Unit I - Random Variables

Practise sheet I

1. A random variable x has the following probl. distn.

x:	-2	-1	0	1	2	3
$p(x)$:	0.1	k	0.2	$2k$	0.3	$3k$

a) Find k b) $P(X<2)$ c) $P(-2<X<2)$ d) CDF of X

Ans: $(\frac{1}{15})$ $\frac{1}{2}$ $\frac{2}{5}$

2. A random variable x has the foll: probl. distn.

x:	0	1	2	3	4	5	6	7
$p(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) $P(1.5<X<4.5 / X>2)$ (iii) the smallest value of λ for which $P(X \leq \lambda) > 1/2$

Ans (i) $\frac{1}{10}$ (ii) $\frac{5}{7}$ (iii) 4

3. If $p(x) = \begin{cases} xe^{-x^2/2} & x \geq 0 \\ 0 & x < 0 \end{cases}$

a) Show that $p(x)$ is a pdf (of a continuous RV X)
b) Find its distribution fn. $P(x)$

Ans: (a) $\int_0^\infty p(x)\,dx = 1$. So is a pdf

(b) $F(x) = P(x) = \int_0^x xe^{-x^2/2}\,dx = 1 - e^{-x^2/2}$; $x \geq 0$

4. A continuous RV has a pdf $f(x) = 3x^2$, $0 \leq x \leq 1$.
Find a and b such that (i) $P(X \leq a) = P(X > a)$ and
(ii) $P(X > b) = 0.05$

Ans (i) $a = 0.7939$ (ii) $b = 0.9830$

5. If the CDF of a RV is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{16} & 0 \le x \le 4 \\ 1 & x \ge 4 \end{cases}$$

Find $P(X > 1 / X < 3)$

Hint: $P(X>1/X<3) = \frac{P(1<X<3)}{P(0<X<3)}$

$= \frac{F(3)-F(1)}{F(3)-F(0)} = \frac{8}{9}$

6. If the pdf of a RV x is $f(x) = 2x$, $0 < x < 1$

find cdf of x

Ans: $F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$

7. X is a continuous RV with pdf given by

$$f(x) = \begin{cases} kx & 0 \le x < 2 \\ 2k & 2 \le x < 4 \\ 6k - kx & 4 \le x < 6 \\ 0 & x \ge 6 \end{cases}$$

Find value of k and $F(x)$

Ans: $k = \frac{1}{8}$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{16} & 0 \le x < 2 \\ \frac{x-1}{4} & 2 \le x < 4 \\ \frac{-1}{16}(20 - 12x + x^2) & 4 \le x < 6 \\ 1 & x \ge 6 \end{cases}$$

8. If X has the dist fn. (i) Find prob dist. of x

(ii) $P(2 < X < 6)$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{3} & 1 \le x < 4 \\ \frac{1}{2} & 4 \le x < 6 \\ \frac{5}{6} & 6 \le x < 10 \\ 1 & x \ge 10 \end{cases}$$

Hint: $P(X = x_i) = F(x_i) - F(x_{i-1})$

$i = 1, 2, 3 \ldots$ where F is constant in $x_{i-1} \le x < x_i$

x	1	4	6	10
$P(x)$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$

(ii) $P(2<X<6) = P(X=4) = \frac{1}{6}$