Steady state anditions and non-zero boundary conditions. A bar, locm long, with insulated mides, has its ends 1 A and B kept at do and 40°C., respectively, until Steady - Atala and itions prevail, that is, until the temperature at any interior point no longer changes with time. The Eemperature at A is then suddenly raised to soic and at the same instant that at B is lowered to 10°C. find the subsequent temperature function u(n,t) at any time. The partial dyferential equalizing one dimensional som : heat flow is ou of one In steady state anditions, the temperature out any time le, u depends particular print does not vary with When the temperatures of only on a and not on time t. $d^2 \frac{\partial u}{\partial n^2} = 0$ [$\frac{\partial u}{\partial t} = 0$ Arince u is a function $\frac{\partial u}{\partial t} = 0$ function $\frac{\partial u}{\partial t} = 0$ Arince u is a function $\frac{\partial u}{\partial t} =$ function of K only Since u is a function of above equation can be written as $\frac{d^2u}{dx^2} = 0$ ($d \neq 0$) Hence when steady state conditions prevails the heat flow equation becomes woods augment of sun and daz ovat mulbred m' tince, or we get some integrating ean 6 w.r to no tune and to require u(n) = antb

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The boundary Conditions are and book and
                                     A box, locm long, with insulated cooking of (i) ends
                                          A and 8 kept at 20 ... 40 C... 40 E (01) (11)
                               Steady - Atote Conditional get of the temporal of the temporal of the pully spirit on the proof of the proof 
       comparation of A is suffered to loc.

Lemperature of A is suffered to 
                                Applying b.c (ii) in ean AP, we get and
                              ucro) = acro) +ao = 40 insorting off
                                                                                                                 10a = 20 \Rightarrow a = 2
                                                sub a=a in agn & , we get a wolf toont
     Shoots of Ju(x) = Ax +20 mod bond what plant of
When the temperatures at A and B are Changed, the
State is no longer steady. Then the temperature
  function u(n/t) satisfies @
                       The boundary anditions in the second state are
                           (0$ b) u(0,t) = 50 ++70 + 50 & 4 som2
                                                                   u(10,t) = 10 4 + 170 mou ad no mutaus
                      The initial temperature of this state is the temperature
                 The initial temperature of Hence the initial in the previous steady - state. Hence the initial
                  Condition
                                                                                 U(1,0) = ax + a0 for OLXL10.
                              Since non-zero boundary conditions have infinite
                                                                                                                                                                              integrating can (6) w.r
                                number of values for A & B]
                                                                                                                                                                       dtap = (N) U
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Therefore, in this case, we split the solution
                           unt drift parts.
 u(n)+) into two parts.
      1e, u(nit) = usin) + ut(nit)
   us(a) is a steady state solution of @
   4+(1,+) & a transient solution which decreases
with increase of t.
To find steady state temperature us(x):
                                    a solution
The Boundary Conditions are
                                   a function of
 (i) Us(0) = 50
 (ii) uo(10,t) = 10.
  The steady state temperature is given by
        Usin) = aix+bi -> (II)
 Applying boundary Condition (i) in eam (ii), we get
          uo(0) = 9,(0) +b1 = 50
                  b1 = 50
   sub. (b) in ear (1), we get u_3(x) = a_1x + 50
  Applying b.c (ii) in em III) we get
          us(10) = a1(10)+50=10
                   1091 = -40
                  top |91 = -4|
 Sub \widehat{\mathbf{a}}_{1} in \widehat{\mathbf{m}}_{2}, we get [u_{3}(\mathbf{n}) = -4\mathbf{n} + 50]
                            MU(X) = dix + 20
To find U+(1/t): 100 000 (III)
                              is a tramient solution of
   we assume that ut (x1t)
   \frac{\partial u}{\partial t} = d^2 \frac{\partial u}{\partial x^2} and satisfying the equation
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4 $u(n_1t)=u_3(n)+u_t(n_1t)$... $u_t(x_1t) = u(x_1t) - u_s(x)$ we have to find the boundary conditions for 4E(11, E) Putting n=0 m IV, we get $u_{\xi}(0,\xi) = u(0,\xi) - u_{\xi}(0) = 50 - 50 = 0$. inf (01+) = 0 . Putting N=l m iv, we get $u_{E}(l,t) = u(l,t) - u_{S}(l) = 10-10 = 0$. :. ut(1,t)=0 . Putting t=0 m (iv), we get u= (110) = u(110) - us(110) = 2x+20 - (-4x+50) = 21+20+411-50 = 6x - 30.. ut(x10) = 6x-30 Now for the function ut (xx) we have the following boundary Conditions. (i) Ut (o,t) =0 +t

(ii) $U_{t}(0,t)=0$ Yt (iii) $U_{t}(1,t)=0$ Yt (iii) $U_{t}(1,t)=0$ for $0 \le 1 \le 10$

The suitable solution is $y(x,t) = (A \cos \lambda x + B \sin \lambda x) e^{-d^2 \lambda^2 t}$

Applying b.c (i) in eqn \mathbb{O} , we get $u_{t}(o_{t}) = Ae^{-d^{2}h^{2}t} = 0.$

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either A=0 for e^{-\alpha^2A^2t}=0. (5)
    e-d2/12t =0 (: It is defined +t)
              (1) A red seem long has its ends A ond O = A
   Sub A=0 m em O, we get landoque so boo son
 u(n,t) = Bonnine - d2/24 - Double and on lines
  Applying b.c (ii) in ear (3), we get his so of homeson
     u(10,t) = B min 10 / e - d^2/2t = 0. British (dir) indirectly
        here, B to ( B=0, we get a birial
                                           soldten)
           e^{-d^2h^2t} \pm o (? it is defined +t).
     Aug To
   abrogat w minio 1 = 0.
              Anno A = Mnnth prov ton 2004 take required
                   only on a and not on time an expos
  Jub \cdot J = \frac{n\pi}{10} in egn \mathfrak{D}, we get
     u(n_1t) = B \sin \frac{n\pi n}{10} e^{-\frac{d^2}{100}}
    u(n_1t) = B_n \sin \frac{n\pi u}{10} e^{-\frac{\lambda^2 n^2 n^2 t}{100}} \text{ where } B = B_n, t
u(n_1t) = B_n \sin \frac{n\pi u}{10} e^{-\frac{\lambda^2 n^2 n^2 t}{100}} \text{ where } B = B_n, t
B_n \text{ is any constant}
                                            Bn is any constant.
       Mirror meralle
     most general som. is __d2n2-n2t
      u(x+t) = 8 Bn min none 100
Applying b.c (iii) in ean 3, we get
    u(n_{10}) = \sum_{n=1}^{\infty} B_n \sin_{n\pi} \frac{n\pi}{10} (1) = 6x - 30 
   find Bn expand 6x-30 in half-range mine
senes m (0,10)
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$$bn = \frac{a}{10} \int_{0}^{10} \varphi(x) \sin \frac{n\pi x}{10} dx$$

$$bn = \frac{a}{10} \int_{0}^{10} \varphi(x) \sin \frac{n\pi x}{10} dx$$

$$Bn = \frac{3}{10} \int_{0}^{10} (6x - 30) \frac{n \ln n \ln n}{10} dx$$

$$=\frac{1}{5}\left[\frac{(6\pi-30)}{\frac{10}{10}}\left(\frac{-\cos\frac{n\pi x}{10}}{\frac{n\pi}{10}}\right)-6\left(\frac{-\sin\frac{n\pi x}{10}}{\frac{(n\pi)^2}{10}}\right)\right]_0$$

$$=\frac{1}{5}\left(30)\left(\frac{10}{n\pi}\right)\left(-(-1)^{n}\right)-\left(-30\right)\left(\frac{10}{n\pi}\right)(-1)\right]$$

$$= \frac{1}{5} \left[\frac{-300}{n\pi} \left(-1 \right)^n - \frac{300}{n\pi} \right]$$

$$=\frac{1}{5}\left[\frac{-360}{n\pi}\right]\left[\left(-1\right)^{n}+1\right]$$

$$=\frac{-60}{n\pi}(1+(-1)^n)$$

sub in ean 3, we get

$$(1+(1)^n)$$
 Ann $(1+(1)^n)$ Ann $(1+(1)^n)$ Ann $(1+(1)^n)$ Ann $(1+(1)^n)$ $(1+(1)^n)$

$$u(n_1t) = u_{\Delta}(n) + u_{t}(n_1t)$$

$$u(n_1t) = 50 - 4nt + \frac{5}{n_1} \frac{-60}{n_1} (1 + (+1)^n) \frac{4n_1n_1n_1}{10} e^{-\frac{3^2n_1^2n_1^2t}{100}}$$

is temperature durtribution.

Exercise Problem

1. A rod AB of length 10 cm. has its ends A and B kept at temperature 30°C and 100°C respectively until the steady-state conditions prevail. At sometime later, the temperature at A is lowered ato 20°C and that at B to 40°C, and then these temperatures are maintained. Find the subsequent temperature distribution.