18ECC204J – Digital Signal processing

CLAT – I (2022–23 ODD)

Answer key

part – A

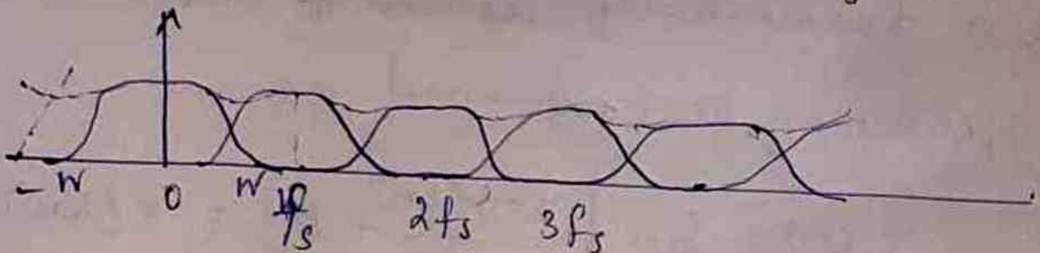
1. $x(n) = 2\cos 150\pi n$.

The freq. of the signal is $f = 75\,Hz$.

Min. sampling rate required is $f_s = 150\,Hz$.

2. Under sampling:

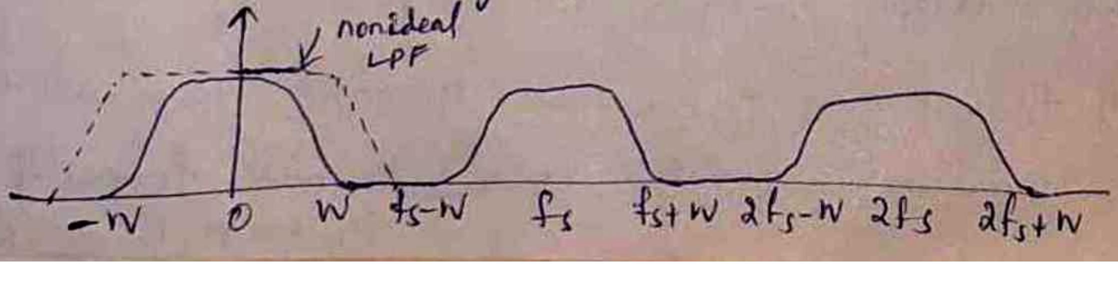
- If the sampling rate is lower than required nyquist rate, i.e. $f_s < 2W$.
- High freq. signals erroneously appear in the base band due to aliasing.



$f_s < 2W$ (or) $f_s < 2f_m$

Over sampling:

- Sampling above min. nyquist rate i.e. $f_s > 2f_m$
- it creates space in the spectrum that can reduce the demands on the analog anti aliasing filter.



$f_s > 2W$ (or) $f_s > 2f_m$

3. Consider the sinusoid $\cos(\omega_0 n+\theta)$.

It follows that

$$\cos[(\omega_0+2\pi)n+\theta] = \cos(\omega_0 n + 2\pi n+\theta)$$
$$= \cos(\omega_0 n+\theta)$$

As a result,

all sequences

$$x_K(n) = A\cos(\omega_K n + \theta), \quad K=0,1,2,\ldots$$

where $\omega_K = \omega_0 + 2K\pi$, $-\pi \le \omega_0 \le \pi$.

are identical.

4. Let $x_k(n)$ be the sinusoidal sgl sequence

$$x_k(n) = \sin\left(\frac{2\pi kn}{N}+\theta\right)$$

this is sinusoid with freq $f_K = K/N$,

which is harmonically related to $x(n)$.

But $x_k(n)$ may be expressed as

$$x_K(n) = \sin\left[\frac{2\pi(kn)}{N}+\theta\right] = x(kn).$$

thus, we observe that $x_K(0) = x(0)$, $x_k(1) = x(k)$,

$x_K(2) = x(2k)$ & so on. Hence the sequence

$x_k(n)$ can be obtained from the values of $x(n)$

by taking every K^{th} value of $x(n)$, beginning with $x(0)$.

By this manner, we can generate the values of

harmonically related sinusoids with freqs $f_K = K/N$

for $K=0,1,\ldots N-1$.

5. $\Delta = \frac{x_{max} - x_{min}}{L-1}$, $x_{max} - x_{min} \rightarrow$ dynamic range

$X_{max} = 1$, $X_{min} = 0$, $L = 11$.

$\Delta = 0.1$.

- If the dynamic range is fixed, increasing the number of quantization levels $L \rightarrow$ results in a decrease of quantization stepsize. Thus quantization error decreases and the accuracy of the quantizer increases.

6. If the quantization method is <u>truncation</u>, the number is approximated by the nearest level that does not exceed it.

The error made by truncating a number to b bits following the binary point satisfies

$$0 \geq x_T - x > -2^{-b}$$

$x_T \rightarrow$ truncated value

<u>ex:</u>

Decimal number 0.12890625.

Its binary eqnt is 0.00100001.

If we truncate the binary no/ to 4 bits, then

$x_T = (0.0010)_2 \rightarrow$ whose decimal is 0.125.

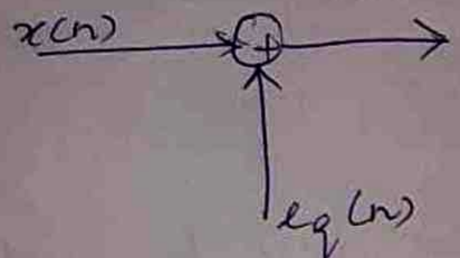
the error $(x_T - x) = -0.00390625$. Which is greater than $-2^{-b} = -2^{-4} = 0.0625$ satisfying the inequality.

Rounding of a number of b-bits is accomplished by choosing the rounded result as the b-bit number closest to the original number unrounded.

ex: 0.11010 rounded to 3 bits is either

0.110 or 0.111.

5.



$x_q(n) = x(n) + e_q(n)$

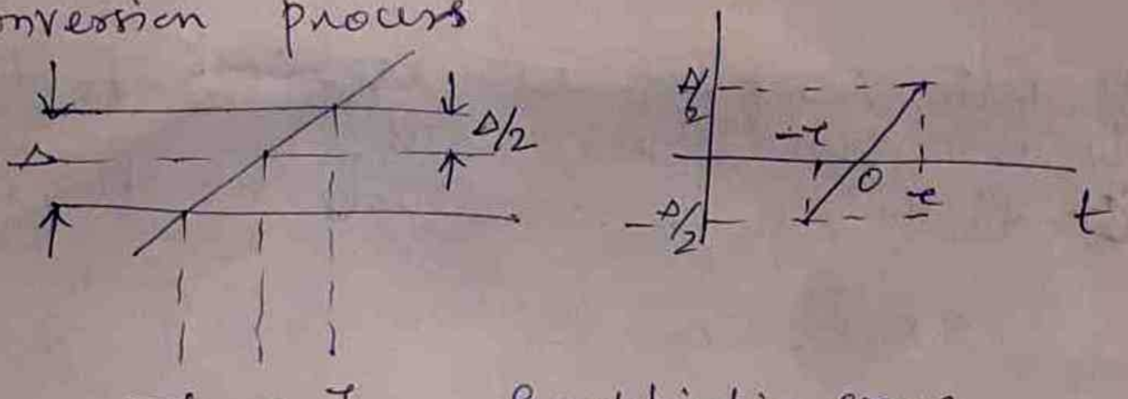
part - B

8. Quantization of sinusoidal signal

- Sampling & quantization of an analog sinusoidal signal $x_a(t) = A\cos\Omega_0 t$.

$x(n) = x_a(nT) \rightarrow$ by sampling & discrete time, discrete amplitude sgl $x_q(nT)$ after quantization.

- If sampling rate F_s satisfies the sampling theorem quantization is the only error in the A/D conversion process



quantization error

$e_q(t) = x_a(t) - x_q(t)$

$\tau \rightarrow$ time that $x_a(t)$ stays within quantization level

- mean error power P_q is

$$P_q = \frac{1}{2\tau}\int_{-\tau}^{\tau} e_q^2(t)\,dt = \frac{1}{\tau}\int_0^{\tau} e_q^2(t)\,dt$$

$e_q(t) = (\Delta/2\tau)\,t, \quad -\tau \le t \le t,$

$$P_q = \frac{1}{\tau}\int_0^{\tau}\left(\frac{\Delta}{2\tau}\right)^2 t^2\,dt = \frac{\Delta^2}{12}$$

→ If the Quantizer has b-bits of accuracy & Quantizer covers 2A range, $\Delta = 2A/2^b$

$$P_q = \frac{A^2/3}{2^{2b}}$$

Avg power $x_a(t)$ is

$$P_x = \frac{1}{T_p}\int_0^{T_p} (A\cos\Omega_0 t)^2 dt = A^2/2$$

$$SQNR = \frac{P_x}{P_q} = \frac{3}{2}\cdot 2^{2b}$$

$$\boxed{SQNR(dB) = 10\log_{10} SQNR = 1.76 + 6.02b}$$

9. a) Min. sampling rate required $F_s = 100\,Hz$

b) $x(n) = 3\cos\frac{100\pi}{200}n = 3\cos\frac{\pi}{2}n$.

c) If the sgl. is sampled at $F_s = 75\,Hz$,

$$x(n) = 3\cos\frac{100\pi}{75}n = 3\cos\frac{4\pi}{3}n.$$

$$= 3\cos\left(2\pi - \frac{2\pi}{3}\right)n = 3\cos\frac{2\pi}{3}n.$$

d) for $F_s = 75\,Hz$,

$$F = fF_s = 75f$$

The freq. of the sinusoid in part (c) is

$f = 1/3$, hence $F = 25\,Hz$.

The sinusoidal sgl is $3\cos 2\pi Ft = 3\cos 50\pi t$ sampled at $F_s = 75$ samples/s → gives identical samples. Hence $F = 50\,Hz$ is an alias of $F = 25\,Hz$ for the sampling rate 75 Hz.