# Week 1 - Problem Set

10/10 points (100%)

Quiz, 10 questions

<b>~</b>	恭喜! :	您通过了!	下一项
	<b>~</b>	1 / 1 points	
	1.		
	Suppo	compression is often used in data storage and transmissionse you want to use data compression in conjunction with otion. Does it make more sense to:	'n.
		The order does not matter neither one will compress data.	the
	0	Compress then encrypt.	
	•	rect nertexts tend to look like random strings and therefore the proportunity for compression is prior to encryption.	e
		The order does not matter either one is fine.	
		Encrypt then compress.	
	<b>~</b>	1 / 1 points	
	2. Let $G$	$: \{0,1\}^s  ightarrow \{0,1\}^n $ be a secure PRG. Which of the follow	wing is a

 $G'(k) = \operatorname{reverse}(G(k))$  where reverse(x) reverses the string x so that the first bit of x is the last bit of reverse(x), the second bit of x is the second to last bit of reverse(x), and so on.

# Correct

a distinguisher for G' gives a distinguisher for G.

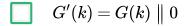
secure PRG (there is more than one correct answer):

Week 1 - Problem Set (here denotes concatenation)

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**Un-selected is correct** 



(here | denotes concatenation)

**Un-selected is correct** 

(here | denotes concatenation)

## Correct

a distinguisher for G' gives a distinguisher for G.

# Correct

a distinguisher for G' gives a distinguisher for G.

$$oxed{ \ \ } G'(k)=G(0)$$

**Un-selected** is correct



1/1 points

3.

Let  $G:K \to \{0,1\}^n$  be a secure PRG.

Week 1 – Problems  $\{k_1,k_2\}=G(k_1) \land G(k_2) \text{ where } \Lambda \text{ is the bit-wise AND}$  function. Consider the following statistical test A on  $\{0,1\}^n$ :

A(x) outputs LSB(x), the least significant bit of x.

What is  $Adv_{\mathrm{PRG}}[A,G']$  ?

You may assume that  $\mathrm{LSB}(G(k))$  is 0 for exactly half the seeds k in K.

Note: Please enter the advantage as a decimal between 0 and 1 with a leading 0. If the advantage is 3/4, you should enter it as 0.75

0.25

## **Correct Response**

for a random string x we have  $Pr[A(x)=1]=1/2\,$  but for a pseudorandom string  $G'(k_1,k_2)$  we have  $Pr_{k_1,k_2}[A(G'(k_1,k_2))=1]=1/4\,.$ 



1/1 points

4.

Let (E,D) be a (one-time) semantically secure cipher with key space  $K=\{0,1\}^\ell$ . A bank wishes to split a decryption key  $k\in\{0,1\}^\ell$  into two pieces  $p_1$  and  $p_2$  so that both are needed for decryption. The piece  $p_1$  can be given to one executive and  $p_2$  to another so that both must contribute their pieces for decryption to proceed.

The bank generates random  $k_1$  in  $\left\{0,1\right\}^\ell$  and sets  $k_1' \leftarrow k \oplus k_1$ . Note that  $k_1 \oplus k_1' = k$ . The bank can give  $k_1$  to one executive and  $k_1'$  to another. Both must be present for decryption to proceed since, by itself, each piece contains no information about the secret key k (note that each piece is a one-time pad encryption of k).

Now, suppose the bank wants to split k into three pieces  $p_1,p_2,p_3$  so that any two of the pieces enable decryption using k. This ensures that even if one executive is out sick, decryption can still succeed. To do so the bank generates two random pairs  $(k_1,k_1')$  and  $(k_2,k_2')$  as in the previous paragraph so that  $k_1\oplus k_1'=k_2\oplus k_2'=k$ .

How should the bank assign pieces so that any two pieces enable decryption using k, but no single piece can decrypt?

Week 1 – Problem Set 
$$p_1=(k_1,k_2), \quad p_2=(k_2,k_2'), \quad p_3=(k_2')$$
  $p_1=(k_1,k_2), \quad p_2=(k_1',k_2), \quad p_3=(k_2')$  Quiz, 10 questions

10/10 points (100%)

### Correct

executives 1 and 2 can decrypt using  $k_1, k_1'$  ,

executives 1 and 3 can decrypt using  $k_2, k_2^\prime$  , and

executives 2 and 3 can decrypt using  $k_2, k_2'$ . Moreover, a single

executive has no information about \$k\$.

$$igcap_1 = (k_1, k_2), \quad p_2 = (k_1'), \quad p_3 = (k_2')$$

$$igcap_1 = (k_1, k_2), \quad p_2 = (k_1, k_2), \quad p_3 = (k_2')$$



1/1 points

5. Let 
$$M=C=K=\{0,1,2,\ldots,255\}$$

and consider the following cipher defined over (K, M, C):

$$E(k,m) = m + k \pmod{256}$$
 ;  $D(k,c) = c - k \pmod{256}$ .

Does this cipher have perfect secrecy?

- No, there is a simple attack on this cipher.
- No, only the One Time Pad has perfect secrecy.
- Yes.

#### Correct

as with the one-time pad, there is exactly one key mapping a given message m to a given ciphertext c.



1 / 1 points

6

Let (E,D) be a (one-time) semantically secure cipher where the

Week 1 - Problems Setciphertext space is  $\{0,1\}^n$ . Which of the following

10/10 points (100%)

Quiz, 10 questions

encryption schemes are (one-time) semantically secure?

$$igcap E'(k,m) = \operatorname{reverse}(E(k,m))$$

## Correct

an attack on E' gives an attack on E.

$$igcap E'(k,m)=0 \parallel E(k,m)$$
 (i.e. prepend 0 to the ciphertext)

#### Correct

an attack on E' gives an attack on E.

**Un-selected is correct** 

$$igcup E'(k,m)=E(0^n,m)$$

**Un-selected** is correct

$$lacksquare E'(\ (k,k'),\ m) = E(k,m) \parallel E(k',m)$$

#### Correct

an attack on  $E^\prime$  gives an attack on E.

$$oxed{igsquare} E'(k,m) = E(k,m) \parallel \mathrm{LSB}(m)$$

**Un-selected** is correct



1/1 points

7.

Suppose you are told that the one time pad encryption of the message "attack at dawn" is *09e1c5f70a65ac519458e7e53f36* 

# Week 1 - Problem Set

10/10 points (100%)

Quiz, 10 questions

(the plaintext letters are encoded as 8-bit ASCII and the given ciphertext is written in <u>hex</u>). What would be the one time pad encryption of the message "attack at dusk" under the same OTP key?

09e1c5f70a65ac519458e7F13B33

**Correct Response** 



1/1 points

8.

The movie industry wants to protect digital content distributed on

DVD's. We develop a variant of a method used to protect Blu-ray disks called <u>AACS</u>.

Suppose there are at most a total of n DVD players in the world (e.g.  $n=2^{32}$ ). We view these n players as the leaves of a binary tree of height  $\log_2 n$ . Each node in this binary tree contains an AES key  $k_i$ . These keys are kept secret from consumers and are fixed for all time. At manufacturing time each DVD player is assigned a serial number  $i \in [0,n-1]$ . Consider the set of nodes  $S_i$  along the path from the root to leaf number i in the binary tree. The manufacturer of the DVD player embeds in player number i the keys associated with the nodes in the set  $S_i$ . A DVD movie m is encrypted as

$$E(k_{\text{root}},k)||E(k,m)||$$

where k is a random AES key called a content-key and

 $k_{
m root}$  is the key

associated with the root of the tree. Since all DVD players have the

key  $k_{
m root}$  all players can decrypt the movie m. We

Week 1 –  $\Pr{\text{perer to}} E(k_{\text{root}}, k)$  as the header and E(k, m) as the

10/10 points (100%)

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body. In what follows the DVD header may contain multiple ciphertexts where each ciphertext is the encryption of the content-key k under some key  $k_i$  in the binary tree.

Suppose the keys embedded in DVD player number r are exposed

by hackers and published on the Internet. In this problem we show that when the movie industry distributes a new

DVD movie, they can encrypt the contents of the DVD using a slightly larger header (containing about  $\log_2 n$  keys) so that all DVD

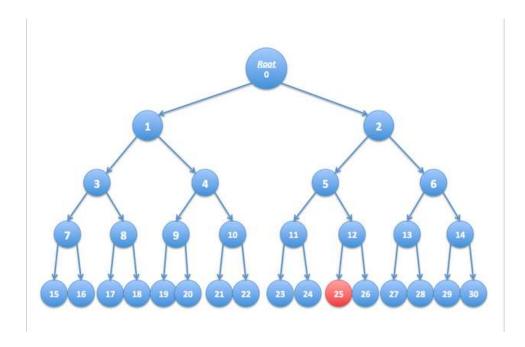
players, except for player number r, can decrypt the movie. In

effect, the movie industry disables player number r without

affecting other players.

As shown below, consider a tree with n=16 leaves. Suppose the leaf node labeled 25 corresponds to an exposed DVD player key. Check the set of keys below under which to encrypt the key k so that *every player* other

than player 25 can decrypt the DVD. Only four keys are needed.



<b>Week 1 – Pro</b> Quiz, 10 questions	Set Set $k$ under any key on the path from the root to node 25. Therefore 26 can only decrypt if you encrypt $k$ under key $k_{26}$ .	10/10 points (100%)
	Un-selected is correct	
	Un-selected is correct	
	Correct You cannot encrypt $k$ under key 5, but 11's children must be able to decrypt $k$ .	
	Correct You cannot encrypt $k$ under 2, but 6's children must be able to decrypt $k$ .	
	Correct You cannot encrypt $k$ under the root, but 1's children must be able to decrypt $k$ .	
	4	

https://www.coursera.org/learn/crypto/exam/6xB4d/week-1-problem-set

**Un-selected is correct** 

13

# Week 1 - Problem Set

**Un-selected is correct** 

10/10 points (100%)

Quiz, 10 questions



9.

Continuing with the previous question, if there are n DVD players, what is the number of keys under which the content key k must be encrypted if exactly one DVD player's key needs to be revoked?

- $\begin{array}{ccc} & 2 & \\ & & n-1 \end{array}$
- $\sqrt{n}$
- n/2
- $\log_2 n$

# Correct

That's right. The key will need to be encrypted under one key for each node on the path from the root to the revoked leaf. There are  $\log_2 n$  nodes on the path.



1/1 points

10.

Continuing with question 8, suppose the leaf nodes labeled 16, 18, and 25 correspond to exposed DVD player keys. Check the smallest set of keys under which to encrypt the key k so that every player other than players 16,18,25 can decrypt the DVD. Only six keys are needed.



4

### Correct

Yes, this will let players 19-22 decrypt.



6

### Correct

# Week 1 - Problemni Switt let players 27-30 decrypt.

10/10 points (100%)

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11

# Correct

Yes, this will let players 23,24 decrypt.

15

### Correct

Yes, this will let player 15 decrypt.

17

# Correct

Yes, this will let player 17 decrypt.

26

### Correct

Yes, this will let player 26 decrypt.

2

**Un-selected** is correct

5

**Un-selected is correct** 

10

**Un-selected is correct** 

27

**Un-selected** is correct

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10/10 points (100%)

Quiz, 10 questions





