Notes:

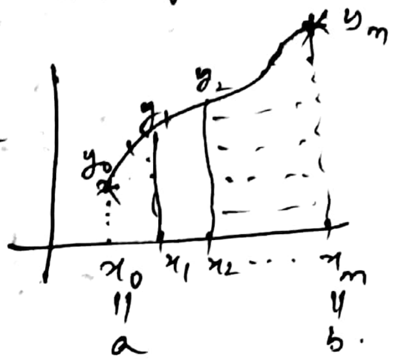
Harmonic Analysis

(*) By doing the experiments, we will get observed data in the form $(x_0, y_0), (x_1, y_1), \ldots (x_m, y_m)$

Where $x_0, x_1 \ldots x_m$ are inputs

$y_0, y_1, \ldots y_m$ are outputs.

(*) If we map these data in the xy axis plane, we will get some curve $y = f(x)$



(*) we can able to get the area under the curve in the given range by integration techniques.

let $y = f(x)$

$x_0 = a$

$x_m = b$

(m - parts in the range).

Ⓐ

$$\therefore \int_a^b f(x)\,dx = (\text{Range}) \times (\text{Mean of } f(x))$$

$$\int_a^b f(x)\,dx = (b-a)\left(\frac{\sum_1^m f(x)}{m}\right)$$

Ⓧ

$$\text{||ly} \quad \int_a^b f(x)\cos\left(\frac{n\pi x}{L}\right)dx = (\text{Range})\left(\text{Mean of } f(x)\cos\left(\frac{n\pi x}{L}\right)\right)$$

$$\int_a^b f(x)\sin\left(\frac{n\pi x}{L}\right)dx = (\text{Range})\left(\text{Mean of } f(x)\sin\left(\frac{n\pi x}{L}\right)\right)$$

(*) Now, our aim is to find the Fourier Harmonics if the observed data $(x_0, y_0), (x_1, y_1) \ldots$ are given.

General Formula: $f(x)$ in $(c, c+2L)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right)$$

$$f(x) = \frac{a_0}{2} + \underbrace{\left(a_1 \cos\frac{\pi x}{L} + b_1 \sin\frac{\pi x}{L}\right)}_{\text{First Harmonic (or) Fundamental Harmonic}} + \underbrace{\left(a_2 \cos\frac{2\pi x}{L} + b_2 \sin\frac{2\pi x}{L}\right)}_{\text{second Harmonic}} + \cdots$$

where

$$a_0 = \frac{1}{L}\int_c^{c+2L} f(x)\,dx = \frac{1}{L}\Big[(\text{Range})(\text{Mean of } f(x))\Big]$$

[By Integration technique (A)]

$$= \frac{1}{L}\left[(2L)\left(\frac{\sum f(x)}{m}\right)\right]$$

$$\boxed{a_0 = \frac{2}{m}\sum f(x)}$$

$$a_1 = \frac{1}{L}\int_c^{c+2L} f(x)\cos\left(\frac{\pi x}{L}\right)dx = \frac{1}{L}\Big[(\text{Range})\left(\text{Mean of } f(x)\cos\left(\frac{\pi x}{L}\right)\right)\Big]$$

$$= \frac{1}{L}\left[(2L)\left(\frac{\sum f(x)\cos\left(\frac{\pi x}{L}\right)}{m}\right)\right]$$

$$\boxed{a_1 = \frac{2}{m}\sum f(x)\cos\left(\frac{\pi x}{L}\right)}$$

Illy,
$$\boxed{b_1 = \frac{2}{m}\sum f(x)\sin\left(\frac{\pi x}{L}\right)}$$

Formula:

If period $= 2L$	If period $= 2\pi$ (or) $360°$ $\boxed{L=\pi}$
(1) $a_0 = \frac{2}{m}\sum f(x)$	
(2) $a_1 = \frac{2}{m}\sum f(x)\cos\left(\frac{\pi x}{L}\right)$	$a_1 = \frac{2}{m}\sum f(x)\cos x$
(3) $b_1 = \frac{2}{m}\sum f(x)\sin\left(\frac{\pi x}{L}\right)$	$b_1 = \frac{2}{m}\sum f(x)\sin x$
(4) $a_2 = \frac{2}{m}\sum f(x)\cos\left(\frac{2\pi x}{L}\right)$	$a_2 = \frac{2}{m}\sum f(x)\cos 2x$
(5) $b_2 = \frac{2}{m}\sum f(x)\sin\left(\frac{2\pi x}{L}\right)$	$b_2 = \frac{2}{m}\sum f(x)\sin 2x$
(6) $a_3 = \frac{2}{m}\sum f(x)\cos\left(\frac{3\pi x}{L}\right)$	$a_3 = \frac{2}{m}\sum f(x)\cos 3x$
(7) $b_3 = \frac{2}{m}\sum f(x)\sin\left(\frac{3\pi x}{L}\right)$	$b_3 = \frac{2}{m}\sum f(x)\sin 3x$

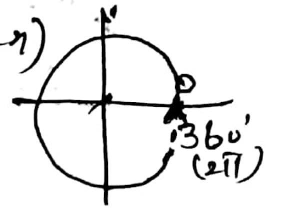
(1) Compute upto three harmonics of the Fourier series from the table,

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1	1.4	1.9	1.7	1.5	1.2	1

Hint: (i) Check always first whether the initial function value and last function value same or not. if both are same, consider only one value for further calculations (since the last value is the starting pt for next rotation).

Period $= 2\pi - 0 = 2\pi \Rightarrow \boxed{L=\pi}$

$\therefore f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + \cdots$



$a_0 = \frac{2}{m} \sum f(x)$

$a_1 = \frac{2}{m} \sum f(x) \cos x$, $b_1 = \frac{2}{m} \sum f(x) \sin x$

$a_2 = \frac{2}{m} \sum f(x) \cos 2x$, $b_2 = \frac{2}{m} \sum f(x) \sin 2x$

$a_3 = \frac{2}{m} \sum f(x) \cos 3x$, $b_3 = \frac{2}{m} \sum f(x) \sin 3x$

(0, and 2π are same)
consider any 0 (or) 2π.

x	$f(x)$	$\cos x$	$\sin x$	$\cos 2x$	$\sin 2x$	$\cos 3x$	$\sin 3x$
0	1	1	0	1	0	1	0
$\frac{\pi}{3}$	1.4	0.5	0.866	−0.5	0.866	−1	0
$\frac{2\pi}{3}$	1.9	−0.5	0.866	−0.5	−0.866	1	0
π	1.7	−1	0	1	0	−1	0
$\frac{4\pi}{3}$	1.5	−0.5	−0.866	−0.5	0.866	1	0
$\frac{5\pi}{3}$	1.2	0.5	−0.866	−0.5	−0.866	−1	0

Here $\boxed{m=6}$

Note: Don't include last value (2π) here. 0 & 2π same

$a_0 = \frac{2}{6} \sum f(x) = \frac{2}{6}[1 + 1.4 + \cdots + 1.2] = 2.9$

$a_1 = \frac{2}{6} \sum f(x) \cos x = \frac{2}{6}[(1)(1) + (1.4)(0.5) + (1.9)(-0.5) + \cdots + (1.2)(0.5)] = -0.37$

$b_1 = \frac{2}{6} \sum f(x) \sin x = 0.17$

$a_2 = \frac{2}{6} \sum f(x) \cos 2x = -0.1$

$b_2 = \frac{2}{6} \sum f(x) \sin 2x = -0.06$

$a_3 = \frac{2}{6} \sum f(x) \cos 3x = 0.03$

$b_3 = \frac{2}{6} \sum f(x) \sin 3x = 0$

$\therefore f(x) = 1.45 + (-0.33 \cos x + 0.17 \sin x) + (-0.1 \cos 2x - 0.06 \sin 2x) + (0.03 \cos 3x) + \cdots$

2. The following table gives the variations of the periodic current over a period.

x	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
Amp	1.98	1.3	1.05	1.3	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current. Also, find the amplitude of the first harmonic.

Sol: period $= 2L = T$ [Calculate upto first harmonic for this problem, no need for higher harmonics]

$\boxed{L = T/2}$

Formula:

$$f(x) = \frac{a_0}{2} + \left(a_1 \cos\frac{\pi x}{L} + b_1 \sin\frac{\pi x}{L}\right) + \cdots$$

Let $\boxed{\theta = \frac{\pi x}{L}} = \frac{\pi x}{T/2} = \frac{2\pi x}{T}$

$a_0 = \frac{2}{m}\sum f(x)$, $a_1 = \frac{2}{m}\sum f(x)\cos\left(\frac{\pi x}{L}\right) = \frac{2}{m}\sum f(x)\cos\theta$

$b_1 = \frac{2}{m}\sum f(x)\sin\left(\frac{\pi x}{L}\right) = \frac{2}{m}\sum f(x)\sin\theta$

x	θ	$f(x)$	$\cos\theta$	$\sin\theta$
0	0	1.98	1	0
$T/6$	$\pi/3$	1.3	0.5	0.866
$T/3$	$2\pi/3$	1.05	-0.5	0.866
$T/2$	π	1.3	-1	0
$2T/3$	$\frac{4\pi}{3}$	-0.88	-0.5	-0.866
$5T/6$	$\frac{5\pi}{3}$	-0.25	0.5	-0.866

$a_0 = \frac{2}{6}\sum f(x) = \frac{2}{6}(4.50) = 1.5$

$a_1 = \frac{2}{6}\sum f(x)\cos\theta$

$= \frac{2}{6}(1.12) = 0.37$

$b_1 = \frac{2}{6}\sum f(x)\sin\theta = 1.005$

$f(x) = \frac{a_0}{2} + (a_1\cos\theta + b_1\sin\theta)$

$f(x) = 0.75 + 0.37\cos\theta + 1.005\sin\theta$

From the given solution, we can conclude that

(i) there is direct part $\frac{a_0}{2} = 0.75$ amp (since it is not related with any θ).

(ii) amplitude for first harmonic $= \sqrt{a_1^2 + b_1^2}$

$= \sqrt{0.37^2 + 1.005^2}$

$= 1.071$