18ECC204J
DIGITAL SIGNAL PROCESSING
Unit 1_Signals and Waveforms_Part2

Syllabus Overview

- **Learning Unit / Module 1: Signals and Waveforms**
- **Learning Unit / Module 2: Frequency Transformations**
- **Learning Unit / Module 3: FIR Filters**
- **Learning Unit / Module 4: IIR Filters**
- **Learning Unit / Module 5: Multirate signal Processing**

Learning Unit / Module 1: Signals and Waveforms

- Basic Elements of DSP , Advantages and applications of DSP
- Continuous Time vs Discrete time signals , Continuous valued vs discrete valued signals.
- Concepts of frequency in analog signals , Continuous and discrete time sinusoidal signals ,
- Sampling of analog signals Sampling theorem
- Aliasing Quantization of continuous amplitude signals,
- Analog to digital conversion Sample and hold, Quantization and coding
- Oversampling A/D converters ,Digital to analog conversion Sample and hold
- Oversampling D/A converters, Quantization noise
- Errors due to truncation IDFT, Probability of error

Topics Covered in this PPT_Unit 1_Part 2

- Analog to digital conversion Sample and hold, Quantization and coding
- Oversampling A/D converters, Digital to analog conversion Sample and hold
- Oversampling D/A converters, Quantization noise
- Errors due to truncation IDFT, Probability of error

Analog to Digital Conversion- Introduction

- Recall that the process of converting a continuous-time (analog) signal to a digital sequence that can be processed by a digital system requires that we quantize the sampled values to a finite number of levels and represent each level by a number of bits.

- The electronic device that performs this conversion from an analog signal to a digital sequence is called an analog-to-digital (A/D) converter (ADC).

- On the other hand, a digital-to-analog (D/A) converter (DAC) takes a digital sequence and produces at its output a voltage or current proportional to the size of the digital word applied to its input.

Analog to Digital Conversion- Sample and Hold

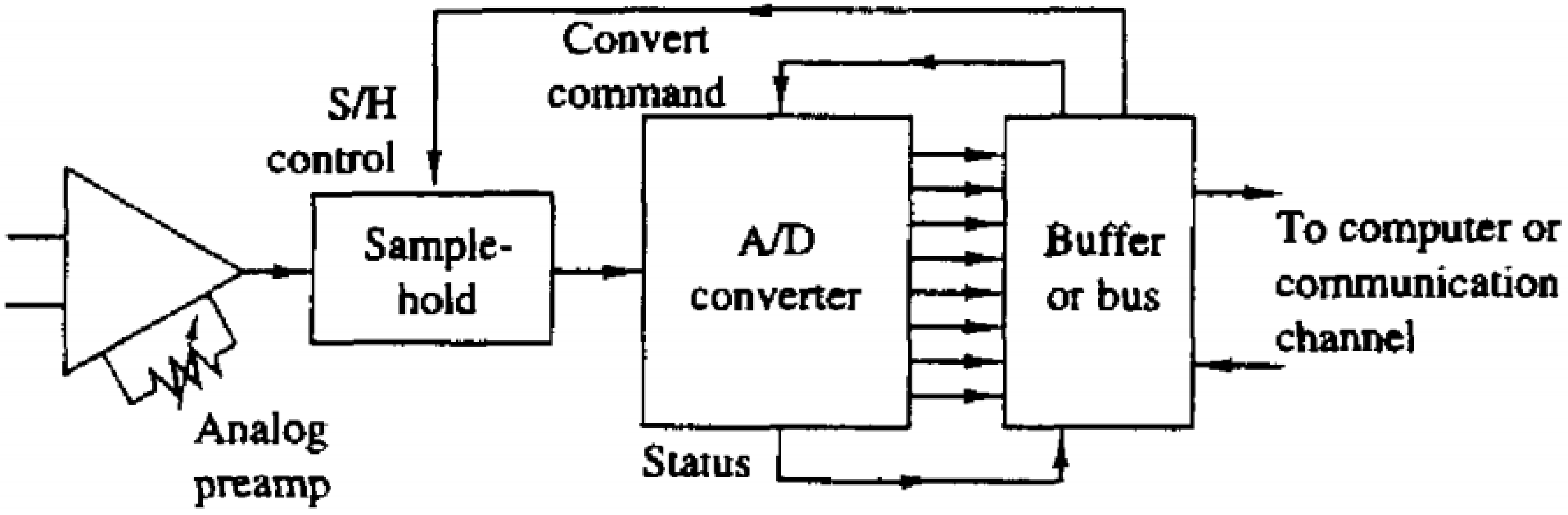
- The sampling of an analog signal is performed by a sample-and-hold (S/H) circuit. The sampled signal is then quantized and converted to digital form.

- Usually, the S/H is integrated into the A /D converter. The S/H is a digitally controlled analog circuit that tracks the analog input signal during the sample mode, and then holds it fixed during the hold mode to the instantaneous value of the signal at the time the system is switched from the sample mode to the hold mode.

- The goal of the S/H is to continuously sample the input signal and then to hold that value constant as long as it takes for the A /D converter to obtain its digital representation. The use of an S/H allows the A /D converter to operate more slowly compared to the time actually used to acquire the sample.

- In the absence of a S/H, the input signal must not change by more than one-half of the quantization step during the conversion, which may be an impractical constraint. Consequently, the S/H is crucial in high-resolution (12 bits per sample or higher) digital conversion of signals that have large bandwidths (i.e., they change very rapidly).

Block diagram of basic elements of an A/D Converter



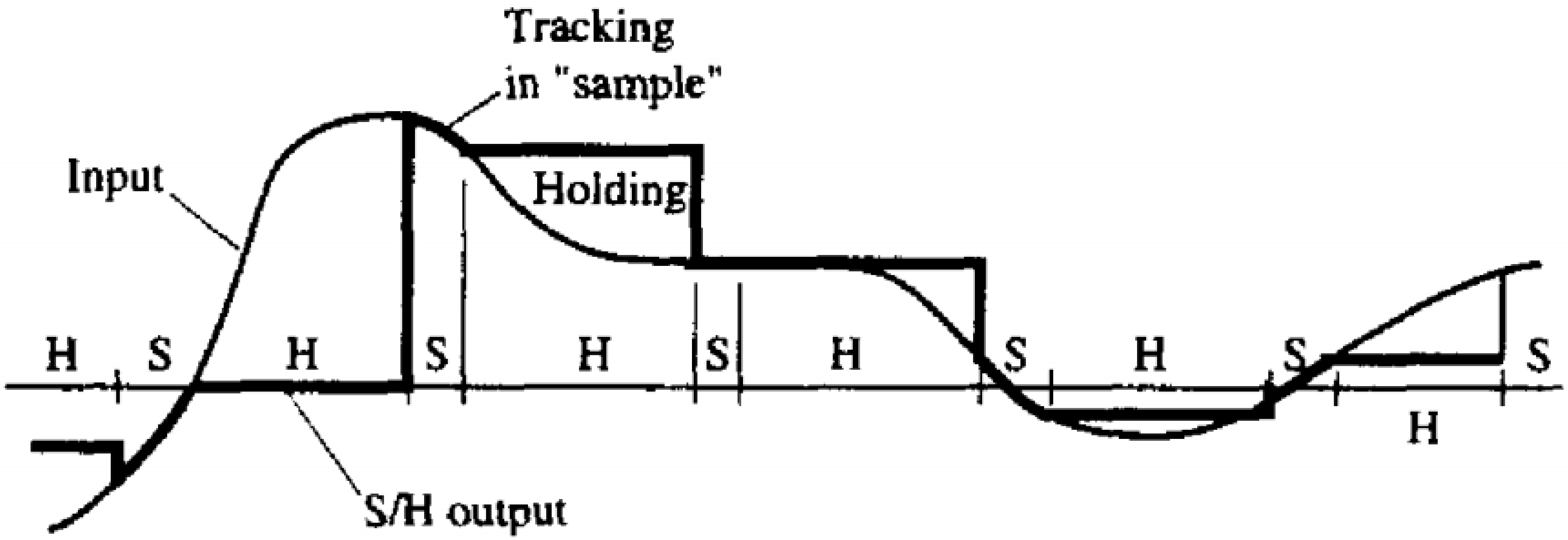
Analog to Digital Conversion- Sample and Hold

- An ideal S/H introduces no distortion in the conversion process and is accurately modeled as an ideal sampler. However, time-related degradations such as errors in the periodicity of the sampling process ("jitter"), nonlinear variations in the duration of the sampling aperture, and changes in the voltage held during conversion ("droop") do occur in practical devices.

- The A /D converter begins the conversion after it receives a convert command. The time required to complete the conversion should be less than the duration of the hold mode o f the S/H.

- Further more, the sampling period T should be larger than the duration of the sample mode and the hold mode.

Time domain response of an ideal S/H circuit



Quantization, Coding and Noise

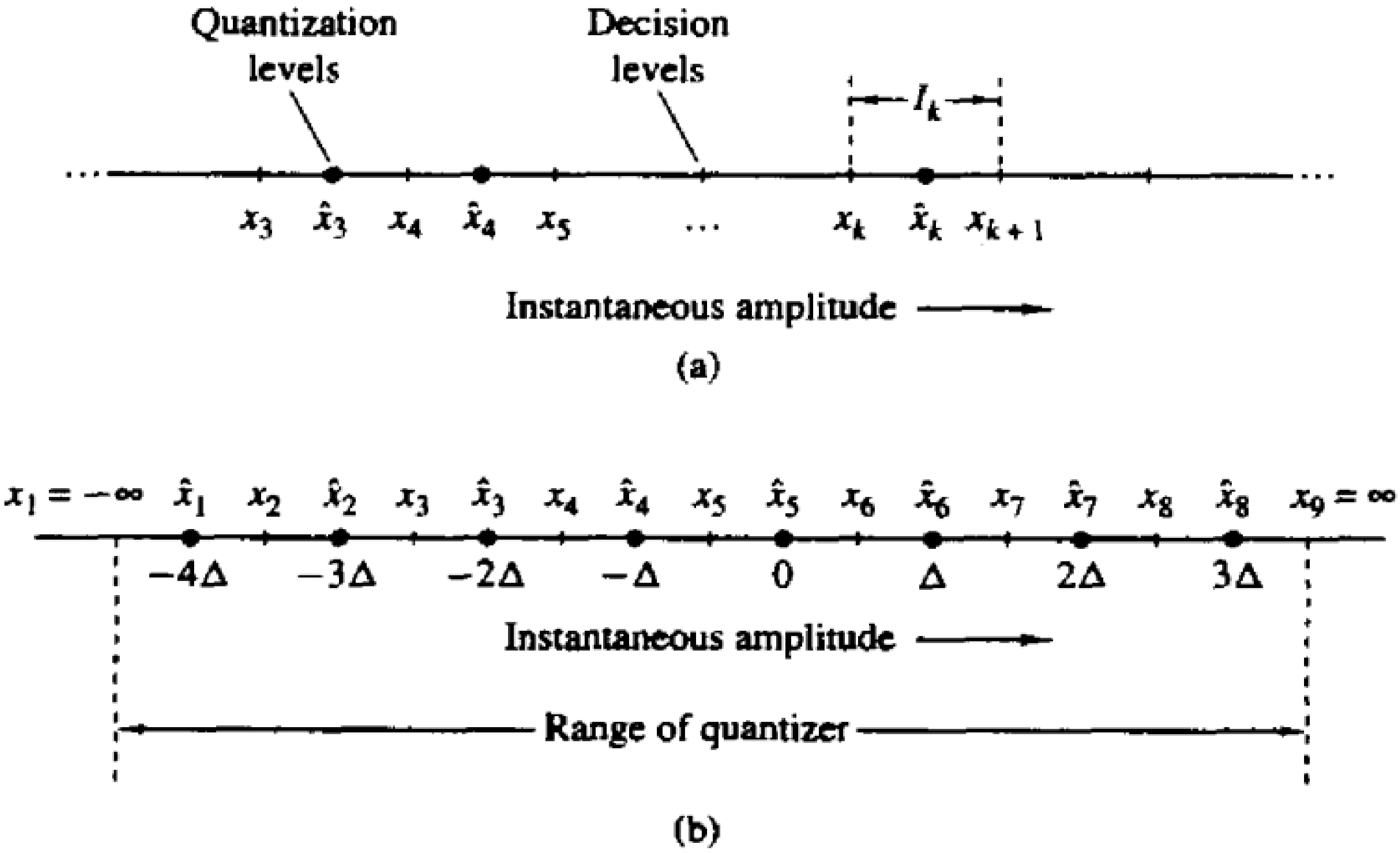
- The basic task o f the A /D converter is to convert a continuous range of input amplitudes into a discrete set of digital code words. This conversion involves the processes of quantization and coding. Quantization is a nonlinear and noninvertible process that maps a given amplitude x(n) = x(nT) at time t = nT into an amplitude x, taken from a finite set of values. where the signal amplitude range is divided into L intervals,

$$I_k = \{x_k < x(n) \leq x_{k+1}\} \qquad k = 1, 2, \ldots, L$$

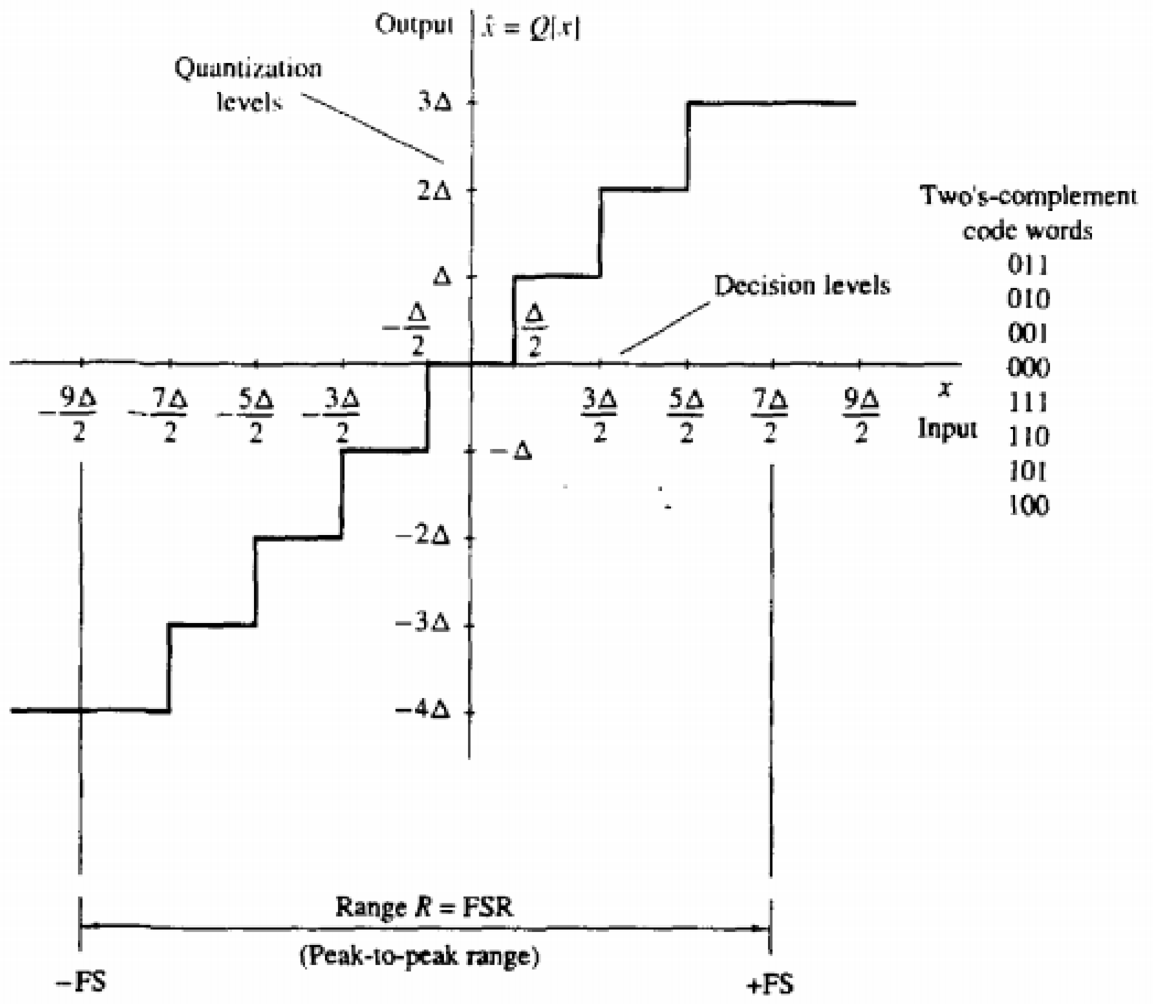
by the $L+1$ *decision levels* $x_1, x_L, \ldots, x_{L+1}$. The possible outputs of the quantizer (i.e., the quantization levels) are denoted as $\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_L$. The operation of the quantizer is defined by the relation

$$x_q(n) \equiv Q[x(n)] = \hat{x}_k \qquad \text{if } x(n) \in I_k$$

Quantization Process and Example of a Midtread process



Example of a Midtread Quantizer



Example of a Midtread Quantizer

- The coding process in an A /D converter assigns a unique binary number to each quantization level. If we have L levels, we need at least L different binary numbers. With a word length of b + 1 bits we can represent 2^{b+1} distinct binary numbers. Hence we should have $2^{b+1} \geq L$ or,
$b + 1 \geq \log_2 L$

- Then the step size or the resolution o f the A /D converter is given by

$$\Delta = \frac{R}{2^{b+1}}$$

where R is the range o f the quantizer

Oversampling of A/D Converter

- The basic idea in oversampling A /D converters is to increase the sampling rate o f the signal to the point where a low-resolution quantizer suffices. By oversampling. we can reduce the dynamic range o f the signal values between successive samples and thus reduce the resolution requirements on the quantizer.

- As we have observed in the preceding section, the variance o f the quantization error in A /D conversion is $\sigma_e^2 = \Delta^2/12$ where $\Delta = R/2^{b+1}$ .Since the dynamic range of the signal, which is proportional to its standard deviation σ_x.

Oversampling of A/D Converter

- Hence for a given number o f bits, the power of the quantization noise is proportional to the variance of the signal to be quantized. Consequently, for a given fixed SQNR , a reduction in the variance of the signal to be quantized allow s us to reduce the number of bits in the quantizer.

- The basic idea for reducing the dynamic range leads us to consider differential quantization. To illustrate this point, let us evaluate the variance of the difference between two successive signal samples. Thus we have

$$d(n) = x(n) - x(n-1)$$

Contd.. .

The variance of $d(n)$ is

$$\begin{aligned}\sigma_d^2 &= E[d^2(n)] = E\{[x(n) - x(n-1)]^2\} \\ &= E[x^2(n)] - 2E[x(n)x(n-1)] + E[x^2(n-1)] \\ &= 2\sigma_x^2[1 - \gamma_{xx}(1)]\end{aligned}$$

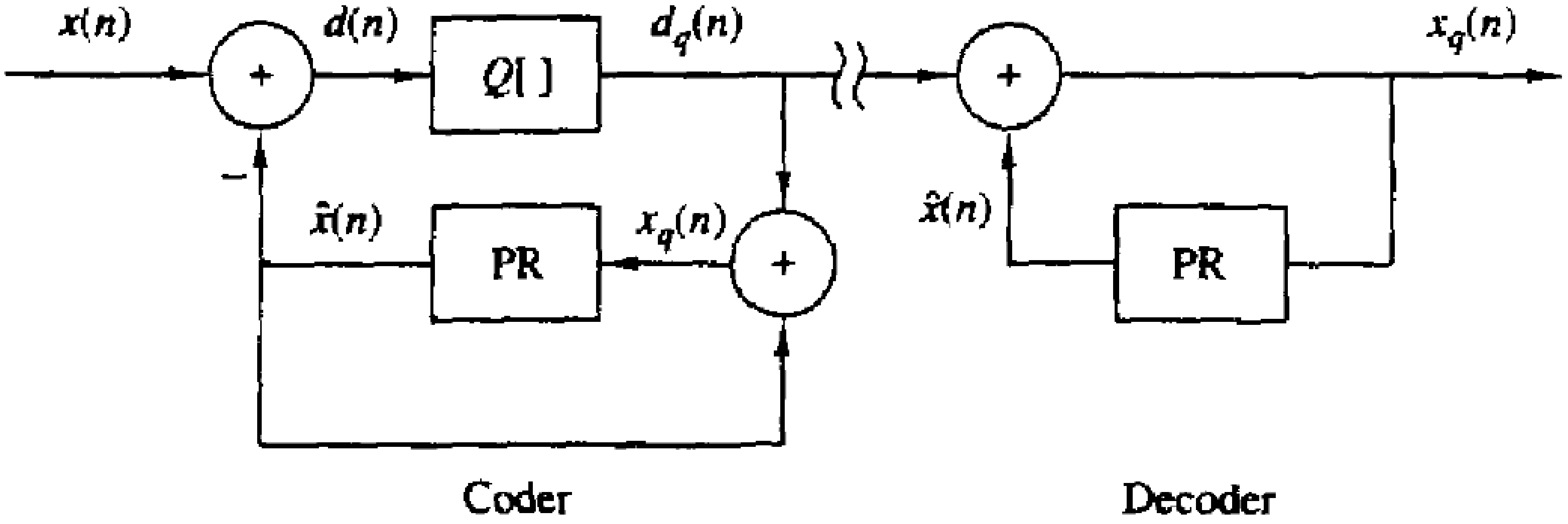
where $\gamma_{xx}(1)$ is the value of the autocorrelation sequence $\gamma_{xx}(m)$ of $x(n)$ evaluated at $m = 1$. If $\gamma_{xx}(1) > 0.5$, we observe that $\sigma_d^2 < \sigma_x^2$. Under this condition, it is better to quantize the difference $d(n)$ and to recover $x(n)$ from the quantized values $\{d_q(n)\}$. To obtain a high correlation between successive samples of the signal, we require that the sampling rate be significantly higher than the Nyquist rate.

An even better approach is to quantize the difference

$$d(n) = x(n) - ax(n-1)$$

Contd.. .

Figure: Encoder and decoder for differential predictive signal quantizer system.



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- The use of the feedback loop around the quantizer as shown in Fig. is necessary to avoid the accumulation o f quantization errors at the decoder. In this configuration, the error $e(n) = d(n) - d_q(n)$ is

$$e(n) = d(n) - d_q(n) = x(n) - \hat{x}(n) - d_q(n) = x(n) - x_q(n)$$

- Thus the error in the reconstructed quantized signal xq(n) is equal to the quantization error for the sample d(n). The decoder for DPCM that reconstructs the signal from the quantized values is also shown in Fig.

Delta Modulation

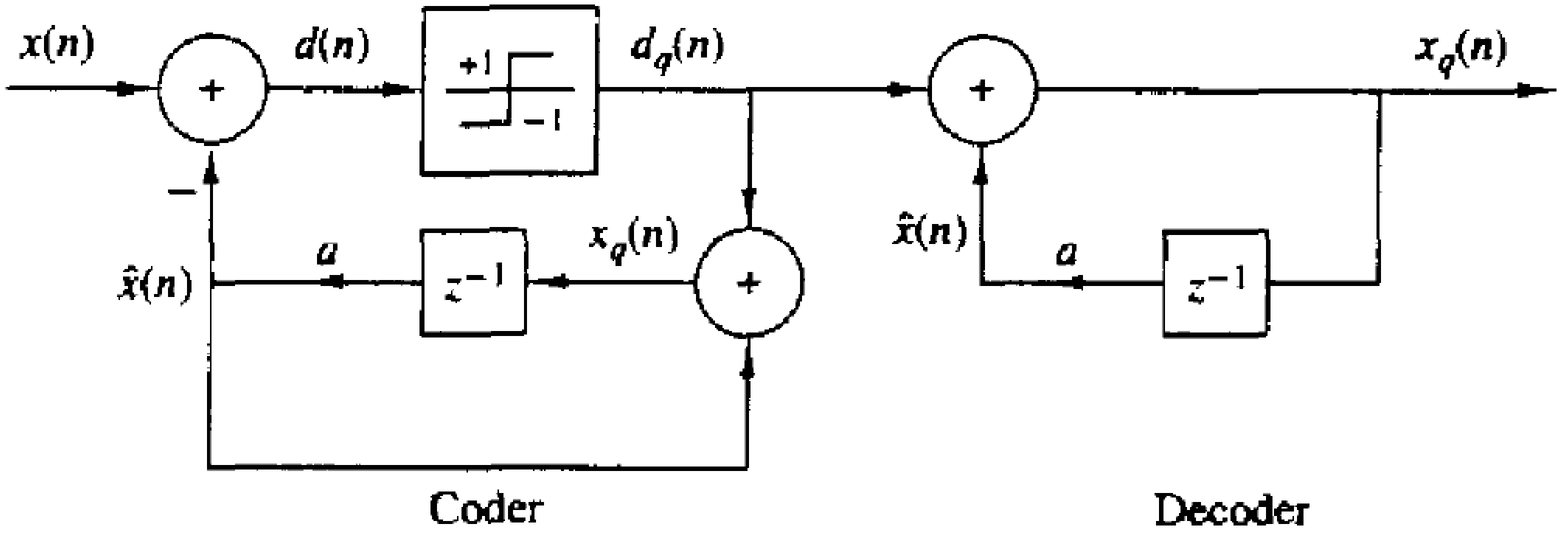
❑The simplest form of differential predictive quantization is called *delta modulation* (DM).

❑In DM , the quantizer is a simple 1-bit (two -level) quantizer and the predictor is a first-order predictor. Basically, DM provides a staircase approximation o f the input signal.

$$\hat{x}(n) = a x_q(n-1)$$

❑At every sampling instant, the sign o f the difference between the input sample *x(n) and its most recent staircase* approximation is determined, and then the staircase signal is updated by a step A in the direction o f the difference.

Delta Modulation system: Fig 1

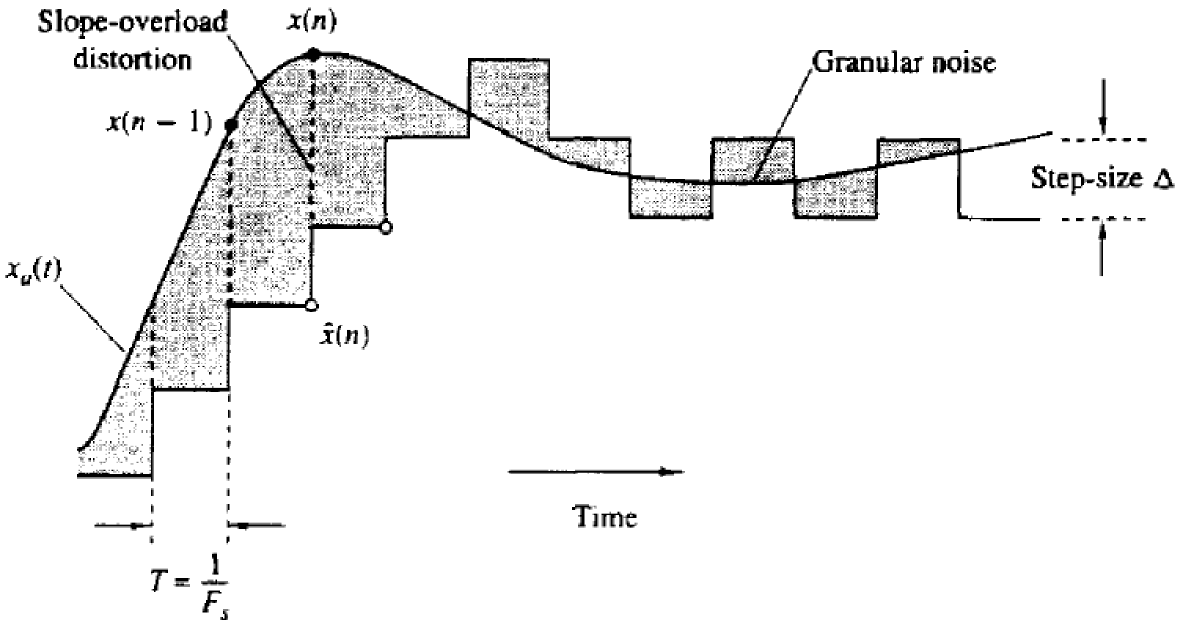


From Fig. 1 we observe that

$$x_q(n) = a x_q(n-1) + d_q(n)$$

which is the discrete-time equivalent of an analog integrator. If *a = 1, we have* an ideal accum ulator (integrator) whereas the choice *a < 1 results in a "leaky* integrator."

Two Types of Quantization errors: Fig 2



❑The crosshatched areas in Fig.2 illustrate two types o f quantization error in DM , slope-overload distortion and granular noise. Since the maximum slope A Δ/T in $x(n)$ *is limited by the step size, slope-overload distortion can be* avoided if $\max |dx(t)/dt| \leq \Delta/T$.

❑ *The granular noise occurs when the DM tracks* a relatively flat (slowly changing) input signal. We note that increasing Δ reduces overload distortion but increases the granular noise, and vice versa.

Two Types of Quantization errors: Fig 3

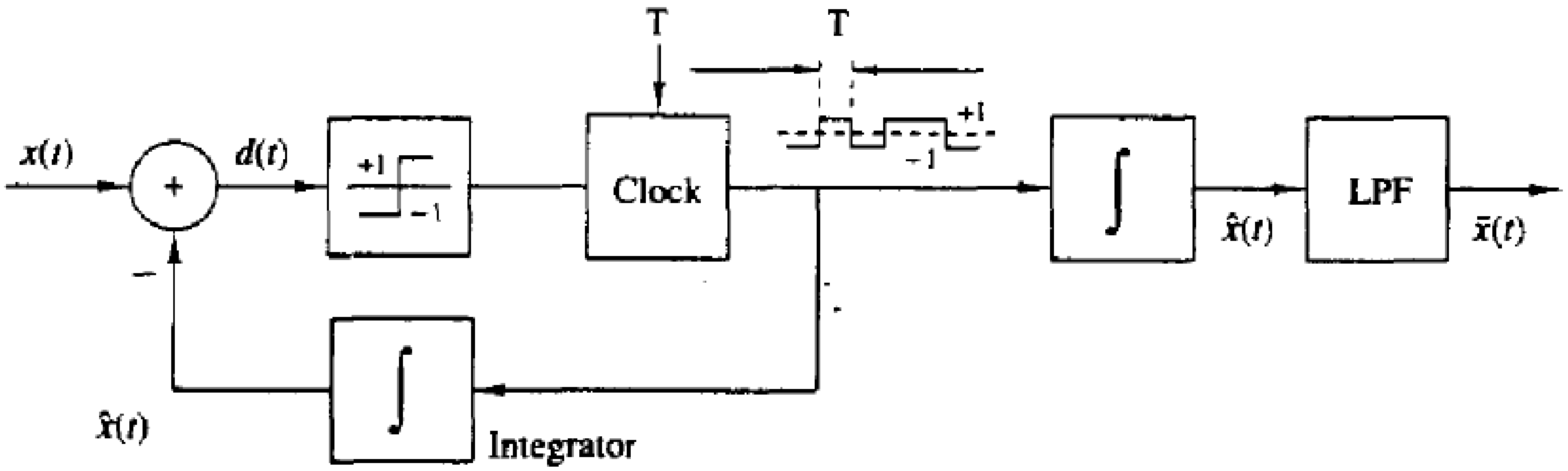
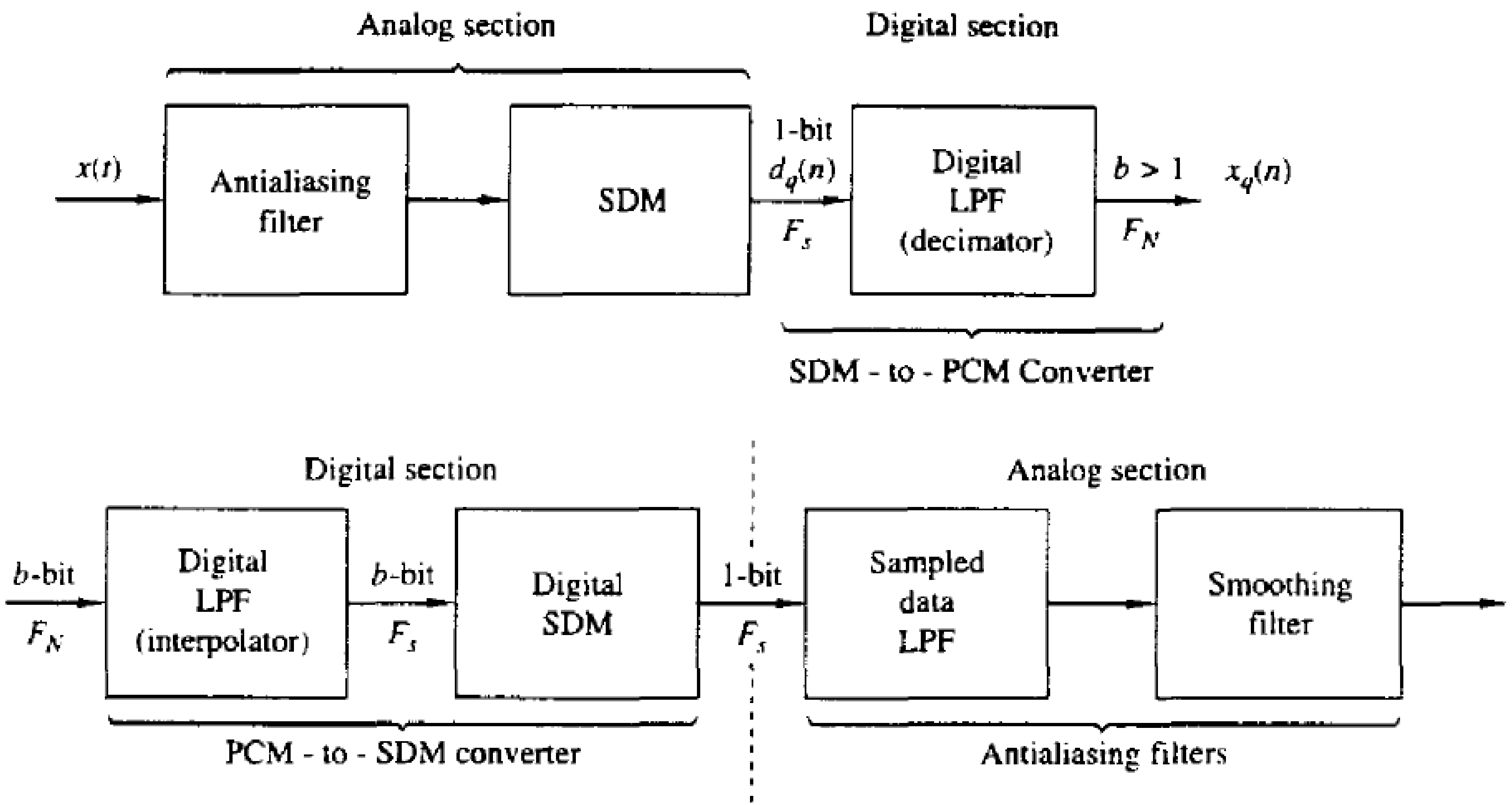


Figure 3 shows an analog model that illustrates the basic principle for the practical implementation of a D M system. The analog low pass filter is necessary for the rejection of out-of-band components in the frequency range between *B and* $F_s/2$, since $F_s >> B$ due to oversampling

Basic Elements of Oversampling A/D Converter: Fig 4



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- If the interpolation factor is I = 256, the A /D converter output can be obtained by averaging successive non-overlapping blocks of 128 bits. This averaging would result in a digital signal with a range of values from zero to 256{b as 8 bits) at the Nyquist rate. The averaging process also provides the required antialiasing filtering.

- Oversampling A/D converters for voice-band (3-kH z) signals are currently fabricated as integrated circuits. Typically, they operate at a 2-M H z sampling rate, downsample to 8 kH z, and provide 16-bit accuracy.

Digital to Analog Conversion

we demonstrated that a bandlimited lowpass analog signal, which has been sampled at the Nyquist rate (or faster), can be reconstructed from its samples without distortion.

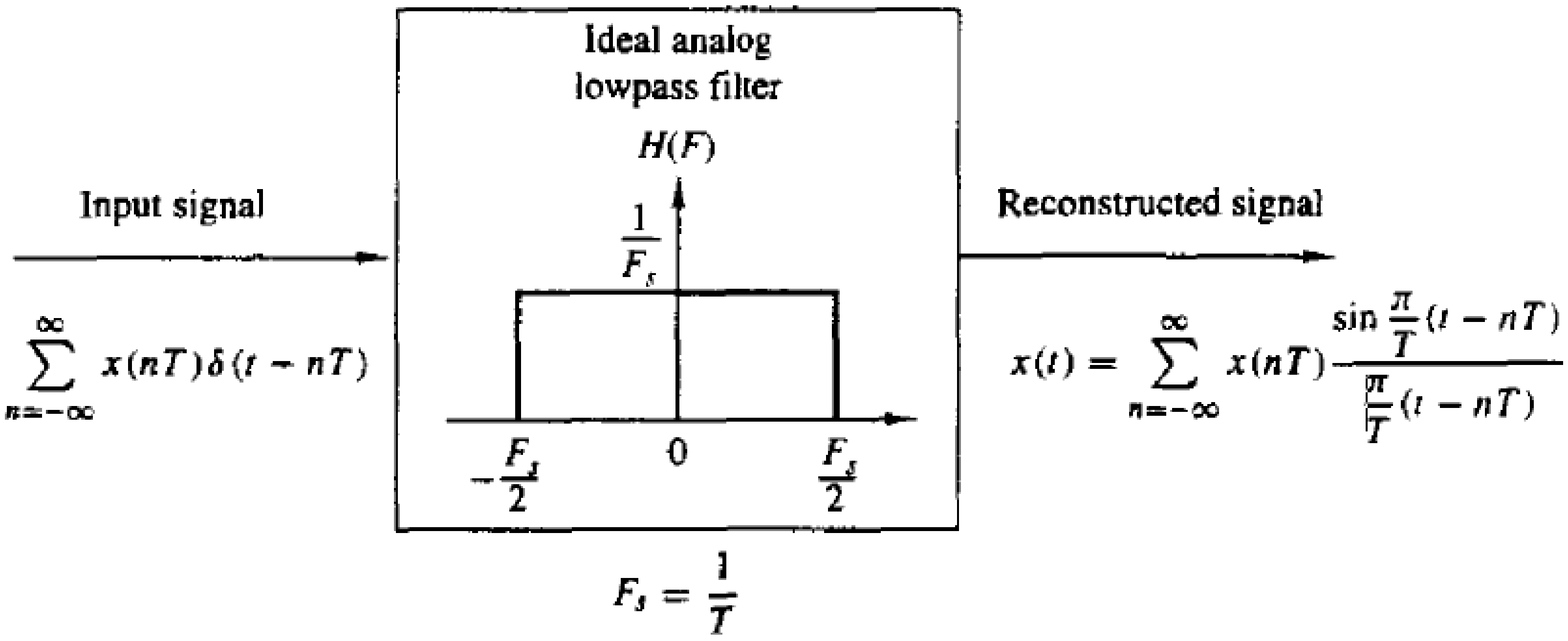
$$x(t) = \sum_{n=-\infty}^{\infty} x(nT) \frac{\sin(\pi/T)(t - nT)}{(\pi/T)(t - nT)}$$

where the sampling interval $T = 1/F_s = 1/2B$, F_s is the sampling frequency and B is the bandwidth of the analog signal.

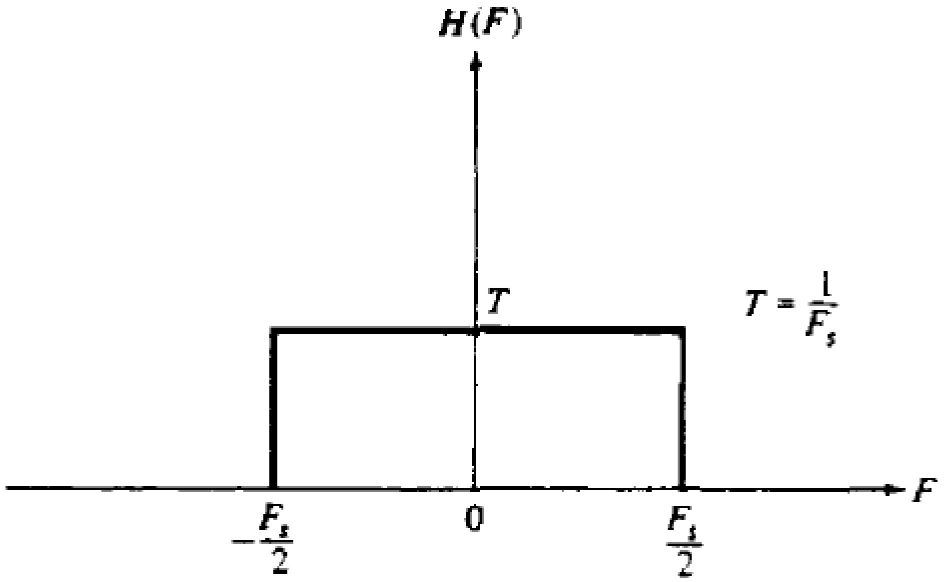
We have viewed the reconstruction of the signal $x(t)$ from its samples as an interpolation problem and have described the function

$$g(t) = \frac{\sin(\pi t/T)}{\pi t/T}$$

Signal reconstruction viewed as a filtering process- fig 5

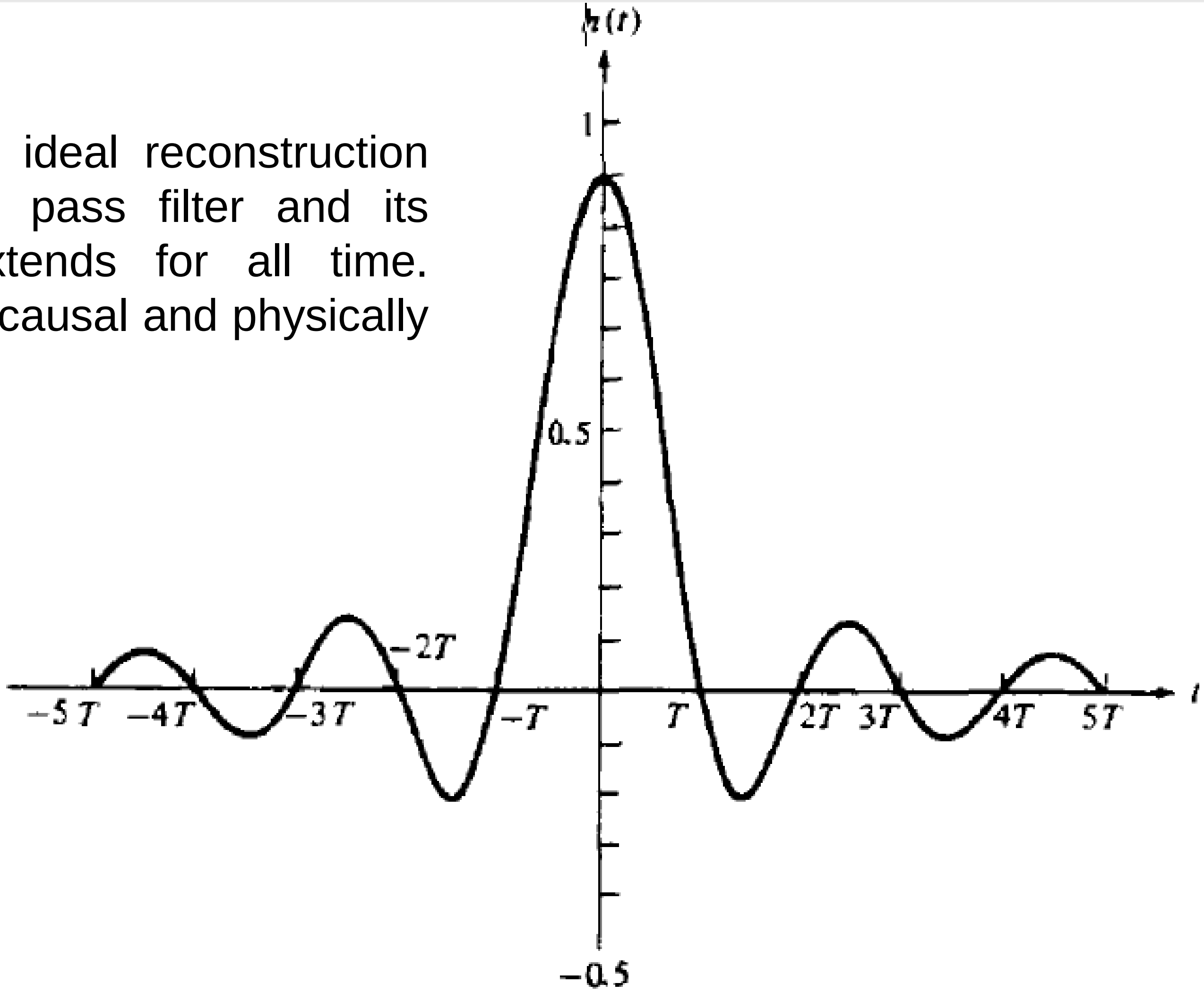


Frequency response (Fig 6.a) impulse response

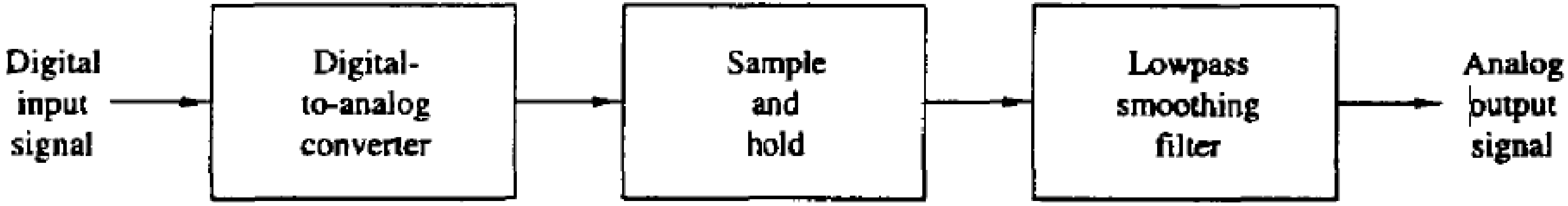


Frequency response of an ideal low-pass filter.

As shown in Fig, the ideal reconstruction filter is an ideal low pass filter and its impulse response extends for all time. Hence the filter is non-causal and physically non-realizable.

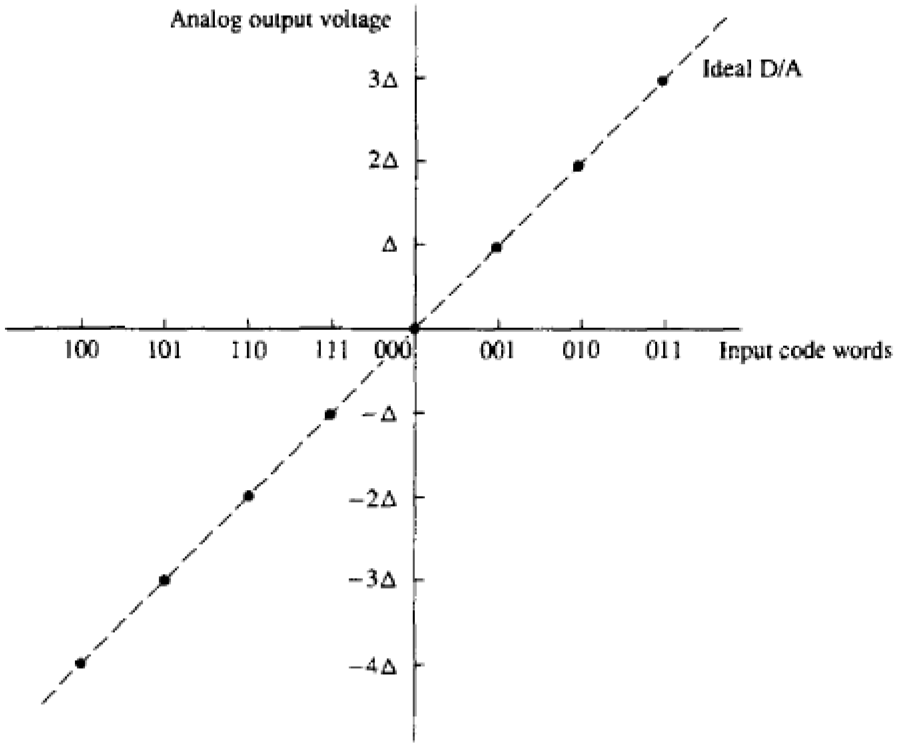


Sample and Hold



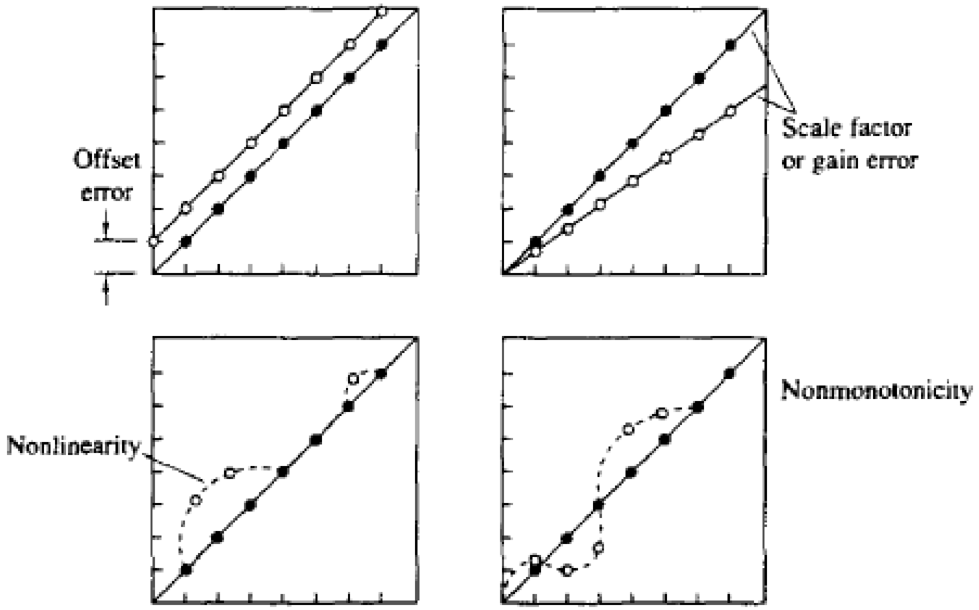
- In practice, D /A conversion is usually performed by combining a D /A converter with a sample-and-hold (S/H) and followed by a low pass (smoothing) filter, as shown.
- T he D /A converter accepts at its input, electrical signals that correspond to a binary word, and produces an output voltage or current that is proportional to the value of the binary word.

Fig(a): Ideal D/A converter characteristic



The D/A converter accepts at its input, electrical signals that correspond to a binary word, and produces an output voltage or current that is proportional to the value of the binary word. Ideally, its input-output characteristic is as shown for a 3-bit bipolar signal.

Fig(b): typical deviations from ideal performance in practical D/A converters



- The line connecting the dots is a straight line through the origin. In practical D /A converters, the line connecting the dots may deviate from the ideal.

- Some of the typical deviations from ideal are offset errors, gain errors, and nonlinearities in the input-output characteristic.

Contd…

- An important parameter o f a D /A converter is its *settling time, which is* defined as the tim e required for the output o f the D /A converter to reach and remain within a given fraction (usually, i^ LSB) of the final value, after application o f the input code word.

- Often, the application of the input code word results in a high-amplitude transient, called a “glitch.” This is especially the case when two consecutive code words to the A /D differ by several bits.

Fig 7.a: Approximation of an analog signal by a staircase

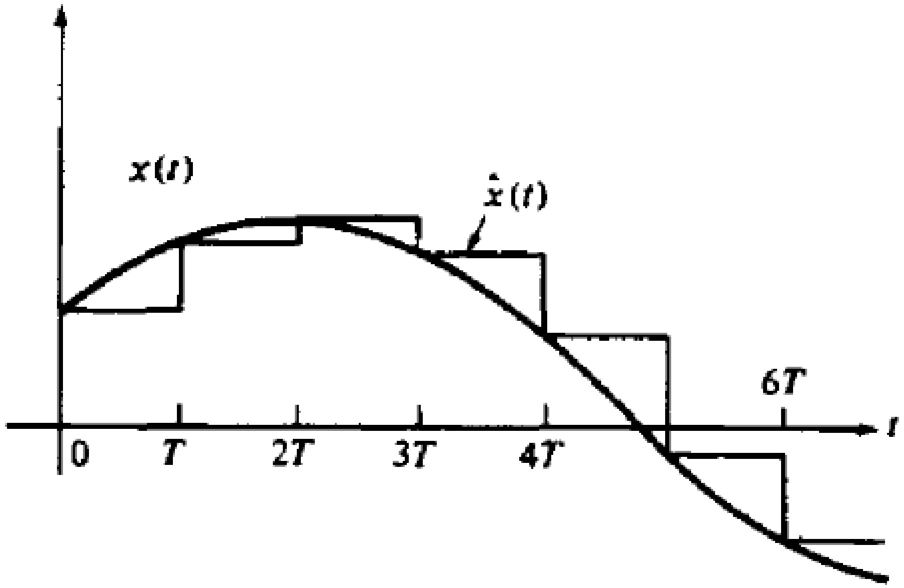
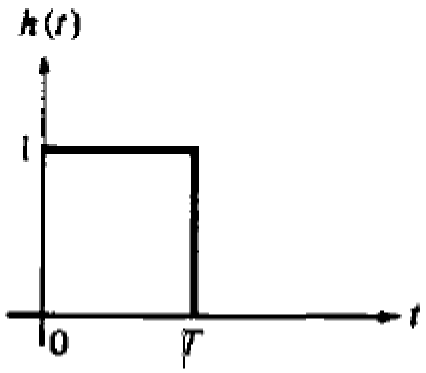


Figure 7.a illustrates the approximation of the analog signal x(t) by a S/H. As shown, the approximation, denoted as x(t), is basically a staircase function which takes the signal sample from the D/A converter and holds it for T seconds. When the next sample arrives, it jumps to the next value and holds it for T seconds, and so on.

Fig 7.b: impulse response of the S/H



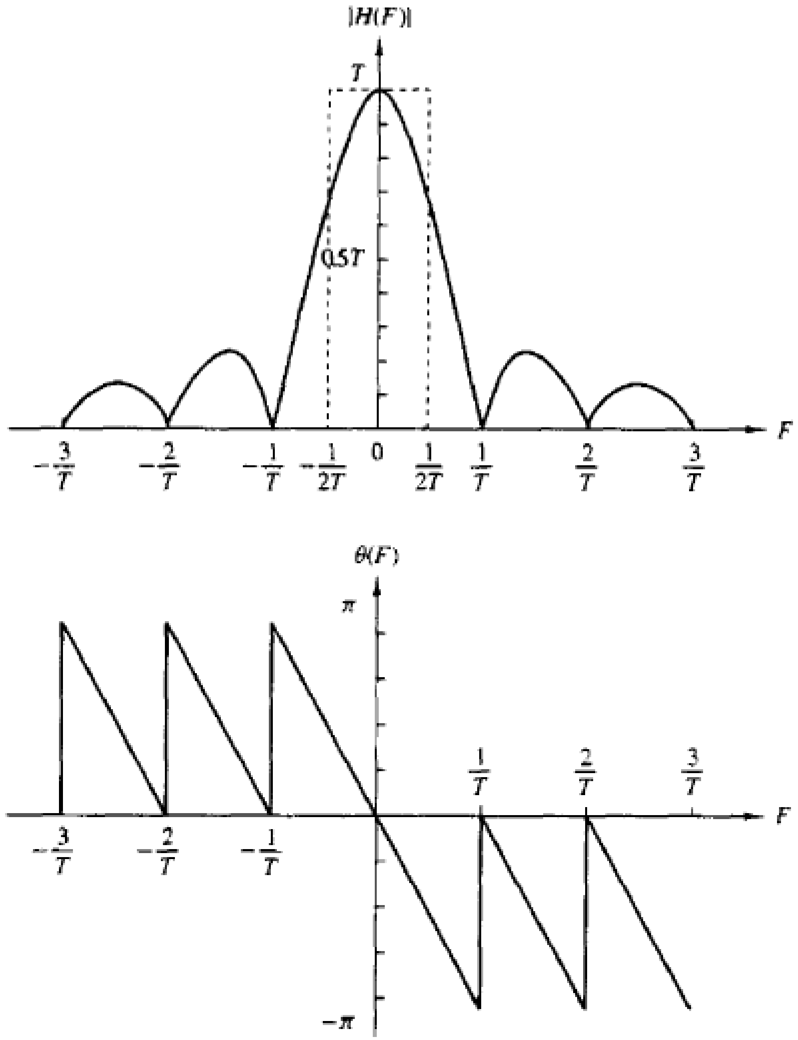
This is illustrated in Fig. 7(b). The corresponding frequency response is

$$H(F) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi Ft}dt$$

$$= \int_{0}^{T} e^{-j2\pi Ft}dt$$

$$= T\left(\frac{\sin \pi FT}{\pi FT}\right)e^{-j\pi FT}$$

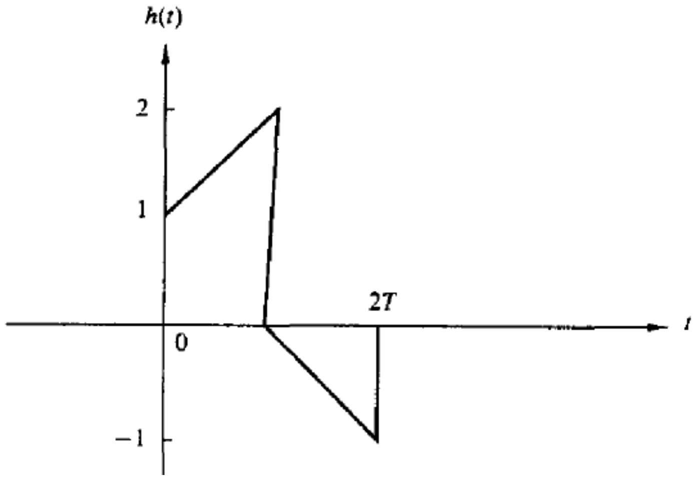
Fig 8: Frequency response Characteristics of the S/H



❑The magnitude and phase of H(F) are plotted in Figs. 8. For comparison, the frequency response of the ideal interpolator is superimposed on the magnitude characteristics.

❑It is apparent that the S/H does not possess a sharp cutoff frequency response characteristic. This is due to a large extent to the sharp transitions of its impulse response h(t).

Fig 9.a: Impulse Response



This impulse response is depicted in Fig. 9.a. The Fourier transform of *h(t)* yields the frequency response, which can be expressed in the form

$$H(F) = T(1 + 4\pi F^2T^2)^{1/2} \left(\frac{\sin \pi FT}{\pi FT}\right)^2 e^{j\Theta(F)}$$

where the phase $\Theta(F)$ is

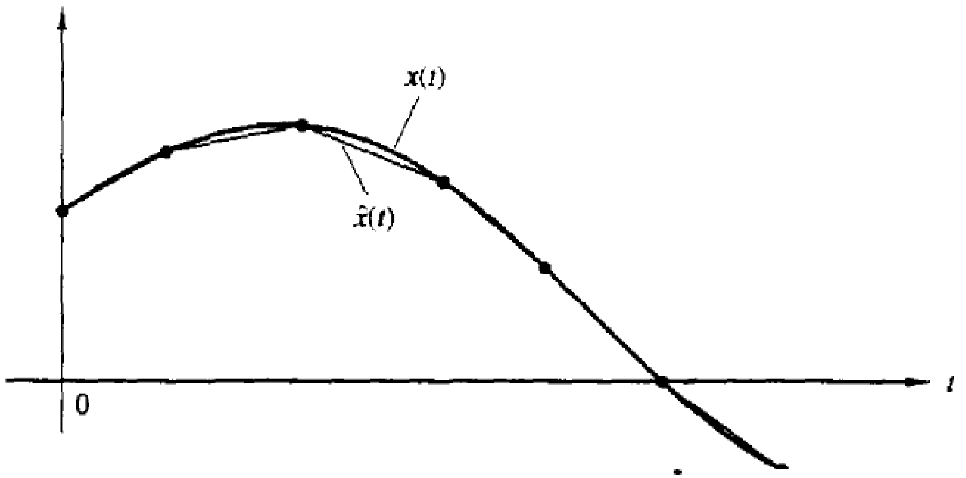
$$\Theta(F) = -\pi FT + \tan^{-1} 2\pi FT$$

Linear Interpolation with delay

- The first-order hold perform s signal reconstruction by computing the slope of the straight line based on the current sample *x(nT) and the past sample x(nT — T) of* the signal. In effect, this technique linearly extrapolates or attempts to linearly predict the next sample of the signal based on the samples *x(nT) and x{n T - T). As* a consequence, the estimated signal waveform i(r) contains jumps at the sample points.

- The jumps in *x(t)* can be avoided by providing a on e-sample delay in the reconstruction process. Then successive sample points can be connected by straight line segments.

Fig 10: Linear interpolation of *x{t) with a 7-second delay.*



Viewed as a linear filter, the linear interpolator with a T-second delay has an impulse response

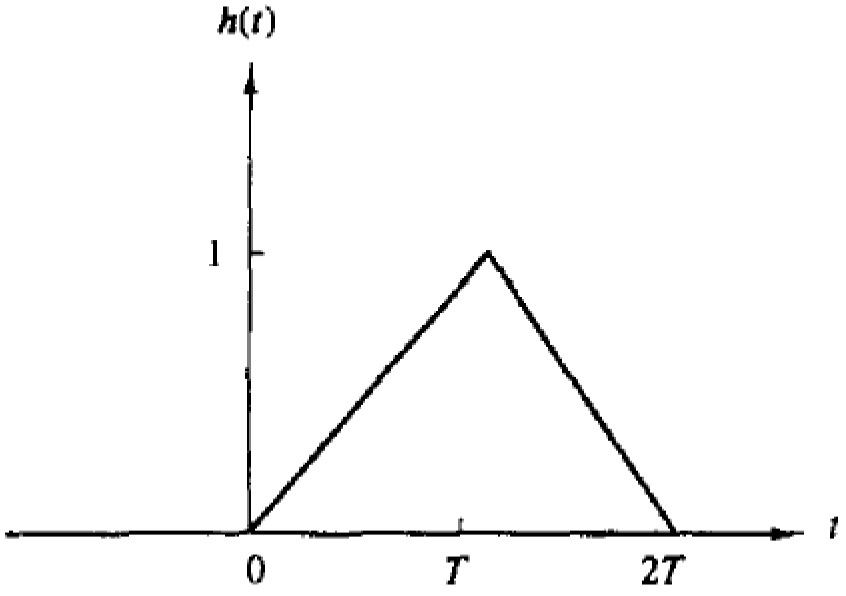
$$h(t) = \begin{cases} t/T, & 0 \leq t < T \\ 2 - t/T, & T \leq t < 2T \\ 0, & \text{otherwise} \end{cases}$$

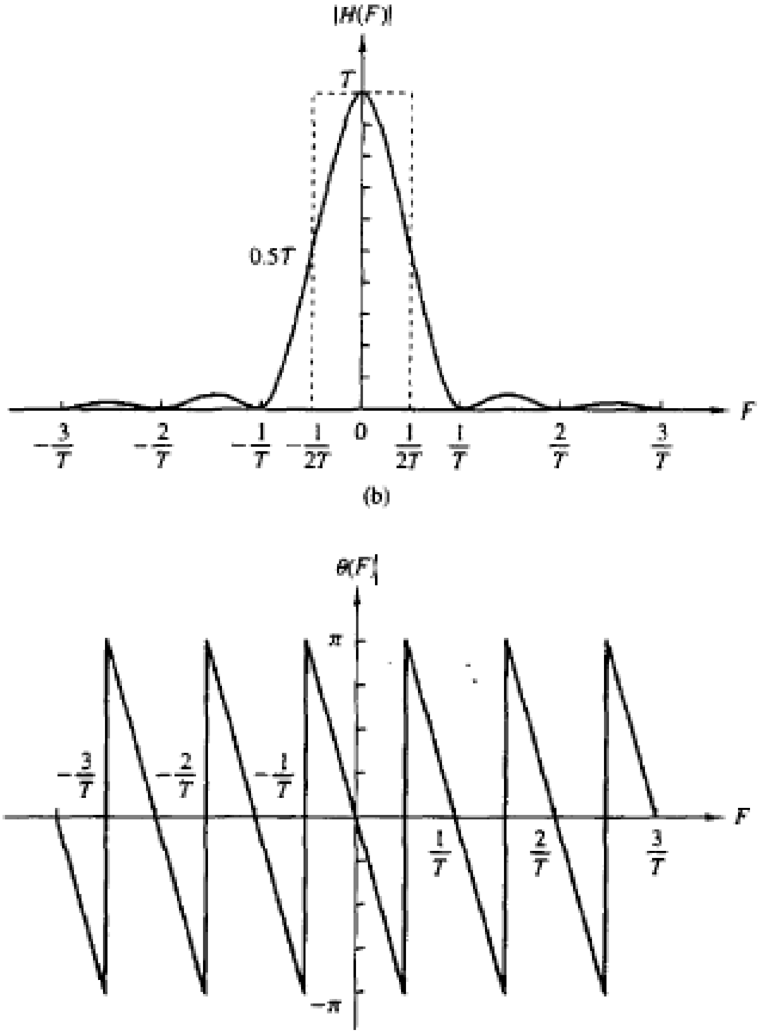
The corresponding frequency response is

$$H(F) = \int_0^T \frac{t}{T} e^{-j2\pi Ft} dt + \int_T^{2T} \left(2 - \frac{t}{T}\right) e^{-j2\pi Ft} dt$$

$$= T \left(\frac{\sin \pi FT}{\pi FT}\right)^2 e^{-j2\pi Ft}$$

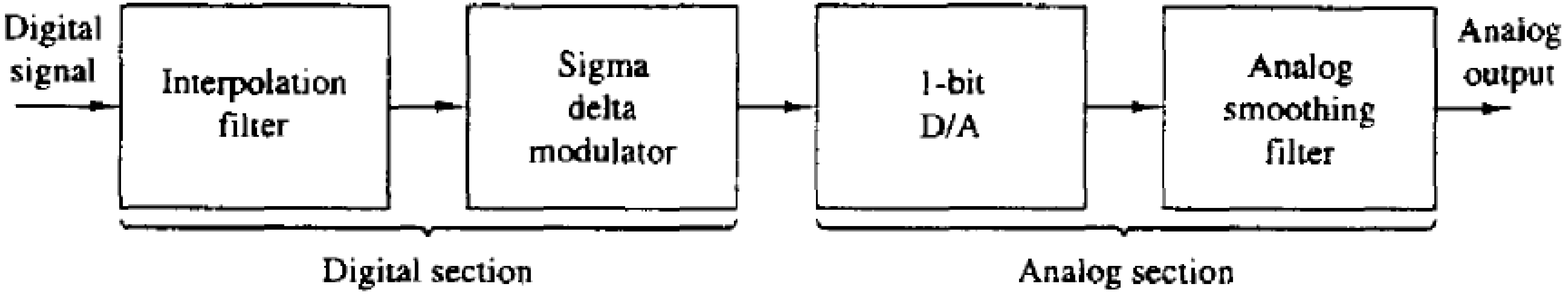
Fig 11: Impulse response (a) and frequency response characteristics (b) and (c) for the linear interpolator with delay.





The impulse response and frequency response characteristics of this interpolation filter are illustrated in Fig. 11.

Oversampling D/A converter



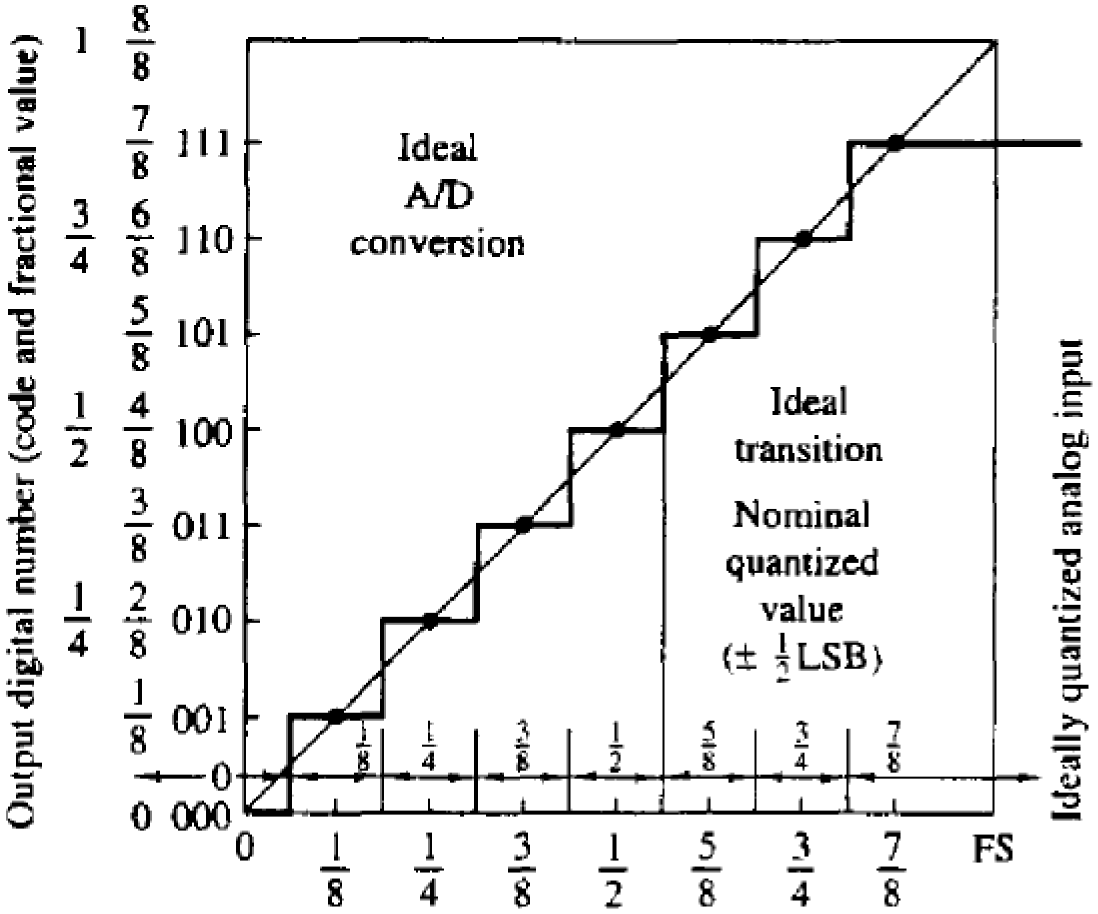
Quantization Errors

- To determine the effects o f quantization on the performance o f an A /D converter, we adopt a statistical approach. The dependence of the quantization error on the characteristics of the input signal and the nonlinear nature of the quantizer make a deterministic analysis intractable.

- In the statistical approach, we assume that the quantization error is random in nature. We model this error as noise that is added to the original (unquantized) signal.

- If the input analog signal is within the range o f the quantizer, the quantization error $e_q(n)$ is bounded in magnitude and the resulting error is called *granular noise. When the input falls outside the range of* $e_q(n)$ the quantizer (clipping), becomes unbounded and results in *overload noise.*
- This type of noise can result in severe signed distortion when it occurs. Our only remedy is to scale the input signal so that its dynamic range falls within the range of the quantizer. The following analysis is based on the assumption that there is no overload noise.

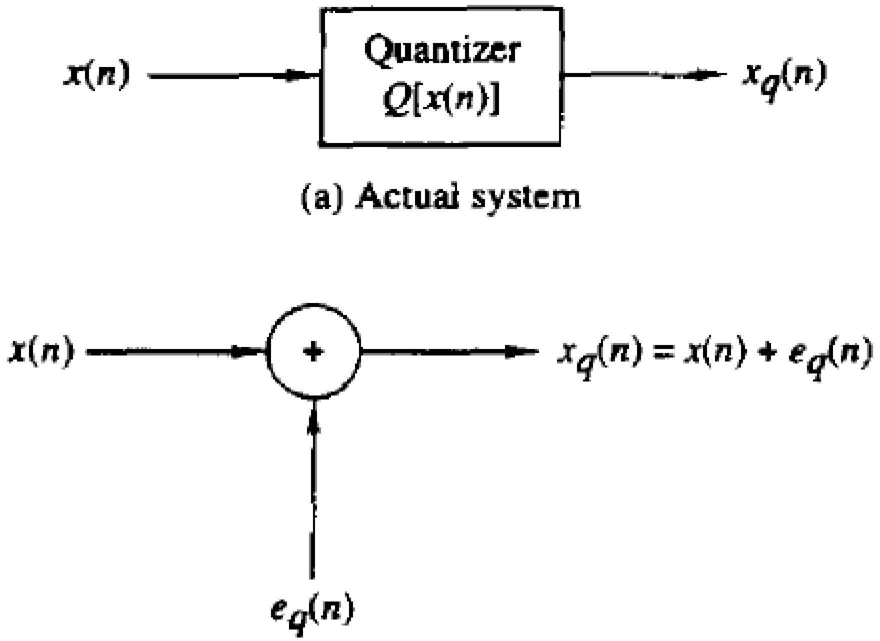
Normalized Analog input



properties of $e_q(n)$:

1. The error $e_q(n)$ is uniformly distributed over the range $-\Delta/2 < e_q(n) < \Delta/2$.
2. The error sequence $\{e_q(n)\}$ is a stationary white noise sequence. In other words, the error $e_a(n)$ and the error $e_a(m)$ for $m \neq n$ are uncorrelated.
3. The error sequence $\{e_q(n)\}$ is uncorrelated with the signal sequence $x(n)$.
4. The signal sequence $x(n)$ is zero mean and stationary.

Mathematical Model of Quantization noise



(a) Actual system

(b) Mathematical model

These assumptions do not hold, in general. However, they do hold when the quantization step size is small and the signal sequence $x(n)$ traverses several quantization levels between two successive samples.

Under these assumptions, the effect of the additive noise $e_q(n)$ on the desired signal can be quantified by evaluating the signal-to-quantization noise (power) ratio (SQNR), which can be expressed on a logarithmic scale (in decibels or dB) as

$$\text{SQNR} = 10 \log_{10} \frac{P_x}{P_n}$$

where $P_x = \sigma_x^2 = E[x^2(n)]$ is the signal power and $P_n = \sigma_e^2 = E[e_q^2(n)]$ is the power of the quantization noise.

If the quantization error is uniformly distributed in the range $(-\Delta/2, \Delta/2)$ as shown in Fig. 9.11, the mean value of the error is zero and the variance (the quantization noise power) is

$$P_n = \sigma_e^2 = \int_{-\Delta/2}^{\Delta/2} e^2 p(e)\,de = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} e^2\,de = \frac{\Delta^2}{12}$$

Probability density function for the quantization error.