Consider Small transverse vibrations of an elastic string of length 1, which is stretched and then fined at

its two ends. Now we will study the transverse vibration of the String when no external forces act on it. the origin and

Take an end of the string as the origin and the string in the equilibrium position as the x-axis and the line through the origin and perpendicular to the 2-axis as the y-axis. I monthat = 30 aronu

We make the following assumptions:

- 1) The motion takes place entirely in one plane. This plane is Chosen as the xy plane.
- a) In this plane, each particle of the string moves in a direction perpendicular to the equilibrium ponition of the string.
- 3) The territor T caused by stretching the string before fixing it at the end points is constant at all times at all points of the deflected string.
- The tension T is very large Compared with the weight of the string and hence the gravitational 4) force may be neglected.

- 5) The effect of friction is negligible.
- The string is perfectly flexible. It can transmit 6) only tennion but not bending or shearing forces.
- The slipe of the deflection curve is small at all points and at all times.

  By wring second law of Newton, we can derive

one of dimensional wave equation:

e,  $\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$  and any only only only le,

where  $\alpha^{\alpha} = \frac{\text{Tention}}{\text{mays}} = \frac{T}{m}$  (positive). the following

Note: The displacement y (MIX) is given by the you by the quation  $\frac{\partial y}{\partial t^2} = a^{2} \frac{\partial y}{\partial x^2}$  gr and so means if

In this plane, each posticle of the string moves in a to the equilibrium pe

direction propondicular sting.

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Solution of the wave equation.  $\frac{\partial^2 y}{\partial t^2} = \alpha^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2$ (By the method of Seperation of Variables) · O= TRAPO TED y = X(x). T(t) be a solution of equality (1), x where X's Let is a function of x only and T(t) is a function of (K) X 02-02/2-0. only.  $\left( p^{2} - \lambda^{2} \right) \chi = 0$ E ay = xi7 proff our ay = xi1  $\frac{\partial y}{\partial x^2} = x'' 7^2 4 30 = 8m \frac{\partial y}{\partial t^2} = x T''$  where 0 = 84 - 8m $X^{11} = \frac{d^2X}{dx^2} + \frac{1}{dx^2} + \frac{1}{dt^2} + \frac{1}{dt^2}$ Hence (D becomes, City) 1e,  $\frac{\chi''}{\chi''} = \frac{\tau''}{a^2\tau}$ The L.H.S of @ 0= 188 pay function of x ... only whereas R.H.s is a function of t only. But and t are independent variables. Hence a is true only is the · 0= XBX + XEP each is equal to a constant.  $\frac{\chi''}{\chi''} = \frac{\chi''}{\chi''} = \frac{\chi'''}{\chi''} = \frac{$ constant. XIII KX =0 + TII - a2KT = 10.8 M Hence : Solit = m Solutions of these equations depend upon the nature  $X = A_{\alpha} \cos \lambda x + B_{\alpha} \sin \lambda x$ the to value of & the color of mandet made of - u.

Case (i) Let k= 12, a positive value. Now, the equation 3 are  $|x|^1 - \lambda^2 x = 0$  &  $|x|^1 - a^2 \lambda^2 T = 0$ .  $X^{11} - \lambda^2 X = 0$  (2)  $T^{11} - a^2 \lambda^2 T = 0$ .  $\frac{d^2X}{dx^8} = \lambda^8X = 0, \quad \frac{d^2T}{dx^2} = \alpha^2\lambda^8T = 0.$  $\left(\frac{d^2}{dx^2} - l^2\right) x = 0.$  D = d/dx  $\left(\frac{d^2}{dt^2} - a^2 l^2\right) T = 0$  $\left(D^{2}-\lambda^{2}\right)\chi=0$ The auniliary egn is ma- 19=0 eshere m = 12 m= ±1. .. X = Ajedx +Bje-lx. :. y = (A1e Ax + B1e-Ax) ( C1e lat + D1e-lat) Case (ii) cet k= -12, the equation 3  $x_{11} + y_3 x = 0$  $\frac{d^2x}{dx^2} + 19x = 0.$  $(D_3 + V_8) X = 0$ The our egn is ma + 12 =0 m9 = -12 me Lit = man depend T= catas Nat + D2 min Nat : X = Aa cos An + Ba sin An u = (Aa cos Ax + B2 min Ax) (C2 cos lat + D2 min lat)

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 $D^2 - a^2 A^2 = 0$ . The auxiliary egn is TX = 100 m8 - a212 =0 ma = a 2 12 m=± la' .. T= Crelat + Dre-lat a negative value. are x'' + 18x = 0 of  $T^{11} + a^2 187 = 0$  $T11 + \alpha^2 \lambda^2 T = 0$ . 0 2 2.4.9  $\frac{d^2T}{dt^2} + a^2I^2T = 0$ (02 + 12 a2) T=0 laupo & Mono The aux egn is m9 + 19a9 =0 0=xy ma = -12a2 m= tila

(3) are x11=0 & 71=0. Now the equations

Out of the above three to types of Connatont wo = 11x

integrating time wr to x,

 $\frac{dx}{dx} = A3$ 

we choose the solution which contains X = A31+ B3

20 AniAu de Tomo De Journes  $\frac{d^2}{dt^2} = 0$ 

int. iv. r to t', (twice)

7 = C3E+D3. 3 100 1 10

trigenometric terms

$$y = (A_3x + B_3)(C_3t + D_3)$$

The various possible solutions of the wave equation are

y(n,t) = (A1eAn + B1e-An) (cle lat + D1e lat) + r(m)A) = (4,n)g

Y(n+t) = (Aacos)n+B2 mnn) (C2 cos lat + D2 mn lat)

 $y(x_1t) = (A_3x + B_3) (c_3t + D_3)$ 

We can choose the correct solution as follows solutions we have Out of the above three types of to Choose the correct one which is connistent with the physical nature of the problem. Since we are dealing with problem on vibrations, y must be a periodic function of x and to Therefore, we choose the solution which contains the Engonometric terms since sine (and to conine functions are solution in nature. Hence the Correct Periodic oldimod

y(n+) = (AcosAn+ BrinAn) (Ccoshat + Drinhat).

g(n,t) = (Ag cosh + Bz hinth) = (Cg coshot + Dz hinhot)

3(x,t) = (A38 + B3) (C3+ + P3)

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