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ECE – A

Physics: Electromagnetic Theory, Quantum Mechanics, Waves and Optics- 18PYB101J

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PHYSICS : ASSIGNMENT - I

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1. Obtain Maxwell's equation for electromagnetism from fundamental laws of electricity and magnetism.

Soln

Maxwells equations Derivation:

Maxwells First Law:

Suppose the charge is distributed over a Volume V. Let ρ be the Volume density of the Charge, then the charge q is given by,

$$q = \int_V \rho\, dv$$

The integral form of Gauss Law is,

$$\psi = \oint_s \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \int_V \rho\, dv \qquad -(1)$$

According to Gauss divergence theorem,

$$\oint_s \vec{E} \cdot \vec{ds} = \int_V (\nabla \cdot \vec{E})\, dv \qquad -(2)$$

From (1) and (2)

$$\int_V (\nabla \cdot \vec{E})\, dv = \frac{1}{\varepsilon_0} \int_V \rho\, dv \qquad -(3)$$

Since this is true for any Volume v, integral must be equal.

$$\therefore \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad -(4)$$

$$\text{div } \vec{E} = \frac{\rho}{\varepsilon_0} \qquad -(5)$$

But electric displacement vector, $\vec{D} = \varepsilon_0 \vec{E}$.

$$(5) \times \varepsilon_0 = \quad \varepsilon_0 \operatorname{div} \vec{E} = \frac{\rho}{\varepsilon_0} \times \varepsilon_0$$

$$\operatorname{div} \varepsilon_0 \vec{E} = \rho$$

$$\operatorname{div} (\vec{D}) = \rho$$

$$\nabla \cdot \vec{D} = \rho \quad - \quad (7)$$

this is the differential form of Maxwells I Law.

From (1)
$$\oint_S \varepsilon_0 \vec{E} \cdot \vec{ds} = \int_V \rho dv$$

$$\oint_S \vec{D} \cdot \vec{ds} = \int_V \rho dv$$

This is the integral form of Maxwells I law

Maxwells Second Law:

From Biot-Savart Law of electromagnetism, the magnetic induction at any point due to a current element,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{idl \sin\theta}{r^2}$$

In Vector notation,

$$\vec{dB} = \frac{\mu_0}{4\pi r^3} (i\vec{dl} \times \vec{r}) = \frac{\mu_0}{4\pi r^2} (i\vec{dl} \times \hat{r})$$

Therefore total induction

$$\vec{B} = \frac{\mu_0 i}{4\pi} \int \left(\frac{1}{r^2} \cdot \vec{dl} \times \hat{r} \right)$$

This is Biot-Savart law,

If we replace the current i by the current density J, the current per area $J = \frac{i}{A}$ then

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{1}{r^2} \cdot (\vec{J} \times \hat{r})\, dv \qquad [\, i = J \cdot A \text{ and } J \cdot dl = J(A\, dl) = J \cdot dV \,]$$

Taking divergence on both sides,

$$\nabla \cdot \vec{B} = \frac{\mu_0}{4\pi} \int_V \nabla \cdot \left(\frac{1}{r^2} \cdot \vec{J} \times \hat{r} \right) dv$$

If the current density is assumed to be constant, then

$$\nabla \cdot \vec{J} = 0$$

$$\therefore \quad \nabla \cdot \vec{B} = 0$$

This is the differential form of Maxwell's second equation.

Experiments to date have shown that magnetic monopoles do not exist. Hence, the number of magnetic lines of force entering any arbitrary closed surface is exactly the same leaving it. Therefore the flux of magnetic induction B across a closed surface is zero.

By Gauss divergence theorem,

$$\int_V (\nabla \cdot \vec{B})\, dv = \int_S B \cdot ds = 0.$$

This is the integral form of Maxwells second law.

Maxwells Third Law.

By Faraday's law of electromagnetic Induction,

$$e = -\frac{d\phi}{dt}.$$

Now, let us consider work done on a charge, moving it through a distance dl.

$$W = \int \vec{E} \cdot dl \quad \text{which is line integral.}$$

The magnetic flux linked with closed area S due to the induction $B = \phi = \oint_S \vec{B} \cdot \vec{ds}$

$\therefore$ emf $= e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[\oint_S \vec{B}\cdot d\vec{s}\right] = \oint_S \frac{d\vec{B}}{dt}\cdot d\vec{s}$

Hence

$$\oint \vec{E}\cdot d\vec{l} = -\oint_S \frac{d\vec{B}}{dt}\cdot ds$$

this is the Maxwells III equation in integral form.

Using Stokes Theorem, the line integral of a Vector function along a closed path $\oint \vec{E}\cdot d\vec{l}$ can be converted to the surface integral of the normal component, the vector $\nabla\times\vec{E}$ of the enclosed surface.

(i.e) $\oint \vec{E}\cdot d\vec{l} = \int_S (\nabla\times\vec{E})\cdot d\vec{s}$

$$\therefore \int_S (\nabla\times\vec{E})\cdot d\vec{s} = -\int_S \frac{d\vec{B}}{dt}\cdot d\vec{s}$$

Hence, $(\nabla\times\vec{E}) = -\frac{\partial B}{\partial t}$

this is the Maxwells third equation in the differential form.

Maxwell's Fourth Law:

By Ampere's Circuital Law,

$$\int_l \vec{B}\cdot d\vec{l} = \mu_0 i$$

But, $\mu_0 = \frac{B}{H}$ or $B = \mu_0 H$

$\therefore$, $\int_l \vec{H}\cdot d\vec{l} = i$

But $i = \int_S \vec{J}\cdot d\vec{s}$

Hence, $\int_l \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{s}$

But $\vec{J} = \sigma\vec{E} + \frac{\partial\vec{D}}{\partial t}$

$$\therefore \oint_l \vec{H} \cdot \vec{dl} = \int_s \sigma \vec{E} \cdot \vec{ds} + \int_s \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$$

This is Maxwells fourth equation in integral form.

Using Stokes theorem,

$$\oint_l \vec{H} \cdot \vec{dl} = \int_s (\nabla \times \vec{H}) \cdot \vec{ds}$$

Hence,

$$\int_s (\nabla \times \vec{H}) \cdot \vec{ds} = \int_s \sigma \vec{E} \cdot \vec{ds} + \int_s \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$$

$$\int_s (\nabla \times \vec{H}) \cdot \vec{ds} = \left[\sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \right] \cdot \vec{ds}$$

(or) $$\nabla \times \vec{H} = \text{curl } \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$$

This is Maxwells fourth equation in integral form.