29. a. The joint PDF of a 2-dimensional RV (X,Y) is given by $f(x,y)=\begin{cases} xy^2+\dfrac{x^2}{8}; & 0\le x\le 2;\ 0\le y\le 1 \\ 0 & elsewhere \end{cases}$

Compute (i) $P(X>1)$ (ii) $P(Y<1/2)$ (iii) $P\left(X>1/Y<\frac{1}{2}\right)$ (iv) $P\left(Y<\frac{1}{2}/X>1\right)$ (v) $P(X<Y)$.

(OR)

b. If X and Y are independent RV's with PDF's $e^{-x}; x\ge 0$ and $e^{-y}; y\ge 0$ respectively, find the density function of $U=\dfrac{X}{X+Y}$ and $V=X+Y$. Are U and V independent?

30. a. The process $\{X(t)\}$ whose probability distribution under certain conditions is given by

$$P\{X(t)=n\}=\frac{(at)^{n-1}}{(1+at)^{n+1}};\ n=1,2\ldots$$
$$=\frac{at}{1+at};\ n=0$$

Show that it is not stationary.

(OR)

b.i. Show that the process $\{X(t)\}=A\cos\lambda t+B\sin\lambda t$ (A and B are RV's) is wide sense stationary if (i) $E(A)=E(B)=0$ (ii) $E(A^2)=E(B^2)$ and (iii) $E(AB)=0$.

ii. A salesman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However, if he sells either B or C. then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in the steady state?

31. a. Consider two random processes $X(t)=3\cos(\omega t+\theta)$ and $y(t)=2\cos\left(\omega t+\theta-\dfrac{\pi}{2}\right)$ where θ is a random variable uniformly distributed in (0, 2π). Prove that $\sqrt{R_{XX}(0).R_{YY}(0)}\ge |R_{XY}(\tau)|$.

(OR)

b.i. If $\{X(t)\}$ is a WSS process with autocorrelation function $R_{XX}(\tau)$ and if $Y(t)=X(t+a)-X(t-a)$, show that $R_{YY}(\tau)=2R_{XX}(\tau)-R_{XX}(\tau+2a)-R_{XX}(\tau-2a)$.

ii. Prove that the cross correlation function of two random processes $\{X(t)\}$ and $\{Y(t)\}$ satisfies the property $R_{XY}(\tau)=R_{YX}(-\tau)$.

32.a.i. Show that the spectral density function of a real random process is an even function.

ii. A circuit has an impulse response given by $h(t)=\begin{cases} \dfrac{1}{t}; & 0\le t\le T \\ 0; & elsewhere \end{cases}$, evaluate $S_{YY}(\omega)$ in terms of $S_{XX}(\omega)$.

(OR)

b. If the power spectral density of a WSS process is given by $S(\omega)=\begin{cases} \dfrac{b}{a}(a-|\omega|) & if\ |\omega|\le a \\ 0 & if\ |\omega|>a \end{cases}$

Find the auto correlation function of the process.

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Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
Third to Seventh Semester

15MA209 – PROBABILITY AND RANDOM PROCESS
(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)
(Statistical table to be provided)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. If the PDF of a random variable X is given by $f(x)=k(1+x);\ 2<x<5$; find k.
   (A) 1/4 (B) 2/27
   (C) 3/5 (D) 4/7

2. The MGF of a binomial random variable is
   (A) $\left(q+e^t\right)^n$ (B) $\left(p+qe^t\right)^n$
   (C) $\left(q+pe^t\right)^n$ (D) $\left(1+pe^t\right)^n$

3. The coefficient of $\dfrac{t^r}{r!}$ in the expansion of MGF is
   (A) μ'_r (B) u_r
   (C) μ (D) $Var(X)$

4. The mean of the exponential distribution is
   (A) 1/2 (B) 1/λ
   (C) λ/2 (D) λ

5. $F(\infty,\infty)$ is equal to
   (A) 0 (B) 1
   (C) 1/2 (D) ∞

6. If $x=\dfrac{u}{v}$ and $y=v$ then $J\left(\dfrac{x,y}{u,v}\right)$ is
   (A) $1/u$ (B) $1/v$
   (C) v (D) uv

7. If X and Y have joint PDF $f(x,y)=\begin{cases} x+y; & 0<x<1 \\ & 0<y<1 \\ 0 & elsewhere \end{cases}$ then $f_X(x)$ is equal to
   (A) $\dfrac{1+2x}{2}$ (B) $\dfrac{x+1}{2}$
   (C) $\dfrac{x^2+2}{2}$ (D) $\dfrac{x^3+2x}{3}$

8. In Lindberg Levy's form S_n follows normal distribution with mean and SD equal to
(A) $n\mu, \sqrt{n}\sigma$
(B) $\mu, n\sigma$
(C) $\mu, \frac{\sigma}{\sqrt{n}}$
(D) $n\mu, n\sigma$

9. If both T and S are continuous, the random process is called a
(A) Discrete random sequence
(B) Continuous random sequence
(C) Discrete random process
(D) Continuous random process

10. The mean of the Poisson process is
(A) $\lambda t+1$
(B) λ
(C) λt^2
(D) λt

11. The sum of all the elements of any row of the tpm is
(A) 0
(B) 1
(C) 0.5
(D) 0.75

12. A state i is said to be periodic with period d_i if
(A) $d_i < 1$
(B) $d_i > 1$
(C) $d_i = 1$
(D) $d_i = 0$

13. $R_{XX}(0)$ is equal to
(A) $E[X(t)]$
(B) $Var[X(t)]$
(C) $E[X^2(t)]$
(D) $(E(X(t)))^2$

14. If the process $\{X(t)\}$ and $\{Y(t)\}$ are orthogonal then $R_{XY}(\tau)$
(A) 1
(B) −1
(C) $R_{YX}(\tau)$
(D) 0

15. $R(\tau)$ is maximum at
(A) $\tau = 1$
(B) $\tau = -1$
(C) $\tau = 0$
(D) $\tau = 2$

16. $R_{XX}(-\tau) =$
(A) $R_{XX}(\tau)$
(B) $-R_{XX}(\tau)$
(C) $\tau R_{XX}(\tau)$
(D) $-\tau R_{XX}(\tau)$

17. Linear impulse response for a causal system h(t) is zero when
(A) $t > 0$
(B) $t = 0$
(C) $t < 0$
(D) Always

18. Let $\{X(t)\}$ be a WSS process which is the input to a linear time invariant system with unit impulse h(t) and output y(t) then $S_{YY}(\omega) =$
(A) $H(\omega)S_{XX}(\omega)$
(B) $|H(\omega)|S_{XX}(\omega)$
(C) $|H(\omega)|^2 S_{XX}(\omega)$
(D) $|H(\omega)|^2 R_{XX}(\omega)$

19. Given the power spectral density function $S(\omega)$, the autocorrelation $R(\tau)$ is given by
(A) Fourier transform of $S(\omega)$
(B) Inverse Fourier transform of $S(\omega)$
(C) Fourier series of $S(\omega)$
(D) Inverse Fourier transform of $R(\tau)$

20. Cross power spectral density of $\{X(t)\}$ and $\{Y(t)\}$ denoted by $S_{XY}(\omega)$ is equal to
(A) $\int_{0}^{\infty} R_{XY}(\tau)e^{-i\omega\tau}d\tau$
(B) $\int_{-\infty}^{\infty} R_{XY}(\tau)e^{-i\omega\tau}d\tau$
(C) $\int_{-\infty}^{\infty} R_{XY}(\tau)e^{i\omega\tau}d\tau$
(D) $\int_{0}^{\infty} R_{XY}(\tau)e^{i\omega\tau}d\tau$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. If the random variable X takes the values 1, 2, 3 and 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$. Find the probability distribution and cumulative distributive function of X.

22. If $f(X)=\begin{cases}1/\pi, & -\pi/2<x<\pi/2\\ 0, & elsewhere\end{cases}$. Find the PDF of Y = tanX.

23. If X_1 X_2,....X_n are Poisson variantes with parameter λ=2, use the Central limit theorem to estimate $P(120 \le S_n \le 160)$ where $Sn = X_1 + X_2 + ...X_n$ and n =75.

24. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between two consecutive arrivals is (i) more than 1 min (ii) between 1 minute and 2 minutes.

25. Find the variance of the stationary process $\{X(t)\}$ whose autocorrelation function is given by $R(\tau)=16+\frac{9}{1+6\tau^2}$.

26. The power spectral density function of a zero mean WSS process $\{X(t)\}$ is given by $S(\omega)=\begin{cases}1; & |\omega|<\omega_0\\ 0 & elsewhere\end{cases}$ find $R(\tau)$.

27. The joint probability distribution of (X, Y) is given by

X \ Y	0	1	2
0	0.1	0.04	0.06
1	0.2	0.08	0.12
2	0.2	0.08	0.12

Examine if X and Y are independent RVs?

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. A random variable X has the following probability distribution.

X = x	−2	−1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

(i) Find K (ii) Evaluate P(X<2) (iii) Evaluate P(-2<X<2) (iv) Find the CDF X and (v) mean of X.

(OR)

b.i. The number of monthly break downs of a computer is a RV having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (A) without a breakdown (B) with only one breakdown (C) with atleast a breakdown.

ii. In a normal distribution 7% of the items are under 35 and 89% are under 63. What are the mean and SD of the distribution?