SRM University
Department of Mathematics
Complex Integration- Multiple Choice questions
UNIT V

Slot-C

1. A contour integral is an integral along a ---------------- curve.
    a. Open Curve
    b. Closed curve
    c. Simple closed curve
    d. Multiple curve

    Answer: c. Simple closed curve

2. If f(z) is analytic inside and on C , the value of $\oint_C f(z)\, dz$,where C is the simple closed curve is
    a. f(a)
    b. $2\pi i$f(a)
    c. πif(a)
    d. 0

    Answer: d. 0

3. If f(z) is analytic inside and on C , the value of $\oint_C \frac{f(z)}{(z-a)^n}\, dz$,where C is the simple closed curve and a is any point within c is
    a. $2\pi i \frac{f^n(a)}{n!}$
    b. $2\pi i\, f(a)$
    c. $2\pi i \frac{f^{n-1}(a)}{(n-1)!}$
    d. 0

    Answer: c. $2\pi i \frac{f^{n-1}(a)}{(n-1)!}$

4. The value of $\oint_C \frac{\sin z}{z+1}\, dz$ where C is the circle $|z| = \frac{1}{3}$ is
    a. 0

b. $2\pi i$
c. $\frac{\pi}{2}i$
d. πi

Answer: a. 0

5. The value of $\oint_C \frac{e^z}{(z-2)^2}\, dz$ where C is the circle $|z| = 3\ is$
   a. 0
   b. $2\pi i e^{-2}$
   c. $2\pi i e^{2}$
   d. $4\pi i e^{-2}$

   Answer: c. $2\pi i e^2$

6. The value of $\oint_C \frac{z}{2z-1}\, dz$ where C is the circle $|z| = 1\ is$
   a. 0
   b. $2\pi i$
   c. $\frac{\pi}{2}i$
   d. πi

   Answer: c. $\frac{\pi}{2}i$

7. The value of $\oint_C \frac{1}{(z-3)^2}\, dz$ where C is the circle $|z| = 1\ is$
   a. 0
   b. $2\pi i$
   c. $\frac{\pi}{2}i$
   d. πi

   Answer: a. 0

8. Let $C_1: |z-a| = R_1$ and $C_2: |z-a| = R_2$be two concentric circles($R_2 > R_1$), the annular region is defined as
   a. Within C_1
   b. Within C_2

c. Within C_2 and outside C_1
d. Within C_1 and outside C_2

Answer: c. Within C_2 and outside C_1

9. $The\ part\ \sum_{n=0}^{\infty} a_n(z-a)^n$ consisting of positive integral powers of (z - a) is called as
   a. The analytic part of the Laurent's series
   b. The principal part of the Laurent's series
   c. The real part of the Laurent's series
   d. The imaginary part of the Laurent's series

   Answer: a. The analytic part of the Laurent's series

10. Let $C_1: |z-a| = R_1$ and $C_2: |z-a| = R_2$ be two concentric circles ($R_2 < R_1$), the f(z) can be expanded as a Laurent's series if
    a. f(z) is analytic within C_2
    b. f(z) is not analytic within C_2
    c. f(z) is analytic in the annular region
    d. f(z) is not analytic in the annular region

    Answer: c. f(z) is analytic in the annular region

11. Expansion of $\frac{1-\cos z}{z}$ in Laurent's series about $z = 0$ is
    a. $\frac{z}{2!} - \frac{z^3}{4!} + \frac{z^5}{6!} - \cdots$
    b. $\frac{z^2}{2!} - \frac{z^4}{4!} + \frac{z^6}{6!} - \cdots$
    c. $\frac{z}{1!} - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots$
    d. $\frac{z}{2!} + \frac{z^3}{4!} - \frac{z^5}{6!} + \cdots$

    Answer: a. $\frac{z}{2!} - \frac{z^3}{4!} + \frac{z^5}{6!} - \cdots$

12. The annular region for the function f(z) = $\frac{1}{z^2-3z+2}$ is

a. $0 < |z| < 1$
b. $1 < |z| < 2$
c. $2 < |z| < 3$
d. $|z| < 3$

Answer :b. $1 < |z| < 2$

13. The Laurent's series expansion $1 + \frac{3}{z}\sum\frac{(-1)^n 2^n}{z^n} - \sum\frac{(-1)^n 3^n}{z^n}$ for the function $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ is valid in the region

a. $|z| < 3$
b. $|z| < 2$
c. $2 < |z| < 3$
d. $|z| > 3$

Answer :d. $|z| > 3$

14. If f(z) is not analytic at $z = z_o$ and there exists a neighborhood of $z = z_o$ containing no other singularity, then

a. The point $z = z_o$ is isolated singularity of f(z)
b. The point $z = z_o$ is a zero point of f(z)
c. The point $z = z_o$ is nonzero of f(z)
d. The point $z = z_o$ is non isolated singularity of f(z)

Answer : a. The point $z = z_o$ is isolated singularity of f(z)

15. If f(z) = $e^{\frac{1}{z+1}}$ then

a. z = -1 is removable singularity
b. z = -1 is pole of order 2
c. z = -1 is an essential singularity
d. z = -1 is zero of f(z)

Answer : c. z = -1 is an essential singularity

16. Let z = a is a simple pole for $f(z) = \frac{P(z)}{Q(z)}$, then the Residue of $f(z)$ is

a. $\frac{P'(a)}{Q(a)}$
b. $\frac{P(a)}{Q(a)}$
c. $\frac{P'(a)}{Q'(a)}$
d. $\frac{P(a)}{Q'(a)}$

Answer : d. $\frac{P(a)}{Q'(a)}$

17. Let z = a is a pole of order 3 for f(z), then the residue is

a. $\lim_{z \to a} [(z - a)f(z)]$
b. $\lim_{z \to a} [(z - a)f''(z)]$
c. $\lim_{z \to a} \frac{1}{2!} \frac{d^2}{dz^2} [(z - a)^3 f(z)]$
d. $\lim_{z \to a} \frac{1}{3!} \frac{d^3}{dz^3} [(z - a)^3 f(z)]$

Answer: c. $\lim_{z \to a} \frac{1}{2!} \frac{d^2}{dz^2} [(z - a)^3 f(z)]$

18. The residue of $f(z) = \frac{z}{(z-2)}$ is

a. $2\pi i$
b. 1
c. 2
d. 0

Answer: c. 2

19. The residue of $f(z) = \frac{1}{(z^2+1)^2}$ at $z = i$ is

a. 4i
b. 1/4i
c. 0
d. 1/2i

Answer :b. 1/4i

20. If f(z) = $\frac{sinz-z}{z^3}$, then

a. z= 0 is a simple pole
b. z= 0 is a pole of order 2
c. z= 0 is a removable singularity
d. z= 0 is a zero of f(z)

Answer: c. z= 0 is a removable singularity

21. The value of the integral $\oint_C \frac{1}{ze^z} dz$ where $|z| = 1$ is

a. $2\pi i$
b. $\frac{\pi}{2} i$
c. πi
d. 0

Answer: a. $2\pi i$

22. If $f(z) = \frac{1}{z} + [2 + 3z + 4z^2 + \cdots]$ then the residue of $f(z)$ at z=0 is

a. 1
b. -1
c. 0
d. -2

Answer: a. 1

23. If the integral $\oint_0^{2\pi} \frac{d\theta}{13+5\cos\theta} = \oint_C f(z)dz$, C is $|z| = 1$, then
(A) $z = -i/5$ lies inside C and
(B) $z = -5i$ lies outside C. Which of the following is true.

a. Both A and B
b. Only A
c. Only B
d. Neither A nor B

Answer: a. Both A and B

24. If the integral $\oint_{-\infty}^{\infty} \frac{\cos mx}{(x^2+1)^2} dx \ , m > 0$, then
(A) $z = i$ double pole lies in the upper half of the z-plane and
(B) $z = -i$ double pole does not lie in the upper half of the z-plane.
Which of the following is true.

a. Both A and B
b. Only A
c. Only B
d. Neither A nor B

Answer: a. Both A and B

25. If $f(z)$ be continuous function such that $|f(z)| \to 0$ as $|z| \to \infty$, for C is the semicircle $|z| = R$ above the real axis, then

a. $\oint_C e^{-imz} f(z) dz \to \infty \ as \ R \to \infty$.
b. $\oint_C e^{imz} f(z) dz \to 0 \ as \ R \to \infty$.
c. $\oint_C e^{imz} f(z) dz \to 0 \ as \ R \to 0$.
d. $\oint_C f(z) dz \to \infty \ as \ R \to 0$.

Answer : b. $\oint_C e^{imz} f(z) dz \to 0 \ as \ R \to \infty$.