SRM INSTITUTE OF SCIENCE & TECHNOLOGY (Deemed to be University u/s 3 of UGC Act, 1956)

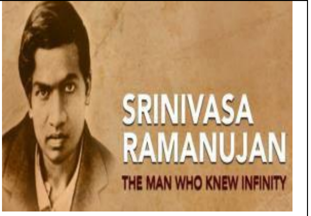
SRM Institute of Science and Technology Kattankulathur

DEPARTMENT OF MATHEMATICS

18MAB203T Probability and Stochastic Processes

Module – II: Two dimensional Random Variables

Tutorial Sheet - IV



Sl.No.	Questions	Answer

Part – B

1 The joint probability distribution of (X, Y) is given below:

X \ Y	1	2	3
0	3k	6k	9k
1	5k	8k	11k
2	7k	10k	13k

(i) Find the value of k?
(ii) Find the Marginal Distribution?

Answer: (i) K=1/72

Marginal distribution of Y
15/72, 24/72, 33/72

X=x	0	1	2
P(X=x)	18/72	24/72	30/72

2 The two-dimensional random variable (X, Y) has the joint density function P (x, y) = $\frac{x+2y}{27}, x=0,1,2;\ y=0,1,2$

(i) Find the conditional distribution of Y for X=x
(ii) Find the conditional distribution of X given Y=1

Answer: (i) 0, 1/3, 2/3
(ii) 2/9, 1/3, 4/9

3 Let X and Y have the following joint distribution

Y \ X	2	4
1	0.10	0.15
3	0.20	0.30
5	0.10	0.15

Show that (X, Y) are independent.

4 The joint p.d.f of the two-dimensional random variable (X, Y) is given by

$$f(x,y)=\begin{cases} e^{-(x+y)} & x\ge 0,\ y\ge 0 \\ 0 & otherwise \end{cases}$$

Find (i) Marginal densities of X and Y.
(iii) P (X< 1)

Answer: (i) $f_X(x)=e^{-x} \quad x>0$
(ii) $f_Y(y)=e^{-y} \quad y>0$
(iii) $1-e^{-1}$

5 The joint p.d.f of the random variable (X, Y) is given by
$f(x,y)=xye^{-(x^2+y^2)} \quad x>0, y>0$
Prove that X and Y are independent

Answer: $f_X(x)=xe^{-x^2} \quad x>0$
$f_Y(y)=ye^{-y^2} \quad y>0$

Part – C

6	If the joint pdf of a two-dimensional random variable (X, Y) is given by $f(x,y)=\begin{cases} x^2+\dfrac{xy}{3} & 0<x<1,\ 0<y<2 \\ 0 & otherwise \end{cases}$ Find (i) P(X>1/2) (ii) P (Y>1) (iii) P(Y<X) (iv) P(Y<1/2 | X<1/2) (v) P (X+Y $\geq$ 1)	(i) 5/6 (ii) 7/12 (iii)7/24 (iv)5/32 (v) 65/72
7	The joint probability mass function of discrete bivariate random variable is given by $P_{XY}(x_i,y_j)=\begin{cases} k(x_i+y_j) & x_i=1,2,3\ ,\ y_j=1,2 \\ 0 & otherwise \end{cases}$ (i) Find the value of k. (ii) Find the marginal probability mass function of X& Y (iii) Are X & Y independent? (iv) Find the conditional distribution $P_{Y/X}(y_j/x_i)\ \&\ P_{X/Y}(x_i,y_j)$ (v) Find $P(X=2\|Y=2)$ & $P(Y=2\|\ X=2)$	(i) 1/21 (ii) 5/21, 7/21, 9/21 (iii)9/21, 12/21 (iv) Not Independent (v) 1/3 , 4/7
8	If the joint pdf of a two-dimensional random variable (X, Y) is given by $f(x,y)=\begin{cases} k(1-x-y), & 0<x<1/2,\ 0<y<1/2 \\ 0 & otherwise \end{cases}$ Find (i) k (ii) the marginal distributions (iii) $P(X<1/3)$ & $P(Y<1/3)$	(i) 8 $f_X(x)=\begin{cases} 3-4x & 0<x<1/2 \\ 0 & otherwise \end{cases}$ $f_Y(y)=\begin{cases} 3-4y & 0<y<1/2 \\ 0 & otherwise \end{cases}$ 7/9 , 7/9
9	Given the joint pdf of (X, Y) as $f(x,y)=\begin{cases} 8xy, & 0<x<y<1 \\ 0 & otherwise \end{cases}$ Find the marginal and conditional probability function of X & Y. (ii) Are X & Y independent? Also find $P(X+Y>1)$	(i) $f_X(x)=\begin{cases} 4x(1-x^2) & 0<x<1 \\ 0 & otherwise \end{cases}$ (ii) $f_Y(y)=\begin{cases} 4y^3 & 0<y<1 \\ 0 & otherwise \end{cases}$ (iii) $f_{X/Y}(x/y)=\begin{cases} \dfrac{2x}{y^2} & ,\ 0<x<y,\ 0<y<1 \\ 0 & otherwise \end{cases}$ (iv) $f_{Y/X}(y/x)=\begin{cases} \dfrac{2y}{(1-x^2)} & ,\ x<y<1,\ 0<x<1 \\ 0 & otherwise \end{cases}$ (v) Not independent (vi) 5/6