Reg. No.															

B.Tech. DEGREE EXAMINATION, MAY 2017

Third / Fourth Semester

15MA209 – PROBABILITY AND RANDOM PROCESS

(For the candidates admitted during the academic year 2015 – 2016 onwards)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. The distribution function of a random variable X is given by $F(x)=1-(1+x)e^{-x}, x\geq 0$ then its density function is
   (A) $(1+x)e^{-x}+e^{-x}, x\geq 0$
   (B) $xe^{-x}, x\geq 0$
   (C) $xe^{x}, x\geq 0$
   (D) $\frac{1}{1+x}e^{-x}, x\geq 0$

2. The random variable X has a binomial distribution with parameters n=20, p=0.4 then its mean is
   (A) 8
   (B) 80
   (C) 0.8
   (D) 50

3. If $\text{var}(X)=4$ then find $\text{var}(4X+5)$ where X is a random variable
   (A) 64
   (B) 16
   (C) 21
   (D) 69

4. If F is the distribution function of the random variable X and if a<b then $P(a<X\leq b)=$
   (A) $p(X=a)+F(b)-F(a)$
   (B) $F(b)-F(a)-p(x=b)$
   (C) $p(a<x<b)+p(x=a)$
   (D) $F(b)-F(a)$

5. If F(x, y) is the cdf of a two dimensional continuous random variable, then f(x, y) is obtained from f by
   (A) $f(x,y)=\frac{\partial F}{\partial x}$
   (B) $f(x,y)=\frac{\partial F}{\partial y}$
   (C) $f(x,y)=\frac{\partial^2 F}{\partial x\partial y}$
   (D) $f(x,y)=\iint F(x,y)\,dx\,dy$

6. Two discrete jointly distributed random variable x and y are independent if
   (A) $p_{ij}=p_{oj}$
   (B) $p_{ij}=p_{i*}\times p_{*j}$
   (C) $p_{ij}=p_{io}$
   (D) $p_{ij}=1$

7. If $x=\frac{u}{v}$ and $y=v$ then $J\left(\frac{x,y}{u,v}\right)$ is

(A) $\frac{1}{u}$
(B) $\frac{1}{v}$
(C) v
(D) u

8. If mean = 2, standard deviation is $\sqrt{2}$ for each of n independent random variables with n=75 then by Central limit theorem S_n follows.

(A) S_n follows N(150, $\sqrt{150}$)
(B) S_n follows N($\sqrt{150}$, 150)
(C) S_n follows N(150, 150)
(D) S_n follows N($\sqrt{2}$, $\sqrt{2}$)

9. If x(t) represents the number of occurrence of a certain event is (0, t) then the discrete random process {x(t)} is called a

(A) Renewal process
(B) Poisson process
(C) Markov process
(D) Exponential process

10. If $p_{ij}^{(n)} > 0$ for same n and for all i and j then every other state can be reached from every other state then the Markov chain is said to be

(A) Irreducible
(B) Reducible
(C) Transient
(D) Persistent

11. If S denotes the state space and T the parameter set then a random process in which T is discrete and S is continuous is called

(A) Continuous random sequence of second order
(B) Continuous random process
(C) Discrete random sequence
(D) Continuous random sequence

12. If certain probability distributions do not depend on t then the random process {x(t)} is called

(A) Strongly stationary process
(B) Stationary process
(C) Wide sense stationary process
(D) Markov process

13. R(τ) is maximum at

(A) $\tau=-1$
(B) $\tau=1$
(C) $\tau=0$
(D) $\tau=2$

14. A stationary process has auto correlation function given by $R(\tau)=\frac{25\tau^2+36}{6.25\tau^2+4}$ then its mean value is

(A) 6
(B) 5
(C) 4
(D) 2

15. $R_{XY}(-\tau)=$

(A) $-R_{XY}(\tau)$
(B) $R_{XY}(\tau)$
(C) $R_{YX}(\tau)$
(D) $-R_{XY}(-\tau)$

16. If {X(t)} and {Y(t)} are independent WSS process with zero mean then the autocorrelation function of {z(t)} where $z(t) = aX(t)y(t)$ is

(A) $aR_{XX}(\tau)R_{YX}(\tau)$
(B) $a^2R_{XX}(\tau)R_{YY}(\tau)$
(C) $\sqrt{a}R_{XX}(\tau)R_{YY}(\tau)$
(D) $\sqrt{a}\, R_{XX}(\tau)$

17. Real $S_{XY}(\omega)$ and $S_{YX}(\omega)$ are _________ functions of ω

(A) Linear
(B) Even
(C) Odd
(D) Complemented

18. The mean square value of the process {X(t)} is

(A) $R_{XX}(\tau)$
(B) $R_{XX}(-\tau)$
(C) $-R_{XX}(-0)$
(D) $R_{XX}(0)$

19. If the value of the output Y(t) at a time t-t_1 depends only on $x(t_1)$ and not an any other value then the system is called

(A) Casual system
(B) Time invariant system
(C) Memoryless system
(D) Time dependent system

20. The power spectral density of a random signal with autocorrelation function $e^{-2\lambda|\tau|}$ is

(A) $\lambda / \lambda^2 + \omega^2$
(B) $\omega / \lambda^2 + \omega^2$
(C) $4\lambda / \lambda^2 + \omega^2$
(D) $4\lambda / \left(4\lambda^2 + \omega^2\right)$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. If X has the probability distribution

x	−1	0	1	2
p(x)	0.3	0.1	0.4	0.2

Find $E(X), E\left(X^2\right)$, var(X) and var$(2X+1)$

22. If the continuous random variable X has the pdf $f_X(x) = \frac{2}{9}(x+1)$ in $-1 < x < 2$ and $= 0$ elsewhere find the pdf of $y=x^2$.

23. The following table against the joint probability distribution of X and Y. Find the marginal probability functions of X and marginal probability function of Y.

Y \ X	1	2	3
1	0.1	0.1	0.2
1	0.2	0.3	0.1

24. Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is not stationary. If A and ω_0 all constants and θ is uniformly distributed random variable in (0, π).

25. Find the mean and variance of the stationary process {X(t)} whose autocorrelation function is given by $R(\tau) = 16 + \frac{9}{1+6\tau^2}$.

26. The power spectral density function of zero mean WSS process {X(t)} is given by $S(\omega)=\begin{cases}1, & |\omega|<\omega_0 \\ 0, & otherwise\end{cases}$. Find $R(\tau)$.

27. For a real random process {X(t)}, show that $S_{XX}(\omega)$ is an even function.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i For a continuous random variable X, the cdf is given by

$$F(x)=\begin{cases}0 & if\ x<2 \\ k(x-2) & 2\le x<6 \\ 1 & if\ x\ge 6\end{cases}$$

Find (i) the pdf of X (ii) the value of k (iii) P(X>4) (iv) P(3<X<5).

ii Find the moment generating function of the random variable X, whose probability function $p(x)=\frac{1}{2^x}$ $x=1,2,3,....$ Hence find the mean and variance.

(OR)

b.i. The life of certain kind of electronic device has a mean of 300hrs and standard deviation of 25hrs. Assuming that the lifetime of the devices follow normal distribution. Find the probability that any one of these devices will have a life time more than 350hrs. What percentage will have life time between 220 and 260 hrs?

ii. The first four moments of a distribution about $x=4$ are $1,4,10,45$. Find mean, variance μ_3 and μ_4.

29. a.i If X and Y each follow an exponential distribution with parameters one and are independent, find the pdf of U = X–Y.

ii. The life time of a certain kind of electric bulb may be considered as a random variable with mean 1200 hrs and standard deviation 250 hrs. Find the probability using Central limit theorem that the average lifetime of 60 bulbs exceeds 1250 hrs.

(OR)

b. The joint pdf of a two dimensional random variable (X, Y) is given by $f(x,y)=xy^2+x^2/8\ \ 0\le x\le 2, 0\le y\le 1$. Compute $P(X>1), P(Y\le 1/2), P(X>1/Y<1/2$ $P(Y<1/2/X>1), P(X<Y)$ and $P(X+Y\le 1)$.

30. a. Show that the process $X(t)=A\cos\lambda t+B\sin\lambda t$ where A and B are random variables is wide sense stationary if (i) $E(A)=E(B)=0$ (ii) $E(A^2)=E(B^2)$ and (iii) $E(AB)=0$.

(OR)

b. The transition probability matrix of a Markov chain $\{X_n\}$ n=1,2,3,.... having three states 1,2,3 is $P=\begin{pmatrix}0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3\end{pmatrix}$ and the initial distribution is $P^{(0)}=\begin{pmatrix}0.7 & 0.2 & 0.1\end{pmatrix}$.

Find (i) $P(X_2=3)$ (ii) $P(X_3=2, X_2=3, X_1=3, X_0=2)$.

31. a. Consider two random processes $X(t) = 3\cos(\omega t + \theta)$ and $y(t) = 2\cos(\omega t + \theta - \pi/2)$ where θ is a random variable uniformly distributed in (0, 2π). Prove that $\sqrt{R_{XX}(0)\ R_{YY}(0)} \geq |R_{XY}(\tau)|$.

(OR)

b.i. The cross power spectrum of a real random process {X(t)} and {Y(t)} is given by $S_{XY}(\omega) = \begin{cases} a + jb\omega, & |\omega| < 1 \\ 0 & , elsewhere \end{cases}$. Find the cross correlation function.

ii. If $y(t) = X(t+a) - X(t-a)$ where X(t) is a WSS process then show that $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau - 2a) - R_{XX}(\tau + 2a)$

32. a. A wide sense stationary process X(t) is the input to a linear system with impulse response $h(t) = 2e^{-7t}, t \geq 0$. If the autocorrelation function of X(t) is $R_{XX}(\tau) = e^{-4|\tau|}$. Find the power spectral density of the output process Y(t).

(OR)

b.i. Find the power spectral density of the random process if its autocorrelation function is given by $R_{XX}(\tau) = e^{-\alpha|\tau|} \cos B\tau$.

ii. The short time moving average of a process {X(t)} is defined as $Y(t) = \frac{1}{T}\int_{t-T}^{t} X(s)\,ds$.

Prove that X(t) and Y(t) are related by means of convolution type integral. Find unit impulse response of the system also.

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