b. In a normal distribution 30% of the items are under 45 and 8% are over 60. Find the mean and S.D of the distribution.

29. a. The joint pdf of (X, Y) is given by

$$f(x,y)=\begin{cases}K\left(x+y^2\right); 0\le x\le 1, 0\le y\le 1;\\ 0, \quad otherwise\end{cases}$$

Find (i) K (ii) marginal pdf of X and Y (iii) $P(1/4\le Y\le 3/4)$ (iv) $P(1/2\le X\le 3/4)$.

(OR)

b. Let X and Y be random variables having the following joint probability distribution. Find the correlation coefficient between X and Y.

X	Y: 0	Y: 1	Y: 2
0	1/16	2/16	1/16
1	2/16	4/16	2/16
2	1/16	2/16	1/16

30. a. If X denotes the sum of the numbers obtained when two dice are thrown, obtain an upper bound for $P\{|X-7|\ge 4\}$.

(OR)

b. The life time of a certain brand of an electric bulb may be considered a random variable with mean 1200h and standard deviation 250h. Find the probability using Central limit theorem that the average life time of 60 bulbs exceeds 1250h.

31. a. Give a random variable Y with characteristic function $\begin{aligned}\phi(\omega)&=E\left(e^{i\omega Y}\right)\\ &=E(\cos\omega Y+i\sin\omega Y)\end{aligned}$

And a random process defined by $X(t)=\cos(\lambda t+Y)$, show that X(t) is stationary in the wide sense if $\phi(1)=\phi(2)=0$.

(OR)

b. Show that the process $X(t)=A\cos\lambda t+B\sin\lambda t$ (where A and B are random variables) is wide sense stationary if (i) $E(A)=E(B)=0$ (ii) $E\left(A^2\right)=E\left(B^2\right)$ and (iii) $E(AB)=0$.

32. a. Given the power spectral density of a continuous process as $S_{XX}(\omega)=\dfrac{\omega^2+9}{\omega^4+5\omega^2+4}$. Find the mean square value of the process.

(OR)

b. A wide sense stationary process X(t) is the input to a linear system with impulse response $h(t)=2e^{-7t}, t\ge 0$. If the autocorrelation function of $X(t)$ is $R_{XX}(\tau)=e^{-4|\tau|}$, find the power spectral density of the output process Y(t).

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Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
Third Semester

18MAB203T – PROBABILITY AND STOCHASTIC PROCESSES
(For the candidates admitted during the academic year 2018 – 2019 onwards)
(Statistical table to be provided)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The probability density function of the random variable X is $f(x)=cx^2, 0<x<2$. The value of C is
(A) 1/4 (B) 3/4
(C) 3/8 (D) 1/8

2. _______ distribution satisfies memoryless property.
(A) Binomial (B) Exponential
(C) Poisson (D) Normal

3. The CDF of a random variable X is defined as $F(x)=\begin{cases}1-\frac{100}{x}, & x>100\\ 0, & elsewhere\end{cases}$
(A) $\dfrac{100}{x^2}$ (B) $\dfrac{200}{x}$
(C) $\dfrac{200}{x^2}$ (D) $\dfrac{100}{x}$

4. For real ω is, $|\phi_X(\omega)|\le$ _______.
(A) 0 (B) −1
(C) 1 (D) 2

5. If $p(x, y)$ is the joint probability distribution of a discrete two dimensional RV, then X and Y are said to be independent if
(A) $p(x, y)=p(x)p(y)$ (B) $p(x, y)=p(x)/p(y)$
(C) $p(x, y)=p(y)/p(x)$ (D) $p(x, y)=p(y)$

6. $F(-\infty, y)=$
(A) 1 (B) −1
(C) 0 (D) $-\infty$

7. If X and Y are two random variables then $f\left(\dfrac{x}{Y=y}\right)=$
(A) $\dfrac{f(x, y)}{f_X(x)}$ (B) $\dfrac{f(x, y)}{f_Y(y)}$
(C) $f_Y(x)f(x,y)$ (D) $f_X(x)f(x,y)$

8. If $Var(X_1)=5, Var(X_2=6), E(X_1)=0, E(X_2)=0, Cov(X_1, X_2)=4$ then $Var(2X_1-3X_2)$ is
(A) 20 (B) 26
(C) 25 (D) 24

9. Central limit theorem provides a simple method for computing approximate probabilities for sums of______ random variables.
(A) Independent (B) Dependent
(C) Correlated (D) Uncorrelated

10. A random variable X has a mean of 9 and a variance of 3. Then an upper bound for $P\{|X-9|\geq 3\}$ is
(A) 1/9 (B) 3
(C) 9 (D) 1/3

11. The inequality $P(X\geq a)\leq \frac{\sigma^2}{\sigma^2+a^2}, a>0$
(A) Chebyshev inequality (B) Jenson's inequality
(C) Markov's inequality (D) One-sided Chebyshev inequality

12. If $X_1, X_2.....$ independent and identically distributed random variables each having finite mean $E(X_i)=\mu$, then $E\left(\frac{X_1+X_2+....+X_n}{n}\right)$ is
(A) μ/n (B) μ
(C) n/μ (D) $n+\mu$

13. A random process is defined by $X(t)=A$ where A is a continuous RV with probability density function $f(a)=1, 0<a<1$. The mean of $X(t)$ is
(A) 0 (B) 1
(C) 2 (D) 1/2

14. If the processes $\{X(t)\}$ and $\{Y(t)\}$ are independent $E(X(t)Y(t))$ is equal to
(A) $E(X^2(t))E(Y^2(t))$ (B) $E(X(t))E(Y(t))$
(C) $E(X(t))$ (D) $E(Y(t))$

15. $R_{XY}(-\tau)=$
(A) $-R_{XY}(\tau)$ (B) $R_{XY}(\tau)$
(C) $R_{YX}(\tau)$ (D) $-R_{XY}(-\tau)$

16. If the parameter set T is discrete and the sample space S is continuous then the random process is a
(A) Discrete random sequence (B) Continuous random sequence
(C) Discrete random process (D) Continuous random process

17. If $Y(t+h)=f(X(t+h))$ where $Y(t)=f(X(t))$, f is called a _______system.
(A) Real (B) Causal
(C) Time invariant (D) Time dependent

18. Real $S_{XY}(\omega)$ and real $S_{YX}(\omega)$ are ______ functions of ω.
(A) Linear (B) Even
(C) Odd (D) Neither even nor odd

19. The mean square value of the process whose power density spectrum $\frac{4}{4+\omega^2}$ is
(A) 1 (B) 1/2
(C) 1/4 (D) 2

20. The power spectral density of a WSS process is always
(A) Finite (B) Zero
(C) Negative (D) Non-negative

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. The probability density function of a continuous random variable X is given by
$f(x)=\begin{cases} Kx, & 0\leq x\leq 5 \\ K(10-x), & 5\leq x\leq 10 \end{cases}$. Find (i) K (ii) $Var(X)$.

22. The following table represents the joint probability distribution of the two dimensional random variable (X, Y). Find the marginal distributions of X and Y.

X	Y		
	1	2	3
1	1/12	0	1/18
2	1/6	1/9	1/4
3	0	1/5	2/15

23. A random variable has the probability density function $f(x)=3e^{-3x}, x>0$. Obtain an upper bound for $P(X\geq 2)$.

24. If $X(t)$ is a WSS process with autocorrelation function $R(\tau)=Ae^{-\alpha|\tau|}$, determine the second order moment of the random variable $X(8)-X(5)$.

25. The power spectral density of a random process $X(t)$ is given by
$S_{XX}(\omega)=\begin{cases} \pi \text{ if } |\omega|<1 \\ 0, \text{ otherwise} \end{cases}$. Find its autocorrelation function.

26. In an electronics laboratory it is found that 10% of transistors are defective. A random sample of 20 transistors are taken for inspection. What is the probability that atleast 4 are defective?

27. If $f(x,y)=\begin{cases} k(1-x-y), & 0<x<1/2, 0<y<1/2 \\ 0, & \text{otherwise} \end{cases}$ is a joint density function, find k.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. The discrete random variable X has the probability distribution given by

x	0	1	2	3	4
p(x)	K	3K	5K	7K	9K

Find (i) K (ii) Mean (iii) Variance (iv) $Var(3X-4)$ (v) $P(0<X<3/X>1)$.

(OR)