DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB101J-Electromagnetic Theory, Quantum Mechanics, Waves and Optics

Module I
Continuity Equation-Derivation

Continuity Equation-Derivation

Consider a closed surface S enclosing a region of volume V. Let the surface enclose some charge within it. The current *i* flowing out through the surface S is given by

$$i = \oint_s \mathbf{J.ds}$$

Where J is the current density vector over a cross-section ds.

Let ρ be the charge density of the charge enclosed within the surface at any instant. Then the total charge enclosed at any instant is given by

$$q = \int_v \rho dv$$

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Continuity Equation-Derivation

The current *i* is the rate of decrease of charge inside the volume that

$$i = -dq/dt = - d/dt \int_v \rho dV = - \int_v (d\rho/dt)dv$$

According to Gauss divergence theorem $\oint_s \mathbf{J.ds} = \int_v (\nabla .\mathbf{J})dv$

Therefore $\int_v (\nabla .\mathbf{J})dv = - \int_v (d\rho/dt)dv$

$$\nabla .\mathbf{J} = - d\rho/dt$$

$$\nabla .\mathbf{J} + d\rho/dt = 0$$

Continuity Equation-Derivation

$$\nabla . \mathbf{J} + (d\rho/dt) = 0$$

This equation is called the equation of continuity. It simply expresses the fact that the charge is conserved.

In most practical situations of interest, the charge density is constant in time at each point. This means $d\rho/dt$ is zero at each point. This is called a steady state. Hence for steady currents

$$\nabla . \mathbf{J} = 0$$