One dimensimal heat flow:

Consider the flow of heat and the accompanying Variation of temperature with position and with time in Conducting solids.

The following empirical laws are taken as the basis of investigation.

- 1. Heat flows from a higher to lower temperature.
- temperature change in a body is proportional to the man of the body and to the temperature change. This constant of proportionality is known as the Specific heat (c) of the Conducting material.
- 3. The rate at which heat flows through an area is proportional to the area and to the temperature gradient normal to the area. This Constant of proportionality is known as the thermal and activity (k) of the normal.

The one dimentional heat flow equation is $\frac{\partial u}{\partial t} = d^2 \frac{\partial^2 u}{\partial x^2}$ where d^2 stands for $\frac{k}{\sqrt{\rho_c}}$ k - thermal Conditactivity $\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$ $\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x}$ $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial$

the heat equation by the method of Solution Seperation of Variables. The Tone-dimensional heat equation is $\frac{\partial u}{\partial t} = d^2 \frac{\partial^2 u}{\partial x^2}$ @5 ourilloug equation Assume a solution of the form $u(n,t) = X(n) \cdot T(t)$ where X is a function of x alone and Y is a function of E alme. dy eventiating & partially w. r to the me get $\frac{\partial u}{\partial t} = 1XT'$ \longrightarrow 3 dufferentiating (5) partially w. r to K twice, we get Ou = X"T = (3, x) = (3, x) Substituting 3 to 4 in eqn (1), we get: XT' = 00 X Tupo bod norelles who

Separating the variables, we get $\frac{T'}{\sqrt{2}} = \frac{x^{10} + \cos^2 x}{x} = \frac{1}{x} \left(\frac{\cos^2 x}{x} \right) = \frac{1}{x} \left(\frac{\cos^2 x}{x} \right)$ 1e) $\frac{T^{1}}{d^{3}T} = k$ and $\frac{x^{11}}{x} = k$ and $\frac{1}{x}$ 1e, $T' - Kq^{a}T = 0$ and 39 x'' - kx = 0 note to are the motion () in the second () call of c The equations (6) to (7) are ordinary duperential equations the solution of which depend on the value of k. Case (i) : Let k=19, a positive number The differential equalities (6) & (7) become stderoder of the heat equation and midwest took of the modulos le, $\frac{d^2x}{dx^2} - \lambda^2x = 0$. le, $\frac{dT}{dt} - \lambda^2x^2T = 0$. (D3-12) x =0 ushere dt dt 1227 of

The auxiliary equation $\frac{dx}{dx} = \frac{d}{dx}$ is $m^{3} - \lambda^{2} = 0$ and $m^{2} = \frac{d^{2}}{dx}$ $m=\pm\lambda$.

X= AjeAx + Bje-Ax

y twice, we get

rodonné ma = 12 bas ands x le, integrating on b.s., we get 1097 = 1842+ + 109 C1 startes Ale 1x + Bie-1x we get promote more yet TXT = UCI e 12d2t

Substituting the values of X and mit in em (2), we get u(n,t) = (A1e 1x + B1e - 1x) (e1e 12d2t)

Case (ii) : Let $k = -\lambda^2$, a negative number. The dufferential equations (+ 7 becomes

$$|x|^{2} + \lambda^{2} x = 0$$

$$|x|^$$

The auxiliary eqns is

$$m^2 + \lambda^2 = 0$$
 and so as

 $m^2 = -\lambda^2$ being $m^2 = -\lambda^2$ and m^2

$$\therefore X = A_2 \cosh x + B_2 \sinh \lambda x$$

$$T = e_2 e$$

$$\frac{d^{2}x}{dx^{2}} + \lambda^{2}x = 0$$

$$\frac{d^{2}x}{dx^{2}} + \lambda^{2}x = 0$$

$$\frac{d^{2}x}{dt} + \lambda^{2}x = 0$$

$$\frac{d^{2$$

 $m^2 + \lambda^2 = 0$ and one sold white south of the sold of $m^2 + \lambda^2 = 0$ and sold of sold of the sold Taking exponential on b.s here, the law the throughout $T = \frac{e_2}{c_{10}} = \frac{12d^2t}{c_{10}}$ then the

Then equations 6 & 9 becomes $x^{(1)} = 0$ $x^{(2)} = 0$ dx = 0 $dx^{(2)} = 0$ $dx^{(2)} = 0$

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the solution (we when the temporalure seath time)

Now, Counter the Solution Case (iii) atom = 1 the sol (Man) = (1 in)

integrating w.r to notation integrating w.r to time get

 $u(x_1 + b_3) = (A_3 x + B_3) + (A_3 x + B_3)$

for our publims on one dimensional heat flow is

u(1) = (A2 60 Ax + B2 1811 Ax) C2e 42 A2 E.

Hence the possible solutions of On are u(nit) = (Ajenx + Bje-Ax) cjed212t U(x,t) = (Ag contin + B2 sin. Ax) C2e-d2/2t (d12 + 12) X=0. $u(x,t) = (A_3x + B_3) C_3.$

Note: top ow and no the Dout of these three solutions we have to choose the correct solution which saturfies the physical nature of the problem. here, we are dealing with the problem on heat conduction. According to the law of Thermodynamics, when time 't' increases the temperature unit will not increase.

Now, Consider the Solution u(n,t) = (A1e1x + B1e-1x) C1e x2/2t

here, if t increases then u(n,t) is also increases. if we allow to on then u(x,t) > 00 This is in contradiction with the law of Thermodynamics. Hence this solution is not suitable for our problems on heat anduction is a padorpodai

- At steady state conditions only we can use the solution (received the temperature no longer varies u(n)+)= (A3x2+B3) (3.1)
- Therefore, the Correct solution which is suitable 3 for our problems on one dimensional heat flow is U(n,t) = (A2 65 Ax + B2 sin Ax) Cze-d2A2t.