DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB101J-Electromagnetic Theory, Quantum Mechanics, Waves and Optics

Module I Lecture-3

Poisson’s and Laplace’s Equation

Poisson's and Laplace's Equation

Gauss's law for a linear medium

$$\nabla . D = \rho$$

$$\nabla . \varepsilon E = \rho$$

Note
$\mathbf{D = \varepsilon E}$

Here E is basically free charge density (Volume) and D is the electric displacement

$$\text{Since } E = -\nabla V,$$

Note
$\nabla . \varepsilon(-\nabla V) = \rho$

the above equation for a homogeneous medium can be written as

$$\nabla^2 V = -\rho/\varepsilon$$

Poisson's and Laplace's Equation

This equation is called as Poisson's equation. For a free charge region, i.e., where $\rho = 0$, the Poisson's equation takes the form $\nabla^2 V = 0$.

This equation is Laplace's Equation. This equation is much useful in solving electrostatic problems where a set of conductors are maintained at different potentials; for example, capacitors and Vacuum tube diodes.

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Poisson's and Laplace's Equation

Using the expressions for Laplace's operator ∇^2 in cartesian, cylindrical and spherical coordinate system, we can write Laplace's Eq. in these coordinate, respectively, as

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$$

$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right)+\frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2}+\frac{\partial^2 V}{\partial z^2}=0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}=0$$