Discrete Mathematics for Engineers
Assignment - II

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ECE - A

Part - A

1 How many distinct four digit integers can one make from the digits 1, 3, 3, 7, 7 and 8?

Soln. For four digit formation, using these digits we have three cases

Case I: All the digits in four digit integer are distinct,

ie $4! = 24$

Case II: Exactly two digits are same in the four digit number,

ie $2 \times 3 \times \frac{4!}{2!} = 3 \times 4! = 72$

Case III: Numbers that only consist of 3 and 7, they are

$\frac{4!}{2! \times 2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 1 \times 1} = 6$

2. How many positive integers not exceeding 1000 that are divisible by 7 (or) 11?

Soln. Let us take two sets A and B

A = Set of no. that are divisible by 7.

B = Set of integers that are divisible by 11

$\therefore n(A)$ = no of integers that are divisible by 7

$\frac{1000}{7} = 142.85 \simeq 142$

$\therefore n(B)$ = no of integers that are divisible by 11,

$\frac{1000}{11} = 90.9 \approx 90$

We have to find $n(A \cap B)$, since, these are some common in the set A and set B. So, we have to exclude them by using $n(A \cap B)$ which is the numbers divisible by both 7 and 11.

$\frac{1000}{7 \times 11} = 12.98 \approx 12$

Wkt,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 142 + 90 - 12$

$= 130 + 90$

$n(A \cup B) = 20$

$\therefore$ There are 220 integers below 1000 that are divisible by 7 or 11.

3 If ${}^nP_4 = 20\,{}^nP_3$, find "n"

Soln. We have, ${}^nP_r = \frac{n!}{(n-r)!}$

$\therefore {}^nP_4 = \frac{n!}{(n-4)!} = \frac{n \times (n-1) \times (n-2) \times (n-3) \times \cancel{(n-4)!}}{\cancel{(n-4)!}}$

Similarly ${}^nP_3 = n \times (n-1) \times (n-2)$

given,

${}^nP_4 = 20\,{}^nP_3$

$\cancel{n \times (n-1)} \times \cancel{(n-2)} \times (n-3) = \cancel{n \times (n-1)} \times \cancel{(n-2)} \times 20$

$n - 3 = 20$

$n = 23$

4 If there are 5 points inside a square of side length 2, prove that two of the points with in a distance of $\sqrt{2}$ each other.

Soln. We have a 2x2 square, we want to split into 4 squares of 1x1 cm, simply by joing the centers of opposite sides, this creats a small grid.

Now we have to insert 5 points in this 4 square. Now, we have to apply pigeon hole principle.

So, $\left\lfloor \frac{5-1}{4} \right\rfloor + 1 = \left\lfloor \frac{4}{4} \right\rfloor + 1 = 2$

So, one square must end up containing at least two points.
Now, we know that given 5 points are inside a square of side length 2, so, two points are inside a square.
∴ The maximum distance b/w the two points lie in the diagonal of the square of side length 1.
That is equal to $\sqrt{2}$. Hence Proved.

5 Which positive integers less than 30, that are relatively prime.

Soln. For the relatively prime to 30, we have to find greatest common divisor (gcd) for all prime number which is less than 30.
Those 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
Now,

$\gcd(1,30) = 1$ ✓, $\gcd(5,30) \neq 1$ ×
$\gcd(2,30) \neq 1$ × $\gcd(7,30) = 1$ ✓
$\gcd(3,30) \neq 1$ × $\gcd(11,30) = 1$ ✓
$\gcd(13,30) = 1$ ✓ $\gcd(23,30) = 1$ ✓
$\gcd(17,30) = 1$ ✓ $\gcd(29,30) = 1$ ✓
$\gcd(19,30) = 1$ ✓

∴ 1, 7, 11, 13, 17, 19, 23, 29 are the possible integers that are less than 30, relatively prime to 30.

Part - B.

6. 250 Students in an Engineering College,

188 → Fortran, 100 → C ; 35 → Java.

88 → Fortran and C, 23 → C and Java, 29 → Fortran and Java.

19 → Fortran, C, Java. How many people didnt take any of three Courses.

Soln.

$n(A) = 188$, $n(B) = 100$, $n(C) = 35$

A → Fortran

B → C

C → Java.

$n(A \cap B) = 88$, $n(B \cap C) = 23$, $n(A \cap C) = 29$

$n(A \cap B \cap C) = 19$.

$\therefore$ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 188 + 100 + 35 - 88 - 23 - 29 + 19$

$= 202$

$\therefore$ no of students who took atleast one Course = 202.

$\therefore$ no of students who didnt take any Course = 250 - 202

= 48.

7. A round table Conference is to be held b/w 10 delegates from 10 Countries. In how many ways can they be seated if,

i) Two particular delegates are always together

ii) Two particular delegates are either side of the chair person.

Soln (i) The two particular delegates who wish to sit together be treated as one unit,

So we have 9 delegates.

In Circular permutation, it is equal to.

$(9-1)! = 8!$

And after this, the two delegates can be permutated b/w themselves, so it will become,

$= 2 \times 8!$

$= 80640$

(ii) Let the person be arranges in between two particular delegates in $^8C_1$ ways,

Remaining arrangements can be done in, $((10-3+1-1)!)$

$= 7!$ ways

Two particular delegates can inter-change among themselves,

$\therefore$ Total ways are $= {}^8C_1 \times 7! \times 2!$

$= 2 \times 8!$

8. Find the integer m and n such that, $28844m + 15712n = 4$

Soln $28844 = 1 \times 15712 + 13132$

$15712 = 1 \times 13132 + 2580$

$13132 = 5 \times 2580 + 232$

$2580 = 11 \times 232 + 28$

$232 = 8 \times 28 + 8$

$28 = 3 \times 8 + 4$

$8 = 4 \times 2 + 0$

$\gcd(28844, 15712) = 4$

$4 = 28 - (3 \times 8)$

$= 28 - (3 \times (232 - 8 \times 28))$

$= 25 \times 28 - 3 \times 232$

$= 25(2580 - 11 \times 232) - 3 \times (232)$

$= 25 \times 2580 - 278(232)$

$\Rightarrow 25(2550) - 278(13132 - 5\times 2550)$

$\Rightarrow 25(2550) - 278(13132) + 1390(2550)$

$\Rightarrow 1415(2550) - 278(13132)$

$\Rightarrow 1415(15712 - 13132) - 278(13132)$

$\Rightarrow 1415(15712) - 1693(13132)$

$\Rightarrow 1415(15712) - 1693(28844 - 15712)$

$\Rightarrow 1415(15712) - 1693(28844) + 1693(15712)$

$\Rightarrow 3108(15712) - 1693(28844)$

$\therefore m = -1693$ and $n = 3108$

9. Using Euclids algorithm find gcd of 12345 and 54321

Soln. So,

$54321 = 4\times 12345 + 4941$

$12345 = 2\times 4941 + 2463$

$4941 = 2\times 2463 + 15$

$2463 = 164\times 15 + 3$ 3 is the last non-zero remainder.

$15 = 5\times 3 + 0$

$\therefore$ GCD of 12345 and 54321 is '3'.

Since 3, is the last non-zero remainder.

10. If $2^n - 1$ is a prime number, then ST "n" is prime.

Soln. Suppose, 2^{n-1} to be prime.

Let it be "p"

$p = 2^n - 1$

Assume, $n = xy$ and n is not prime $(x, y > 0)$

$p = 2^{xy} - 1$

$p = (2^x)^y - 1$

$$p = \frac{(2^x)^y - 1}{2^x - 1} \times (2^x - 1) \quad \text{—— (1)}$$

So this can be written as

$$1 + 2^x + (2^x)^2 + (2^x)^3 + \cdots + (2^x)^{y-1}$$

$$= \frac{(2^x)^y - 1}{2^x - 1}$$

[$\because a + ar + ar^2 + \cdots ar^{n}$

$\Rightarrow \dfrac{a(r^n - 1)}{r - 1}$

Since $a = 1$.

$1 + r + r^2 + \cdots r^{n-1}$

$= \dfrac{r^{n} - 1}{r - 1}$]

Sub the above in eq (1)

$$p = \left(1 + 2^x + (2^x)^2 + \cdots + (2^x)^{y-1}\right) \times (2^x - 1) \quad \text{—— (2)}$$

from eqn (2) it is clear that "p" can be written as a product of two numbers. This implies that "p" is not a prime number, which is contradiction to our assumptions.

$\therefore$ our assumption "n" is not a prime number is wrong. Thus we can say $(2^n - 1)$ is prime and then "n" is also prime.

Hence Proved.