DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB101J-Electromagnetic Theory, Quantum Mechanics, Waves and Optics

Module I Lecture-15

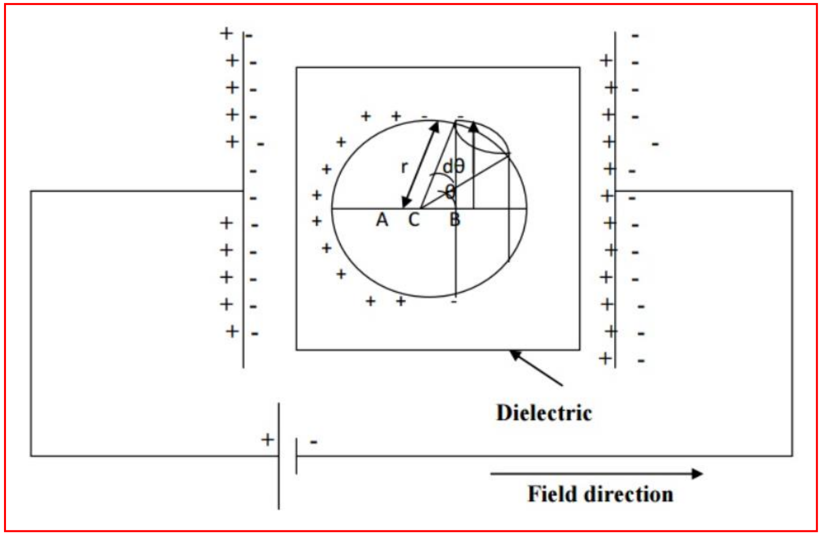
Internal Field in a dielectric and Clausius-Mossotti equation-Derivation

Internal Field in a dielectric

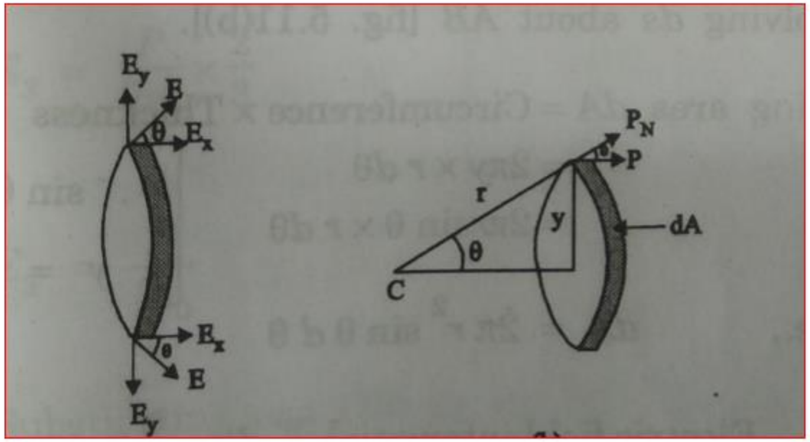
- The electric field acting at an atom in a dielectric know as the internal field or local field E_{int}.

- $E_{int} = E_1 + E_2 + E_3 + E_4$

- E_1 = EF due to charges on the plates of the capacitor.
- E_2 = EF due to polarized charges on the plane surface of the dielectric.
- E_3 = EF due to polarized charges induced on the surface of imaginary spherical cavity.
- E_4 = EF due to permanent dipoles of atoms inside the spherical cavity .



r
dθ
θ
A
C
B
Dielectric
Field direction



E_y
E
θ
E_x
P_N
P
r
y
θ
C
dA

E_1 = EF due to charges on the plates of the capacitor.

When a dielectric medium is polarized due to an electric field E, the displacement vector D is given by

$D = E\varepsilon_0 + P$

Charge density on plates is given by $D = E_1\varepsilon_0$

$E_1\varepsilon_0 = E\varepsilon_0 + P$

$E_1 = E + P/\varepsilon_0$

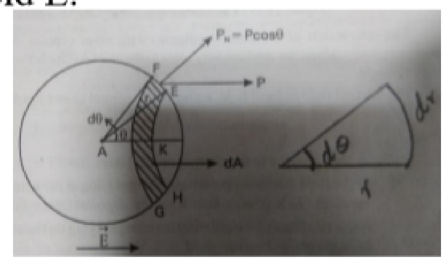
E_2 = EF due to polarized charges on the plane surface of the dielectric.

E_2 is the field due to polarization charges on the external surface of the dielectric . This field acts in a direction opposite to the external field. Therefore, it is given by

$$E_2 = -P/\varepsilon_0$$

E_3 = EF due to polarized charges induced on the surface of imaginary spherical cavity.

Let us consider a small area *dA* on the surface of spherical cavity. It is confined within an angle $d\theta$ at an angle θ in the direction of the Electric field E.



P = q/A (surface charge per unit area). ………1

or

q = P.dA ……………... 2

Polarization is parallel to E. P_N is the components of polarization to the area dA.

$q = P\cos\theta.dA$ …………3

EF intensity at A due to charge q is given by (Coulomb's law)

$E_3 = q/4\pi\varepsilon_0 r^2$ ……………4

$E_3 = P\cos\theta.dA/4\pi\varepsilon_0 r^2$ ……5

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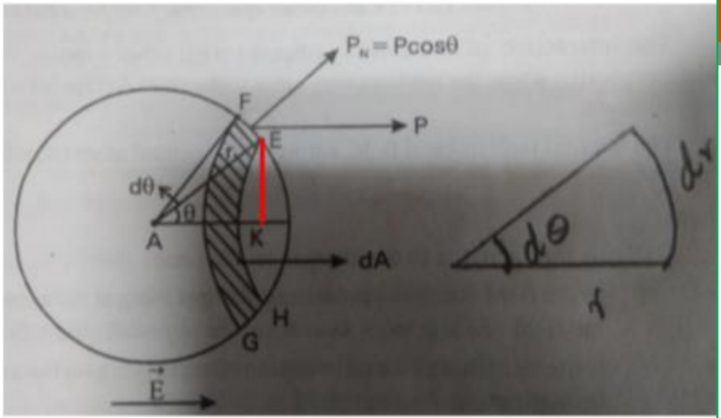
The EF intensity is along the radius r and it is resolved into two components E_x and E_y

The component of intensity parallel to the EF direction $E_x = E\cos\theta$ ……… *6*

Substituting E value in the above equation

$E_x = P\cos\theta dA\cos\theta/4\pi\varepsilon_0 r^2$ ……………. 7

$E_x = P\cos^2\theta dA/4\pi\varepsilon_0 r^2$ …………………..8



Ring area dA = circumference × thickness
Ring area dA = $2\pi \times EK \times EF$ …………9

$\sin\theta = EK/r$
$EK = r\sin\theta$ ……………………………10

$d\theta = dr/r$ = arc/radius
Arc (EF) = $r\,d\theta$ …………………………..11

Therefore $dA = 2\pi r \sin\theta\, r\, d\theta = 2\pi r^2 \sin\theta\, d\theta$ …… 12

The EF intensity due to elemental ring dA is given by = $E_x = P\cos^2\theta dA/4\pi\varepsilon_0 r^2$

Sub 12 in 8 equations

$E_x = P\cos^2\theta \, .2\pi r^2 \sin\theta \, d\theta / 4\pi\varepsilon_0 r^2$ …………………… 13

$E_x = P\cos^2\theta \sin\theta \, d\theta / 2\varepsilon_0$ ……………………………14

EF intensity due to charges present in the whole sphere is obtained by integrating the above equation within the limits 0 to π.

$$= \int_0^{\pi} \frac{P}{2\varepsilon_0} \cos^2\theta . \sin\theta . d\theta$$

$$= \frac{P}{2\varepsilon_0} \int_0^{\pi} \cos^2\theta . \sin\theta . d\theta$$

$$let .. x = \cos\theta \rightarrow dx = -\sin\theta d\theta$$

$$= \frac{P}{2\varepsilon_0} \int_1^{-1} x^2 . dx$$

$$= \frac{-P}{2\varepsilon_0} (\frac{x^3}{3})_1^{-1} \Rightarrow \frac{-P}{2\varepsilon_0} (\frac{-1-1}{3})$$

Note

$\theta = 0, x = 1$

$\theta = \pi, x = -1$

$E_3 = P/3\varepsilon_0$ ………………15

E_4 = EF due to permanent dipoles of atoms inside the spherical cavity.

We have considered that the specimen in non polar dielectric material, at the centre of the specimen the dipole moment is zero and hence the EF intensity at the center is zero due to symmetric structure.

$$E_4 = 0$$

The total internal field is

$$E_{int} = E + P/\varepsilon_0 + -P/\varepsilon_0 + P/3\varepsilon_0 + 0 = E + P/3\varepsilon_0$$

Clausius-Mossotti equation

Consider a dielectric material having cubic structure and assume ionic polarizability and orientation polarizability are zero.

$$\alpha_i = \alpha_0 = 0$$

$$polarization..P = N\mu$$

$$P = N\alpha_e E_i ......where., \mu = \alpha_e E_i$$

$$where., E_i = E + \frac{P}{3\varepsilon_0}$$

$$P = N\alpha_e E_i$$

$$P = N\alpha_e (E + \frac{P}{3\varepsilon_0})$$

$$P = N\alpha_e E + N\alpha_e \frac{P}{3\varepsilon_0}$$

$$P - N\alpha_e \frac{P}{3\varepsilon_0} = N\alpha_e E$$

$$P(1 - \frac{N\alpha_e}{3\varepsilon_0}) = N\alpha_e E$$

$$P = \frac{N\alpha_e E}{(1 - \frac{N\alpha_e}{3\varepsilon_0})} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

We know that the polarization vector P

$$P = \varepsilon_0 E(\varepsilon_r - 1)............(2)$$

from eq^{n}s(1) & (2)

$$\frac{N\alpha_e E}{(1 - \frac{N\alpha_e}{3\varepsilon_0})} = \varepsilon_0 E(\varepsilon_r - 1)$$

$$1 - \frac{N\alpha_e}{3\varepsilon_0} = \frac{N\alpha_e E}{\varepsilon_0 E(\varepsilon_r - 1)}$$

$$1 = \frac{N\alpha_e}{3\varepsilon_0} + \frac{N\alpha_e E}{\varepsilon_0 E(\varepsilon_r - 1)}$$

$$1 = \frac{N\alpha_e}{3\varepsilon_0} + \frac{N\alpha_e}{\varepsilon_0 (\varepsilon_r - 1)}$$

$$1 = \frac{N\alpha_e}{3\varepsilon_0}(1 + \frac{3}{\varepsilon_r - 1})$$

$$\frac{N\alpha_e}{3\varepsilon_0} = \frac{1}{(1 + \frac{3}{\varepsilon_r - 1})}$$

$$\frac{N\alpha_e}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} ...... \rightarrow \text{Classius Mosotti relation}$$