SRM Institute of Science and Technology
College of Engineering and Technology

SET-B

DEPARTMENT OF ECE

SRM Nagar, Kattankulathur – 603203, Chengalpattu District, Tamilnadu

Academic Year: 2022-2023 (ODD)

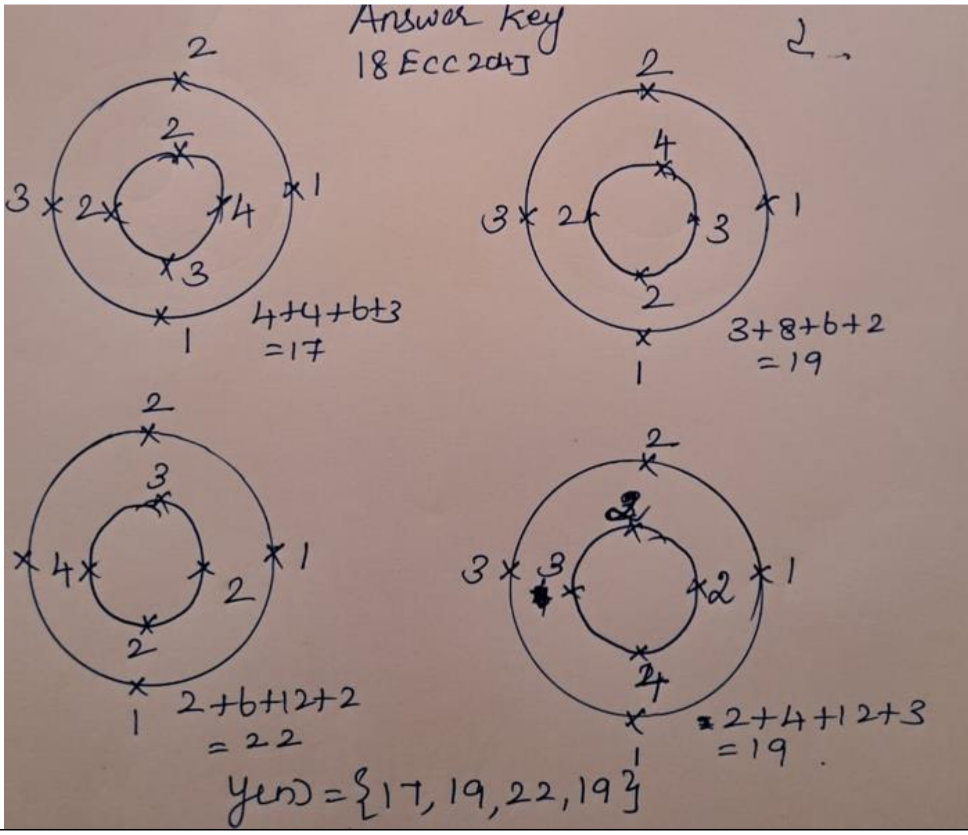
Test: CLAT-2 | **Date: 17/10/22**
Course Code & Title: 18ECC204J-Digital Signal Processing | **Duration:**8.00-9.40am
Year & Sem: III /VI | **Max. Marks:** 50

Course Articulation Matrix: ***(to be placed)***

	18ECC204J – Digital Signal Processing	Program Outcomes (POs)														
		Graduate Attributes												PSO		
S. No.	**Course Outcomes (COs)**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**1**	**2**	**3**
1	Summarize the concepts of A//D and D/A converters.	3	-	-	1	-	-	-	-	-	-	-	-	-	-	2
2	Explain the concepts of DFT with its efficient computation by using FFT algorithm.	-	2	-	-	-	-	-	-	-	-	-	-	-	1	
3	Develop FIR filters using several methods	-	2	3	-	-	-	-	-	-	-	-	-	-	-	3
4	Construct IIR filters using several methods	-		3	-	-	-	-	-	-	-	-	-	-	-	3
5	Discuss the basics of multirate DSP and its applications.	-	2	-	-	-	-	-	-	-	-	-	-	-	1	-
6	Design digital filter and multi rate signal processing for real time signals	-	2	-	-	-	-	-	-	-	-	-	-	2	-	-

ANSWER KEY

Part-A (5 x 10 marks= 50 Marks)
Answer any 5

Q. No	Question	Marks	BL	CO	PO
1					

(ii) $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{+j2\pi kn/N}$

$x(0) = \frac{1}{4}[2+1-j+0+1+j] = 1/4(4) = 1$

$x(1) = \frac{1}{4}[4 \cdot \cos 0 \; 2 + j\sin 0]$

$= \frac{1}{4}[4+j+1(-1)-j] = 1/4(4) = 1.$

$x(4) = \cancel{5}\,1$

$x(2) = \frac{1}{4}[X(k)e^{j4\pi/4}]$

$= \frac{1}{4}[2+(1-j)(-1)+0(1+j)(-1)]$

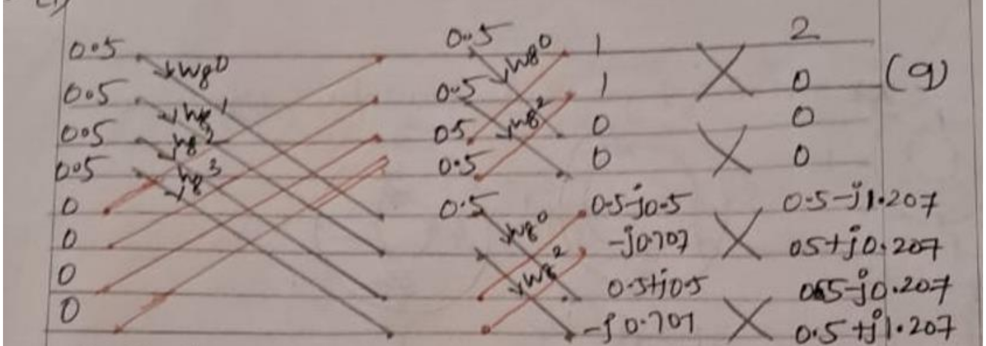
Illy $= 0$

$x(3) = 0.$

$x(n) = \{1, 1, 0, 0\}.$

(iii)-0.707-j0.707

2(i)

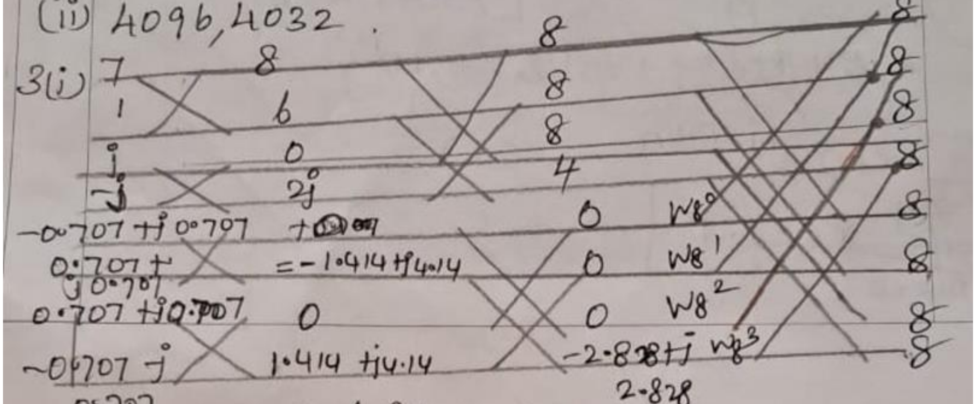


X(K) = {2, 0.5-j1.207, 0, 0.5-j0.207, 0, 0.5+ 0.207, 0, 0.5+j1.207}

Each stage 3 marks.

(ii) 4096, 4032.

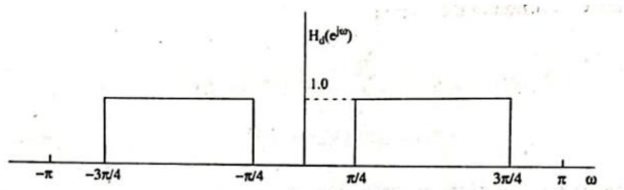
3(i)



each stage 3 marks

x(n)=1/8(8,8,8,8,8,8,8,8);x(n)=(1,1,1,1,1,1,1,1)

(ii) (1,4,3,2,2,3,4,1).



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n} \quad = \frac{1}{2\pi}\left[\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{4}} e^{j\omega n}d\omega + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} e^{j\omega n}d\omega\right]$$

$$= \frac{1}{2\pi jn}\left[e^{-j\pi n/4} - e^{-j3\pi n/4} + e^{j3\pi n/4} - e^{j\pi n/4}\right] \quad = \frac{1}{\pi n}\left[\sin\frac{3\pi}{4}n - \sin\frac{\pi}{4}n\right]$$

$$-\infty \le n \le \infty$$

Truncating $h_d(n)$ to 11 samples, we have

$$h(0) = \frac{1}{2\pi}\left[\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{4}} d\omega + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\omega\right]$$
$$= \frac{1}{2\pi}\left[-\frac{\pi}{4} + \frac{3\pi}{4} + \frac{3\pi}{4} - \frac{\pi}{4}\right] = \frac{1}{2} = 0.5$$

$$h(1) = h(-1) = \frac{\sin\frac{3\pi}{4} - \sin\frac{\pi}{4}}{\pi} = 0$$

$$h(2) = h(-2) = \frac{\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}}{2\pi} = \frac{-2}{2\pi} = -0.3183$$

$$h(3) = h(-3) = \frac{\sin\frac{9\pi}{4} - \sin\frac{3\pi}{4}}{3\pi} = 0$$

$$h(4) = h(-4) = \frac{\sin 3\pi - \sin\pi}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin\frac{15\pi}{4} - \sin\frac{5\pi}{4}}{5\pi} = 0$$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n)\left(z^n + z^{-n}\right)\right.$$
$$= 0.5 - 0.3183(z^2 + z^{-2})$$

$$H'(z) = z^{-5}\left[0.5 - 0.3183(z^2 + z^{-2})\right]$$
$$= -0.3183z^{-3} + 0.5z^{-5} - 0.3183z^{-7}$$

(ii) windowing Technique

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$
$$= \frac{1}{2\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} e^{j\omega n}d\omega$$
$$= \frac{1}{2\pi jn}e^{j\omega n}\Big|_{-\pi/2}^{\pi/2}$$
$$= \frac{1}{\pi n(2j)}\left[e^{j\pi n/2} - e^{-j\pi n/2}\right]$$
$$= \frac{\sin\frac{\pi}{2}n}{\pi n} \quad -\infty \leq n \leq \infty$$

$$h(0) = h_d(0) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} d\omega = \frac{1}{2\pi}\omega\Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{2\pi} = \frac{1}{2}.$$

For $n = 1$ $\quad h(1) = h(-1) = \frac{\sin\frac{\pi}{2}}{\pi} = \frac{1}{\pi} = 0.3183.$

$$h(2) = h(-2) = \frac{\sin\pi}{2\pi} = 0$$
$$h(3) = h(-3) = \frac{\sin\frac{3\pi}{2}}{3\pi} = -\frac{1}{3\pi} = -0.106$$
$$h(4) = h(-4) = \frac{\sin\frac{4\pi}{2}}{4\pi} = 0$$
$$h(5) = h(-5) = \frac{\sin\frac{5\pi}{2}}{5\pi} = \frac{1}{5\pi} = 0.06366.$$

Hamming window

The Hamming window sequence is given by

$$w_H(n) = 0.54 + 0.46\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -(N-1)/2 \leq n \leq (N-1)/2$$
$$= \quad 0 \quad \text{otherwise}$$

The window sequence for $N = 11$ is given by

$$w_H(n) = 0.54 + 0.46\cos\frac{\pi n}{5} \quad \text{for} \quad -5 \leq n \leq 5$$
$$= \quad 0 \quad \text{otherwise}$$

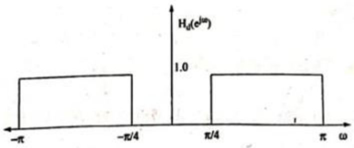
$W_H(0)$=1, $W_1(0)$=0.912, $W_H(2)$=0.682, $W_H(3)$=0.398, $W_H(4)$=0.1678, $W_H(5)$=0.08

h(0) =0.5;h(1)=0.290;h(2)=0;h(3)=0.0421;h(4)=0;h(5)=0.005.

$$H(z) = h(0) + \sum_{n=1}^{5} h(n)(z^{-n}+z^{n})$$
$$= h(0) + h(1)(z^{-1}+z) + h(2)(z^{-2}+z) + h(3)(z^{-3}+z^{3})$$
$$+ h(4)(z^{-4}+z^{4}) + h(5)(z^{-5}+z^{5})$$
$$= 0.5 + 0.290(z^{-1}+z) + 0 + 0.0421(z^{-3}+z^{3}) + 0$$
$$+ 0.005(z^{-5}+z5)$$
$$H(z) = 0.5 + 0.290z^{-1} + 0.290z + 0.0421z^{-3}$$
$$+ 0.0421z^{3} + 0.005z^{-5} + 0.005z^{5}.$$

(ii)Constant

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$
$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\frac{\pi}{4}} e^{j\omega n}d\omega + \int_{\frac{\pi}{4}}^{\pi} e^{j\omega n}d\omega\right]$$
$$= \frac{1}{2\pi jn}\left[e^{j\omega n}\Big|_{-\pi}^{-\pi/4} + e^{j\omega n}\Big|_{\pi/4}^{\pi}\right]$$



$$= \frac{1}{\pi n(2j)}\left[e^{-j\pi n/4} - e^{-j\pi n} + e^{j\pi n} - e^{j\pi n/4}\right] \qquad = \frac{1}{\pi n}\left[\sin\pi n - \sin\frac{\pi}{4}n\right]$$

For $n = 0$

$$h(0) = \lim_{n\to 0}\frac{\sin\pi n}{\pi n} - \lim_{n\to 0}\frac{\sin\frac{\pi}{4}n}{\pi n}$$
$$= \left((1 - \frac{1}{4}\right)$$

$$\boxed{\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1 \quad \lim_{\theta\to 0}\frac{\sin n\theta}{\theta} = n}$$

For $n = 1$

$$h(1) = h(-1) = \frac{\sin\pi - \sin\frac{\pi}{4}}{\pi} = -0.225$$

$$h(2) = h(-2) = \frac{\sin 2\pi - \sin\frac{\pi}{2}}{2\pi} = -0.159$$
$$h(3) = h(-3) = \frac{\sin 3\pi - \sin\frac{3\pi}{4}}{3\pi} = -0.075$$
$$h(4) = h(-4) = \frac{\sin 4\pi - \sin\pi}{4\pi} = 0$$
$$h(5) = h(-5) = \frac{\sin 5\pi - \sin\frac{5\pi}{4}}{5\pi} = 0.045$$

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for } -(N-1)/2 \le n \le (N-1)/2$$
$$= \quad 0 \quad \text{otherwise}$$

For $N = 11$

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{\pi n}{5} \quad -5 \le n \le 5$$
$$= \quad 0 \quad \text{otherwise}$$

$$w_{Hn}(0) = 0.5 + 0.5 = 1$$
$$w_{Hn}(1) = w_{Hn}(-1) = 0.5 + 0.5\cos\frac{\pi}{5} = 0.9045$$
$$w_{Hn}(2) = w_{Hn}(-2) = 0.5 + 0.5\cos\frac{2\pi}{5} = 0.655$$
$$w_{Hn}(3) = w_{Hn}(-3) = 0.5 + 0.5\cos\frac{3\pi}{5} = 0.345$$
$$w_{Hn}(4) = w_{Hn}(-4) = 0.5 + 0.5\cos\frac{4\pi}{5} = 0.0945$$
$$w_{Hn}(5) = w_{Hn}(-5) = 0.5 + 0.5\cos\pi = 0$$

The filter coefficients using Hanning window are

$$h(n) = h_d(n)w_{Hn}(n) \quad \text{for} \quad -5 \le n \le 5$$
$$= \quad 0 \quad \text{otherwise}$$

$$h(0) = h_d(0)w_{Hn}(0) = (0.75)(1) = 0.75$$
$$h(-1) = h(1) = h_d(1)w_{Hn}(1) = (-0.225)(0.905) = -0.204$$
$$h(-2) = h(2) = h_d(2)w_{Hn}(2) = (-0.159)(0.655) = -0.104$$
$$h(-3) = h(3) = h_d(3)w_{Hn}(3) = (-0.075)(0.345) = -0.026$$
$$h(-4) = h(4) = h_d(4)w_{Hn}(4) = (0)(0.8145) = 0$$
$$h(-5) = h(5) = h_d(5)w_{Hn}(5) = (0.045)(0) = 0$$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} [h(n)(z^n + z^{-n}] \quad = 0.75 + \sum_{n=1}^{5} [h(n)(z^n + z^{-n}]$$
$$= 0.75 - 0.225(z + z^{-1}) - 0.159(z^2 + z^{-2}) - 0.075(z^3 + z^{-3})$$
$$+ 0.045(z^5 + z^{-5})$$

The transfer function of the realizable filter is $H'(z) = z^{-5}H(z)$

$$= z^{-5}[0.75 - 0.225(z + z^{-1}) - 0.159(z^2 + z^{-2}) - 0.075(z^3 + z^{-3})$$
$$+ 0.045(z^5 + z^{-5})]$$

(ii) 8π/N

(i)

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$
$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-2\pi/3} e^{j\omega n}d\omega + \int_{-\pi/3}^{\pi/3} e^{j\omega n}d\omega + \int_{2\pi/3}^{\pi} e^{j\omega n}d\omega\right]$$
$$= \frac{1}{2\pi jn}\left[e^{-j2\pi n/3} - e^{-j\pi n} + e^{j\pi n/3} - e^{-j\pi n/3} + e^{j\pi n} - e^{j2\pi n/3}\right]$$
$$= \frac{1}{\pi n}\left[\sin\pi n + \sin\frac{\pi}{3}n - \sin\frac{2\pi}{3}n\right] \qquad -\infty \le n \le \infty$$

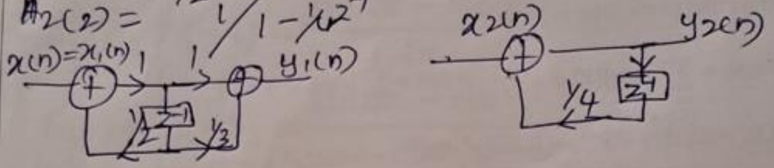
7(iii) b. Hanning

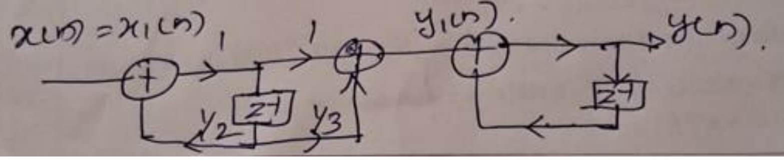
(ii) $H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-1}}$

$= \frac{1+\frac{1}{3}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} = H_1(z)H_2(z).$

$H_1(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$

$H_2(z) = 1/1-\frac{1}{4}z^{-1}$





(iii) hanning