# DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
# SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

## 18PYB101J-Electromagnetic Theory, Quantum Mechanics, Waves and Optics

## Module 3- Lecture-8

**Time-dependent Schrodinger's wave equation**

SRM
Institute of Science and Technology


# *Derivation of Time-Dependent Schroedinger Equation*

## *Time-Dependent Schrodinger Wave Equation*

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t)$$

PHYSICS NOTATION

Total E term

K.E. term

P.E. term

$$\Psi(x,t) = e^{-iEt/\hbar}\psi(x)$$

## *Time-Independent Schrodinger Wave Equation*

$$E\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x)$$


*Let us consider a particle of mass 'm', moving with a velocity 'v'. The de Broglie wavelength associated with it is given by,*

$$\lambda = \frac{h}{mv} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where h = Planck's constant = $6.626 \times 10^{-34}$ J s.

Let $\psi$ be the wave function of the particle along *x, y* and *z* coordinate at any time '*t*' .The classical differential equation of a progressive wave moving with a wave velocity '*v*' can be written as,

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The solution for the equation (2) is given by,

$$\psi = \psi_\circ e^{-i\omega t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where

$\psi_0$ = Amplitude of the wave at the point (*x,y,z*)

$\omega$ = *Angular frequency of the wave*


Differentiating eqn. (3) with respect to '$t$',

$$\frac{\partial \psi}{\partial t} = (-i\omega)\psi_{\square} e^{-i\omega t} = (-i\omega)\psi \quad \text{................(4)}$$

But, $\omega = 2\pi f = 2\pi\left(\frac{E}{h}\right)$ $\quad \square \; f = \frac{E}{h} \quad$ ................(5)

where E = energy of a photon

<u>Substituting eqn (5) in eqn (4),</u>

$$\frac{\partial \psi}{\partial t} = (-i)\left[2\pi\left(\frac{E}{h}\right)\right]\psi = -i\left(\frac{E}{\hbar}\right)\psi$$

$$(OR),\; i\square\left(\frac{\partial \psi}{\partial t}\right) = E\psi \quad \text{................(6)}$$

Multiply $i$ both sides

SRM Institute of Science and Technology


**Substituting the equation(6), in the time independent wave equation**

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[i\hbar\left(\frac{\partial\psi}{\partial t}\right) - V\psi\right] = 0$$

(OR)

$$\nabla^2\psi = -\frac{2m}{\hbar^2}\left[i\hbar\frac{\partial}{\partial t} - V\right]\psi$$

Rearrange and Common term

SRM
Institute of Science and Technology


$$-\frac{\hbar^2}{2m}\bullet\nabla^2\psi=\left[i\hbar\frac{\partial}{\partial t}-V\right]\psi$$

Rearrange and Multiply ‘–’ both sides

Keeping Time derivative one side we have

$$-\frac{\hbar^2}{2m}\bullet\nabla^2\psi+V\psi=i\hbar\frac{\partial\psi}{\partial t}$$

rearrange

**This equation is known as *Schrödinger time dependent wave equation*.**


Further simplification and re-arrangement lead to hamilton form  Time-Dependent Schroedinger Equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2\psi + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

where H = $\frac{-\hbar^2}{2m}\nabla^2 + V$ =Hamiltonian operator


## Time-Dependent Schrodinger Wave Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t)$$

PHYSICS NOTATION

Total E term

K.E. term

P.E. term

$$\Psi(x,t) = e^{-iEt/\hbar}\psi(x)$$