b. If the joint pdf of a two dimensional RV(X, Y) is given by

$f(x,y) = x^2 + \frac{xy}{3}; 0 < x < 1; 0 < y < 2$

$= 0; \quad otherwise$

Find (i) $P(X > 1/2)$ (ii) $P(Y < 1/2)$ (iii) $P(Y < X)$ and (iv) $P(Y < 1/2 \;/\; X < 1/2)$

30. a. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

(OR)

b.i. Consider the random process $X(t) = \cos(t + \phi)$ where ϕ is a RV with pdf $f(\phi) = \frac{1}{\pi}$; $\frac{-\pi}{2} < \phi < \frac{\pi}{2}$. Show that the process is not stationary.

ii. Show that the process $X(t) = A\cos\lambda t + B\sin\lambda t$ where A and B are RV's is a WSS if $E(A) = E(B) = 0, E(A^2) = E(B^2)$ and $E(AB) = 0$.

31. a.i. The autocorrelation function of a stationary process is given by $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$, find the mean and variance of x(t).

ii. Prove that the mean of the output of a linear system is given by $b_Y = H(0)b_X$, where X(t) is a WSS.

(OR)

b.i A circuit has unit impulse response given by $h(t) = \begin{cases} \frac{1}{T}; 0 \le t \le T \\ 0; otherwise \end{cases}$

Evaluate $S_{YY}(\omega)$ in terms of $S_{XX}(\omega)$.

ii. Prove that (i) $R_{XY}(-\tau) = R_{YX}(\tau)$ (ii) if X(t) and Y(t) are independent, then $R_{XY}(\tau) = R_{YX}(\tau)$.

32. a.i The autocorrelation function of an ergodic process

X(t) is $R_{xx}(\tau) = \begin{cases} 1 - |\tau|; & |\tau| \le 1 \\ 0; & otherwise \end{cases}$

Obtain the power spectral density of X.

ii. Find the autocorrelation function corresponding to the power density spectrum. Also find the average power.

$$S_{XX}(\omega) = \frac{157 + 12\omega^2}{(16 + \omega^2)(9 + \omega^2)}$$

(OR)

b. If X(t) is a WSS process with autocorrelation $R_{XX}(\tau)$ and if $Y(t) = X(t+a) - X(t-a)$, then prove that $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau + 2a) - R_{XX}(\tau - 2a)$. Hence show that $S_{YY}(\omega) = 4\sin^2 a\omega . S_{XX}(\omega)$

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Reg. No.														

B.Tech. DEGREE EXAMINATION, DECEMBER 2017

Third/ Fourth/ Fifth Semester

15MA209 – PROBABILITY AND RANDOM PROCESS

(For the candidates admitted during the academic year 2015 – 2016 onwards)

(Statistical tables are to be permitted)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. If $f(x)$ is a cumulative distribution function of random variable, then $f(x)$ is
   (A) Decreasing (B) Increasing
   (C) Alternating (D) Non-increasing

2. $\text{var}(AX + B)$ is
   (A) $A^2 \text{var}(X)$ (B) $A\text{var}(X)$
   (C) $A\text{var}(X) + B$ (D) $A + B$

3. Let one copy of a magazine out of 10 copies bears a special prize following geometric distribution. What is its variance?
   (A) 10 (B) 20
   (C) 60 (D) 90

4. Poisson distribution is the limiting case of
   (A) Geometric distribution (B) Normal distribution
   (C) Binomial distribution (D) Exponential distribution

5. If $F(x, y)$ is the joint cumulative distribution function, then $F(\infty, \infty) =$
   (A) 0 (B) 1
   (C) ∞ (D) $-\infty$

6. In central limit theorem (Lindberg and Levy's form), S_n following normal distribution with mean and standard deviation equal to
   (A) $N\mu, \sqrt{N}\sigma$ (B) $\mu, N\sigma$
   (C) $\mu, \frac{\sigma}{\sqrt{N}}$ (D) $N\mu, N\sigma$

7. The marginal probability function of X from $F_{xy}(x, y)$ is
   (A) $\int F(X,Y)dY$ (B) $\int F(X,Y)dX$
   (C) $\iint_R F(X,Y)dXdY$ (D) $\frac{\partial F}{\partial X}(X,Y)$

8. If X, Y are jointly distributed two dimensional continuous random variables which are transformed to two other random variables U and V, then
   (A) $f_{UV} = f_{XY} + |J|$ (B) $f_{UV} = f_{XY} - |J|$
   (C) $f_{XY} = f_{UY}|J|$ (D) $f_{UV} = f_{XY}|J|$

9. $R_{XX}(0)$ is equal to

(A) $E\{X(T)\}$ (B) $\text{var}\{X(T)\}$

(C) $E\{X^2(T)\}$ (D) $[E\{X(T)\}]^2$

10. If the process $\{X(T)\}$ and $\{Y(T)\}$ are orthogonal, then $R_{XY}(\tau)$=

(A) 1 (B) −1

(C) $R_{XX}(\tau)$ (D) 0

11. If the process $\{X(T)\}$ and $\{Y(T)\}$ are independent, then $R_{XX}(\tau)$=

(A) $E\{X^2(T)\}.E\{Y^2(T)\}$ (B) $E\{X(T)\}.E\{Y(T)\}$

(C) 0 (D) 1

12. If $\{X(T)\}$ and $\{Y(T)\}$ are independent WSS processes with zero mean, then the autocorrelation function of $\{Z(T)\}$ where $Z(T)=AX(T)Y(T)$ is

(A) $AR_{XX}(\tau)R_{YY}(\tau)$ (B) $A^2R_{XX}(\tau)R_{YY}(\tau)$

(C) $\sqrt{A}R_{XX}(\tau)R_{YY}(\tau)$ (D) $\sqrt{AR_{XX}(\tau).R_{YY}(\tau)}$

13. A discrete parameter markov process is called a

(A) Markov process (B) Morkov chain

(C) Stationary process (D) Independent increment

14. The sum of all the elements of any row of the transition probability matrix is

(A) 0 (B) 1

(C) 0.5 (D) 0.75

15. Mean of the Poisson process is

(A) $\lambda T+1$ (B) λ

(C) λT^2 (D) λT

16. If T is continuous and S is discrete then the random process is called

(A) Discrete random sequence (B) Continuous random sequence

(C) Discrete random process (D) Continuous random process

17. The average power of a random process $\{X(t)\}$ is defined by

(A) $R_{XX}(\tau)$ (B) $R_{XX}(0)$

(C) $R_{XX}(-\tau)$ (D) $S_{XX}(0)$

18. The convolution form of the output of linear time invariant system is

(A) $Y(T)=\int_{-\infty}^{\infty} H(U)X(T-U)DU$ (B) $Y(T)=\int_{-\infty}^{\infty} H(T)X(T-U)DU$

(C) $Y(T)=\int_{0}^{\infty} H(U)X(T-U)DU$ (D) $Y(T)=\int_{-\infty}^{\infty} H(T)X(U)DU$

19. The cross correlation of two processes $\{X(T)\}$ and $\{Y(T)\}$ is denoted by $R_{XY}(\tau)$, then

(A) $|R_{XY}(\tau)|\le R_{XX}(0)R_{YY}(0)$ (B) $|R_{XY}(\tau)|\le \sqrt{R_{XX}(0)R_{YY}(0)}$

(C) $\sqrt{|RXY(\tau)|}\le R_{XX}(0)R_{YY}(0)$ (D) $|RXY(\tau)|^2\le \sqrt{R_{XX}(0)R_{YY}(0)}$

20. Let X(T) be a WSS process which is the input to a linear time invariant time system with unit impulse H(T) and output Y(T), then $S_{YY}(\omega)$=

(A) $H(\omega)S_{XX}(\omega)$ (B) $|H(\omega)|S_{XX}(\omega)$

(C) $|H(\omega)|^2 S_{XX}(\omega)$ (D) $|H(\omega)|^2 R_{XX}(\omega)$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. The cdf of a continuous random variable X is given by

$F(x)=0; x<0$

$=x^2; 0\le x\le 1/2$

$=1-\frac{3}{25}(3-x)^2; \frac{1}{2}\le x<3$

$=1; x\ge 0$

Find the pdf of X and evaluate $P\left(\frac{1}{3}\le X<4\right)$.

22. State and prove the exponential distribution.

23. The joint probability distribution of a 2- dimensional random variables X and Y is given by $P(X=0,Y=1)=\frac{1}{3}$, $P(X=1,Y=-1)=\frac{1}{3}$ and $P(X=1,Y=1)=\frac{1}{3}$. Find the marginal distributions of X and Y and conditional probability distribution of X given Y=1.

24. If $X_1, X_2,.....X_n$ are Poisson random variables with parameters λ=2, use the central limit theorem to estimate $P(120\le S_n\le 160)$, where $S_n=X_1+X_2+....+X_n$ and n = 75.

25. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between that the interval between 2 consecutive arrivals is (i) more than 1 min (ii) between 1 min and 2 mins.

26. If the autocorrelation function of a stationary process is $R_{XX}(\tau)=36+\frac{4}{1+3\tau^2}$. Find the mean and variance of process.

27. If $R(\tau)=e^{-2\lambda|\tau|}$ is the autocorrelation function of a random process X(t), obtain the spectral density of X(t).

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. A manufacturer, who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes form the producer of bottles. Using Poisson distribution, find how many boxes will contain (i) no defective and (ii) at least two defectives.

ii. A and B shoot independently until each has hit his own target. The probabilities of their hitting the target at each shot are 3/5 and 5/7 respectively. Find the probability that B will require more shots than A.

(OR)

b.i. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and the standard deviation of the distribution.

ii. Find the MGF of a binomial distribution. Hence find the mean and variance.

29. a. If X and Y are independent random variables with pdf $e^{-x}; x\ge 0$ and $e^{-y}; y\ge 0$ respectively, find the pdf of $U=\frac{X}{X+Y}$ and $V=X+Y$. Show that U and V are independent random variables.

(OR)