SRM Institute of Science and Technology
College of Engineering and Technology
DEPARTMENT OF ECE
SRM Nagar, Kattankulathur – 603203, Chengalpattu District, Tamilnadu.

Academic Year: 2022-2023 (ODD)

Test: CLAT-3 **Date: 19/11/22**
Course Code & Title: 18ECC204J-Digital Signal Processing **Duration: 10:00-11:40 AM**
Year & Sem: III /V **Max. Marks: 50**

Course Articulation Matrix: ***(to be placed)***

	18ECC204J – Digital Signal Processing	Program Outcomes (POs)														
		Graduate Attributes												PSO		
S. No.	Course Outcomes (COs)	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3
1	Summarize the concepts of A//D and D/A converters.	3	-	-	1	-	-	-	-	-	-	-	-	-	-	2
2	Explain the concepts of DFT with its efficient computation by using FFT algorithm.	-	2	-	-	-	-	-	-	-	-	-	-	-	1	
3	Develop FIR filters using several methods	-	2	3	-	-	-	-	-	-	-	-	-	-	-	3
4	Construct IIR filters using several methods	-		3	-	-	-	-	-	-	-	-	-	-	-	3
5	Discuss the basics of multirate DSP and its applications.	-	2	-	-	-	-	-	-	-	-	-	-	-	1	-
6	Design digital filter and multi rate signal processing for real time signals	-	2	-	-	-	-	-	-	-	-	-	-	2	-	-

Part-A (5 x 10= 50 Marks)

<table>
<tr><th colspan="6">Answer any 5</th></tr>
<tr><th>Q. No</th><th>Question</th><th>Marks</th><th>BL</th><th>CO</th><th>PO</th></tr>
<tr><td>1</td><td>

i) Design an analog Butterworth filter that satisfies the following constraints

$0.9 \leq |H(j\Omega)| \leq 1; \quad 0 \leq \Omega \leq 0.2\pi$

$|H(j\Omega)| \leq 0.2; \quad 0.4\pi \leq \Omega \leq \pi$

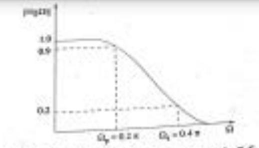

Fig. 5.8 Magnitude response of example 5.5

$\varepsilon = 0.484$ and $\lambda = 4.898$

$$N \geq \frac{\log\left(\frac{\lambda}{\varepsilon}\right)}{\log\frac{\Omega_s}{\Omega_p}} = \frac{\log\frac{4.898}{0.484}}{\log\left(\frac{0.4\pi}{0.2\pi}\right)} = 3.34$$

i.e., $N = 4$

From the table 5.1, for $N = 4$, the transfer function of normalised Butterwort filter is

$$H(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.8477s+1)}$$

4 Mark

we know $\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}} = \frac{\Omega_p}{\varepsilon^{1/N}} = \frac{0.2\pi}{(0.484)^{1/4}} = 0.24\pi.$

$H(s)$ for $\Omega_c = 0.24\pi$ can be obtained by substituting $s \to \frac{s}{0.24\pi}$ in $H(s)$ i.e

$$H(s) = \frac{1}{\left\{\left(\frac{s}{0.24\pi}\right)^2 + 0.76537\left(\frac{s}{0.24\pi}\right)+1\right\}} \times \frac{1}{\left(\frac{s}{0.24\pi}\right)^2 + 1.8477\left(\frac{s}{0.24\pi}\right)+1}$$

5 Mark

$$= \frac{0.323}{(s^2+0.577s+0.0576\pi^2)(s^2+1.393s+0.0576\pi^2)}$$

</td><td>9</td><td>3</td><td>4</td><td>3</td></tr>
<tr><td></td><td>ii) The magnitude response of Butterworth filter ______ as the frequency increases.
c)decreases monotonically</td><td>1</td><td>1</td><td>4</td><td>1</td></tr>
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<tr><td>2</td><td>

i) Apply bilinear transformation to $H(s) = \frac{2}{(s+1)(s+2)}$ with T=1 sec and find $H(Z)$.

Given $H(s) = \frac{2}{(s+1)(s+2)}$

Substitute $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ in $H(s)$ to get $H(z)$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{2}{(s+1)(s+2)}\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

4 Mark

Given $T = 1$ sec

$$H(z) = \frac{2}{\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right\}\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right\}} = \frac{2(1+z^{-1})^2}{(3-z^{-1})(4)} = \frac{(1+z^{-1})^2}{6-2z^{-1}} = \frac{0.166(1+z^{-1})^2}{(1-0.33z^{-1})}$$

5 Mark

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ii) The poles of Chebyshev filter lie on
b) ellipse

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<tr><td>3</td><td>

i) Determine the order and transfer function of the filter using Chebyshev approximation for the following specifications.

$\propto_p = 3bB, \propto_s = 16dB, f_p = 1KHz\ and$
$f_s = 2KHz.$

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Step 1:

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\frac{\Omega_s}{\Omega_p}} = \cosh^{-1}\frac{\sqrt{\frac{10^{1.6}-1}{10^{0.3}-1}}}{\cosh^{-1}\frac{4000\pi}{2000\pi}} = 1.91$$

Step 2: Rounding N to next higher value we get $N = 2$.

For N even, the oscillatory curve starts from $\frac{1}{\sqrt{1+\varepsilon^2}}$.

Step 3: The values of minor axis and major axis can be found as below

$$\varepsilon = (10^{0.1\alpha_p}-1)^{0.5} = (10^{0.3}-1)^{0.5} = 1$$

4 Mark

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.414$$

$$a = \Omega_p\frac{[\mu^{1/N}-\mu^{-1/N}]}{2} = 2000\pi\frac{[(2.414)^{1/2}-(2.414)^{-1/2}]}{2} = 910\pi$$

$$b = \Omega_p\frac{[\mu^{1/N}+\mu^{-1/N}]}{2} = 2000\pi\frac{[(2.414)^{1/2}+(2.414)^{-1/2}]}{2} = 2197\pi$$

Step 4: The poles are given by

$$s_k = a\cos\phi_k + jb\sin\phi_k, \quad k = 1,2$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1,2$$

$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{4} = 135^\circ$$

$$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = 225^\circ$$

$$s_1 = a\cos\phi_1 + j\,b\sin\phi_1 = -643.46\pi + j1554\pi$$

$$s_2 = a\cos\phi_2 + j\,b\sin\phi_2 = -643.46\pi - j1554\pi$$

Step 5: The denominator of $H(s) = (s + 643.46\pi)^2 + (1554\pi)^2$

Step 6: The numerator of $H(s) = \frac{(643.46\pi)^2 + (1554\pi)^2}{\sqrt{1+\varepsilon^2}} = (1414.38)^2\pi^2$

The transfer function $H(s) = \frac{(1414.38)^2\pi^2}{s^2 + 1287\pi s + (1682)^2\pi^2}$.

5 Mark

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ii) The impulse invariance method is unsuccessful for implementing _________
b) High pass filter

</td><td>1</td><td>1</td><td>4</td><td>1</td></tr>
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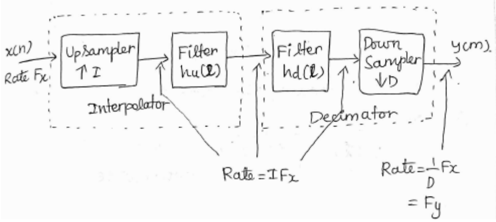
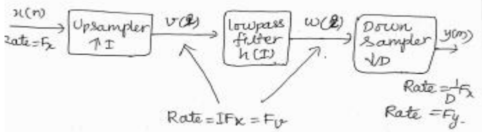
4	i) Discuss the steps involved in converting sampling rate by a factor of I/D.	**9**	**3**	**5**	**2**

Sample rate conversion by a rational factor I/D





In the domain, the output of the upsampler is the sequence

$$v(l) = \begin{cases} x(l/I), & l = 0, \pm I, \pm 2I, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

and

the output of the linear time-invariant filter is

$$w(l) = \sum_{K=-\infty}^{\infty} h(l-k)\, v(K)$$

$$= \sum_{K=-\infty}^{\infty} h(l-KI)\, x(K)$$

Thus,

the spectrum at the output of the linear filter with impulse response $h(l)$ is

$$V(\omega_v) = H(\omega_v) X(\omega_v I)$$

$$= \begin{cases} I X(\omega_v I), & 0 \le |\omega_v| \le \min(\pi/D, \pi/I) \\ 0, & \text{otherwise.} \end{cases}$$

The spectrum of the output sequence $y(m)$ obtained by decimating the sequence $v(n)$ by a factor of D is

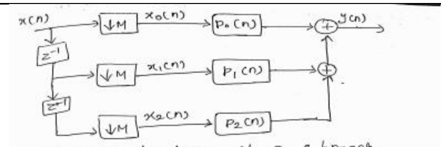
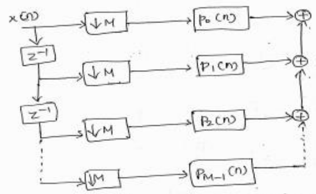
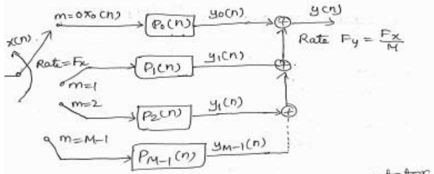
$$Y(\omega_y) = \frac{1}{D} \sum_{K=0}^{D-1} V\left(\frac{\omega_y - 2\pi K}{D}\right)$$

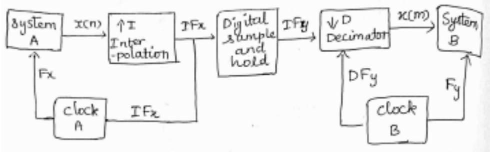
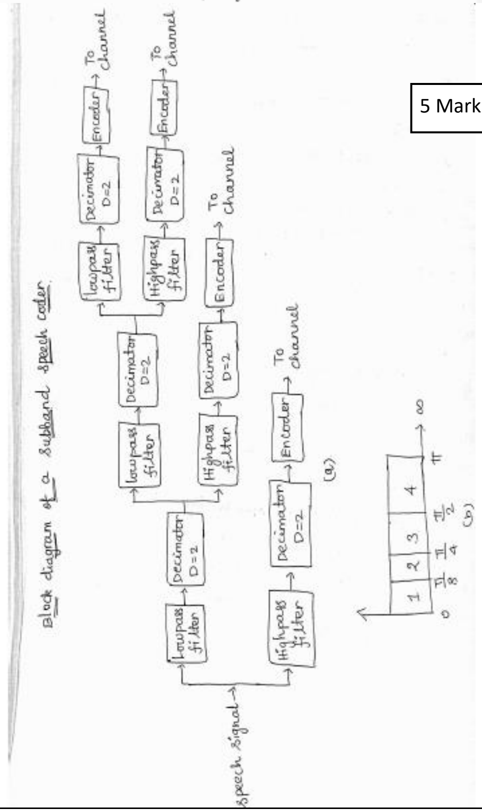
where $\omega_y = D\omega_v$.

Since, the linear filter prevents aliasing

$$Y(\omega_y) = \begin{cases} \frac{1}{D} X\left(\frac{\omega_y}{D}\right), & 0 \le |\omega_y| \le \min\left(\pi, \frac{\pi D}{I}\right) \\ 0, & \text{otherwise} \end{cases}$$

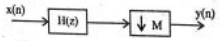
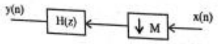
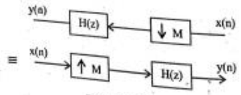
	ii) If x[n] = [1, 2, 3, 4] then y[n]=x[n/2] will be b) [1, 0, 2, 0, 3, 0, 4, 0]	**1**	**1**	**5**	**1**

5	i) Realize M-branch decimator using polyphase structure. Polyphase structure of [illegible] $$H(z) = \sum_{m=0}^{M-1} z^{-m} P_m(z^M)$$ here $P_m(z) = \sum_{n=0}^{[(N+1)/M]} h(Mn+m) z^{-n}$ The z-transform of an infinite sequence given by $$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$ $$H(z) = \sum_{m=0}^{M-1} z^{-m} P_m(z^m)$$ here $P_m(z) = \sum_{r=-\infty}^{\infty} h(rM+m) z^{-r}$ $$H(z) = \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} z^{-m} h(rM+m) z^{-rM}$$ $$= \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} h(rM+m) z^{-(rM+m)}$$ let $h(rM+m) = P_m(r)$ $$\Rightarrow H(z) = \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} P_m(r) z^{-(rM+m)}$$ $$Y(z) = \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} P_m(r) X(z) z^{-(rM+m)}$$ $$y(n) = \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} P_m(r) x[n-(rM+m)]$$ let $x_m(r) = x(rM-m)$ then $$y(n) = \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} P_m(r) x_m(n-r)$$ $$= \sum_{m=0}^{M-1} P_m(n) * x_m(n)$$ $$= \sum_{m=0}^{M-1} y_m(n).$$ where $y_m(n) = P_m(n) * x_m(n)$ The operation $P_m(n) * x_m(n)$ is known as polyphase convolution and the overall process is polyphase filtering. $$y(n) = \sum_{m=0}^{2} y_m(n)$$ $$= y_0(n) + y_1(n) + y_2(n)$$ $$= P_0(n) * x_0(n) + P_1(n) * x_1(n) + P_2(n) * x_2(n)$$	**9**	**3**	**6**	**2**
	 polyphase structure of a 3 branch decimator  polyphase structure of a M-branch decimator  Polyphase decimator with a commutator ii) A two-channel subband coding filter bank is also called as a) Quadrature-mirror filter bank				
		1	**1**	**6**	**1**
6	i) Discuss two practical applications of multirate DSP with suitable block diagram.	**9**	**3**	**6**	**2**

	ii) Interfacing of digital systems with different sampling Rates. 4 Mark  Figure:- Interfacing of two digital systems with different sampling rates. 5 Mark 				

	ii) Anti-alias filter is to be kept a) before down sampler	**1**	**1**	**5**	**1**
7	i) Convert the single pole lowpass filter with system function $H(Z) = \frac{0.5(1+Z^{-1})}{1-0.302Z^{-2}}$ into bandpass filter with upper and lower cutoff frequencies ω_u and ω_l respectively. The lowpass filter has 3 dB bandwidth $\omega_p=\frac{\pi}{6}$, $\omega_u= \frac{3\pi}{4}$ and $\omega_l=\frac{\pi}{4}$. $$z^{-1} \longrightarrow \frac{-\left(z^{-2} - \frac{2\alpha k}{1+k}z^{-1} + \frac{k-1}{k+1}\right)}{\frac{k-1}{k+1}z^{-2} - \frac{2\alpha k}{k+1}z^{-1} + 1}$$ $$k = \cot\left[\frac{\omega_u - \omega_l}{2}\right]\tan\frac{\omega_p}{2} = \cot\left(\frac{\frac{3\pi}{4} - \frac{\pi}{4}}{2}\right)\tan\frac{\pi}{12} = \cot\left(\frac{\pi}{4}\right)\tan\frac{\pi}{12} = 0.268$$	**5**	**3**	**4**	**3**

	$$\alpha = \frac{\cos\frac{\omega_u+\omega_l}{2}}{\cos\frac{\omega_u-\omega_l}{2}} = \frac{\cos\left(\frac{\frac{3\pi}{4}+\frac{\pi}{4}}{2}\right)}{\cos\left(\frac{\frac{3\pi}{4}-\frac{\pi}{4}}{2}\right)} = \frac{\cos\frac{\pi}{2}}{\cos\frac{\pi}{4}} = 0$$ Substituting the values of α and k in the transformation $$z^{-1} \to \frac{-\left(z^{-2}+\frac{0.268-1}{0.268+1}\right)}{\frac{0.268-1}{0.268+1}z^{-2}+1}$$ i.e., $$z^{-1} \to \frac{-(z^{-2}-0.577)}{-0.577z^{-2}+1}$$ Now the transfer function of bandpass filter can be obtained by substituting the above transformation in $H(z)$. $$H(z) = 0.5\frac{\left[1+\frac{-z^{-2}+0.577}{1-0.577z^{-2}}\right]}{1-0.302\left(\frac{-z^{-2}+0.577}{1-0.577z^{-2}}\right)}$$ $$= 0.5\left[\frac{1.577(1-z^{-2})}{0.82575-0.275z^{-2}}\right]$$ $$= \frac{0.955(1-z^{-2})}{(1-0.333z^{-2})}$$ ii) Show that the transpose of a factor-of-M decimator is a factor-of-M interpolator if the transpose of a factor-of-M downsampler is a factor-of-M upsampler.	**4**	**3**	**5**	**2**

	Solution The transpose of a digital filter is obtained by reversing all paths, in changing the input and the output nodes, replacing the pick-off node with an ad and vice-versa. The factor-of-M decimator is shown in Fig.8.41.  **Fig. 8.41** Interchanging the input and output nodes and reversing the paths we obtain  **Fig. 8.42** If the transpose of a factor-of-M down-sampler is a factor-of-M upsampler have  **Fig. 8.43** Hence the transpose of a factor-of-M decimator is a factor-of-M interpolator iii) _____ filter is used to remove the image spectrum that is introduced due to the addition of zero samples between successive data points. d) Anti-imaging	**1**	**1**	**6**	**1**

Signature of the Course Teacher **Signature of the Course Co-ordinator**

Signature of the Academic Advisor