Harmonic Analysis

Monday, November 29, 2021 8:19 AM

Harmonic Analysis – F.S [discrete form] – periodic fns.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \Rightarrow f(x) = \frac{a_0}{2} + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \dots$$

I – fundamental; II

$$a_0 = \frac{2}{n} \sum_{i=1}^{n} f(x_i)$$

$$a_1 = \frac{2}{n} \sum_{i=1}^{n} f(x_i) \cos x_i$$

$$b_1 = \frac{2}{n} \sum f(x_i) \sin x_i \quad \cdots$$

→ degree
→ radians
→ T
→ change of interval

① Find the F.S. upto the I harmonic for $y = f(x)$ in $(0, 2\pi)$ defined by the table of values given below. [Radians]

x	0	π/3	2π/3	π	4π/3	5π/3	2π (x)
y	1	1.4	1.9	1.7	1.5	1.2	1.0

Note: Periodic fn: Omit the last value (ie. 2π)

Min $n = 6$.

Calculator change to radians.

$$a_0 = \frac{2}{n} \sum f(x_i) \qquad a_2 = \frac{2}{n} \sum f(x_i) \cos 2x_i$$

$$a_1 = \frac{2}{n} \sum f(x_i) \cos x_i \qquad b_2 = \frac{2}{n} \sum f(x_i) \sin 2x_i$$

$$b_1 = \frac{2}{n} \sum f(x_i) \sin x_i$$

$$f(x) = \frac{a_0}{2} + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x.$$

	x	$y = f(x)$	$\cos x_i$	$\sin x_i$	$\cos 2x_i$	$\sin 2x_i$	$f(x_i)\cos x_i$	$f(x_i)\sin x_i$	$f(x_i)\cos 2x_i$	$f(x_i)\sin 2x_i$
0°	0	1	1	0	1	0	1	0	1	0
60°	π/3	1.4	0.5	0.866	−0.5	0.866	0.7	1.2124	−0.7	1.2124
120°	2π/3	1.9	−0.5	0.866	−0.5	−0.866	−0.95	1.6454	−0.95	−1.6454
180°	π	1.7	−1	0	1	0	−1.7	0	1.7	0
240°	4π/3	1.5	−0.5	−0.866	−0.5	0.866	−0.75	−1.299	−0.75	1.299
300°	5π/3	1.2	0.5	−0.866	−0.5	−0.866	0.6	−1.0392	−0.6	−1.0392
		$\sum f(x_i) = 8.7$					$\sum f(x_i)\cos x_i = -1.1$	$\sum f(x_i)\sin x_i = 0.5196$	$\sum f(x_i)\cos 2x_i = -0.3$	$\sum f(x_i)\sin 2x_i = -0.1732$

$$a_0 = \frac{2}{6} \times 8.7 = 2.9$$

$$a_1 = \frac{2}{6}[-1.1] = -0.37$$

$$b_1 = \frac{2}{6} \times 0.5196 = 0.17$$

$$a_2 = \frac{2}{6} \times -0.3 = -0.1$$

$$b_2 = \frac{2}{6} \times -0.1732 = -0.06$$

$$f(x) = \frac{2.9}{2} - 0.37 \cos x + 0.17 \sin x - 0.1 \cos 2x - 0.06 \sin 2x \ //.$$

② The value of x and the cor. values of f(x) over a period T are given below

Show that $f(x) = 0.75 + 0.37 \cos\theta + 1.004 \sin\theta$ where $\theta = \frac{2\pi x}{T}$ ["T" problem]

x	0	T/6	T/3	T/2	2T/3	5T/6	T (x)
y	1.4	1.7	1.9	1.6	0.7	0.2	0.0

Show that $f(x) = 0.75 + 0.37\cos\theta + 1.004\sin\theta$ where $\theta = \frac{2\pi x}{T}$ [T problem]

x	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
$y=f(x)$	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

X (0, T)

Soln: $n=6$

x	$\theta = \frac{2\pi x}{T}$		$f(x)=y$	$\cos\theta_i$	$\sin\theta_i$	$f(x_i)\cos\theta_i$	$f(x_i)\sin\theta_i$
0	0	0°	1.98	1	0	1.98	0
$T/6$	$\pi/3$	60°	1.30	0.5	0.866	0.65	1.1258
$T/3$	$2\pi/3$	120°	1.05	-0.5	0.866	-0.525	0.9093
$T/2$	π	180°	1.30	-1	0	-1.3	0
$2T/3$	$4\pi/3$	240°	-0.88	-0.5	-0.866	0.44	0.762
$5T/6$	$5\pi/3$	300°	-0.25	0.5	-0.866	-0.125	0.2165
			$\Sigma f(x_i)=4.5$			$\Sigma f(x_i)\cos\theta_i = 1.12$	$\Sigma f(x_i)\sin\theta_i = 3.0136$

$a_0 = \frac{2}{6} \times 4.5 = 1.5$ $\quad b_1 = \frac{2}{6} \times 3.0136 = 1.0045$

$a_1 = \frac{2}{6} \times 1.12 = 0.3733$

$f(x) = 0.75 + 0.37\cos\theta + 1.004\sin\theta$.

Absentees!

50, 53, 56, [redacted], 64, 67

115, [redacted], 118, 130

③ Find the constant term and the co-eff of the first sine and cosine terms in the fourier expansion of y as given in the foll. table.

x	0	1	2	3	4	5
y	9	18	24	28	26	20

$n=6$ [Change of interval]

$y = \frac{a_0}{2} + \Sigma\left[a_n\cos\frac{n\pi x}{l} + b_n\sin\frac{n\pi x}{l}\right]$

Soln: Length of interval is $2l = 6$ $\quad l = 3$

$f(x) = y = \frac{a_0}{2} + a_1\cos\frac{\pi x}{3} + b_1\sin\frac{\pi y}{3}$

x	$y=f(x)$	$\cos\frac{\pi x_i}{3}$	$\sin\frac{\pi x_i}{3}$	$f(x_i)\cos\frac{\pi x_i}{3}$	$f(x_i)\sin\frac{\pi x_i}{3}$
0	9	1	0	9	0
1	18	0.5	0.866	9	15.588
2	24	-0.5	0.866	-12	20.785
3	28	-1	0	-28	0
4	26	-0.5	-0.866	-13	-22.517
5	20	0.5	-0.866	10	-17.321
	$\Sigma f(x_i)=125$			-25	-3.465

$a_0 = \frac{2}{6} \times 125 = 41.67$ $\quad b_1 = \frac{2}{6} \times (-3.465) = -1.16$

$a_1 = \frac{2}{6} \times (-25) = -8.33$

$y = \frac{41.67}{2} - 8.33\cos\frac{\pi x}{3} - 1.16\sin\frac{\pi x}{3}$

Try!!

$\frac{41.61}{2}$ $\frac{0.33\ \omega}{3}$ $\frac{1.10\ \sin x}{3}$ //

Try!!

① Find an empirical formula of the form $f(x) = a_0 + a_1 \cos x + b_1 \sin x$ for the foll. data such that $f(x)$ is periodic with period 2π.

x	0°	60°	120°	180°	240°	300°	360°
$y = f(x)$	40.00	31.0	-13.7	20.0	3.7	-21.0	40.0

$n = 6$.