SRM University
Department of Mathematics
Complex Integration- Multiple Choice questions
UNIT V

Slot-B

1. A continuous curve which does not have a point of self-intersection is called a
   a. Curve
   b. Closed curve
   c. Simple closed curve
   d. Multiple curve

   Answer: c. Simple closed curve

2. The zero's of $f(z) = \frac{z^2+1}{1-z^2}$ are
   a. 0
   b. $\pm i$
   c. ± 1
   d. 1

   Answer: b. $\pm i$

3. If $f(z)$ is analytic inside and on C, then the value of $\oint_C \frac{f(z)}{z-a} dz$, where C is the simple closed curve and a is any point within C is

   a. $f(a)$
   b. $2\pi i f(a)$
   c. $\pi i f(a)$
   d. 0

   Answer: b. $2\pi i$f(a)

4. If $f(z)$ is analytic inside and on C, then the value of $\oint_C \frac{f(z)}{(z-a)^5} dz$, where C is the simple closed curve and a is any point within C is

    a. $2\pi i \frac{f^{v}(a)}{5!}$
    b. $2\pi i f(a)$
    c. $2\pi i \frac{f^{iv}(a)}{4!}$
    d. 0

Answer:c. $2\pi i \frac{f^{iv}(a)}{4!}$

5. The value of $\oint_C \frac{e^{-z}}{z+1} dz$ where C is the circle $|z| = \frac{1}{3}$ is
    a. 0
    b. $2\pi ie$
    c. $\frac{\pi}{2} ie$
    d. πie

Answer: a. 0

6. The value of $\oint_C \frac{e^{2z}}{(z+1)^3} dz$ where C is the circle $|z| = 2$ is
    a. 0
    b. $2\pi ie^{-2}$
    c. $8\pi ie^{-2}$
    d. $4\pi ie^{-2}$

Answer: d. $4\pi ie^{-2}$

7. The value of $\oint_C \frac{1}{2z-3} dz$ where C is the circle $|z| = 1$ is
    a. 0
    b. $2\pi i$
    c. $\frac{\pi}{2} i$
    d. πi

Answer: a. 0

8. The value of $\oint_C \frac{z^2}{(z-2)^2} dz$ where C is the circle $|z| = 3\ is$
   a. 0
   b. $2\pi i$
   c. $4\pi i$
   d. $8\pi i$

   Answer: d. $8\pi i$

9. Let $C_1: |z-a| = R_1$ and $C_2: |z-a| = R_2$ be two concentric circles $(R_2 < R_1)$, the annular region is defined as
   a. Within C_1
   b. Within C_2
   c. Within C_2 and outside C_1
   d. Within C_1 and outside C_2

   Answer: d. Within C_1 and outside C_2

10. The part $\sum_{n=1}^{\infty} b_n(z-a)^{-n}$ consisting of negative integral powers of $(z-a)$ is called as
    a. The analytic part of the Laurent's series
    b. The principal part of the Laurent's series
    c. The real part of the Laurent's series
    d. The imaginary part of the Laurent's series

    Answer: b. The principal part of the Laurent's series

11. Let $C: |z-a| = r$ be a circle, the $f(z)$ can be expanded as a Taylor's series if
    a. $f(z)$ is a function on C
    b. $f(z)$ is an analytic function within C
    c. $f(z)$ is not an analytic function within C
    d. $f(z)$ is an analytic function outside C

    Answer: b. f(z) is an analytic function within c

12. Expansion of $\frac{\sin z}{(z-\pi)}$ in Taylor's series about $z = \pi$ is

a. $\frac{(z-\pi)}{1!} - \frac{(z-\pi)^3}{3!} + \frac{(z-\pi)^5}{5!} - \cdots$
b. $\frac{(z-\pi)^2}{2!} - \frac{(z-\pi)^4}{4!} + \frac{(z-\pi)^6}{6!} - \cdots$
c. $-1 + \frac{(z-\pi)^2}{3!} - \frac{(z-\pi)^4}{5!} + \cdots$
d. $\frac{(z-\pi)}{2!} + \frac{(z-\pi)^3}{4!} - \frac{(z-\pi)^5}{6!} + \cdots$

Answer :c. $-1 + \frac{(z-\pi)^2}{3!} - \frac{(z-\pi)^4}{5!} + \cdots$

13. The annular region for the function $f(z) = \frac{1}{z^2-z-6}$ is

a. $0 < |z| < 1$
b. $1 < |z| < 2$
c. $2 < |z| < 3$
d. $|z| < 3$

Answer :c. $2 < |z| < 3$

14. The Laurent's series expansion $-\frac{1}{2}\sum\frac{(z+2)^n}{4^n} - \sum\frac{3^n}{(z+2)^n}$ for the function $f(z) = \frac{z}{(z-1)(z-2)}$ is valid in the region

a. $|z+2| < 3$
b. $1 < |z+2| < 2$
c. $3 < |z+2| < 4$
d. $|z+2| > 4$

Answer :c. $3 < |z+2| < 4$

15. If $f(z)$ is not analytic at $z = z_0$ and there exists $\lim_{z\to z_0} f(z)$ and is finite then,

a. The point $z = z_0$ is isolated singularity of $f(z)$
b. The point $z = z_0$ is a removable singularity of $f(z)$
c. The point $z = z_0$ is essential singularity of $f(z)$
d. The point $z = z_0$ is non isolated singularity of $f(z)$

Answer : b. The point $z = z_0$ is a removable singularity of $f(z)$

16. Let $z = a$ is a simple pole for $f(z)$ and $b = \lim_{z \to a} (z - a)f(z)$, then

a. b is a simple pole
b. b is removable singularity
c. b is a residue at a of order n
d. b is a residue at $z = a$

Answer : d. b is a residue at $z = a$

17. Let z = a is a pole of order m for $f(z)$, then the residue is

a. $\lim_{z \to a} [(z - a)f(z)]$
b. $\lim_{z \to a} [(z - a)f''(z)]$
c. $\lim_{z \to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} [(\mathrm{z} - \mathrm{a})^m f(z)]$
d. $\lim_{z \to a} \frac{1}{m!} \frac{d^m}{dz^m} [(\mathrm{z} - \mathrm{a})^m f(z)]$

e. Answer: c. $\lim_{z \to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} [(\mathrm{z} - \mathrm{a})^m f(z)]$

18. The residue of $f(z) = \frac{z}{(z-1)^2}$ at $z = 1$ is

a. π
b. 1
c. -1
d. 0

Answer: b. 1

19. The residue of $f(z) = \frac{z}{z^2+1}$ at $z = i$ is

a. 1
b. -1
c. 0
d. 1/2

Answer : d. 1/2

20. If $f(z) = \frac{sinz}{z}$, then

a. $z = 0$ is a simple pole
b. $z = 0$ is a pole of order 2
c. $z = 0$ is a removable singularity
d. $z = 0$ is a zero of $f(z)$

Answer: c. $z = 0$ is a removable singularity

21. The value of the integral $\oint_C e^z dz$ where $|z| = 1$ is

a. $2\pi i$
b. $\frac{\pi}{2} i$
c. πi
d. 0

Answer: d. 0

22. If $f(z) = \frac{-1}{(z-1)} - 2[1 + (z-1) + (z-1)^2 + \cdots]$ then the residue of $f(z)$ at $\mathrm{z} = 1$is

a. 1
b. -1
c. 0
d. -2

Answer: b. -1

23. If the integral $\oint_0^{2\pi} \frac{d\theta}{5+3\cos\theta} = \oint_C f(z)dz$, C is $|z| = 1$, then
(A) $z = -\frac{1}{3}$ lies inside C and
(B) $z = 3$ lies outside C. Which of the following is true.

a. Both A and B
b. Only A
c. Only B
d. Neither A nor B

Answer: a. Both A and B

24. In Cauchy's Lemma for contour integration, if $f(z)$ be continuous function such that $|zf(z)| \to 0$ as $|z| \to \infty$, for C is the circle $|z| = R$, then

a. $\oint_C f(z)dz \to \infty \; as \; R \to \infty.$

b. $\oint_C f(z)dz \to 0 \; as \; R \to \infty.$

c. $\oint_C f(z)dz \to 0 \; as \; R \to 0.$

d. $\oint_C f(z)dz \to \infty \; as \; R \to 0.$

Answer : b. $\oint_C f(z)dz \to 0 \; as \; R \to \infty.$

25. If $\oint_C \frac{e^z}{z^2} dz = 0$, then C is

a. $|z| = 1$
b. $|z - 1| = 2$
c. $|z - 2| = 1$
d. $|z| = 2$

Answer: c. $|z - 2| = 1$