Digital Signal Processing Lab

Exp 10: Design of Analog Butterworth and Chebyshev Filters

I Pre-Lab Questions:

1 List the differences between butterworth and Chebyshev filter

Soln.

Butterworth Filter	Chebyshev Filter
- For a particular desired specification of a digital filter the order of butterworth filter will be higher than Chebyshev filter.	- For a particular desired specification of a digital filter, the order of chebyshev filter will be lower as compared to butterworth filter.
- For a particular specification, butterworth filter requires more hardware.	- For a particular specification, Chebyshev filter requires less hardware.
- Cutoff freq is not equal to passband frequency. $\Omega_c = \Omega_p\left(\frac{1}{A_p^2}-1\right)^{1/2N}$	- Cutoff frequency is equal to passband freq. $\Omega_c = \Omega_p$

2 Define the term prewarping and mention its importance.

Soln Frequency warping follows a known pattern, and there is a known relationship between the wrapped freq and the known freq we can use a technique called prewrapping to account for the nonlinearity and produce a more faithful wrapping.

3 To design a discrete time low pass filter the specifications are
passband $F_P = 4$ kHz with 0.8 dB ripple.
Stopband $F_S = 4.5$ kHz with 50 dB attenuation
sampling freq $F_s = 22$ kHz.

Soln

(i) the mapping from analog to digital frequency.

$\omega = \frac{2\pi f}{F_s}$ with F_s being the sampling frequency.

Then passband and stop band becomes

$$\omega_p = \frac{2\pi \times 4}{22} = 0.36\pi \text{ rad}$$

$$\omega_s = \frac{2\pi \times 4.5}{22} = 0.41\pi \text{ rad}$$

(ii) The freq response in the passband is within the interval $1 \pm \delta$ where δ is such that $20 \log_{10}(1+\delta) = 0.8$.

this yields $\delta = 1.0024 - 1 = 1.0096$

$\therefore$ The frequency response within the passband is within the interval $0.9033 \leq$ modulus of $H(\omega) \leq 1.096$.

Similarly in stop band, value of $|H(\omega)| \leq 10^{-\frac{50}{20}}$

is $H(\omega) = 0.0031$

II Post-Lab Questions:

1. List the properties of Chebyshev filter.

Soln. The chebyshev filter has a passband optimization to minimize the maximum error over the complete passband frequency range and a stop band controlled by the frequency response being marginally flat at $\omega = \infty$. The passband ripple and the filter order are the two parameters to be determined by the specification.

2. In your CD the data is samples at 44.1 kHz, and we want to have a good sound quality upto 21 kHz. If you had to use an analog butterworth filter as reconstruction filter, what would be the order of the filter?

Soln. Since we want the filter to pass the signal and reject all frequency above $F_s/2$, we can see that passband and stop band frequencies are

$\Omega_p = 2\pi(21k)$ rad/sec.

$\Omega_s = \frac{2\pi(44100)}{2} = 2\pi(22050)$ rad/sec.

Assuming at 1dB passband ripple and 40dB attenuation in the stop band this would yields a frequency response of the form.

$$|H(\omega)| = \sqrt{\frac{1}{1+\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}}}$$

With $\varepsilon = 0.509$ and Ω_p as given. For an attenuation of 40dB we obtain N: use of the Solving

$$|H(\Omega)| = \sqrt{\frac{1}{1+\varepsilon\left(\frac{\Omega}{\Omega_1}\right)^{2N}}} \leq 0.01$$ for the order N.

3 Give the normalized low pass butterworth denominator polynomials for N = 8, 9, 10

Soln. For N = 8

$(1+0.390s+s^2)(1+1.11s+s^2)$

$=(1+1.663s+s^2)(1+1.962s+s^2)$

For N = 9

$(1+s)(1+0.3+7s+s^2)(1+s+s^2)$

$(1+1.532s+s^2)(1+1.879s+s^2)$

For N = 10

$(1+0.313s+s^2)(1+0.908s+s^2)$

$(1+1.414s+s^2)(1+1.782s+s^2)(1+1.975s+s^2)$