SRM Institute of Science and Technology

DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB101J - Electromagnetic Theory, Quantum Mechanics, Waves and Optics
Module-IV (Waves and Optics) Lecture-3

Fraunhofer Single Slit and Double Slit Diffraction

Fraunhofer Single Slit Diffraction

To obtain a Fraunhoffer diffraction pattern, the incident wave front must be plane and the diffracted light is collected on the screen with the help of a lens. Thus, the source of light should either be at a large distance from the slit or a collimating lens must be used.

18.2. FRAUNHOFFER DIFFRACTION AT A SINGLE SLIT

In Fig. 18.1 S is a narrow slit perpendicular to the plane of the paper and illuminated by monochromatic light. L_1 is the collimating lens and AB is a slit of width *a*. XY is the incident spherical wave front. The light passing through the slit AB is incident on the lens L_2 and the final refracted beam is observed on the screen MN. The screen is perpendicular to the plane of the paper. The line SP is perpendicular to the screen. L_1 and L_2 are achromatic lenses.

A plane wave front is incident on the slit AB and each point on this wave front is a source of secondary disturbance. The secondary waves traveling in the direction parallel to OP *viz*. AQ and BV come to focus at P and a bright central image is observed. The secondary waves from points equidistant from O and situated in the upper and lower halves OA and OB of the wave front travel the same distance in reaching P and hence the path difference is zero. The secondary waves reinforce one another and P will be a point of maximum intensity.

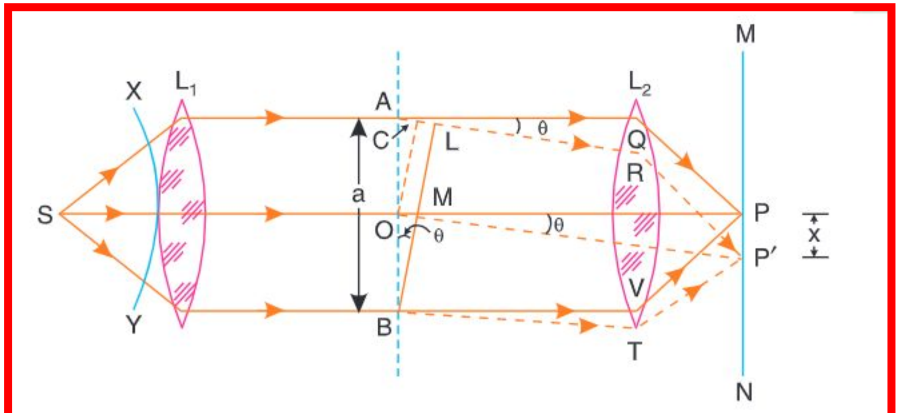


Fig. 18.1

Fraunhofer Single Slit Diffraction

Now, consider the secondary waves traveling in the direction AR, inclined at an angle è to the direction OP. All the secondary waves traveling in this direction reach the point P′ on the screen. The point P′ will be of maximum or minimum intensity depending on the path difference between the secondary waves originating from the corresponding points of the wave front. Let us draw OC and BL perpendicular to AR.

Then, in ΔABL

$$\sin\theta = \frac{AL}{AB} = \frac{AL}{a}$$

or

$$AL = a\sin\theta \tag{18.1}$$

where a is the width of the slit and AL is the path difference between the secondary waves originating from A and B. If this path difference is equal to λ the wavelength of the light used, then P′ will be a point of minimum intensity. The whole wave front can be considered to be of two halves OA and OB and if the path difference between the secondary waves from A and B is λ, then the path difference between the secondary waves from A and O will be $\lambda/2$. Similarly, for every point in the upper half OA, there is a corresponding point in the lower half OB, and the path difference between the secondary waves from these points is $\lambda/2$ .Thus, destructive interference takes place and the point P′ will be of minimum intensity. If the direction of the secondary waves is such that $AL = 2\lambda$, then also the point where they meet the screen will be of minimum intensity. This is so because the secondary waves from the corresponding points of the lower half differ in path by $\lambda/2$. And this again gives the position of minimum intensity. In general,

$$a\sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{n\lambda}{a} \tag{18.2}$$

Fraunhofer Single Slit Diffraction

where θ_n gives the direction of the n^{th} minimum. Here n is an integer. If, however, the path difference is odd multiples of $\lambda / 2$, the directions of the secondary maxima can be obtained. In this case,

$$a \sin \theta_n = (2n + 1) \lambda / 2$$

$$\sin \theta_n = \frac{(2n+1)\lambda}{2a} \qquad (18.3)$$

where $n = 1,2,3,\ldots$

Thus, the diffraction pattern due to a single slit consists of a central bright maximum at P followed by secondary maxima and minima on both the sides, as shown in Fig. 18.2. P corresponds to the position of the central bright maximum and the points on the screen for which the path difference between the points A and B is λ, 2λ etc correspond to the position of secondary minima. The secondary maxima are of much less intensity. The intensity falls off rapidly from the point P outwards.

If the lens L_2 is very near the slit or the screen is far away from the lens L_2, then,

$$\sin \theta = \frac{x}{f} \qquad (18.4)$$

where f is the focal length of the lens L_2.

But

$$\sin \theta = \frac{\lambda}{a} \qquad (18.5)$$

$$\therefore \quad \frac{x}{f} = \frac{\lambda}{a} \quad \text{or} \quad x = \frac{f\lambda}{a}$$

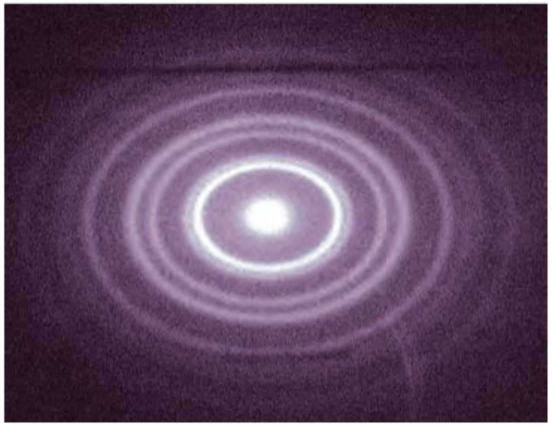
where x is the distance of the secondary minimum from the point P.

Thus, the width of the central maximum $W = 2x$

or

$$W = \frac{2f\lambda}{a} \qquad (18.6)$$

Fraunhofer Single Slit Diffraction



Diffraction pattern.

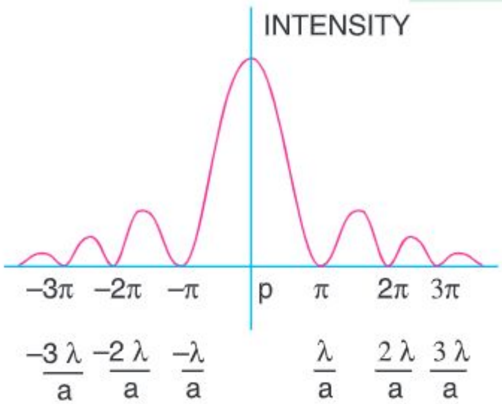
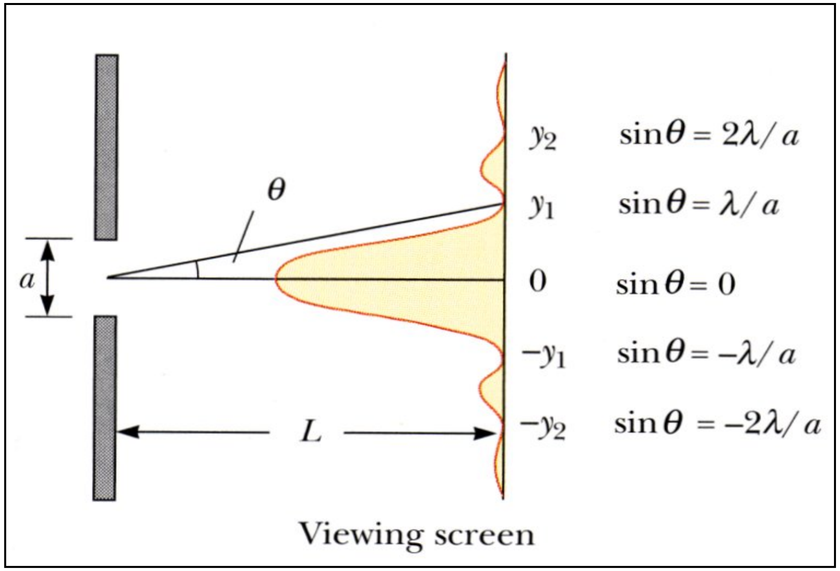


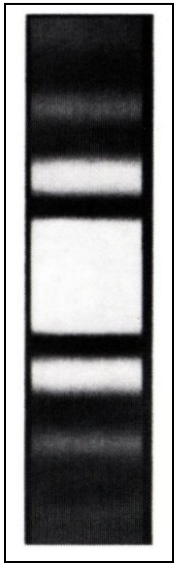
Fig. 18.2

The width of the central maximum is proportional to the wavelength of the light. With red light (longer wavelength), the width of the central maximum is more than with violet light (shorter wavelength). With a narrow slit, the width of central maximum is more. The diffraction pattern consists of alternate bright and dark bands with monochromatic light. With white light, the central maximum is white and the rest of the diffraction bands are coloured. From equation (18.5) if the width a of the slit is large, $\sin\theta$ is small and hence θ is small. The maxima and minima are very close to the central maximum at P. But with a narrow slit, a is small and hence θ is large. This results in a distinct diffraction maxima and minima on both the sides of P.

Fraunhofer Single Slit Diffraction Intensity

The general features of that distribution are shown below.





Most of the intensity is in the central maximum. It is twice the width of the other (secondary) maxima.

Intensity :

$$I^2 = R^2 = A^2 \frac{\sin^2\alpha}{\alpha^2}$$

Fraunhofer Double Slit Diffraction

18.4. FRAUNHOFFER DIFFRACTION AT DOUBLE SLIT

In Fig. 18.8, AB and CD are two rectangular slits parallel to one another and perpendicular to the plane of the paper. The width of each slit is a and the width of the opaque portion is b. L is a collecting lens and MN is a screen perpendicular to the plane of the paper. P is a point on the screen such that OP is perpendicular to the screen. Let a plane wave front be incident on the surface of XY. All the secondary waves traveling in a direction parallel to OP come to focus at P. Therefore, P corresponds to the position of the central bright maximum.

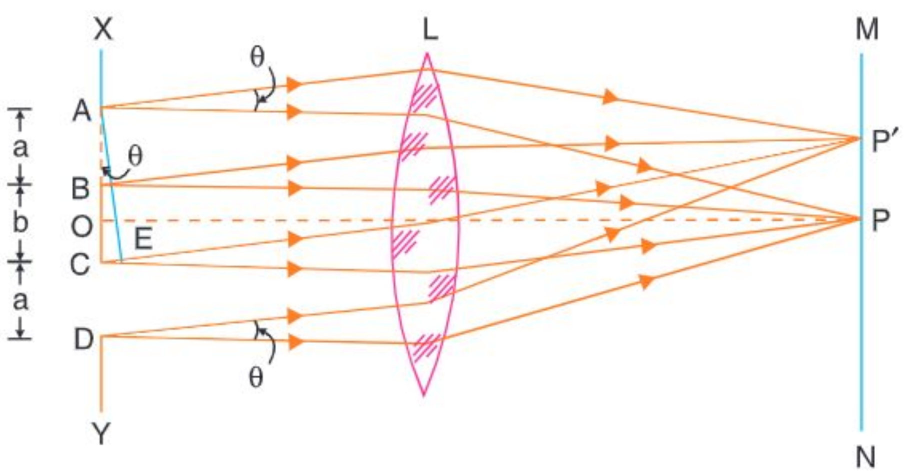


Fig. 18.8

Fraunhofer Double Slit Diffraction

In this case, the diffraction pattern has to be considered in two parts (*i*) the interference phenomenon due to the secondary waves emanating from the corresponding points of the two slits and (*ii*) the diffraction pattern due to the secondary waves from the two slits individually. For calculating the positions of interference maxima and minima, the diffracting angle is denoted as θ and for the diffraction maxima and minima it is denoted as ϕ. Both the angles θ and ϕ refer to the angle between the direction of the secondary waves and the initial direction of the incident light.

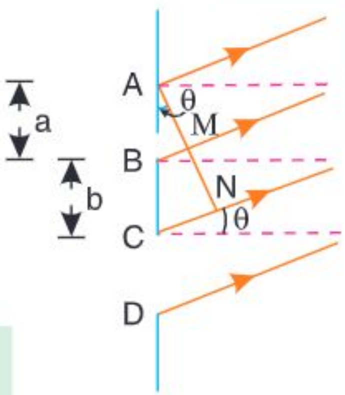


Fig. 18.9

(*i*) Interference maxima and minima:

Let us consider the secondary waves traveling in a direction inclined at an angle θ with the initial direction.

In the Δ CAN (Fig. 18.9) $\sin\theta = \frac{CN}{AC} = \frac{CN}{a+b}$

or $CN = (a+b)\sin\theta$

Fraunhofer Double Slit Diffraction

If this path difference is equal to odd multiples of $\lambda/2$, θ gives the direction of minima due to interference of the secondary waves from the two slits.

$$\therefore \qquad CN = (a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2} \qquad (18.28)$$

Putting $n = 1, 2, 3$, etc, the values of θ_1, θ_2, θ_3 etc, corresponding to the directions of minima can be obtained.

From equation (18.28)

$$\sin\,\theta_n = \frac{(2n+1)\lambda}{2(a+b)} \qquad (18.29)$$

On the other hand, if the secondary waves travel in a direction θ' such that the path difference is even multiples of $\lambda / 2$, then θ' gives the direction of the maxima due to interference of light waves emanating from the two slits.

$$\therefore \qquad CN = (a+b)\ \sin\theta'_n = 2n\frac{\lambda}{2}$$

or

$$\sin\theta'_n = \frac{n\lambda}{(a+b)} \qquad (18.30)$$

Fraunhofer Double Slit Diffraction

Putting $n = 1, 2, 3$ etc, $\theta'_1, \theta'_2, \theta'_3$ etc corresponding to the directions of the maxima can be obtained. From equation(18.29), we get

$$\sin\theta_1 = \frac{3\lambda}{2(a+b)}$$

and
$$\sin\theta_2 = \frac{5\lambda}{2(a+b)}$$

$$\therefore \qquad \sin\theta_2 - \sin\theta_1 = \frac{\lambda}{(a+b)} \qquad (18.31)$$

Thus, the angular separation between any two consecutive minima (or maxima) is equal to $\frac{\lambda}{a+b}$. The angular separation is inversely proportional to $(a+b)$, the distance between the two slits.

(*ii*) Diffraction maxima and Minima:

Let us consider the secondary waves traveling in a direction inclined at an angle ϕ with the initial direction of the incident light.

If the path difference BM is equal to λ the wavelength of the light used, then ϕ will give the direction of the diffraction minimum (Fig. 18.9). That is, the path difference between secondary waves emanating from the extremities of a slit (i.e., points A and B) is equal to λ. Considering the wave front on AB to be made up of the two halves, the path difference between the corresponding points of the upper and lower halves is equal to $\lambda/2$. The effect at P′ due to the wave front incident on AB is zero. Similarly, for the same direction of the secondary waves, the effect at P′ due to the wave front incident on the slit CD is also zero. In general,

$$a\sin\phi_n = n\lambda.$$

Putting $n = 1, 2, 3$, etc, the values of ϕ_1, ϕ_2, ϕ_3 etc, corresponding to the directions of diffraction minima can be obtained.

Fraunhofer Double Slit Diffraction

The intensity at P_1 is

$$I^2 = R^2 = 4A^2 \frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

$$= 4I_0 \frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

Where $I_0 = A^2$

- The above represents the intensity distribution on the screen. The intensity at any point on the screen depends on α, β, and the intensity of central maximum is $4I_o$.
- The term $\cos^2\beta$ corresponds to interference and $\sin^2\alpha$ corresponds to diffraction.

Fraunhofer Double Slit Diffraction

The intensity distribution on the screen due to double slit diffraction is shown in Fig.