Numbers

SUBTOPICS:

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CLASSIFICATION OF NUMBERS:

NATURAL NUMBERS: {1, 2, 3, 4, 5, ………….}

WHOLE NUMBERS: {0, 1, 2, 3, 4, 5, ………….}

RATIONAL NUMBERS: Numbers are in the form of p/q (q is not equal to 0)
Examples: 4, 2/5, 1/3, 22/7, …….

IRRATIONAL NUMBERS: Numbers which are not rational but can be represented by points on the number line.
Examples: √2, ∏, e, ……

Alternate definition:
Terminating decimals and recurring decimals are both rational numbers. Any non-terminating, non-recurring decimal is an irrational number.

REAL NUMBERS: Both rational and irrational numbers are real numbers.

SET OF INTEGERS: All negative integers, zero and all positive integers.

PRIME NIMBERS: Numbers with exactly 2 factors are prime numbers. Or numbers which are divisible by 1 and itself are prime numbers.
Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97,.....

COMPOSITE NUMBERS: Numbers with more than 2 factors.
Examples: 4, 6, 8,……

Co primes / Relative primes: If the HCF of 2 numbers is 1, the numbers are called co primes or relative primes.
Examples: (8, 9), (9, 10), (2, 3),……

Twin primes: Primes which are differ by 2 are twin primes.
Examples: (3, 5), (5, 7)……

SUMMATION FORMULE:

Sum of first n natural numbers= $1+2+3+\ldots\ldots..+n =[n(n+1)]/2$

Sum of the squares of first n natural numbers= $1^2+2^2+3^2+\ldots\ldots..+n^2 = [n(n+1)(2n+1)]/6$

Sum of the cubes of first n natural numbers= $1^3+2^3+3^3+\ldots\ldots..+n^3 = [n^2(n+1)^2]/4$

Sum of even numbers = $n(n+1)$, where n is number of even numbers.

Sum of odd numbers = n^2, where n is number of odd numbers.

1.What is the sum of first 80 natural numbers?

a) 3140 b) 3240 c) 3340 d) 3440

Explanation:

Sol: Sum = [n (n+1)]/2 = (80) (80+1)/2 =3240

2.What is the sum of the squares of first 20 even natural numbers?

a) 9480 b) 10480 c) 11480 d) 12480

Explanation:

Sol: $2^2+4^2+6^2+8^2+\ldots\ldots + 40^2 = 2^2\,(1^2+2^2+3^2+\ldots\ldots+20^2)$

$= 4 \times [n\,(n+1)(2n+1)]/6$

$= 4 \times (20 \times 21 \times 41)/6 = 11480$

3. A wants to type first 1000 natural numbers. How many times he has to press the buttons of a computer key board?

a) 2893 b) 2987 c) 3000 d) 2500

Explanation:

For 1 to 9, number of times to be pressed = 9

For 10 to 99, number of times to be pressed = 90 x 2 = 180

For 100 to 999, number of times to be pressed = 900 x 3 = 2700

For 1000, number of times to be pressed = 4

Hence total = 9+180+2700+4 = 2893

4. A printer numbers the pages of a book starting with 1 and uses 3089 digits in all. How many pages does the book have?

a) 1040 b) 1048 c) 1049 d) 1050

Explanation:

For pages 1 to 9, number of digits used = 9

For pages 10 to 99, number of digits used = 90 x 2 = 180

For pages 100 to 999, number of digits used = 900 x 3 = 2700

So far the digits used = 9+180+2700 = 2889

The remaining digits = 3089 – 2889 = 200, with these next 50 pages can be numbered.

So total = 999+ 50 = 1049.

5. One page is torn from a booklet whose pages are numbered in the usual manner starting from the first page 1. The sum of the numbers on the remaining pages is 195. The torn page contains which of the following numbers?

a) 5, 6 b) 7, 8 c) 9, 10 d) 11, 12

Explanation:

Here, basically, our sum of first n natural numbers should be slightly greater than 195.

By trial and error, if $n = 10$, then $[n (n+1)]/2 = (10 \times 11)/2 = 55$

if $n = 15$, then $[n (n+1)]/2 = (15 \times 16)/2 = 120$

if $n = 20$, then $[n (n+1)]/2 = (20 \times 21)/2 = 210$

So, 210 – 195=15.That is the torn page contains pages 7 and 8

DIVISIBILITY RULES

By 2: Check the last digit
By 4: Check the last 2 digit number
By 8: Check the last 3 digit number
By 16: Check the last 4 digit number, etc

By 3: Check the sum of the digits
By 9: Check the sum of the digits

BY 5: Last digit should be 0 or 5

BY 11: A number is divisible by 11, if the difference between the sum of the digits in odd places and the sum of the digits in even places is 0 or a multiple of 11.

In case of composite numbers:
Divisible by 6: Check with 2 and 3
D Divisible by 12: Check with 3 and 4
Divisible by 18: Check with 2 and 9
Divisible by 24: Check with 8 and 3
That is, for composite numbers we need to check with co prime pair

6.If 6896x45 is divisible by 9 then x is -

a) 4 b) 5 c) 6 d) 7

Explanation:

Here sum of the digits = 38+x, so x= 7

7.If 481A769B is divisible by 5, 6 and 9 then A+B is -

a) 0 b) 1 c) 2 d) 3

Explanation:

Given, 481A769B is divisible by 5 and 6, implies B = 0

By 9, sum of the digits = 35+ A, so A = 1 and A+B = 1

8. An 8 digit number 4252746B leaves a remainder 0 when divided by 3. How many values are possible for B?

a) 2 b) 3 c) 4 d) 6

Explanation:

Sum of the digits of 4252746B = 30+B

So, B can take 0, 3, 6 and 9, that is 4 values.

9. What is the remainder when the 100 digit number starting with 1, writing the consecutive natural numbers next to it, is divided by 5?

a) 1 b) 2 c) 4 d) 0

Explanation:

Here we need to know the last digit of this 100 digit number.

The 100 digit number is 1234…..9101112…….545 (that is 9 single digit numbers, then 45 two digit numbers, 10 to 54 and then 5)

So the remainder is 0

10 .If the 8 digit number 5668x25y is divisible by 48, find the least value of x+y

a)10 b) 9 c) 8 d) 7

Explanation:

Divisibility by 48 means we need to check with 3 and 16.

Sum of the digits of 5668x25y = 32+(x + y)

Among the options if (x + y) is 7 or 10 then only it is divisible by 3.

Divisibility by 16 means we need to check last 4 digit number and for 8 we have to check the last 3 digit number.

25y is divisible by 8, implies y = 6

If x= 1, then the 4 digit number 1256 is not divisible by 16.

Hence the value of (x+y) is 10

Def: A decimal in which a digit or a set of digits is repeated continuously is called a recurring decimal.

Examples are given below

1/3 = 0.333333…. = 0.**3** (read highlighted 3 as bar 3, to denote repetition)

1/7 = 0.142857142857142857 = 0. **142857**

Let us see how to convert a recurring decimal in to fraction

Let x = 0.3333333…… (1)

10 x = 3.333333…….(2)

(2) – (1) gives 9x = 3, implies x = 1/3

Let x = 0.454545…….. (1)

100 x = 45.454545…… (2)

(2) – (1) gives 99x = 45, implies x = 45/99= 5/11

11. The value of 0.057057057057……. is -

a) 57/99 b) 57/999 c) 57/990 d) 57/909

Explanation:

Let x = of 0.057057057057…….(1)

1000 x = 057.057057…………..(2)

(2) – (1) gives 999x = 057, implies x = 57/999

Short cut: Take the repeated digits once in the numerator and the number of 9's corresponding to the number of repeated digits in the denominator.

That is = 057/999 = 57/999

12.The value of 0.1254545454………. is (that is 0.12**54)**

a) 1242/(9900) b) 621/(2950) c) 207/(1650) d) 69/(550)

Explanation:

Answer = (1254 – 12)/(9900) = 1242/(9900)

(Numerator: Take the whole number and subtract the non recurring part.)

(Denominator: Number of 9's corresponding to the number of repeated digits, followed by number of 0's corresponding to the number of non repeated digits)

13.The recurring decimal representation 1.27272727.......... is -

a) 13/11 b) 14/11 c) 127/99 d) 137/99

Explanation:

Answer= 1+ (27/99) = 126/99 = 14/11

FACTORS
:

Factors of 6 are 1, 2, 3, and 6
Factors of 12 are 1, 2, 3, 4, 6, and 12
Factors of 16 are 1, 2, 4, 8 and 16
Factors of 25 are 1, 5 and 25

NOTE:
Prime numbers contain exactly 2 factors.
Squares of primes contain exactly 3 factors.
Any perfect square contains odd number of factors.

Example: Prime factorization of 400 is?

$400 = 4 \times 100 = 4 \times 10 \times 10 = (2x2)\ (2x5)(2x5) = 2^4 \times 5^2$

Here 2, 5 are prime factors of 400.

FORMULE:

If N is a composite number such that $N = a^p \times b^q \times c^r$.........where a, b, c... are prime factors of N and p, q, r...... are positive integers.

Then, number of factors on N = $(p+1)(q+1)(r+1)$ ......

Number of ways of writing N as product of 2 factors

$= (1/2)\,[(p+1)(q+1)(r+1)....]$

Sum of all the factors = $[a^{p+1} - 1] / (a-1) \times [b^{q+1} - 1] / (b-1) \times$ ....

14. Find the number of factors 1225

a) 5 b) 6 c) 8 d) 9

Explanation:

$1225 = 25 \times 49 = 5^2 \times 7^2$,

Number of factors = (p+1) (q+1) (r+1) ……

= (2+1) (2+1) = 9

15. Find the number of factors 19404, excluding 1 & the number itself?

a) 52 b) 54 c) 58 d) 59

Explanation:

$19404 = 11 \times 1764 = 11 \times 9 \times 196 = 11^1 \times 3^2 \times 2^2 \times 7^2$

Therefore number of factors = $2 \times 3 \times 3 \times 4 = 54$

Answer = $54 - 2 = 52$.

16. In how many ways can 3420 be written be written as product of 2 factors?

a) 12 b) 14 c) 18 d) 36

Explanation:

$3420 = 10 \times 342 = 10 \times 9 \times 38 = (2\times5)\ (3\times3)\ (2\times19) = 2^2 \times 3^2 \times 5^1 \times 19^1$

Answer = $(1/2)\ [(p+1)\ (q+1)\ (r+1)....]$

$= (1/2)\ [3 \times 3 \times 2 \times 2\ ] = 18$

17. Find the number of odd & even number of factors of 1680?

a) 8, 32 b) 8, 9 c) 10, 9 d) none

Explanation:

$1680 = 10 \times 168 = 10 \times 4 \times 42 = (2x5)(2x2)(2x3x7) = 2^4 \times 5^1 \times 3^1 \times 7^1$

Number of odd factors = All the factors of $(5^1 \times 3^1 \times 7^1)$ = 2x2x2 = 8

$2^4 \times 5^1 \times 3^1 \times 7^1 = 2\,[2^3 \times 5^1 \times 3^1 \times 7^1]$

Number of even factors = All the factors of $[2^3 \times 5^1 \times 3^1 \times 7^1]$ = 4x2x2x2 = 32

18. Find the number of factors of 243243 which are multiples of 21?

a) 20 b) 23 c) 25 d) none

Explanation:

$243243 = 243 (1001) = 3^5 \times 11 \times 13 \times 7 = 21[3^4 \times 11 \times 13] = 5 \times 2 \times 2 = 20$

19.Find the sum of all the factors of 120?

a) 240 b) 280 c) 360 d) 400

Explanation:

$120 = 40 \times 3 = 8 \times 5 \times 3 = 2^3 \times 5^1 \times 3^1$

Sum of all the factors = $[a^{p+1} – 1] / (a-1) \times [b^{q+1} – 1] / (b-1) \times$ ……….

$= (2^4 – 1)/(2-1) \times (5^2 – 1)/(5-1) \times (3^2 – 1)/(3-1)$

$= 360$

20. What is the smallest number that should multiply 840 to make it a perfect square and 2940 to make it a perfect cube respectively?

a) 200, 3100 b) 210, 3150 c) 210, 3250 d) None

Explanation:

Note:

To make it a perfect square, make the powers of prime factors a multiple of 2

To make it a perfect cube, make the powers of prime factors a multiple of 3

$840 = 2^3 \times 5^1 \times 3^1 \times 7^1$, answer = $2 \times 5 \times 3 \times 7 = 210$

$2940 = 2^2 \times 5^1 \times 7^2 \times 3^1$, answer = $2 \times 5^2 \times 7 \times 3^2 = 3150$

LCM AND HCF

LCM is the least common multiple

Ex: LCM 0f 6, 8

Multiples of 6 = 6, 12, 18, 24, 30, 36, 42, 48………

Multiples of 8 = 8, 16, 24, 32, 40, 48 ………………..

Common multiples of 6, 8 = 24, 48 ……………

In these least one = 24. That is LCM of 6, 8 = 24

HCF is the highest common factor

Ex: HCF 12, 18

Factors of 12 = 1, 2, 3, 6 and 12

Factors of 18 = 1, 2, 3, 6, 9 and 18

Common factors are 1, 2, 3 and 6

So the highest common factor is 6. That is HCF of 12, 18 = 6

FORMULE:

1 . Product of 2 numbers = LCM X HCF

2. LCM of fractions = (LCM of numerators)/(HCF of denominators)

3. HCF of fractions = (HCF of numerators)/(LCM of denominators)

21. Find the respective LCM and HCF of the following

(i) 42, 72, 90

a) 1200, 4 b) 7200, 16 c) 2520, 6 d) 1000, 35

(ii) $\frac{1}{2}$, 2/3, $\frac{3}{4}$, 4/5

a) 12, 1/60 b) 24, 1/30 c) 1/24, 30 d) 24, 30

Explanation:

(i) 42, 72, 90, HCF = 6 & LCM = 2520

Explanation:

(ii) $\frac{1}{2}$, 2/3, $\frac{3}{4}$, 4/5,

LCM = (LCM of 1, 2, 3 ,4)/(HCF of 2, 3, 4, 5) = 12/1 = 12

HCF = (HCF of 1, 2, 3 ,4)/(LCM of 2, 3, 4, 5) = 1/60

22. The HCF of 2 numbers is 16 and their LCM is 160. If one of the numbers is 32, what is the other?

a) 60 b) 80 c) 40 d) 20

Explanation:

We have, Product of 2 numbers = LCM X HCF

$32 * x = 16 * 160$, implies $x = 80$

23. Find the least number which when divided by 48 and 72 leaves a remainder of 9 in each case and is greater than 9?

a) 144 b) 152 c) 151 d) 153

Explanation:

Required number = LCM of (48, 72) +9

= 144 + 9 = 153

24. What is least 4 digit number which when divided by 3, 4, 5 and 6 leaves remainder of 2 in each case?

a) 1012 b) 1022 c) 1122 d) 1222

Explanation:

Required number will be of the form = k [LCM of (3, 4, 5, 6)] + 2

= 60k +2

When k = 17, we get the least 4 digit number, which is 1020 +2 = 1022

25. Find the smallest number which when divided by 3,5,7,9 and 11 leaves respective remainder of 2,4,6,8 and 10?

a) 2265 b) 2275 c) 2274 d) 3464

Explanation:

Here the difference between the divisor and the remainder is 1

Required number = LCM of (3, 5, 7, 9 and 11) – 1

= 3365 – 1

= 3364

26. Find the smallest number which leaves a remainder of 7 when divided by 11 and leaves a remainder of 12 when divided by 13?

a) 51 b) 62 c) 72 d) 36

Explanation:

Using the division algorithm (N = dq +r) we can write the number as

$N = 11a + 7 = 13b + 12$

$11a = 13b + 5$

Now find the least value of 'b' so that 'a' is an integer.

We can easily see that when b = 3, 'a' is an integer.

So the required number = 13b +12 = 51

27. Find the smallest and largest 3 digit number which when divided by 22, 33 and 55 leave a remainder of 5 in each case?

a) 330, 990 b) 345, 980 c) 335, 995 d) 325, 925

Explanation:

Number form = k (LCM of 22, 33 and 55) + 5

= 330k +5

For smallest 3 digit number, put k = 1, N = 335

For greatest 3 digit number, put k = 3, N = 995

28. Find the smallest and the largest 4 digit number which when decreased by 12 is exactly divisible by 16, 24 and 40?

a) 1208, 9848 b) 1200, 9840 c) 1212, 9852 d) 1188, 9828

Explanation:

Number form = k (LCM of 16, 24 and 40) + 12

= 240k +12

For smallest 4 digit number, put k = 17, N = 1212

For greatest 4 digit number, put k = 41, N = 9852

29. Six bells ring together at 11am and after that they ring at intervals of 5, 10, 15, 25, 30 seconds. How many times will they ring together from 11.00 am to 12.30 pm on the same day?

a) 18 b) 19 c) 20 d) 17

Explanation:

LCM of (5, 10, 15, 25, 30) = 300 seconds = 5 minutes

So, answer = (90/5) + 1 = 19

30. Find the greatest number which when 110 and 99 are divided it leaves a remainder of 2 and 3 respectively?

a) 24 b) 6 c) 12 d) 36

Explanation:

Required number = HCF of [(110 – 2), (99 – 3)

= HCF of (108, 96)

= 12

31. Find the largest number which when 145, 121, 97 are divided the remainders are same?

a) 48   b) 12   c) 36   d) 24

Explanation:

Required number = HCF of [(145 – 121), (121 – 97)]

= HCF of (24, 24)

= 24

32. What is the greatest length x such that 3½ m and 8¾ m are integral multiples of x?

a)1 ½ m b) 1 1/3 m c) 1 ¼ m d) 1 ¾ m

Explanation:

3½ = 7/2

8¾ = 35/4

HCF of fractions = (HCF of numerators)/(LCM of denominators)

= (HCF of 7, 35)/(LCM of 2, 4)

= 7/4

= 1 ¾ m

LARGEST POWER OF A NUMBER IN N!

33. Find the highest power of 2 in 100!

Explanation:

100/2 = 50 + 25 + 12 + 6 + 3 + 1 = 97

34. Find the highest power of 3 in 100!

Explanation:

100/3 = 33 + 11 + 3 + 1 = 48

35. Find the highest power of 6 in 100!

Explanation:

Highest power of 6 in 100!
= smaller of [highest power of 2 in 100!, highest power of 3 in 100!]
= smaller of (97, 48)
= 48

36. Find the highest power of 24 in 500!

a) 166 b) 165 c) 164 d) 163

Explanation:

$24 = 8 \times 3 = 2^3 \times 3$.

In 500!,number of 8's are lesser compare to number of 3's. So we need to find only number of 8's

Number of 2's in 500!, 500/2 = 250+125+62+31+15+7+3+1 = 494

Therefore number of 8's in 500! = 494/3 =164

37. Find the number of zeros at the end of 200!

a) 48 b) 49 c) 50 d) 51

Explanation:

With a combination of 2 and 5 we get 1 zero.

For number of zero's at the end of any factorial, in general, we find the number of 5's

200/5 = 40 + 8 + 1=49

38. Find the number of zeros at the end of (150)! x (80)!

a) 50 b) 55 c) 56 d) 58

Explanation:

150/5 = 30 + 6 + 1 = 37

80/5 = 16 +3 = 19

Answer = 37 + 19 =56

39. Find the number of zeros in the product 1 x 5 x 10 x 15 x 20 x 25 x…….x 60

a) 10 b) 12 c) 14 d) 15

Explanation:

	1	5	10	15	20	25	30	35	40	45	50	55	60
5’s	0	1	1	1	1	2	1	1	1	1	2	1	1
2’s	0	0	1	0	2	0	1	0	3	0	1	0	2

Number of 5’s in the product = 14 & number of 2’s = 10

Therefore number of zero’s = smaller of (14, 10) = 10

THE LAST DIGIT/ UNIT'S OF ANY POWER

Let us look at the powers of 2

2^1 ends with 2
2^2 ends with 4
2^3 ends with 8
2^4 ends with 6
2^5 ends with 2
2^6 ends with 4
2^7 ends with 8
2^8 ends with 6

Let us look at the powers of 3

3^1 ends with 3
3^2 ends with 9
3^3 ends with 7
3^4 ends with 1
3^5 ends with 3
3^6 ends with 9
3^7 ends with 7
3^8 ends with 1

We can notice that the last digits repeat after every 4 steps for both 2 and 3.
In other words whenever the power is a multiple of 4, the last digit of the number will be same as the last digit of 2^4 and for powers of 3 it is 3^4.

NOTE:
Last digit of (even number)^{4k} = 6
Last digit of (odd number)^{4k} = 1

If the number ends with 0 and raised to any power, the last digit = 0

If the number ends with 5 and raised to any power, the last digit = 5

40. Find the last digit of 2^{99}

a) 2 b) 3 c) 4 d) 6

Explanation:

$2^{99} = 2^{96} \times 2^3 = 4 \times 8 = 2$ (2^{96} is in 2^{4k} form, the last digit is 6 & 2^3 ends with 8)

41. Find the units digit of $14^{124} \times 29^{123}$

a) 2 b) 3 c) 4 d) 6

Explanation:

$14^{124} = 4^{124} = (\text{even number})^{(4k)}$ form, so last digit is 6

$29^{123} = 9^{123} = 9^{120} \times 9^3 = 1 \times 9 = 9$

In the product last digit = $6 \times 9 = 4$

42. Find the units digit of $(518)^{163} + (142)^{157}$

a) 2 b) 3 c) 4 d) 6

Explanation:

$(518)^{163} = 8^{163} = 8^{160} \times 8^3 = 6 \times 2 = 2$

$(142)^{157} = 2^{157} = 2^{156} \times 2^1 = 6 \times 2 = 2$

So unit digit = 2 + 2 = 4

43. Find the units digit of $(1567)^{143} \times (1239)^{197} \times (2566)^{1027}$

a) 2 b) 3 c) 4 d) 6

Explanation:

$(1567)^{143} = 7^3 = 3$

$(1239)^{197} = 9^1 = 9$

$(2566)^{1027} = 6$

Answer = 3 x 9 x 6 = 6

FINDING THE REMAINDERS

44. Find the remainder when 2^{55} is divided by 9?

a) 1 b) 2 c) 3 d) 4

Explanation:

We need to find the remainder when 2^{55} is divided by 9.

Check which power of 2 leaves a remainder of +1 or – 1 when divided by 9.

Clearly it is 2^3 (since 8 by 9 the remainder is -1)

So $2^{55} = (2^3)^{18} \times 2^1 = (-1)^{18} \times 2 = 1 \times 2 = 2$

45. Find the remainder when 3^{147} is divided by 11?

a) 8 b) 9 c) 7 d) None

Explanation:

Check which power of 3 leaves a remainder of +1 or – 1 when divided by 11.

Clearly it is 3^5 (since $3^5/11 = 243/11$, implies remainder = 1)

So $3^{147} = (3^5)^{29} \times 3^2 = (1)^{29} \times 9 = 1 \times 9 = 9$

46. Find the remainder when 3^{86} is divided by 8?

a) 1 b) 6 c) 7 d) 2

Explanation:

Check which power of 3 leaves a remainder of +1 or – 1 when divided by 8.

Clearly it is 3^4 (since $3^4/8 = 81/8$, implies remainder = 1)

So $3^{86} = (3^4)^{20} \times 3^2 = (1)^{20} \times 9 = 1 \times 9 = 9$

When 9 is divided by 8 the remainder is 1

47. Find the remainder when (1251 x 1252 x 1253) is divided by 11?

a) 6 b) 9 c) 8 d) 7

Explanation:

we have to find the remainder when (1251 x 1252 x 1253) is divided by 11?

Now Remainder of [(1251)/11] = 8

Remainder of [(1252)/11] = 9

Remainder of [(1253)/11] = 10

Therefore Remainder of [(8 x 9 x 10)/11] = (-3) x (-2) x (-1) = 6

PROBLEMS ON DIVISION ALGORITHM (N = d x q +r)

48. A number when divided by 161 leaves a remainder of 57. Find the remainder when the same number is divided by 7?

a) 0 b) 1 c) 2 d) 3

Explanation:

A number when divided by 161 leaves a remainder of 57.

Using the division algorithm we can write the number as, N = 161q + 57

Now R (N/7) = R [(161q +57)]/7 = 1 (since 161 is divisible by 7 & when 57 is divided by 7, the remainder is 1)

49. A number when divided by a certain divisor leaves a remainder of 19. When twice the number is divided by the same divisor, the remainder is 7. Find the divisor?

a) 14 b) 21 c) 31 d) cannot be determined

Explanation:

A number when divided by a certain divisor leaves a remainder of 19.

Using the division algorithm we can write the number as, N = d x q + 19

Now twice the number, 2N = 2 d x q + 38

Given, R (2N)/d = 7

That is R (2 d x q + 38)/d = 7, implies d must be 31

50. A number when divided by a certain divisor leaves a remainder of 11.When the Square of the same number is divided by the same divisor the remainder is 1. How many values are possible for the divisor?

a)10 b) 11 c) 12 d) None

Explanation:

Given, a number N, when divided by a certain divisor d leaves a remainder of 11.

Using the division algorithm we can write the number as, $N = d \times q + 11$

Now square of the number, $N^2 = d^2q^2 + 22 \times d \times q + 121$

Here d^2q^2 is divisible by d and $(22 \times d \times q)$ is also divisible by d.

That means R (121/d) = 1, means 120 is divisible by d
That means d can be any factor of 120, greater than 11.
Therefore d can be 12, 15, 20, 24, 30, 40, 60 and 120.
So, 8 values are possible for d.