DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB101J-Electromagnetic Theory, Quantum Mechanics, Waves and Optics

Module 3- Lecture-7

Time-independent Schrodinger’s wave equation

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Topics to be Discussed:-

Time-independent Schrodinger’s wave equation

Second derivative with respect to X

Shrodinger Wave Function

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

Position

Energy

Potential Energy

Time-Independent Schroedinger Equation in One Dimension

A variable quantity which characterizes de-Broglie waves is known as Wave function and is denoted by the symbol ψ. The value of the wave function associated with a moving particle at a point (x, y, z) and at a time ‘t’ gives the probability of finding the particle at that time and at that point.

Schrodinger Wave Equation

- ***It is one of the basic equations in quantum mechanics.***
- ***a particle exhibits wave properties; then there should be some sort of wave equation associated with the particle describing the behavior of the particle.***
- ***Schrodinger derived a mathematical equation to describe the dual nature of matter waves.***
- ***The equation that describes the wave nature of a particle in mathematical form is known as Schrodinger wave equation***
- ***Schrodinger connected expression for the de Broglie wavelength into the classical wave equation for a moving particle and obtained a new wave equation.***
- ***The Schrodinger is applicable for both microscopic and macroscopic particles.***

DERIVATION Of Time-Independent Schroedinger Equation

Let us consider a particle of mass 'm', moving with a velocity 'v'. The de Broglie wavelength associated with it is given by,

$$\lambda = \frac{h}{mv} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where h = Planck's constant = 6.626 × 10 $^{-34}$ J s.

Let *ψ* be the wave function of the particle along *x, y* and *z* coordinate at any time '*t*' .The classical differential equation of a progressive wave moving with a wave velocity '*v*' can be written as,

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The solution for the equation (2) is given by,

$$\psi = \psi_\circ e^{-i\omega t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where

ψ □= **Amplitude of the wave at the point (*x,y,z*)**

ω = *Angular frequency of the wave*

Differentiating eqn. (3) with respect to '*t*',

$$\frac{\partial \psi}{\partial t} = (-i\omega)\psi_{\circ} e^{-i\omega t} \qquad \text{..................................(4)}$$

Differentiating eqn. (4) with respect to '*t*',

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)^2 \psi_{\circ} e^{-i\omega t} = -\omega^2 \psi \qquad \text{..................................(5)}$$

Substituting eqn. (5) in eqn. (2),

$$\nabla^2\psi = -\left(\frac{\omega^2}{v^2}\right)\psi \quad \text{-------------------------------------(6)}$$

where ∇^2 = Laplacian operator

$$\omega = 2\pi f = 2\pi\left(\frac{v}{\lambda}\right) \qquad (OR), \frac{\omega}{v} = \frac{2\pi}{\lambda}$$

$$\frac{\omega^2}{v^2} = \frac{4\pi^2}{\lambda^2} \quad \text{-------------------------------------(7)}$$

Substituting eqn. (7) in eqn. (6),

$$\nabla^2\psi = -\left(\frac{4\pi^2}{\lambda^2}\right)\psi \qquad \textbf{\underline{(OR),}}$$

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \quad \text{--(8)}$$

<u>Substituting eqn (1) in eqn (8),</u>

$$\nabla^2\psi + \frac{4\pi^2}{\left(h/mv\right)^2}\psi = 0 \quad \text{--(9)}$$

$$\nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = o$$

If '*E*' is the total energy of the particle, '*V*', the potential energy, then total energy of the particle = *E* = PE + KE

$$E = V + \frac{1}{2} mv^2 \text{---}(10)$$

(OR), 2(E −V) =

Multiplying both sides by '*m*', in equation (10), we have

$$2m\,(E - V) = m^2 v^2 \text{------------------------------}(11)$$

Substituting eqn. (11) in eqn (9),

$$\nabla^2\psi + \frac{8\pi^2 m}{\lambda^2}(E - V)\psi = 0 \qquad \text{------------------------}(12)$$

This equation is known as *Schrödinger's time independent wave equation*

Introducing, in the above equation (12)

$$\hbar = \frac{h}{2\pi}$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

This is called One Dimensional Schroedinger Equation (Time-Independent)

For three dimension

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$