

Table 3.1 : Summary of frequency response characteristics of linear phase FIR filters

Impulse Response $h(n)$	Number of samples of $h(n)$ N	The magnitude function of $H(\omega)$
Symmetric	Odd	Symmetric
Symmetric	Even	Antisymmetric
Antisymmetric	Odd	Antisymmetric
Antisymmetric	Even	Symmetric

Table 3.2 : Summary of $|H(\omega)|$ for linear phase FIR filters

Nature of impulse response	Magnitude function
Case I : Impulse response is symmetric and N is odd	$ H(\omega) = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \cos \omega n$
Case II : Impulse response is symmetric and N is even	$ H(\omega) = \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) \cos\left(\omega\left(n - \frac{1}{2}\right)\right)$
Case III : Impulse response is antisymmetric and N is odd	$ H(\omega) = \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \sin \omega n$
Case IV : Impulse response is antisymmetric and N is even	$ H(\omega) = \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) \sin\left(\omega\left(n - \frac{1}{2}\right)\right)$

① When impulse response
Sym. & odd

$$\text{Mag of } |H(\omega)| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \cos \omega n$$

$$\text{Phase } \angle H(\omega) = -\omega\left(\frac{N-1}{2}\right) = -\omega d$$

$$d = \frac{N-1}{2}$$

② When impulse response
Sym. & even

$$\text{Mag } |H(\omega)| = \left[\sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) \cos\left[\omega\left(n - \frac{1}{2}\right)\right] \right]$$

$$\text{Phase } \angle H(\omega) = -\omega\left(\frac{N-1}{2}\right) = -\omega d;$$

$$d = \frac{N-1}{2}$$

③ Antisym. & odd

$$|H(\omega)| = \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \sin \omega n$$

$$\angle H(\omega) = \frac{\pi}{2} - \omega \frac{N-1}{2} = \beta - \omega d$$

$$\beta = \frac{\pi}{2} \text{ \& } d = \frac{N-1}{2}$$

④ Antisym. & even

$$|H(\omega)| = \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) \sin\left[\omega\left(n - \frac{1}{2}\right)\right]$$

$$\angle H(\omega) = \frac{\pi}{2} - \omega \frac{N-1}{2} = \beta - \omega d$$

$$\beta = \frac{\pi}{2} \text{ \& } d = \frac{N-1}{2}$$