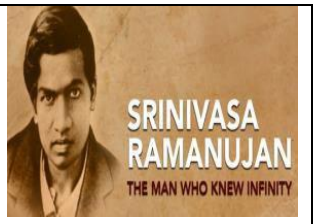
SRM Institute of Science and Technology
Kattankulathur

DEPARTMENT OF MATHEMATICS

18MAB203T Probability and Stochastic Processes

Module – IV
Tutorial Sheet - 11



Sl.No.	Questions	Answer
	Part – B	
1	X(t) is a random telegraph signal process with $E(X(t))=0$ & $R(\tau)=e^{-2\lambda\|\tau\|}$ .Find mean and variance of the time average of X(t) over (-T, T). Is it mean ergodic?	$(i)\,E(X(t))=E(\overline{X_T})=0$ $(ii)\,V(\overline{X_T})=0\ as\ T\to\infty.$ Mean ergodic
2	The random binary transmission process X(t) is a WSS process with mean zero and autocorrelation function is $R(\tau)=1-\frac{\|\tau\|}{T}$ where T is a Constant. (i) Find the mean and variance of the time average of X(t) over (0, T). (ii) Is X(t) mean ergodic?	(i) $E(X(t))=E(\overline{X_T})=0$ $V(\overline{X_T})=2/3\ as\ T\to\infty.$ (ii) X(t) is not Mean ergodic
3	If the auto covariance of a stationary random process X(t) is $C(\tau)=ae^{-\alpha\|\tau\|}$ then prove that X(t) is mean ergodic and also find $V(\overline{X}_T)$,where $\overline{X}_T$ is the time average of X(t) over (-T, T)	$E(X(t))=E(\overline{X_T})=0$ $V(\overline{X_T})=0\ as\ T\to\infty.$
4	Find the mean and variance of a stationary random process whose auto correlation function is given by $R_{XX}(\tau)=9+2e^{-\|\tau\|}$	(i) Mean= 3 (ii) Variance= 2
5	Find the mean square value of the auto correlation $\frac{A^2}{2}\cos\omega\tau$	$\frac{A^2}{2}$
	PART - C	
6	Prove that the random process $X(t)=A\cos(\omega t+\theta)$, where A, ω are constants and θ is a uniformly distributed random variable in $(0,2\pi)$ is correlation ergodic.	To prove (i) X(t) is WSS (ii) Time autocorrelation function is equal to auto correlation function
7	Consider two random process $X(t)=3\cos(wt+\theta)$ and $Y(t)=2\cos\left(wt+\theta-\frac{\pi}{2}\right)$ where θ is a random variable uniformly distributed in $(0,2\pi)$.Prove that $\|R_{XY}(\tau)\|\le\sqrt{R_{XX}(0)R_{YY}(0)}$	(i) $\|R_{XY}(\tau)\|\le 3$ (ii) $\sqrt{R_{XX}(0)R_{YY}(0)}=\sqrt{\frac{9}{2}\times 2}=3$
8	If $\{X(t),t\in T\}$ is a WSS process with $E(X(t))=2$ and	(i) Mean = 2 (ii) Variance = 1

	$R_{XX}(\tau)=4+e^{-\frac{\lvert\tau\rvert}{10}}$. Find the mean of $S=\int_0^1 X(t)dt$ and Variance of X(t)	
9	If $\{X(t),t\in T\}$ is a WSS process with auto correlation function $R_{XX}(\tau)$ and if $Y(t)=X(t+a)-X(t-a)$, Show that $R_{YY}(\tau)=2R_{XX}(\tau)-R_{XX}(\tau+2a)-R_{XX}(\tau-2a)$	
10	If $S=\int_0^{10} X(t)dt$ with $E(X(t))=8$ and $R_{XX}(\tau)=64+10e^{-2\lvert\tau\rvert}$. Find the mean & variance of S .	(i) E(S) = 80 (ii) V(S) = 5 $(19+e^{-20})$