Signals and Systems

Unit-1
Classification of Signals and Systems

Contents

Introduction to signal and system

Signal:

- A signal is defined as any physical quantity that that varies with time, space or any other independent variable or variables.
- Mathematically, we describe a signal as a function of one or more independent variables.
- For e.g. $s_1(t) = 5t$ ----------------Depend upon time

  $s_2(t) = 20t^2$ ----------------Depend upon time

$S(x, y) = 3x + 2xy + 10y^2$ -----Depend upon independent variables

Here x, y are independent variables.

Introduction to signal and system (Cont.)

System:

- A system is defined as a physical device or software realization that performs an operation on a signal.

- For e.g.

Filtering:- removal of noise from the signal.

noise is unwanted signal which is added to the desired signal.

Application of signals and systems

- **Control application:** Used in industrial control and automation.

e.g:- Controlling the position of a valve or shaft of amotor.

Important techniques used in it are : time-domain solution of differential equations

Laplace transformation,

Stability estimation

Application of signals and systems (Cont.)

Communication applications:

- Communication is transformation of information (signal) over a channel. The channel may be free space, coaxial cable, fiber optic cable
- A Key component of transmission is modulation:

analog modulation   digital modulation

- Signal and system is applied in communication for transmission, storage and display of information.

Speech and audio processing:

- Cancellation of noise
- Extraction of features
- Analysis of signal.

Types of signals

- Continuous-time signal (analog signal)

- Discrete-time signal

- Digital signal

- In signal and system course, we will cover operation on analog signal and discrete time signal.

Continuous-time signal (analog signal)

- Continuous-time signals are defined for all values of time t and is represented by $x(t)$.
- A continuous time signal is also called an analog signal.
- E.g. are ECG (electrocardiogram signal), AC power supply

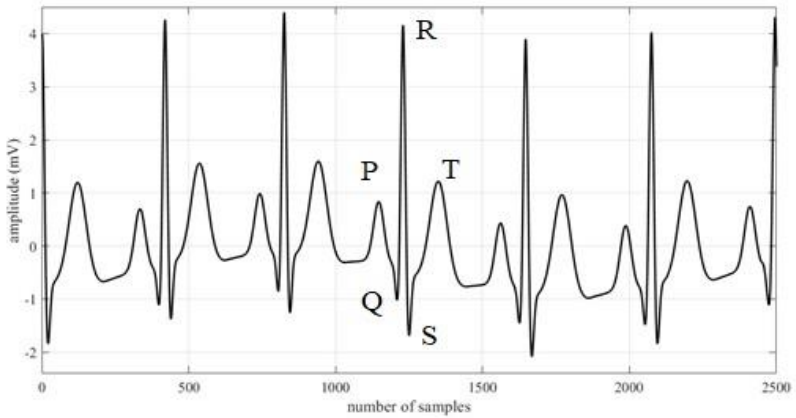


Fig.1. ECG signal

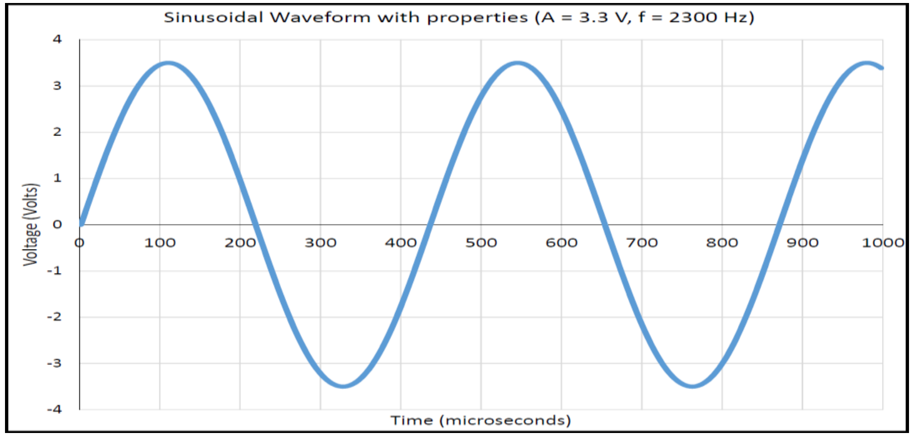
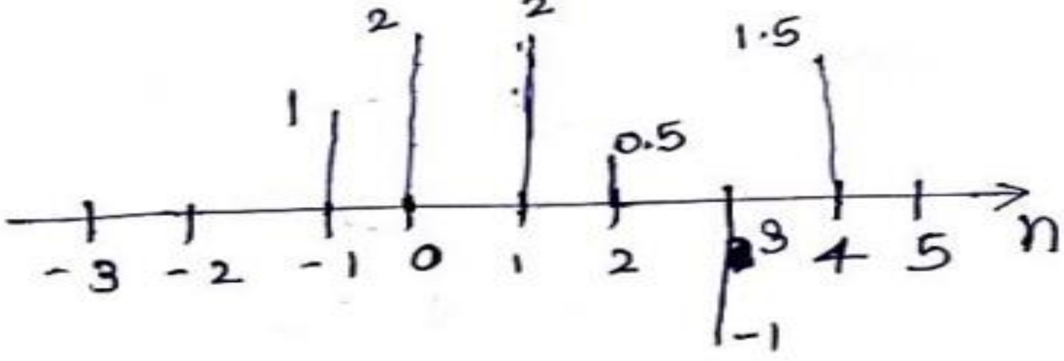


Fig. 2. AC power supply

Discrete-time signal

- The discrete-time signals are defined at a discrete instant of time and is represented by $x(n)$ where n is index.
- Some signals are discrete in nature
- Some signals may be discrete representation of continuous-time signal. (the amplitude at each interval)

$x(n)$



Digital signals

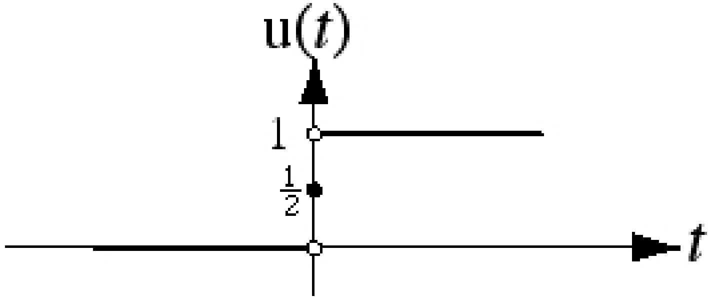
- A signal that is discretized in time and quantized in amplitude is known as digital signal.
- The signal consists of binary values (zeros or ones).

Elementary continuous time signals

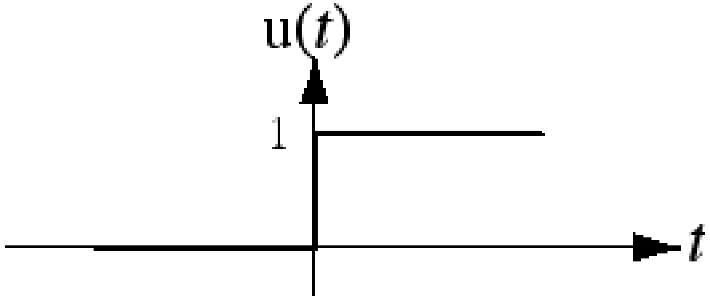
1. **Unit step function** The unit step function is defined as

$$u(t) = 1 \quad for\ t \geq 0$$
$$= 0 \quad for\ t < 0$$

Precise Graph



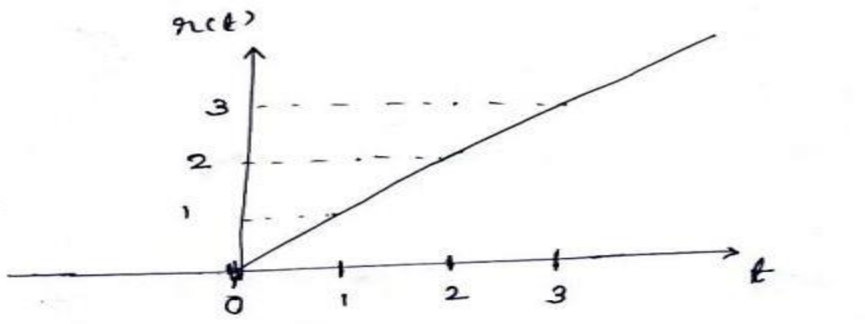
Commonly-Used Graph



Elementary continuous time signals (cont.)

2. **Unit ramp function** The unit ramp function is defined as

$$r(t) = t \quad for\ t \geq 0$$
$$\quad\ = 0 \quad for\ t < 0$$



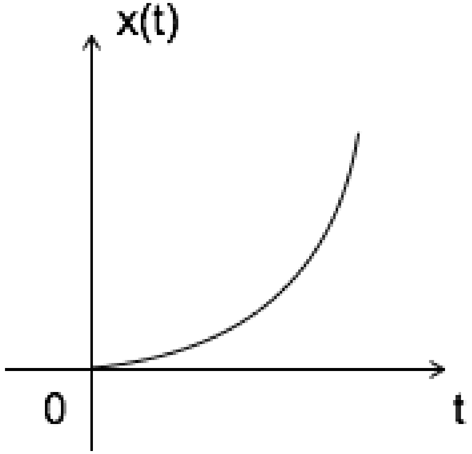
- Unit ramp function can be obtained by applying unit step function to an integrator. $r(t) = \int u(t) \quad dt$
- Unit step function can be obtained by applying unit ramp function to a differentiator. $u(t) = \frac{d\, r(t)}{dt}$

Elementary continuous time signals (cont.)

3. **<u>Unit parabolic function:</u>** The unit parabolic function is given by

$$p(t) = \frac{t^2}{2} \quad for\ t \geq 0$$

$$= 0 \quad for\ t < 0$$



$$p(t) = \frac{t^2}{2} u(t)$$

Elementary continuous time signals (cont.)

- 4. **Unit impulse function:** Unit impulse function is defined as

$$\delta(t) = 1 \quad for\ t = 0$$
$$= 0 \quad for\ t \neq 0$$

Area of unit impulse function is 1

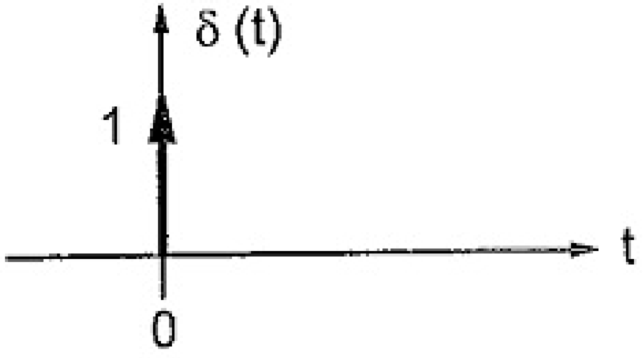


Fig. Unit impulse function

Elementary continuous time signals (cont.)

Properties of unit impulse:-

I. $\int_{-\infty}^{\infty} x(t) \quad \delta(t) \quad dt = x(0)$

Proof:
$$\delta(t) = 1 \quad for\ t = 0$$
$$= 0 \quad for\ t \neq 0$$

For value of t other than 0 is 0. When t=0, $\delta(t) = 1$ and $x(t)$= $x(0)$

$$\int_{-\infty}^{\infty} x(t) \quad \delta(t) \quad dt =$$

$$\int_{-\infty}^{-1} x(t) \quad \delta(t) \quad dt + \int_{0}^{0} x(t) \quad \delta(t) \quad dt + \int_{1}^{\infty} x(t) \quad \delta(t) \quad dt$$

= 0+$x(0)\delta(0) + 0 = x(0)$ (proved)

Elementary continuous time signals (cont.)

II. $x(t)\delta(t - t_0) = x(t_0)\,\delta(t - t_0)$

Proof: At $t = t_0$ $\delta(t - t_0) = 1$,

otherwise $\delta(t - t_0) = 0$

So, $x(t)\delta(t - t_0) = x(t_0)\,\delta(t - t_0)$

Elementary continuous time signals (cont.)

III. $\int_{-\infty}^{\infty} x(t) \quad \delta(t - t_0) \quad dt = x(t_0)$

Proof: At $t = t_0$ $\delta(t - t_0) = 1$, otherwise $\delta(t - t_0) = 0$

So, $\int_{-\infty}^{\infty} x(t) \quad \delta(t - t_0) \quad dt = \int_{-\infty}^{\infty} x(t_0) \quad \delta(t - t_0) \quad dt$

= $x(t_0) \int_{-\infty}^{\infty} \delta(t - t_0) \quad dt$ Let $\lambda = t - t_0$ $dt = d\lambda$

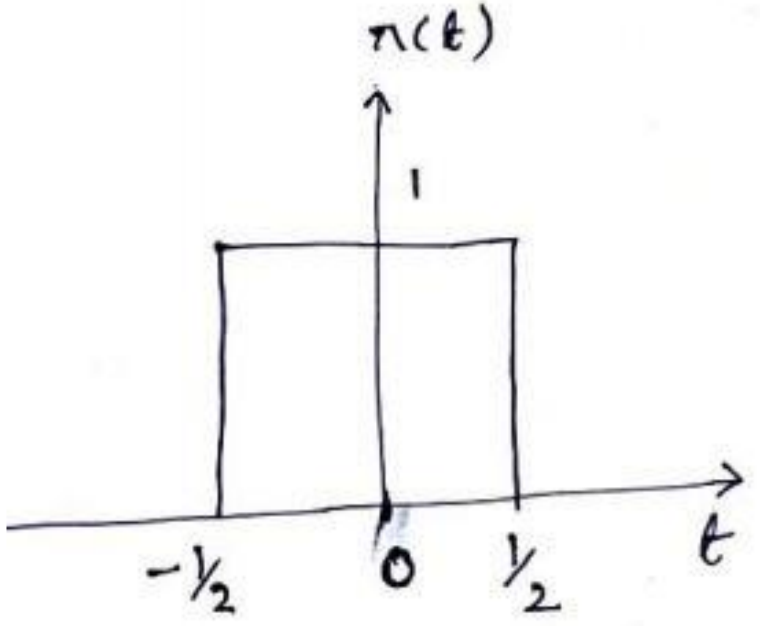
= $x(t_0) \int_{-\infty}^{\infty} \delta(\lambda) \quad d\lambda$= $x(t_0)$.1= $x(t_0)$ (proved)

IV. $\delta(at) = \frac{1}{|a|} \delta(t)$

Elementary continuous time signals (cont.)

5.**<u>Rectangular Pulse function</u>**

The rectangular pulse function is defined as
$$\pi(t) = 1 \quad for\ |t| \leq \frac{1}{2}$$
$$= 0 \quad otherwise$$



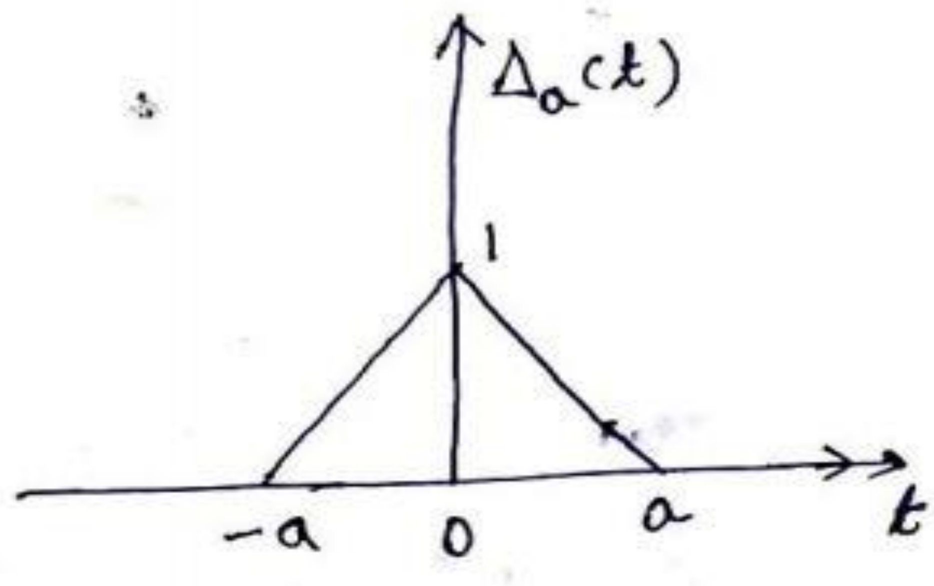
Elementary continuous time signals (cont.)

6. <u>Triangular pulse function</u>

The unit triangular pulse function is defined as

$$\Delta_a(t) = 1 - \frac{|t|}{a} \quad for\ |t| \le a$$

$$= 0 \quad otherwise$$



Elementary continuous time signals (cont.)

7. Sinusoidal function

- A continuous time sinusoidal signal is given by

$$x(t) = A sin(\Omega t + \theta)$$

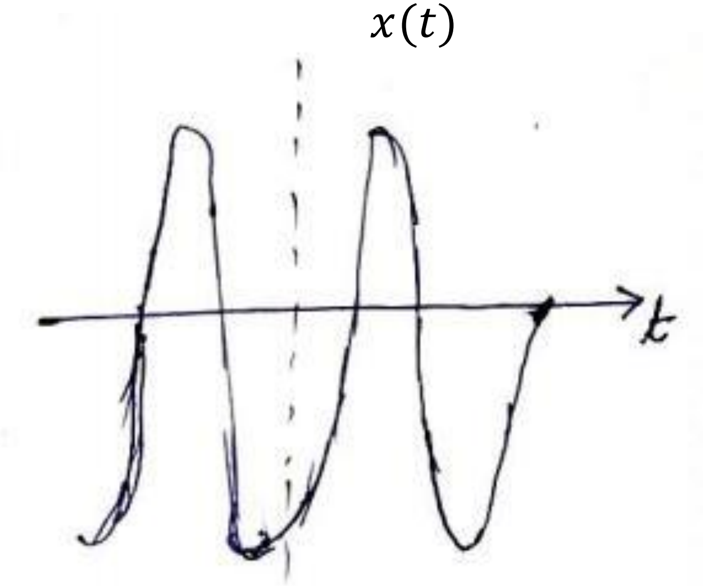
$A =$ amplitude

Ω= frequency

θ= phase

$\Omega = \frac{2\Pi}{T}$

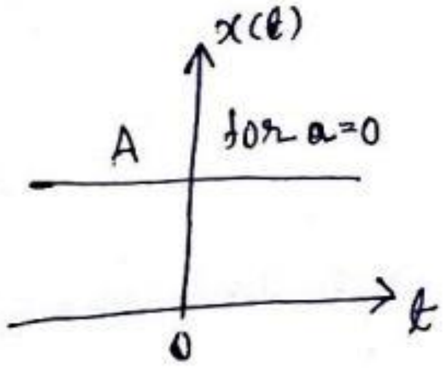
T=*time period*



Elementary continuous time signals (cont.)

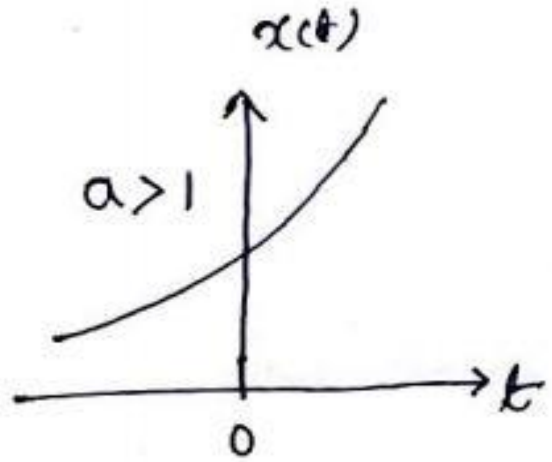
8. **<u>Real exponential signals:</u>** A real exponential signal is defined as $x(t) = Ae^{at}$

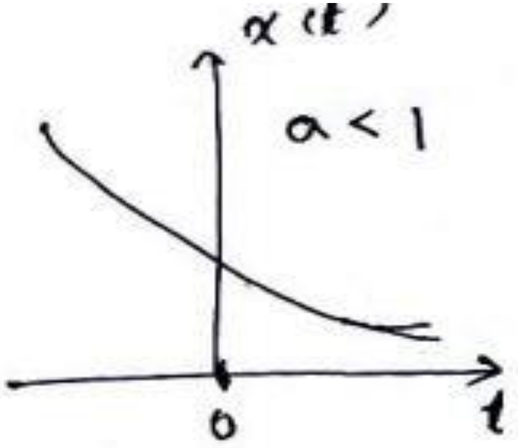
If a=0 $x(t) = A$



If a>1 $x(t)$ exponentially growing signal

If a<1 $x(t)$ exponentially decaying signal





Elementary continuous time signals (cont.)

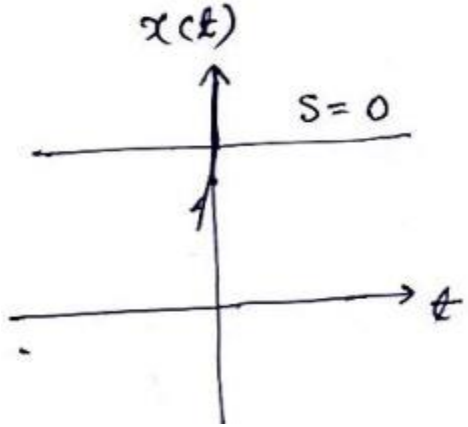
9. **<u>Complex exponential signal:</u>** The general representation of complex exponential signal is given by $x(t) = e^{st}$

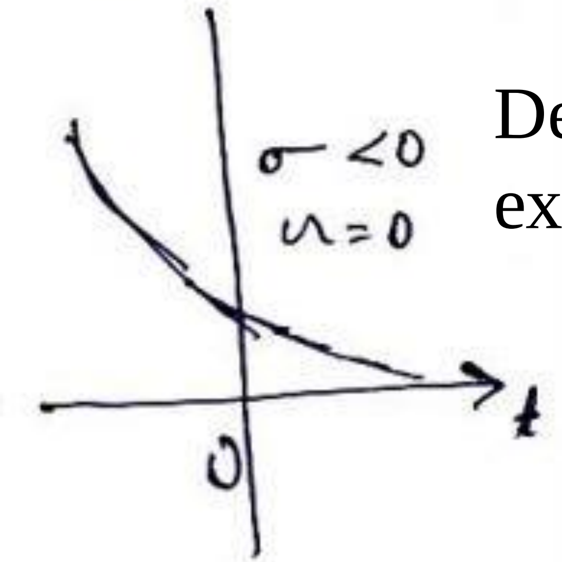
where $s = \sigma + j\Omega$ is complex variable

$x(t) = e^{st}$=$e^{(\sigma+j\Omega)t}$=$e^{\sigma t}e^{j\Omega t}$ As $e^{j\Omega t}$=$(cos\Omega t + jsin\Omega t)$

$x(t) = e^{\sigma t}(cos\Omega t + jsin\Omega t)$

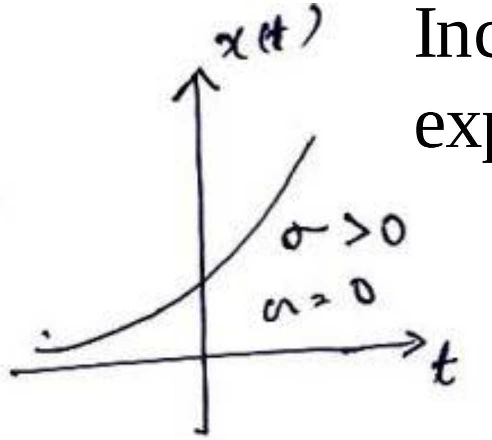
If, s =0, then $x(t) = 1$(pure Dc signal)





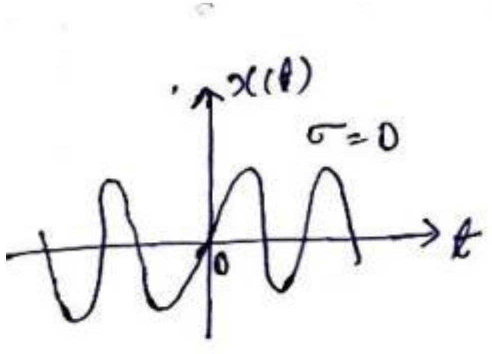
Decays exponentially

Elementary continuous time signals (cont.)

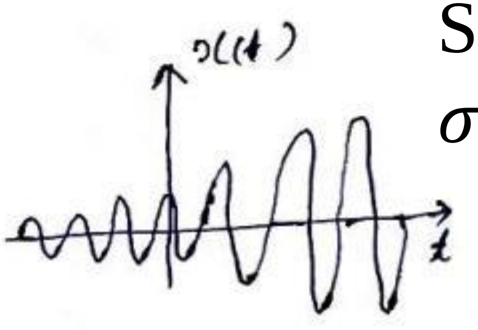


Increases exponentially

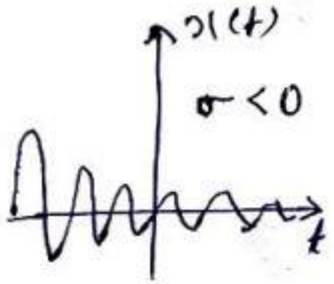
sinusoidal



Growing Sinusoidal $\sigma > 0$

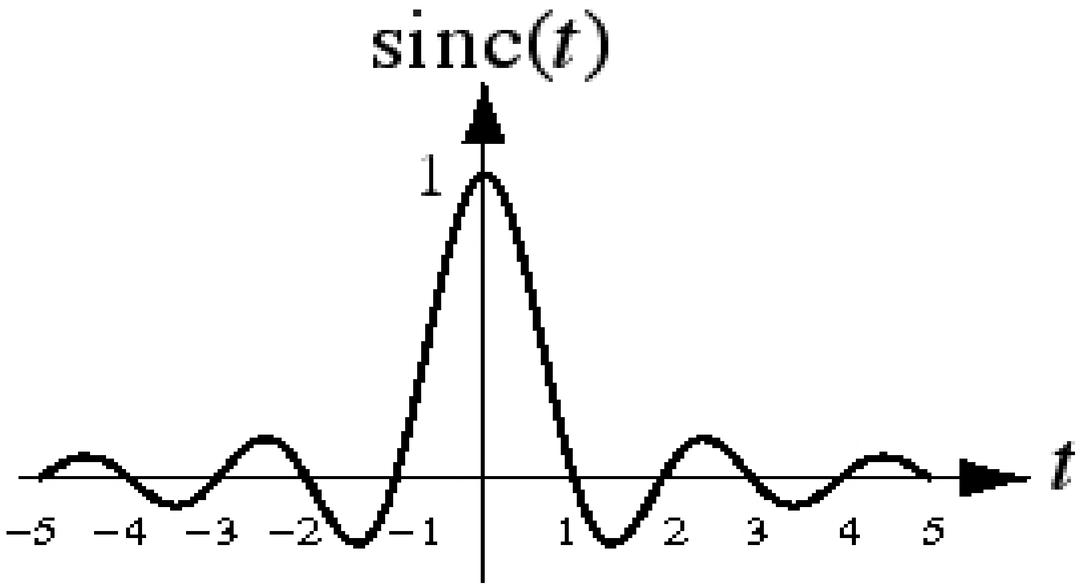


Decaying Sinusoidal



Sinc Function

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



Problems:

Calculate the value of the followings:

1.Q-$\int_{-\infty}^{\infty} \boldsymbol{e^{-\alpha t^2}} \quad \boldsymbol{\delta(t-10)} \quad \boldsymbol{dt}$

Ans- As we know $\begin{aligned} \delta(t) &= 1 \quad for\ t = 0 \\ &= 0 \quad for\ t \neq 0 \end{aligned}$

At t=10, $\delta(t-10)$=1, for other value of t, $\delta(t-10)$=0

SO $\int_{-\infty}^{\infty} e^{-\alpha t^2} \quad \delta(t-10) \quad dt$=$e^{-\alpha 100}$

2. Q- $\int_{-\infty}^{\infty} \boldsymbol{t^2} \quad \boldsymbol{\delta(t-3)} \quad \boldsymbol{dt}$

Ans- At t=3, $\delta(t-3)$=1, for other value of t, $\delta(t-3)$=0

So $\int_{-\infty}^{\infty} t^2 \quad \delta(t-3) \quad dt$=9

Problems (cont.):

3 Q-$\boldsymbol{\int_0^5 \delta(t) \quad sin(\frac{\pi t}{2}) \quad dt}$

A- At t=0, $\delta(t)$=1, for other value of t, $\delta(t)$=0

sin0=0, $\int_0^5 \delta(t) \quad sin\pi t \quad dt$ =0

4 Q- $\boldsymbol{\int_{-\infty}^{\infty} e^{-t} \quad \delta(t+3) \quad dt}$

A-At t=-3, $\delta(t+3)$=1, for other value of t, $\delta(t+3)$=0

$\int_{-\infty}^{\infty} e^{-t} \quad \delta(t+3) \quad dt$= e^3

5 Q- $\boldsymbol{\int_{-\infty}^{\infty} (t-5)^2 \quad \delta(t-3) \quad dt}$

A-At t=3, $\delta(t-3)$=1, for other value of t, $\delta(t-3)$=0

$$\int_{-\infty}^{\infty} (t-5)^2 \quad \delta(t-3) \quad dt = 4$$

Problems (cont.):

6 Q-$\int_{-\infty}^{\infty}[\boldsymbol{\delta(t)} \quad \mathbf{cos}(\boldsymbol{t}) + \quad \boldsymbol{\delta(t-1)} \quad \mathbf{sin}(\boldsymbol{t})]$

A- At t=0, $\delta(t)$=1, for other value of t, $\delta(t)$=0

At t=1, $\delta(t-1)$=1, for other value of t, $\delta(t)$=0

$\int_{-\infty}^{\infty}[\delta(t) \quad \cos(t) + \quad \delta(t-1) \quad \sin(t)]$=1+sin 1

7 **Q-**$\int_{-\infty}^{\infty} \boldsymbol{\delta(t)} \quad \boldsymbol{e^{-j\Omega t}} \quad \boldsymbol{dt}$

A- At t=0, $\delta(t)$=1, for other value of t, $\delta(t)$=0

$\int_{-\infty}^{\infty} \delta(t) \quad e^{-j\Omega t} \quad dt$=1

Representation of discrete-time signal

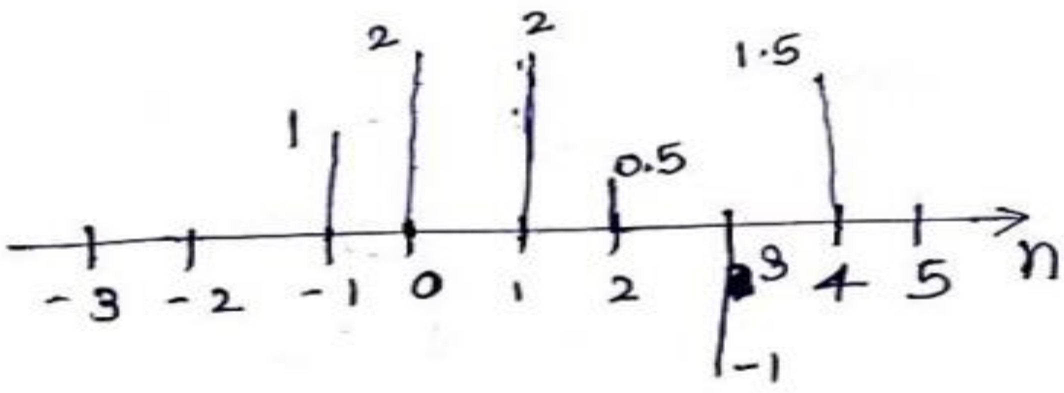
- The discrete-time signals are defined at a discrete instant of time and is represented by $x(n)$ where n is index.
- There are different types of representation of discrete time signals. They are
- (i) Graphical representation
- (ii) Functional representation
- (iii) Tabular representation
- (iv) Sequence representation

Representation of discrete-time signal (cont.)

Let us consider a discrete time signal $\boldsymbol{x(n)}$.
$\boldsymbol{x(-1) = 1, x(0) = 2, x(1) = 2, x(2) = 0.5, x(3) = -1, x(4) = 1.5}$

(i) <u>Graphical representation:-</u> The graphical representation of given $x(n)$ is

$x(n)$



Representation of discrete-time signal (cont.)

- **Let us consider a discrete time signal** $\boldsymbol{x(n)}$.
- $\boldsymbol{x(-1) = 1, x(0) = 2, x(1) = 2, x(2) = 0.5, x(3) = -1, x(4) = 1.5}$

- <u>(ii) Functional representation:</u> Functional representation of given data is
- $$x(n) = \begin{cases} 1 & for\ n = -1 \\ 2 & for\ n = 0,1 \\ 0.5 & for\ n = 2 \\ -1 & for\ n = 3 \\ 1.5 & for\ n = 4 \\ 0 & otherwise \end{cases}$$

Representation of discrete-time signal (cont.)

- **Let us consider a discrete time signal $\boldsymbol{x(n)}$.**

$\boldsymbol{x(-1) = 1, x(0) = 2, x(1) = 2, x(2) = 0.5, x(3) = -1, x(4) = 1.5}$

(iii) Tabular representation

n	-1	0	1	2	3	4
$x(n)$	1	2	2	0.5	-1	1.5

Representation of discrete-time signal (cont.)

- **Let us consider a discrete time signal** $\boldsymbol{x(n)}$.

$\boldsymbol{x(-1) = 1, x(0) = 2, x(2) = 0.5, x(3) = -1, x(4) = 1.5}$

<u>(iv) Sequence representation</u>

A finite duration sequence with time origin (n=0) indicated by symbol ↑

(Write the sequence of $x(n)$, when n=0 put ↑

$x(n)$={1, 2,0, 0.5, -1, 1.5}
$\quad\quad\quad\quad$↑

If a sequence written without ↑ symbol, then 1^{st} location is n=0.

If $x(n)$={2, 4, 6, 0,8, -3}

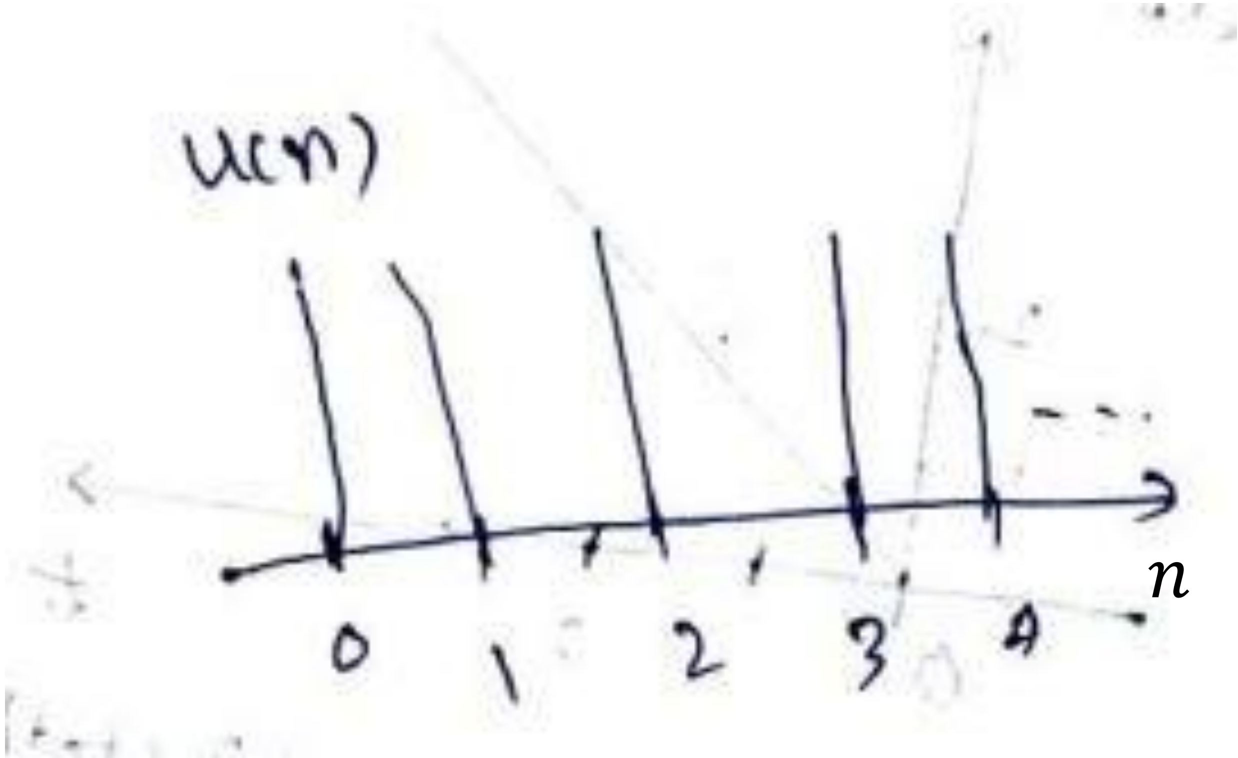
$x(0) = 2, x(1) = 4\ x(2) = 6\ x(4) = 8, x(5) = -3, x(6) = 0\ x(3) = 0$

Elementary discrete time signals

1. **Unit step sequence:-**

The unit step sequence is defined as

$$u(n) = 1 \quad for\, n \geq 0$$
$$= 0 \quad for\, n < 0$$

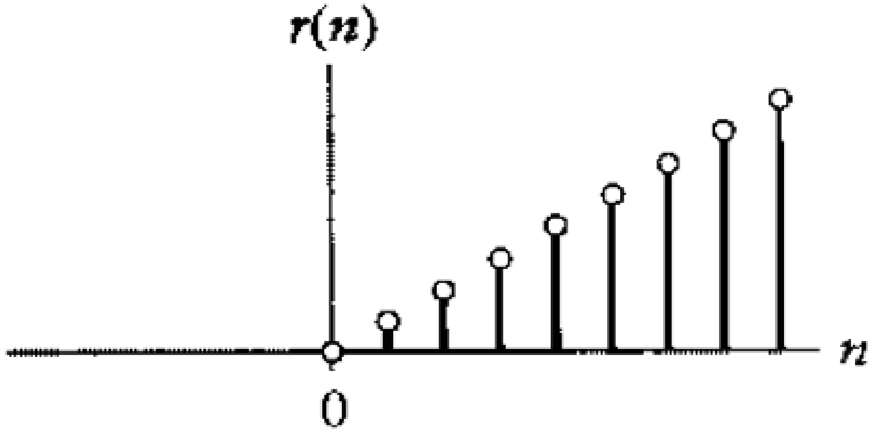


Elementary discrete time signals (Cont.)

2. Unit ramp sequence:-

The unit ramp sequence is defined as

$$r(n) = n \quad for\ n \geq 0$$
$$= 0 \quad for\ n < 0$$



Elementary discrete time signals (Cont.)

- 3. **Unit impulse sequence (unit sample sequence):** Unit impulse sequence is defined as

$$\delta(n) = 1 \quad for\ n = 0$$
$$= 0 \quad for\ n \neq 0$$

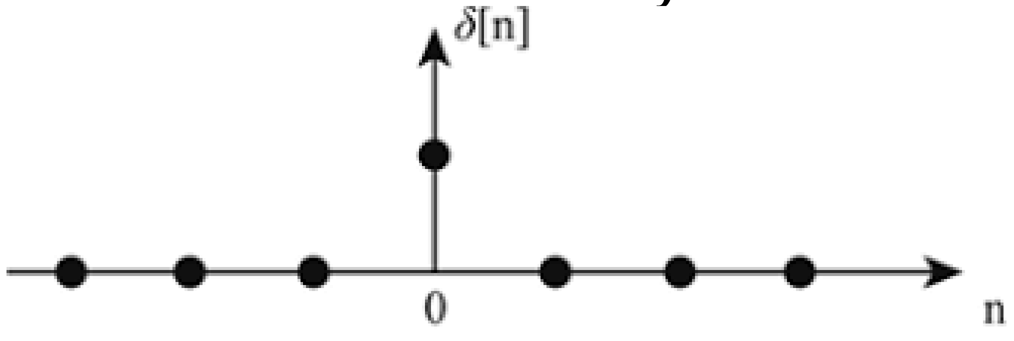
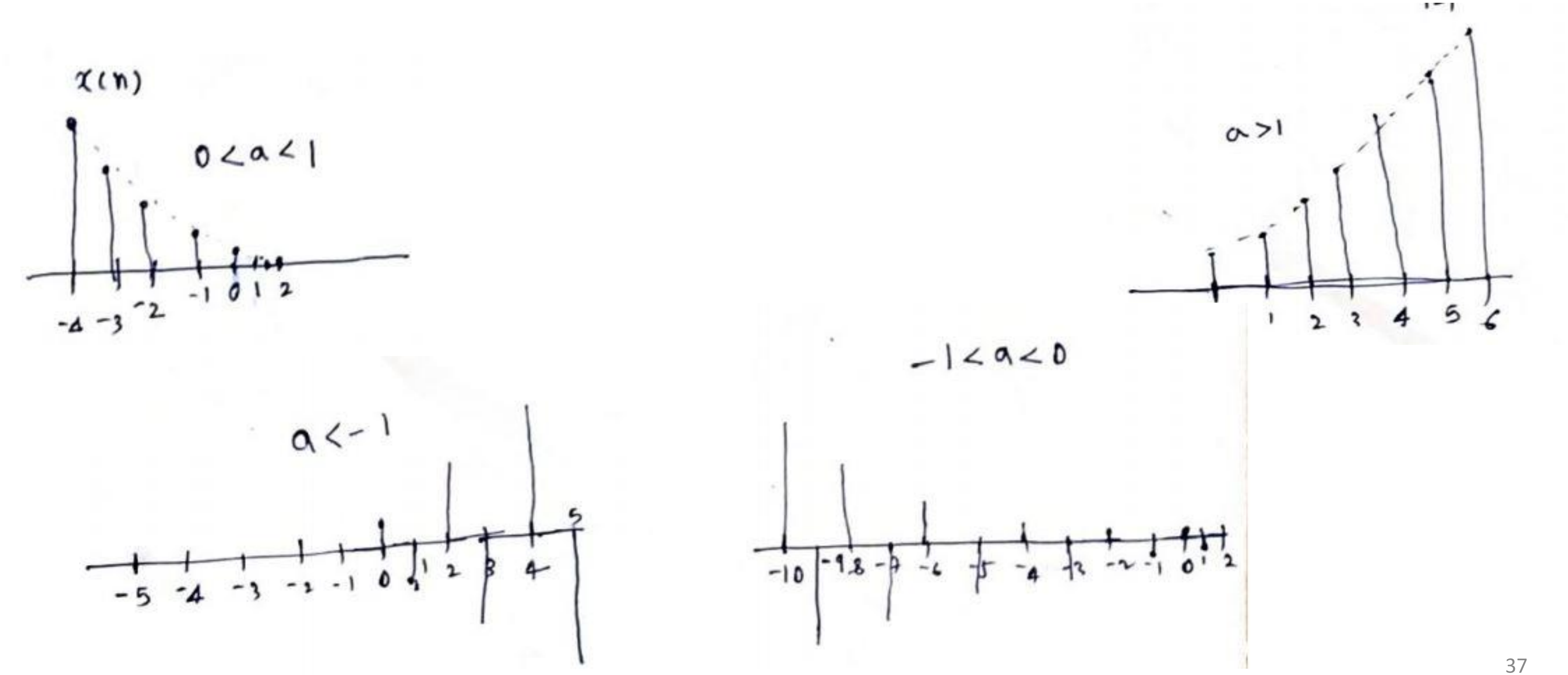


Figure 1

Elementary discrete time signals (Cont.)

- 4. <u>Exponential sequence</u>

The exponential signal is a sequence of the form $x(n) = a^n$ for all n

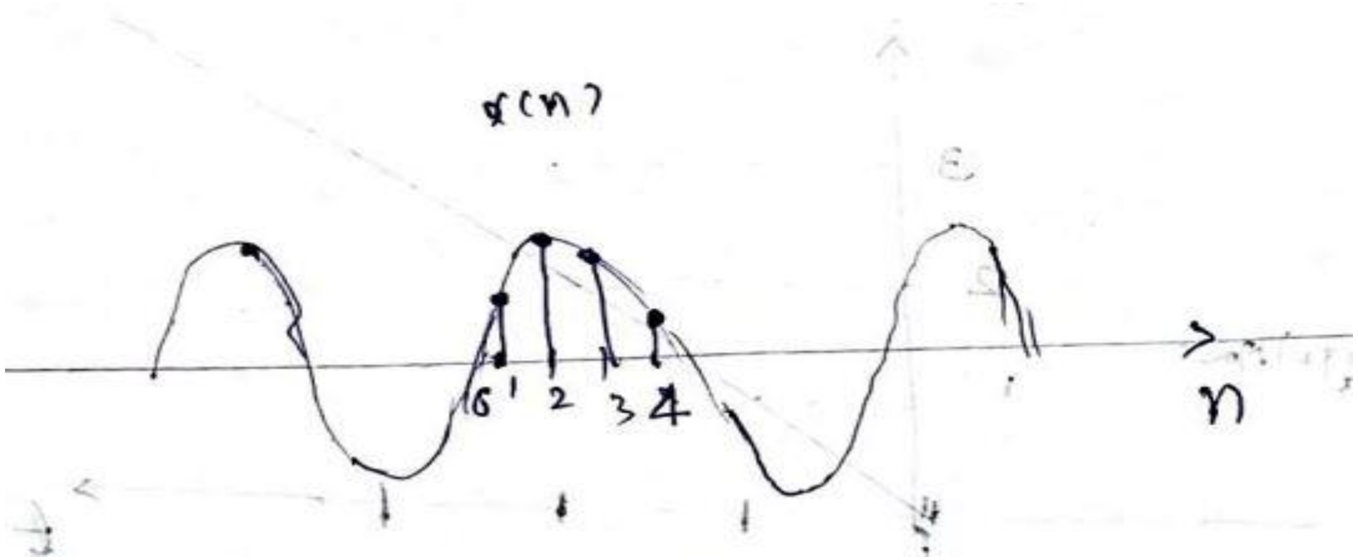


Elementary discrete time signals (Cont.)

<u>5. Sinusoidal signal:</u> The discrete time sinusoidal signal is given by

$$x(n) = Acos(w_0 n + \emptyset)$$

$\emptyset$ =phase differencre



Elementary discrete time signals (Cont.)

6. Complex exponential signal:-

The discrete time comlex exponential signal is given by

$$x(n) = a^n e^{j(w_0 n + \emptyset)}$$

- For $|a| < 1$ → The amplitude of the sinusoidal sequence decays exponentially.

- For $|a| > 1$ → the amplitude of sinusoidal sequence increases exponentially.

Problems:

Calculate the value of the followings:

1. Q. $\sum_{\boldsymbol{n}=-\infty}^{\infty} \boldsymbol{e}^{\boldsymbol{2n}} \quad \boldsymbol{\delta}(\boldsymbol{n}-\boldsymbol{2})$

Ans:- As we know
$$\delta(n) = 1 \quad for\ n = 0$$
$$= 0 \quad for\ n \neq 0$$

At n=2 or n-2=0, $\delta(n-2)$=1,

for other value of n, $\delta(n-2)$=0

So, $\sum_{-\infty}^{\infty} e^{2n} \quad \delta(n-2)$=$e^{2n}|_{n=2}$

=e^{4}

Problems (cont.)

2. Q. $\boldsymbol{\sum_{n=-\infty}^{\infty} \delta(n-1) \quad sin2n}$

Ans:-At n=1 or n-1=0, $\delta(n-1)$=1,

for other value of n, $\delta(n-1)$=0

So, $\sum_{n=-\infty}^{\infty} \delta(n-1) \quad sin2n$

= $sin2n|_{n=1} = sin2$

3. Q. $\boldsymbol{\sum_{n=-\infty}^{\infty} n^2 \quad \delta(n+2)}$

Ans: At n=-2 or $n+2$=0, $\delta(n+2)$=1,

for other value of n, $\delta(n+2)$=0

$$\sum_{n=-\infty}^{\infty} n^2 \quad \delta(n+2)$$

= $n^2|_{n=-2} = 4$

Problems (cont.)

4. Q. $\sum_{\boldsymbol{n}=-\infty}^{\infty} \boldsymbol{\delta}(\boldsymbol{n}-\mathbf{1}) \quad \boldsymbol{e}^{\boldsymbol{n}^{2}}$

Ans: At n=1 or n-1=0, $\delta(n-1)$=1,

for other value of n, $\delta(n-1)$=0

So, $\sum_{n=-\infty}^{\infty} \delta(n-1) \quad e^{n^{2}}$

$=e^{n^{2}}\Big|_{n=1}$=e

5. Q. $\sum_{\boldsymbol{n}=\mathbf{0}}^{\mathbf{5}} \boldsymbol{\delta}(\boldsymbol{n}+\mathbf{1}) \quad \mathbf{2}^{\boldsymbol{n}}$

Ans:- At n=-1 or n +1=0, $\delta(n+1)$=1,

for other value of n, $\delta(n-1)$=0

Here in summation starts n= -1 is not available, so result will be 0

Problems (cont.)

6Q $\boldsymbol{\sum_{n=2}^{\infty} \delta(n-1) \quad sin2n}$

Ans:- 0

7Q $\sum_{n=0}^{\infty} x(n) \quad \delta(n-2)$

Ans: $x(2)$

8Q $\sum_{n=-\infty}^{\infty} a^{n-2} \quad \delta(n+3)$

Ans: a^{-5}

Basic operations on signal

The basic set of operations on a signal are:

1. Time shifting
2. Time reversal
3. Amplitude scaling
4. Time scaling
5. Signal addition
6. Signal multiplier

1.Time shifting

- Let us consider a continuous time signal $x(t)$.
- Time shifting of the signal may be delay or advance
- Let T is positive.
- $x(t-T) \longrightarrow$ delaying (right shift by T unit).
- $x(t+T) \longrightarrow$ advancing (left shift by T unit) .

Tips:
Plot the signal do not put X axis. Then $x(t-T)$ means add T unit to the X axis

E.g.1: $x(t)$ is shown in Fig. 1(a), calculate $x(t-2)$ and $x(t+1)$

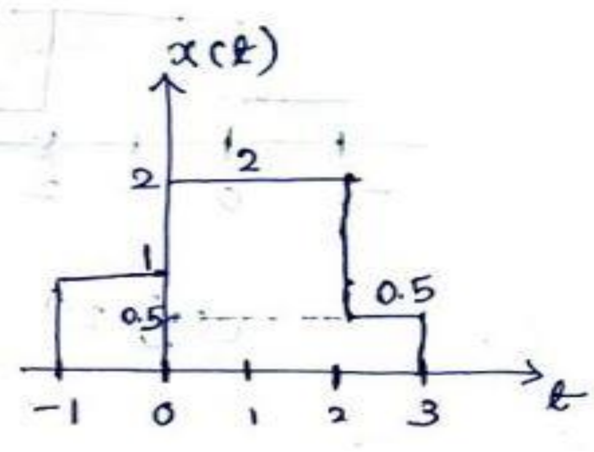


Fig.1.(a)

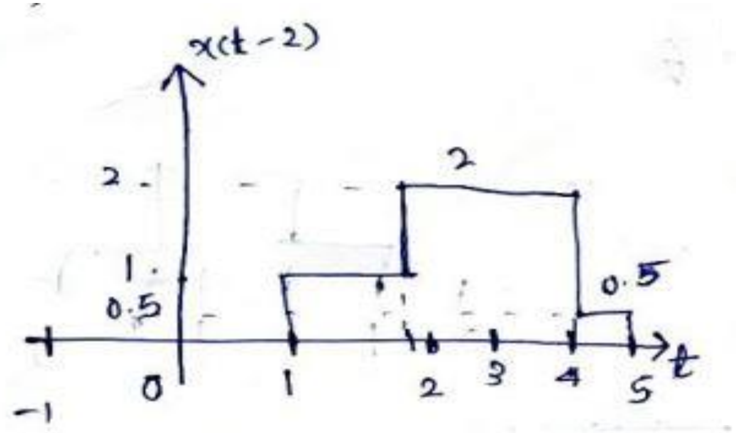


Fig.1.(b)

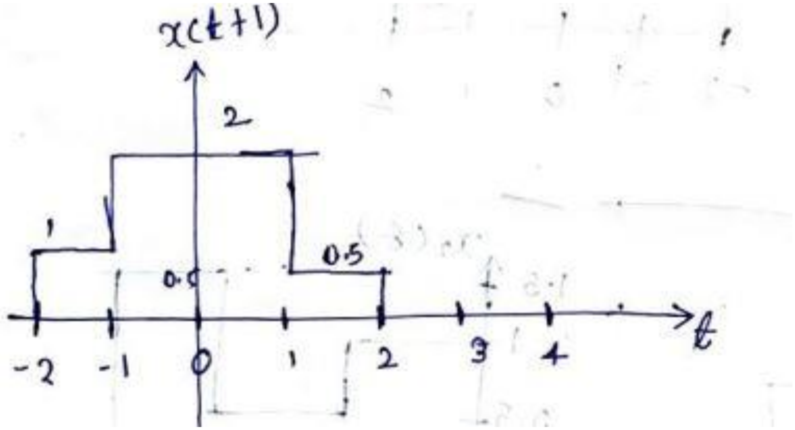


Fig.1.(c)

1.Time shifting (cont.)

- Similarly, for a discrete time signal $x(n)$

Let k is positive.

- $x(n-k) \longrightarrow$ delaying (right shift by k unit).
- $x(n+k) \longrightarrow$ advancing (left shift by k unit).

1.Time shifting (cont.)

- E.g. Given $x(-3) = 4, x(-2) = -3, x(-1) = 2, x(0) = 1, x(1) = -1, x(2) = 3, x(3) = 4$.
- Do the graphical representation of $x(n), x(n-2),\ x(n+3)$.

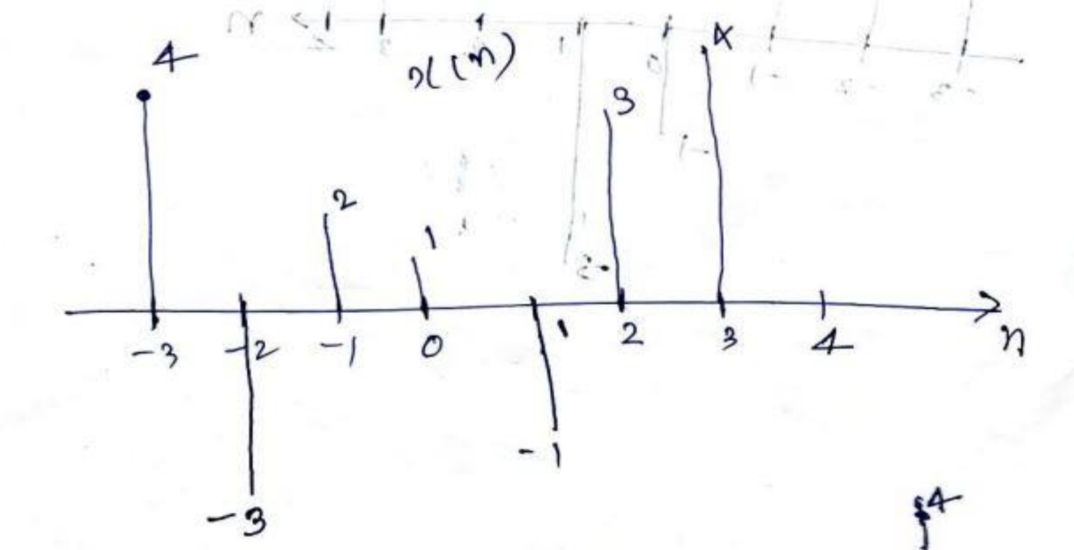


Fig. 2.a

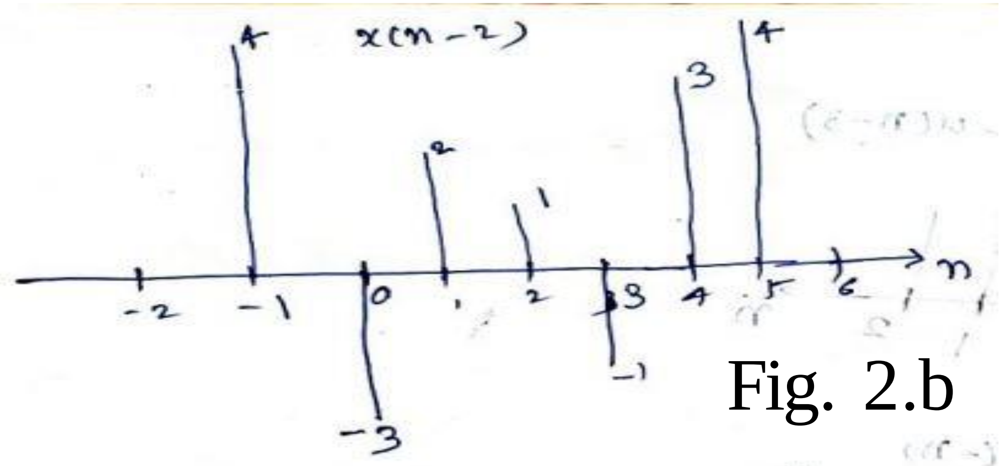


Fig. 2.b

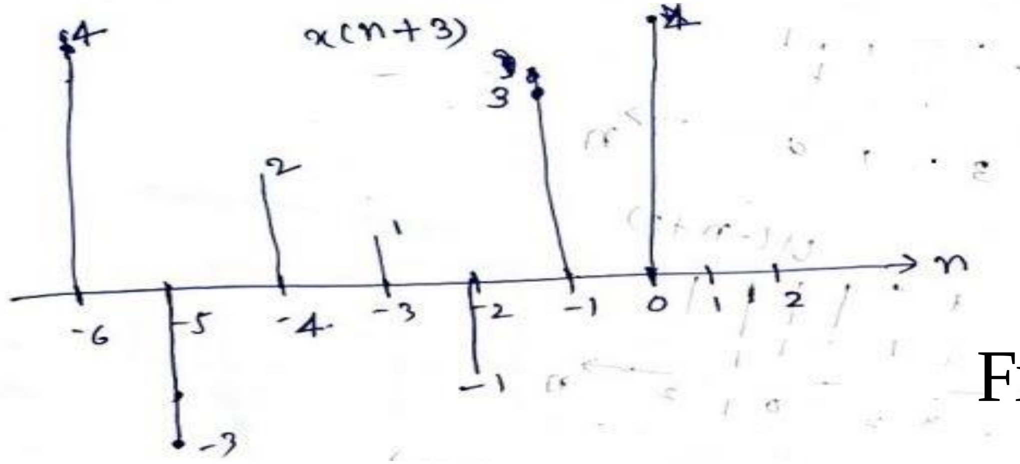
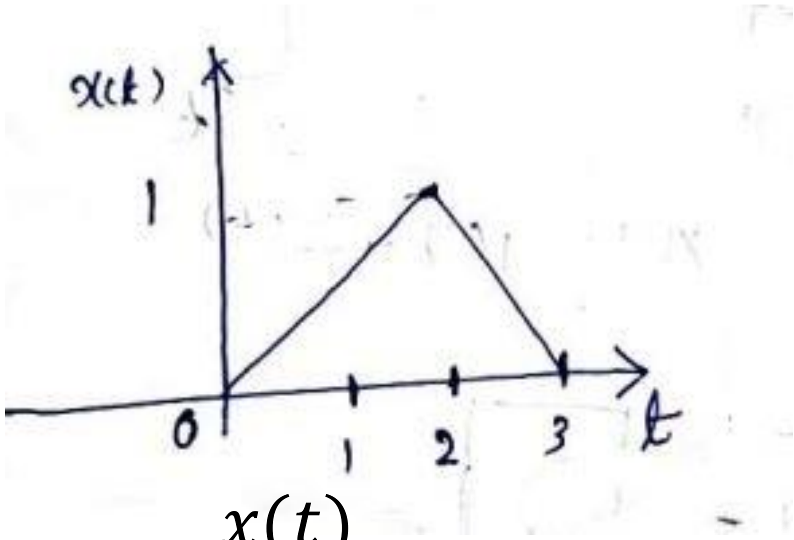


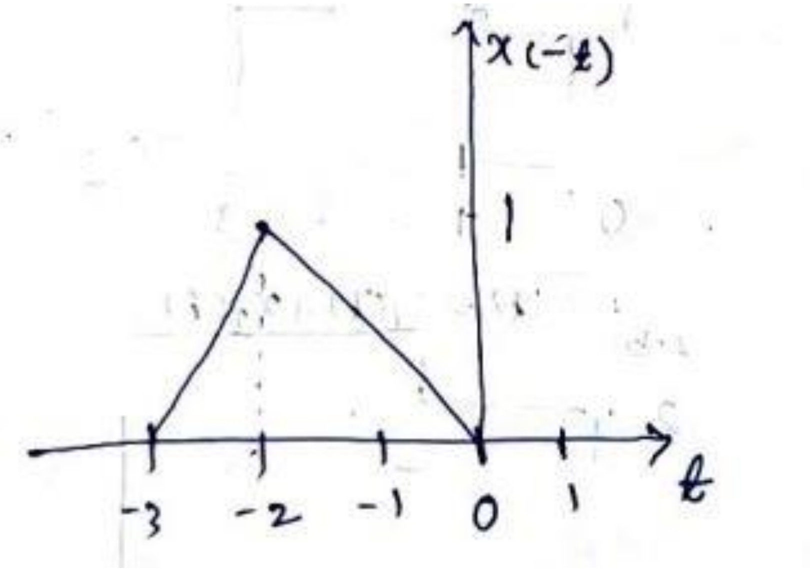
Fig. 2.c

2. Time reversal (folding)

- Time reversal of $x(t)$ can be obtained by folding the signal about t=0. It is denoted by $x(-t)$ as shown in Fig. 3.b.
- Let k is positive
- $x(-t+k) \longrightarrow$ delaying (right shift of $x(-t)$ by k unit)
- $x(-t-k) \longrightarrow$ advancing (left shift $x(-t)$ by k unit.

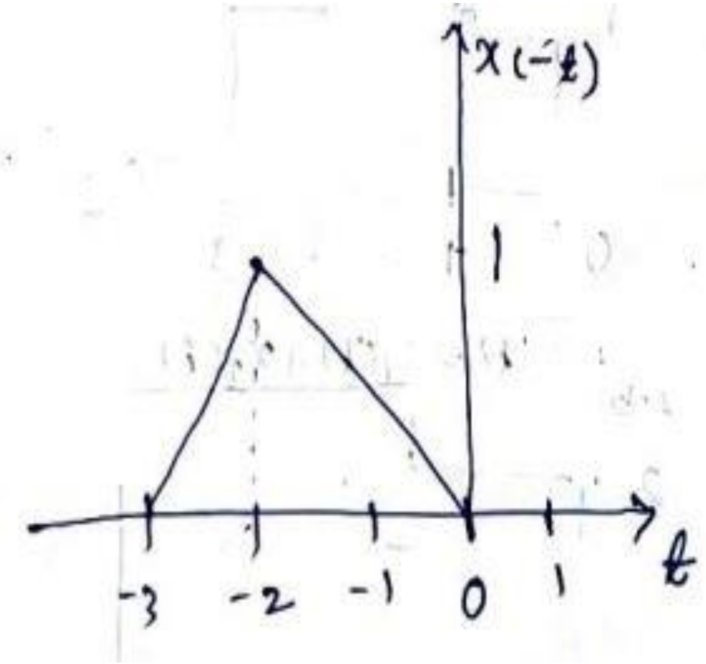


$x(t)$

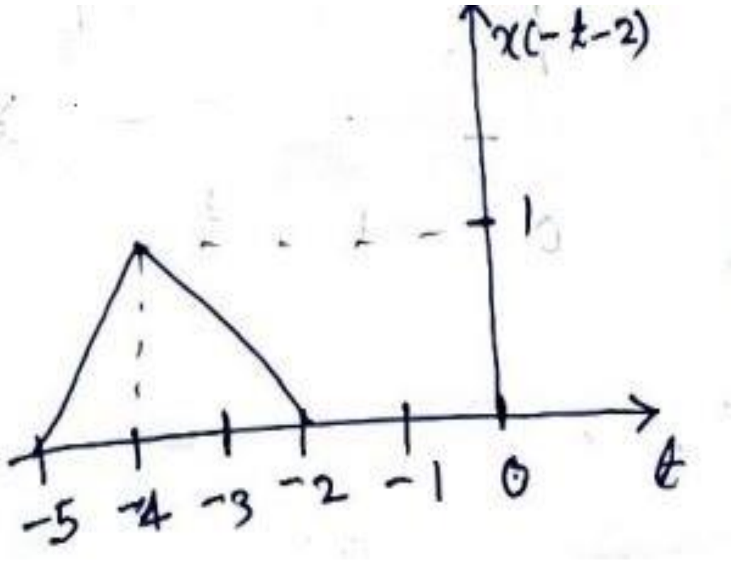


$x(-t)$

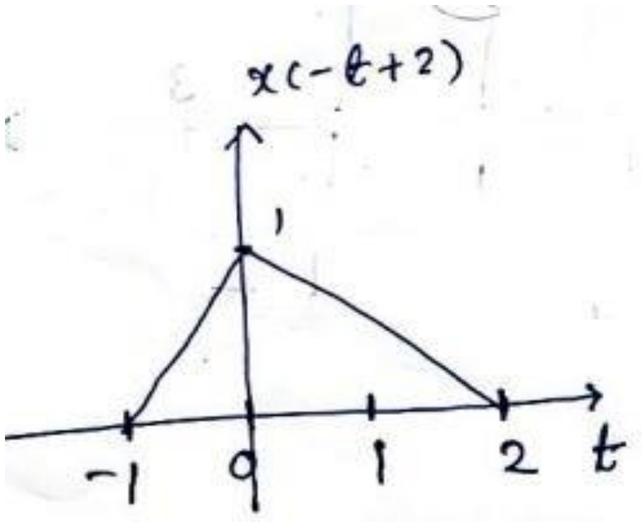
2. Time reversal (cont.)



$x(-t)$



$x(-t-2)$



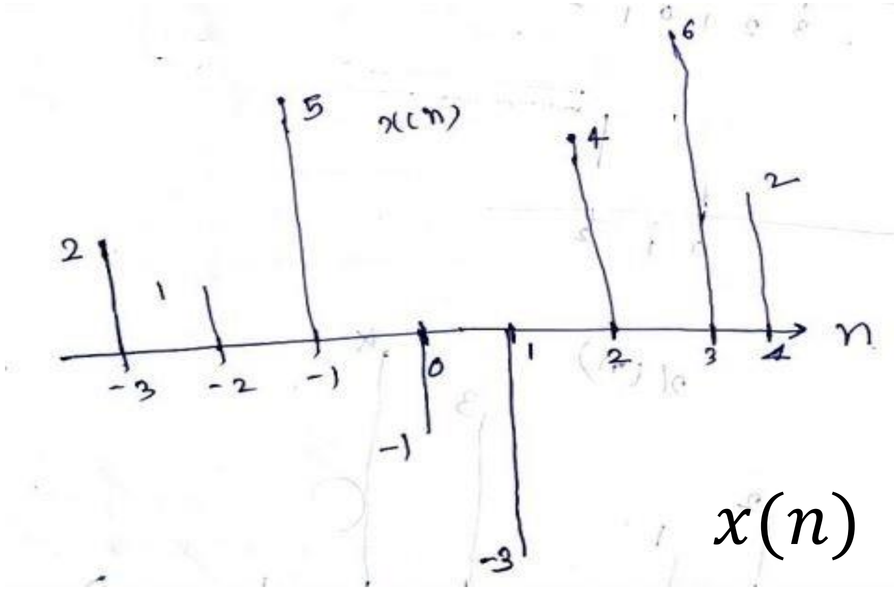
$x(-t+2)$

2. Time reversal (cont.)

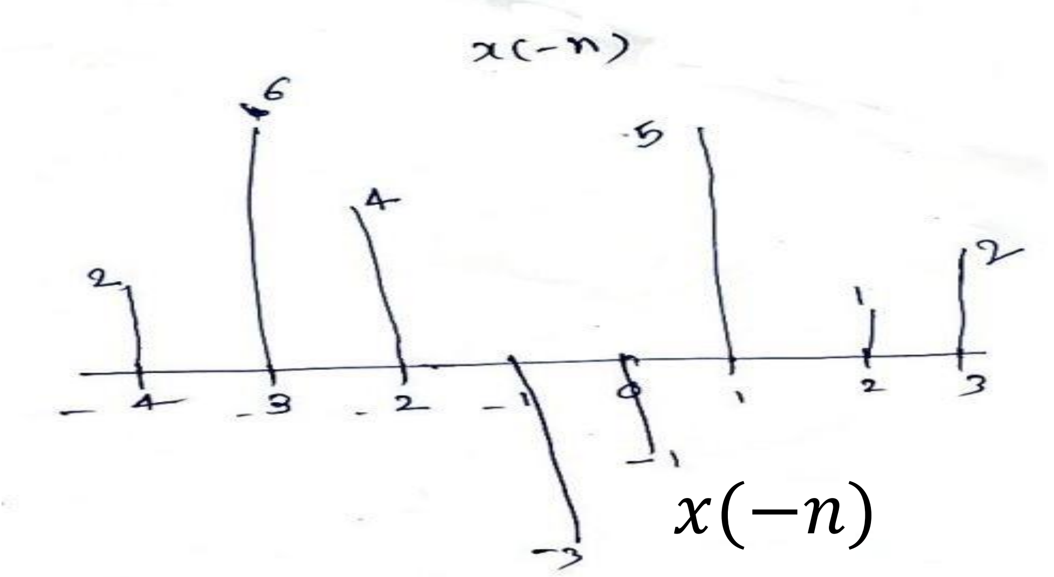
- Similarly, for a discrete time signal $x(n)$. Time reversal of $x(n)$ can be obtained by folding the signal about n=0. It is denoted by $x(-n)$
- Let k is positive
- $x(-n+k) \longrightarrow$ delaying (right shift of $x(-n)$ by k unit)
- $x(-n-k) \longrightarrow$ advancing (left shift $x(-n)$ by k unit)

- E.g. Given $x(-3) = 2, x(-2) = 1, x(-1) = 5, x(0) = -1, x(1) = -3, x(2) = 4, x(3) = 6, x(4) = 2$. Do the graphical representation of $x(n),\ x(-n), x(-n+2),\ x(-n-3)$.

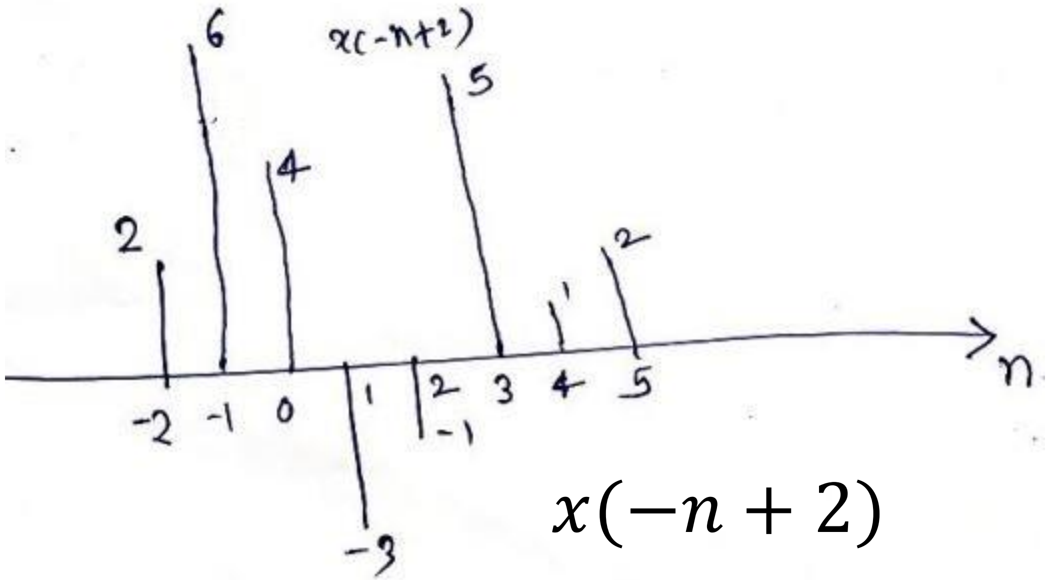
2. Time reversal (cont.)



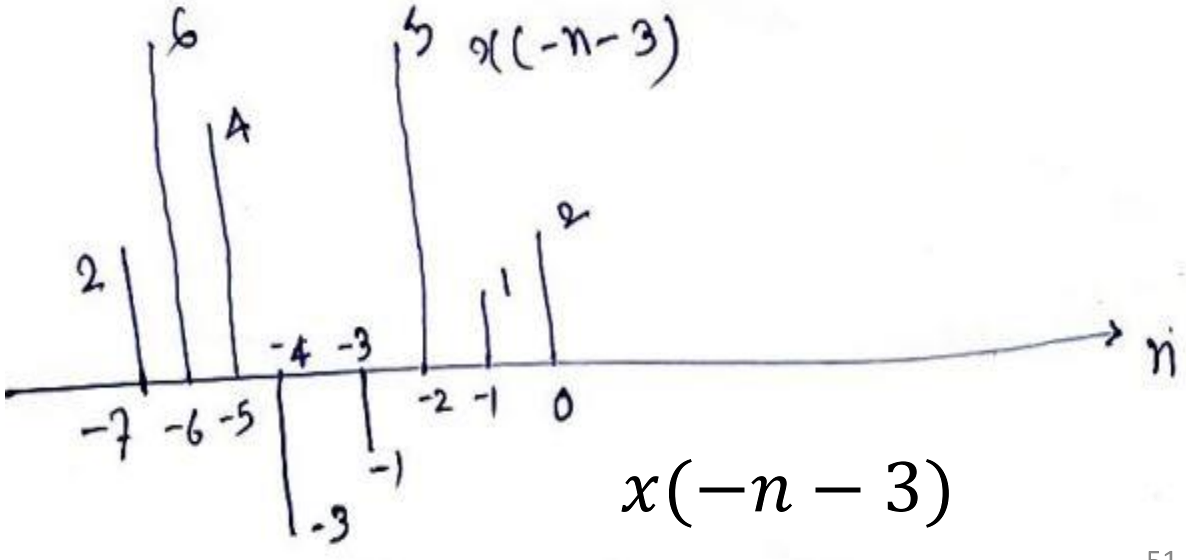
$x(n)$



$x(-n)$



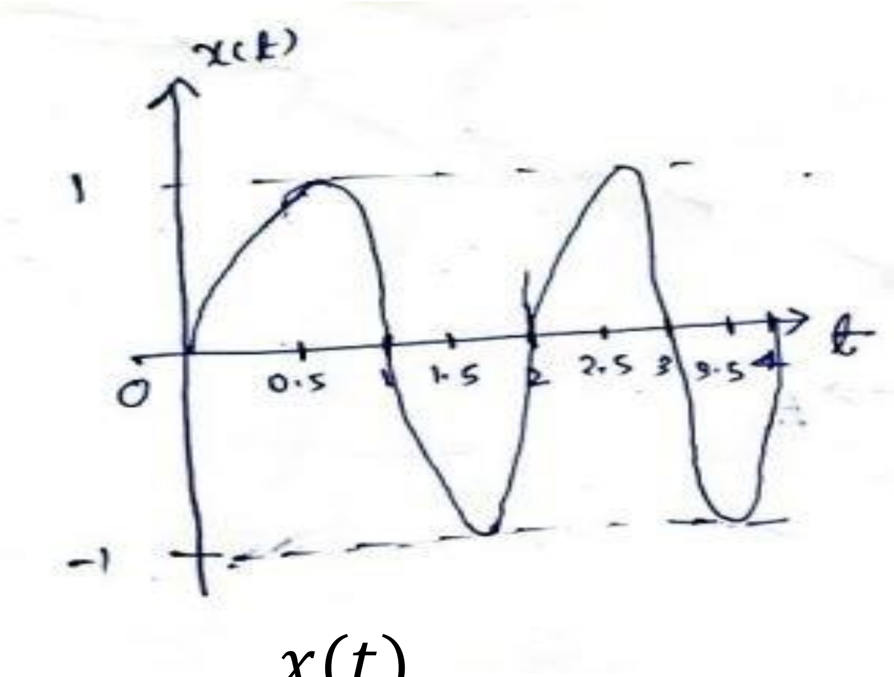
$x(-n+2)$



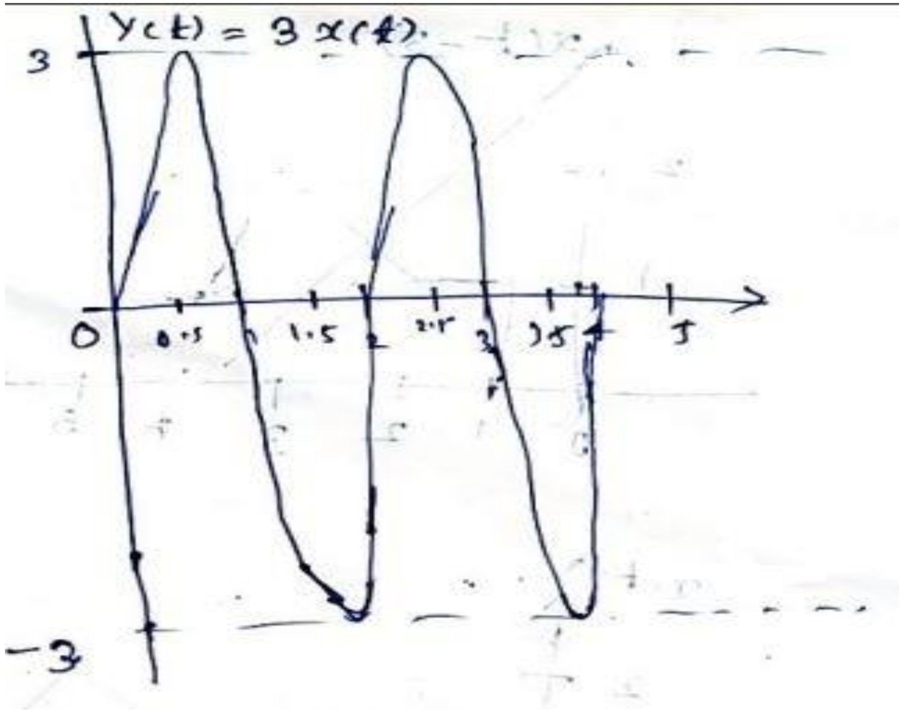
$x(-n-3)$

3. Amplitude scaling (constant multiplication)

- Amplitude scaling means multiply constant to whole signal.
- $y(t) = 3\,x(t)$ is shown in figure.



$x(t)$



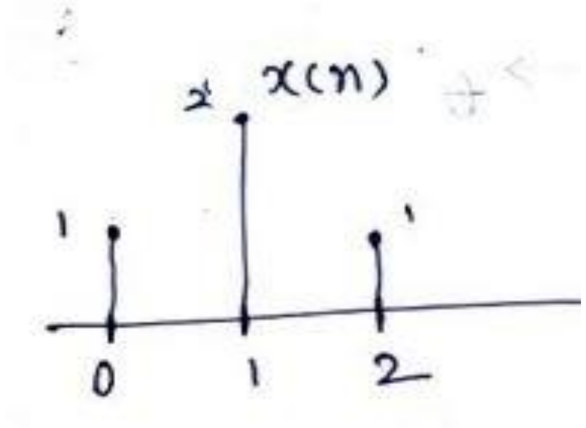
$3x(t)$

3.Amplitude scaling (cont.)

- Similarly, the amplitude of discrete time signal can be represented

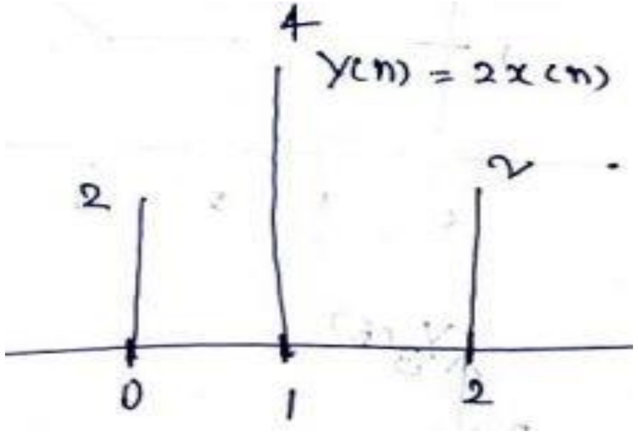
$y(n) = ax(n)$. Here, a is constant.

Eg. E.g. Given $x(0) = 1, x(1) = 2, x(2) = 1$Do the graphical representation of $x(n),\ y(n), where\ y(n) = 2x(n)$.



x(n)

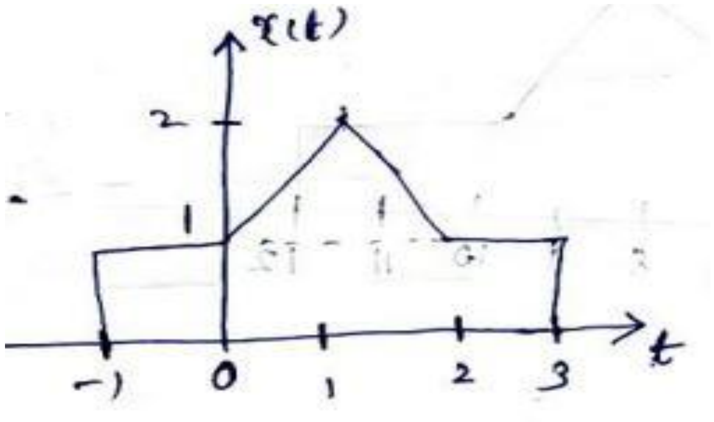
$$y(n) = 2x(n)$$
$$y(0) = 2x(0) = 2 * 1 = 2$$
$$y(1) = 2x(1) = 2 * 2 = 4$$
$$y(2) = 2x(2) = 2 * 1 = 2$$



$2x$(n)

4. Time scaling

- In the time scaling, replace ‘’t’’ by ‘’at’’ in $x(t)$. for, $x(at + k)$, first perform $x(t + k)$, then perform time scaling. Similarly for $x(at - k)$, first perform $x(t - k)$, then perform time scaling.



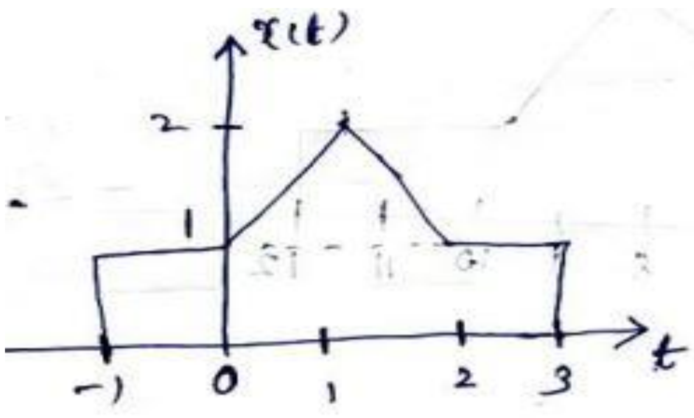
$x(t)$

(a)$y_1(t) = x(2t)$ (b) $y_2(t) = x\left(\frac{t}{2}\right)$ (c) $y_3(t) = x\left(\frac{t}{2} - 3\right)$

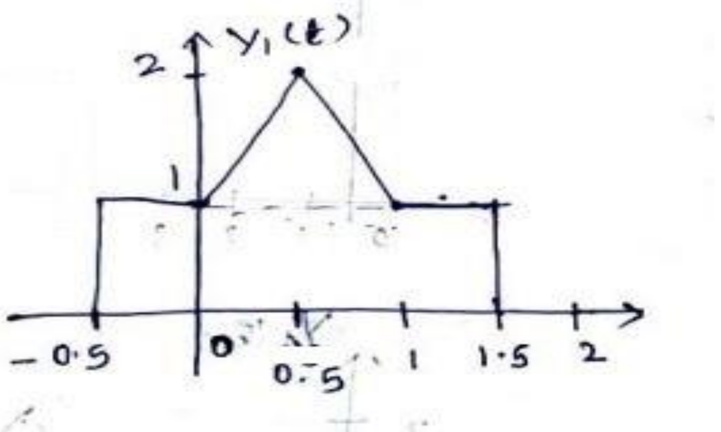
(d) $y_1(t) = x(2t - 3)$

4. Time scaling (cont.)

$$y_1(t) = x(2t)$$
$$y_1(-0.5) = x(-1) = 1$$
$$y_1(0) = x(0) = 1$$
$$y_1(0.5) = x(1) = 2$$
$$y_1(1.5) = x(3) = 1$$
$$y_1(2) = x(4) = 0$$



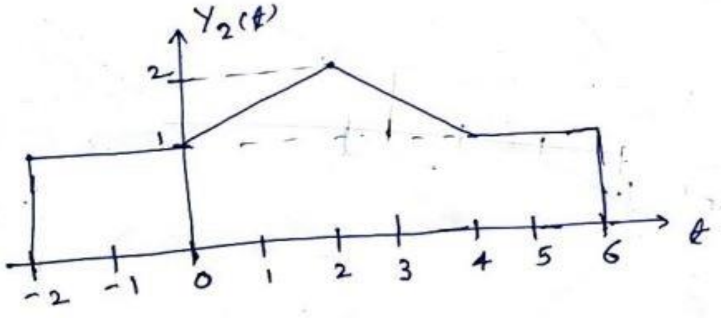
$x(t)$



$x(2t)$

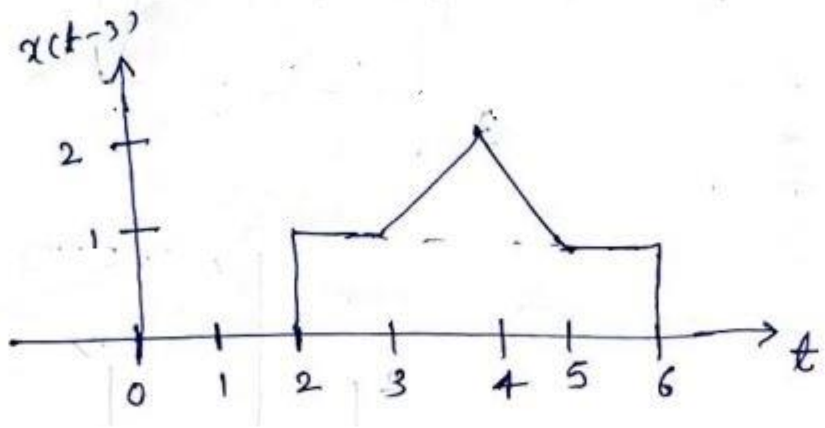
$$y_2(t) = x\left(\frac{t}{2}\right)$$
$$y_2(-2) = x(-1) = 1, y_2(-1) = x(-0.5) = 1$$
$$y_2(0) = x(0) = 1$$

$y_2(1) = x(0.5), y_2(2) = x(1) = 2$
$y_2(3) = x(1.5), y_2(4) = x(2) = 1$
$y_2(5) = x(2.5)$=1, $y_2(6) = x(3) = 1$, $y_2(5) = x(3.5)$=0

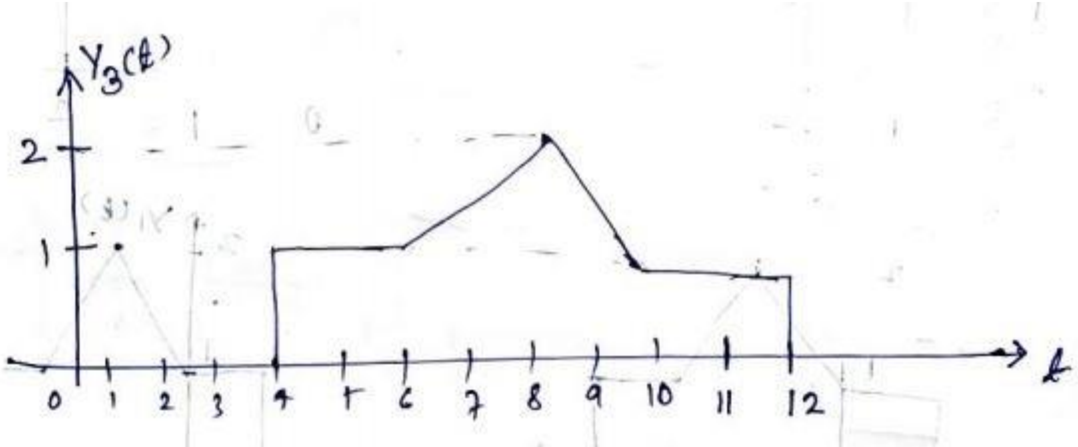


$x\left(\frac{t}{2}\right)$

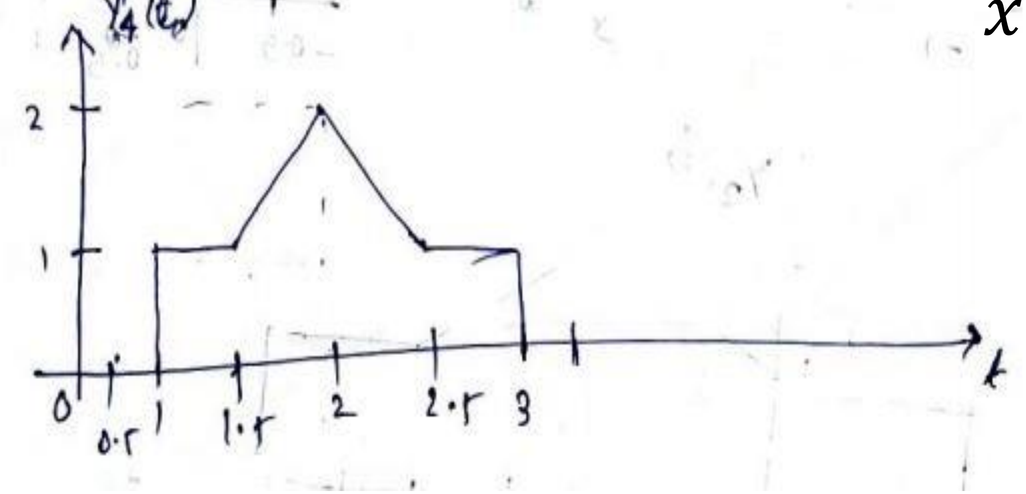
4. Time scaling (cont.)



$x(t-3)$



$x(t/2-3)$

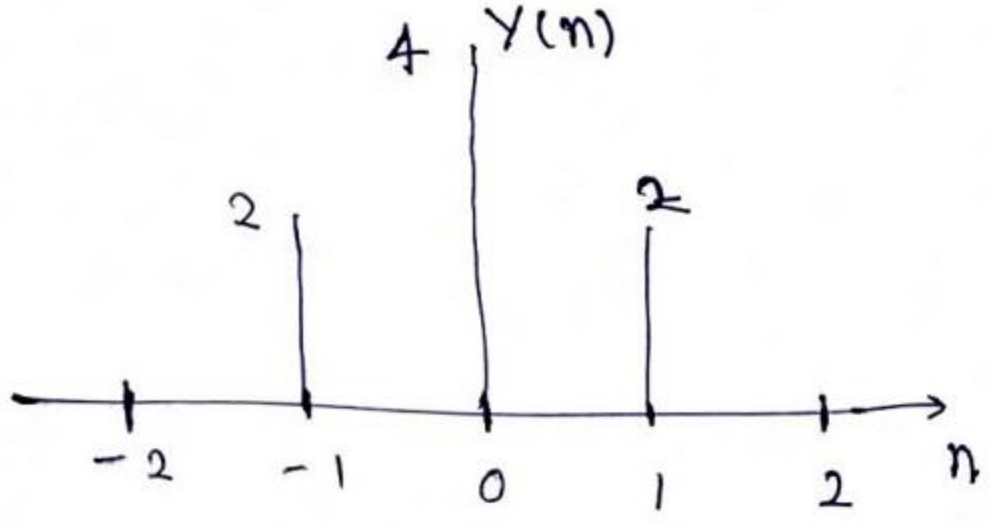


$x(2t-3)$

4. Time scaling (cont.)

- Similarly, for discrete time signal, In the time scaling, replace ‘’n” by ‘’an” in $x(n)$
- E.g. Given $x(-3) = 1, x(-2) = 2, x(-1) = 2, x(0) = 4, x(1) = 3, x(2) = 2, x(3) = 1$. Do the graphical representation of $y(n)$, $where\ y(n) = x(2n)$.
- Answer: $y(n) = x(2n)$

$$y(-2) = x(-4) = 0$$
$$y(-1) = x(-2) = 2$$
$$y(0) = x(0) = 4$$
$$y(1) = x(2) = 2$$
$$y(2) = x(4) = 0$$



5. Signal addition

- $y(t) = x_1(t) + x_2(t)$ where $x_1(t)$ and $x_2(t)$ are continuous time signal.
- Similarly for discrete time signal
- $y(n) = x_1(n) + x_2(n)$
- The addition of 2 signals can be obtained by adding their values at every instant

For interval $0 \leq t \leq 1$

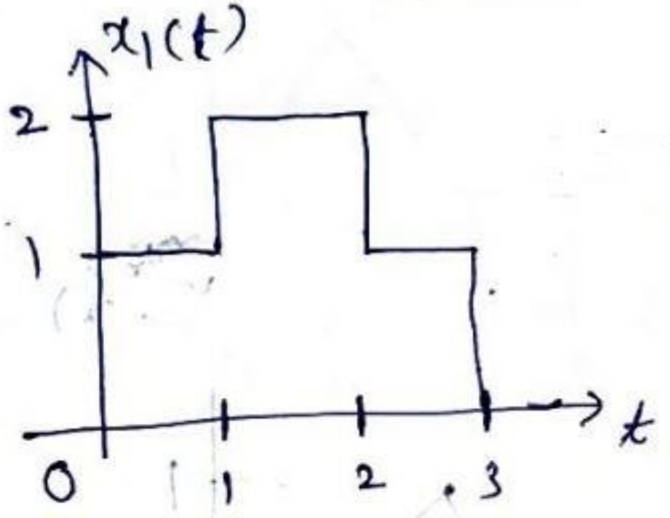
$$x_1(t) = 1;\ x_2(t) = 1$$

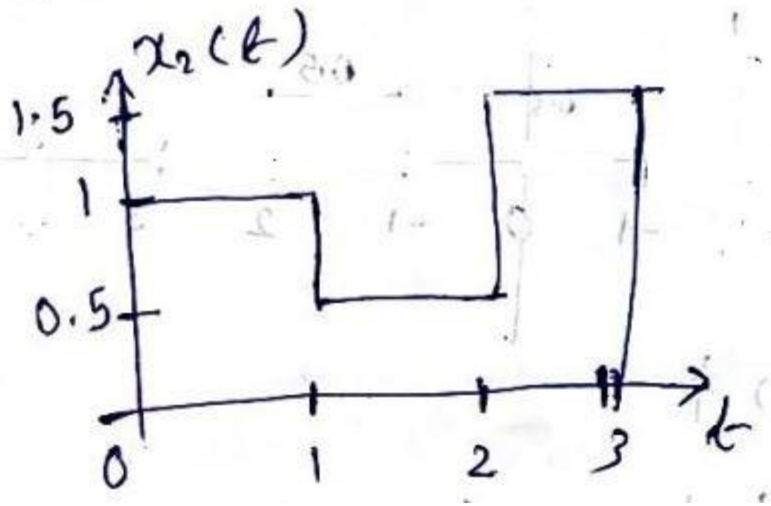
For interval $1 \leq t \leq 2$

$$x_1(t) = 2;\ x_2(t) = 0.5$$

For interval $2 \leq t \leq 3$

$$x_1(t) = 1;\ x_2(t) = 1.5$$





5. Signal addition (Cont.)

$y(t) = x_1(t) + x_2(t)$

For interval $0 \le t \le 1$

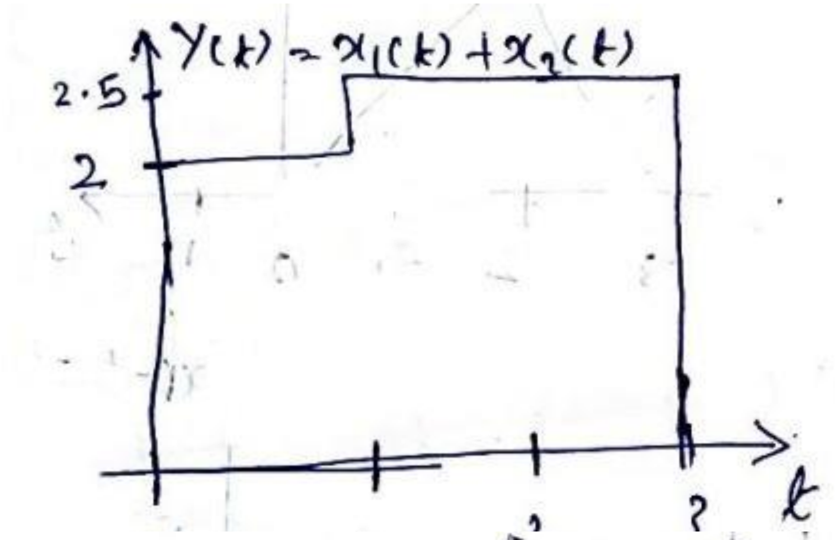
$$y(t) = 1 + 1 = 2$$

For interval $1 \le t \le 2$

$$y(t) = 2 + 0.5 = 2.5$$

For interval $2 \le t \le 3$

$$y(t) = 1 + 1.5 = 2.5$$



$y(t) = x_1(t) - x_2(t)$

For interval $0 \le t \le 1$

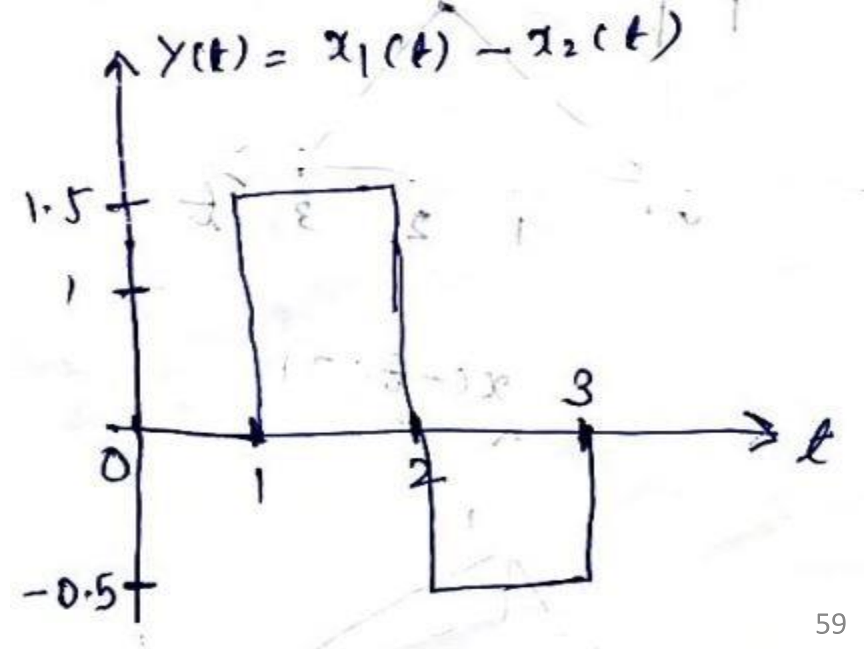
$$y(t) = 1 - 1 = 0$$

For interval $1 \le t \le 2$

$$y(t) = 2 - 0.5 = 1.5$$

For interval $2 \le t \le 3$

$$y(t) = 1 - 1.5 = -0.5$$



5. Signal addition (Cont.)

- E.g. $x_1(n) = \{1,3,2,1\}$ $\qquad$ $x_2(n) = \{1,-2,\underset{\uparrow}{3},2\}$

$y_1(n) = x_1(n) + x_2(n)$ $\qquad$ $y_1(n) = \{0+1, 0-2, 1\underset{\uparrow}{+}3, 3+2, 2+0, 1+0\}$

$$y_1(n) = \{1, -2, \underset{\uparrow}{4}, 5, 2, 1\}$$

$y_2(n) = x_1(n) - x_2(n)$ $\qquad$ $y_2(n) = \{0-1, 0+2, 1\underset{\uparrow}{-}3, 3-2, 2-0, 1-0\}$

$$y_2(n) = \{-1, 2, \underset{\uparrow}{-2}, 1, 2, 1\}$$

6. Signal multiplication

The multiplication of 2 signals can be obtained by multiplying their values at every instant.

For interval $0 \leq t \leq 1$

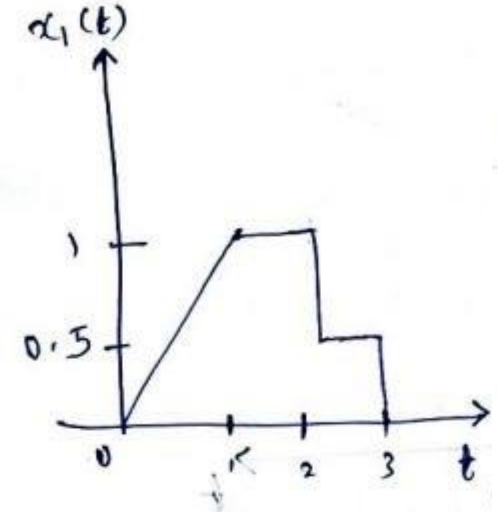
$$x_1(t) = t;\ x_2(t) = 1$$

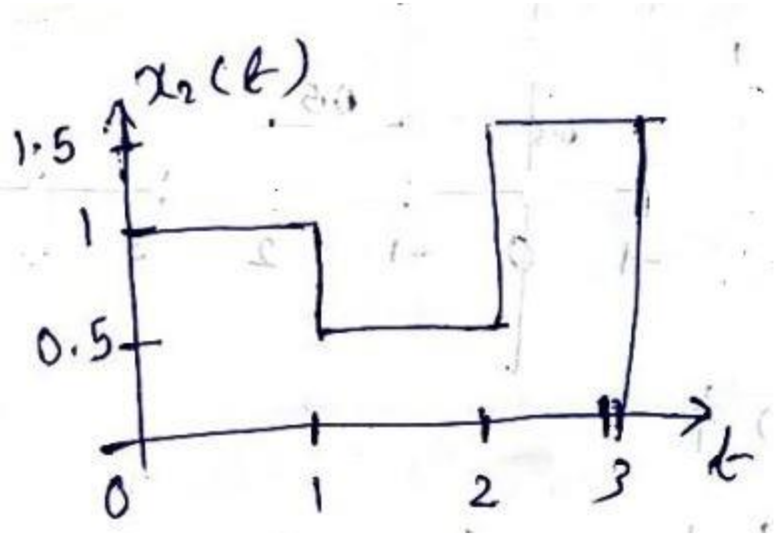
For interval $1 \leq t \leq 2$

$$x_1(t) = 1;\ x_2(t) = 0.5$$

For interval $2 \leq t \leq 3$

$$x_1(t) = 0.5;\ x_2(t) = 1.5$$



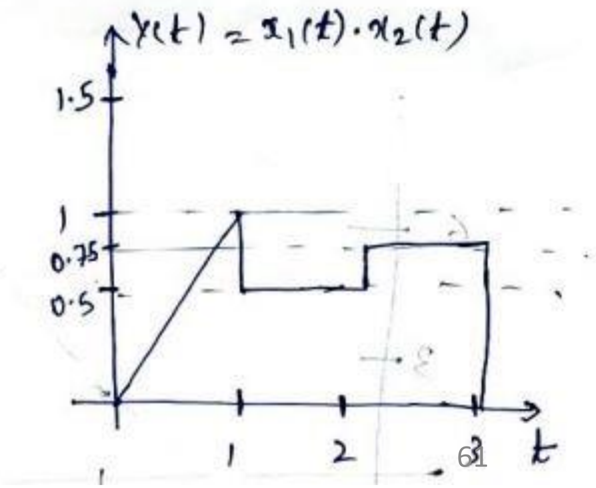


$$y(t) = x_1(t).x_2(t)$$

For interval $0 \leq t \leq 1$ $\quad y(t) = t * 1 = t$

For interval $1 \leq t \leq 2$ $\quad y(t) = 1 * 0.5 = 0.5$

For interval $2 \leq t \leq 3$ $\quad y(t) = 0.5 * 1.5 = 0.75$



6. Signal multiplication (cont.)

- E.g. $x_1(n) = \{1,2,-2,3,2\}$ $\qquad x_2(n) = \{-1,\underset{\uparrow}{1},0.5,0.5,1\}$

$y(n) = x_1(n) * x_2(n)$

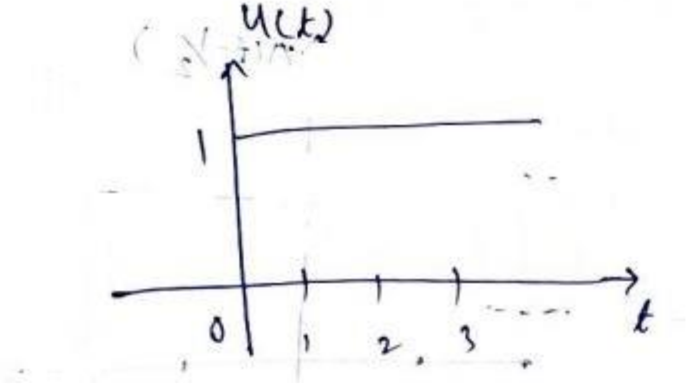
$$y(n) = \{0 * -1, 1 \underset{\uparrow}{*} 1, 2 * 0.5, -2 * 0.5, 3 * 1, 2 * 0\}$$

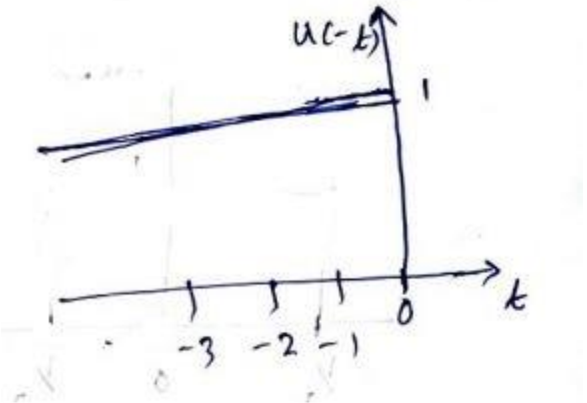
$$y(n) = \{0,\underset{\uparrow}{1},1,-1,3,0\}$$

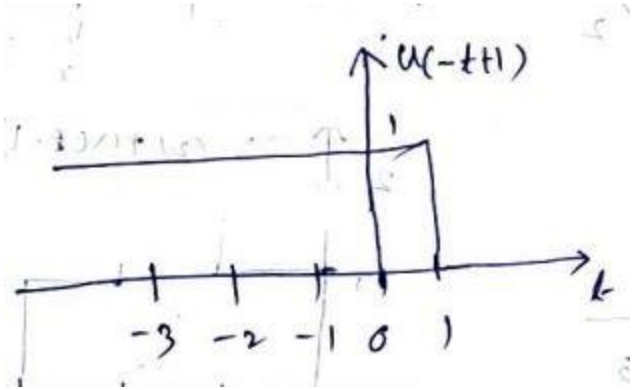
Problems:

- Sketch the followings:

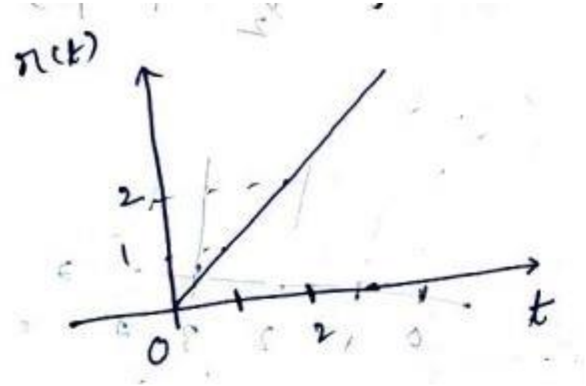
1. $u(-\mathrm{t}+1)$

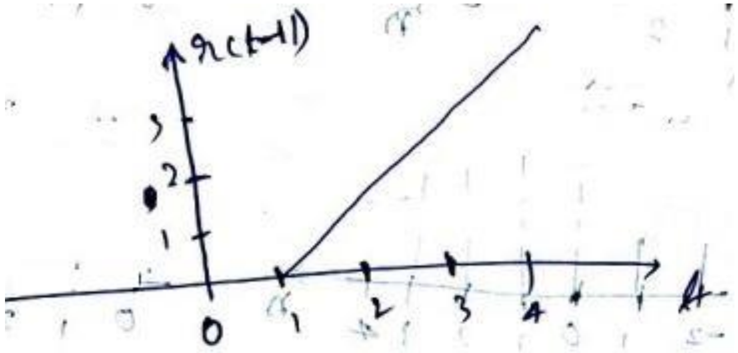


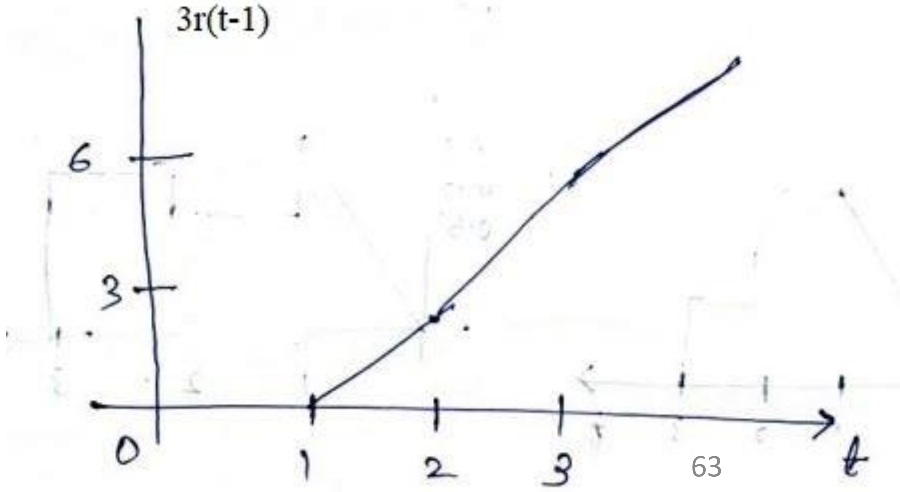




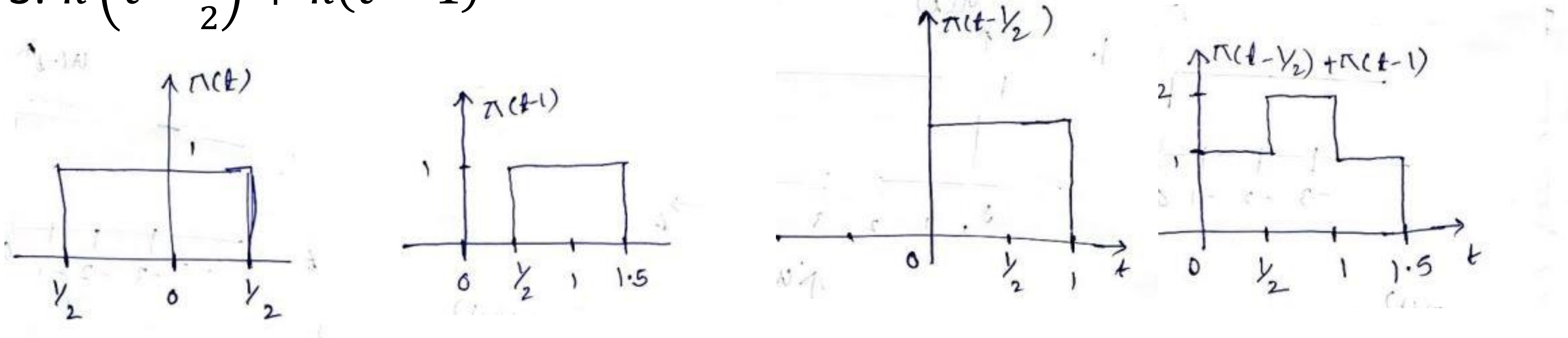
2. $3r(\mathrm{t}-1)$



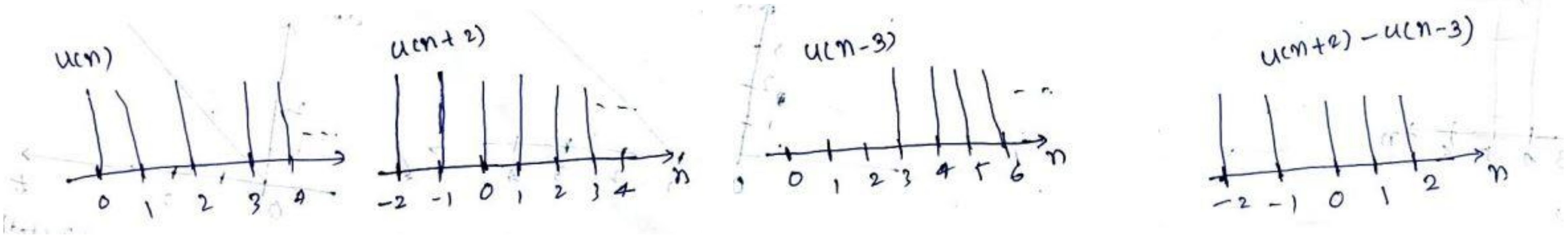




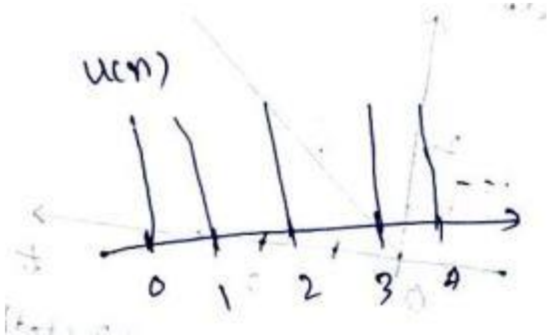
3. $\pi\left(t-\frac{1}{2}\right)+\pi(t-1)$

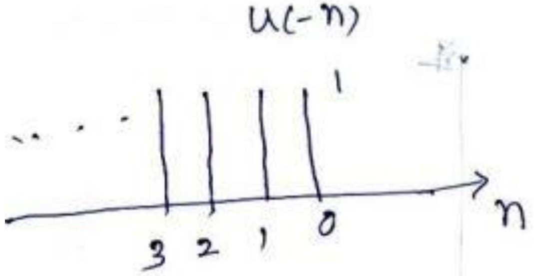


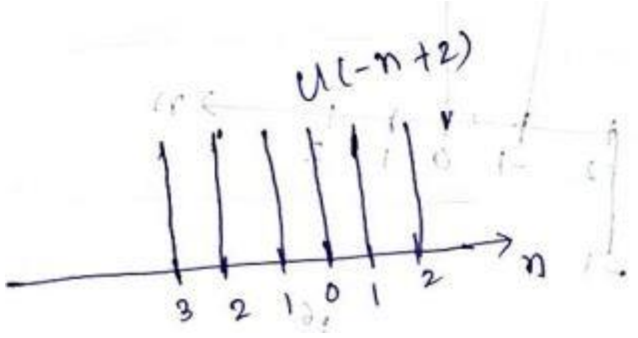
4. $u(n+2)-u(n-3)$

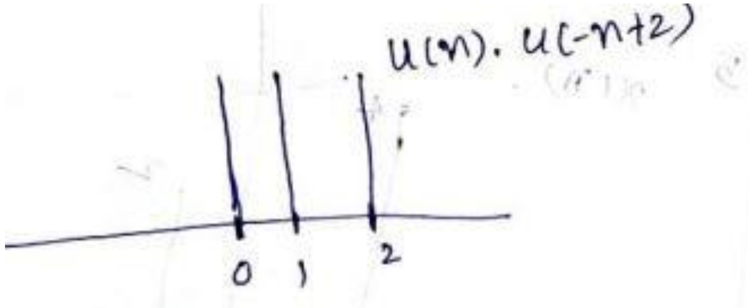


5. $u(-n + 2)u(n)$









Classification of Signals

The signals are classified according to their characteristic.

1. Continuous time and discrete time signal

2. Deterministic and random signals

3. Periodic and aperiodic signals

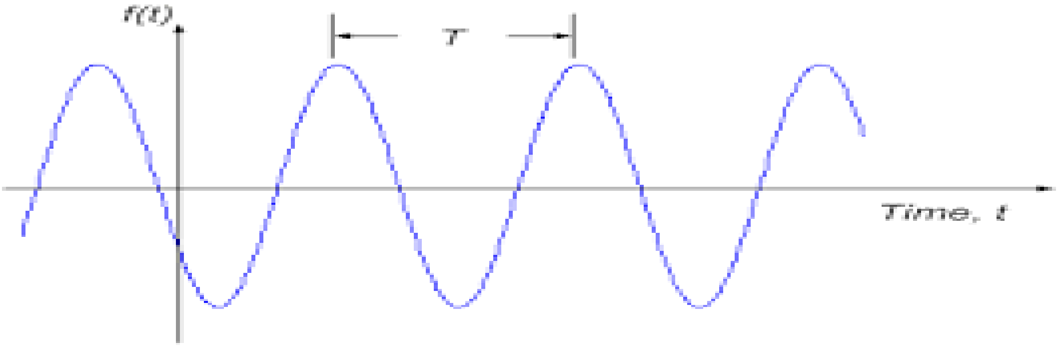
4. Even and odd signals

5. Energy and power signals

2. Deterministic and Random Signals

<u>Deterministic signal:</u>

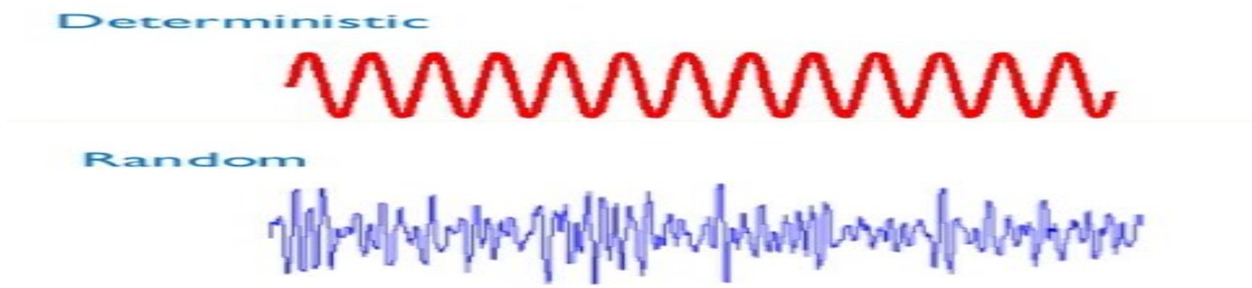
- A deterministic signal is a signal exhibiting no uncertainty of value at any given instant of time
- Its instantaneous value can be predicted by mathematical equation
- For example $x(t) = \sin(3t)$ is deterministic signal.



2. Deterministic and Random Signals (cont.)

Random signal:

- A random signal is characterized by uncertainty before its actual occurrence.
- Behavior of these signals are random i.e. not predictable with respect to time.
- These signals can’t be expressed mathematically.
- For example Thermal Noise generated is non deterministic signal.



3. Periodic and aperiodic signals

- A continuous time signal $x(t)$ is said to be periodic if it satisfies the condition

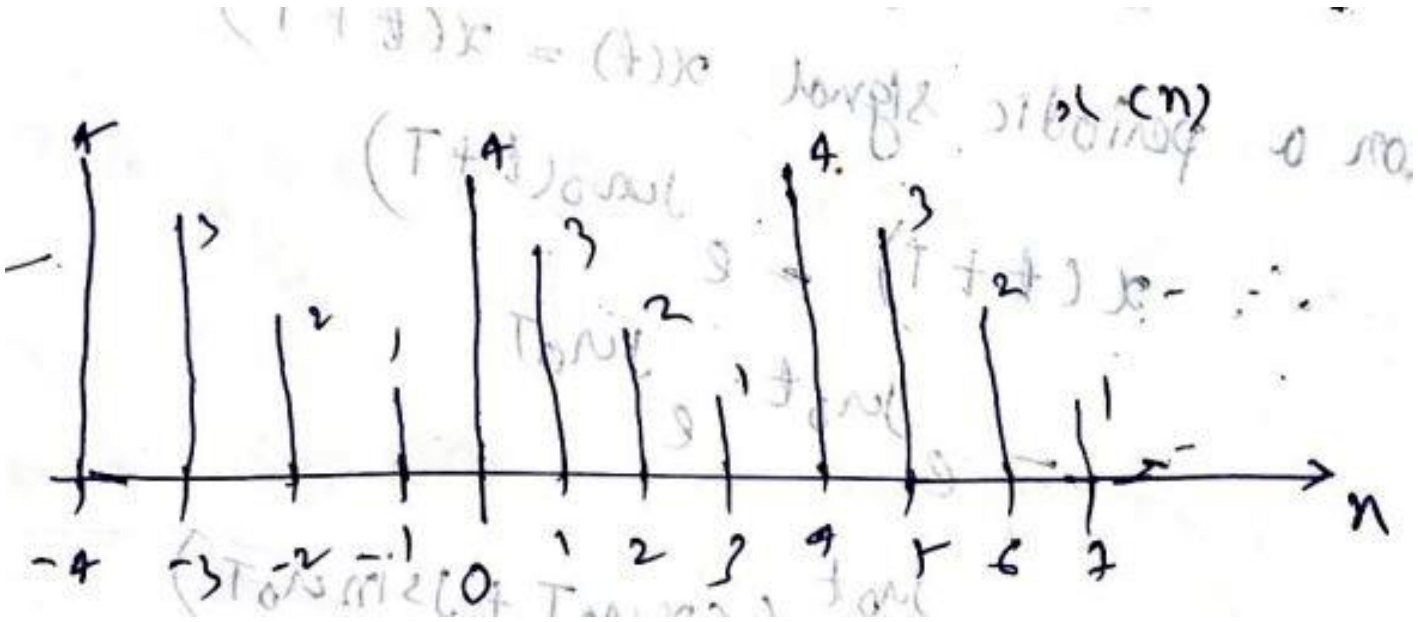
$$x(t + T) = x(t) \text{ for all t. Otherwise aperiodic}$$

- The smallest value of T that satisfies the above condition is known as fundamental period.
- A discrete time signal $x(n)$ is said to be periodic if it satisfies the condition

$$x(n + N) = x(n) \text{ for all n. Otherwise aperiodic}$$

- The smallest value of N that satisfies the above condition is known as fundamental period.

3. Periodic and aperiodic signals (cont.)



- The above sequence is repeating after every 4 samples. So, fundamental period=4.

3. Periodic and aperiodic signals (cont.)

<u>For Sinusoidal signal</u>

Let $x(t) = Asin(\Omega_0 t + \theta)$----------------(1)

- For periodic signal $x(t + T) = x(t)$

$x(t + T) = Asin(\Omega_0(t + T) + \theta)$

$= Asin(\Omega_0 t + \Omega_0 T + \theta)$-----------------------(2)

Equation (1) and (2) are equal if

$\Omega_0 T$=2π,

so $T = \frac{2\pi}{\Omega_0}$

T =fundamental period or time period

Ω_0=Fundamental frequency

3. Periodic and aperiodic signals (cont.)

<u>For complex exponential signal</u>

Let $x(t) = e^{j\Omega_0 t}$

For periodic signal $x(t+T) = x(t)$

$x(t+T) = e^{j\Omega_0 (t+T)} = e^{j\Omega_0 t}\, e^{j\Omega_0 T} = e^{j\Omega_0 t}(cos\Omega_0 T + jsin\ \Omega_0 T)$

As for periodic $x(t+T) = x(t)$

$(cos\Omega_0 T + jsin\ \Omega_0 T)$=1, If $\Omega_0 T$=2π, $(cos\Omega_0 T + jsin\ \Omega_0 T)$=1

So for periodic

$T = \frac{2\pi}{\Omega_0}$ $\quad$ T =fundamental period or time period

Ω_0=Fundamental frequency

Condition for sum of 2 periodic signal to be periodic

- The sum of 2 periodic signal $x_1(t)$ and $x_2(t)$ with periods T_1 and T_2 may or may not be periodic depending on the relation between T_1 and T_2.

- If the sum to be periodic, the ratio of periods $\frac{T_1}{T_2}$ must be rational number or ratio of 2 integers. Otherwise sum is not periodic.
- If, $\frac{T_1}{T_2}=\frac{a}{b}$
- Fundamental time period=$T = bT_1$

Problem

Find the fundamental period T of the following continuous-time signal, if, they are periodic.

1. Q. $\boldsymbol{x(t) = je^{j5t}}$

Ans:- Given $x(t) = je^{j5t}$. The signal is periodic.

$T = \frac{2\pi}{\Omega_0}$ Here Ω_0=5.

$T = \frac{2\pi}{5}$=0.4π second.

2. **Q.** $\boldsymbol{x(t) = \sin(50\pi t)}$

Ans:- $T = \frac{2\pi}{\Omega_0}$ Here Ω_0=50 $\boldsymbol{\pi}$. The signal is periodic

$T = \frac{2\pi}{50\boldsymbol{\pi}} = \frac{1}{25}$

Problem (Cont.)

3 Q. $\boldsymbol{x(t) = 20\cos(10\pi t + \frac{\pi}{6})}$

Ans:- $T = \frac{2\pi}{\Omega_0}$ Here $\Omega_0 = 10\pi$. The signal is periodic

$T = \frac{2\pi}{10\pi} = \frac{1}{5}$ second

4 Q. $\boldsymbol{x(t) = 4\cos(5\pi t)}$

$T = \frac{2\pi}{\Omega_0}$ Here $\Omega_0 = 5\pi$. The signal is periodic

$T = \frac{2\pi}{5\pi} = \frac{2}{5}$ second

5. Q. $\boldsymbol{x(t) = \sin(50\pi t)}$**u(t)**

Ans:- $x(t+T) = \sin(50\pi(t+T))$u(t+T).

As u(t)≠u(t+T) The signal is aperiodic.

Problem (Cont.)

6.Q. $\boldsymbol{x(t) = e^{-|t|}}$

Ans:- $x(t+T) = e^{-|t+T|}$

As $x(t) \neq x(t+T)$.

So the $x(t) = e^{-|t|}$ is aperiodic

Problem (Cont.)

Find whether the following signals are periodic or not. Also, find the fundamental time period T.

1 Q. $\boldsymbol{x(t) = 2\cos(10t + 1) - \sin(4t - 1)}$

Ans:-$T_1 = \frac{2\pi}{\Omega_0}$ Here $\Omega_0 = 10$, So $T_1 = \frac{2\pi}{10} = \frac{\pi}{5}$

$T_2 = \frac{2\pi}{\Omega_0}$ Here $\Omega_0 = 4$, So $T_2 = \frac{2\pi}{4} = \frac{\pi}{2}$

So, $\frac{T_1}{T_2} = \frac{\frac{\pi}{5}}{\frac{\pi}{2}} = \frac{2}{5}$

As the ratio is rational number so the sum of 2 signals are periodic.

Fundamental period=T= $5T_1 = 2T_2 = 5\frac{\pi}{5} = \pi$ sec.

Problem (Cont.)

2. Q. $\boldsymbol{x(t) = \cos(60\pi t) - \sin(50\pi t)}$

Ans:-$T_1 = \frac{2\pi}{\Omega_0}$ Here $\Omega_0 = 60\ \pi$, So $T_1 = \frac{2\pi}{60\ \pi} = \frac{1}{30}$

$T_2 = \frac{2\pi}{\Omega_0}$ Here $\Omega_0 = 50\ \pi$, So $T_2 = \frac{2\pi}{50\pi} = \frac{1}{25}$

So, $\frac{T_1}{T_2} = \frac{\frac{1}{30}}{\frac{1}{25}} = \frac{5}{6}$

As the ratio is rational number so the sum of 2 signals are periodic.

Fundamental period=T= $6T_1 = 5T_2 = 6\frac{1}{30} = \frac{1}{5}\,sec$

Problem (Cont.)

3. Q. $\boldsymbol{x(t) = 2u(t) + 2\sin(2t)}$

Ans:- $u(t)$ is aperiodic, so total sum will be aperiodic

4. Q. $\boldsymbol{x(t) = 3\cos(4t) + 2\sin(2\pi t)}$

Ans:-$T_1 = \frac{2\pi}{\Omega_0}$ Here Ω_0= 4, So $T_1 = \frac{2\pi}{4} = \frac{\pi}{2}$

$T_2 = \frac{2\pi}{\Omega_0}$ Here Ω_0= 2π, So $T_2 = \frac{2\pi}{2\pi}$=1

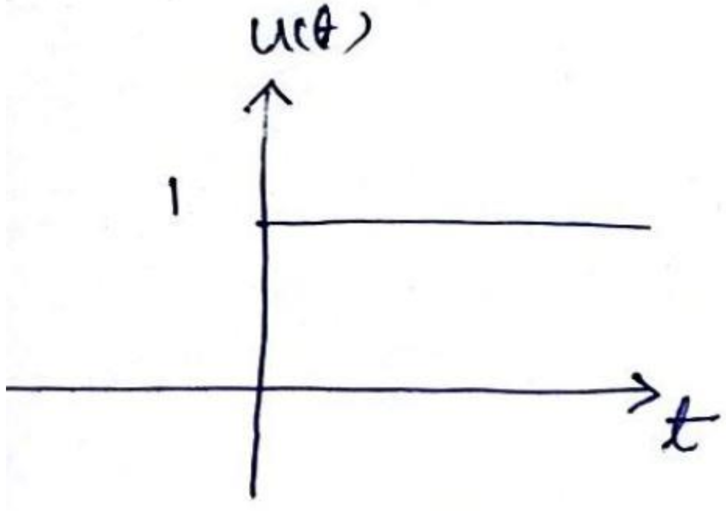
So, $\frac{T_1}{T_2} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$

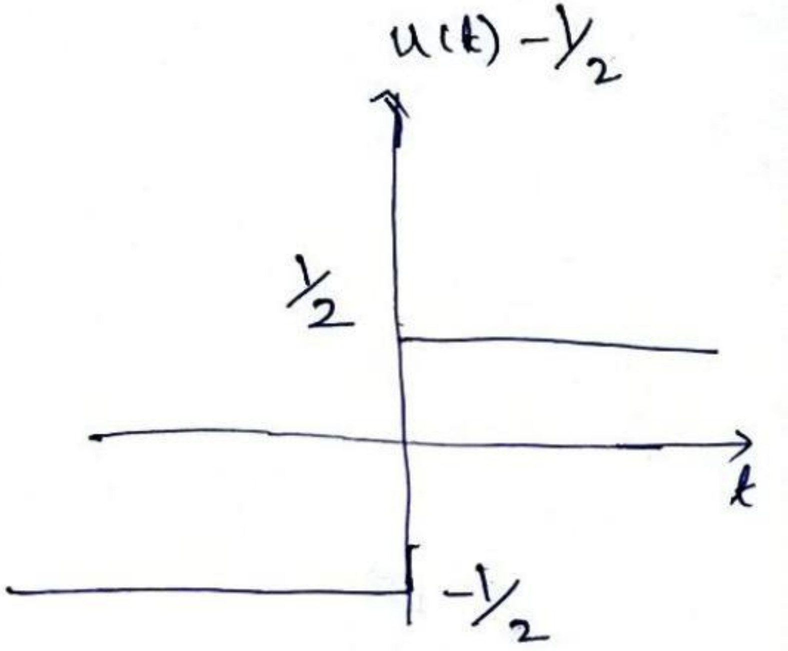
As the ratio is not rational number so the sum of 2 signals are aperiodic.

Problem (Cont.)

5. Q. $\boldsymbol{x(t) = u(t) - \frac{1}{2}}$

Ans:- The signal is aperiodic, because it can not be repeated.





Problem (Cont.)

6. Q. $\boldsymbol{x(t) = sin^2(t)}$

Ans:-$sin^2(t)$=$\frac{1-\cos(2t)}{2}$

$T = \frac{2\pi}{\Omega_0}$ Here Ω_0=2. $T = \frac{2\pi}{2}$=π second.

The signal is periodic

Condition for discrete time signal to be periodic

- A discrete time signal $x(n)$ is said to be periodic if it satisfies the condition

  $x(n+N) = x(n)$ for all n. Otherwise aperiodic

- The smallest value of N that satisfies the above condition is known as fundamental period.

Let $x(n) = Asin(w_0 n + \theta)$----------------(1)

- For periodic signal $x(n+N) = x(n)$

$x(n+N) = Asin(w_0(n+N) + \theta)$

$= Asin(w_0 n + w_0 N + \theta)$-----------------------(2)

Equation (1) and (2) are equal for periodic if

$w_0 N$=$2\pi m$, so $N = \frac{2\pi m}{w_0}$ $\quad$ *m is a value so that N is integer*

N =fundamental period or time period

w_0=Fundamental frequency

Problems

Find the whether the following are periodic or not. If periodic, find the fundamental period.

1. **Q.** $\boldsymbol{x(n) = \cos(0.1\pi n)}$

Ans:- $N = \frac{2\pi m}{w_0}$, Here $w_0 = 0.1\pi$,

So N= $\frac{2\pi m}{0.1\pi}$=20m.

To convert N as integer minimum value of m=1

Fundamental period=N=20. signal is periodic

Problems (Cont.)

2. Q. $\boldsymbol{x(n) = e^{j6\pi n}}$

Ans:- $N = \frac{2\pi m}{w_0}$, Here $w_0 = 6\pi$,

So N= $\frac{2\pi m}{6\pi} = \frac{m}{3}$.

To convert N as integer minimum value of m=3

Fundamental period=N=1. signal is periodic

3. Q. $\boldsymbol{x(n) = \sin\left(\frac{6\pi n}{7} + 1\right)}$

Ans:- $N = \frac{2\pi m}{w_0}$, Here $w_0 = \frac{6\pi}{7}$,

So N= $\frac{2\pi m}{\frac{6\pi}{7}} = \frac{7m}{3}$.

To convert N as integer minimum value of m=3

Fundamental period=N=7. signal is periodic

Problems (cont.)

4. Q. $\boldsymbol{x(n) = e^{j\frac{2\pi}{3}n} + e^{j\frac{3\pi}{4}n}}$

Ans:- $N_1 = \frac{2\pi m}{w_0}$, Here $w_0 = \frac{2\pi}{3}$,

So $N_1 = \frac{2\pi m}{\frac{2\pi}{3}} = 3\text{m}$.

To convert N_1 as integer minimum value of m=1 $\quad N_1$=3.

$N_2 = \frac{2\pi m}{w_0}$, Here $w_0 = \frac{3\pi}{4}$,

So $N_2 = \frac{2\pi m}{\frac{3\pi}{4}} = \frac{8m}{3}$.

To convert N_2 as integer minimum value of m=3 $\quad N_2$=8.

$N = \frac{N_1}{N_2} = \frac{3}{8}$ so N =8 N_1= 8 N_2=24 signal is periodic

Problems (cont.)

5. Q. $\boldsymbol{x(n) = \frac{3}{5} e^{j3\pi(n+\frac{1}{2})}}$

Ans:- $N = \frac{2\pi m}{w_0}$, Here $w_0 = 3\pi$,

So N= $\frac{2\pi m}{3\pi} = \frac{2m}{3}$.

To convert N as integer minimum value of m=3

Fundamental period=N=2. signal is periodic

6. Q. $\boldsymbol{x(n) = 12\cos(20n)}$

Ans:- $N = \frac{2\pi m}{w_0}$, Here $w_0 = 20$,

So N= $\frac{2\pi m}{20} = \frac{\pi m}{10}$.

To convert N as integer minimum value of m as a integer can not be determined. So, signal is aperiodic

Problems (cont.)

7. Q. $\boldsymbol{x(n) = \cos\left(\frac{1}{4}n\right)}$

Ans:- $N = \frac{2\pi m}{w_0}$, Here $w_0 = \frac{1}{4}$,

So $N = \frac{2\pi m}{\frac{1}{4}} = 8\pi m.$

To convert N as integer minimum value of m as a integer can not be determined. So, signal is aperiodic

4. Symmetric (even) and anti-symmetric (odd) signal

Symmetric (even) signal

- A continuous time signal $x(t)$ is said to be symmetric (even) signal if it satisfies the condition $x(-t) = x(t)$ for all t
- A discrete time signal $x(n)$ is said to be symmetric (even) signal if it satisfies the condition $x(-n) = x(n)$ for all n.

Anti-symmetric (odd) signal

- A continuous time signal $x(t)$ is said to be anti-symmetric (odd) signal if it satisfies the condition $x(-t) = -x(t)$ for all t
- A discrete time signal $x(n)$ is said to be anti-symmetric (odd) signal if it satisfies the condition $x(-n) = -x(n)$ for all n.

Even and odd part of signal

<u>Even part of signal</u>

Even part of continuous time signal $x(t)$=$x_e(t) = \frac{x(t)+x\,(-t)}{2}$

Even part of discrete time signal $x(n)$=$x_e(n) = \frac{x(n)+x\,(-n)}{2}$

<u>Odd part of signal</u>

Odd part of continuous time signal $x(t)$=$x_o(t) = \frac{x(t)-x\,(-t)}{2}$

Odd part of discrete time signal $x(n)$=$x_o(n) = \frac{x(n)-x\,(-n)}{2}$

Problems

- Identify the following signals are even or odd.

1. Q. $\boldsymbol{x(t) = cost}$

Ans:- $x(-t) = \cos(-t)$
$= cost$

As $x(-t) = x(t)$.

So $x(t) = cost$ is even signal.

2. Q. $\boldsymbol{x(t) = cost\ sint}$

Ans:- $x(-t) = \cos(-t)\sin(-t)$

$=cost\ (-sint)=-costsint$

As $x(-t) = -x(t)$.

So, $x(t) = costsint$ is odd signal.

Problems

- **Find the even and odd components of the following signals**

1. Q. $\boldsymbol{x(t) = cost + sint + cost\ sint}$

Ans:- Even part of continuous time signal $x(t)$=$x_e(t) = \frac{x(t)+x\,(-t)}{2}$

$x(-t)$= $\cos(-t) + \sin(-t) + \cos(-t)\sin(-t)$

= $cost - sint - cost\ sint$

$x_e(t) = \frac{x(t)+x\,(-t)}{2}$ = $\frac{cost+sint+cost\ sint+cost-sint-cost\ sint}{2}$ =$\frac{2cost}{2}$ = $cost$

Odd part of continuous time signal $x(t)$=$x_o(t) = \frac{x(t)-x\,(-t)}{2}$

$x_o(t) = \frac{x(t)-x\,(-t)}{2}$ = $\frac{cost+sint+cost\ sint-cost+sint+cost\ sint}{2}$

=$\frac{2sint+2cost\ sint}{2}$ =$sint + cost\ sint$

Problems (cont.)

2. Q. $\boldsymbol{x(n)} = \{\boldsymbol{-2, 1, \underset{\uparrow}{2}, -1, 3}\}$

Ans:- $x(-n) = \{3, -1, \underset{\uparrow}{2}, 1, -2\}$

Even part of discrete time signal $x(n)$=$x_e(n) = \frac{x(n)+x\,(-n)}{2}$

$x_e(n) = \frac{1}{2}\{-2+3, 1-1, \underset{\uparrow}{2+2}, -1+1, 3-2\}$= $\frac{1}{2}\{1, 0, 4, 0, 1\}$

= $\{0.5, 0, \underset{\uparrow}{2}, 0, 0.5\}$

Odd part of discrete time signal $x(n)$=$x_o(n) = \frac{x(n)-x\,(-n)}{2}$

$x_o(n) = \frac{1}{2}\{-2-3, 1+1, \underset{\uparrow}{2-2}, -1-1, 3+2\}$= $\frac{1}{2}\{-5, 2, \underset{\uparrow}{0}, -2, 5\}$

= $\{-2.5, 1, \underset{\uparrow}{0}, -1, 2.5\}$

Problems (cont.)

3. Q. $\boldsymbol{x(t) = sint + 2sint + 2sin^2(t)\ cost}$

Ans:- $x(-t) = \sin(-t) + 2\sin(-t) + 2sin^2(-t)\cos(-t)$

$= -sint - 2sint + 2sin^2(t)\ cost$

Even part of continuous time signal $x(t)$=$x_e(t) = \frac{x(t)+x\,(-t)}{2}$

$x_e(t) = \frac{x(t)+x\,(-t)}{2} = \frac{sint+2sint+2sin^2(t)\ cost-sint-2sint+2sin^2(t)\ cost}{2}$

$= \frac{4sin^2(t)\ cost}{2} = 2sin^2(t)\ cost$

Odd part of continuous time signal $x(t)$=$x_o(t) = \frac{x(t)-x\,(-t)}{2}$

$x_o(t) = \frac{x(t)-x\,(-t)}{2} = \frac{sint+2sint+2sin^2(t)\ cost+sint+2sint-2sin^2(t)\ cost}{2}$

$= \frac{2sint+4sint}{2} = sint + 2sint$

Problems (cont.)

4. Q. $\boldsymbol{x(n)} = \{\mathbf{1, 0, -1, 2, 3}\}$

Ans:- $x(-n) = y(n) = \{3, 2, -1, 0, 1\}$

Even part of discrete time signal $x(n)$=$x_e(n) = \frac{x(n)+x\,(-n)}{2}$

$x_e(n) = \frac{1}{2}\{3, 2, -1,0,1 + 1,0, -1,2,3\}$= $\frac{1}{2}\{3, 2, -1,0,2,0, -1,2,3\}$

=$\{1.5, 1, -0.5,0,1,0, -0.5,1,1.5\}$

Odd part of discrete time signal $x(n)$=$x_o(n) = \frac{x(n)-x\,(-n)}{2}$

$x_e(n) = \frac{1}{2}\{-3, -2,1,0,1 - 1,0, -1,2,3\}$= $\frac{1}{2}\{-3, -2,1,0,0,0, -1,2,3\}$

=$\{-1.5, -1,0.5,0,0,0, -0.5,1,1.5\}$

5. Energy and power signal

- Energy and average power of any signal is defined as:

	Continuous Time Signal $x(t)$	**Discrete Time Signal** $x(\boldsymbol{n})$
Energy E (Joules)	$\lim_{T\to\infty} \int_{-T}^{T} \lvert x(t)\rvert^2 \quad dt = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$\sum_{n=-\infty}^{\infty} \lvert x(n)\rvert^2$
Average Power P (watts)	$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \lvert x(t)\rvert^2 dt$	$\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x(n)\rvert^2$

RMS value of the signal $x(t) = \sqrt{P}$

5. Energy and power signal (cont.)

- A signal $x(t)$ is called an **energy signal** if the energy satisfies $0 < E < \infty$. For an energy signal $P = 0$.

- A signal $x(t)$ is called a **power signal** if the power satisfies $0 < P < \infty$. For a power signal $E = \infty$.

- If either of conditions are not satisfied, the signal is neither energy nor power signal.

Energy Signal	Energy is finite	Power is zero
Power Signal	Power is finite	Energy is infinite

Some Formulas of math

$$cos^2 A = \frac{1 + \cos 2A}{2} \qquad sin^2 A = \frac{1 - \cos 2A}{2}$$

$|e^{j\theta}|$=1

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}$$

Problems

Find which of the signals are energy signals, power signals, neither energy or nor power signals.

1. Q. $\boldsymbol{x(t) = e^{-3t}u(t)}$

Ans:-$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \quad dt = \lim_{T\to\infty} \int_{-T}^{T} |e^{-3t}u(t)\,|^2 \quad dt$

As $\begin{aligned} u(t) &= 1 \quad for\ t \geq 0 \\ &= 0 \quad for\ t < 0 \end{aligned}$ So, $E = \lim_{T\to\infty} \int_{0}^{T} |e^{-3t}\,|^2 \quad dt$

$E = \lim_{T\to\infty} \int_{0}^{T} e^{-6t} \quad dt = \lim_{T\to\infty} \left.\frac{e^{-6t}}{-6}\right|_{0}^{T} = \left.\frac{e^{-6t}}{-6}\right|_{t=0}^{\infty} = 0 - \frac{1}{-6} = \frac{1}{6}$ $\quad \color{red}{E = \frac{1}{6}}$

$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \quad dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |e^{-3t}u(t)\,|^2 \quad dt$

$P = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} |e^{-3t}\,|^2 \quad dt = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} e^{-6t} \quad dt = \lim_{T\to\infty} \frac{1}{2T} \left.\frac{e^{-6t}}{-6}\right|_{t=0}^{T}$

$= \lim_{T\to\infty} \frac{1}{2T} \frac{e^{-6T}-1}{-6} = 0$, P=0 $\qquad$ E= finite and P=0 So it is a energy signal.

Problems (Cont.)

2. Q. $\boldsymbol{x(t) = e^{j\left(2t+\frac{\pi}{4}\right)}}$

Ans:- $E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \quad dt = \lim_{T\to\infty} \int_{-T}^{T} \left|e^{j\left(2t+\frac{\pi}{4}\right)}\right|^2 \quad dt$

As, $\left|e^{j\theta}\right|$=1, So, $E = \lim_{T\to\infty} \int_{-T}^{T} 1 \quad dt = \lim_{T\to\infty} t|_{t=-T}^{T} = \lim_{T\to\infty} 2\text{T}$

E=∞

$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \quad dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left|e^{j\left(2t+\frac{\pi}{4}\right)}\right|^2 \quad dt$

$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1 \quad dt = \lim_{T\to\infty} \frac{2T}{2T} = 1$

As P finite and E infinite so it is a power signal

Problems (Cont.)

3. Q. $x(t) = cost$

Ans:- $E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \quad dt = \lim_{T\to\infty} \int_{-T}^{T} cos^2(t) \quad dt = \lim_{T\to\infty} \int_{-T}^{T} \frac{1}{2}(1 + cos2t) \quad dt = \infty$

$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \quad dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} cos^2(t) \quad dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1}{2}(1 + cos2t) \quad dt = \frac{1}{2}$

As P finite and E infinite so it is a power signal

Problems (Cont.)

4\. Q. $x(n) = \left(\frac{1}{3}\right)^n u(n)$

Ans:- E= $\sum_{n=-\infty}^{\infty} |x(n)|^2$ =$\sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{3}\right)^n u(n)\right|^2$

$u(n) = 1 \quad for\ n \geq 0$

$\quad\ \ = 0 \quad for\ n < 0$

So E=$\sum_{n=0}^{\infty} \left|\left(\frac{1}{3}\right)^n\right|^2$ =$\sum_{n=0}^{\infty} \frac{1}{9}^n$

$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$ So E=$\frac{1}{1-\frac{1}{9}}$=$\frac{9}{8}$

P= $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{\mathrm{N}} |x(n)|^2$= $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{\mathrm{N}} \left|\left(\frac{1}{3}\right)^n u(n)\right|^2$ =

$\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{\mathrm{N}} \left|\left(\frac{1}{3}\right)^n\right|^2$ = $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{\mathrm{N}} \frac{1}{9}^n$

$\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}$, So P= $\lim_{N\to\infty} \frac{1}{2N+1} \frac{1-\frac{1}{9}^{N+1}}{1-\frac{1}{9}}$=0 As E is finite and p is zero so energy signal

Problems (Cont.)

5.Q. $x(n) = e^{j\left(\frac{\pi}{2}n+\frac{\pi}{8}\right)}$

Ans:- $x(n) = e^{j\left(\frac{\pi}{2}n+\frac{\pi}{8}\right)}$

E= $\sum_{n=-\infty}^{\infty}|x(n)|^2$

$= \sum_{n=-\infty}^{\infty}\left|e^{j\left(\frac{\pi}{2}n+\frac{\pi}{8}\right)}\right|^2$

$= \sum_{n=-\infty}^{\infty} 1=\infty$

P= $\lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N}|x(n)|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N}\left|e^{j\left(\frac{\pi}{2}n+\frac{\pi}{8}\right)}\right|^2$

$=\lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=0}^{N}(2N+1) \; = 1$

Power is finite. So the signal is power signal.

Problems (Cont.)

6. Q. $x(n) = cos\left(\frac{\pi}{4}n\right)$

$$cos^2A = \frac{1 + \cos 2A}{2}$$

Ans:-E= $\sum_{n=-\infty}^{\infty}|x(n)|^2$

$= \sum_{n=-\infty}^{\infty}\left|cos\left(\frac{\pi}{4}n\right)\right|^2$

$=\sum_{n=-\infty}^{\infty}\frac{1+cos\left(\frac{\pi}{2}n\right)}{2}=\infty$

P= $\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{\mathrm{N}}|x(n)|^2$

$= \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{\mathrm{N}}\frac{1+cos\left(\frac{\pi}{2}n\right)}{2}$

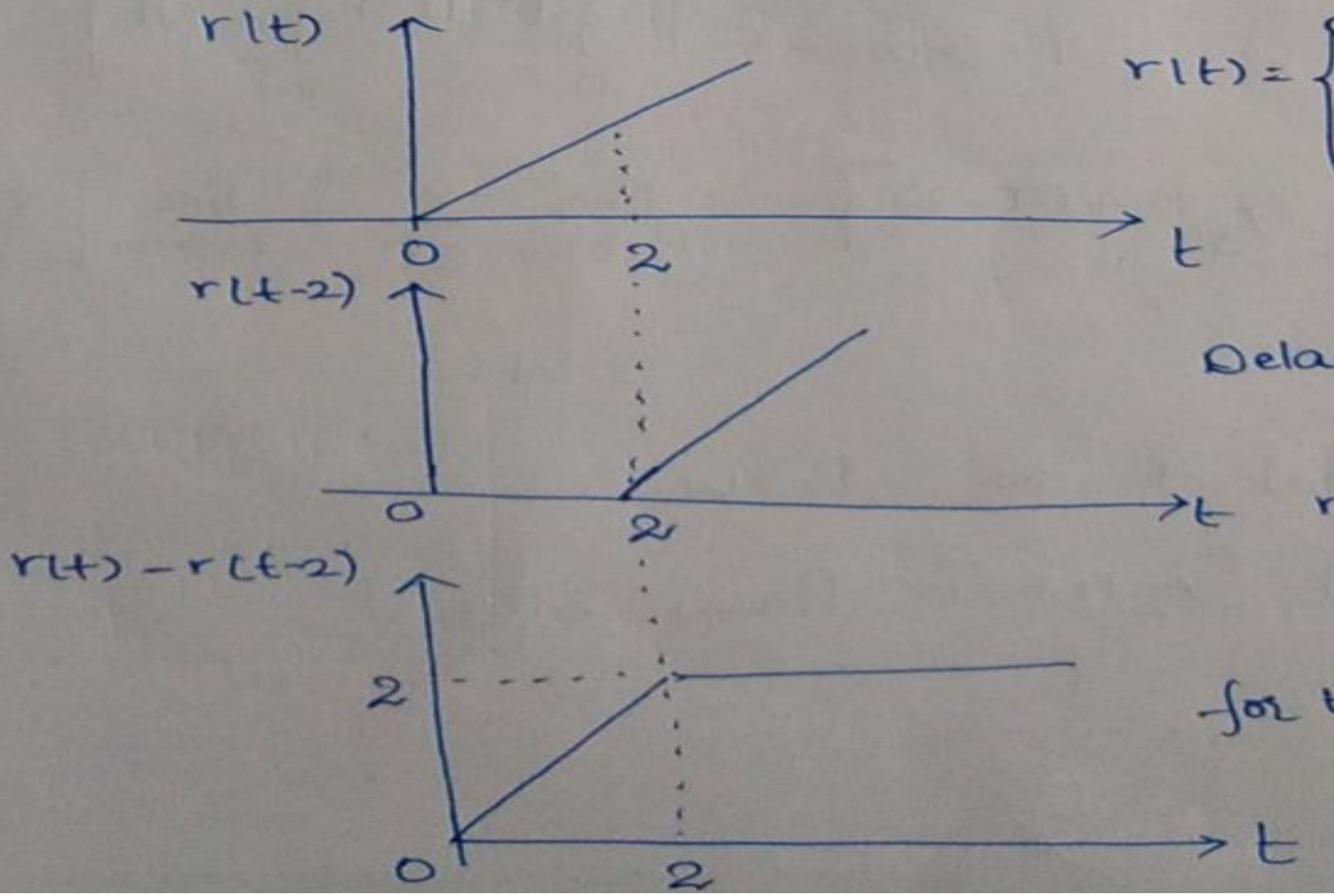
$= \frac{1}{2N+1}\frac{1}{2}(2N + 1) = \frac{1}{2}$

Power is finite. So the signal is power signal.

Problems (Cont.)

7. Q. $x(t) = r(t) - r(t-2)$

(ii) $x_2(t) = r(t) - r(t-2)$.



$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Delayed ramp by 2 units

$$r(t-2) = \begin{cases} t-2 & t-2 \geq 0 \\ & t \geq 2 \\ 0 & t < 2 \end{cases}$$

for $t \geq 2$, $r(t) - r(t-2)$

$= t - (t-2)$

$= 2$

$$\therefore\ x_2(t) = r(t) - r(t-2)$$

$$= \begin{cases} t & 0 \le t < 2 \\ 2 & t \geq 2 \end{cases}$$

Energy of $x_2(t)$ $\quad E = \int_{-\infty}^{\infty} |x_2(t)|^2 dt$

$$= \int_0^2 t^2 dt + \int_2^\infty 2^2 dt$$

$$= \left[\frac{t^3}{3}\right]_0^2 + 4[T]_2^\infty = 8/3 + 4(\infty - 2)$$

$$= \infty$$

Average power $P = \lim_{T \to \infty} \frac{1}{2T} \left[\int_0^2 t^2 dt + \int_2^T 4\, dt \right]$

$= \lim_{T \to \infty} \frac{1}{2T} \left[8/3 + 4(T-2) \right] = \lim_{T \to \infty} \frac{8}{6T} + \lim_{T \to \infty} \left[\frac{4T}{2T} - \frac{2}{2T} \right]$

$= 2W.$

For given signal $E = \infty$, $P = 2W$.

$\therefore$ The signal $x_2(t)$ is Power signal.

Problems (Cont.)

8. Q. Determine the power and RMS value of the signal

$x(t) = Acos(\Omega_0 t + \theta)$

Ans:- P= $\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$

$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |Acos(\Omega_0 t + \theta)|^2 dt$

$$cos^2 A = \frac{1 + \cos 2A}{2}$$

$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A^2 cos^2(\Omega_0 t + \theta) dt$

$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{A^2}{2}(1 + cos\ 2(\Omega_0 t + \theta)) dt$

$= \lim_{T\to\infty} \frac{A^2}{4T}\left[\int_{-T}^{T} dt + \int_{-T}^{T} cos\ 2(\Omega_0 t + \theta) dt\right]$

$= \lim_{T\to\infty} \frac{A^2}{4T}$(T-(-T))+0 $= \frac{A^2}{2}$

The RMS value $=\sqrt{p} = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$

Classification of systems

System: A system is defined as a physical device or software realization that performs an operation on a signal.

Classification of systems: The system may be classified as follows:

1. Continuous time and discrete time system
2. Static and dynamic system
3. Causal and non-causal system
4. Linear and non-linear system
5. Time invariant and time variant systems
6. Stable and unstable system.

1. Continuous time and discrete time system

<u>Continuous time system:</u> A continuous time system is one which operates on a continuous time signal and produces continuous time output signal.

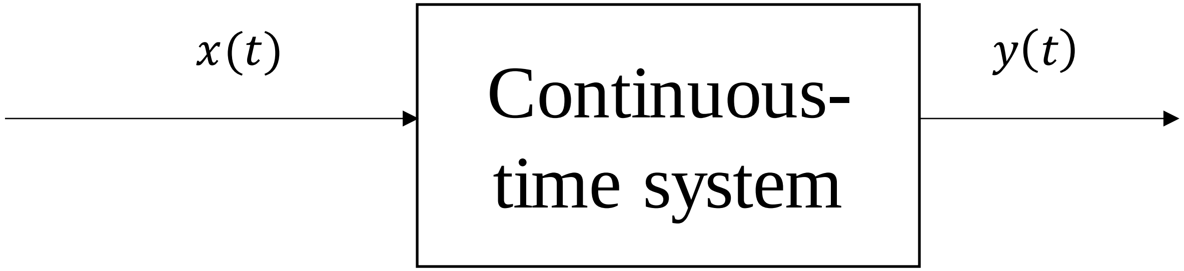


Fig. 1. Continuous time system

<u>Here</u> $x(t)$= input, $y(t) = output$ Eg. $y(t) = x(2t - 43)$=$T[x(t)]$

$y(t) = T[x(t)]$

T is transformation

1. Continuous time and discrete time system (Cont.)

<u>Discrete time system:</u> A discrete time system is one which operates on a discrete time signal and produces discrete time output signal.

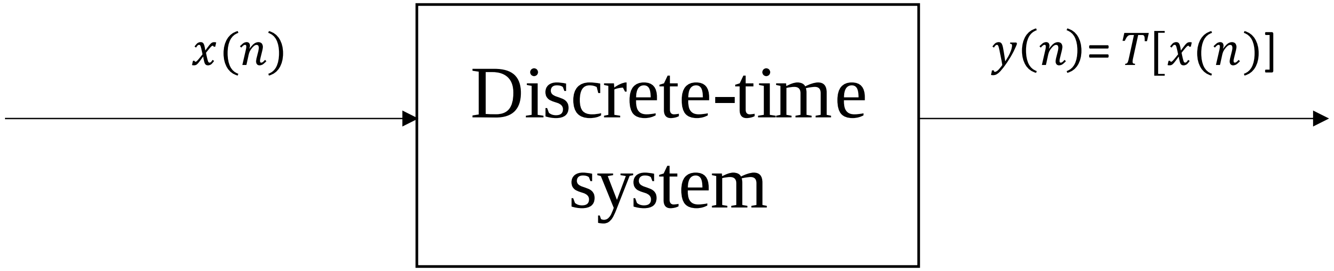


Fig. 1. Discrete time system

<u>Here</u> $x(n)$= input, $y(n) = output$ E.g. $y(n) = 2x(-n)$

$y(n) = T[x(n)]$

T is transformation

2. Static and dynamic system

- A system is called static or memoryless if its output at any instant depends on the input at that instant only but not on past or future value of input.
- Otherwise system is said to be dynamic or with memory.
- E.g. $y(t) = x^2(t)$ ----------------Static
- $y(n) = nx(n)$ -------------------static
- $y(t) = \frac{dx}{dt}$ -------------------------As it is differentiation depend up on past so dynamic
- $y(n) = x(n-1)$ ----- $y(0) = x(-1)$ ----- As depend upon past so dynamic.

Problems:

Find the following system are static or dynamic

1 Q. $\boldsymbol{y(t) = x(t-3)}$

Ans:- at $t = 0$

$y(0) = x(-3)$

As it depend upon past so it is dynamic

2. Q. $\boldsymbol{y(n) = x(-n)}$

Ans:- at $n = -1$

$y(-1) = x(1).$

As it depend upon future so it is dynamic

Problems (cont.)

3. Q. $\boldsymbol{y(t) = x^3(t)}$

Ans:-at $t = 1$
$y(1) = x(1)$

As it depend upon past so it is Static

4. Q. $\boldsymbol{y(t) = \frac{d^2y}{dt^2}}$

Ans:- As differentiation means depend upon past

So the system is dynamic

3. Causal and non-causal system

- A system is said to be causal if its output depends upon present and past inputs only, and does not depend upon future input.
- For non causal system, the output depends upon future inputs also

Example for Continuous time causal system.

- $y(t) = 2\ x(t) + 3\ x(t-3)$ For present value $t=1$, the system output is $y(1) = 2x(1) + 3x(-2)$. Here, the system output only depends upon present and past inputs. Hence, the system is causal.

Similarly, Eg for Discrete time causal system

- $y(n)=nx(n)+x(n-3)$ where the system depends only on the present and past inputs

3. Causal and non-causal system (Cont.)

- Example for Non-causal CT system.

  $y(t)=x(t+3)+x^2(t)$

  And a DT Non causal system is given by

  $y(n)=x(2n)$

Problems

Check the following systems are causal or not.

1. **Q.** $\boldsymbol{y(n) = x(n) + \frac{1}{x(n-1)}}$

Ans:- For $n = 0$, $y(0) = x(0) + \frac{1}{x(-1)}$

For $n = 1$, $y(1) = x(1) + \frac{1}{x(0)}$

For $n = -1$, $y(-1) = x(-1) + \frac{1}{x(-2)}$

For all value of n, output depends upon present and past not future. So Causal system

Problems

2. Q. $\boldsymbol{y(t) = x^2(t) + x(t-2)}$

Ans:- For $t = 0$, $y(0) = x^2(0) + x(-2)$

For $t = 1$, $y(1) = x^2(1) + x(-1)$

For $t = -1$, $y(-1) = x^2(-1) + x(-3)$

- For all value of t, output depends upon present and past not future. So Causal system

3. **Q.** $\boldsymbol{y(t) = x(t-2) + x(2-t)}$

For $t = 0$ $y(0) = x(-2) + x(2)$

The output depends upon future value so system is non-causal

Problems

4. Q. $\boldsymbol{y(t) = \int_{-\infty}^{2t} x(\tau) \quad d\tau}$

Ans:- $y(t)$ =Z(τ) $|_{\tau=-\infty}^{2t}$

$y(t)$ =Z(2t)-Z($-\infty$)

T=1 $y(1)$ =Z(2)-Z($-\infty$)

So non-causal

5.Q $\boldsymbol{y(n) = x(-n)}$

n=-1 $y(-1) = x(1)$

So non-causal

Problems (cont.)

6.Q $\boldsymbol{y(n) = x(n^2)}$

n=-1 $y(-1) = x(1)$

So non-causal

4. Linear and non-linear system

- A system is called linear if it satisfies the superposition theorem
- Otherwise system is called non-linear.

<u>Superposition Theorem</u> Superposition theorem states that response of a linear system to a sum of signal is the sum of the response to each individual input signal.

- If an arbitrary input $x_1(t)$ produces output $y_1(t)$, and an arbitrary input $x_2(t)$ produces output $y_2(t)$.
- Then, Superposition theorem states that

$T[a_1x_1(t) + a_2x_2(t)]= a_1T[x_1(t)] + a_2T[x_2(t)]$ (for continuous time) $= a_1y_1(t) + a_2y_2(t)$

$T[a_1x_1(n) + a_2x_2(n)]= a_1T[x_1(n)] + a_2T[x_2(n)]$ (for discrete time)

- $= a_1y_1(n) + a_2y_2(n)$

Problems

Check whether the following system are linear or not.

1 Q. $\boldsymbol{y(n) = Ax(n) + B}$

Ans:- $y(n) = T[x(n)] = Ax(n) + B$

$T[x_1(n)] = Ax_1(n) + B$

$T[x_2(n)] = Ax_2(n) + B$

$a_1T[x_1(n)]= a_1(Ax_1(n) + B)$

$a_2T[x_2(n)]= a_2(Ax_2(n) + B)$

<u>RHS</u>

$a_1T[x_1(n)]+ a_2T[x_2(n)]=a_1(Ax_1(n) + B)+ a_2(Ax_2(n) + B)$

<u>LHS</u>

$T[a_1x_1(n) + a_2x_2(n)]=\mathrm{A}([a_1x_1(n) + a_2x_2(n)]) + \mathrm{B}$

As LHS≠RHS so system is non-linear

Problems

2. Q. $\boldsymbol{y(n) = 2x(n) + \frac{1}{x(n-1)}}$

$T[x(n)] = 2x(n) + \frac{1}{x(n-1)}$ $T[x_1(n)] = 2x_1(n) + \frac{1}{x_1(n-1)}$

$a_1T[x_1(n)]= a_1(2x_1(n) + \frac{1}{x_1(n-1)})$

$a_2T[x_2(n)]= a_2(2x_2(n) + \frac{1}{x_2(n-1)})$

<u>RHS</u>

$a_1T[x_1(n)]+ a_2T[x_2(n)]=a_1(2x_1(n) + \frac{1}{x_1(n-1)})+a_2(2x_2(n) + \frac{1}{x_2(n-1)})$

<u>LHS</u>

$T[a_1x_1(n) + a_2x_2(n)]= 2(a_1x_1(n) + a_2x_2(n)) + \frac{1}{a_1x_1(n-1)+a_2x_2(n-1))}$

As LHS$\neq$ RHS, system is non-linear

Problems (cont.)

3. Q. $\boldsymbol{y(n) = nx(n)}$

Ans:- $T[x(n)] = nx(n)$ $T[x_1(n)] = nx_1(n)$

$a_1T[x_1(n)] = a_1(nx_1(n))$

$a_2T[x_2(n)] = a_2(nx_2(n))$

<u>RHS</u>

$a_1T[x_1(n)] + a_2T[x_2(n)] = a_1(nx_1(n)) + a_2(nx_2(n))$

<u>LHS</u>

$T[a_1x_1(n) + a_2x_2(n)] = n(a_1x_1(n) + a_2x_2(n))$

As LHS= RHS, system is linear

Problems (cont.)

4. Q. $\boldsymbol{y(t) = x^2(t)}$

Ans:- $T[x(t)] = x^2(t) \quad T[x_1(t)] = x_1{}^2(t)$

$a_1 T[x_1(t)] = a_1\, x_1{}^2(t)$

$a_2 T[x_2(t)] = a_2\, x_2{}^2(t)$

RHS

$a_1 T[x_1(t)] + a_2 T[x_2(t)] = a_1\, x_1{}^2(t) + a_2\, x_2{}^2(t)$

LHS

$T[a_1 x_1(t) + a_2 x_2(t)] = (a_1 x_1(t) + a_2 x_2(t))^2$

As LHS$\neq$ RHS, system is non-linear

Problems (cont.)

5. Q. $\boldsymbol{y(t) = e^{x(t)}}$

Ans:- $T[x(t)] = e^{x(t)}$ $T[x_1(t)] = e^{x_1(t)}$

$a_1 T[x_1(t)] = a_1\ e^{x_1(t)}$

$a_2 T[x_2(t)] = a_2\ e^{x_2(t)}$

<u>RHS</u>

$a_1 T[x_1(t)] + a_2 T[x_2(t)] = a_1\ e^{x_1(t)} + a_2\ e^{x_2(t)}$

<u>LHS</u>

$T[a_1 x_1(t) + a_2 x_2(t)] = e^{a_1 x_1(t) + a_2 x_2(t)}$

As LHS$\neq$ RHS, system is non-linear

Problems (cont.)

6. Q. $\boldsymbol{y(t) = t^2 x(t)}$

$$Ans{:}- \quad T[x(t)] = t^2 x(t) \qquad\qquad T[x_1(t)] = t^2 x_1(t)$$

$a_1 T[x_1(t)]= a_1(t^2 x_1(t))$

$a_2 T[x_2(t)]= a_2(t^2 x_2(t))$

<u>RHS</u>

$a_1 T[x_1(t)]+ a_2 T[x_2(t)]=a_1(t^2 x_1(t))+a_2(t^2 x_2(t))$

<u>LHS</u>

$T[a_1 x_1(t) + a_2 x_2(t)]=t^2(a_1 x_1(t) + a_2 x_2(t))$

As LHS= RHS, system is linear

Problems (cont.)

7.Q. $\boldsymbol{\frac{dy(t)}{dt}+3ty(t)=t^2x(t)}$

Ans:- Let $y_1(t)$ is output for input $x_1(t)$, $y_2(t)$ is output for input $x_2(t)$

$\frac{dy_1(t)}{dt}+3ty_1(t)=t^2x_1(t)$ ------------------------(1)

$\frac{dy_2(t)}{dt}+3ty_2(t)=t^2x_2(t)$ ------------------------(2)

Multiply equation (1) by a_1 and equation (2) by a_2 and adding both equations.

$a_1\frac{dy_1(t)}{dt}+3a_1ty_1(t)+a_2\frac{dy_2(t)}{dt}+3a_2ty_2(t)=a_1t^2x_1(t)+a_2t^2x_2(t)$

$\frac{d(a_1y_1(t)+a_2y_2(t))}{dt}+3t\,(a_1y_1(t)+a_2y_2(t))=t^2(a_1x_1(t)+a_2\,x_2(t))$ --(3)

Equation---(3) shows weighted sum of inputs produces weighted sum of output so system is linear

Problems (cont.)

8.Q. $\boldsymbol{\frac{dy(t)}{dt}+2y(t)=x^2(t)}$

Ans:- Let $y_1(t)$ is output for input $x_1(t)$, $y_2(t)$ is output for input $x_2(t)$

$\frac{dy_1(t)}{dt}+2y_1(t)=x_1{}^2\,(t)$ ------------------------(1)

$\frac{dy_2(t)}{dt}+2y_2(t)=x_2{}^2\,(t)$ ------------------------(2)

Multiply equation (1) by a_1 and equation (2) by a_2 and adding both equations.

$a_1\,\frac{dy_1(t)}{dt}+2a_1y_1(t)+a_2\,\frac{dy_2(t)}{dt}+2a_2y_2(t)=a_1x_1{}^2\,(t)+a_2x_2{}^2\,(t)$

$=\frac{d(a_1y_1(t)+a_2y_2(t))}{dt}+2\,(a_1y_1(t)+a_2y_2(t))=a_1x_1{}^2\,(t)+a_2x_2{}^2\,(t)$ ---------(3)

Equation---(3) shows input is sum nonlinear function to produce output so, system is non-linear

Problems (cont.)

9. Q. $\boldsymbol{\frac{dy(t)}{dt} + 2y(t) = x(t)\frac{dx(t)}{dt}}$

Ans:- Let $y_1(t)$ is output for input $x_1(t)$, $y_2(t)$ is output for input $x_2(t)$

$$\frac{dy_1(t)}{dt}+2y_1(t) = x_1(t)\,\frac{dx_1(t)}{dt} \text{ -----------------------(1)}$$

$$\frac{dy_2(t)}{dt}+2y_2(t) = x_2(t)\,\frac{dx_2(t)}{dt} \text{-----------------------(2)}$$

Multiply equation (1) by a_1 and equation (2) by a_2 and adding both equations.

$$a_1\,\frac{dy_1(t)}{dt}+2a_1y_1(t)+a_2\,\frac{dy_2(t)}{dt}+2a_2y_2(t) = a_1x_1(t)\,\frac{dx_1(t)}{dt}+a_2x_2(t)\,\frac{dx_2(t)}{dt}$$

$$= \frac{d(a_1y_1(t)+a_2y_2(t))}{dt}+2(a_1y_1(t)+a_2y_2(t)) = a_1x_1(t)\,\frac{dx_1(t)}{dt}+a_2x_2(t)\,\frac{dx_2(t)}{dt} \text{ -(3)}$$

Equation---(3) input is sum nonlinear function to produce output so, system is non-linear

Problems (cont.)

10.Q. $\boldsymbol{y(t) = \int_{-\infty}^{t} x(\tau) \quad d\tau}$

Ans:- Let $y_1(t)$ is output for input $x_1(t)$, $y_2(t)$ is output for input $x_2(t)$

$y_1(t) = \int_{-\infty}^{t} x_1(\tau) \quad d\tau$ -----------------------------------(1)

$y_2(t) = \int_{-\infty}^{t} x_2(\tau) \quad d\tau$ -----------------------------------(2)

Multiply equation (1) by a_1 and equation (2) by a_2 and adding both equations.

$a_1 y_1(t) + a_2 y_2(t) = a_1 \int_{-\infty}^{t} x_1(\tau) \quad d\tau + a_2 \int_{-\infty}^{t} x_2(\tau) \quad d\tau$ --------(3)

Equation---(3) shows weighted sum of inputs produces weighted sum of output so system is linear

5. Time invariant and time variant systems

- A system is said to be time or shift invariant if its input output characteristics do not change with respect to time.
- A system is said to be time or shift variant if its input output characteristic changes with time.
- If, $x(t)$ is input, $y(t)$ is output of a continuous-time system.
- $y(t) = T[x(t)]$
- The system is time invariant if

$y(t - T_1) = T[x(t - T_1)]$

$y(t - T_1)$= $y(t, T_1)$
$Where\ y(t - T_1)\ is\ in\ y(t), put\ \ t = t - T_1$

$y(t, T_1) = T[x(t - T_1)]$, here in place of $x(t)$ put $x(t - T_1)$

5. Time invariant and time variant systems (cont.)

If, $x(n)$ is input, $y(n)$ is output of a continuous-time system.

$y(n) = T[x(n)]$

- The system is time invariant if

$y(n-k) = T[x(n-k)]$

$y(n-k)= y(n,k)$

$here\ y(n-k)$ is $in\ y(n), put\ \ n = n-k$

$y(n,k)$=$T[x(n-k)]$, here in place of $x(n)$ put $x(n-k)$

5. LTI (linear time invariant system)(cont.)

<u>LTI (linear time invariant system)</u>

- For a LTI system the coefficients of differential equation describing the system are constant.
- If constants are function of time then the system is a linear time variant system.

E.g.1. $2\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+5y(t)=5x(t)$

Here all coefficients are constant, so system is LTI.

E.g.1. $\frac{d^2y(t)}{dt^2}+4t\frac{dy(t)}{dt}+5y(t)=5x(t)$

Here all coefficients is function of time, so system is time variant system.

Problems

For each of the following systems, determine whether or not the system is time-invariant.

1.Q. $\boldsymbol{y(t) = tx(t)}$

Ans:-$T[x(t)] = tx(t),\ T[x(t-T_1)] = tx(t-T_1)$
$y(t-T_1) = (t-T_1)x(t-T_1)$

As, $y(t-T_1) \neq T[x(t-T_1)]$, so system is time-variant

2.Q. $\boldsymbol{y(t) = x(t)cos50\pi t}$

Ans:- $T[x(t)] = x(t)cos50\pi t,\ T[x(t-T_1)] = x(t-T_1)cos50\pi t$

$y(t-T_1) = x(t-T_1)cos50\pi(t\text{-}T_1)$

As, $y(t-T_1) \neq T[x(t-T_1)]$, so system is time-variant

Problems (Cont.)

3.Q. $\boldsymbol{y(t) = x(t^2)}$

Ans:-$T[x(t)] = x(t^2)$,

$T[x(t - T_1)] = x(t^2 - T_1)$

$y(t - T_1) = x((t - T_1)^2)$

As, $y(t - T_1) \neq T[x(t - T_1)]$, so system is time-variant

4.Q. $\boldsymbol{y(t) = x(-t)}$

Ans:-:-$T[x(t)] = x(-t)$,

$T[x(t - T_1)] = x(-t - T_1)$

$y(t - T_1) = x(-(t - T_1))$

As, $y(t - T_1) \neq T[x(t - T_1)]$, so system is time-variant

Problems (Cont.)

5. Q. $\boldsymbol{y(t) = e^{x(t)}}$

Ans:- $T[x(t)] = e^{x(t)}$,

$T[x(t - T_1)] = e^{x(t-T_1)}$

$y(t - T_1) = e^{x(t-T_1)}$

As, $y(t - T_1) = T[x(t - T_1)]$, so system is time-invariant

6.Q. $\boldsymbol{y(n) = x(2n)}$

Ans:- $T[x(n)] = x(2n)$

$T[x(n - k)] = x(2n - k)$

$y(n - k) = x\big(2(n - k)\big)$

As, $y(n - k) \neq T[x(n - k)]$ so system is time-variant

Problems (Cont.)

7.Q. $\boldsymbol{y(n) = x(n) + nx(n-1)}$

Ans:- $T[x(n)] = x(n) + nx(n-1)$
$T[x(n-k)] = x(n-k) + nx(n-k-1)$
$y(n-k) = x(n-k) + (n-k)x(n-k-1)$

As, $y(n-k) \neq T[x(n-k)]$ so system is time-variant

8.Q. $\boldsymbol{y(n) = x^2(n-1)}$

Ans:- $T[x(n)] = x^2(n-1)$
$T[x(n-k)] = x^2(n-k-1)$
$y(n-k) = x^2(n-k-1)$

As, $y(n-k) = T[x(n-k)]$ so system is time-invariant

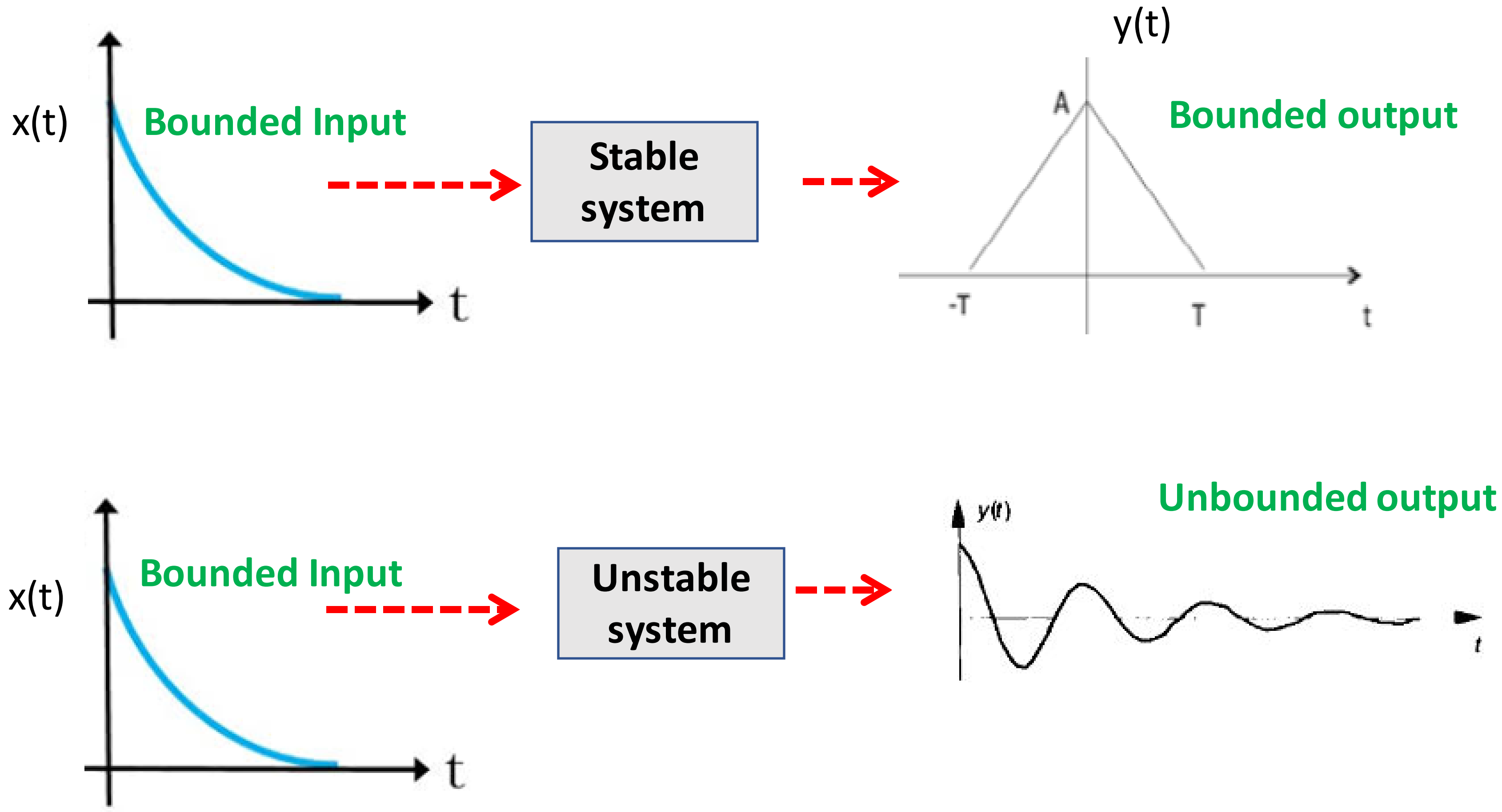
Stable versus unstable system

- An arbitrary relaxed system is said to be bounded input and bounded output (BIBO) stable if and only if bounded input produces a bounded output otherwise unstable.

- An signal $x(t)$ is said to be bounded if it satisfies the condition $|x(t)| \leq M_x < \infty$ for all t.

- Similarly, the output signal is bounded if it satisfies the condition $|y(t)| \leq M_y < \infty$ for all t.

Typical stable and unstable systems



Condition for stability for a system having transfer function

- For continuous time system $h(t)$ is stable

If $\int_{-\infty}^{\infty} |h(t)| \quad dt < \infty$

- For discrete time system $h(n)$ is stable

If $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

Problems

Find whether the system are stable or not?

1.Q. $\boldsymbol{h(n) = 2^n u(-n)}$

Ans:- $\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |2^n u(-n)|$

$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$ for a<1

$= \sum_{n=-\infty}^{0} |2^n| = \sum_{n=0}^{\infty} |2^{-n}| = \frac{1}{1-\frac{1}{2}} = 2$

$$u(n) = 1 \quad for\, n \geq 0$$
$$= 0 \quad for\, n < 0$$

As $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$ so system is stable.

2.Q. $\boldsymbol{h(n) = 5^n u(3-n)}$

Ans:- $\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |5^n u(3-n)|$

$= \sum_{n=-\infty}^{3} |5^n| = \sum_{n=-\infty}^{0} |5^n| + \sum_{n=1}^{3} |5^n| = \sum_{n=0}^{\infty} |5^{-n}| + \sum_{n=1}^{3} |5^n|$

$= \frac{1}{1-\frac{1}{5}} + 155 < \infty$ so system is stable.

Problems (Cont.)

$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$ for a<1

3.Q. $\boldsymbol{h(n) = e^{2n}u(n-1)}$

Ans:- $\sum_{n=-\infty}^{\infty}|h(n)| = \sum_{n=-\infty}^{\infty}|e^{2n}u(n-1)|$

$= \sum_{n=1}^{\infty}|e^{2n}|$

Here a is e^2 more than 1. so summation will be infinite

So, the system is unstable.

$u(n) = 1 \quad for\ n \geq 0$

$ = 0 \quad for\ n < 0$

4.Q. $\boldsymbol{h(n) = e^{-6|n|}}$

Ans:- $\sum_{n=-\infty}^{\infty}|h(n)| = \sum_{n=-\infty}^{\infty}\left|e^{-6|n|}\right|$

$= \sum_{n=-\infty}^{0} e^{6n} + \sum_{n=1}^{\infty} e^{-6n} = \sum_{n=0}^{\infty} e^{-6n} + \sum_{n=1}^{\infty} e^{-6n}$

$= \frac{1}{1-e^{-6}} + \frac{1}{1-e^{-6}} - 1 = \frac{1+e^{-6}}{1-e^{-6}}$

$\sum_{n=-\infty}^{\infty}|h(n)| < \infty$ so system is stable.

Problems (Cont.)

5.Q. $\boldsymbol{h(t) = e^{-2t}u(t-1)}$

Ans:- $\int_{-\infty}^{\infty} |h(t)| \quad dt = \int_{-\infty}^{\infty} e^{-2t}u(t-1) \quad dt$

$=\int_{1}^{\infty} e^{-2t} \quad dt = \frac{e^{-2t}}{-2} |_{t=1}^{\infty} = \frac{e^{-2}}{2}$

As, $\int_{-\infty}^{\infty} |h(t)| \quad dt < \infty$ so system is stable.

$u(t) = 1 \quad for\ t \geq 0$
$= 0 \quad for\ t < 0$

6.Q. $\boldsymbol{h(t) = te^{-t}u(t)}$

Ans:- $\int_{-\infty}^{\infty} |h(t)| \quad dt = \int_{-\infty}^{\infty} te^{-t}u(t) \quad dt$

$= t\int_{0}^{\infty} e^{-t} \ dt - [\int_{0}^{\infty}(\frac{dt}{dt}\int_{0}^{\infty} e^{-t} \ dt)\mathrm{dt}\ ]=\text{-t}\ e^{-t} |_{t=0}^{\infty}\text{-}\ e^{-t} |_{t=1}^{\infty}$

$=1$, As, $\int_{-\infty}^{\infty} |h(t)| \quad dt < \infty$ so system is stable.

$\lim_{t \to \infty} \mathrm{t}\ e^{-t} = 0$

$(According\ to\ L\ hospital\ rule$

Problems (Cont.)

$$u(t) = 1 \quad for\ t \geq 0$$
$$= 0 \quad for\ t < 0$$

7.Q. $\boldsymbol{h(t) = e^{-2|t|}}$

Ans:- $\int_{-\infty}^{\infty} |h(t)| \quad dt = \int_{-\infty}^{\infty} e^{-2|t|} \quad dt$

$= \int_{-\infty}^{0} e^{2t} \quad dt + \int_{0}^{\infty} e^{-2t} \quad dt = \frac{e^{2t}}{2} \Big|_{t=-\infty}^{0} + \frac{e^{-2t}}{-2} \Big|_{t=0}^{\infty} = \frac{1}{2} + \frac{1}{2} = 1$

As, $\int_{-\infty}^{\infty} |h(t)| \quad dt < \infty$ so system is stable.

8.Q. $\boldsymbol{h(t) = e^{2t} u(t-1)}$

Ans:- $\int_{-\infty}^{\infty} |h(t)| \quad dt = \int_{-\infty}^{\infty} e^{2t} u(t-1) \quad dt = \int_{1}^{\infty} e^{2t} \quad dt$

$= \frac{e^{2t}}{2} \Big|_{t=1}^{\infty} = \infty$

so system is unstable.

Example 1

Check whether stable or not : $y(t)=tx(t)$

- Here, for a finite input, we cannot expect a finite output.
- For example, if we will put $x(t)=2 \Rightarrow y(t)=2t$
- This is not a finite value because we do not know the value of t.
- So, it can be ranged from anywhere.
- **Therefore, this system is not stable. It is an unstable system**

Example 2

Check whether stable or not : $y(t)=x(t)/\sin t$

- The sine function has a definite range from -1 to +1
- But here, it is present in the denominator.
- So, in worst case scenario, if we put t = 0 and sine function becomes zero.
- Then the whole system will tend to infinity. Therefore, this type of system is not at all stable. Obviously, this is an unstable system.

Example 3

Check whether $y(t)=\sin t.x(t)$ is stable or not.

- Suppose, we have taken the value of $x(t)$ as 3.

- Here, sine function has been multiplied with it and maximum and minimum value of sine function varies between -1 to +1.

- Therefore, the maximum and minimum value of the whole function will also vary between -3 and +3.

- **Thus, the system is stable because here we are getting a bounded input for a bounded output.**

Example 4

- ***Check whether stable or not:*** **$y(t)=x(t)+10$**
- Here, for a definite bounded input, we can get definite bounded output. i.e. if we put $x(t)=2, y(t)=12$ which is bounded in nature.
- **Therefore, the system is stable.**

Example 5

- ***Check whether stable or not:***

$y(t) = A\,x(t)$

Reason: let us assume $x(t) = u(t)$, then at every instant $u(t)$ will keep on multiplying with A and hence it will not be bonded.

Obviously, this is an unstable system