18ECC104T - Signals and Systems

Assignment - 3

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Q1. $x(n) = (0.6)^n u(n) + (2.1)^n u(-n-1)$

Soln. $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (0.6)^n u(n) e^{-j\omega n} + \sum_{n=-\infty}^{\infty} (2.1)^n u(-n-1) e^{-j\omega n}$

$= \sum_{n=0}^{\infty} [(0.6) e^{-j\omega}]^n + \sum_{n=0}^{\infty} [(2.1)^{-1} e^{j\omega}]^n - 1$

$= \dfrac{1}{1-0.6e^{-j\omega}} + \dfrac{1}{1-\frac{e^{j\omega}}{2.1}} - 1.$

$\therefore X(e^{j\omega}) = \dfrac{1}{1-(0.6)e^{-j\omega}} + \dfrac{\frac{e^{j\omega}}{2.1}}{1-\frac{e^{j\omega}}{2.1}}$

Q2. $x(n) = 3^n u(n)$

Soln $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} 3^n u(n) e^{-j\omega n}$

$= \sum_{n=0}^{\infty} 3^n e^{-j\omega n}$

$= \sum_{n=0}^{\infty} (3e^{-j\omega})^n$

$X(e^{j\omega}) = \dfrac{1}{1-3e^{-j\omega}}$

The solution of the Fourier transform is incorrect as the given sequence is not absolutely summable therefore FT does not exist.

Q3. FT $x(n) = \{1, -1, 2, 2\}$

Soln. $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

$= 1 - e^{-j\omega} + 2e^{-2j\omega} + 2e^{-3j\omega}$

Q4. $x(n) = \sin(an) \cdot u(n)$.

Soln.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sin(an)\, u(n)\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left[\frac{e^{jan} - e^{-jan}}{2j}\right] e^{-j\omega n}$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} e^{n(ja - j\omega)} - \frac{1}{2j} \sum_{n=0}^{\infty} e^{n(-ja - j\omega)}$$

$$= \frac{1}{2j}\left[\frac{1}{1 - e^{jan} e^{-j\omega}} - \frac{1}{1 - e^{-jan} e^{-j\omega}}\right]$$

$$= \frac{1}{2j}\left[\frac{e^{-j\omega}[e^{jan} - e^{-jan}]}{1 - e^{-j\omega}(e^{jan} + e^{-jan} + e^{-2j\omega})}\right]$$

$$X(e^{j\omega}) = \left[\frac{e^{-j\omega} \sin(an)}{1 - 2\cos(an) + e^{-2j\omega}}\right].$$

Q. Find IDFT

$$X(e^{j\omega}) = \begin{cases} 1j & \frac{\pi}{4} < |\omega| < \frac{3\pi}{4} \\ 0; & \frac{3\pi}{4} < |\omega| \le \pi \\ \cancel{\ }; & 0 \le |\omega| \le \frac{\pi}{4} \end{cases}$$

Soln.

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \left\{ \int_{-3\pi/4}^{-\pi/4} e^{j\omega n}\, d\omega + \int_{\pi/4}^{3\pi/4} e^{j\omega n}\, d\omega \right\}$$

$$= \frac{1}{2\pi j n}\left[e^{-j\pi/4 n} - e^{-3j\pi/4 n} + e^{\frac{3jn}{4\pi}} - e^{jn\pi/4}\right]$$

$$= \frac{1}{\pi n}\left[\sin\left(\frac{3\pi}{4}n\right) - \sin\left(\frac{\pi}{4}n\right)\right]$$

Q.7. IDTFT of $X(e^{j\omega}) = e^{-j\omega}\left[0.5 + 0.5\cos\omega\right]$

Soln.

$$x(n) = e^{-j\omega}\left[\frac{1}{2} + \frac{1}{2}\left[\frac{e^{j\omega} + e^{-j\omega}}{2}\right]\right]$$

$$= e^{-j\omega}\left[\frac{1}{2} + \frac{e^{j\omega} + e^{-j\omega}}{4}\right]$$

$$= 0.5e^{-j\omega} + 0.25 + 0.25e^{-2j\omega}$$

$x(0) = x(2) = 0.25$; $x(1) = 0.5$

$x(n) = 0$, otherwise.

Q. $x_1(n) = \left(\frac{1}{2}\right)^n u(n)$

$x_2(n) = \left(\frac{1}{3}\right)^n u(n)$.

Soln.

$$F\left[x_1(n) * x_2(n)\right] = \left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right]\left[\frac{1}{1-\frac{1}{3}e^{-j\omega}}\right]$$

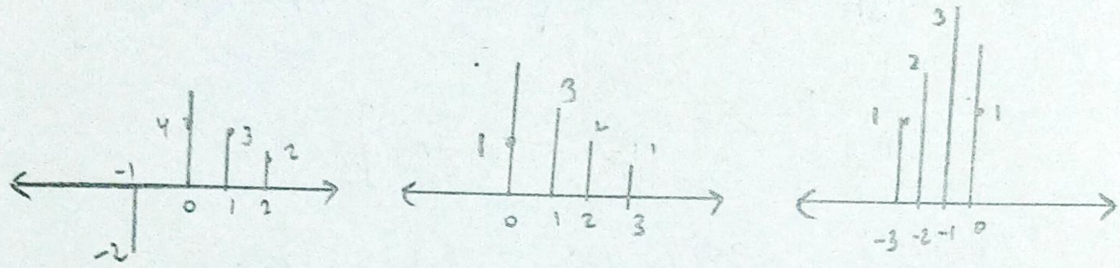
$$= \frac{1}{1 - \frac{1}{3}e^{-j\omega} - \frac{1}{2}e^{-j\omega} + \frac{1}{6}e^{-2j\omega}}$$

Q. Convolution sum of 2 sequences using graphical, matrix and tabular methods.

$x(n) = \{-2, \underset{\uparrow}{1}, 3, 2\}$

$h(n) = \{1, 3, 2, 1\}$

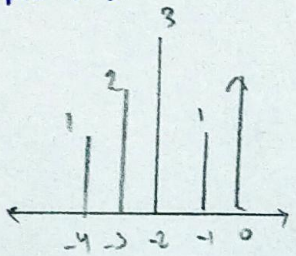
Graphical method



$x(n)$ $h(n)$ $h(-n)$

When $n = -1$

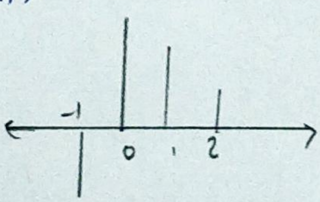
$h(-1-k)$



$y(-1) = -2 \cdot 1 = -2.$

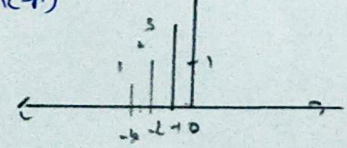
When $n = 0.$

$x(n).$



$y(0) = 3 \times -2$
$= -6.$

$h(-n)$

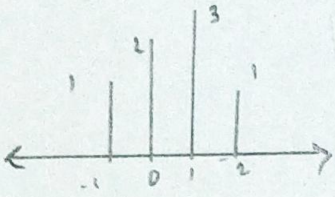


When $n = 1$

$y(1) = 2(-2) + 4(3) + 3(1).$
$= -4 + 12 + 3$
$= 11$

When $n = 2$.

$h(2-k)$

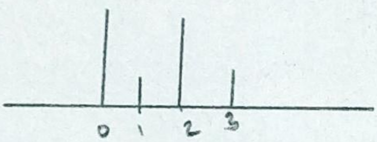


$y(2) = -2(1) + 4(2) + 3(3) + 2(1)$

$= 8 + 1 = 9$

When $n = 3$

$h(3-k)$

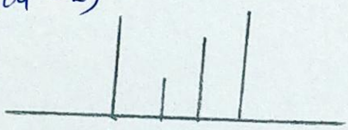


$y(3) = 4(1) + 3(2) + 3(2)$

$= 16.$

When $n = 4$

$h(4-k)$

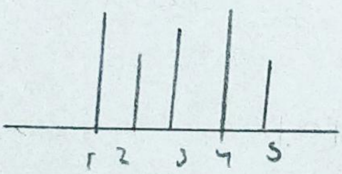


$y(4) = 3(1) + 2(2)$

$= 7.$

When $n = 5$

$h(5-k)$



$y(5) = 2(1)$

$= 2.$

$y(n) = \{-2, \underset{\uparrow}{-2}, 11, 17, 16, 7, 2\}$

Tabular Method

$\overrightarrow{x(n)}$ / $\downarrow h(n)$	-2	4	3	2
1	-2	4	3	2
3	-6	12	9	6
2	-4	8	6	4
1	-2	4	3	2

$y(n) = \{-2, -2, 4, 17, 16, 7, 2\}$

Matrix Method

$x(n) \qquad\qquad h(n)$

$$\begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} -2 & 2 & 3 & 4 \\ 4 & -2 & 2 & 3 \\ 3 & 4 & -2 & 2 \\ 2 & 3 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6+6+4 \\ 4-6+4+3 \\ 3+12-4+2 \\ 2+9+8-2 \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \\ 3 \\ 17 \end{bmatrix} = y(n).$$

Q. Z-transform and ROC $\quad x(n) = a^n u(n)$.

Soln.

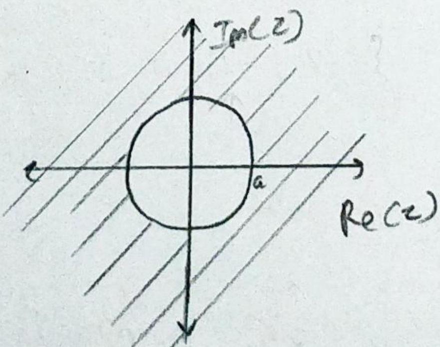
$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}.$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n = \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a}.$$

$z - a = 0$

$z = a > 0 \qquad |z| > |a|$



Q. Z - Transform;

(i) $\{1, 2, 3, 2\}$

Sol. $x(n) = \{1, 2, 3, 2\}$

$x(n) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$.

$1z^{0} + 2z^{-1} + 3z^{-2} + 2z^{-3}$.

ROC = Entire z plane except $z = 0$.

(ii) $x(n) = \{3, 2, 1, 0, 3, 1, -2\}$

Sol. $x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$.

$= 3z^{0} + 2z^{-1} + 1z^{-2} + 3z^{-4} + z^{-5} - 2z^{-6}$.

ROC = Entire z-plane exept at $z = 0$.

(iii) $x(n) = \{3, 2, 1, 0, 3, 1, -2\}$

$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$.

$x(z) = 3z^{6} + 2z^{5} + z^{4} + 3z^{2} + z - 2$

ROC = Entire z-plane except at $z = 0$.

Q. $X(z) = \log(1 - az^{-1})$; $|z| > |a|$.

Sol. $\frac{d}{dz} x(z) = \frac{1}{1 - az^{-1}} \cdot (az^{-2})$

$= \frac{az^{-2}}{1 - az^{-1}}$

$-z \frac{d}{dz} X(z) = \frac{-az^{-1}}{1 - az^{-1}} = -az^{-1}\left[\frac{1}{1 - az^{-1}}\right]$

$$= az^{-1}\left[z(a^n u(n))\right]$$

$$= az\left[a^{n-1} u(n-1)\right].$$

$$\left[x(n)\right] = -z \frac{d}{dz}\left[X(z)\right].$$

$$x\left[x(n)\right] = -a\left\{a^{n-1} u(n-1)\right\}$$

$$x(n) = \frac{-a^n u(n-1)}{n}$$

Q. $x(z) = \dfrac{1+z^{-1}}{1-0.25z^{-2}}$ $z \to \infty$.

Sol. $\lim\limits_{z\to\infty} \dfrac{1+z^{-1}}{1-0.25^{-1}} = \dfrac{1+0}{1-0} = 1.$

Q. $x(z) = 1/2 \quad (n^2+n)\left(\frac{1}{3}\right)^{n-1} u(n-1).$

Sol. $\frac{1}{2} n^2 \left(\frac{1}{3}\right)^{n-1} u(n-1) + \frac{1}{2} n \left(\frac{1}{3}\right)^{n-1} u(n-1).$

$$z\left[\left(\frac{1}{3}\right)^n u(n)\right] = \frac{z}{z-\frac{1}{3}}$$

$$z\left[\left(\frac{1}{3}\right)^{n-1} u(n-1)\right] = \frac{1}{z-\frac{1}{3}}.$$

Using multiplication property.

$$z\left[n\left[\left(\frac{1}{3}\right)^{n-1} u(n-1)\right]\right] = -z\frac{d}{dz}\left[\frac{1}{z-\frac{1}{3}}\right]$$

$$= \frac{z}{\left(z-\frac{1}{3}\right)^3}$$

ROC $|z| > \frac{1}{3}$.

$$x(z) = \frac{1}{2}\left[\frac{z\left(z+\frac{1}{3}\right)}{\left(z-\frac{1}{3}\right)^2} + \frac{z}{\left(z-1/3\right)^2}\right] = \frac{z^2}{\left(z-1/3\right)}$$