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ECE – A

Physics: Electromagnetic Theory, Quantum Mechanics, Waves and Optics- 18PYB101J

18PYB101J - Electromagnetic Theory, Quantum Mechanics, Waves and Optics

06.07.21 Assignment - 2.

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Year, Branch & Section: ECE - A.

Date: 6th July 2021.

Question:

1. Derive Schrodinger time independent equation.

Soln: Let us consider a particle of mass 'm' moving with a velocity 'v'. The de Broglie wavelength associated with it is given by,

$$\lambda = \frac{h}{mv} \quad -(1)$$

where h is the plancks constant
$h = 6.626 \times 10^{-34}$ JS.

Let ψ be the wave function of the particle along x, y and z coordinate at any time 't'. The classical differential equation of a progressive wave moving with a wave velocity 'v' can be written as,

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2} \quad -(2)$$

The solution for eqn (2) is given by,

$$\psi = \psi_0 e^{-i\omega t} \quad -(3)$$

where ψ_0 = Amplitude of the wave at the point (x, y, z).
ω = Angular frequency of the wave.

Differentiating eqs ③ with respect to t,

$$\frac{\partial \psi}{\partial t} = (-i\omega)\, \psi_0 e^{-i\omega t} \quad — ④$$

Differentiating eqs ④ with respect to 't'.

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)^2 \psi_0 e^{-i\omega t} = -\omega^2 \psi \quad — ⑤$$

Substituting eqs ⑤ in eqs ②

$$\nabla^2 \psi = -\left[\frac{\omega^2}{v^2}\right] \psi \quad — ⑥$$

where ∇^2 = Laplacian operator.

$$\omega = 2\pi f = 2\pi \left[\frac{v}{\lambda}\right] \quad \text{or} \quad \frac{\omega}{v} = \frac{2\pi}{\lambda}$$

then

$$\frac{\omega^2}{v^2} = \frac{4\pi^2}{\lambda^2} \quad — ⑦$$

Substituting ⑦ in ⑥.

$$\nabla^2 \psi = -\left[\frac{4\pi^2}{\lambda^2}\right] \psi$$

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0 \quad — ⑧$$

Substituting ① in ⑧.

$$\nabla^2 \psi + \frac{4\pi^2}{(h/mv)^2} \psi = 0$$

$$\nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0.$$

If E is the total energy of the particle, 'V' is the potential energy, then the total energy of the particle, E = PE + KE

(or) $E = V + \frac{1}{2} m v^2$

$$m v^2 = 2(E - V)$$

$$m^2 v^2 = 2m(E - V) \quad \text{— (10)}$$

Substituting (10) in (9)

$$\boxed{\nabla^2 \psi + \frac{8\pi^2 m}{\lambda^2}(E - V)\psi = 0}$$

This equation is known as Schrodingers time independent wave equation.