Reg. No.	R	A	2	0	1	1	0	0	4	0	1	0	0	5	1

B.Tech. DEGREE EXAMINATION, JULY 2022

Fourth Semester

18MAB203T – PROBABILITY AND STOCHASTIC PROCESSES

(For the candidates admitted from the academic year 2020 – 2021 and 2021 – 2022)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

(ii) **Part - B** should be answered in answer booklet.

Time: 2½ Hours Max. Marks: 75

PART – A (25 × 1 = 25 Marks)

Answer **ALL** Questions

Q. No.	Question	Marks	BL	CO	PO
1.	The second order moment about the mean is (A) 1 (B) 0 (C) Var(X) (D) Mean(X)	1	1	1	1
2.	If X and Y are independent RV's then $\phi_{x+y}(\omega)=$ (A) $\phi_x(\omega)+\phi_y(\omega)$ (B) $\phi_x(\omega)\cdot\phi_y(\omega)$ (C) $\phi_x(\omega)-\phi_y(\omega)$ (D) $\phi_x(\omega)/\phi_y(\omega)$	1	1	1	1
3.	If the cdf of a random variable X is given by $F(x)=1-e^{-\lambda x};x\geq 0$ and 0 for x<0, then the pdf of X for x≥0 is (A) $e^{-\lambda x}$ (B) $\lambda e^{-\lambda x}$ (C) $-e^{-\lambda x}$ (D) $\lambda e^{\lambda x}$	1	2	1	2
4.	The variance of Poisson distribution defined by $P(X=x)=\dfrac{e^{-2}2^x}{x!};x:0,1,\ldots\infty$ is (A) 2 (B) 1/2 (C) e^2 (D) e^{-2}	1	2	1	2
5.	The pdf of Y given *Y=g(X)* where X is a continuous RV with pdf $f_X(x)$ and *g(x)* is strictly monotonic function of *x* is (A) $f_X(x)\left\lvert\dfrac{dy}{dx}\right\rvert$ (B) $\dfrac{1}{f_X(x)}\left\lvert\dfrac{dy}{dx}\right\rvert$ (C) $\dfrac{1}{f_X(x)}\left\lvert\dfrac{dx}{dy}\right\rvert$ (D) $f_X(x)\left\lvert\dfrac{dx}{dy}\right\rvert$	1	1	1	1
6.	If the joint probability distribution function is *F(x,y)* then *f(x,y)*= (A) $\dfrac{\partial F(x,y)}{\partial x}$ (B) $\dfrac{\partial F(x,y)}{\partial y}$ (C) $\dfrac{\partial^2 F(x,y)}{\partial x\partial y}$ (D) $\iint F(x,y)\,dxdy$	1	1	2	1

7. If X and Y are two independent RV's then COV(X,Y)= 1 1 2 1

(A) 1 (B) −1
(C) 0 (D) ∞

8. If the mean of X and Y is 5 and −3 respectively then E(X+Y)= 1 2 2 2

(A) 8 (B) −2
(C) −15 (D) 2

9. If X and Y have joint pdf $f(x,y)=x+y; 0<x<1, 0<y<1$, *then* $f_X(x)=$ 1 2 2 2

(A) $\frac{1+2y}{2}$ (B) $\frac{1+x}{2}$
(C) $\frac{1+2x}{2}$ (D) $\frac{1+y}{2}$

10. In the transformation of two dimensional random variable if $U=x+y$ and $V=\frac{x}{x+y}$ then y= 1 2 2 2

(A) UV (B) U/V
(C) 1−U (D) U(1−V)

11. Moment generating function is used to find the bounds of a distribution in 1 1 3 1

(A) Chebychev's inequality (B) Jensen's inequality
(C) Chernoff bounds (D) Cauchy-Schwartz inequality

12. If X is a random variable with mean 100, then by Markov inequality $P(X \geq 10) \leq ?$ 1 2 3 2

(A) 0 (B) 1
(C) 10 (D) 100

13. If $Var(X)=0$, *then* $P\{X=E(X)\}=$ 1 1 3 1

(A) 0 (B) 1
(C) ½ (D) 1/4

14. For any two RVs X and Y then $\{E(XY)\}^2 \leq E(X^2)\cdot E(Y^2)$ represents the 1 1 3 1

(A) Markov inequality (B) Jensen's inequality
(C) Chebychev's inequality (D) Cauchy-Schwartz inequality

15. If X is a random variable with mean zero and variance σ^2, then for any value a>0, 1 1 3 1

(A) $P\{X \geq a\} \leq \frac{\sigma}{\sigma+a}$ (B) $P\{X \geq a\} \leq \frac{\sigma^2}{\sigma+a}$
(C) $P\{X \geq a\} \leq \frac{\sigma^2}{\sigma^2+a}$ (D) $P\{X \geq a\} \leq \frac{\sigma^2}{\sigma^2+a^2}$

16. If X(t) is ergodic with zero mean and has no periodic components then $\lim_{|\tau|\to\infty} R_{XX}(\tau)=$ 1 1 4 1

(A) Constant (B) Function of τ
(C) Zero (D) one

17. $R_{XY}(\tau)=$ 1 1 4 1

(A) $R_{YX}(-\tau)$ (B) $R_{XY}(-\tau)$
(C) $R_{YX}(\tau)$ (D) $\sqrt{R_{XX}(\tau)\cdot R_{YY}(\tau)}$

18.	If the marginal and joint density function of the process do not depend on the choice of the time origin, the process is (A) Linear (B) Time variant (C) Stationary (D) Constant	1	1	4	1
19.	Given the autocorrelation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau)=25+\frac{4}{1+6\tau^2}$, the value of mean is (A) 5 (B) 25 (C) 4 (D) 2/3	1	2	4	2
20.	Two random processes X(t) and Y(t) are uncorrelated with mean 5 and 4 respectively. Then their cross correlation $R_{XY}(t,t+\tau)=$ (A) 1 (B) 0 (C) 9 (D) 20	1	2	4	2
21.	If the output $Y(t_1)$ at a given time $t=t_1$, depends only on $X(t_1)$ and not on any other past or future values of X(t), then the system is called a (A) Memoryless system (B) Causal system (C) Time-invariant system (D) Linear system	1	1	5	1
22.	The mean-square value of the process {X(t)} if its autocorrelation function is given by $R(\tau)=e^{-\tau^2/2}$ is (A) ½ (B) 0 (C) 2 (D) 1	1	2	5	2
23.	If $R(\tau)=e^{-2\lambda\lvert\tau\rvert}$ is the autocorrelation function of a random process X(t) then the spectral density of X(t) is (A) $\frac{2\lambda}{4\lambda^2+\omega^2}$ (B) $\frac{2\lambda^2}{4\lambda^2+\omega^2}$ (C) $\frac{4\lambda}{4\lambda^2+\omega^2}$ (D) $\frac{\lambda}{4\lambda^2+\omega^2}$	1	2	5	2
24.	The mean square value of a WSS process is equal to the total area under the graph of (A) Auto correlation (B) Cross correlation (C) Spectral density (D) Cross power spectral density	1	1	5	1
25.	The power spectral density S(ω) of a continuous time random process X(t) is defined as the (A) Fourier inverse transform of R(τ) (B) Fourier transform of R(τ) (C) Fourier sine transform of R(τ) (D) Fourier cosine transform of R(τ)	1	1	5	1

PART – B (5 × 10 = 50 Marks)

Answer **ALL** Questions

		Marks	BL	CO	PO
26. a.	Consider the experiment of tossing a fair coin 4 times. Define X=0 if 0 or 1 head appears, X=1 if 2 head appears, X=2 if 3 or 4 head appears. Analyze the situation and define a suitable probability function for the above experiment. Compute the mean, variance and the distribution function for the defined probability function.	10	4	1	1,2

(OR)

b. After correcting the proof up to 50 pages of a book, the proof reader found that there are three errors per 5 pages. Use Poisson distribution and estimate the number of pages with 0,1,2,3 errors and more than 3 errors in a book of 1000 pages.	10	3	1	1,2
27. a. Let X and Y each follows an exponential distribution with parameter 1 and are independent. Assume X and Y are transformed to the random variables U and V by means of the transformation U=X−Y and V=Y. Estimate the pdf of U for the given transformation.	10	3	2	1,2
(OR)				
b. In producing gallium-arsenide microchips, it is known that the ratio between gallium and arsenide is independent of producing a high percentage of workable wafer, which are main components of microchips. Let X denote the ratio of gallium to arsenide and Y denote the percentage of workable micro wafers retrieved during a 1-hour period. X and Y are independent random variables with the joint density being known as $f(x,y)=\begin{cases}\frac{x(1+3y^2)}{4}; & 0<x<2, 0<y<1\\ 0 & otherwise\end{cases}$ Estimate the average ratio of gallium to arsenide E(x) and the average percentage of workable microwafer retrieved during a 1-hour period E(Y). Compute the average of the product of X and Y and check if E(XY)=E(X)·E(Y).	10	4	2	1,2
28. a. If X denotes the sum of the numbers obtained when 2 dices are thrown, obtain an upper bound for $P\{\|X-7\|\geq 4\}$. Compare the answer with the exact probability.	10	4	3	1,2
(OR)				
b. The life time of a certain brand of a battery may be considered a RV with mean 1200h and standard deviation 250h. Apply central limit theorem to estimate the probability that the average lifetime of 60 batteries exceeds 1250h.	10	3	3	1,2
29. a. Show that the process $X(t)=A\cos\lambda t+B\sin\lambda t$, where A and B are RVs is wide sense stationary only on applying the condition (i) E(A)=E(B)=0 (ii) $E(A^2)=E(B^2)$ and (iii) E(AB)=0	10	3	4	1,2
(OR)				
b. If {X(t)} and {Y(t)} are independent wide sense stationary processes with zero mean, find the auto-correlation function of {Z(t)}, when (i) $Z(t)=a+bX(t)+cY(t)$ (ii) $Z(t)=aX(t)Y(t)$	10	4	4	1,2
30. a. The auto correlation of the random binary transmission is given by $R_{XX}(\tau)=\begin{cases}1-\frac{\|\tau\|}{T} & for\ \|\tau\|\leq T\\ 0 & for\ \|\tau\|>T\end{cases}$ Determine the power spectrum of the binary transmission.	10	3	5	1,2
(OR)				
b. A wide sense stationary process X(t) is the input to a linear system with impulse response $h(t)=2e^{-7t}; t\geq 0$. If the auto correlation function of X(t) is $R_{XX}(\tau)=e^{-4\|\tau\|}$, estimate the power spectral density of the output process.	10	4	5	1,2

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