SOLUTIONS FOR PRACTICE EXERCISE:

1. The prime factorization of 1936 is,

   a) $2^2 \times 3 \times 11^3$ b) $2^3 \times 11^3$ c) $2^4 \times 11^2$ d) None of these

   $1936=11x11x2x2x2x2=11^2x2^4$ (Option c)

2. The prime factorization of 1240 is,

   a) 2^3x5x31 b) 2^2x5x31 c) 3^2x5x31 d) None of these

   $1240=124 \times 10 =4 \times 31 \times 5 \times 2=2^3x5x31$ (Option a)

3. Find the number of divisors or factors of 1800?

   a) 24 b) 32 c) 36 d) 40

   $1800=18 \times 100 =9 \times 2 \times 25 \times 4 =2^3 \times 3^2 \times 5^2$

   No of factors = (3+1) x (2+1) x(2+1)=4 x 3 x3=36(Option c)

4. Find the number of odd divisors or factors of 1800?

   a)8 b) 10 c) 9 d) 6

   $1800=18 \times 100 =9 \times 2 \times 25 \times 4 =2^3 \times 3^2 \times 5^2$

   For odd divisors we find divisors of $3^2 \times 5^2$

   No of odd factors = (2+1) x(2+1)= 3x3=9 (Option c)

5. Find the number of even divisors or factors of 1800?

   a) 24 b)32 c)30 d)27

   $1800=18 \times 100 =9 \times 2 \times 25 \times 4 =2^3 \times 3^2 \times 5^2$

   No of factors = (3+1) x (2+1) x (2+1)=4 x 3 x3=36

   No of odd factors = (2+1) x(2+1)= 3x3=9

   No of even factors = total factors – odd factors=36-9=27 (Option d)

6. Find the sum of all the factors of 600?

   a) 2400 b) 1280 c) 1360 d) 1860

$600 = 3 \times 2 \times 25 \times 4 = 2^3 \times 3 \times 5^2$

Sum of all factors of 600 = $\{(2^{3+1} - 1)/(2-1)\} \times \{(3^{1+1} - 1)/(3-1)\} \times \{(5^{2+1} - 1)/(5-1)\} = 15 \times 4 \times 31 = 1860$ (Option d)

7. Find the sum of all odd factors of 600?

a)120 b)124 c)360 d)240

$600 = 3 \times 2 \times 25 \times 4 = 2^3 \times 3 \times 5^2$

To find sum of odd factors drop the even factors.

Sum of odd factors of 600 $= \{(3^{1+1} - 1)/(3-1)\} \times \{(5^{2+1} - 1)/(5-1)\} = 4 \times 31 = 124$ (Option b)

8. Find the sum of all even factors of 600?

a)1240 b)1736 c)3452 d)1346

$600 = 3 \times 2 \times 25 \times 4 = 2^3 \times 3 \times 5^2$

(Sum of factors of a number is basically summation of geometric progressions. This approach is used here to solve the question. Student can solve this problem using sum of factors formula too)

Sum of all factors of 600 = $(2^0 + 2^1 + 2^2 + 2^{3)}) \times (3^0 + 3^{1)}) \times (5^0 + 5^1 + 5^2)$ $= 15 \times 4 \times 31 = 1860$

Sum of odd factors of 600 $= (1+3) \times (1+5+25) = 4 \times 31 = 124$

Sum of even factors of 600 = total sum of factors – odd factors sum $= 1860 - 124 = 1736$ (Option b)

9. Find the number of factors of 1800 that are divisible by 5?

a) 24 b) 23 c)32 d)20

$1800 = 18 \times 100 = 9 \times 2 \times 25 \times 4 = 2^3 \times 3^2 \times 5^2$

Factors which are all divisible by 5 = $5\,(2^3 \times 3^2 \times 5)$

Factors which are all divided by 5 = (3+1) x (2+1) x (1+1) =4x3x2=24 (Option a)

10. Find the number of factors of 1200 which are divisible by 15?
    a) 20   b) 12   c) 10   d) none of these

$1200=4x3x25x4=2^4 \times 3 \times 5^2$
Factors which are all divisible by 15 = $3 \times 5 \times (2^4 \times 5^1)$

Factors which are all divisible by 15 = (4+1) x (1+1) =5 x2 =10 (Option c)

11. Find the number of factors of 1800 that are divisible by 5 but not by 25?
    a) 24   b) 30   c)12   d)15

$1800=18 \times 100 =9 \times 2 \times 25 \times 4 =2^3 \times 3^2 \times 5^2$

Factors which are all divisible by 5 = $5 (2^3 \times 3^2 \times 5)$

Factors which are all divisible by 5 = (3+1) x (2+1) x (1+1) =4x3x2=24
Factors which are all divisible by 25= $25 (2^3 \times 3^2) = (3+1) \times (2+1) = 4 \times 3$

=12

Therefore factors which are divisible by 5 but not by 25 = 24 – 12 = 12 (Option c)

12. Find the number of factors of 1200 which are perfect squares?
    a) 4   b)6   c)10   d)8

$1200= 4 \times 3 \times 25 \times 4 =2^4 \times 3 \times 5^2$
Examine the powers of the prime factors & carry out the following exercise.
2^0 , 2^2 , 2^4 are perfect squares of 2 so total 3
3^0 is a perfect squares of 3 so total 1
5^0 , 5^2 are perfect squares of 5 total 2
Hence number of factors of 1200 which are perfect squares =3 x 1 x2 =6 (Option b)

13. Find the number of factors of 1500 which are perfect squares?

a) 4 b)6 c)10 d)8

$1500 = 5 \times 3 \times 25 \times 4 = 2^2 \times 3 \times 5^3$

2^0 , 2^2 are prefect squares total 2

3^0 is prefect squares total 1

5^0 , 5^2 are prefect squares total 2

number of factors of 1500 which are perfect squares $=2 \times 1 \times 2 = 4$ (option a)

14.Find the number of factors of 5400 which are perfect cube ?

a) 4 b)6 c)10 d)8

$5400 = 54 \times 25 \times 24 = 27 \times 2 \times 25 \times 4 = 2^3 \times 3^3 \times 5^2$

2^0 , 2^3 are perfect cube of 2 so total 2

3^0 , 3^3 are perfect cube of 3 so total 2

5^0 is prefect cube of 5 so total 1

Hence number of factors of 5400 which are perfect cube $=2 \times 2 \times 1 = 4$ (option a)

15. Find the no of divisors of 19404 excluding 1 and the no itself?

a) 54 b) 53 c) 52 d) 50

$19404 = 11 \times 4 \times 21 \times 21 = 11 \times 4 \times 7 \times 3 \times 7 \times 3 = 11 \times 2^2 \times 3^2 \times 7^2$

Number of factors $= (1+1) \times (2+1) \times (2+1) \times (2+1) = 54$

Find the number of divisors of 19404 excluding 1 and the number itself = total factors – 2 =54-2=52 (option c)

16. In how many ways can 2744 be resolved as a product of 2 factors?

a) 16 b) 8 c) 12 d) 4

$2744 = 8 \times 343 = 2^3 \times 7^3$

Number of factors of 2744 is $=(3+1) \times (3+1) = 4 \times 4 = 16$

Number of ways in which 2744 can be resolved as a product of two factors is = ½ (total factors)=1/2 x 16 =8 (Option b)

17. In how many ways can 1296 be expressed as a product of 2 distinct factors and product of 2 factors respectively is -

a) 25, 25 b) 13, 12 c) 12, 13 d) 15, 10

$1296=4x324=4x4x81=2^4\ x3^4$

Total factors = 5 x 5 = 25
But 25 is not divisible by 2.
So as a product of 2 distinct factors we can write in (25 -1)/2 ways=12
And as a product of 2 factors in= (25 + 1)/2 ways. = 13.
Hence answer is (Option c)

Note to student: Whenever the number is a perfect square you will encounter this situation, because any perfect square has odd number of factors.

18. What is the smallest number that should be multiplied with 840 to make it a perfect square and 1200 to make it a perfect cube respectively?

a) 200, 3100 b) 210, 3150 c) 210, 3250 d) None of these

$840=8x105=8x5x21=2^3\ x3\ x5\ x7$

The smallest number that should be multiplied with 840 to make it a perfect square is found by converting all powers on the prime factors to even numbers,thus $=(2^3\ x3\ x5\ x7)\ x\ (2x3x5x7\ ) = 2^4\ x\ 3^2\ x\ 5^2\ x\ 7^2$

So ans =2x3x5x7=210

$1200=2^2\ x\ 3\ x\ 5^2\ x\ 2^2 =2^4\ \ x\ 3\ x5^2$

The smallest number that should be multiplied with 1200 to make it a perfect cube is found by converting all powers on the prime factors into multiples of 3 ,thus $=(2^4\ \ x\ 3^2\ x5^2)\ x\ \ 2^2\ x\ 3x5$

So ans is 2^2 x 3 x 5 = 60.
(Option d)

19. What is the smallest number that should be multiplied with 3600 to make it a perfect square?

a) 1 b) 6 c)10 d) 2

$3600 = 9 x 4 x 25 x 4 = 2^4 x 3^2 x 5^2$

To make it a perfect square = $(2^4 x3^2 x5^2)$ x 1 we have to multiply it with 1 ,because the powers of prime factors are already even. Also note that 3600 is already a perfect square.
(Option a)

20. What is the smallest number that should be multiplied with 3600 to make it a perfect cube?
a) 60 b) 6 c) 10 d) 8

To make prefect cube = $(2^4 x3^2 x5^2)\ 2^2$ x 3 x 5.
We have to multiply it with 60 to make cube (Option a)

21. Express 0.81818181…….= 0.**81(bold faced to denote repetition,read as 0.81 bar)** in form of a fraction?
   a) 9/11 b)6/11 c)10/11 d)8/11

   0.81818181……. = 81/99 = 9/11
   (Two digits repeat after decimal point so put two 9's in denominator. Remove decimal point and bar you are left with the number 81 which is numerator.
   (Simplify in cases where it is possible & then report the answer)
   (Option a)

22. Express 0.27777777……=0.2**7** (read as 0.27 with bar on 7) in form of a fraction?
   a) 5/18 b)6/17 c) 7/18 d)8/18

   0.27777777… = (27 – 2)/(90) = 25/90 = 5/18
   [(Numerator: Remove decimal & bar you end up with 27.From this subtract the non repeating digit which is 2 .)
   [Denominator: One digit repeats after decimal so put one 9 in denominator. One digit doesn't repeat after the decimal point hence put one 0 in the denominator correspondingly] (Option a)

23. Express 0.279797979…….= 0.2**79** in form of a fraction?

a) 277/990 b) 377/990 c) 277/999 d) 377/999

0.279797979……. = (279 – 2)/(990) = 277/990 (Option a)

24. Express 1. 116161616……. in form of a fraction?
a) 223/198 b) 367/330 c) 62/55 d) 221/198

1. 116161616……= 1.1**16** = 1 +[(116 – 1)/(990)] = (990 +115)/(990)
= 223/990 (Option a)

25. Which of the following is a prime number?
a) 429 b) 307 c) 428 d) 851

a) 429, sum of the digits = 15, divisible by 3
b) 307, approximate square root of 307 is 18.
List out all primes below 18, i.e 2, 3, 5, 7, 11, 13 and 17
We observe that 307 is not divisible by any one of these primes.
So it is a prime number (if it is divisible by any one of these primes then it is not a prime)
c) Divisible by 4
d) Divisible by 23. Using the same method as described in option b.
(Option b)

26. Which of the following is not a prime?
a) 113 b) 161 c) 223 d) 181

a) 113 is not divisible by 2,3,5,7 and 11. So it is a prime.
b) 161 is divisible by 7
c) 223 is not divisible by 2,3,5,7,11,13.So it is prime
d) 181 is not divisible by 2,3,5,7,11,13.So it is prime.
(Option b)

27. Find the value of 50+51+52+53+………+99
a) 3627 b) 8510 c) 3725 d) 3075

50+51+52+53+………+99, an AP
Sum of n terms = (n/2) [a + l], a & l are first and last terms
= (50/20 [50 + 99]

$= 3725$

Alternate method:
$50+51+52+53+.........+99 = (1+2+3+........+99) - (1+2+.....+49)$
Then apply sum of first n terms formula (Option c)

28. What is the sum of first 80 natural numbers?

a) 3140 b) 3240 c) 3340 d) 3440

Sum = $[n(n+1)]/2 = (80)(80+1)/2 = 3240$ (Option b)

29. What is the sum of the squares of first 20 even natural numbers?

a) 9480 b) 10480 c) 11480 d) 12480

$2^2+4^2+6^2+8^2+....... + 40^2 = 2^2(1^2+2^2+3^2+........+20^2)$
$= 4 \times [n(n+1)(2n+1)]/6$
$= 4 \times (20 \times 21 \times 41)/6 = 11480$ (Option c)

30. A wants to type first 1000 natural numbers on a desk top. How many times he has to press the keys of the computer key board?

a) 2893 b) 2987 c) 3000 d) 2500

To enter 1 to 9, number of times key to be pressed = 9
To enter10 to 99, number of times keys to be pressed = 90 x 2 = 180
To enter 100 to 999, number of times keys to be pressed = 900 x 3 = 2700
To enter 1000, number of times keys to be pressed = 4
Hence total = 9+180+2700+4 = 2893 (Option a)

31. A printer numbers the pages of a book starting with 1 and uses 3089 digits in all. How many pages does the book have?

a) 1040 b) 1048 c) 1049 d) 1050

For pages 1 to 9, number of digits used by printer = 9

For pages 10 to 99, number of digits used by printer = 90 x 2 = 180

For pages 100 to 999, number of digits used by printer = 900 x 3 = 2700

So far the digits used = 9+180+2700 = 2889
The remaining digits to be used = 3089 – 2889 = 200, with these next 50 pages can be numbered.

So total = 999+ 50 = 1049 pages can be numbered. (Option c)

32. One sheet is torn from a book ,in which both sides of the sheet have page numbers, starting from page number 1. The sum of the numbers on the remaining pages is 195. The sheet that is removed contains which of the following page numbers?
a) 5, 6 b) 7, 8 c) 9, 10 d) 11, 12

Here, basically, our sum of first n natural numbers should be slightly greater than 195.

By trial and error, if n = 10, then [n (n+1)]/2 = (10 x 11)/2 = 55
if n = 15, then [n (n+1)]/2 = (15 x 16)/2 = 120
if n = 20, then [n (n+1)]/2 = (20 x 21)/2 = 210
So, 210 – 195=15. ,i.e, the removed sheet contains pages 7 and 8 (Option b)

33. If 6896x45 is divisible by 9 then x is ,
a) 4 b) 5 c) 6 d) 7
Here sum of the digits = 38+x = 45, so x= 7 (Option d)

34. If 481A769B is divisible by 5, 6 and 9 then A+B is,
a) 0 b) 1 c) 2 d) 3

Given, 481A769B is divisible by 5, 6 and 9, implies B = 0
For 9, sum of the digits = 35+ A = 36, so A = 1 and A+B = 1 (Option b)

35. An 8 digit number 4252746B leaves a remainder 0 when divided by 3. How many values are possible for B?

a) 2 b) 3 c) 4 d) 6

Sum of the digits of 4252746B = 30+B,
30,33,36 & 39 are all multiples of 3,
So, B can take 0, 3, 6 and 9, that is 4 values. (Option c)

36. What is the remainder, when the 100 digit formed by writing consecutive natural numbers side by side starting with 1, is divided by 5?

a) 1 b) 2 c) 4 d) 0

Here we need to know the last digit of this 100 digit number.
The 100 digit number is 1234…..9101112…….545 (that is 9 single digit numbers, then 45 two digit numbers, 10 to 54 and then 5)
So the remainder is 0. (Option d)

37. If the 8 digit number 5668x25y is divisible by 48, find the least value of x+y?

a) 10 b) 9 c) 8 d) 7

Divisibility by 48 means we need to check divisibility with 3 and 16.
Sum of the digits of 5668x25y = 32+(x + y)
Among the options if (x + y) is 7 or 10 then only it is divisible by 3.
Divisibility by 16 means we need to check last 4 digit number and for 8 we have to check the last 3 digit number.
25y is divisible by 8, implies y = 6

If x= 1, then the 4 digit number 1256 is not divisible by 16.
Hence the value of (x + y) is 10 (Option a)

38. The value of 0.057057057057……. is,

a) 57/99 b) 57/999 c) 57/990 d) 57/909

Let x = of 0.057057057057…… .(1)

1000 x = 057.057057………….(2)

(2) – (1) gives 999x = 057, implies x = 57/999

Short cut: As explained in Q.No 21 & 22. (Option b)

39. The value of 0.1254545454………. is (that is 0.12**54)**

a) 1242/(9900) b) 621/(2950) c) 207/(1650) d) 69/(550)

Answer = (1254 – 12)/(9900) = 1242/(9900)
(Numerator: Take the whole number and subtract the non recurring part.) (Denominator: Number of 9's corresponding to the number of repeated digits after decimal point, followed by number of 0's corresponding to the number of non repeated digits after decimal point) (Option a)

40. The recurring decimal representation 1.27272727……….. is,

a) 13/11 b) 14/11 c) 127/99 d) 137/99

Answer= 1+ (27/99) = 126/99 =14/11 (Option b)

41. Find the number of factors 1225

a) 5 b) 6 c) 8 d) 9

1225 = 25 x 49 = 5^2 x 7^2,

Number of factors = (p+1) (q+1) (r+1) ……

= (2+1) (2+1) = 9 (Option d)

42. In how many ways can 3420 be written be written as product of 2 factors?

a) 12 b) 14 c) 18 d) 36

3420 = 10 x 342 = 10 x 9 x 38 = (2x5) (3x3) (2x19)

= 2^2 x 3^2 x 5^1 x 19^1

Answer = (1/2) [(p+1) (q+1) (r+1)….]
= (1/2) [3 x 3 x 2 x 2] = 18 (Option c)

43. Find the number of odd & even number of factors of 1680?
a) 8, 32   b) 8, 9   c) 10, 9   d) none

$1680 = 10 \times 168 = 10 \times 4 \times 42 = (2x5)(2x2)(2x3x7) = 2^4 \times 5^1 \times 3^1 \times 7^1$

a) Number of odd factors = All the factors of $(5^1 \times 3^1 \times 7^1) = 2x2x2 = 8$

b) For even factors

$2^4 \times 5^1 \times 3^1 \times 7^1 = 2\,[2^3 \times 5^1 \times 3^1 \times 7^1]$

Number of even factors = All the factors of $[2^3 \times 5^1 \times 3^1 \times 7^1]$

= 4x2x2x2 = 32

Answer is (Option a)

44. Find the number of factors of 243243 which are multiples of 21?
a) 20   b) 23   c) 25   d) none

$243243 = 243\,(1001) = 3^5 \times 11 \times 13 \times 7 = 21[3^4 \times 11 \times 13]$
=5 x 2 x 2
= 20 (Option a)

45. Find the sum of all the factors of 120?
a) 240   b) 280   c) 360   d) 400

$120 = 40 \times 3 = 8 \times 5 \times 3 = 2^3 \times 5^1 \times 3^1$
Sum of all the factors = $[\,a^{p+1} - 1] / (a-1) \times [b^{q+1} - 1] / (b-1) \times ..$

$= (2^4 - 1)/(2-1) \times (5^2 - 1)/(5-1) \times (3^2 - 1)/(3-1)$
= 360 (Option c)

46. Find the smallest four digit number which when increased by 3 is divisible by 4,5 & 6?

a) 1090 b) 1027 c) 1017 d)1005

Proceed by options,
Increase of 3 gives options as 1093,1030,1020,1008 in that order. Only 1020 & 1008 are divisible by 4 of which only 1020 is divisible by 5. Now check 1020 for divisibility by 6.

Sum of digits is 3 so divisible by 3 & the number ends in 0 , so Even. Hence answer is (Option c)

47. Which smallest natural number should be added to 5312468 to make the result divisible by 11?

a) 6 b) 4 c) 8 d) 2

Proceed by options,
5312468 + 6 = 5312474, (5 + 1 + 4 + 4 = 14)
(3 + 2 + 7 = 12)
Both sums don't match & their difference is not a multiple of 11. So eliminate first option.
5312468 + 4 = 5312472, (5 + 1 + 4 + 2 = 12)
(3 + 2 + 7 = 12)
The sums match hence this is the answer.
The reader is expected to check other two options as explained above. (Option b)

48. Which number amongst the following is divisible by 15 & 24?

a) 4680 b) 3630 c) 2460 d) 5460

We test divisibility by 3, 5 & 8.
Only (Option a) satisfies.

49. Which number among the following is divisible by 144?

a) 23764 b) 428888 c) 195320 d) 66528

We test divisibility by 9 & 16.
If you proceed by options only (Option d) is divisible by 9.
Hence divisibility by 16 becomes an optional check.

50. Which of the following is a prime number?

a)1567893 b) 89394811
c) 96314283 d) None of these

Option a & c are divisible by 3 & Option b by 11.
So answer in none of these

51. A person belonging to a charitable organization had some money with him. The amount available with him could be divided equally among 7 or 9 or 11 people. Find the least amount in rupees he must have had, if it was a four digit number?

a) 1212 b) 1386 c) 1425 (d) 1584

We should choose that number from the options that is divisible by 7,9 & 11.
Options b & d are divisible by both 9 & 11,of which 7 divides only Option b.

52. If the number 23576X is divisible by 36, Find value of the digit denoted as X?

a) 8 b) 6 c) 4 d) 0

We test divisibility by 4 & 9 .Recall that they are co-primes.
When divisible by 4 X can take 0 or 4 or 8.
When divisible by 9 ,check sum of digits = 23 + x.
Divisibility by 9 gets established only when X=4. (Option c)

53. Let N be a natural number. If N^2 is divisible by 8 , then which of The following is true

(a) N is always divisible by 4

(b) N is always divisible by 8
(c) N is always divisible by 16.
(d) N is always divisible by 64.

As N^2 is divisible by 8 it is of the type 8k where k is a positive integer, but 8k should also be a perfect square.

Some values are worked out to explain the method as in the table below.

k	$8k = N^2$	N^2	N
1	8	Not a perfect square	
2	16	Perfect square	4
3	24	Not a perfect square	
4	32	Not a perfect square	
5	40	Not a perfect square	
6	48	Not a perfect square	
7	56	Not a perfect square	
8	64	Perfect square	8
18	144	Perfect square	12

Hence we observe N is divisible by 4.
Alternate method:
$N^2 = 8k = 2^3 \times k$, k should be chosen such that it has a factor 2 in it multiplied by a prime factor with an even power, only then N gets defined.
$N^2 = 2^3 \times 2^1 \times p^x$, where p is prime number & x is even.
Then $N = 2^2 \times \sqrt{(p)^x}$
Which implies N will always be a multiple of 4. (Option a)

54. Let "A" be a three digit number with digits "abc" that are distinct. Let "B" be another number "cba" formed by reversing the digits of A. Then the highest number, that divides , the absolute difference of A & B is,
a) 96 b) 99 c) 11 d) 98

Consider for example the number 45 whose value is = 10 x 4 +5 x 1 , because 4 occupies place value 10 & 5 occupies place value 1.

Like wise,

Value of A = abc = 100a + 10b + c

Value of B = cba = 100c + 10b + a

The absolute difference of A & B = |99 (a-c)| = 99 |a – c|

Such a number is divisible by 9, 11 & 99 .

Out of which 99 is the highest number. (Option b)

55. How many numbers from 300 to 500 (both inclusive) are divisible by 4?

a) 52 b) 49 c) 50 d) 51

300 = 4 x 75 & 500 = 4 x 125

From 75 to 125 we have 125 – 75 + 1 = 51 numbers.

Hence answer is (Option d)