D2- Slot

PART-A

1. b. $\frac{1}{3}[2+2\cos t+3\sin t]$

2. a. $\frac{55}{3}$

3. c. $\frac{1}{1+4\tau^2}$

4. d. $a^2 R_{XX}(\tau)\ R_{YY}(\tau)$

5. a. $\underset{T\to\infty}{Lt}\ \operatorname{var} \bar{X}_T = 0$

6. d. non- negative

7. a. Odd function

8. a. 0, 0

9. c. $\frac{1}{\alpha^2+\omega^2} S_{XX}(\omega)$

10. b. $\frac{2\lambda}{\lambda^2+\omega^2}$

PART-B

11. θ is a R.V uniformly distributed in $(0, 2\pi)$

$$\therefore\ f(\theta) = \frac{1}{2\pi},\ 0 < \theta < 2\pi \quad \text{— (2M)}$$

$$E[X(t)] = \int_0^{2\pi} A\cos(\omega t+\theta)\frac{1}{2\pi}\,d\theta$$

$$= \frac{A}{2\pi}\left[\sin(\omega t+\theta)\right]_0^{2\pi}$$

$$E[X(t)] = 0,\ \text{constant} \quad \text{— (2M)}$$

$$R_{xx}(\tau) = E[x(t)\, x(t+\tau)]$$

$$= E[A\cos(\omega t+\theta)\, A\cos(\omega t+\omega\tau+\theta)]$$

$$= \frac{A^2}{2}\left\{E[\cos(2\omega t+\omega\tau+2\theta)] + E[\cos(-\omega\tau)]\right\} \quad \text{— (2M)}$$

$$E[\cos(2\omega t+\omega\tau+2\theta)] = \int_0^{2\pi} \cos(2\omega t+\omega\tau+2\theta)\frac{1}{2\pi}\,d\theta$$

$$= 0 \quad \text{— (2M)}$$

$$\therefore R_{xx}(\tau) = \frac{A^2}{2}\cos\omega\tau \quad \text{— (2M)}$$

$\therefore$ $x(t)$ is a WSS process.

12. $$E[x(t)=n] = \sum_{n=} n\, p(n)$$

$$= \frac{1}{(1+at)^2} + 2\cdot\frac{at}{(1+at)^3} + 3\frac{(at)^2}{(1+at)^4} + \cdots$$

$$= \frac{1}{(1+at)^2}\left[1 + 2\left(\frac{at}{1+at}\right) + 3\left(\frac{at}{1+at}\right)^2 + \cdots\right]$$

$$= \frac{1}{(1+at)^{2}} \times (1+at)^2$$

$$= 1, \text{ a constant} \quad \text{(3M)}$$

$$E[x^2(t)] = \sum_{n=0}^{\infty} n^2 p(n)$$

$$= \sum_{n=0}^{\infty} n(n+1)\, p(n) - \sum_{n=0}^{\infty} n\, p(n)$$

$$= \frac{1}{(1+at)^2}\sum_{n=1}^{\infty} n(n+1)\left(\frac{at}{1+at}\right)^{n-1} - 1$$

$$= 2(1+at) - 1 \quad \text{— (3M)}$$

$$= 1+2at$$

$\therefore \operatorname{var}[x(t)] = E[x^2(t)] - (E[x(t)])^2$

$= 1 + 2at - 1$

$= 2at$, which is function of 't'

$\therefore x(t)$ is not a stationary process. —— (4M)

13. $R_{xx}(\tau) = E[x(t)\, x(t+\tau)]$

$= E[(U\cos t + V\sin t)(U\cos(t+\tau) + V\sin(t+\tau))]$

$= E[U^2 \cos t \cos(t+\tau) + UV \cos t \sin(t+\tau) + V^2 \sin t \sin(t+\tau) + VU \sin t \cos(t+\tau)]$

$= E[U^2] \cos t \cos(t+\tau) + E(V^2) \sin t \sin(t+\tau) + E[UV] \sin(2t+\tau)$ —— (4M)

since U & V are independent

$\therefore E(UV) = E(U)\, E(V)$

Value:	-2	1
P :	$\cancel{1}/3$	$2/3$

$E(U) = -2(1/3) + 1(2/3) = 0$

$E(V) = 0$ —— (2M)

$\Rightarrow E(UV) = 0$

$E(U^2) = \sum u^2 P(u) = (-2)^2 \cdot \frac{1}{3} + 1^2\left(\frac{2}{3}\right)$

$= 2$ —— (2M)

$E(V^2) = 2$

$$\therefore R_{xx}(\tau) = 2\cos t \cos(t+\tau) + 2\sin t \sin(t+\tau)$$
$$= 2\cos\tau \quad \text{—— (2M)}$$

14. $$\frac{\omega^2+9}{(\omega^2+1)(\omega^2+4)} = \frac{A}{\omega^2+1} + \frac{B}{\omega^2+4}$$
$$\Rightarrow \omega^2+9 = A(\omega^2+4) + B(\omega^2+1)$$
Put $\omega^2 = -4$, $B = -\frac{5}{3}$

$\omega^2 = -1$, $A = \frac{8}{3}$

$$\frac{\omega^2+9}{(\omega^2+1)(\omega^2+4)} = \frac{8/3}{\omega^2+1} + \frac{-5/3}{\omega^2+4} \quad \text{—— (4M)}$$

$$F^{-1}[S_{xx}(\omega)] = \frac{8}{3} F^{-1}\left[\frac{1}{\omega^2+1}\right] - \frac{5}{3} F^{-1}\left[\frac{1}{\omega^2+4}\right]$$
$$= \frac{8}{6} F^{-1}\left[\frac{2}{\omega^2+1}\right] - \frac{5}{12} F^{-1}\left[\frac{4}{\omega^2+4}\right]$$
$$R_{xx}(\tau) = \frac{8}{6} e^{-|\tau|} - \frac{5}{12} e^{-2|\tau|} \quad \text{—— (4M)}$$
$$\text{—— (2M)}$$
$$\cancel{B}\ P_{xx} = R_{xx}(0) = \frac{11}{12}$$

15. $$S_{xx}(\omega) = \int_{-1}^{1} (1-|\tau|)\, e^{-i\omega\tau} d\tau$$
$$= 2\int_0^1 (1-\tau)\cos\omega\tau\, d\tau \quad \text{—— (4M)}$$
$$= 2\left[(1-\tau)\underbrace{\frac{\sin\omega\tau}{\omega}}_{\to 0} - (-1)\,\frac{-\cos\omega\tau}{\omega^2}\right]_0^1$$
$$= -\frac{2}{\omega^2}\left[\cos\omega - 1\right]$$
$$= \frac{2}{\omega^2}\, 2\sin^2\frac{\omega}{2}$$

$$S_{xx}(\omega) = \left(\frac{\sin \omega/2}{\omega/2}\right)^2 \quad \text{— (6M)}$$

16. $$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau)\, e^{-i\omega\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-4|\tau|}\, e^{-i\omega\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-4|\tau|} (\cos\omega\tau - i\sin\omega\tau)\, d\tau$$

$$= 2\int_{0}^{\infty} e^{-4\tau} \cos\omega\tau\, d\tau \quad \| \because \int_{-\infty}^{\infty} e^{-4|\tau|} \sin\omega\tau\, d\tau = 0$$

$$= 2\left[\frac{e^{-4\tau}}{(-4)^2+\omega^2}(-4\cos\omega\tau + \omega\sin\omega\tau)\right]_0^{\infty}$$

$$S_{xx}(\omega) = \frac{8}{4^2+\omega^2} \quad \text{— (4M)}$$

$$H(\omega) = F[h(t)] = \int_{-\infty}^{\infty} h(t)\, e^{-i\omega t}\, dt$$

$$= \int_{0}^{\infty} 2e^{-7t}\, e^{-i\omega t}\, dt$$

$$= 2\int_{0}^{\infty} e^{-(7+i\omega)t}\, dt$$

$$= 2\left[\frac{e^{-(7+i\omega)t}}{-(7+i\omega)}\right]_0^{\infty}$$

$$= \frac{2}{7+i\omega}$$

$$|H(\omega)|^2 = \frac{4}{49+\omega^2} \quad \text{— (4M)}$$

$$\therefore S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$$

$$= \frac{32}{(16+\omega^2)(49+\omega^2)}$$

— (2M)