DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB101J-Electromagnetic Theory, Quantum Mechanics, Waves and Optics

Module I

Basic definition of Del operator, Basic derivation and Physical significance of Div, Curl, and Grad operation and Geometrical interpretation-Gauss-divergence and Stoke's theorem, Statement and Physical Significance

Del Operator (∇):

☐ The del operator is defined through the partial derivatives with respect to space variables. In Cartesian coordinates, the del operator is written as,
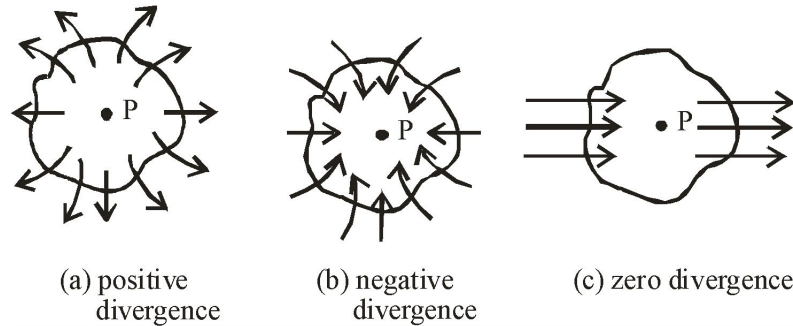
$$\nabla = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z}$$

☐ It is a vector operator and it may be applied on scalars, vectors or tensors. The del operator is important since it provides a number of indications as to how a vector or scalar functions vary with position. It shows up in the gradient, curl, divergence and Laplacian.

Divergence

The divergence of a vector **V** written as div **V** represents the scalar quantity.

$$\text{div } \mathbf{V} = \nabla \cdot \mathbf{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

(a) positive divergence   (b) negative divergence   (c) zero divergence

Physically the divergence of a vector quantity represents the rate of change of the field strength in the direction of the field.

If the divergence of the vector field is positive at a point then something is diverging from a small volume surrounding that point and that point is acting as a source

If it negative, then something is converging into the small volume surrounding that point is acting as sink.

The divergence at a point is zero then the rate at which something entering a small volume surrounding that point is equal to the rate at which it is leaving that volume. The vector field whose divergence is zero is called *solenoidal.*
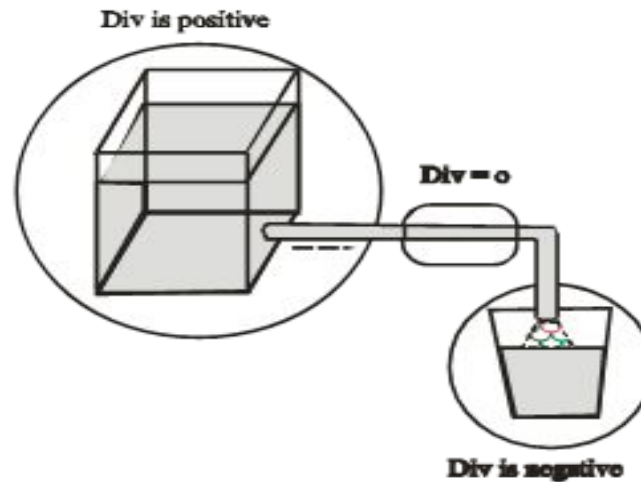

Fig. Examples for Divergence

Curl

The *curl* of **V** is written as curl **V** represents a vector quantity.

$$\text{curl } \mathbf{V} = \quad \nabla \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

- Physically, the curl of a vector field represents the rate of change of the field strength in a direction at right angles to the field and is a measure of rotation of something in a small volume surrounding a particular point.
- For streamline motions and conservative fields, the curl is zero while it is maximum near the whirlpools.
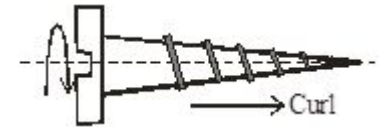
For vector fields whose curl is zero there is no rotation of the paddle wheel when it is placed in the field. Such fields are called *irrotational.*
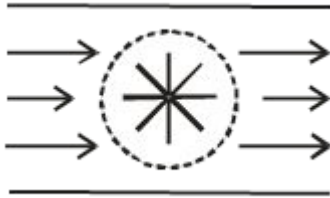
i. No rotation of the paddle wheel means the curl of the field is zero.
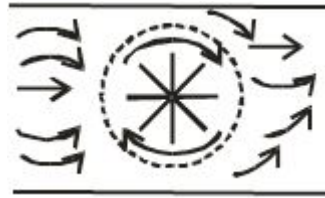
ii. Rotation of the paddle wheel means where the curl of the field exists.

(iii) direction of curl

Fig. Example for curl

The Gradient

The gradient of a scalar function φ is a vector whose cartesian components are

$$\mathrm{grad}\,\phi = \nabla\phi = \vec{i}\,\frac{\partial\phi}{\partial x} + \vec{j}\,\frac{\partial\phi}{\partial y} + \vec{k}\,\frac{\partial\phi}{\partial z}$$

For example, the electric field intensity at any point is given by,

$$\boldsymbol{E} = -\,\mathbf{grad}\,V = \text{negative gradient of potential}$$

The negative sign implies that the direction of $\boldsymbol{E}$ opposite to the direction in which V increases.

Statement: It relates the volume integral of the divergence of a vector V to the surface integral of the vector itself. According to this theorem, if a closed S bounds a volume τ, then

$$\int_{\tau} (\text{div } V)\, d\tau = \oint_{S} V \bullet ds \quad (\text{or}) \quad \int_{\tau} (\nabla \bullet V) d\tau = \int_{S} V \bullet ds$$

Statement: It relates the surface integral of the curl of a vector to the line integral of the vector itself. According to this theorem, if a closed path C bounds a surface S,

$$\int_S (\text{curl } V) \bullet ds = \oint_C V \bullet dl$$