



DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB101J-Electromagnetic Theory, Quantum Mechanics, Waves and Optics

Module 3- Lecture-7

Time-independent Schrodinger's wave equation



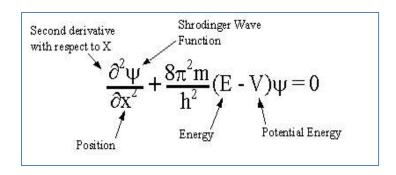


Topics to be Discussed:-

Time-independent Schrodinger's wave equation







Time-Independent Schroedinger Equation in One Dimension





A variable quantity which characterizes de-Broglie waves is known as Wave function and is denoted by the symbol ψ . The value of the wave function associated with a moving particle at a point (x, y, z) and at a time 't' gives the probability of finding the particle at that time and at that point.

Schrodinger Wave Equation

- It is one of the basic equations in quantum mechanics.
- a particle exhibits wave properties; then there should be some sort of wave equation associated with the particle describing the behavior of the particle.
- Schrodinger derived a mathematical equation to describe the dual nature of matter waves.
- The equation that describes the wave nature of a particle in mathematical form is known as Schrodinger wave equation
- Schrodinger connected expression for the de Broglie wavelength into the classical wave equation for a moving particle and obtained a new wave equation.
- •The Schrodinger is applicable for both microscopic and macroscopic particles.

 18PYB101J Module-III Lecture-4





DERIVATION Of Time-Independent Schroedinger Equation





Let us consider a particle of mass 'm', moving with a velocity 'v'. The de Broglie wavelength associated with it is given by,

$$\lambda = \frac{h}{mv}....(1)$$

where h = Planck's constant = $6.626 \times 10^{-34} \, \mathrm{J s}$.

Let ψ be the wave function of the particle along x, y and z coordinate at any time 't'. The classical differential equation of a progressive wave moving with a wave velocity 'v' can be written as,

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial v^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} \qquad (2)$$

The solution for the equation (2) is given by,

$$\psi = \psi_{\circ} e^{-i\omega t} \qquad (3)$$

where

 ψ_{ω} Amplitude of the wave at the point (x,y,z) $\omega = Angular frequency of the wave$





Differentiating eqn. (3) with respect to 't.

$$\frac{\partial \Psi}{\partial t} = (-i\omega)\Psi \circ e^{-i\omega t} \qquad(4)$$

Differentiating eqn. (4) with respect to 't.

$$\frac{\partial^2 \Psi}{\partial t^2} = (-i\omega)^2 \Psi_{\circ} e^{-i\omega t} = -\omega^2 \Psi \qquad (5)$$

Substituting eqn. (5) in eqn. (2),





$$\nabla^2 \psi = -\left(\frac{\omega^2}{v^2}\right) \psi \qquad ----(6)$$
where $\nabla^2 = \text{Laplacian operator}$

$$\omega = 2\pi f = 2\pi \left(\frac{v}{\lambda}\right) \quad (OR), \frac{\omega}{v} = \frac{2\pi}{\lambda}$$

$$\frac{\omega^2}{v^2} = \frac{4\pi^2}{\lambda^2} \qquad (7)$$





Substituting eqn. (7) in eqn. (6),

$$\nabla^2 \psi = -\left(\frac{4\pi^2}{\lambda^2}\right) \psi \qquad (OR).$$

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0$$
 (8)

Substituting eqn (1) in eqn (8),

$$\nabla^{2} \psi + \frac{4\pi^{2}}{\left(\frac{h}{mv}\right)^{2}} \psi = 0$$

$$\nabla^{2} \psi + \frac{4\pi^{2} m^{2} v^{2}}{h^{2}} \psi = 0$$
(9)





If 'E' is the total energy of the particle, 'V', the potential energy, then total energy of the particle = E = PE + KE

$$E = V + \frac{1}{2}$$
 $(OR), \qquad 2(E \qquad -V) = mV^2$

Multiplying both sides by 'm', in equation (10), we have

$$2m (E - V) = m^2 v^2$$
 (11)

Substituting eqn. (11) in eqn (9),

$$\nabla^2 \psi + \frac{8\pi^2 m}{\lambda^2} (E - V) \psi = 0$$

This equation is known as Schrödinger's time independent wave equation





Introducing, in the above equation (12)

$$\Box = \frac{h}{2\pi}$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{2\pi}(E - V)\psi = 0$$

This is called One Dimensional Schroedinger Equation (Time-Independent)

For three dimension

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$