UNIT IV TUTORIAL 3

Answer all the questions

PART B

1. Prove that F(x^nf(x)) = $(-i)^n \frac{d^n}{ds^n}$(F(s)).

2. Prove that F($f^n(x)$)= $(-is)^n$F(s).

3. StateParseval's identity for both sine and cosine transforms.

4. If $F_c(e^{-a^2x^2}) = \frac{1}{a\sqrt{2}} e^{-\frac{s^2}{4a^2}}$,find $F_s(f'$(x)) and $F_c(f''$(x)) .

5. If f(x)= $e^{-3|x|}$, find F(x$e^{-3|x|}$) and F($x^2 e^{-3|x|}$) .

PART C

6. f(x)=$\begin{cases} a-|x|, & |x|<a \\ 0, & otherwise \end{cases}$, find the value of $\int_0^\infty (\frac{\sin t}{t})^4$dt.

7. UsingParseval's identity evaluate $\int_0^\infty (\frac{1}{x^2+a^2})(\frac{1}{x^2+b^2})$dx and $\int_0^\infty \frac{x^2}{(x^2+a^2)^2}$dx

8. Solve the integral equation $\int_0^\infty f(x)\, cos\alpha x\, dx = \begin{cases} 1-\alpha, & 0\le \alpha \le 1 \\ 0, & \alpha > 1 \end{cases}$.

9. Solve the integral equation $\int_0^\infty f(x)\, cos\alpha x\, dx$=$e^{-\alpha}$

10. Solve for f(x) if $\int_0^\infty f(x) sinsx\, dx$=$\begin{cases} 1, & 0\le s<1 \\ 2, & 1\le s<2. \\ 0, & s\ge 2 \end{cases}$

11 .Prove that $e^{-\frac{x^2}{2}}$ is self reciprocal under Fourier transforms.

12. Find the Fourier sine and cosine transforms of x^{n-1} and hence show that $\frac{1}{\sqrt{x}}$ is self reciprocal under both the transforms.

13. Find the Fourier transform of e^{-2x^2} and hence find F($e^{-2(x-3)^2}$) and F($e^{-2x^2} cos3x$).

ANSWERS FOR THE QUESTIONS IN TUTORIAL 3

4. $F_c(e^{-a^2x^2}) = \frac{1}{a\sqrt{2}}e^{-\frac{s^2}{4a^2}}$

F_s(f'(x))= -s$\frac{1}{a\sqrt{2}}e^{-\frac{s^2}{4a^2}}$ and F_c(f''(x))= $-s^2F_c$(s)- $\frac{\sqrt{2}}{\sqrt{\pi}}f'(0)$= $-s^2\frac{1}{a\sqrt{2}}e^{-\frac{s^2}{4a^2}}$

5. If f(x)=$e^{-3|x|}$,then F($e^{-3|x|}$)=$\frac{1}{\sqrt{2\pi}}\frac{6}{9+s^2}$

F($xe^{-3|x|}$)=$\frac{i}{\sqrt{2\pi}}\frac{12s}{(9+s^2)^2}$ and F($x^2e^{-3|x|}$)= $\frac{1}{\sqrt{2\pi}}\frac{108-36s^2}{(9+s^2)^3}$

6. $\int_0^\infty(\frac{\sin t}{t})^4$dt= $\frac{\pi}{3}$

7. $\int_0^\infty(\frac{1}{x^2+a^2})(\frac{1}{x^2+b^2})$dx =$\frac{\pi}{2ab(a+b)}$ and $\int_0^\infty\frac{x^2}{(x^2+a^2)^2}$dx= $\frac{\pi}{4a}$

8. f(x)= $\frac{4sin^2(\frac{x}{2})}{\pi x^2}$

9. f(x)= $\frac{2}{\pi}\frac{1}{1+x^2}$

10 .f(x)= $\frac{2}{\pi}\frac{1+\cos x-2cos2x}{x}$

13. F(e^{-2x^2})=$\frac{1}{2}$ $e^{-\frac{s^2}{8}}$.

F($e^{-2(x-3)^2}$)=$\frac{e^{i3s}}{\sqrt{2}}$ $e^{-\frac{s^2}{8}}$ and F($e^{-2x^2}\cos 3x$)= $\frac{1}{2}\left[\frac{1}{2}\ e^{-\frac{(s+3)^2}{8}} - \frac{1}{2}\ e^{-\frac{(s-3)^2}{8}}\right]$.