DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB101J-Electromagnetic Theory, Quantum Mechanics, Waves and Optics

Module I

Electric Field and Electrostatic Potential, Volume, Surface Charge and Line Charge, Point Charge, Gauss Law and Applications

Electric field:

Electric charges affect the space around them. The space around the charge within which its effect is felt or experienced is called electric field.

Electric field Intensity (or) Strength of the electric field(E)

The electrostatic field intensity due to a point charge q_a at a given point is defined as the force per unit charge exerted on a test charge q_b placed at that point in the field.

$$\vec{E}_a = \frac{\vec{F}_{ba}}{q_b} = \frac{q_a \;\hat{r}_a}{4\pi\varepsilon_0 r^2} \quad \text{volt m}^{-1} \text{ (or) N C}^{-1}$$

Coulomb's Inverse Square Law

Coulomb's inverse square law gives the force between the two charges. According to this law, the force *(F)* between two electrostatic point charges (q_1 and q_2) is proportional to the product of the charges and inversely proportional to the square of the distance *(r)* separating the charges.

i.e. $F \propto q_1 q_2$

$$F \propto \frac{1}{r^2}$$

(or) $$F = K \frac{q_1 q_2}{r^2}$$

$$K = \frac{1}{4\pi\varepsilon_0\varepsilon_r} = \frac{1}{4\pi\varepsilon}$$

where ε = permittivity of the medium.

ε_0 = permittivity of free space $= 8.854 \times 10^{-12}$ F m^{-1}

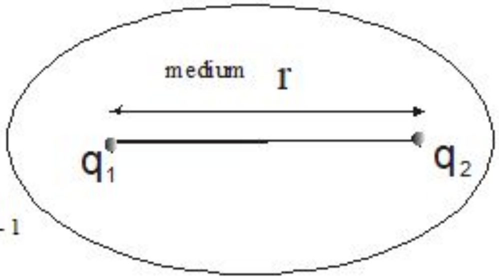

Fig. 2.1 Coulomb Inverse Square Law

For air medium, $\varepsilon_r = 1$

In the scalar form, the force between the electric charges is given by,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

where $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ Nm^2C^{-2}$

Electrostatic Potential (V)

Just as the heat flows from a higher temperature to lower temperature, water flows from higher level to lower level and air flows from higher pressure to lower pressure, electric charge flows from a body where electrical level is more to a body where it is less. This electrical level is called electric potential.

The electric potential is defined as the amount of work done in moving unit positive charge from infinity to the given point of the field of the given charge against the electrical force.

Unit: volt (or) joule / coulomb

The electric potential at any point is equal to the work done in moving the unit positive charge from infinity to that point.

$$\text{Therefore Potential} = V = -\int_{\infty}^{r} E \, . \, dx = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_0 x^2} \, dx$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{q}{4\pi\varepsilon_0 r}$$

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$$\int \frac{1}{x^2}\,dx.$$

Now, we can rewrite this because

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx$$

and proceed to use the power rule of integration, i.e.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,$$

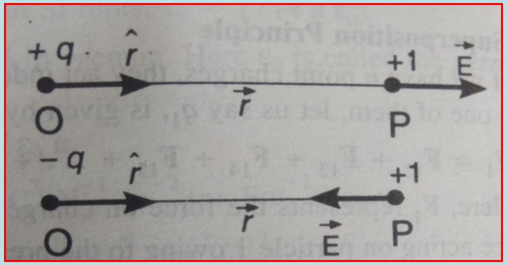
Therefore,

$$\int x^{-2}\,dx = \frac{x^{-1}}{-1} + C$$

and this is obviously equal to $-\frac{1}{x} + C$.

Electric Field due to Point Charge

Let P be a point lying in vacuum at a distance r form a point charge q lying at O. Let a test charge q_0 be placed at P. According to Coulomb's Law, the force acting on q_0 due to q is



$F = qq_0/4\pi\varepsilon_0 r^2$

The EF at a point P is, by definition, given by the force per unit test charge.

$E = F/q_0$

$E = q/4\pi\varepsilon_0\, r^2$

The direction of E is along the line joining O and P, pointing outward if q is positive and inward if q is negative.

Charge density

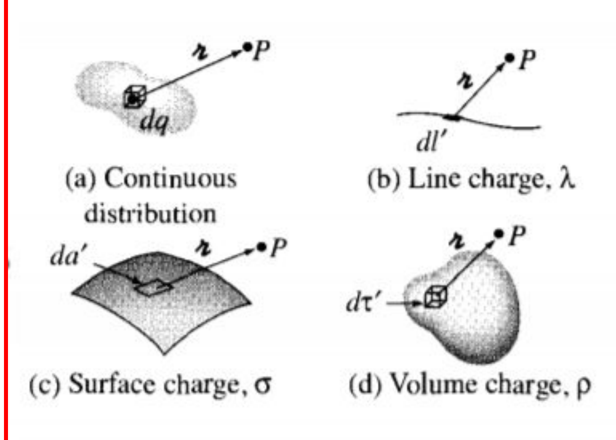
If the charge is distributed continuously in medium called Charge density

Line Charge density

If the charge is spread out along a line, with charge per unit length λ, then $dq = \lambda dl$

Thus, the electric field of a line charge is

$E = 1/4\pi\varepsilon_0 \int (\lambda/r^2)\, dl$



Surface Charge density

If the charge is smeared out over a surface, with charge per unit area σ, then $dq = \sigma da$

For a Surface charge

$$E = 1/4\pi\varepsilon_0 \int (\sigma/r^2)\, da$$

Volume Charge density

If the charge fills a volume, with charge per unit volume ρ, then $dq = \rho d\tau$

Thus, the electric field of a volume charge is

$$E = 1/4\pi\varepsilon_0 \int (\rho/r^2)\, d\tau$$

Gauss theorem (or) Gauss law

This law relates the flux through any closed surface and the net charge enclosed within the surface. The electric flux ($1/\varepsilon_0$) through a closed surface is equal to the $1/\varepsilon_0$ times the net charge q enclosed by the surface.

$$\phi = \left(\frac{1}{\varepsilon_0}\right) q \qquad \text{or} \qquad \phi = \left(\frac{q}{\varepsilon_0}\right) = \oint E\, ds \cos\theta$$

Electric flux

The electric flux is defined as the number of lines of force that pass through a surface placed in the electric field.

The electric flux ($d\phi$) through an elementary area ds is defined as the product of the area and the component of electric field strength normal to the area.

$\therefore$ The electric flux normal to the area $ds = d\phi = \overline{E} \cdot \overline{ds}$

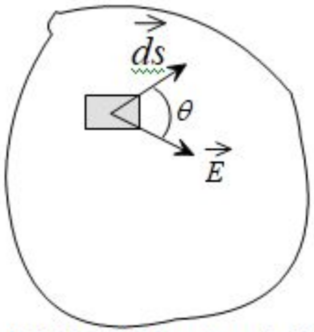


Fig.2.3 Flux of the electric field

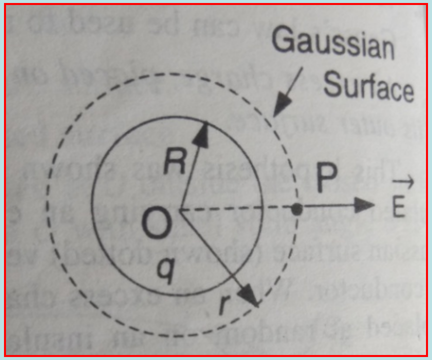
$$d\phi = E\, ds \cos\theta = (E \cos\theta) \cdot ds$$

= (Component of E along the direction of the normal $\times$ area)

The flux over the entire surface $= \phi = \oint_S d\phi = \oint_S E \cos\theta \cdot ds$ Unit: $Nm^2\, C^{-1}$

Electric field due to a uniformly charged sphere When the point P lies outside the sphere

P is a point at a distance r from the centre O. We have to find the electric Field E at P. Draw a concentric sphere of radius OP with centre O. This is the Gaussian surface. At all points of this sphere, the magnitude of the electric field is same and its direction is perpendicular to the surface. Angle between E and dS is zero. The flux through this surface is given by



Electric field due to a uniformly charged sphere When the point P lies outside the sphere

The flux through this surface is given by $\varphi = \oint E.ds = \oint Eds = E(4\Pi r^2)$

By Gauss's Law

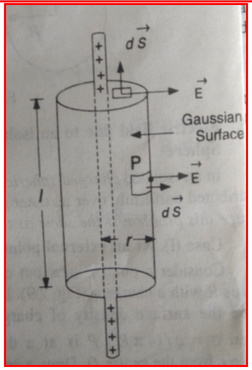
$E(4\Pi r^2) = q/\varepsilon_0$

$E = 1/(4\Pi\ \varepsilon_0\ )q/r^2$

Hence the EF at an external point due to a uniformly charged sphere is the same as if the total charge is concentrated at its centre.

Electric field due to an Infinite line of charge

Consider a uniformly charged wire of infinite length having a constant linear charge density λ. Let P be a point at a distance r from the wire. Let us find an expression for E at P. As a Gaussian surface, we choose a circular cylinder of radius r and length *l*, closed at each end by plane caps normal to the axis. By symmetry, the magnitude of EF will be the same at all points on the curved surface of the cylinder, and directed radially outward. Also E and ds are along the same direction.



Electric field due to an Infinite line of charge

Electric flux due to the curved surface $\varphi = \oint E.ds = E(2\Pi rl)$

Electric flux due to each plane face = 0 (E and ds are at right angle)

Therefore total flux through the Gaussian surface = $\varphi = E(2\Pi rl)$

The net charge enclosed by the Gaussian surface = $q = \lambda l$

By Gauss law $E(2\Pi rl) = \lambda l / \varepsilon_0$

$$E = \lambda / 2\Pi\varepsilon_0 r$$