

Experiment 10(a) Design of Analog Butterworth filter

Aim: To Design and analyze the function of an analog Butterworth filter

Software Requirement: SCI Lab

Theory: The signal processing filter which is having a flat frequency response in the passband can be termed as Butterworth filter and is also called as a maximally flat magnitude filter. In 1930 physicist and the British engineer Stephen Butterworth described about a Butterworth filter in his “on the theory of filter amplifiers” paper for the first time. Hence, this type of filter named as Butterworth filter. There are various types of Butterworth filters such as low pass Butterworth filter and digital Butterworth filter. The frequency response of the nth order Butterworth filter is given as

$$H_{(j\omega)} = \frac{1}{\sqrt{1 + \epsilon^2 \left(\frac{\omega}{\omega_p} \right)^{2n}}}$$

Where ‘n’ indicates the filter order, ‘ ω ’ = $2\pi f$, Epsilon ϵ is maximum pass band gain, (A_{max}).

Algorithm:

Step 1: For the given Cutoff frequency, band ripple and stopband ripple find the order of the filter

Step 2: Round the order of the filter to the next higher order integer

Step 3: Determine the magnitude and phase response of the analog Butterworth filter to Plot

Program:

```
// To Design an Analog Butterworth Filter
//For the given cutoff frequency and filter order
//Wc = 500 Hz
omegap = 500; //pass band edge frequency
omegas = 1000; //stop band edge frequency
delta1_in_dB = -3; //PassBand Ripple in dB
delta2_in_dB = -40; //StopBand Ripple in dB
delta1 = 10^(delta1_in_dB/20)
```

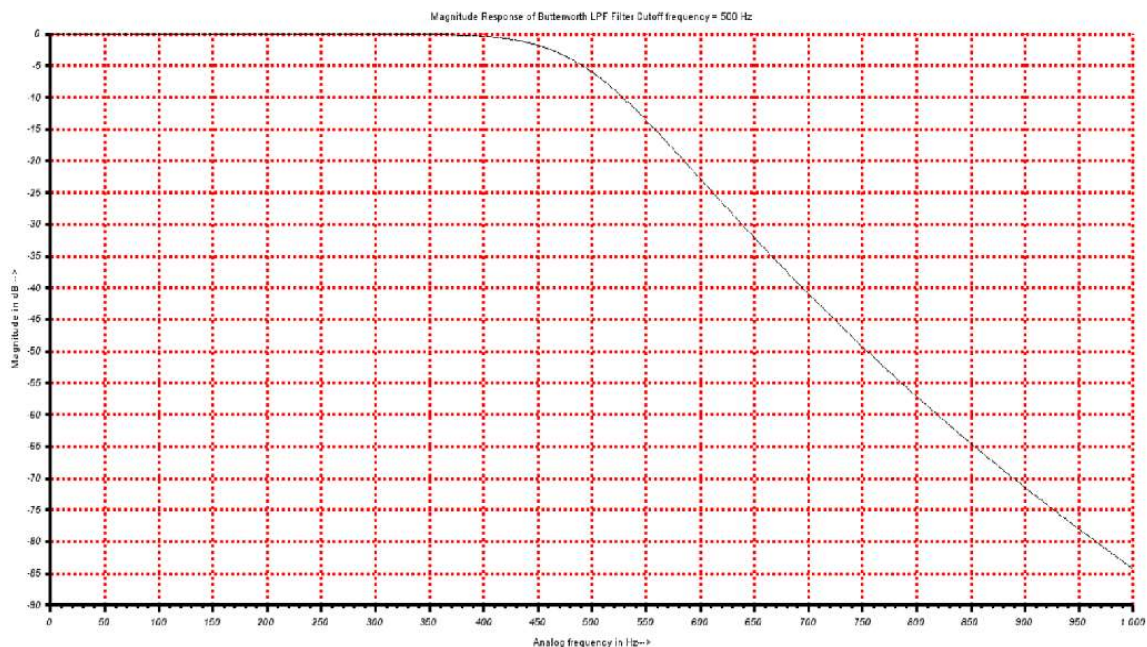
```

delta2 = 10^(delta2_in_dB/20)
//Calculation of filter order
N = log10((1/(delta2^2))-1)/(2*log10(omegas/omegap))
N = ceil(N) //Rounding off nearest integer
omegac = omegap;
h=buttmag(N,omegac,1:1000); //Analog Butterworth filter magnitude response
mag=20*log10(h); //Magnitude Response in dB
plot2d((1:1000),mag,[0,-180,1000,20]);
a=gca();
a.thickness = 3;
a.foreground = 1;
a.font_style = 9;
xgrid(5)

xtitle('Magnitude Response of Butterworth LPF Filter Cutoff frequency = 500 Hz','Analog
frequency in Hz--->','Magnitude in dB -->');

```

Simulation Output:



Experiment 10(b) Design of Analog Chebyshev filter

Aim: To Design and analyze the function of an analog Chebyshev filter

Software Requirement: SCI Lab

Theory: Chebyshev filters are used for distinct frequencies of one band from another. They cannot match the windows-sinc filter's performance and they are suitable for many applications. The main feature of Chebyshev filter is their speed, normally faster than the windowed-sinc. Because these filters are carried out by recursion rather than convolution. The designing of the Chebyshev and Windowed-Sinc filters depends on a mathematical technique called as the Z-transform. Types of Chebyshev Filters

Chebyshev filters are classified into two types, namely type-I Chebyshev filter and type-II Chebyshev filter.

Type-I Chebyshev Filters

This type of filter is the basic type of Chebyshev filter. The amplitude or the gain response is an angular frequency function of the nth order of the LPF (low pass filter) is equal to the total value of the transfer function $H_n(j\omega)$

$$G_n(\omega) = |H_n(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 T_n^2(\omega/\omega_0)}}$$

Where, ϵ = ripple factor

ω_0 = cutoff frequency

T_n = Chebyshev polynomial of the nth order

$$s_{pm}^{\pm} = \pm \sinh \left(\frac{1}{n} \operatorname{arsinh} \left(\frac{1}{\epsilon} \right) \right) \sin(\theta_m) \\ + j \cosh \left(\frac{1}{n} \operatorname{arsinh} \left(\frac{1}{\epsilon} \right) \right) \cos(\theta_m)$$

Here $m = 1, 2, 3, \dots, n$

$$\theta_m = \frac{\pi}{2} \frac{2m-1}{n}$$

The above equation produces the poles of the gain G . For each pole, there is the complex conjugate, & for each and every pair of conjugate there are two more negatives of the pair. The TF should be stable, transfer function (TF) is given by

$$H(s) = \frac{1}{2^{n-1} \epsilon} \prod_{m=1}^n \frac{1}{(s - s_{pm}^-)}$$

Algorithm:

Step 1: Convert the given analog frequencies to digital domain

Step 2: Prewarp the frequency components and find the prewarping frequency

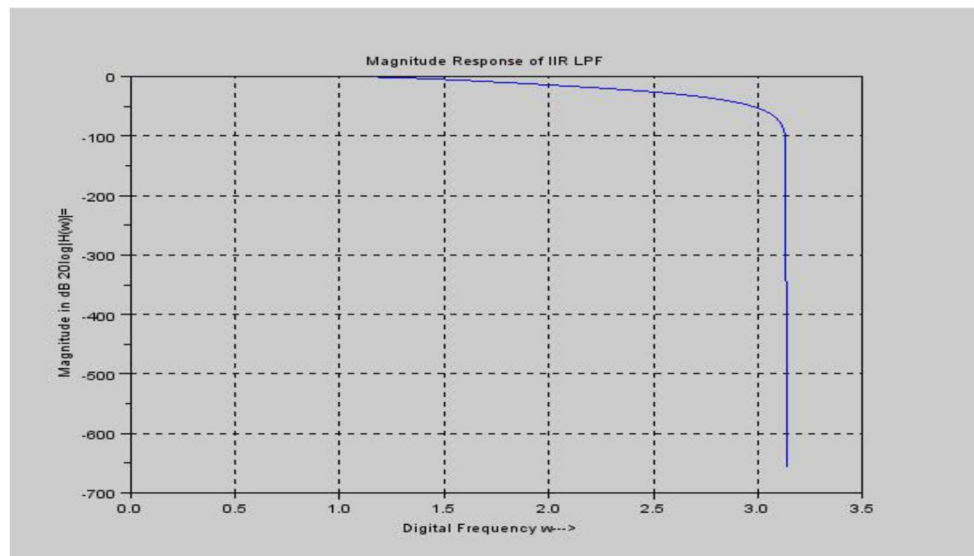
Step 3: Determine the order of the filter and round to next higher order integer

Step 4: Convert the analog transfer function to digital by using Bilinear transformation technique and plot the magnitude and frequency response

Program:

```
// Caption : To obtain Digital IIR Chebyshev low pass
filter
// Frequency response
clc;
clear;
close;
fp= input ( ' Enter the pass band edge (Hz ) = ' );
fs= input ( ' Enter the stop band edge (Hz ) = ' );
kp= input ( ' Enter the pass band attenuation (dB) = ' );
ks= input ( ' Enter the stop band attenuation (dB) = ' );
Fs= input ( ' Enter the sampling rate samples / s e c = ' );
d1 = 10^( kp /20) ;
d2 = 10^( ks /20) ;
d = sqrt ((1/( d2 ^2)) -1);
E = sqrt ((1/( d1 ^2)) -1);
// Digital filter specifications (rad / samples)
wp =2* %pi *fp *1/ Fs;
ws =2* %pi *fs *1/ Fs;
// Pre warping
op =2* Fs* tan (wp /2);
os =2* Fs* tan (ws /2);
N = acosh (d/E)/ acosh (os/op);
oc = op /(( E ^2) ^(1/(2* N)));
N = ceil (N); // rounded to nearest integer
disp (N, ' IIR Filter order N = ' );
disp (oc , ' Cut off Frequency in rad / s e c o n d s OC = ' )
[pols ,gn] = zpchl (N,E,op);
HS = poly (gn , ' s ' , ' c o e f f ' )/ real ( poly (pols , ' s ' ));
z = poly(0,'z')
Hz = horner (HS ,(2* Fs *(z -1) /(z +1) ))
num = coeff (Hz (2) )
den = coeff (Hz (3) )
Hz (2)= Hz (2) ./ den (3) ;
Hz (3)= Hz (3) ./ den (3) ;
disp (Hz , ' Transfer function of Digital IIR Chebyshev LPF H(Z)= ' )
[Hw ,w] = frmag (Hz ,256) ;
figure (1)
plot (2*w*%pi ,20* log10 ( abs (Hw)));
xlabel ( ' Di g i t a l Frequency W>' )
ylabel ( ' Magni tude i n dB 20 l o g jH(w) j= ' )
title ( ' Magni tude Re spons e o f IIR LPF ' )
xgrid (1)
```

Simulation Output



Prelab Questions:

1. List the differences between butterworth and Chebyshev filter
2. Define the term Prewarping and mention its importance
3. To design a Discrete time Low pass filter the specifications are
Passband $F_p = 4$ KHz with 0.8 dB ripple
Stopband $F_p = 4.5$ KHz with 50 dB attenuation
Sampling frequency $F_s = 22$ KHz
 - (i) Determine the passband and stopband frequencies
 - (ii) Determine the maximum and minimum values of $|H(\omega)|$ in the pass band and stopband

Postlab Questions:

1. List the properties of Chebyshev filter
2. In your CD the data is sampled at 44.1kHz, and we want to have a good sound quality upto 21kHz. If you had to use an analog butterworth filter as reconstruction filter, What would be the order of the filter?
3. Give the Normalized Low pass Butterworth Denominator polynomials for $N=8,9,10$.