29.a. A two dimensional discrete RV (X,Y) has the joint pmf,

X \ Y	0	1	2
0	0	1/27	2/27
1	2/27	3/27	4/27
2	4/27	5/27	6/27

Find (i) $P(X \le 1)$ (ii) $P(Y \le 1)$ (iii) $P(X \ge Y)$ (iv) $P(X + Y \le 3)$ (v) $P(X \le 1, Y \le 1)$ (vi) $P(X \le 2 / Y \le 1)$.

(OR)

b.i. If the joint pdf of (X,Y) is given by $f(x,y)=\begin{cases}\frac{2}{5}(2x+5y), 0\le x\le 1,\ 0\le y\le 1\\ 0, \quad otherwise\end{cases}$

Find the marginal density functions of X and Y.

ii. If the joint pdf of (X,Y) is given by $f(x,y)=\begin{cases}x+y, 0\le x\le 1\\ \quad 0\le y\le 1\\ 0, \quad otherwise\end{cases}$

Find the pdf of U=XY.

30.a.i. If {X(t)} is a WSS process with autocorrelation function $R(\tau)=Ae^{-\alpha|\tau|}$. Determine the second order moment of the RV X(8)–X(5).

ii. A radioactive source emits particles at a rate of 6 per minute in accordance with Poisson process. Each particle emitted has a probability 1/3 of being recorded. Find the probability atleast 4 particles are recorded in 5 min period.

(OR)

b. A gambler's luck follows a pattern. If he wins a game, the probability of his winning the next game is 0.6. However, if he loses a game, the probability of his losing the next game is 0.7. There is an even chance that the gambler wins the first game. What is the probability that he wins (i) the second game (ii) in the long run?

31.a.i. Prove: $R_{XY}(\tau)=R_{XX}(-\tau)$.

ii. Prove: $|R_{XY}(\tau)|\le\sqrt{R_{XX}(0)\cdot R_{YY}(0)}$.

iii. If {X(t)} and {Y(t)} are independent processes, then prove that $R_{XY}(\tau)=R_{YX}(\tau)=\mu_X\cdot\mu_Y$.

(OR)

b. If $X(t)=A\cos(\omega t+\theta)$ and $Y(t)=B\sin(\omega t+\theta)$ where θ is a RV uniformly distributed in $(0,2\pi)$ where A and B are constant, prove that {X(t)} and {Y(t)} are jointly WSS.

32.a.i. Find the autocorrelation function corresponding to $S_{XX}(\omega)=\frac{157+12\omega^2}{(16+\omega^2)(9+\omega^2)}$. Find also the average power.

ii. If $S_{XX}(\omega)=\begin{cases}\pi, |\omega|<1\\ 0, otherwise\end{cases}$

Find $R_{XX}(\tau)$.

(OR)

b. A WSS process *{X(t)}* is the input to a linear system with impulse response $h(t)=2e^{-7t}, t\ge 0$. If the autocorrelation function of *{X(t)}* is $R_{XX}(\tau)=e^{-4|\tau|}$, find the power spectral density of the output process {X(t)}.

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Reg. No.															

B.Tech. DEGREE EXAMINATION, DECEMBER 2019

First to Eighth Semester

15MA209 – PROBABILITY AND RANDOM PROCESS

(For the candidates admitted during the academic year 2015-2016 to 2017-2018)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B and Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. Given that the pdf of a RV X is $f(x)=2x, 0<x<1$. What is *P(X>0.5)*?
 (A) 1/2 (B) 2/3
 (C) 3/4 (D) 4/5

2. The mean and SD of a binomial distribution are 5 and 2. What is the value of n?
 (A) 5 (B) 10
 (C) 15 (D) 25

3. The mean of the exponential distribution is
 (A) $\frac{1}{\lambda}$ (B) $\frac{2}{\lambda^2}$
 (C) λ (D) $\frac{\lambda^2}{2}$

4. A coin is tossed repeatedly. What is the probability of getting a head for the first time in 6th toss?
 (A) $\left(\frac{1}{2}\right)^5$ (B) $\left(\frac{1}{2}\right)^3$
 (C) $\frac{1}{2}$ (D) $\left(\frac{1}{2}\right)^6$

5. The joint pmf of (X,Y) is given by

X \ Y	0	1	2
0	0	k	2k
1	k	2k	3k
2	2k	3k	4k

 Then the value of K is
 (A) 1/18 (B) 1/12
 (C) 3/4 (D) 1/3

6. The marginal probability function of X from $f_{xy}(x,y)$ is
 (A) $\int f(x,y)dx$ (B) $\int f(x,y)dy$
 (C) $\iint_R f(x,y)dxdy$ (D) $\frac{d}{dx}f(x,y)$

7. If X and Y are independent random variables with density functions $f_X(x)=e^{-x}, x\ge 0$ and $f_Y(y)=e^{-y}, y\ge 0$ then the joint pdf of (X,Y) is
 (A) $e^{-x}+e^{-y}$ (B) $e^{-(x+y)}$
 (C) $\frac{e^{-x}}{e^{-y}}$ (D) $\frac{e^{-y}}{e^{-x}}$

8. If $S_n \sim N(200,5)$ *then* $P(200 < S_n < 210)$ is equal to
(A) $P(0<z<2)$
(B) $P(-2<z<2)$
(C) $P(0<z<5)$
(D) $P(-2<z<1.6)$

9. If both S and T are discrete then the random process is called
(A) Discrete random sequence
(B) Continuous random sequence
(C) Discrete random process
(D) Continuous random process

10. The mean of the Poisson process is
(A) $1+\lambda t$
(B) λ
(C) λt^2
(D) λt

11. If the tpm of the Markov chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$, find the steady state distribution of the chain
(A) $\pi_1 = \frac{1}{5},\ \pi_2 = \frac{4}{5}$
(B) $\pi_1 = \frac{1}{3},\ \pi_2 = \frac{2}{3}$
(C) $\pi_1 = \frac{1}{2},\ \pi_2 = \frac{1}{2}$
(D) $\pi_1 = \frac{1}{4},\ \pi_2 = \frac{3}{4}$

12. The sum of all the elements in any row of tpm is
(A) 0
(B) 1
(C) 0.5
(D) 0.75

13. $R_{XX}(0)$ is equal to
(A) $E[X(t)]$
(B) $Var(X(t))$
(C) $E[X^2(t)]$
(D) $[E(X(t))]^2$

14. Two random processes X(t) and Y(t) are orthogonal if $R_{xy}(\tau)$ is
(A) 1
(B) -1
(C) $R_{yx}(\tau)$
(D) 0

15. If the random processes $\{X(t)\}$ and $\{Y(t)\}$ are independent then R_{XY}=?
(A) $E\left[X^2(t)\ Y^2(t)\right]$
(B) $E(X(t))\cdot E(Y(t))$
(C) 0
(D) 1

16. Let $\{X(t)\}$ be a random process. Then the time average of the process over (–T,T) is defined as
(A) $\overline{X}_T = \frac{1}{2T}\int_{-T}^{T} X(t)dt$
(B) $\overline{X}_T = \frac{1}{T}\int_{-T}^{T} X(t)dt$
(C) $E[X(t)]$
(D) $E[X(t)]^2$

17. Real $(S_{XY}(\omega))$ and real $(S_{YX}(\omega))$ are ______functions of ω.
(A) Always linear
(B) Even
(C) Odd
(D) Neither even nor odd

18. The power spectral density function of a random signal with $R_{XX}(\tau) = e^{-\alpha|\tau|}$ is
(A) $\frac{\alpha}{\alpha^2+\omega^2}$
(B) $\frac{\omega}{\alpha^2+\omega^2}$
(C) $\frac{2\alpha}{\alpha^2+\omega^2}$
(D) $\frac{2\omega}{\alpha^2+\omega^2}$

19. The convolution form of the output of linear time invariant system is,
(A) $Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$
(B) $Y(t) = \int_{-\infty}^{\infty} h(t)X(t-u)du$
(C) $Y(t) = \int_{0}^{\infty} h(t)X(t-u)du$
(D) $Y(t) = \int_{-\infty}^{\infty} h(t)X(u)du$

20. The power spectral density function of a WSS process is always
(A) Zero
(B) Finite
(C) Non negative
(D) Negative

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. If a RV X has MGF $M_X(t) = \frac{2}{2-t}$, find Var(x).

22. If X is a continuous RV with pdf, $f(x) = \begin{cases} \frac{x}{6}, 2<x<4 \\ 0,\ otherwise \end{cases}$

Find the pdf of Y=2X+3.

23. If $f(x,y) = \begin{cases} x+y,\ 0<x<1 \\ \quad\quad 0<y<1 \\ 0,\quad otherwise \end{cases}$

Find the marginal density functions of X and Y.

24. If $X_1, X_2, \ldots X_n$ are independent Poisson variables with parameter λ=2, use CLT to estimate $P(120 < S_n < 160)$ where $S_n = S_1 + S_2 + \ldots + S_{75}$.

25. If patients arrive randomly and independently at a clinic according to a Poisson process with mean rate of 2/min. find the probability that in a 2 min interval exactly one patient arrives.

26. A stationary random process X(t) has autocorrelation function given by, $R_{XX}(\tau) = \frac{25\tau^2+36}{6.25\tau^2+4}$

Find the mean and variance of X(t).

27. If $R_{XX}(\tau) = \begin{cases} 1,\ |\tau| \le 1 \\ 0,\ otherwise \end{cases}$

Find the power spectral density function of the process.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28.a.i. A continuous RV X has the pdf $f(x) = kx^2e^{-x}, x \ge 0$. Find its mean and variance.

ii. The number of accidents in a year attributed to taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the number of drivers with (i) no accidents in a year (ii) more than 2 accidents in a year.

(OR)

b.i. The probability that an applicant for a driver's license will pass the road test on any given trail is 0.7. find the probability that he will pass the test finally (a) On the 3rd trial (b) Fewer than four trials

ii. If a RV X follows normal distribution with mean 28 and SD 25, find (i) P(32<X<68) (ii) P(X>32).