



DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB101J-Electromagnetic Theory, Quantum Mechanics, Waves and Optics

Module 3- Lecture-8

Time-dependent Schrodinger's wave equation



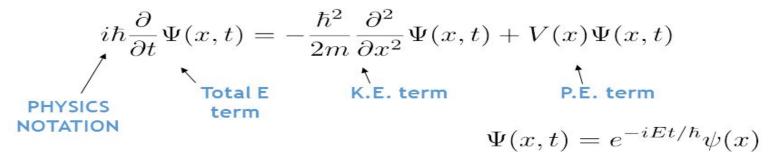


Derivation of Time-Dependent Schroedinger Equation





Time-Dependent Schrodinger Wave Equation



Time-Independent Schrodinger Wave Equation

$$E\psi(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x)\psi(x)$$





Let us consider a particle of mass 'm', moving with a velocity 'v'. The de Broglie wavelength associated with it is given by,

$$\lambda = \frac{h}{mv} \dots (1)$$

where h = Planck's constant = $6.626 \times 10^{-34} \, \mathrm{J s}$.

Let ψ be the wave function of the particle along x, y and z coordinate at any time 't'. The classical differential equation of a progressive wave moving with a wave velocity 'v' can be written as,

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial v^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} \qquad (2)$$

The solution for the equation (2) is given by,

$$\psi = \psi_{\circ} e^{-i\omega t} \qquad (3)$$

where

 ψ_{ω} Amplitude of the wave at the point (x,y,z) $\omega = Angular frequency of the wave$





Differentiating eqn. (3) with respect to 't',

$$\frac{\partial \psi}{\partial t} = (-i\omega)\psi_{\Box}e^{-i\omega t} = (-i\omega)\psi_{\Box}(4)$$

But,
$$\omega = 2\pi f = 2\pi \left(\frac{E}{h}\right)$$
 \Box $f = \frac{E}{h}$ (5)
where E = energy of a photon

Substituting eqn (5) in eqn (4),

$$\frac{\partial \Psi}{\partial t} = (-i) \left[2\pi \left(\frac{E}{h} \right) \right] \Psi = -i \left(\frac{E}{\hbar} \right) \Psi$$

$$(OR), i \left(\frac{\partial \psi}{\partial t} \right) = E\psi \qquad (6)$$
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Multiply *i* both sides





Substituting the equation(6), in the time

independent wave equation
$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

$$\nabla^2 \Psi + \frac{2m}{\hbar^2} \left[i \hbar \left(\frac{\partial \Psi}{\partial t} \right) - V \Psi \right] = 0$$

(OR)

$$\nabla^2 \Psi = -\frac{2m}{\hbar^2} \left[i \hbar \frac{\partial}{\partial t} - V \right] \Psi$$

Rearrange and Common term





$$-\frac{\hbar^2}{2m} \bullet \nabla^2 \psi = \left[i\hbar \frac{\partial}{\partial t} - V \right] \psi$$
Rearrange and Multiply '–' both sides

Keeping Time derivative one side we have

$$-\frac{\hbar^2}{2m} \bullet \nabla^2 \psi + V \psi = i\hbar \frac{\partial \psi}{\partial t}$$
 rearrange

This equation is known as Schrödinger time dependent wave equation.





Further simplification and re-arrangement lead to hamilton form Time-Dependent Schroedinger Equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2\psi + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

where H =
$$\frac{-\Box^2}{2m}\nabla^2 + V$$
 =Hamiltonian operator





Time-Dependent Schrodinger Wave Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t)=-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)+V(x)\Psi(x,t)$$
 Total E K.E. term P.E. term
$$\Psi(x,t)=e^{-iEt/\hbar}\psi(x)$$