# Quadratic Equation

Week-3

# Quadratic Equation
# $2^{nd}$ Degree Polynomial Equation in X = 0
# The coefficient of $X^2 \neq 0$

## General Form

- $ax^2 + bx + c = 0$ where $a \neq 0$ and a, b, c are real
- 2 Values of x satisfy the Equation.
- They are called Roots (Real or Imaginary)

## General Method

- Roots can be found by
- (i) Factorising the expression
- (ii) Using standard formula

# $ax^2 + bx + c = 0$ where $a \neq 0$ and a, b, c are real

## Sum of the Roots = -b/a

## Product of the Roots = c/a

### Finding Roots by Factorisation

- Write in the form $(x-\alpha)(x-\beta) = 0$
- The roots of the equation are $\alpha, \beta$
- Eg: $x^2 - 5x + 6 = 0$
- $(x-3)(x-2) = 0$
- The Roots are 3, 2

### Finding Roots by Formula

- $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- Eg: $x^2 - 5x + 6 = 0$
- $x = \frac{-(-5) \pm \sqrt{(-5)^2-4(1)(6)}}{2(1)}$
- $x = \frac{5 \pm 1}{2} \rightarrow x = 3, 2$

# Nature of the Roots

## $ax^2 + bx + c = 0$ where $a \neq 0$

## $b^2 - 4ac$ is called "DISCRIMINANT"

| S.No. | Discriminant | Roots (when a, b, c are real) | Roots (when a, b, c are rational) |
|---|---|---|---|
| 1 | $b^2 - 4ac < 0$ | Complex Conjugates | Complex and Unequal |
| 2 | $b^2 - 4ac = 0$ | Real and Equal | Rational and Equal |
| 3 | $b^2 - 4ac > 0$ and a perfect Square | Real and Distinct | Rational and Unequal |
| 4 | $b^2 - 4ac > 0$ but not a perfect Square | Real and Distinct | Irrational and Unequal |

# Find the roots of the Equation

$x^2 + 10x + 24 = 0$

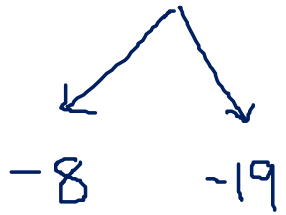

$$x(x+4) + 6(x+4) = 0$$

$$(x+4)(x+6) = 0$$

$$x = -4, -6$$

# Find the roots of the Equation

$1x^2 - 27x + 152 = 0$




Roots = $\frac{8}{1}, \frac{19}{1}$

| | |
|---|---|
| 2 | 152 |
| 2 | 76 |
| 2 | 38 |
| 19 | 19 |
| | 1 |

$152 = 2^3 \times 19^1$

$\downarrow \quad \downarrow$

$8 \quad 19$

# Find the roots of the Equation

$x^2 + 38x + 360 = 0$

$360 = 3^2 \times 5 \times 2^3$

$3^2, 5 \times 2^3 = 9, 40$

$3^2 \times 2, 5 \times 2^2 = 18, 20$

$-18, -20$

# Find the roots of the Equation

$15x^2 - 44x + 21 = 0$

$15 \times 21 = 5 \times 3 \times 7 \times 3$

$= -35 \times -9$

Roots $= \frac{35}{15}, \frac{9}{15}$

$= \frac{7}{3}, \frac{3}{5}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{44 \pm \sqrt{1936 - 1260}}{30}$$

$$= \frac{44 \pm 26}{30}$$

$$= \frac{7}{3}, \frac{3}{5}$$

# A and B are the roots of the Equation $x^2 - 22x + 120 = 0$. Find the value of $A^2 + B^2$

$S = 22 \quad P = 120$

$A, B = 10, 12$

$A + B = 22 \quad AB = 120$

$$A^2 + B^2 = (A+B)^2 - 2AB$$

$$= 22^2 - 2(120)$$

$$= 244$$

# A and B are the roots of a Quadratic Equation. (A+B=22, A*B=120. Find the value of $A^2 + B^2$

$$(A+B)^2 = 22^2$$

$$A^2 + B^2 + 2AB = 22^2$$

$$A^2 + B^2 = 484 - 240 = 244$$

# Comment on the nature of the roots of $8x^2 - 2x - 4 = 0$

$b^2 - 4ac = 132 > 0$ Irrational, Unequal

↑ Not a perf square

# Comment on the nature of the roots of $3x^2 - x - 4 = 0$

$b^2 - 4ac = 49 > 0$ Rational, Equal

↑ perf square

# Form a Quadratic Equation whose roots are 7 and 3

$x^2 - 10x + 21 = 0$

Sum = 10

Product = 21

If p + $\sqrt{q}$ is one root of a quadratic equation with rational coefficients, then find the other root.

$p - \sqrt{q}$

# Form a Quadratic Equation with rational coefficients one of whose roots is $4 + \sqrt{7}$

$4 - \sqrt{7}$

Sum = 8

Product = 9

$x^2 - 8x + 9 = 0$