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ECE – A

Control Systems –
18ECS201T

18ECS201T - CONTROL SYSTEMS

ASSIGNMENT - I

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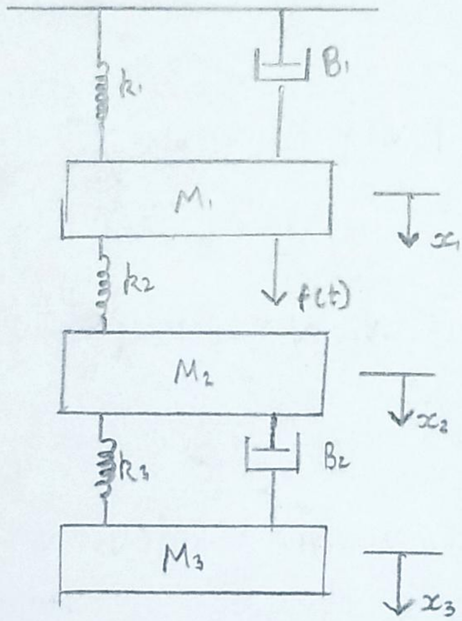
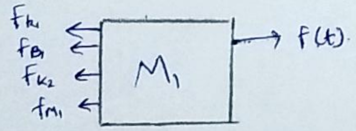
ECE - A.

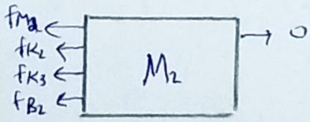
1. Write the differential equations governing the mechanical system in Force - Voltage and Force - Current analogy.

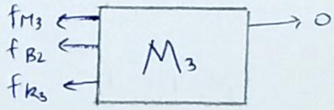


Soln. Given System is a Mechanical Translational System, where the number of nodes = 3.

Free body diagram for each node is,







$$f(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + K_1 x_1(t) + B_1 \frac{d x_1(t)}{dt} + K_2 (x_1(t) - x_2(t)). \quad -(1)$$

$$0 = M_2 \frac{d^2 x_2(t)}{dt^2} + K_2 (x_2(t) - x_1(t)) + K_3 (x_2(t) - x_3(t)) + B_2 \frac{d}{dt} (x_2(t) - x_3(t)). \quad -(2)$$

$$0 = M_3 \frac{d^2 x_3(t)}{dt^2} + B_2 \frac{d}{dt} (x_3(t) - x_2(t)) + K_3 (x_3(t) - x_2(t)) \quad -(3).$$

In eqs (1), (2), and (3) can be rewritten as,

$$\frac{d^2x}{dt^2} = \frac{dv}{dt},$$

$$\frac{dx}{dt} = V, \text{ and}$$

$$x = \int v\,dt.$$

(1), becomes,

$$f(t) = M_1\frac{dV_1}{dt} + B_1V_1 + K_1\int V_1 \cdot dt + K_2\int (V_1 - V_2)\cdot dt$$

(2) becomes,

$$0 = M_2\frac{dV_2}{dt} + B_2(V_2 - V_3) + K_2\int (V_2 - V_1)\cdot dt + K_3\int (V_2 - V_3)\cdot dt$$

(3) becomes,

$$0 = M_3\frac{dV_3}{dt} + B_2(V_3 - V_2) + K_3\int (V_3 - V_2)\cdot dt.$$

Converting the above equations to Force - Voltage Analogy:

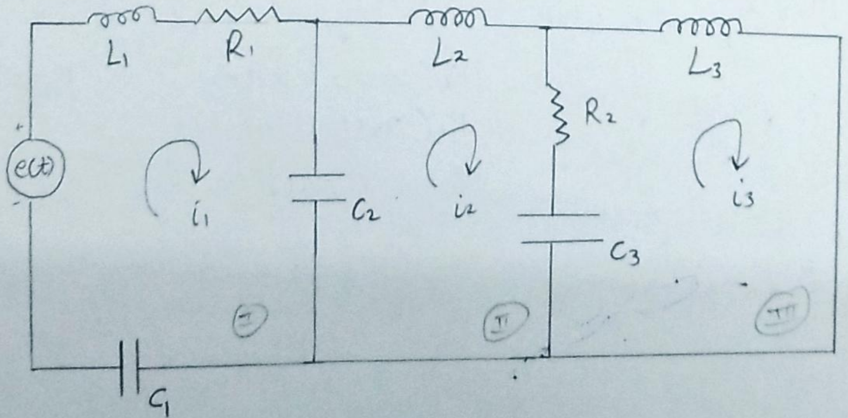
$f(t)$ - $e(t)$.

B_1, B_2 - R_1, R_2

K_1, K_2, K_3 - $1/C_1, 1/C_2, 1/C_3$

M_1, M_2, M_3 - L_1

V_1, V_2, V_3 - i_1, i_2, i_3



By applying KVL, across all three meshes, we get

In mesh (I)

$$e(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1 \cdot dt + \frac{1}{C_2}\int (i_1 - i_2)\cdot dt \quad -(4)$$

In mesh (II)

$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int (i_2 - i_1)\,dt + R_2 (i_2 - i_3) + \frac{1}{C_3}\int (i_2 - i_3)\cdot dt \quad -(5).$$

In mesh - III

$$0 = L_3 \frac{di_3}{dt} + \frac{1}{C_3}\int (i_3 - i_2)\,dt + R_2 (i_3 - i_2). \quad -(6)$$

Comparing Equations (1), (2), (3) and (4), (5), (6) we get the Force - Voltage Analogy.

Converting the Force equations to Force - Current analogy.

$F(t) \rightarrow i(t)$.

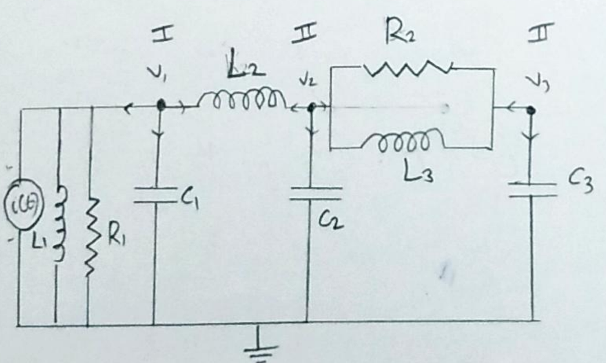
M_1, M_2, M_3 - C_1, C_2, C_3

B_1, B_2, B_3 - $1/R_1, 1/R_2, 1/R_3$

K_1, K_2, K_3 - $1/L_1, 1/L_2, 1/L_3$.

V_1, V_2, V_3 - V_1, V_2, V_3

(velocity) (voltage)



Applying KCL across all nodes we get..

In node (I).

$$i(t) = C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int V_1\,dt + \frac{1}{L_2}\int (V_1 - V_2)\,dt \quad -(7)$$

In node (II)

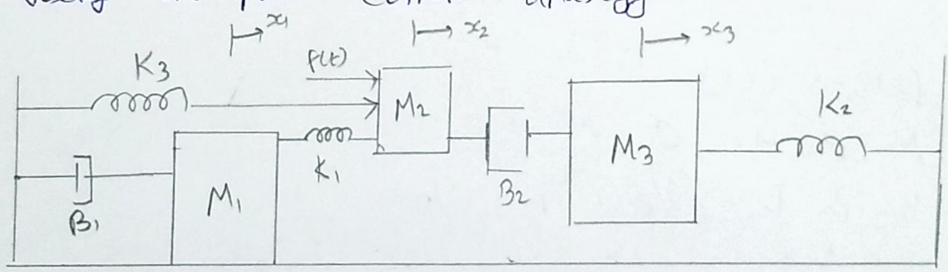
$$\cancel{i(t)=}\ 0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_2}\int (V_2 - V_1)\,dt + \frac{V_2 - V_3}{R_2} + \frac{1}{L_3}\int (V_2 - V_3)\,dt \quad -(8)$$

In node (III)

$$0 = C_3 \frac{dV_3}{dt} + \frac{V_3 - V_2}{R_2} + \frac{1}{L_3}\int (V_3 - V_2)\cdot dt \quad -(9)$$

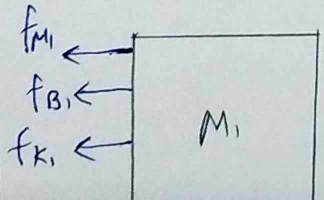
Comparing Eqs (1), (2) (3) and (7), (8), (9) we get the force-current analogy.

2 Obtain the mathematical model of the mechanical System in Force Voltage and Force Current analogy.

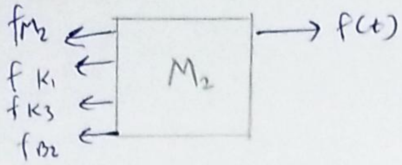


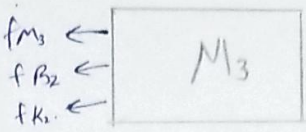
Soln. No of nodes = 3. ∴ no of equations = 3.

Free body diagram of each node is given as.



$$0 = M_1 \frac{d^2 x_1}{dt} + B_1 \frac{dx_1}{dt} + K_1 (x_1 - x_2) \quad -(1)$$





$$f(t) = M_2\frac{d^2 x_2}{dt} + K_3 x_2 + K_1(x_2 - x_1) + B_2\frac{d}{dt}(x_2 - x_3) \quad -(2)$$

$$0 = M_3\frac{d^2 x_3}{dt} + K_2 x_3 + B_2\frac{d}{dt}(x_3 - x_2) \quad -(3)$$

Eqs (1), (2) and (3) can be rewritten as.

(1) becomes.

$$0 = M_1\frac{dV_1}{dt} + B_1 V_1 + K_1\int(V_1 - V_2)\,dt$$

(2) becomes.

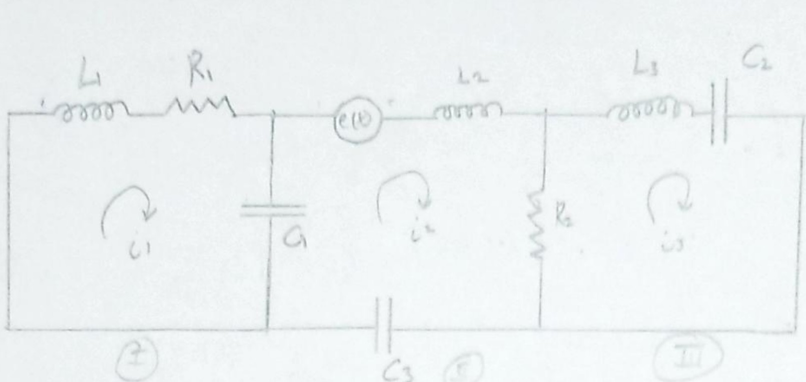
$$f(t) = M_2\frac{dV_2}{dt} + K_3\int V_2\,dt + K_1\int(V_2 - V_1)\,dt + B_2(V_2 - V_3).$$

(3) becomes.

$$0 = M_3\frac{dV_3}{dt} + K_2\int V_3\,dt + B_2(V_3 - V_2).$$

Converting the above equations to force-voltage analogy.

$M_1, M_2, M_3 - L_1, L_2, L_3$
$B_1, B_2 - R_1, R_2$
$K_1, K_2, K_3 - 1/C_1, 1/C_2, 1/C_3$
$f(t) - e(t).$
$v_1, v_2, v_3 - i_1, i_2, i_3.$



Applying KVL to all the meshes, we get.

In Mesh (I).

$$0 = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int (i_1 - i_2)\, dt \quad - (4)$$

In Mesh (II).

$$e(t) = L_2 \frac{di_2}{dt} + \frac{1}{C_3}\int i_2 \cdot dt + \frac{R_2}{\cancel{R_4}}(i_2 - i_3) \quad - (5)$$
$$+ \frac{1}{C_1}\int (i_2 - i_1)\, dt$$

In Mesh (III)

$$0 = L_3 \frac{di_3}{dt} + \frac{1}{C_2}\int i_3 \cdot dt + R_2(i_3 - i_2) \quad - (6)$$

Comparing eqs (1), (2), (3) and (4), (5), (6), we get the force - voltage analogy.

Converting the force equations to <u>Force Current Analogy</u>

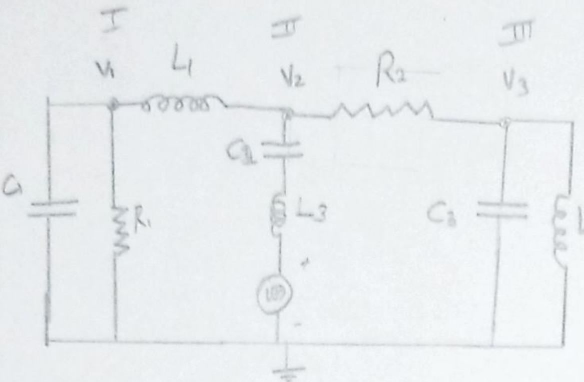
M_1, M_2, M_3 – C_1, C_2, C_3

B_1, B_2 – $1/R_1$, $1/R_2$

K_1, K_2, K_3 – $1/L_1, 1/L_2, 1/L_3$.

$f(t)$ – $i(t)$.

V_1, V_2, V_3 (velocity) – V_1, V_2, V_3 (Voltage).



Applying KCL across all 3 nodes, we get.

in node (I).

$$0 = C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int (V_1 - V_2)\cdot dt \quad -(7)$$

In node (II)

$$i(t) = C_2 \frac{dV_2}{dt} + \frac{1}{L_3}\int V_2\, dt + \frac{1}{L_1}\int (V_2 - V_1)\, dt + \frac{1}{R_2}(V_2 - V_3) \quad -(8)$$

In node (III)

$$0 = C_3 \frac{dV_3}{dt} + \frac{1}{L_2}\int V_3\, dt + \frac{1}{R_2}(V_3 - V_2) \quad -(9)$$

Comparing eqs (1), (2), (3) and (7), (8), (9). we get the force - current analogy.