Exercise Problems.

① Find the Fourier transform of $f(x) = \begin{cases} x & \text{if } |x| \le a \\ 0 & \text{if } |x| > a \end{cases}$

Solution : $i\sqrt{2/\pi}\left[\dfrac{\sin sa - as\cos sa}{s^2}\right]$

② Show that the Fourier transform of $f(x) = \begin{cases} |x| & \text{if } |x| < a \\ 0 & \text{if } |x| > a,\ a > 0 \end{cases}$

is $\sqrt{\dfrac{2}{\pi}}\left[\dfrac{as\sin sa + \cos sa - 1}{s^2}\right]$

③ Find the Fourier transform of the function $f(x)$ defined by

$f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| \geq 1. \end{cases}$ Hence prove that

$$\int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos\left(s/2\right)\,ds = \frac{3\pi}{16}.$$

④. Find the Fourier transform of $f(x) = \begin{cases} a-|x| & \text{for } |x| < a \\ 0 & \text{for } |x| > a > 0 \end{cases}$

hence deduce that $\displaystyle\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}.$

⑤ Show that the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$

is $2\sqrt{\dfrac{2}{\pi}}\left(\dfrac{\sin as - as\cos as}{s^3}\right)$. Hence, deduce that

$\displaystyle\int_0^\infty \frac{\sin t - t\cos t}{t^3}\,dt = \frac{\pi}{4}.$ Using Parseval's identity, Show that

$$\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt = \frac{\pi}{15}.$$