# SRM Institute of Science and Technology
## Faculty of Engineering and Technology

**SET-A**

## DEPARTMENT OF ECE

SRM Nagar, Kattankulathur – 603203, Chengalpattu District, Tamilnadu

**Academic Year: 2022-2023 (ODD)**

**Test: CLAT-3** | **Date: 19/11/22**

**Course Code & Title: 18ECC204J-Digital Signal Processing** | **Duration: 08:00-09:40 AM**

**Year & Sem: III /V** | **Max. Marks: 50**

**Course Articulation Matrix:** ***(to be placed)***

| | 18ECC204J – Digital Signal Processing | Program Outcomes (POs) | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Graduate Attributes | | | | | | | | | | | | PSO | | |
| S. No. | Course Outcomes (COs) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 1 | 2 | 3 |
| 1 | Summarize the concepts of A//D and D/A converters. | 3 | - | - | 1 | - | - | - | - | - | - | - | - | - | - | 2 |
| 2 | Explain the concepts of DFT with its efficient computation by using FFT algorithm. | - | 2 | - | - | - | - | - | - | - | - | - | - | - | 1 | |
| 3 | Develop FIR filters using several methods | - | 2 | 3 | - | - | - | - | - | - | - | - | - | - | - | 3 |
| 4 | Construct IIR filters using several methods | - | | 3 | - | - | - | - | - | - | - | - | - | - | - | 3 |
| 5 | Discuss the basics of multirate DSP and its applications. | - | 2 | - | - | - | - | - | - | - | - | - | - | - | 1 | - |
| 6 | Design digital filter and multi rate signal processing for real time signals | - | 2 | - | - | - | - | - | - | - | - | - | - | 2 | - | - |

| Q. No | Answer with choice variable | Marks | BL | CO | PO |
|---|---|---|---|---|---|
| 1 | i)<br>$\varepsilon = 1 \qquad \lambda = 9.95$<br>Step 1: $N \geq \frac{\cosh^{-1} \lambda/\varepsilon}{\cosh^{-1} \frac{\Omega_s}{\Omega_p}} = \frac{\cosh^{-1} 9.95}{\cosh^{-1} 2} = 2.269$<br>(2 marks)<br>Step 2: $N \approx 3$ N odd, so oscillatory curve starts from unity<br>Step 3: $\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.414$<br>$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 2\left[\frac{(2.414)^{1/3} - (2.414)^{-1/3}}{2}\right] = 0.596$<br>$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 2\left[\frac{(2.414)^{1/3} + (2.414)^{-1/3}}{2}\right] \doteq 2.087$<br>(2 marks)<br>Step 4: To calculate poles of chebyshev filter<br>$\phi_k = \pi/2 + \frac{(2k-1)\pi}{2N} \quad k = 1, 2, 3$ | 9 | 3 | 4 | 3 |

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$\phi_1 = \pi/2 + \frac{\pi}{6} = 120, \quad \phi_2 = \pi/2 + \frac{3\pi}{6} = 180^\circ, \quad \phi_3 = \pi/2 + \frac{5\pi}{6} = 240^\circ$

$S_k = a\cos\phi_k + jb\sin\phi_k$

$S_1 = a\cos\phi_1 + jb\sin\phi_1 = -0.298 + j1.807$

$S_2 = -0.596$

$S_3 = -0.298 - j1.807$ (2 marks)

Step5: Denominator polynomial is given by

$(S+0.298-j1.807)(s+0.596)(s+0.298+j1.807)$

$= \left[(S+0.298)^2 + (1.807)^2\right](s+0.596)$

$= (s+0.596)(s^2+0.596s+3.265)$ (2 marks)

Step6: Numerator of H(s) obtained by substituting s=0 for Nodd in denom.

Numerator $= 0.596 \times 3.265 = 1.945 \approx 2$

TF $H(s) = \dfrac{2}{(s+0.596)(s^2+0.596s+3.265)}$ (1 mark)

ii) b) Unstable IIR filter
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i)

$H(s) = \dfrac{1}{(s+1)(s^2+s+1)}$

$s^2+s+1$

$\dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$= \dfrac{-1 \pm \sqrt{3}j}{2}$

$= -0.5 \pm j0.866$

$= \dfrac{A}{s+1} + \dfrac{B}{s+0.5+j0.866} + \dfrac{C}{s+0.5-j0.866}$

$A\left[(s+0.5+j0.866)(s+0.5-j0.866)\right] + B\left[(s+1)(s+0.5-j0.866)\right]$

$+ C\left[(s+1)(s+0.5+j0.866)\right] = 1$

If $s=-1$

$A\left[(-0.5+j0.866)(-0.5-j0.866)\right] = 1$

$A\left[(-0.5)^2 + (0.866)^2\right] = 1 \Rightarrow A = 1$

If $s = -0.5 - j0.866$

$B\left[(-0.5-j0.866+1)(-0.5-j0.866+0.5-j0.866)\right] = 1$

$B\left[(0.5-j0.866)(-1.732j)\right] = 1$

$B = -0.5 + 0.288j$ (3 marks)

$C = B^* = -0.5 - j0.288$
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|---|---|---|---|---|---|
| | $H(s) = \frac{1}{s+1} + \frac{-0.5+0.288j}{s+0.5+j0.866} + \frac{-0.5-j0.288}{s+0.5-j0.866}$<br>$= \frac{1}{s-(-1)} + \frac{-0.5+0.288j}{s-(-0.5-j0.866)} + \frac{-0.5-0.288j}{s-(-0.5+j0.866)}$ (2 marks)<br>Impulse invariant technique<br>If $H(s) = \sum_{k=1}^{N} \frac{C_k}{s-P_k}$ then $H(z) = \sum_{k=1}^{N} \frac{C_k}{1-e^{P_k T} z^{-1}}$<br>$H(z) = \frac{1}{1-0.368z^{-1}} + \frac{-1+0.66z^{-1}}{1-0.786z^{-1}+0.368z^{-2}}$ (4 marks)<br>ii) Entirely inside the unit circle | **1** | **1** | **4** | **1** |
| **3** | **i)**<br>Magnitude function of is given by<br>$\lvert H(j\Omega)\rvert = \frac{1}{[1+(\Omega/\Omega_c)^{2N}]^{1/2}}$<br>$\lvert H(j\Omega)\rvert^2 = \frac{1}{1+(\Omega)^{2N}}$ $N=1,2,3,\ldots$ Normalized Butterworth filter<br>To derive transfer function, substitute $\Omega = \frac{s}{j}$<br>$\lvert H(j\Omega)\rvert^2 = H(-\Omega^2) = H(-s^2) = H(s)H(-s)$<br>$H(s)H(-s) = \frac{1}{1+\left(\frac{s}{j}\right)^{2N}} = \frac{1}{1+(-1)^N s^{2N}} = \frac{1}{1+(-s^2)^N}$ (2 marks)<br>This function has poles in both LHP & RHP because of $H(s)$ & $H(-s)$<br>$1+(-s^2)^N = 0$<br>For N odd, $1-s^{2N}=0 \Rightarrow s^{2N}=1=e^{j2\pi k}$ corresponding roots $S_k = e^{j\pi k/N}$ $k=1,2,\ldots 2N$<br>For N even $s^{2N}=-1=e^{j(2k-1)\pi}$, $S_k = e^{j(2k-1)\pi/2N}$ for $K=1,2,\ldots 2N$ (2 marks)<br>Poles also can be found using formula $S_k = e^{j\phi_k}$ where<br>$\phi_k = \pi/2 + \frac{(2k-1)\pi}{2N}$ for $k=1,2,\ldots N$<br>$N=4$ $S_1 = e^{j5\pi/8} = -0.3827+j0.9239$<br>$S_2 = e^{j7\pi/8} = -0.9239+j0.3827$<br>$S_3 = e^{j9\pi/8} = -0.9239-j0.3827$<br>$S_4 = e^{j11\pi/8} = -0.3827-j0.9239$ (2 marks) | **9** | **3** | **4** | **3** |

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denominator

$$H(s) = (s+0.3827-j0.9239)(s+0.9239-j0.3827)(s+0.9239+j0.3827)(s+0.3827+j0.9239)$$

$$= \{(s+0.3827)^2+(0.9239)^2\}\{(s+0.9239)^2+(0.3827)^2\}$$

$$= (s^2+0.76536s+1)(s^2+1.84776s+1)$$

For fourth order Butterworth filter, TF with $\Omega_c = 1\,rad/sec$ is

$$H(s) = \frac{1}{(s^2+0.76536s+1)(s^2+1.84776s+1)}$$ (3 marks)

**ii)** b) Impulse invariant method

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**i)**

→ $y(n)$ can be obtained by multiplying $x(n)$ with a pulse train of period D

$$y(n) = x(mD)\,p(mD) = x(mD)$$ (1 mark)

Apply Z-transform

$$Y(z) = \sum_{m=-\infty}^{\infty} y(m) z^{-m} = \sum_{m=-\infty}^{\infty} \hat{x}(mD) z^{-m}$$

$$= \sum_{m=-\infty}^{\infty} \hat{x}(m) z^{-m/D}$$

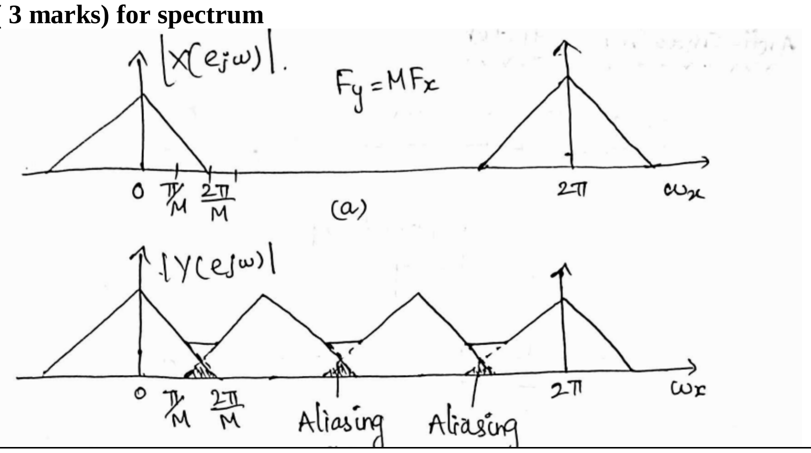
$$Y(z) = \sum_{m=-\infty}^{\infty} x(m) \left[\frac{1}{D}\sum_{k=0}^{D-1} e^{j2\pi mk/D}\right] z^{-m/D}$$

$$= \frac{1}{D}\sum_{k=0}^{D-1}\sum_{m=-\infty}^{\infty} x(m)\left(e^{-j2\pi k/D} z^{1/D}\right)^{-m}$$

$$= \frac{1}{D}\sum_{k=0}^{D-1} X\left(e^{-j2\pi k/D} z^{1/D}\right)$$ (3 marks)

$$Y(e^{j\omega}) = \frac{1}{D}\sum_{k=0}^{D-1} X\left(e^{-j2\pi k/D} e^{j\omega/D}\right) = \frac{1}{D}\sum_{k=0}^{D-1} X\left(e^{j\left(\frac{\omega-2\pi k}{D}\right)}\right)$$

**( 3 marks) for spectrum**

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| | (b) Anti-aliasing. (2 marks)<br>* The spectrum obtained after downsampling will overlap if the original spectrum is not band limited to $\omega = \frac{\pi}{M}$. This overlap causes aliasing.<br>* Therfore aliasing due to downsampling a signal by a factor of M is absent if and only if the signal $x(n)$ is band limited to $\pm\frac{\pi}{M}$. If the signal $x(n)$ is not band limited to $\pm\frac{\pi}{M}$, then a lowpass filter with cut-off frequency $\frac{\pi}{M}$ is used prior to downsampling. This filter is known as anti-aliasing filter.<br>**ii)** c) [1, 3, 5, 7] | **1** | **1** | **5** | **1** |
| **5** | **i)**<br>The z-transform of an infinite sequence given by<br>$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$<br>$H(z) = \sum_{m=0}^{M-1} z^{-m} P_m(z^M)$<br>where $P_m(z) = \sum_{r=-\infty}^{\infty} h(rM+m) z^{-r}$<br>$H(z) = \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} z^{-m} h(rM+m) z^{-rM}$<br>$= \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} h(rM+m) z^{-(rM+m)}$ (2 marks)<br>let $h(rM+m) = P_m(r)$<br>$\Rightarrow H(z) = \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} P_m(r) z^{-(rM+m)}$<br>$Y(z) = \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} P_m(r) X(z) z^{-(rM+m)}$<br>$y(n) = \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} P_m(r) x[n-(rM+m)]$ | **9** | **3** | **6** | **2** |

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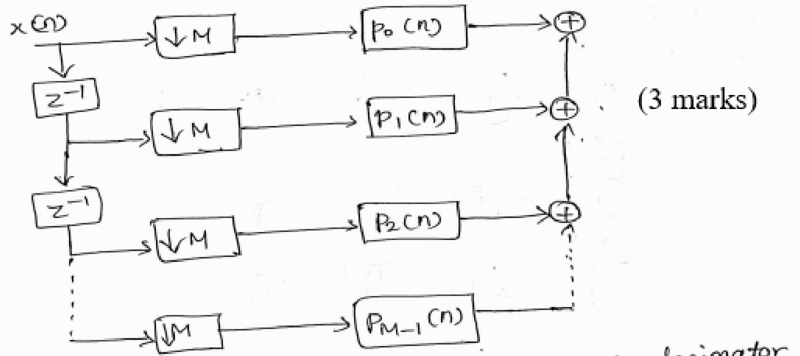
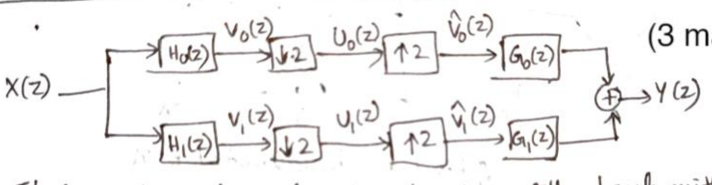
| | let $x_m(r) = x(rM-m)$ then<br>$y(n) = \sum_{m=0}^{M-1} \sum_{r=-\infty}^{\infty} p_m(r)\, x_m(n-r)$<br>$= \sum_{m=0}^{M-1} p_m(n) * x_m(n)$<br>$= \sum_{m=0}^{M-1} y_m(n).$ (2 marks)<br>where $y_m(n) = p_m(n) * x_m(n)$<br>The operation $p_m(n) * x_m(n)$ is known as polyphase convolution and the overall process is polyphase filtering.<br>$y(n) = \sum_{m=0}^{2} y_m(n)$<br>$= y_0(n) + y_1(n) + y_2(n)$ (2 marks)<br>$= p_0(n) * x_0(n) + p_1(n) * x_1(n) + p_2(n) * x_2(n)$<br><br>(3 marks)<br>polyphase structure of a M-branch decimator<br>**ii)** b) Interpolation, Decimation | 1 | 1 | 6 | 1 |
| 6 | **i)**<br>Quadrature-mirror Filter (QMF) bank<br><br>(3 marks)<br>It is a two channel subband coding filter bank with complementary freq responses. It consists of two sections → Analysis → Synthesis | 9 | 3 | 6 | 2 |

O/P of LP & HPF $\quad V_0(z) = X(z)\, H_0(z) \quad \text{--- (1)}$

$$V_1(z) = X(z)\, H_1(z) \quad \text{--- (2)}$$

Down sample with $M=2$, subband signals are

$$U_0(z) = \frac{1}{2}\left[V_0(z^{1/2}) + V_0(-z^{1/2})\right] \quad \text{--- (3)}$$

$$U_1(z) = \frac{1}{2}\left[V_1(z^{1/2}) + V_1(-z^{1/2})\right] \quad \text{--- (4)}$$

Substitute (1) in (3)

$$U_0(z) = \tfrac{1}{2}\left[X(z^{1/2})\, H_0(z^{1/2}) + X(-z^{1/2})\, H_0(-z^{1/2})\right]$$

Similarly $U_1(z) = \frac{1}{2}\left[X(z^{1/2})\, H_1(z^{1/2}) + X(-z^{1/2})\, H_1(-z^{1/2})\right]$

In matrix form **(2 marks)**

$$\begin{bmatrix} U_0(z) \\ U_1(z) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} H_0(z^{1/2}) & H_0(-z^{1/2}) \\ H_1(z^{1/2}) & H_1(-z^{1/2}) \end{bmatrix}\begin{bmatrix} X(z^{1/2}) \\ X(-z^{1/2}) \end{bmatrix}$$

$$Y(z) = G_0(z)\, \hat{V}_0(z) + G_1(z)\, \hat{V}_1(z) \qquad \hat{V}_0(z) = U_1(z^2)$$

$$= G_0(z)\, U_0(z^2) + G_1(z)\, U_1(z^2) \qquad \hat{V}_p(z) = U_1(z^2)$$

$$Y(z) = \left[G_0(z) \quad G_1(z)\right]\begin{bmatrix} U_0(z^2) \\ U_1(z^2) \end{bmatrix}$$

$$\begin{bmatrix} U_0(z^2) \\ U_1(z^2) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix}\begin{bmatrix} X(z) \\ X(-z) \end{bmatrix}$$

$$Y(z) = \left[G_0(z) \quad G_1(z)\right] \frac{1}{2}\begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix}\begin{bmatrix} X(z) \\ X(-z) \end{bmatrix}$$

$$= \tfrac{1}{2}\left[G_0(z)\, H_0(z) + G_1(z)\, H_1(z)\right] X(z) + \tfrac{1}{2}\left[G_0(z)\, H_0(-z) + G_1(z)\, H_1(-z)\right] X(-z)$$

$Y(z) = T(z)\, X(z) + A(z)\, X(-z)$ **(2 marks)**

where $T(z) = \frac{1}{2}\left[G_0(z) H_0(z) + G_1(z) H_1(z)\right]$ Distortion TF

$A(z) = \frac{1}{2}\left[G_0(z)\, H_0(-z) + G_1(z)\, H_1(-z)\right]$ Aliasing components

**Alias free filter bank**

To obtain alias-free filter bank, choose synthesis filter such that

$$A(z) = 0$$

i.e. $G_0(z)\, H_0(-z) + G_1(z)\, H_1(-z) = 0$ **(2 marks)**

simple Sufficient condition for alias cancellation is

$$G_0(z) = H_1(z) \quad \& \quad G_1(z) = -H_0(z)$$

$$Y(z) = T(z)\, X(z) \qquad Y(e^{j\omega}) = T(e^{j\omega})\, X(e^{j\omega})$$

$$= |T(e^{j\omega})|\, e^{j\theta(\omega)}\, X(e^{j\omega})$$

| | | | | | |
|---|---|---|---|---|---|
| | **ii)** c) after up sampler | **1** | **1** | **5** | **1** |

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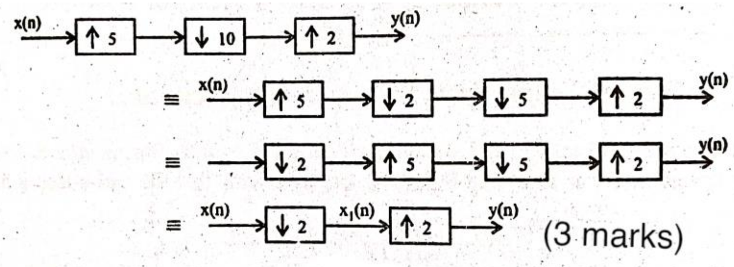
| 7 | LPF<br>$\Omega_c = \Omega_p = 500$ rad/sec ; $\Omega_s = 1000$ rad/sec ; $\alpha_p = 3$dB ; $\alpha_s = 15$dB<br>$N \geq \dfrac{\log\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log \Omega_s/\Omega_p} = \dfrac{\log\sqrt{\dfrac{10^{1.5}-1}{10^{0.3}-1}}}{\log\dfrac{1000}{500}} = 2.47$<br>$\therefore$ $N = 3$ (2 marks)<br>H(s) for $\Omega_c = 1$ rad/sec & $N=3$ is<br>$H(s) = \dfrac{1}{(s+1)(s^2+s+1)}$<br>To get HPF with cutoff frequency $\Omega_c = \Omega_p = 1000$ rad/sec<br>substitute $s \to \dfrac{1000}{s}$<br>$H_a(s) = H(s)\Big\|_{s \to \frac{1000}{s}} = \dfrac{1}{(s+1)(s^2+s+1)}\Big\|_{s \to \frac{1000}{s}}$<br>$= \dfrac{1}{\left(\frac{1000}{s}+1\right)\left(\frac{(1000)^2}{s^2}+\frac{1000}{s}+1\right)}$ (2 marks)<br>$H_a(s) = \dfrac{s^3}{(s+1000)\,(s^2+1000s+(1000)^2)}$ (1 mark) | 5 | 3 | 4 | 3 |
| | **ii)**<br><br>(3 marks)<br>$x_1(n) = x(2n)$<br>and $y(n) = x_1\left(\frac{n}{2}\right)$ for $n = 2k$<br>$= 0$ otherwise (1 mark)<br>$\Rightarrow$ $y(n) = x(n)$ for $n = k$<br>$= 0$ otherwise | 4 | 3 | 5 | 2 |
| | **iii)** b) 2/3 | 1 | 1 | 6 | 1 |

**Question Paper Setter** **Signature of the Course Coordinator**

**Signature of the Academic Advisor**