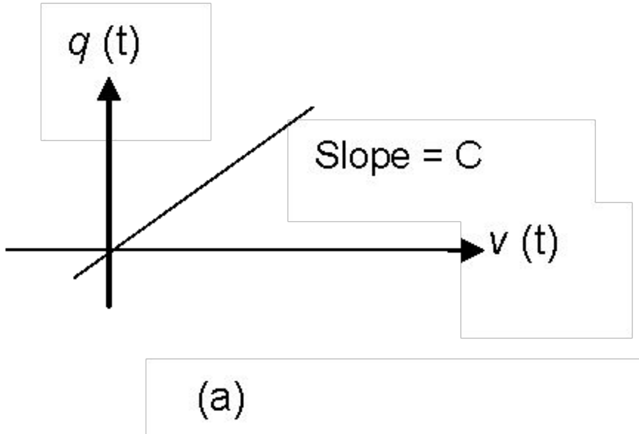
RL Circuit Transient Analysis
RC Transient Analysis

2.5.2 CAPACITOR

Any two conducting surfaces separated by an insulating medium form a capacitor. A two terminal element will be called a CAPACITOR, if at any instant time t, the charge in it q(t) and the voltage across it v(t) satisfy a relation defined by the curve in the v-q plane as shown in Fig. 2.14(a). In the circuit diagram, a capacitor is shown in Fig. 2.14(b). CAPACITANCE is the property of the capacitor.



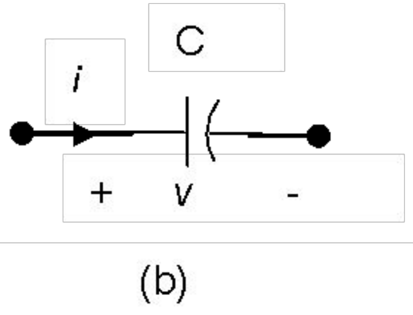


Fig. 2.14 Characteristics and representation of capacitor.

The capacitance is the rate of change of charge with respect to voltage. The relation between the charge $q(t)$ and the voltage $v(t)$ can be expressed as

$$q(t) = C\ v(t) \tag{2.14}$$

where C is a constant which is equal to the slope of the characteristic. This constant is called as the CAPACITANCE. The unit of capacitance is Farad. It is to be noted that capacitance is charge per unit voltage.

The equation relating the terminal voltage and the element current can now be obtained. Knowing that the rate of change of charge (with respect to time) is the current, differentiating both sides of Eq. (2.14), we get

$$\frac{dq(t)}{dt} = i(t) = C\frac{dv(t)}{dt}$$

Therefore $$i(t) = C\frac{dv(t)}{dt} \tag{2.15}$$

Therefore $$i(t) = C \frac{dv(t)}{dt} \tag{2.15}$$

From the above equation $$\frac{dv(t)}{dt} = \frac{1}{C}\, i(t)$$

Integrating both sides of the above equation yield

$$v(t) = v(0) + \frac{1}{C} \int_0^t i(t)\, dt \tag{2.16}$$

where v(0) is the initial voltage across the capacitor.

The above equation can be written as

$$v(t) = v(0) + S \int_0^t i(t)\, dt \tag{2.17}$$

where $S = 1 / C$ which is called the ELASTANCE.

A capacitor is completely specified as a circuit element only if the capacitance C and the initial voltage v(0) are given.

2.5.3 INDUCTOR

A wire of certain length, when twisted into a coil becomes a basic inductor. A two terminal element will be called an INDUCTOR, if at any instant of time, the flux linkage in it $\Psi(t)$ and the current passing though it $i(t)$ satisfy a relation defined by the curve in i - Ψ plane as shown in Fig. 2.15(a). In the circuit diagram, an inductor is shown as in Fig. 2.15(b). INDUCTANCE is the property of the inductor.

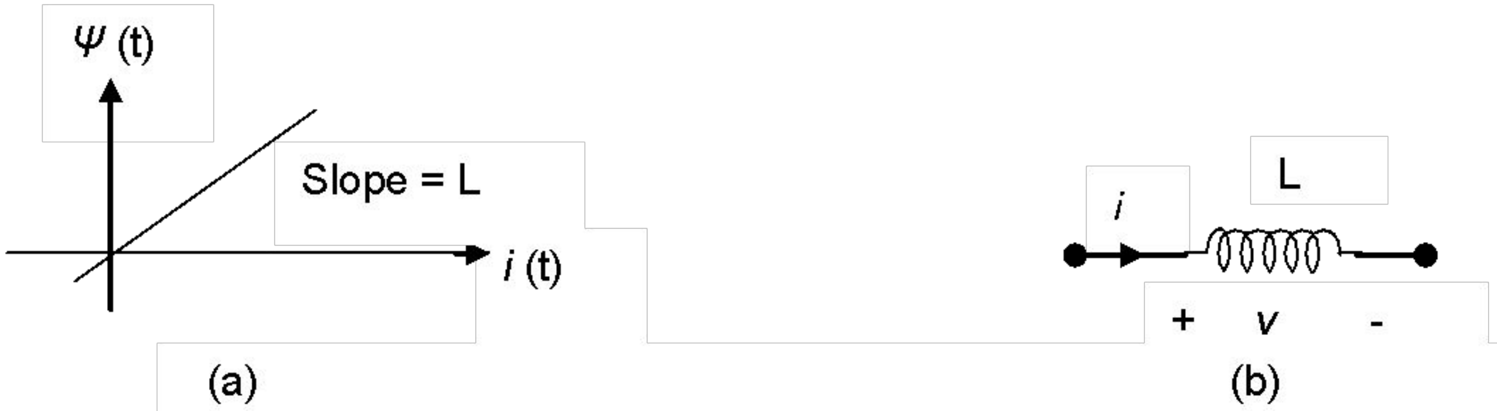


Fig. 2.15 Characteristics and representation of inductor.

Inductance is the rate of change of flux linkage with respect to the current.The relation between the flux linkage $\Psi(t)$ and the current $i(t)$ can be expressed as

$$\Psi(t) = L\ i(t) \tag{2.18}$$

where L is a constant which is equal to the slope of the characteristic. This constant is called the INDUCTANCE. The unit of inductance is Henry. It is to be noted that inductance is flux linkage per unit current.

The equation relating the terminal voltage and the element current can now be obtained. Knowing that the rate of change of flux linkage is the voltage, differentiating both sides of Eq. (2.18) we get

$$\frac{d\psi(t)}{dt} = v(t) = L\frac{di(t)}{dt}$$

$$\text{Therefore } v(t) = L\frac{di(t)}{dt} \tag{2.19}$$

From the above equation $\frac{di(t)}{dt} = \frac{1}{L} v(t)$

Integrating both sides of the above equation yield

$$i(t) = i(0) + \frac{1}{L} \int_0^t v(t)\, dt \tag{2.20}$$

Here $i(0)$ is the initial current through the inductor.

The above equation can be written as

$$i(t) = i(0) + \Gamma \int_0^t v(t)\, dt \tag{2.21}$$

where $\Gamma = 1 / L$ which is called the RECIPROCAL INDUCTANCE.

A inductor is completely specified as a circuit element only if the inductance L and the initial current $i(0)$ are given.

7.1 INTRODUCTION

So far steady state analysis of electric circuits was discussed. Electric circuits will be subjected to sudden changes which may be in the form of opening and closing of switches or sudden changes in sources etc. Whenever such a change occurs, the circuit which was in a particular steady state condition will go to another steady state condition. Transient analysis is the analysis of the circuits during the time it changes from one steady state condition to another steady state condition.

Transient analysis will reveal how the currents and voltages are changing during the transient period. To get such time responses, the mathematical models should necessarily be a set of differential equations. Setting up the mathematical models for transient analysis and obtaining the solutions are dealt with in this chapter.

A quick review on various test signals is presented first. Transient response of simple circuits using classical method of solving differential equations is then discussed. Laplace Transform is a very useful tool for solving differential equations. After introducing the Laplace Transform, its application in getting the transient analysis is also discussed.

7.3 CERTAIN COMMON ASPECTS OF RC AND RL CIRCUITS

While doing transient analysis on simple RC and RL circuits, we need to make use of the following two facts.

1. **The current in an inductor as well as the voltage across a capacitor cannot have discontinuity.**

2. **With dc excitation, at steady state, capacitor will act as an open circuit and inductor will act as a short circuit.**

These two aspects can be explained as follows.

Voltage across an inductor is $v_L = L\ (di\ /\ dt)$. If the current through the inductor has discontinuity, then at the time when the discontinuity occurs, $di\ /\ dt$ becomes infinity resulting the voltage v_L to become infinity. However, in physical system, we exclude the possibility of infinite voltage. Then, we state that in an inductor, the current cannot have discontinuity. Suppose, if the circuit condition is changed at time $t = 0$, the inductor current must be continuous at time $t = 0$ and hence

$$i_L(0^+) = i_L(0^-) \tag{7.14}$$

where time 0^+ refers the time just after $t = 0$ and time 0^- refers the time just before $t = 0$.

Similarly, the current through a capacitor is given by $i_C = C\ (dv / dt)$. If the voltage across the capacitor has discontinuity, this will result in infinite current through the capacitance. In physical system, we exclude the possibility of infinite current. Then, we state that the voltage across the capacitor cannot have discontinuity. Suppose, if the circuit condition is changed at time $t = 0$, then

$$v_C(0^+) = v_C(0^-) \tag{7.15}$$

With dc excitation, at steady state condition, all the element currents and voltages are of dc in nature. Therefore, both di / dt and dv / dt will be zero. Since $i_C = C\ (dv / dt)$ and $v_L = L\ (di / dt)$, with dc excitation, at steady state condition, the current through the capacitor as well as the voltage across the inductor will be zero. In other words, with dc excitation, at steady state condition, the capacitor will act as an open circuit and the inductor will act as a short circuit.

7.4 TRANSIENT IN RC CIRCUIT

While studying the transient analysis of RC and RL circuits, we shall encounter with two types of circuits namely, source free circuit and driven circuit.

Source free circuit

A circuit that does not contain any source is called a source free circuit. Consider the circuit shown in Fig. 7.7 (a). Let us assume that the circuit was in steady state condition with the switch is in position S_1 for a long time. Now, the capacitor is charged to voltage E and will act as open circuit.

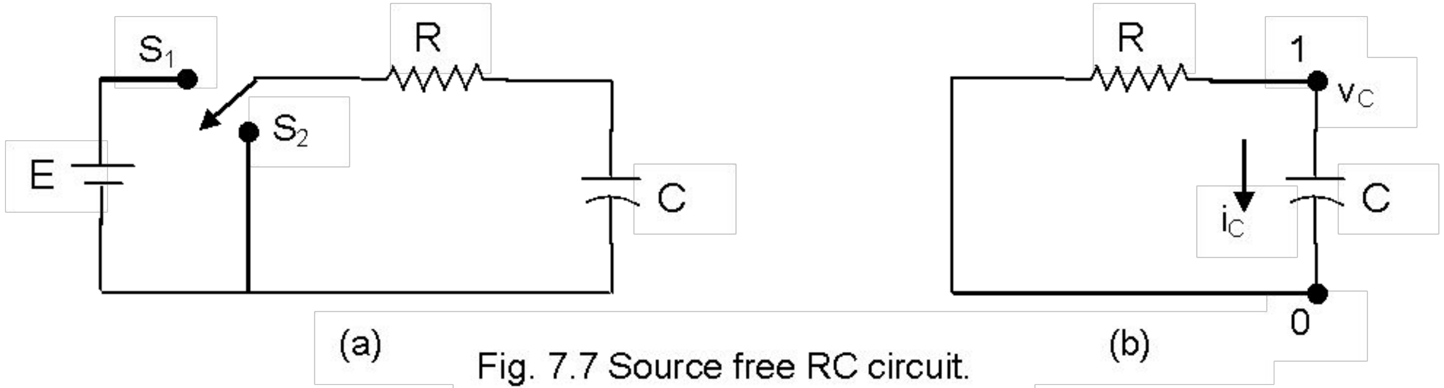


Fig. 7.7 Source free RC circuit.

Suddenly, at time t = 0, the switch is moved to position S_2. The voltage across the capacitor and the current through the capacitor are designated as v_C and i_C respectively. The voltage across the capacitor will be continuous. Hence

$$v_C(0^+) = v_C(0^-) = E \tag{7.16}$$

The circuit for time t > 0 is shown in Fig. 7.7 (b). We are interested in finding the voltage across the capacitor as a function of time. Later, if required, current through the capacitor can be calculated from $i_C = C\dfrac{dv}{dt}$. Voltage at node 1 is the capacitor voltage v_C. The node equation for the node 1 is

$$\frac{v_C}{R} + C\frac{dv_C}{dt} = 0 \qquad (7.17)$$

i.e. $$\frac{dv_C}{dt} + \frac{v_C}{RC} = 0 \qquad (7.18)$$

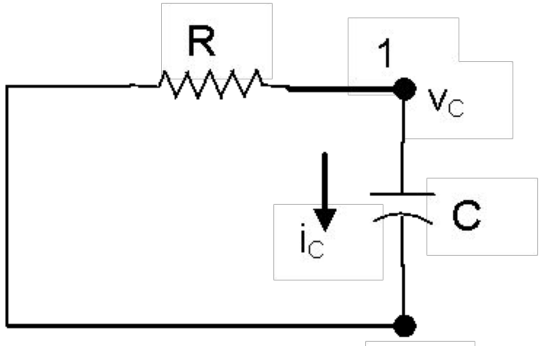


Fig. 7.7 (b)

We have to solve this first order differential equation (DE) with the initial condition

$$v_C(0^+) = E \qquad (7.19)$$

We notice that DE in Eq. (7.18) is a **homogeneous equation** and hence will have **only complementary solution**. Let us try $v_C(t) = K\,e^{st}$ (7.20)

as a possible solution of Eq. (7.18).

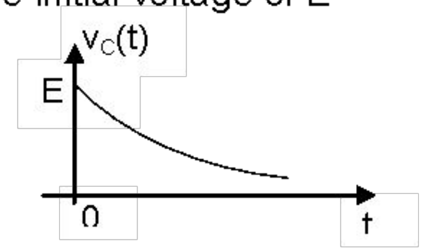
Taking the initial capacitor voltage as $v_C(0)$ and the final capacitor voltage as $v_C(\infty)$ capacitor voltage is can be derived as $v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]\ e^{-\frac{1}{R_2 C}t}$ (7.47)

<u>Summary of formulae useful for transient analysis on RC circuits</u>

1. Time constant $\tau = RC$

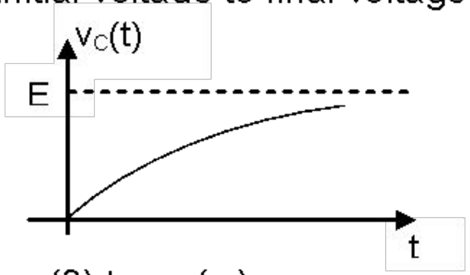
2. When the capacitor is discharging from the initial voltage of E

$$v_C(t) = E\ e^{-\frac{1}{RC}t}$$



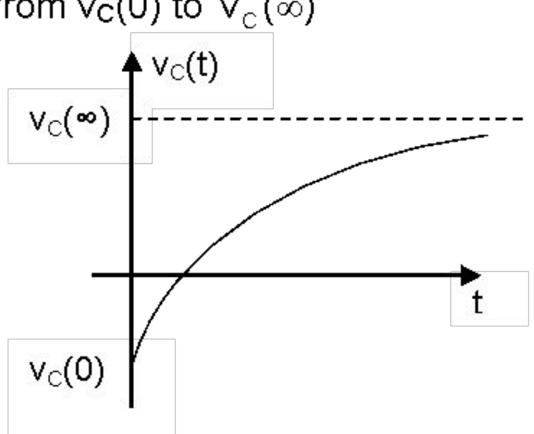
3. When the capacitor is charged from zero initial voltage to final voltage of E

$$v_C(t) = E\ (1 - e^{-\frac{1}{RC}t})$$



4. When the capacitor voltage changes from $v_C(0)$ to $v_C(\infty)$

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]\ e^{-\frac{1}{RC}t}$$



5. Capacitor current $i_C(t) = C\dfrac{dv_C(t)}{dt}$

7.5 TRANSIENT IN RL CIRCUIT

Now we shall now consider RL circuit for the transient analysis. As stated earlier,

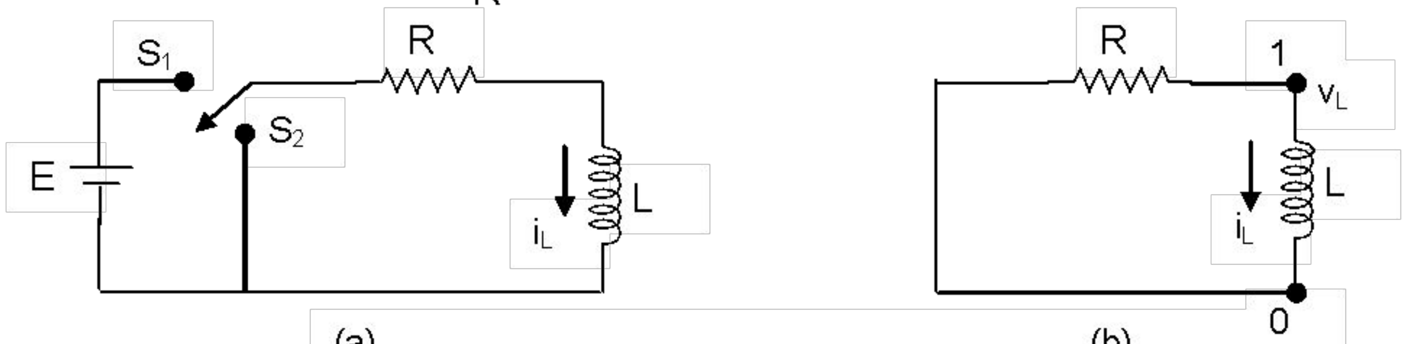
1. The current in an inductor cannot have discontinuity.

2. With dc excitation, at steady state, inductor will act as a short circuit.

Now also we shall end up with first order DE whose solution will be exponential in nature.

<u>Source free circuit</u>

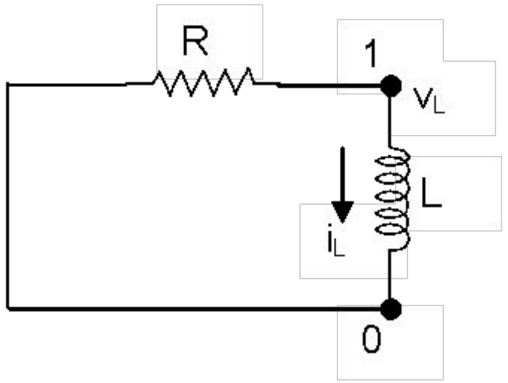
A circuit that does not contain any source is called a source free circuit. Consider the circuit shown in Fig. 7.35 (a). Let us assume that the circuit was in steady state condition with the switch is in position S_1 for a long time. Now the inductor acts as short circuit and it carries a current of $\frac{E}{R}$.



(a) (b)

Fig. 7.35 Source free RL circuit.

Suddenly, at time t = 0, the switch is moved to position S_2. The current through the inductor and the voltage across the inductor are designated as i_L and v_L respectively. The current through the inductor will be continuous. Hence



$$i_L(0^+) = i_L(0^-) = \frac{E}{R} \tag{7.49}$$

The circuit for time t > 0 is shown in Fig.above. We are interested in finding the current through the inductor as a function of time. Later, if required, voltage across the inductor can be calculated from $v_L = L\frac{di}{dt}$. The mesh equation for the circuit is

$$R\, i_L + L\frac{di_L}{dt} = 0 \tag{7.50}$$

$$\text{i.e. } \frac{di_L}{dt} + \frac{R}{L} i_L = 0 \tag{7.51}$$

We need to solve the above equation with the initial condition

$$i_L(0^+) = \frac{E}{R} \tag{7.52}$$

The structure of the equation 7.51 is the same as Eq. 7.18. In this case, the time constant, τ is $\frac{L}{R}$. The inductor current exponentially decays from the initial value of $\frac{E}{R}$ to the final value of zero. Thus the solution of equation 7.51 yields

$$i_L(t) = \frac{E}{R} e^{-\frac{R}{L}t} \tag{7.53}$$

The plot of inductor current is shown in Fig. 7.36 (a).

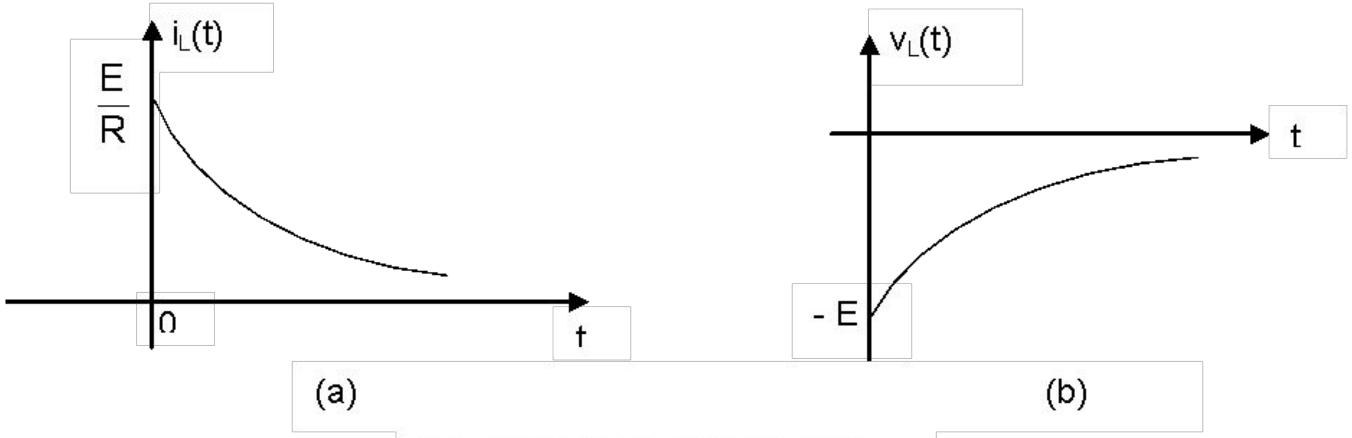


Fig. 7.36 Plot of $i_L(t)$ and $v_L(t)$.

It can be seen that the dimension of L / R is time. Dimensionally

$$\frac{L}{R} = \frac{\text{Flux linkage}}{\text{amp.}} \frac{\text{amp.}}{\text{volt}} = \frac{\text{Flux linkage}}{\text{Flux linkage / sec}} = \text{sec.}$$

The voltage across the inductor is: $v_L(t) = L\frac{di}{dt} = L\,\frac{E}{R}\left(-\frac{R}{L}\right) e^{-\frac{R}{L}t} = -E\, e^{-\frac{R}{L}t}$ (7.54)

The plot of the voltage across the inductor is shown in Fig. 7.36 (b).

Taking the initial inductor current as $i_L(0)$ and the final inductor current as $i_L(\infty)$, inductor current can be obtained as

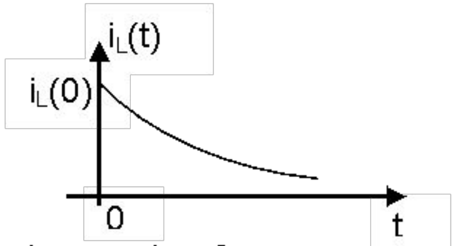
$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\, e^{-\frac{R_2}{L}t} \qquad (7.63)$$

<u>Summary of formulae useful for transient analysis on RL circuits</u>

1. Time constant $\tau = L / R$

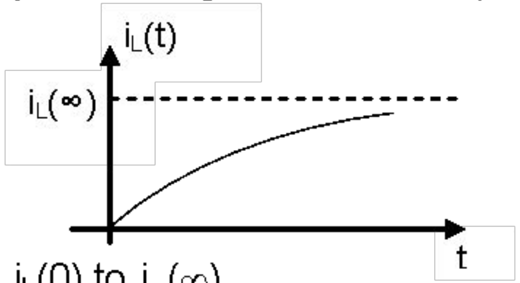
2. When the inductor current is decaying from the initial value of $i_L(0)$ to zero

$$i_L(t) = i_L(0)\, e^{-\frac{R}{L}t}$$



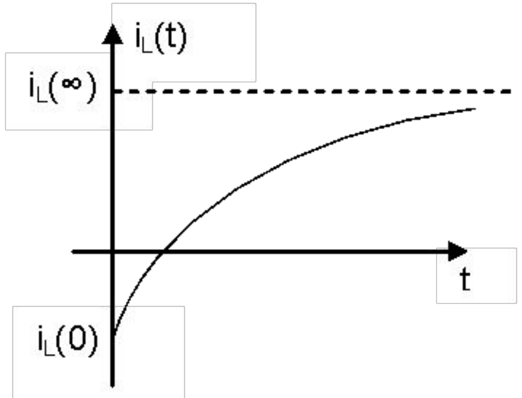
3. When the inductor current is exponentially increasing from zero to $i_L(\infty)$

$$i_L(t) = i_L(\infty)\,(1 - e^{-\frac{R}{L}t})$$



4. When the inductor current changes from $i_L(0)$ to $i_L(\infty)$

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\, e^{-\frac{R}{L}t}$$



5. Inductor voltage $v_L(t) = L\dfrac{di_L(t)}{dt}$