UNIT IV TUTORIAL 2

Answer all the questions

PART B

1. Find $F_s(e^{-5x}\sin 2x)$

2. Find $F_s(e^{-5x}\cos 2x)$

3. Find $F_c(e^{-5x}\sin 2x)$

4. Find $F_c(e^{-5x}\cos 2x)$

5. Find $F_s(xe^{-5x})$ and $F_c(xe^{-5x})$

6. State convolution of two functions in Fourier transforms.

7. If $F_c(e^{-a^2x^2}) = \frac{1}{a\sqrt{2}}e^{-\frac{s^2}{4a^2}}$,Find $F_c(e^{-9a^2x^2})$ using change of scale property.

PART C

8. Find $F_s(e^{-ax})$ and $F_c(e^{-ax})$ and hence derive the inversion formula.

9. Find $F_c(e^{-a^2x^2})$ and hence find If $F_s(xe^{-a^2x^2})$.

10. Find $F_s(\frac{e^{-ax}}{x})$ and use it to evaluate $\int_0^\infty tan^{-1}(\frac{x}{a})$ sinx dx.

11. State and prove convolution theorem in fourier transforms.

12. Find the function if its sine transform is $\frac{e^{-as}}{s}$.

13. Find $F_c(\frac{1}{1+x^2})$.

14. Prove that $F_s(xf(x))= -\frac{d}{ds}(F_c(s))$ and $F_c(xf(x))= \frac{d}{ds}(F_s(s))$

ANSWERS FOR THE QUESTIONS IN TUTORIAL 2.

1. $F_s(e^{-5x}\sin 2x) = \frac{1}{2}\{F_c(s-2) - F_c(s+2)\} = \frac{1}{2}\left\{\frac{\sqrt{2}}{\sqrt{\pi}}\frac{5}{(s-2)^2+25} - \frac{\sqrt{2}}{\sqrt{\pi}}\frac{5}{(s+2)^2+25}\right\}$

2. $F_s(e^{-5x}\cos 2x) = \frac{1}{2}\{F_s(s+2) + F_s(s-2)\} = \frac{1}{2}\left\{\frac{\sqrt{2}}{\sqrt{\pi}}\frac{s+2}{(s+2)^2+25} + \frac{\sqrt{2}}{\sqrt{\pi}}\frac{s-2}{(s-2)^2+25}\right\}$

3. $F_c(e^{-5x}\sin 2x) = \frac{1}{2}\{F_s(s+2) - F_s(s-2)\} = \frac{1}{2}\left\{\frac{\sqrt{2}}{\sqrt{\pi}}\frac{s+2}{(s+2)^2+25} + \frac{\sqrt{2}}{\sqrt{\pi}}\frac{s-2}{(s-2)^2+25}\right\}$

4. $F_c(e^{-5x}\cos 2x) = \frac{1}{2}\{F_c(s+2) + F_c(s-2)\} = \frac{1}{2}\left\{\frac{\sqrt{2}}{\sqrt{\pi}}\frac{5}{(s+2)^2+25} + \frac{\sqrt{2}}{\sqrt{\pi}}\frac{5}{(s-2)^2+25}\right\}$

7. $F_c\left(e^{-(3a)^2x^2}\right) = \frac{1}{3}F\left(\frac{s}{3}\right) = \frac{1}{3}\frac{1}{a\sqrt{2}}e^{-\frac{s^2}{36a^2}}$.

9. $F_c\left(e^{-a^2x^2}\right) = \frac{1}{a\sqrt{2}}e^{-\frac{s^2}{4a^2}}$ and $F_s\left(xe^{-a^2x^2}\right) = \frac{s}{2\sqrt{2}a^3}e^{-\frac{s^2}{4a^2}}$

10. $F_s\left(\frac{e^{-ax}}{x}\right) = \frac{\sqrt{2}}{\sqrt{\pi}}\tan^{-1}\left(\frac{s}{a}\right)$ and $\int_0^\infty \tan^{-1}\left(\frac{s}{a}\right)\sin x\,dx = \frac{\pi}{2}e^{-a}$

12. $F^{-1}\left(\frac{e^{-as}}{s}\right) = \frac{\sqrt{2}}{\sqrt{\pi}}\tan^{-1}\left(\frac{x}{a}\right)$.

13. $F_c\left(\frac{1}{1+x^2}\right) = \frac{\sqrt{\pi}}{\sqrt{2}}e^{-s}$