Steady state Conditions and zero boundary Conditions.

① A rod 30cm long has its ends A and B kept at do'c and 80°c respectively, until steady state Conditions

Dievail. The temperature at each end is then suddenly brevail. The temperature at each end is then suddenly reduced to o'c, and kept so. Find the resulting temperature reduced to o'c, and kept so. Find the resulting temperature.

Som: The P.D.E of one dimensional heat flow is $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x^2} \longrightarrow \widehat{\partial}$

In Steady state and any the temperature at any particular point does not vary with time. It, u depends only on a and not on time t.

 $d^{2} = 0$ [$\frac{\partial u}{\partial t} = 0$ mince $u \ddot{u} = 0$ function $\frac{\partial u}{\partial t} = 0$ mince $u \ddot{u} = 0$

Since u is a function of x only, the above equation can be written as $\frac{d^2u}{dx^2} = 0$. $(d \neq 0)$

Hence when steady steady state conditions prevails the heat flow equation be comes $\frac{d^2u}{dx^2} = 0 \quad \longrightarrow \quad \textcircled{2}$

int. egn @ w. rto 'n' twice, we get

$$\boxed{u(x) = ax+b} \longrightarrow \textcircled{1}$$

The boundary conditions in steady state are Conditions (n), (n) and (n) and (n) (n) (n)

- onet solution 3 T & g the form (ii) u(30) = 80

Applying b.c (i) in eqn (1), we get

u(o)= a(o)+b= 20 $a(0,t) = b = an / n e^{-a/2}$

Sub. b= do in eqn & we get u(x) = ax + 20 \longrightarrow

Applying b.c (ii) in ean and, we get

u(30) = a(30) + 20 = 80 300 = 60 19/06 = 0 MARIO 8 = (JIV) U a=2.

sub a=2 in en en we get " (11) u(80, 4) = 18 130 1 0-42

: (u(n) = an + do

and B are cheduced to the temperature at A and Zero, the temperature distribution changes and the no more steady state. This transient the boundary conditions are - Accord state

- u(0,t)=0 + +1/0 (i)
- u(30,t)=0 + +7,0.

The initial temperature of this state is the temperature the previous steady-sate. Hence the initial auth) = B wo diff Condition is

(iii) u(1,0) = antao, for orne30.

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Now, we have to find u(1,t) satisfying the
 Conditions (i), (ii) and (iii) and the P.D.E. (a). The
 Correct solution of O & of the form
      u(n_1t) = (A\cos\lambda n + B\sin\lambda n) e^{-d^2\lambda^2 t} \rightarrow 0
Applying b.c (i) in em (), we get
      u(0,t) = A(1) e^{-d^2 h^2 t} = 0
      either A=0 or e^{-d^2h^2t}=0.
             but e^{-d^2h^2t} \neq 0 (: It is defined +t)
                 A=0 on (B) mo m (11) and implified the
   sub. A=0 in eqn (), we get
       u(nit) = B ainAn e - d2/2t -> 2
   Applying b.c (ii) in em @, we get
       u(30, t) = B Mn30 / e-d2/2t =0.
  B + 0 ( ii 76 B=0, we get trivial solution).
       e^{-d^2/2t} \neq 0 ( i. it is defined +t) arrangement and many
           8in30 \lambda = 0
Sin30 \lambda = 0
n\pi

where n is an integer
                30/ = nT
                                   a 4 + 1 0= (1'0) (1)
                   1 = nitt
                                  act 1. ac(10871)
   temperature.
          1= nT in em 3, we get
       lodini 30 at someti sta = \frac{d^2n^2\pi^2}{900} and \frac{d^2n^2\pi^2}{30}
       u(\eta t) = Bn sin that e^{-\frac{d^2n^2\eta^2t}{900}} where B = Bn and a
                                       Bn is any constant.
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The most general solution can be written as
$$u(x_1t) = \frac{9}{39} \quad \text{Bn Ain} \frac{nnx}{30} \quad e^{-\frac{2nx}{12}t^2}$$

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Applying b. (iii) in eqn (3), we get
$$u(x_10) = \frac{9}{30} \quad \text{Bn Ain} \frac{nnx}{30} \quad (1) = \frac{4nx+40}{30}.$$
To find Bn, expand ax+40 in half range nine.

An +30 = \frac{9}{30} \left \text{fin minnx} \quad \text{where}

$$b_1 = \frac{3}{30} \int_0^{30} (4x) \frac{nnnnx}{30} \, dx$$

$$= \frac{3}{15} \left(\frac{3nx+40}{30} \right) \frac{nnnnx}{30} \, dx$$

$$= \frac{1}{15} \left(\frac{3nx+40}{30} \right) \frac{(-\frac{3nnx}{30})}{(-\frac{3nnx}{30})} - \frac{(3)}{(3)} \frac{(-\frac{3nnx}{30})}{(3)} = \frac{3}{15} \left(\frac{30}{30} \right) \frac{(-1)}{(-1)^{n+1}} + \frac{600}{nn} \right) (-1)$$

$$= \frac{1}{15} \left(\frac{3400}{30} \left(\frac{(-1)^{n+1}}{nn} \right) + \frac{600}{nn} \right) (-1)$$

$$= \frac{40}{15} \left(\frac{1+4}{(-1)^{n+1}} + \frac{1}{15} \right)$$

$$= \frac{40}{nn} \left(\frac{1+4}{(-1)^{n+1}} + \frac{1}{15} \right)$$

Sab. in em sub. the value of Bn in em 3, u(1)= Bn com non e (1) we get The temperature distribution is where temperature dustribution is $u(n_1t) = \frac{3}{n-1} \frac{40}{n\pi} \left(1+4(+1)^{n+1}\right) \sin \frac{n\pi n}{n\pi} = \frac{-\alpha^2 n^2 \pi^2 t}{900}$ degrees.

A rod of length 'l' has its ends A and B kept 2) at o'c and 100'c until steady state condition prevail. If the temperature at B is reduced suddenly to o'c and kept no while that & A is maintained, find the temperature u(x,t) at a dustance in from A and at time to Fam (A & S), we get

The partial differential quatern of one dimensional heat flow is in small (064x8)

In Steady state Conditions, the temperature at any particular point does not vary with time. 10, u depends only on a and not on time to

Since u is a function of nonly, the above equation com be written as $\frac{d^{12}}{dx^{2}} = 0$ (d ± 0)

Hence when steady state Conditions prevails the heat flow equation becomes +1)

integrating eqn @ w. r to n' twice, we get

$$u(x) = ax + b \longrightarrow \textcircled{n}$$

The boundary anditions are

Applying b.c (i) in acm (ii) were get (ii) analytime and

Sub b=0 in $Q_n \in \mathcal{A}$, $u(x) = ax + 0 \rightarrow \mathcal{A}$

Applying b.c (ii) in ear PA, we get ((d,t)) of an Aco) 6 - 42 / ti

$$u(l) = a(l) = \frac{100}{100}$$
 $a = \frac{100}{100}$
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(Jy havijeh
$$\alpha = \frac{100}{41}$$
.

Sub $a = \frac{100}{2}$ in em (a), we get

When the temperature at B is reduced to zero, the temperature distribution changes and the state is no more steady state. For this transient state, the boundary conditions are state, the boundary anditions are

Sub A=D in opn (1), we got

cii)
$$u(l,t) = 0$$
 $4 + 7/0$

The initial temperature of this state is the temperature in the previous steady-state. Hence the initial about the dist Condition is Ciii) $u(x_{10}) = 100x$ for $0 \le x \ge 1000$ moderned off Now, we have to find u(n+) satisfying the anditions (i), (ii) and (iii) and the P.D.E a). The Suitable solution & 1) is a the form u(x1t) = (AcosAn+BainAn) e-d2/2t -Applying b.c (i) in ean O, we get $u(o_1t) = A(1) e^{-d^2h^2t} = 0$. (11) either A=0 or $e^{-d^2 h^2 t} = 0$. = (9)e-Nalat +o (: it is defined +t) $(\mathcal{B}) = \frac{1}{2} = \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$: (A=0) sub A=0 in egn (1), we get $a(n_1t) = B minhne - d^2h^2t \longrightarrow ②$ Applying b.c cii) in em ②, we get u(let) = Brinkle = d2/2t modudrights entragment here, B=0 (B=0, we get trivial solution) e-d2/12t +0 (; it is defined for all t) integer in integer

$$\lambda l = n\pi$$

$$\lambda = n\pi$$

Sub
$$\lambda = \frac{\pi \Pi}{1}$$
 in eqn \otimes , we get $u(x_1t) = B \sin \frac{\pi \Pi X}{1}$ e $\frac{-d^2 h^2 \Pi^2 t}{d^2}$ where $B = B n$, $B n$ is any constant. The most general solution can be written as $u(x_1t) = \sum_{h=1}^{\infty} B_h \sin \frac{\pi \Pi X}{L}$ e $\frac{-d^2 h^2 \Pi^2 t}{d^2}$ where $B = B n$, $B n$ is $a u(x_1t) = \sum_{h=1}^{\infty} B_h \sin \frac{\pi \Pi X}{L}$ e $\frac{-d^2 h^2 \Pi^2 t}{d^2}$ where $B = B n$, $B n$ is $a u(x_1t) = \sum_{h=1}^{\infty} B_h \sin \frac{\pi \Pi X}{L}$ e $\frac{-d^2 h^2 \Pi^2 t}{d^2}$ where $\frac{d^2 h^2 \Pi^2 t}{d^2}$ is $a u(x_1t) = \sum_{h=1}^{\infty} B_h \sin \frac{\pi \Pi X}{L}$ (1) = $\frac{100 x}{L}$.

To find $B n n$ expand $\frac{1}{1} \cos x$ in $\frac{1}{1} \cos x$ where $\frac{1}{1} \cos x$ in $\frac{1}{1} \cos x$ is $\frac{1}{1} \cos x$ in $\frac{1} \cos x$ in $\frac{1}{1} \cos x$ in $\frac{1}{1} \cos x$ in $\frac{1}{1} \cos x$ in $\frac{$

$$= \frac{aoo}{l^{2}} \left[2\left(\frac{l}{n\pi}\right) \left(-conn\pi\right) \right]$$

$$= \frac{aoo}{l^{2}} \left[\frac{l}{n\pi} \left(-l\right)^{n} \right] = \frac{aoo}{n\pi} \left(-l\right)^{n+1}$$

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$$= \frac{aoo}{l^{2}} \left[\frac{l}{n\pi} \left(-l\right)^{n} \right] = \frac{aoo}{l^{2}} \left(-l\right)^{n+1}$$

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$$= \frac{aoo}{l^{2}} \left[\frac{l}{n\pi} \left(-l\right)^{n+1} \left(-l\right)^{n} \right] = \frac{aoo}{l^{2}} \left(-l\right)^{n+1}$$

$$= \frac{aoo}{l^{2}} \left(-l\right)^{n+1} \left(-l$$

Exercise problems.

A rod of length I has its ends A and B

kept at o'c and lao'c respectively, until

steady state conditions prevail. If the temperature

at B is reduced to o'c and kept no

while that of A is maintained. find the

temperature distribution in the rod.

temperature distribution in the rod.

Ans: $u(n,t) = \frac{240}{n} \stackrel{2}{\sim} \frac{C+1)^{n+1}}{n}$ sin min e