测验, 10 个问题

9/10 分 (90%)

$x, y$  are a collision for  $H_2$ .

☒ Either  $x, y$  are a collision for  $H_1$  or

$H_1(x), H_1(y)$  are a collision for  $H_2$ .

正确

If  $H_2(H_1(x)) = H_2(H_1(y))$  then

either  $H_1(x) = H_1(y)$  and  $x \neq y$ , thereby giving us

a collision on  $H_1$ . Or  $H_1(x) \neq H_1(y)$  but

$H_2(H_1(x)) = H_2(H_1(y))$  giving us a collision on  $H_2$ .

Either way we obtain a collision on  $H_1$  or  $H_2$  as required.

☐ Either  $x, y$  are a collision for  $H_2$  or

$H_1(x), H_1(y)$  are a collision for  $H_1$ .



1 / 1 分

8.

In this question you are asked to find a collision for the compression function:

$$f_1(x, y) = \text{AES}(y, x) \oplus y,$$

where  $\text{AES}(x, y)$  is the AES-128 encryption of  $y$  under key  $x$ .

Your goal is to find two distinct pairs  $(x_1, y_1)$  and  $(x_2, y_2)$  such that  $f_1(x_1, y_1) = f_1(x_2, y_2)$ .

Which of the following methods finds the required  $(x_1, y_1)$  and  $(x_2, y_2)$ ?

☐ Choose  $x_1, y_1, y_2$  arbitrarily (with  $y_1 \neq y_2$ ) and let  $v := \text{AES}(y_1, x_1)$ .

$$\text{Set } x_2 = \text{AES}^{-1}(y_2, v \oplus y_2)$$

☐ Choose  $x_1, y_1, x_2$  arbitrarily (with  $x_1 \neq x_2$ ) and let  $v := \text{AES}(y_1, x_1)$ .

$$\text{Set } y_2 = \text{AES}^{-1}(x_2, v \oplus y_1 \oplus x_2)$$

☐ Choose  $x_1, y_1, y_2$  arbitrarily (with  $y_1 \neq y_2$ ) and let  $v := \text{AES}(y_1, x_1)$ .

$$\text{Set } x_2 = \text{AES}^{-1}(y_2, v \oplus y_1)$$

☒ Choose  $x_1, y_1, y_2$  arbitrarily (with  $y_1 \neq y_2$ ) and let  $v := \text{AES}(y_1, x_1)$ .

$$\text{Set } x_2 = \text{AES}^{-1}(y_2, v \oplus y_1 \oplus y_2)$$

正确

You got it !



1 / 1 分

## Week 3 - Problem Set

9/10 分 (90%)

测验, 10 个问题 Repeat the previous question, but now to find a collision for the compression function

$$f_2(x, y) = \text{AES}(x, x) \oplus y.$$

Which of the following methods finds the required  $(x_1, y_1)$  and  $(x_2, y_2)$ ?



Choose  $x_1, x_2, y_1$  arbitrarily (with  $x_1 \neq x_2$ ) and set

$$y_2 = y_1 \oplus \text{AES}(x_1, x_1) \oplus \text{AES}(x_2, x_2)$$



正确

Awesome!



Choose  $x_1, x_2, y_1$  arbitrarily (with  $x_1 \neq x_2$ ) and set

$$y_2 = \text{AES}(x_1, x_1) \oplus \text{AES}(x_2, x_2)$$



Choose  $x_1, x_2, y_1$  arbitrarily (with  $x_1 \neq x_2$ ) and set

$$y_2 = y_1 \oplus \text{AES}(x_1, x_1)$$



Choose  $x_1, x_2, y_1$  arbitrarily (with  $x_1 \neq x_2$ ) and set

$$y_2 = y_1 \oplus x_1 \oplus \text{AES}(x_2, x_2)$$



1 / 1 分

10.

Let  $H : M \rightarrow T$  be a random hash function where  $|M| \gg |T|$  (i.e. the size of  $M$  is much larger than the size of  $T$ ).

In lecture we showed

that finding a collision on  $H$  can be done with  $O(|T|^{1/2})$

random samples of  $H$ . How many random samples would it take

until we obtain a three way collision, namely distinct strings  $x, y, z$

in  $M$  such that  $H(x) = H(y) = H(z)$ ?



$$O(|T|^{2/3})$$



正确

An informal argument for this is as follows: suppose we

collect  $n$  random samples. The number of triples among the  $n$

samples is  $n$  choose 3 which is  $O(n^3)$ . For a particular

triple  $x, y, z$  to be a 3-way collision we need  $H(x) = H(y)$

and  $H(x) = H(z)$ . Since each one of these two events happens

with probability  $1/|T|$  (assuming  $H$  behaves like a random

function) the probability that a particular triple is a 3-way



collision is  $O(1/|T|^2)$ . Using the union bound, the probability

## Week 3 – Problem Set

that some triple is a 3-way collision is  $O(n^3/|T|^2)$  and since

9/10 分 (90%)



测验, 10 个问题

we want this probability to be close to 1, the bound on  $n$

follows.

- ☐  $O(|T|^{3/4})$
- ☐  $O(|T|)$
- ☐  $O(|T|^{1/4})$

