Most common & famous Logical Equivalen

Name of the law	Primal form	Dual form
1. Identity law	$P \vee F \equiv P$	$P \wedge T \equiv P$
2. Domination " (Null Law)	$P \vee T \equiv T$	$P \wedge F \equiv F$
3. Idempotent " (self Repeat)	$P \vee P \equiv P$	$P \wedge P \equiv P$
4. Double Negation " (Involution law)	$\neg(\neg P) \equiv P$	
5. Commutative law	$P \vee q \equiv q \vee P$	$P \wedge q \equiv q \wedge P$
6. Associative law	$(P \vee q) \vee r \equiv P \vee (q \vee r)$	$(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$
7. Distributive law	$P \vee (q \wedge r) \equiv (P \vee q) \wedge (P \vee r)$	$P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$
8. De morgan's law	$\neg(P \vee q) \equiv \neg P \wedge \neg q$	$\neg(P \wedge q) \equiv \neg P \vee \neg q$
9. Absorption law	$P \vee (P \wedge q) \equiv P$	$P \wedge (P \vee q) \equiv P$
10. Negation law (complement law)	$P \vee \neg P \equiv T$	$P \wedge \neg P \equiv F$

$P \vee (P \wedge q) = (P \vee P) \wedge (P \vee q)$ — Distri

$= P \wedge (P \vee q)$ $\because P \vee P = P$

$= (P \wedge P) \vee (P \wedge q)$ — Distri

$= P \vee (P \wedge q)$ $\because P \wedge P = P$

Taking P as common

$= P \wedge (1 \vee q)$

$= P \wedge 1$ $\because 1 \vee q = 1$

$= P$ $\because P \wedge 1 = P$

Equivalence involving conditionals :-

1. $\boxed{p \rightarrow q \equiv \neg p \vee q}$
2. $\boxed{p \rightarrow q \equiv \neg q \rightarrow \neg p}$ → contrapositive
3. $p \vee q \equiv \neg p \rightarrow q$
4. $p \wedge q \equiv \neg(p \rightarrow \neg q)$
5. $\neg(p \rightarrow q) \equiv p \wedge \neg q$
6. $\boxed{(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)}$
7. $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
8. $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
9. $\boxed{(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r}$

Equivalence Involving Bi-conditionals :-

1. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
2. $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
3. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
4. $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

$A \Rightarrow B$

$A \to B$ is tau

Implications :-

p	q	$p \wedge q$	p
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

1. $p \wedge q \Rightarrow p$
2. $p \wedge q \Rightarrow q$
3. $p \Rightarrow p \vee q$
4. $\neg p \Rightarrow p \to q$
5. $q \Rightarrow p \to q$
6. $\neg(p \to q) \Rightarrow p$.
7. $\neg(p \to q) \Rightarrow \neg q$
8. $p \wedge (p \to q) \Rightarrow q$
9. $\neg q \wedge (p \to q) \Rightarrow \neg p$.
10. $\neg p \wedge (p \vee q) \Rightarrow q$
11. $(p \to q) \wedge (q \to r) \Rightarrow p \to r$
12. $(p \vee q) \wedge (p \to r) \wedge (q \to r) =$

Problems :-

① Without using truth tables prove the following, (or) using the laws of logic

P.T $(P \wedge q) \longrightarrow P \vee q$ is a tautology

i.e. $(P \wedge q) \longrightarrow P \vee q \Longleftrightarrow T.$ $(\equiv)$

Sol

$(P \wedge q) \rightarrow P \vee q \equiv \neg(P \wedge q) \vee (P \vee q)$ → logical Equivalence $P \rightarrow q \equiv \neg P \vee q$

$\equiv (\neg P \vee \neg q) \vee (P \vee q)$ — DeMorgon's law $\neg(P \wedge q) \equiv \neg P \vee \neg q$

$\equiv \neg P \vee (\neg q \vee (P \vee q))$ — Associative $(P \vee q) \vee r \equiv P \vee (q \vee r)$

$\equiv \neg P \vee ((\neg q \vee P) \vee q)$ — " $P \vee (q \vee r) \equiv (P \vee q) \vee r$

$\equiv \neg P \vee ((P \vee \neg q) \vee q)$ — commutative law $(P \vee q) \equiv (q \vee P)$

$\equiv \neg P \vee (P \vee (\neg q \vee q))$ — Assowative

$\equiv \neg P \vee (P \vee T)$ — complement law $\neg P \vee P \equiv T$

$\equiv \neg P \vee T$ ← Dominant or null $P \vee T \equiv T$ or P

$\equiv T$ ← null

② using Laws of logic, Prove that

$$(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$$

$(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv (\neg p \vee q) \wedge ((p \wedge p) \wedge q)$ — associative

$\equiv (\neg p \vee q) \wedge (p \wedge q)$ → idempotent

$\equiv \underbrace{(p \wedge q)}_{a} \wedge \underbrace{(\neg p \vee q)}_{(b \vee c)}$ — commutative

$\equiv ((p \wedge q) \wedge \neg p) \vee ((p \wedge q) \wedge q)$ — Distributive

$\equiv ((q \wedge p) \wedge \neg p) \vee (p \wedge (q \wedge q))$ — commutative & associative

$\equiv (q \wedge (p \wedge \neg p)) \vee (p \wedge q)$ — associative complement

$\equiv (q \wedge F) \vee (p \wedge q)$ — complement

$\equiv F \vee (p \wedge q)$ — Null or Dominant

$\equiv p \wedge q$ — Identity

$[((\neg p \vee q) \wedge p] \wedge q$

$[(\neg p \wedge p) \vee (q \wedge p)] \wedge q$

$[F \vee (q \wedge p)] \wedge q$

$(q \wedge p) \wedge q$

$(p \wedge q) \wedge q$

$p \wedge (q \wedge q)$

$p \wedge q$

③ using Laws of logics, show that

$$p \rightarrow (q \rightarrow p) \equiv \neg p \rightarrow (p \rightarrow q)$$

$p \rightarrow (q \rightarrow p) \equiv \neg p \vee (q \rightarrow p)$ Logical Conditional Equivalence $p \rightarrow q \equiv \neg p \vee q$

$\equiv \neg p \vee (\neg q \vee p)$ "

$\equiv \neg p \vee (p \vee \neg q)$ commutative

$\equiv (\neg p \vee p) \vee \neg q$ associative

$\equiv T \vee q$ — complement $p \vee T \equiv T$

$\equiv T$ — ① $\because p \vee T \equiv T$.

$\neg P \to (P \to q) \equiv \neg(\neg P) \vee (P \to q)$ $\quad p \to q \equiv \neg p \vee q$

$\equiv p \vee (p \to q)$ — double negation

$\equiv p \vee (\neg p \vee q)$ — conditional equivalence

$\equiv (p \vee \neg p) \vee q$ — associative

$\equiv T \vee q$ — complement

$\equiv T$ — complement

— ②

From ① & ② it follows that R.H.S = L.H.S. as all tautologies are equivalent to one another. All contradictions are equivalent to one another.

① Determine which of the following statements are tautologies or contradictions without using truth tables.

(i) $(p \to \neg p) \longrightarrow \neg p$.

$(p \to \neg p) \to \neg p \equiv (\neg p \vee \neg p) \to \neg p$ — $p \to q \equiv \neg p \vee q$

$\equiv \neg p \to \neg p$

$\equiv \neg(\neg p) \vee \neg p$

$\equiv p \vee \neg p$ — Double negation

It is a tautology $\equiv T$ — Complement.

(2) $p \to (p \vee q)$

$p \to (p \vee q) \equiv \neg p \vee (p \vee q)$ — conditional Equivalence $p \to q \equiv \neg p \vee q$

$\equiv (\neg p \vee p) \vee q$ — Associative.

$\equiv T \vee q$ — complement

$\equiv T$ — Dominant (or) Null.

(3) $(q \to p) \wedge (\neg p \wedge q)$

$\underbrace{(q \to p)}_{a} \wedge (b \wedge c)$

$(q \to p) \wedge (\neg p \wedge q) \equiv (\neg q \vee p) \wedge (\neg p \wedge q)$

$\equiv ((\neg q \vee p) \wedge \neg p) \wedge q$ — associative

$\equiv (\neg p \wedge (\neg q \vee p)) \wedge q$ — Commutative

$\equiv [(\neg p \wedge \neg q) \vee (\neg p \wedge p)] \wedge q$ — Distributive

$\equiv [(\neg p \wedge \neg q) \vee F] \wedge q$ — Complemen or Negation

$\equiv (\neg p \wedge \neg q) \wedge q$ — identity

$\equiv \neg p \wedge (\neg q \wedge q)$ — complement.

$\equiv \neg p \wedge F \equiv F$. — Null.

It is a Contradiction

Example :-

① prove the following implications without using truth tables.

(i) $p \wedge q \Rightarrow p \rightarrow q$

To prove $p \wedge q \longrightarrow (p \rightarrow q) \equiv T$.

$p \wedge q \longrightarrow (p \rightarrow q) \equiv \neg(p \wedge q) \vee (\neg p \vee q)$ $a \rightarrow b \equiv \neg a \vee b$

$\equiv (\neg p \vee \neg q) \vee (\neg p \vee q)$ (De Morgan's L

$\equiv (\neg q \vee \neg p) \vee (\neg p \vee q)$ (commutative law)

$\equiv [(\neg q \vee \neg p) \vee \neg p] \vee q$ (associative law)

$\equiv [\neg q \vee (\neg p \vee \neg p)] \vee q$ (")

$\equiv (\neg q \vee \neg p) \vee q$ (idempotent law)

$\equiv$ ~~¬p ∨ (¬q∨~~ $(\neg p \vee \neg q) \vee q$ (commutative law)

$\equiv \neg p \vee (\neg q \vee q)$ (associative law)

$\equiv \neg p \vee T$ (complement law)

$\equiv T$ (null law)

$p \Rightarrow (q \to p)$

To prove $p \to (q \to p) \equiv T$.

$p \to (q \to p) \equiv \neg p \vee (\neg q \vee p)$ $\quad a \to b \equiv \neg a \vee b$

$\equiv \neg p \vee (p \vee \neg q)$ (commutative law)

$\equiv (\neg p \vee p) \vee \neg q$ (associative law)

$\equiv T \vee \neg q$ (complement law)

$\equiv T$ (null law)

③ $((p \vee \neg p) \to q) \to ((p \vee \neg p) \to r) \Rightarrow q \to r$.

To prove

$[((p \vee \neg p) \to q) \to ((p \vee \neg p) \to r)] \to (q \to r) \equiv T$.

$\equiv [(T \to q) \to (T \to r)] \to (q \to r)$

$[(\neg T \vee q) \to (\neg T \vee r)] \to (\neg q \vee r)$

$\equiv [(\neg T \vee q) \to (F \vee r)] \to (\neg q \vee r)$

$\equiv [(F \vee q) \to (q \to r)]$

$= (q \to r) \to (q \to r)$

$= \neg(q \to r) \vee (q$

$\neg(q \to r) \vee (q \to r)$

$\neg(\neg q \vee r) \vee (\neg q \vee r)$

$(q \wedge \neg r) \vee (r \vee \neg q)$

$[q \wedge (\neg r \vee r)] \vee \neg q$

$T \vee \neg q$

$\equiv T$

Note :-

Write down the duals ... the following equivalence and prove ... duals without using truth ...

(i) $\neg(p\wedge q) \longrightarrow (\neg p \vee (\neg p \vee q)) \equiv \neg p \vee q$

L.H.S $\neg(\neg(p\wedge q) \vee (\neg p \vee (\neg p \vee q))$

Dual : $(p\vee q) \wedge (\neg p \wedge (\neg p \wedge q)) \equiv \neg p \wedge q$.

L.H.S $(p\vee q) \wedge (\neg p \wedge (\neg p \wedge q))$

$\equiv (p\vee q) \wedge [(\neg p \wedge \neg p) \wedge q]$ Distributive

$\equiv (p\vee q) \wedge (\neg p \wedge q)$ $(A \vee B) \wedge C$. $A\wedge C \vee B \wedge C$.

$\equiv (q\vee p) \wedge (\neg p \wedge q)$

$\equiv [q \wedge (\neg p \wedge q)] \vee (p \wedge \neg p \wedge q)$

$\equiv (q \wedge \neg p) \vee (F \wedge q)$

$\equiv (q\wedge \neg p) \vee F$ (identity laws $p \vee F \equiv p$.

$\equiv q \wedge \neg p$.

Note :- If a statement formula contains $\rightarrow$ or $\leftrightarrow$, we replace them by equivalent formula involving $\wedge$ and $\vee$ and then write the dual.