D1 - slot

PART-A

1. a. 2
2. b. 0.
3. d. $|R_{xy}(\tau)| \le \frac{1}{2}[R_{xx}(0)+R_{yy}(0)]$
4. a. $R_{xx}(\tau)+R_{yy}(\tau)+R_{xy}(\tau)+R_{yx}(\tau)$
5. b. 149
6. d. $\frac{\sin\tau}{\tau}$
7. b. ~~Odd function~~ Even function
8. b. $e^{-3|\tau|}$
9. c. $f[x(t+\tau)]$
10. b. 1

PART-B

11. $\phi(\omega) = E[\cos\omega Y] + i\,E[\sin\omega Y]$

$\phi(1) = \phi(2) = 0 \Rightarrow E[\cos Y] = 0 = E[\sin Y]$ — (2M)

$E[\cos 2Y] = 0 = E[\sin 2Y]$

$E[X(t)] = E[\cos(\lambda t + Y)]$

$= E[\cos\lambda t \cos Y - \sin\lambda t \sin Y]$

$= \cos\lambda t\, E[\cos Y] - \sin\lambda t\, E[\sin Y]$ — (2M)

$= 0$, a constant

$R_{xx}(\tau) = E[X(t)X(t+\tau)]$

$= E[\cos(\lambda t+Y)\cos(\lambda t+\lambda\tau+Y)]$

$= \frac{1}{2}E[\cos(2\lambda t+\lambda\tau+2Y) + \cos\lambda\tau]$

$= \frac{1}{2}E[\cos(2\lambda t+\lambda\tau)\cos 2Y - \sin(2\lambda t+\lambda\tau)\sin 2Y + \cos\lambda\tau]$

$= \frac{1}{2}\{\cos(2\lambda t+\lambda\tau)E[\cos 2Y] - \sin(2\lambda t+\lambda\tau)E[\sin 2Y] + \cos\lambda\tau\}$ — (4M)

$R_{xx}(\tau) = \frac{\cos\lambda\tau}{2}$, $\{X(t)\}$ is a WSS process.

— (2M)

12. $R_{YY}(\tau) = E[Y(t)\, Y(t+\tau)]$

$= E[(X(t+a) - X(t-a))(X(t+\tau+a) - X(t+\tau-a))]$

$= E[X(t+a)\, X(t+\tau+a) - X(t+a)\, X(t+\tau-a)$
$\quad - X(t-a)\, X(t+\tau+a) + X(t-a)\, X(t+\tau-a)]$

$= E[X(t+a)\, X(t+\tau+a)] - E[X(t+a)\, X(t+\tau-a)]$
$\quad - E[X(t-a)\, X(t+\tau+a)] + E[X(t-a)\, X(t+\tau-a)]$ —— (5M)

$= E[X(t+a)\, X(t+a+\tau)] - E[X(t+a)\, X(t+a+\tau-2a)]$
$\quad - E[X(t-a)\, X(t-a+\tau+2a)] + E[X(t-a)\, X(t-a+\tau)]$

$= R_{XX}(\tau) - R_{XX}(\tau-2a) - R_{XX}(\tau+2a) + R_{XX}(\tau)$

$= 2R_{XX}(\tau) - R_{XX}(\tau-2a) - R_{XX}(\tau+2a)$ —— (5M)

13. $R_{XX}(0) = E[X^2(t)]$

$= E[9\cos^2(\omega t+\theta)]$

$= \frac{9}{2} E[1 + \cos 2(\omega t+\theta)]$

$= \frac{9}{2} \{ E[1] + E[\cos(2\omega t + 2\theta)] \}$

$E[\cos(2\omega t+2\theta)] = \int_0^{2\pi} \cancel{X(t)} \cos(2\omega t+2\theta) \frac{1}{2\pi} d\theta$

$= 0$

$\therefore \quad R_{XX}(0) = \frac{9}{2}$ —— (2M)

$$R_{yy}(0) = E[y^2(t)]$$

$$= E[4 \sin^2(\omega t+\theta)]$$

$$= \frac{4}{2} E[1 - \cos(2\omega t+2\theta)]$$

$$= 2\{ E[1] - E[\cos(2\omega t+2\theta)]\}$$

$$E[\cos(2\omega t+2\theta)] = \int_0^{2\pi} \cos(2\omega t+2\theta)\frac{1}{2\pi}\, d\theta = 0$$

$$\therefore \quad R_{yy}(0) = 2 \qquad \text{(2M)}$$

$$\Rightarrow \quad R_{xx}(0)\, R_{yy}(0) = \frac{9}{2} \times 2 = 9$$

$$\Rightarrow \quad \sqrt{R_{xx}(0)\, R_{yy}(0)} = 3 \qquad \text{(1M)}$$

$$R_{xy}(\tau) = E[x(t)\, y(t+\tau)]$$

$$= E[3\cos(\omega t+\theta) \cdot 2\sin(\omega t+\omega\tau+\theta)]$$

$$= 3\, E[\sin(2\omega t+\omega\tau+2\theta) - \sin(-\omega\tau)]$$

$$= 3\{E[\sin(2\omega t+\omega\tau+2\theta)] + E[\sin\omega\tau]\}$$

$$E[\sin(2\omega t+\omega\tau+2\theta)] = \int_0^{2\pi} \sin(2\omega t+\omega\tau+2\theta)\frac{1}{2\pi}\, d\theta$$

$$= 0$$

$$R_{xy}(\tau) = 3\sin\omega\tau \qquad \text{(4M)}$$

$$\Rightarrow \quad |R_{xy}(\tau)| = 3|\sin\omega\tau| \le 3 = \sqrt{R_{xx}(0)\, R_{yy}(0)}$$

$$\therefore \quad |R_{xy}(\tau)| \le \sqrt{R_{xx}(0)\, R_{yy}(0)} \qquad \text{(1M)}$$

14. $R_{xx}(\tau) = \frac{1}{2\pi}\int_{-a}^{a} \frac{b}{a}(a-|\omega|)\, e^{i\omega\tau}\, d\omega$ —— (2M)

$= \frac{b}{\pi a}\int_{0}^{a} (a-\omega) \cos\omega\tau \, d\omega$

$\because \int_{-a}^{a}(a-|\omega|) \sin\omega\tau \, d\omega = 0$

$= \frac{b}{\pi a}\left[(a-\omega) \underbrace{\frac{\sin\omega\tau}{\tau}}_{\to 0} - (-1)\left(-\frac{\cos\omega\tau}{\tau^2}\right)\right]_0^a$ —— (2M)

$= -\frac{b}{\pi a\tau^2}\left[\cos\omega\tau\right]_0^a$

$= -\frac{b}{\pi a\tau^2}\left[\cos a\tau - 1\right]$

$= \frac{b}{\pi a\tau^2}(1-\cos a\tau)$ —— (6M)

$R_{xx}(\tau) = \frac{2b}{\pi a\tau^2}\sin^2\frac{a\tau}{2}$

15. $S_{xx}(\omega) = F[R_{xx}(\tau)]$ —— (2M)

$= \int_{-\infty}^{\infty} e^{-\lambda|\tau|}\cos\beta\tau \; e^{-i\omega\tau}\, d\tau$

$= \int_{-\infty}^{0} e^{(\lambda-i\omega)\tau}\cos\beta\tau\, d\tau + \int_{0}^{\infty} e^{-(\lambda+i\omega)\tau}\cos\beta\tau\, d\tau$ (2M)

$= \left[\frac{e^{(\lambda-i\omega)\tau}}{(\lambda-i\omega)^2+\beta^2}\left((\lambda-i\omega)\cos\beta\tau+\beta\sin\beta\tau\right)\right]_{-\infty}^{0}$

$+ \left[\frac{e^{-(\lambda+i\omega)\tau}}{(\lambda+i\omega)^2+\beta^2}\left(-(\lambda+i\omega)\cos\beta\tau+\beta\sin\beta\tau\right)\right]_{0}^{\infty}$ —— (4M)

$$S_{XX}(\omega) = \frac{\lambda - i\omega}{(\lambda-i\omega)^2+\beta^2} + \frac{\lambda+i\omega}{(\lambda+i\omega)^2+\beta^2} \quad \text{— (2M)}$$

16. $$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-i\omega t}\, dt$$

$$= \int_0^{\infty} 2e^{-t}\, e^{-i\omega t}\, dt$$

$$= 2\int_0^{\infty} e^{-(1+i\omega)t}\, dt$$

$$= 2\left[\frac{e^{-(1+i\omega)t}}{-(1+i\omega)}\right]_0^{\infty}$$

$$H(\omega) = \frac{2}{1+i\omega} \Rightarrow |H(\omega)|^2 = \frac{4}{1+\omega^2} \quad \text{— (4M)}$$

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-i\omega\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-2|\tau|}(\cos\omega\tau - i\sin\omega\tau)\, d\tau$$

$$= 2\int_0^{\infty} e^{-2\tau}\cos\omega\tau\, d\tau$$

$$\left\| \because \int_{-\infty}^{\infty} e^{-2|\tau|}\sin\omega\tau\, d\tau = 0\right.$$

$$= 2\left[\frac{e^{-2\tau}}{(-2)^2+\omega^2}(-2\cos\omega\tau + \omega\sin\omega\tau)\right]_0^{\infty}$$

$$S_{XX}(\omega) = \frac{4}{4+\omega^2} \quad \text{— (4M)}$$

$$S_{YY}(\omega) = |H(\omega)|^2\, S_{XX}(\omega)$$

$$S_{YY}(\omega) = \frac{16}{(1+\omega^2)(4+\omega^2)} \quad \text{— (2M)}$$