Principle of wind Energy Conversion and wind power.

wind Energy Mill works on the principle of converting kinetic energy of the wind to mechanical energy.

U_w = velocity of wind km/h

ρ = density of air $(1.225 \, kg/m^3)$

A = Air density

(mass) $M = \rho A U_w$.

$K.E = \frac{1}{2} m u_w^2$

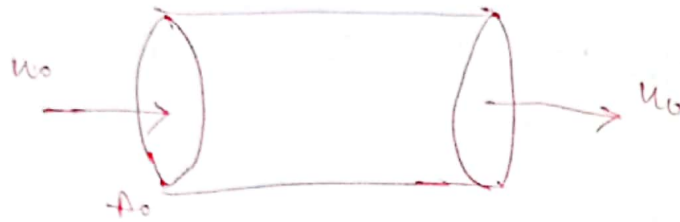$\qquad = \frac{1}{2} (\rho A U_w) \times U_w^2 = \frac{1}{2} \rho A U_w^3$ watts

$$\boxed{Total \ power = \frac{1}{2} \rho A U_w^3}$$

power output $\propto U_w^3$

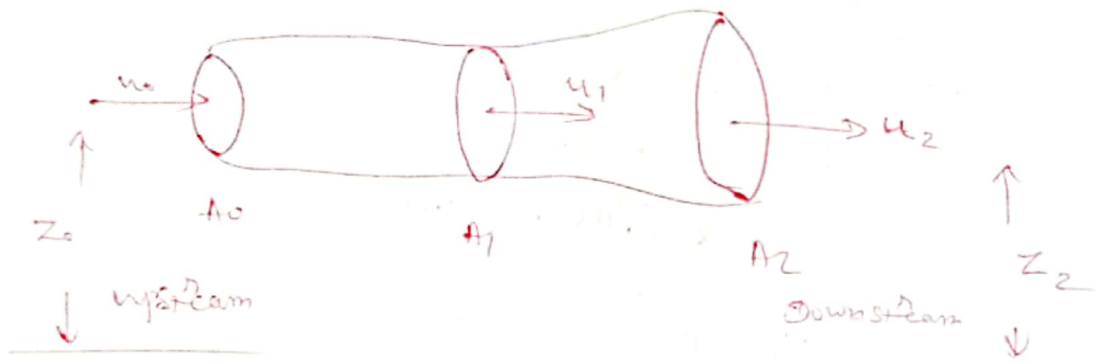$$P_{Total} = \frac{1}{2} \rho \times \frac{\pi}{4} D^2 \times U_w^3 = \frac{1}{8} \rho \pi D^2 U_w^3.$$

D = Diameter in meters

Power Extraction from wind :

the unperturbed wind stream tube in
absence of turbine.

u_0 and u_2 are wind speed of
upstream and downstream respectively.

$$F = \dot{m}u_0 - \dot{m}u_2 \quad (\text{mass} \times \text{velocity})$$

At u_1

$$P_T = \frac{\text{Power} = \text{force} \times \text{velocity}}{}$$

$$P_T = F \cdot u_1 = \dot{m}(u_0 - u_2)u_1$$

$$\boxed{u_1 = \frac{u_0 + u_2}{2}}$$

$$P_T = \frac{1}{2}\dot{m}(u_0 - u_2)(u_0 + u_1)$$

$$= \frac{1}{2}\dot{m}(u_0^2 - u_2^2)$$

The interference (or perturbance factor) is defined as the fractional in wind speed at turbine.

$$a = \frac{u_0 - u_1}{u_0}$$

$$a = \frac{u_0 - u_2}{2u_0}$$

$$a = 1 - \frac{b}{2}$$

$$b = \frac{u_2}{u_0}$$

$$u_2 = u_0(1-2a)$$

$$u_1 = u_0(1-a)$$

$$P_T = \frac{1}{2}\dot{m}\left(u_0^2 - u_0^2(1-2a)^2\right)$$

$$= \frac{1}{2}\dot{m}\,u_0^2\left(1 - 1 - 4a^2 + 4a\right)$$

$$= \frac{1}{2}\dot{m}\,u_0^2\left(4a - 4a^2\right)$$

$$= \frac{1}{2}\dot{m}\,u_0^2(4a)(1-a)$$

$$\boxed{\dot{m} = \rho A u_1}$$

$$= \frac{1}{2}\left[\rho A u_1\right]u_0^2(4a)(1-a)$$

$$\boxed{u_1 = u_0(1-a)}$$

$$\boxed{P_T = \frac{1}{2}\left[\rho A\,u_0^3\right]4a(1-a)^2}$$

$$\boxed{P_T = C_p P_0}$$

$$\boxed{C_p = 4a\,(1-a)^2}$$

$C_p =$ Power Coefficient

$$\boxed{u_1 = (1-a)\,u_0}$$

when $\boxed{u_1 = u_0}$

$a = 0$

The Turbine doesn't generate any power

→ The air just passes through the turbine.

$u_0 \quad u_1 \quad u_2$

$\xrightarrow{u_0} \qquad \qquad \xrightarrow{u_2}$
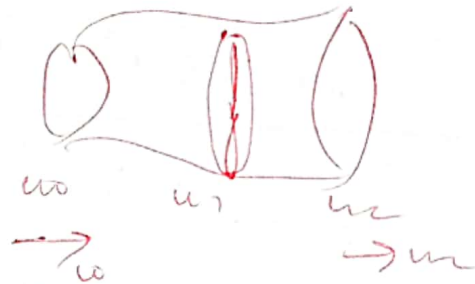
differential with respect to a

$$\frac{dC_p}{da} = 0$$

$a = \frac{1}{3}$

$$\boxed{C_p = \frac{16}{27} = 0.593}$$

$$\boxed{P_T = 0.593 \left[\begin{array}{l} \text{Power on} \\ \text{Free wind} \end{array} \right]}$$