SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

18MAB201T/Transforms and Boundary value problems

UNIT II – FOURIER SERIES

TUTORIAL SHEET -2

PART B Questions

1. Expand $f(x)=(x-1)^2$ in $0<x<1$ in a Fourier series of sine series only.

2.The Fourier series of the function $f(x)=x+x^2, -\pi<x<\pi$ is

$\frac{\pi^2}{3}+\sum_{n=1}^{\infty}(-1)^n\left(\frac{4}{n^2}\cos nx-\frac{2}{n}\sin nx\right)$. Then deduce that $\sum_{1}^{\alpha}\frac{1}{n^2}=\frac{\pi^2}{6}$

3. Find b_1 for the function $f(x)=\begin{cases}\sin x, 0<x<\pi \\ 0, \pi<x<2\pi\end{cases}$.

4.The Fourier series of the function $f(x)=(\pi-x)^2, 0<x<2\pi$ is

$\frac{\pi^2}{3}+4\sum^{\infty}\left(\frac{1}{n^2}\cos nx\right)$. Then deduce the sum $\sum_{1}^{\alpha}\frac{1}{n^2}$

5. .Express $f(x)=x(\pi-x), 0<x<\pi$, as a Fourier series of periodicity 2π containing sine terms only.

PART C Questions

6.Find the Fourier series of $f(x)=x+x^2, -2<x<2$ .Hence find the sum of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+........$

7.Find the half –range cosine series for the function $f(x)=(x-1)^2, in 0<x<1$ .Hence show that $\pi^2=6\{\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+........\}$.

8.If $f(x)=\begin{cases}\frac{x}{l}, 0<x<l \\ \frac{2l-x}{l}, l<x<2l\end{cases}$ .Express f(x) as a Fourier series of periodicity $2l$.

9.Find the Fourier series of periodicity 2 for $f(x)=\begin{cases}x, -1<x<0 \\ x+2, 0<x<1\end{cases}$ and deduce the sum of $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+........\alpha$

10.Express f(x) = x in half-range cosine series and sine series of periodicity $2l$ in the range $0<x<l$ and deduce the value of $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+........$

Tutorial Sheet-2

Answers

Part-A

1. f(x)= $\sum_{n=1}^{\infty}\left[\frac{4(-1)^n}{n^3\pi^3}+\frac{2}{n\pi}-\frac{4}{n^3\pi^3}\right]\sin n\pi x$

2. Take $x=\pi \quad and \ \sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$

3. $b_1 = 1/2$

4. Take $x=0 \quad and \ \sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$

5. f(x)= $\frac{8}{\pi}\sum_{1}^{\infty}\frac{\sin\ x}{n^3}$

Part- B

6. f(x)= $\frac{4}{3}+\sum_{1}^{\infty}\frac{16(-1)^{n+1}}{n^2\pi^2}\cos\left(\frac{n\pi}{2}\right)x+\sum_{1}^{\infty}\frac{4(-1)^{n+1}}{n\pi}\sin\frac{n\pi}{2}x \ \& take \ x=2,$

$we\ get\ \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+...=\frac{\pi^2}{4}$

7. f(x) $=\frac{1}{3}+\sum_{1}^{\infty}\frac{-4}{n^2\pi^2}\cos n\pi x \ \ and\ \ take\ x=0, we\ get\sum_{1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$

8. f(x) $=\frac{1}{3}+\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi x$

9. f(x) $=1+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[1-2(-1)^n\right]\sin n\pi x$ & take x = $1/2$, we get $1-1/3+1/5-1/7+...=\frac{\pi}{4}$

10. cosine series: f(x) $=\frac{l}{2}-\frac{4l}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{n\pi}{l}\right)x}{n^2}$

sine series f(x) $=\sum_{1}^{\infty}b_n\sin\left(\frac{n\pi}{l}\right)x \ and \ \frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+...=\frac{\pi^2}{96}.$