

SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

18MAB201T/Transforms and Boundary value problems

UNIT IV-FOURIER TRANSFORMS

TUTORIAL SHEET -1

PART-B QUESTIONS

1. If $F\{f(x)\} = F(s)$, then $F\{f(ax)\} = \frac{1}{|a|}F\left(\frac{s}{a}\right)$.
2. State and Prove Modulation theorem.
3. If $F\{f(x)\} = F(s)$, then $F\{x^n f(x)\} = (-i)^n \frac{d^n}{ds^n} F(s)$.
4. Find the complex Fourier transform of $f(x) = \begin{cases} x, & \text{for } |x| \leq a \\ 0, & \text{for } |x| > a \end{cases}$

PART-C QUESTIONS

5. Show that the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ is $2\sqrt{\frac{2}{\pi}} \left(\frac{\sin as - as \cos as}{s^3} \right)$. Hence deduce that $\int_0^\infty \frac{\sin t - t \cos t}{t^3} dt = \frac{\pi}{4}$. Using Parseval's identity show that $\int_0^\infty \left(\frac{\sin t - t \cos t}{t^3} \right)^2 dt = \frac{\pi}{15}$.
6. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$
7. Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$ and $\int_{-\infty}^\infty \frac{\sin as \cos sx}{s} ds$.
8. Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ and using Parseval's identity, prove $\int_0^\infty \left(\frac{\sin t}{t} \right)^2 dt = \frac{\pi}{2}$.
9. Show that the transform of $e^{\left(\frac{-x^2}{2}\right)}$ is $e^{\left(\frac{-s^2}{2}\right)}$ by finding the Fourier transform of $e^{-a^2 x^2}, a > 0$.
10. Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence find the value of $\int_0^\infty \frac{\sin^4 t}{t^4} dt$.

***** ALL THE BEST *****

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ANSWERS-TUTORIAL SHEET -1

PART-B QUESTIONS

$$5. F\{f(x)\} = \frac{2i}{s^2} \frac{1}{\sqrt{2\pi}} [\sin sa - as \cos sa]$$

PART-C QUESTIONS

$$7. F\{f(x)\} = \frac{-4}{\sqrt{2\pi}} \left[\frac{s \cos s - \sin s}{s^3} \right] \text{ and } \int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx = -\frac{3\pi}{16}$$

$$8. F\{f(x)\} = \sqrt{\frac{2}{\pi}} \frac{\sin as}{s}$$
$$\int_{-\infty}^{\infty} \frac{\sin as \cos sx}{s} ds = \begin{cases} \frac{\pi}{2} & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

$$\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$$

$$12. F\{f(x)\} = \sqrt{\frac{2}{\pi}} \left(\frac{1 - \cos s}{s^2} \right) \text{ and } \int_0^\infty \frac{\sin^4 t}{t^4} dt = \frac{\pi}{3}$$