

1. Show that $(\neg A \vee B)$ can be derived from the premisses $P \rightarrow Q, Q \rightarrow \neg R, R, P \vee (\neg A \vee B)$. ①

<u>Step No</u>	<u>Statement</u>	<u>Reason</u>
1.	$P \rightarrow Q$	P
2.	$Q \rightarrow \neg R$	P
3.	$P \rightarrow \neg R$	1, 2 Hypothetical Syllogism
4.	$R \rightarrow \neg P$	3, and $P \rightarrow Q \equiv \neg Q \rightarrow \neg P$
5.	R	P
6.	$\neg P$	4, 5 and modus ponens
7.	$P \vee (\neg A \vee B)$	P
8.	$\neg A \vee B$	6, 7, and Disjunctive Syllogism.

2. Show that $(A \vee B)$ follows logically from the premisses $P \vee Q, (P \vee Q) \rightarrow \neg R, \neg R \rightarrow (S \wedge \neg E)$ and $(S \wedge \neg E) \rightarrow (A \vee B)$.

<u>Step No</u>	<u>Statement</u>	<u>Reason</u>
1.	$(P \vee Q) \rightarrow \neg R$	P
2.	$\neg R \rightarrow (S \wedge \neg E)$	P
3.	$(P \vee Q) \rightarrow (S \wedge \neg E)$	1, 2 and Hypothetical Syllogism
4.	$P \vee Q$	P
5.	$S \wedge \neg E$	3, 4 and modus ponens
6.	$(S \wedge \neg E) \rightarrow (A \vee B)$	P
7.	$A \vee B$	5, 6 and modus ponens.

3. Derive $r \rightarrow s$ using CP rule from the premises
 $P \rightarrow (q \rightarrow s), (\neg r \vee P), q \Rightarrow (r \rightarrow s)$

Step No	Statement	Reason
1.	$P \rightarrow (q \rightarrow s)$	P
2.	$\neg r \vee P$	P
3.	q	P
4.	r	AP [Additional premise]
5.	$r \rightarrow P$	2, $P \rightarrow q \equiv \neg P \vee q$
6.	$r \rightarrow (q \rightarrow s)$	5 & 1, Hypothetical Syllogism.
7.	$\neg r \vee (\neg q \vee s)$	[6, $P \rightarrow q \equiv \neg P \vee q$]
8.	$q \wedge (\neg r \vee (\neg q \vee s))$	[3 and 7]
9.	$q \wedge (\neg r \vee (\neg q \vee s))$	
9.	$q \wedge ((\neg r \vee s) \vee \neg q)$	[Associative and commutative, 8]
10.	$(q \wedge (\neg r \vee s)) \vee (q \wedge \neg q)$	[9, Distributive]
11.	$(q \wedge (\neg r \vee s)) \vee f$	
12.	$(q \wedge (\neg r \vee s))$	[11, $P \vee f \equiv P$]
13.	$\neg r \vee s$	12, Simplification.
14.	$r \rightarrow s$	[13]
15.	s	[4, & 14].

(3)

4. Derive $P \rightarrow (q \rightarrow s)$ using CP-rule from the premises $P \rightarrow (q \rightarrow r)$ and $q \rightarrow (r \rightarrow s)$.

<u>Step No</u>	<u>Statement</u>	<u>Reason</u>
1.	<u>P</u>	Additional premise
2.	$P \rightarrow (q \rightarrow r)$	P
3.	$q \rightarrow r$	1, 2, Modus ponens
4.	$\neg q \vee r$	[3, equivalence]
5.	$q \rightarrow (r \rightarrow s)$	P
6.	$\neg q \vee (r \rightarrow s)$	5, equivalence
7.	$\neg q \vee (r \wedge (r \rightarrow s))$	4, 6, distributive
8.	$\neg q \vee s$	7, modus ponens
9.	<u>$q \rightarrow s$</u>	8, equivalence
10.	$P \rightarrow (q \rightarrow s)$	9, CP rule.

5. Use the indirect method to show that $r \rightarrow \neg q$, $r \vee s$, $s \rightarrow \neg q$, $P \rightarrow q \Rightarrow \neg P$.

To use indirect method we will include $\neg(P)$ as an additional premise and prove a contradiction.

<u>Step No</u>	<u>Statement</u>	<u>Reason</u>
1.	P	P (additional)
2.	$P \rightarrow q$	P
3.	q	T, 1, 2 and modus ponens
4.	$r \rightarrow \neg q$	P
5.	$s \rightarrow \neg q$	P
6.	$r \vee s \rightarrow \neg q$	4, 5 and equivalence.
7.	$r \vee s$	P
8.	$\neg q$	6, 7 and modus ponens.
9.	$q \wedge \neg q$	3, 8 and conjunction
10.	F	Contradiction.

6. Show that b can be derived from the premises $a \rightarrow b$, $c \rightarrow b$, $d \rightarrow (a \vee c)$, d , by the indirect method.

Let us include $\neg b$ as an additional premise and prove a contradiction.

<u>Step No</u>	<u>Statement</u>	<u>Reason</u>
1.	$a \rightarrow b$	P
2.	$c \rightarrow b$	P
3.	$(a \vee c) \rightarrow b$	1, 2 and equivalence
4.	$d \rightarrow (a \vee c)$	P
5.	$d \rightarrow b$	3, 4 and hypothetical syllogism

6.	d	P
7.	$\neg b$	5, 6 modus ponens
8.	$\neg b$	P (additional)
9.	$b \wedge \neg b$	Conjunction of 7 and 8
10.	F	Contradiction, (9).

7. Prove that the premises $P \rightarrow q$, $q \rightarrow r$, $S \rightarrow \neg r$ and $q \wedge S$ are inconsistent.

<u>Step no</u>	<u>Statement</u>	<u>Reason</u>
1.	$P \rightarrow q$	P
2.	$q \rightarrow r$	P
3.	$P \rightarrow r$	1, 2 and Hypothetical syllogism
4.	$S \rightarrow \neg r$	P
5.	$r \rightarrow \neg S$	4, Contrapositive
6.	$q \rightarrow \neg S$	2, 5, Hypothetical syllogism
7.	$\neg q \vee \neg S$	6, equivalence
8.	$\neg(q \wedge S)$	7 and De Morgan's law
9.	$(q \wedge S)$	P
10.	$(q \wedge S) \wedge \neg(q \wedge S)$	8, 9 and conjunction.
11.	F	10, Contradiction

8. Prove that the premises $a \rightarrow (b \rightarrow c)$, $d \rightarrow (b \wedge \neg c)$ and $(a \wedge d)$ are inconsistent.

(6)

<u>Step NO</u>	<u>Statement</u>	<u>Reason</u>
	$a \wedge d$	P
1.		
2.	a	1, Simplification
3.	d	1, Simplification
		P
4.	$a \rightarrow (b \rightarrow c)$	
5.	$b \rightarrow c$	2, 4 and modus ponens
6.	$\neg b \vee c$	5, equivalence
		P
7.	$d \rightarrow (b \wedge \neg c)$	
8.	$\neg(b \wedge \neg c) \rightarrow \neg d$	7 and Contrapositive
9.	$\neg b \vee c \rightarrow \neg d$	8, equivalence
10.	$\neg d$	6, 9, modus ponens
11.	$d \wedge \neg d$	3, 10, conjunction
12.	f	11, Contradiction.

9. Construct an argument to show that the following premises imply the conclusion "it rained".

"If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on".

"If the sports day is held, the trophy will be awarded" and "the trophy was not awarded".

Let us symbolise the statement as follows:

P : It rains

q : There is traffic dislocation

r : Sports day will be held.

s : Cultural programme will go on.

t : The trophy will be awarded.

$$\neg P \vee \neg q \rightarrow r \wedge s, \quad r \rightarrow t, \quad \neg t \Rightarrow P.$$

Step no

Statement

Reason

- | | | |
|-----|--|-------------------------|
| 1. | $\neg P \vee \neg q \rightarrow r \wedge s$ | P |
| 2. | $(\neg P \rightarrow (r \wedge s)) \wedge (\neg q \rightarrow (r \wedge s))$ | 1, equivalence |
| 3. | $\neg (r \wedge s) \rightarrow P$ | 2, contrapositive. |
| 4. | $r \rightarrow t$ | P |
| 5. | $\neg t \rightarrow \neg r$ | 4, contrapositive of 4. |
| 6. | $\neg t$ | P |
| 7. | $\neg r$ | 5, 6 and modus ponens |
| 8. | $\neg r \vee \neg s$ | 7, and addition |
| 9. | $\neg (r \wedge s)$ | 8, Demorgan's law |
| 10. | P | 3, 9, modus ponens. |

10. Show that the following set of premises is inconsistent: (8)

If Rama gets his degree, he will go for a job.

If he goes for a job, he will get married soon.

If he goes for higher study, he will not get married.

Rama gets his degree and goes for higher study.

Let the statements be symbolised as follows:

P : Rama gets his degree

q : He will go for a job

r : He will get married soon

s : He goes for higher study.

$P \rightarrow q$, $q \rightarrow r$, $s \rightarrow \neg r$, $P \wedge s$ are inconsistent.

Step No

Statement

Reason

1.

$P \rightarrow q$

P

2.

$q \rightarrow r$

P

3.

$P \rightarrow r$

1, 2. and hypothetical syllogism

4.

$P \wedge s$

P

5.

P

4 and simplification

6.

s

4 and simplification

7.

$s \rightarrow \neg r$

P

8.

$\neg r$

6, 7, modus ponens

9.

r

3, 5 and modus ponens

10.

$r \wedge \neg r$

8, 9, conjunction

11.

F

10, Contradiction.