SRM Institute of Science and Technology
Ramapuram Campus

Department of Mathematics

Year / Sem: I / II

Branch: Common to ALL Branches of B.Tech. except B.Tech. (Business Systems)

UNIT IV - ANALYTIC FUNCTIONS

Part – A

1.	The critical point of the transformation $w = z^2$ is (A) $z = 0$ (B) $z = -i$ (C) $z = 1$ (D) $z = -1$	ANS **A**	(CLO-4, Apply)
2.	If $w = f(z) = u + iv$ is analytic, then the family of curves $u = C_1$ and $v = C_2$ (A) cut orthogonally (B) intersect each other (C) are parallel (D) coincide	ANS **A**	(CLO-4, Remember)
3.	If a function $u(x, y)$ satisfies the equation $u_{xx} + u_{yy} = 0$, then u is called (A) analytic function (B) harmonic function (C) differential function (D) continuous function	ANS **B**	(CLO-4, Remember)
4.	Cauchy-Riemann equations in Polar co-ordinates are (A) $u_r = \frac{1}{r} v_\theta,\ v_r = -\frac{1}{r} u_\theta$ (B) $u_r = -\frac{1}{r} v_\theta,\ v_r = \frac{1}{r} u_\theta$ (C) $u_r = -\frac{1}{r} v_\theta,\ v_r = -\frac{1}{r} u_\theta$ (D) $u_r = \frac{1}{r} v_\theta,\ v_r = \frac{1}{r} u_\theta$	ANS **A**	(CLO-4, Remember)
5.	The critical point of the transformation $w = z^4$ is (A) $z = 2$ (B) $z = -2$ (C) $z = 0$ (D) $z = 1$	ANS **C**	(CLO-4, Apply)
6.	If $w = f(z) = u + i\ v$ is an analytic function of z, then (A) u and v are not harmonic (B) u is not harmonic (C) both u and v are harmonic (D) u and v are constants	ANS **C**	(CLO-4, Remember)

7.	An analytic function with constant modulus is (A) zero (B) analytic (C) harmonic (D) constant	ANS **D**	(CLO-4, Remember)
8.	Cauchy – Riemann equation in Cartesian co-ordinates are (A) $u_x = v_y, u_y = -v_x$ (B) $u_x = -v_y, u_y = v_x$ (C) $u_x = v_y, u_y = v_x$ (D) $u_x = -v_y, u_y = -v_x$	ANS **A**	(CLO-4, Remember)
9.	The invariant point of the transformation $w = \frac{1}{z-2i}$ is (A) $z = 0$ (B) $z = 1$ (C) $z = -1$ (D) $z = i$	ANS **D**	(CLO-4, Apply)
10.	The transformation $w = a\,z$, where a is a real constant represents (A) magnification (B) rotation (C) reflection (D) inversion	ANS **A**	(CLO-4, Apply)
11.	The fixed points of the transformation $w = \frac{z-1}{z+1}$ are (A) $\pm i$ (B) ± 1 (C) ± 2 (D) ± 3	ANS **A**	(CLO-4, Apply)
12.	An analytic function with constant real part is (A) zero (B) analytic (C) harmonic (D) constant	ANS **D**	(CLO-4, Remember)
13.	An analytic function with constant imaginary part is (A) zero (B) analytic (C) harmonic (D) constant	ANS **D**	(CLO-4, Remember)
14.	The transformation $w = a\,z$, where a is a complex constant represents (A) magnification (B) reflection (C) magnification and rotation (D) inversion	ANS **C**	(CLO-4, Remember)
15.	If $f(z) = e^z$, then $f(z)$ is (A) zero function (B) analytic function (C) discontinuous function (D) constant function	ANS **B**	(CLO-4, Remember)

16.	$f(z) = \frac{1}{z^2+1}$ is analytic everywhere except at (A) $z = \pm i$ (B) $z = \pm 1$ (C) $z = \pm 2$ (D) $z = \pm 3$	ANS **A**	(CLO-4, Apply)
17.	The invariant points of the transformation $w = \frac{2z+6}{z+7}$ are (A) $6, -1$ (B) $3, 2$ (C) $-3, 2$ (D) $-6, 1$	ANS **D**	(CLO-4, Apply)
18.	The fixed points of the transformation $w = \frac{z-1}{z+1}$ are (A) $\pm i$ (B) ± 1 (C) ± 2 (D) ± 3	ANS **A**	(CLO-4, Apply)
19.	The image of $\lvert z - 2i \rvert = 2$ under the transformation $w = \frac{1}{z}$ is (A) $x^2 + y^2 = 0$ (B) $x^2 + y^2 + 4y = 0$ (C) $x^2 + y^2 - 4y = 0$ (D) $x^2 + y^2 + y = 0$	ANS **C**	(CLO-4, Apply)
20.	The image of $\lvert z \rvert = 2$ under the transformation $w = 3z$ is (A) $x^2 + y^2 = 0$ (B) $x^2 + y^2 = 4$ (C) $x^2 - y^2 = 0$ (D) $x^2 - y^2 = 4$	ANS **B**	(CLO-4, Apply)
21.	The image of $\lvert z + 1 \rvert = 1$ under the transformation $w = \frac{1}{z}$ is (A) $x^2 + y^2 + 2x = 0$ (B) $x^2 + y^2 + 2y = 0$ (C) $x^2 + y^2 - 2x = 0$ (D) $x^2 - y^2 - 2y = 0$	ANS **A**	(CLO-4, Apply)
22.	The transformation $w = \frac{1}{z}$ is known as (A) magnification (B) reflection (C) rotation (D) inversion	ANS **D**	(CLO-4, Remember)
23.	If the image of a point z under the transformation $w = f(z)$ is itself, then the point is called (A) fixed point (B) critical point (C) singular point (D) regular point	ANS **A**	(CLO-4, Remember)
24.	The function $f(z) = \bar{z}$ is (A) nowhere differentiable (B) analytic (C) constant (D) singular	ANS **A**	(CLO-4, Apply)

25.	The function $f(z) = \sin z$ is (A) nowhere differentiable (B) analytic (C) not analytic (D) constant	ANS **B**	(CLO-4, Apply)
26.	A mapping that preserves angles between oriented circles both in magnitude and in sense is called a ______________ mapping. (A) isogonal (B) conformal (C) regular (D) formal	ANS **B**	(CLO-4, Remember)
27.	A transformation that preserves angles between every pair of curves through a point only in magnitude, but not in direction is said to be ____________ at that point. (A) isogonal (B) conformal (C) regular (D) formal	ANS **A**	(CLO-4, Remember)
28.	The real part of $f(z) = e^{2z}$ is (A) $e^x \cos y$ (B) $e^x \sin y$ (C) $e^{2x} \cos 2y$ (D) $e^{2x} \sin 2y$	ANS **C**	(CLO-4, Apply)
29.	The points at which the function $f(z) = \frac{1}{z^2-1}$ fails to be analytic are (A) $z = \pm i$ (B) $z = \pm 1$ (C) $z = \pm 2$ (D) $z = \pm 3$	ANS **B**	(CLO-4, Apply)
30.	The transformation $w = z + a$, where a is a complex constant represents (A) magnification (B) reflection (C) translation (D) inversion	ANS **C**	(CLO-4, Remember)
31.	The fixed points of the transformation $w = \frac{5z+4}{z+5}$ are (A) $\pm i$ (B) ± 1 (C) ± 2 (D) ± 3	ANS **C**	(CLO-4, Apply)
32.	The harmonic conjugate of $u = e^x \cos y$ is (A) $e^x \sin y$ (B) $e^{2x} \sin y$ (C) $e^{2x} \cos 2y$ (D) $e^{2x} \sin 2y$	ANS **A**	(CLO-4, Apply)
33.	The invariant points of the transformation $w = \frac{1-iz}{z-i}$ are (A) $\pm i$ (B) ± 1 (C) ± 2 (D) ± 3	ANS **B**	(CLO-4, Apply)

34.	The real part of $f(z) = \log z$ is (A) $u = \log r$ (B) $u = \log x$ (C) $u = \log y$ (D) $u = \log \theta$	ANS **A**	(CLO-4, Apply)
35.	If $f(z) = x + y + i\,(cy - x)$ is analytic, then the value of c is (A) $\pm i$ (B) 1 (C) 2 (D) -1	ANS **B**	(CLO-4, Apply)
36.	The critical points of the transformation $w = z + \frac{1}{z}$ are (A) $\pm i$ (B) ± 1 (C) ± 2 (D) ± 3	ANS **B**	(CLO-4, Apply)

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Unit 5 – Complex Integration

Part – B (Each question carries 3 Marks)

1. Evaluate $\int_C e^{\frac{1}{z}}\, dz$ where C is | z – 2 |=1 by Cauchy's integral theorem.

(A) π i (B) 4 π i (C) 0 (D) 2 π i

Solution

$e^{\frac{1}{z}}$ is analytic inside and on C.

Hence by Cauchy's Integral theorem, $\int_C e^{\frac{1}{z}}\, dz = 0$.

Answer: (C)

2. Evaluate $\int_C \frac{1}{2z-3} dz$ where C is | z | = 1 by Cauchy's integral formula.

(A) 1 (B) 4 π i (C) 0 (D) 2 π i

Solution

Here $a = \frac{3}{2}$ lies outside | z | = 2.

By Cauchy's Integral formula,

$$\int_C \frac{1}{2z-3} dz = 0$$

Answer: (C)

3. Evaluate $\int_C \frac{1}{(z-3)^2} dz$ where C is | z | = 1 by Cauchy's integral formula.

(A) 1 (B) 4π i (C) 0 (D) 2π i

Solution

Here $a = 3$ lies outside | z | = 1.

By Cauchy's Integral formula,

$$\int_C \frac{1}{(z-3)^2} dz = 0$$

Answer: (C)

4. Evaluate $\int_C \frac{2z}{z-1} dz$ where C is | z | = 2 by Cauchy's integral formula.

(A) 1 (B) 4π i (C) 0 (D) 2π i

Solution

Here $f(z) = 2z$ and $a = 1$ lies inside | z | = 2.

By Cauchy's Integral formula,

$$\int_C \frac{2z}{z-1} dz = 2\pi i \; f(1) = 2\pi i(2) = 4\pi i$$

Answer: (B)

5. Evaluate $\int_C \frac{\cos \pi z}{z-1} dz$ where C is | z | = 3.

(A) -2π i (B) 4π i (C) 0 (D) 2π i

Solution

Here $f(z) = \cos \pi z$ and $a = 1$ lies inside | z | = 3.

By Cauchy's Integral formula,

$$\int_C \frac{\cos \pi z}{z-1} dz = 2\pi i \; f(1) = 2\pi i(-1) = -2\pi i$$

Answer: (A)

6. Evaluate $\int_C \frac{e^{-z}}{z+1}\,dz$ where C is | z | = 1.5.

(A) $-2\pi i e$ (B) 4π i (C) 0 (D) $2\pi i e$

Solution

Here $f(z) = e^{-z}$ and $a = -1$ lies inside | z | = 1.5.

By Cauchy's Integral formula,

$$\int_C \frac{e^{-z}}{z+1}\,dz = 2\pi i\ f(-1) = 2\pi i e$$ **Answer: (D)**

7. Evaluate $\int_C \frac{1}{z e^{z}}\,dz$ where C is | z | = 1.

(A) $-2\pi i e$ (B) 2π i (C) 0 (D) $2\pi i e$

Solution

Here $f(z) = \frac{1}{e^{z}}$ and $a = 0$ lies inside | z | = 1.

By Cauchy's Integral formula,

$$\int_C \frac{\frac{1}{e^{z}}}{z}\,dz = 2\pi i\ f(0) = 2\pi i 1 = 2\pi i$$ **Answer: (B)**

8. Evaluate $\int_C \frac{z+1}{z(z-2)}\,dz$ where C is | z | = 1.

(A) $-2\pi i e$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $2\pi i e$

Solution

Here $f(z) = \frac{z+1}{z-2}$ and $a = 0$ lies inside | z | = 1.

By Cauchy's Integral formula,

$$\int_C \frac{\frac{z+1}{z-2}}{z}\,dz = 2\pi i\ f(0) = -\frac{1}{2}$$

Answer: (C)

9. Evaluate $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)}\,dz$ where C is | z | = 1.5.

(A) 1 (B) 4π i (C) 0 (D) 2π i

Solution

Here $f(z) = \frac{\cos \pi z^2}{z-2}$ and $a = 1$ lies inside | z | = 1.5.

By Cauchy's Integral formula,

$$\int_C \frac{\frac{\cos \pi z^2}{z-2}}{z-1}\,dz = 2\pi i\ f(1) = 2\pi i\ \frac{\cos \pi}{1-2} = 2\pi i$$

Answer: (D)

10. Evaluate $\int_C \frac{1}{(z+1)(z-2)^2}\,dz$ where C is | z | = 1.5.

(A) 1 (B) $\frac{4\pi i}{9}$ (C) 0 (D) $\frac{2\pi i}{9}$

Solution

Here $f(z) = \frac{1}{(z-2)^2}$ and $a = -1$ lies inside | z | = 1.5.

By Cauchy's Integral formula,

$$\int_C \frac{\frac{1}{(z-2)^2}}{z+1}\,dz = 2\pi i\ f(-1) = 2\pi i\ \frac{1}{9} = \frac{2\pi i}{9}$$

Answer: (D)

11. Evaluate $\int_C \frac{z}{(z-1)^3} dz$ where C is | z | = 2 by Cauchy's integral formula for derivatives.

(A) 1 (B) 4π i (C) 0 (D) 2π i

Solution

Here $f(z) = z$ and $a = 1$ lies inside | z | = 2.

By Cauchy's Integral formula for derivatives,

$$\int_C \frac{z}{(z-1)^3} dz = \frac{2\pi i}{2!} f''(1) = \pi i(0) = 0$$

Answer: (C)

12. Calculate the residue at z = 0 for the function $f(z) = \frac{3-e^{2z}}{z}$.

(A) 1 (B) 2 (C) 3 (D) – 2

Solution

$$\operatorname{Re}s[f(z), a] = \lim_{z\to a} (z-a) f(z)$$

$$\operatorname{Re}s[f(z), 0] = \lim_{z\to 0} (z-0) \frac{(3-e^{2z})}{z} = 2$$

Answer: (B)

13. Calculate the residue at z = i for the function $f(z) = \frac{1}{z^2+1}$.

(A) 1 (B) 2 (C) $\frac{1}{2i}$ (D) – 2

Solution

$$\operatorname{Re}s[f(z), a] = \lim_{z\to a} (z-a) f(z)$$

$$\operatorname{Re}s[f(z), i] = \lim_{z\to i} (z-i) \frac{1}{(z+i)(z-i)} = \frac{1}{2i}$$

Answer: (C)

14. Calculate the residue at z = $-i$ for the function $f(z) = \dfrac{z}{z^2+1}$.

(A) 1 (B) 2 (C) 1/2 (D) – 2

Solution

$$\text{Re}\,s\left[f(z),a\right] = \lim_{z\to a}(z-a)\,f(z)$$

$$\text{Re}\,s\left[f(z),-i\right] = \lim_{z\to -i}\ (z+i)\frac{z}{(z+i)(z-i)} = \frac{1}{2}$$

Answer: (C)

15. Calculate the residue of the function $f(z) = \dfrac{e^{2z}}{(z+1)^2}$ at its pole.

(A) 2e (B)3e (C) 2e^{-2} (D)2e^2

Solution

z = – 1 is a pole of order 2.

$$\text{Re}\,s\left[f(z),a\right] = \frac{1}{(n-1)!}\lim_{z\to a}\frac{d^{n-1}}{dz^{n-1}}(z-a)^n\ f(z)$$

$$\text{Re}\,s\left[f(z),-1\right] = \frac{1}{(2-1)!}\lim_{z\to -1}\frac{d^{2-1}}{dz^{2-1}}(z+1)^2\ \frac{e^{2z}}{(z+1)^2} = \frac{1}{1!}\lim_{z\to -1}\frac{d}{dz}e^{2z} = 2e^{-2}$$

Answer: (C)

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DEPARTMENT OF MEATHEMATICS

18MAB102T ADVANCED CALCULUS & COMPLEX ANALYSIS

UNIT –IV ANALYTIC FUNCTIONS

Sl.No.	Tutorial Sheet -1	Answers
Part – A		
1	Test whether $f(z)=z^3$ is analytic.	Analytic everywhere
2	If $f(z)$ and $f(\bar{z})$ are analytic function of z, then prove that $f(z)$ is constant.	
3	Show that the function $e^x(\cos y+i\sin y)$ is analytic and find its derivative.	$f'(z)=e^z$
4	Prove that if v is harmonic conjugate of u and u is harmonic conjugate of v, then $f(z)$ is constant.	
5	Show that the function $u=2\log(x^2+y^2)$ is harmonic.	
Part – B		
6	Show that an analytic function with (i) constant real part is constant (ii) constant modulus is constant.	
7	If $f(z)=u+iv$ is an analytic function of z, show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2=4\lvert f'(z)\rvert^2$	
8	If $f(z)=u+iv$ is an analytic function of z, show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\lvert f(z)\rvert=0$	
9	Show that the function $u=e^x\cos y$ is harmonic and find the harmonic conjugate of u.	$v=e^x\sin y$

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	18MAB102T ADVANCED CALCULUS & COMPLEX ANALYSIS	
	UNIT –IV ANALYTIC FUNCTIONS	
	Tutorial Sheet -2	**Answers**
	Part – A	
1	Find the image of the circle |z|=3 under the transformation w=2z	6
2	Find a function w such that w=u+iv is analytic, if $u = e^x \sin y$	$f(z) = -ie^z + c$
3	Determine the analytic function u+iv whose real part $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$	$f(z) = z^3 + 3z^2 + c$
	Part – B	
4	Find the analytic function $f(z) = u + iv$ if $u - v = e^x(cosy - siny)$	$f(z) = e^z + c$
5	Find the analytic function $f(z) = u + iv$ if $u - v = \frac{sin2x}{cosh2y - cos2x}$	$f(z) = \frac{\cot z}{1+i} + c$
6	Determine the region D' of the w-plane into which the triangular region D enclosed by the lines x=0, y=0, x+y=1 is transformed under the transformation w=2z	
7	Find an analytic function $f(z) = u + iv$, given that $2u + 3v = \frac{\sin 2x}{\cos h2y - \cos x}$	$f(z) = \frac{(2+3i)\cot z}{13} + c$

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	18MAB102T ADVANCED CALCULUS & COMPLEX ANALYSIS	
	UNIT –IV Mapping and Bilinear Transformation	
Sl.No.	**Tutorial Sheet -3**	**Answers**
	Part – A	
1	Find the images of the $\|z+1\|=1$ where the map $w=\frac{1}{z}$	$u=-\frac{1}{2}$
2	Find the images of the $\|z-2i\|=2$ where the map $w=\frac{1}{z}$	$v=-\frac{1}{4}$
3	Describe about w=$\frac{1}{z}$ transformation.	
4	Define Bilinear Transformation	
	Part – B	
5	Find the bilinear map which maps the points $z=1,i,-1$ onto the points $w=i,0,-i$	$\frac{-z+i}{z+i}$
6	Find the bilinear map which maps the points $z=\infty,i,0$ onto the points $w=0,-i,\infty$	$\frac{1}{z}$
7	Find the bilinear map which maps the points $z=0,1,\infty$ onto the points w = i, 1, -i	$\frac{z+i}{1+iz}$

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DEPARTMENT OF MEATHEMATICS

18MAB102T ADVANCED CALCULUS & COMPLEX ANALYSIS

UNIT - V :
Complex Integration
Tutorial Sheet
13

Sl.No.	Questions	Answer
	Part – A	
1	Evaluate $\oint_C \frac{e^{-z}}{z-1}dz$ where C is a circle (i) $\lvert z\rvert = 2$ (ii) $\lvert z\rvert = \frac{1}{4}$.	(i) $2\pi i e^{-1}$ (ii) 0
2	Evaluate $\oint_C \frac{dz}{z^2-2z}$ where C is a circle $\lvert z-2\rvert = 1$.	πi
3	Evaluate $\oint_C z^2.e^{\frac{1}{z}}\,dz$ where C is the circle $\lvert z\rvert = 1$.	$\frac{\pi i}{3}$
4	Obtain Taylor's series of $f(z) = \frac{z-1}{z^2}$ in powers of $z-1$.	$\sum_{n=1}^{\infty}(-1)^{n-1}n(z-1)^n$
5	Obtain Laurent's series of $f(z) = \frac{1}{z(z-1)}$ in $\lvert z\rvert < 1$ and $\lvert z\rvert > 1$.	(i) $-\frac{1}{z}-\sum_{n=0}^{\infty}z^n$ (ii) $-\frac{1}{z}+\sum_{n=1}^{\infty}\left(\frac{1}{z}\right)^n$
	Part – B	
6	Evaluate $\oint_C \frac{\cos \pi z^2}{(z-1)(z-2)}dz$ where C is the circle $\lvert z\rvert = \frac{3}{2}$.	$2\pi i$
7	Evaluate $\oint_C \frac{z+4}{z^2+2z+5}dz$ where C is the circle $\lvert z+1+i\rvert = 2$.	$\frac{\pi}{2}(2i-3)$
8	Expand $\frac{1}{(z-1)(z-2)}$ in the region $0<\lvert z-1\rvert<1$	$-\frac{1}{z-1}-\sum_{n=0}^{\infty}(z-1)^n$
9	Expand $\frac{7z-2}{(z+1)z(z-2)}$ in the region $1<\lvert z+1\rvert<3$	$\frac{-2}{z+1}+\sum_{n=2}^{\infty}\frac{1}{(z+1)^n}-\frac{2}{3}\sum_{n=0}^{\infty}\left(\frac{z+1}{3}\right)^n$
10	Represent the function $\frac{4z+3}{z(z-3)(z+2)}$ in Laurent's series (i) Within $\lvert z\rvert = 2$ (ii) in the annular region between $\lvert z\rvert = 2$ and $\lvert z\rvert = 3$ and (iii) exterior to $\lvert z\rvert = 3$.	(i) $\frac{z^{-1}}{2}-\frac{5}{3}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n-\frac{1}{4}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{2}\right)^n$ (ii) $\frac{z^{-1}}{2}-\frac{5}{3}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n-\frac{1}{2z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{z}\right)^n$ (iii) $\frac{z^{-1}}{2}-\frac{5}{z}\sum_{n=0}^{\infty}\left(\frac{3}{z}\right)^n-\frac{1}{2z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{z}\right)^n$

	SRM Institute of Science and Technology Kattankulathur **DEPARTMENT OF MEATHEMATICS** **18MAB102T ADVANCED CALCULUS & COMPLEX ANALYSIS** **UNIT - V :** **Taylor's & Laurent' series, Singularity, Poles and Residue** **Tutorial Sheet 14**	
Sl.No.	**Questions**	**Answer**
	Part – A	
1	Find the Taylor's series expansion of $f(z)=\dfrac{z+3}{(z-1)(z-4)}$ about $z=2$ and also determine the region of convergence.	$\sum_{n=0}^{\infty}\left\{\frac{4}{3}(-1)^{n+1}-\frac{7}{6}.\frac{1}{2^n}\right\}(z-2)^n$
2	Obtain the series for $\dfrac{1}{z-3}$ valid in (i)$\lvert z\rvert<3$,(ii)$\lvert z\rvert>3$.	(i) $-\frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n$ (ii) $\frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{3}{z}\right)^n$
3	Expand $f(z)=\dfrac{z}{(z-1)(z-3)}$ as Laurent's series valid in the region $1<\lvert z\rvert<3$	$-\frac{1}{2z}\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^n-\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n$
4	Find the residues of $\dfrac{e^z}{z^8}$.	$\frac{1}{7!}$
5	Find the residue of $\dfrac{1-\cos(z)}{z^3}$.	1
	Part – B	
6	Find the Laurent's series of $f(z)=\dfrac{1}{z(1-z)}$ valid in the region (i)$\lvert z+1\rvert<1$,(ii)$\lvert z+1\rvert>2$.	(i) $-\sum_{n=0}^{\infty}(z+1)^n+\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{z+1}{2}\right)^n$ (ii) $\sum_{n=1}^{\infty}\frac{1}{(z+1)^n}-\frac{1}{(1+z)}\sum_{n=0}^{\infty}\left(\frac{2}{z+1}\right)^n$
7	Find the Laurent's series of $f(z)=\dfrac{z}{(z^2+1)(z^2+4)}$ in the region $1<\lvert z\rvert<2$.	$\frac{1}{3z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{z^2}\right)^n-\frac{z}{12}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z^2}{4}\right)^n$
8	Find the residue at $z=0$ for $f(z)=\dfrac{1+e^z}{\sin z+z\cos z}$ and $f(z)=\dfrac{1}{z^2e^z}$	1,-1
9	Find the residue at each pole of $f(z)=\dfrac{z^2}{(z-1)^2(z+2)}$.	$\frac{4}{9},\frac{5}{9}$
10	Find the residue at $z=0$ for $\operatorname{cosec}^2 z$	0

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Sl.No.	**Questions**	**Answer**
	Part – A	
1	Find the residues of $f(z)=\frac{z}{(z-1)^2}$ at the poles	1
2	Find the residues of $f(z)=\frac{e^z}{z^2+a^2}$ at $z=ai$	$2aie^{ai}$
3	Find the residue of $f(z)=\frac{1-e^z}{z^2}$.	-1
4	Find the residues of $f(z)=\frac{1}{(z^2+1)^2}$.	$-\frac{i}{4},-\frac{i}{4}$
5	Find the residue of $f(z)=\frac{1}{(z^2+a^2)^2}$ at $z=ai$	$-\frac{i}{4a^3}$
	Part – B	
6	Evaluate $\oint_C \frac{z-3}{z^2+2z+5}dz$ where C is the circle $\lvert z+1-i\rvert=2$	$\pi(i-2)$
7	Using Cauchy's residue theorem evaluate $\oint_C \frac{z\sec z}{1-z^2}dz$ where C is the ellipse $4x^2+9y^2=9$	$-2\pi i\sec 1$
8	Show that $\int_0^{2\pi}\frac{d\theta}{1+a\cos\theta}=\frac{2\pi}{\sqrt{1-a^2}},(a^2<1)$.	
9	Evaluate $\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$	$\frac{\pi}{6}$
10	Evaluate $\int_0^{2\pi}\frac{d\theta}{1-2a\cos\theta+a^2}$, $a^2<1$	$\frac{2\pi}{1-a^2}$