S. Kunal Keshan

RA2011004010051

ECE – A

Control Systems –

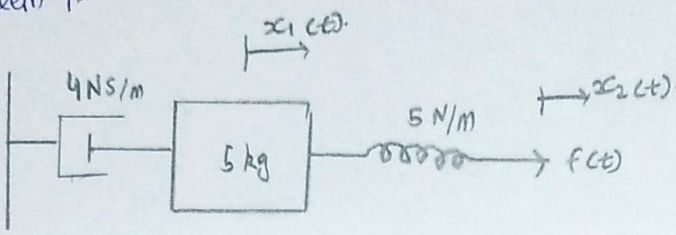
18ECS201T

18ECE201T - CONTROL SYSTEMS

MTS Assignment

KUNAL KESHAN
RA2011004010051
ECE - A

1 Problem 1.



Find $\frac{X_2(s)}{F(s)}$

Soln:

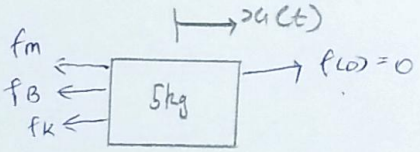
Given;

For dashpot, B = 4 NS/m

Mass, M = 5kg No of nodes = 1

Spring Constant, K = 5 N/m

Force along Mass of 5 Kg



$$0 = 5\frac{d^2x_1(t)}{dt^2} + 4\frac{dx_1(t)}{dt} + 5(x_1(t) - x_2(t)) \quad \text{— (1)}$$

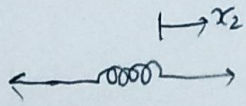
Taking Laplace of (1), we get

$$L[(1)] = 0 = 5s^2X_1(s) + 4sX_1(s) + 5X_1(s) - 5X_2(s) \quad \text{— (2)}$$

Simplifying (2)

$$(5s^2 + 4s + 5)X_1(s) = 5X_2(s).$$

Force along Spring



$$f(t) = 5\,(x_2(t) - x_1(t)) \quad - (3)$$

Taking Laplace of (3), we get

$$L[(3)] = F(s) = 5\left[X_2(s) - X_1(s)\right] \quad - (4)$$

From (2), we get

$$X_1(s) = \frac{5\,X_2(s)}{(5s^2 + 4s + 5)} \quad - (5)$$

Substituting (5) in (4), we get

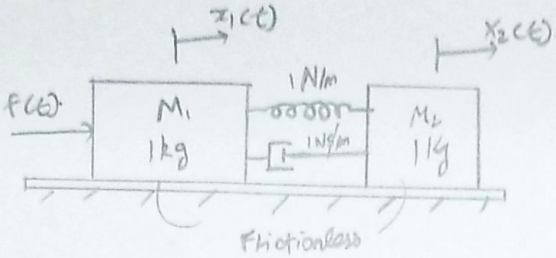
$$F(s) = \frac{5\,X_2(s) \times (5s^2+4s+5) - 25\,X_2(s)}{(5s^2 + 4s + 5)}$$

$$\frac{F(s)}{X_2(s)} = \frac{5(5s^2+4s+5) - 25}{(5s^2 + 4s + 5)}$$

$$\frac{X_2(s)}{F(s)} = \frac{5s^2 + 4s + 5}{25s^2 + 20s + 25 - 25}$$

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{5s^2 + 4s + 5}{25s^2 + 20s}}$$

2 Problem 2.



Find $\frac{X_2(s)}{F(s)}$.

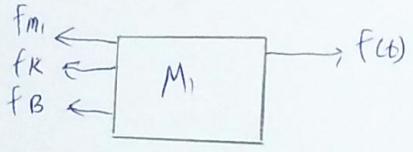
Soln. Given.

Dashpot, B = 1 N s/m

Spring Constant, K = 1 N/m No of nodes = 2.

Mass, $M_1 = M_2 = 1$ kg.

Free body diagram for M_1



$$f(t) = 1 \cdot \frac{d^2 x_1(t)}{dt^2} + 1 \cdot (x_1(t) - x_2(t)) + 1 \cdot \frac{d(x_1(t) - x_2(t))}{dt} \quad -(1)$$

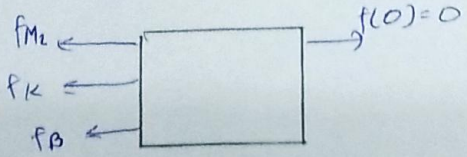
Taking Laplace of (1), we get

$$L[(1)]: \quad F(s) = s^2 X_1(s) + X_1(s) - X_2(s) + s X_1(s) - s X_2(s) \quad -(2)$$

Simplifying (2),

$$F(s) = (s^2 + s + 1) X_1(s) - (1 + s) X_2(s).$$

Free body diagram for M_2



$$0 = 1\,\frac{d^2x_2(t)}{dt^2} + 1\,(x_2(t) - x_1(t)) + 1\,\frac{d}{dt}(x_2(t) - x_1(t)) \quad -(3)$$

Taking Laplace of (3), we get

$$L[(3)] = 0 = s^2X_2(s) + X_2(s) - X_1(s) + sX_2(s) - sX_1(s)$$

Simplifying (3),

$$(s^2 + s + 1)X_2(s) - (1+s)X_1(s) = 0$$

$$X_1(s) = \frac{(s^2+s+1)X_2(s)}{(1+s)} \quad -(5)$$

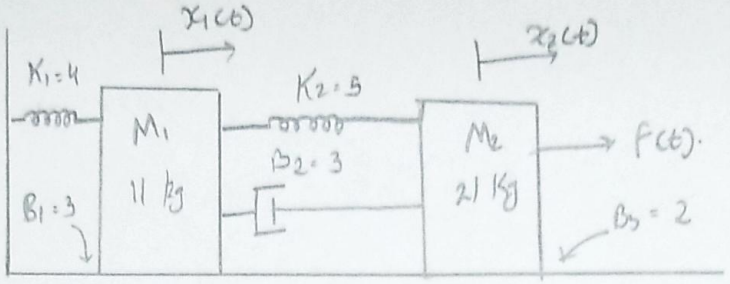
Substituting (5) in (2), we get

$$F(s) = \frac{(s^2+s+1)^2}{(1+s)}X_2(s) - (1+s)X_2(s)$$

$$\frac{F(s)}{X_2(s)} = \frac{(s^2+s+1)^2 - (1+s)^2}{(1+s)}$$

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{(1+s)}{(s^2+s+1)^2 - (1+s)^2}}$$

3. Problem 3.



Find $\frac{X_2(s)}{F(s)}$

Soln.

Given

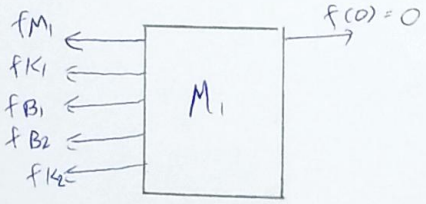
Spring Constant, $K_1 = 4$
$K_2 = 5$

No of Nodes = 2

Dashpot Constant, $B_1 = 3$
$B_2 = 3$
$B_3 = 2$

Mass, $M_1 = 11$ kg
$M_2 = 21$ kg

Free body Diagram for M_1,



$$0 = 11 \cdot \frac{d^2 x_1(t)}{dt^2} + 4 \cdot x_1(t) + 5(x_1(t) - x_2(t)) + 3\frac{d x_1(t)}{dt} + 3\frac{d}{dt}(x_1(t) - x_2(t)) \quad -(1)$$

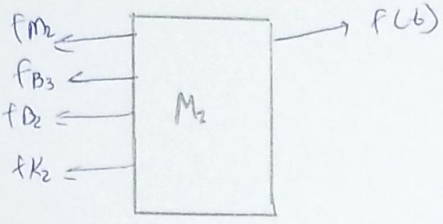
Taking Laplace of (1), we get

$$L[0] = 0 = 11 s^2 X_1(s) + 4 X_1(s) + 5 X_1(s) - 5(X_2(s)) + 3s X_1(s) + 3s X_1(s) - 3s X_2(s)$$

$$11S^2 X_1(S) + 9X_1(S) + 6S X_1(S) - (S+3S)X_2(S) = 0$$

$$X_1(S) = \frac{(S+3S)\,X_2(S)}{(11S^2+6S+9)} \quad - (2)$$

Free body diagram of M_2,



$$f(t) = 21\frac{d^2x_2(t)}{dt^2} + 2\frac{d}{dt}x_2(t) + 3\frac{d}{dt}(x_2(t) - x_1(t)) + 5(x_2(t) - x_1(t)) \quad - (3)$$

Taking Laplace of (3), we get.

$$L[(3)] = F(S) = 21S^2 X_2(S) + 2S X_2(S) + 3S X_2(S) - 3S X_1(S) + 5X_2(S) + 5X_1(S).$$

$$F(S) = (21S^2 + 5S + 5)X_2(S) - (3S+5)X_1(S) \quad - (4)$$

Substituting (2) in (4)

$$F(S) = (21S^2 + 5S + 5)X_2(S) - \frac{(3S+5)^2 X_2(S)}{(11S^2+6S+9)}$$

$$\boxed{\frac{X_2(S)}{F(S)} = \frac{(11S^2+6S+9)}{(21S^2+5S+5)(11S^2+6S+9) - (3S+5)^2}}$$