DEPARTMENT OF PHYSICS AND NANOTECHNOLOGY
SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

18PYB101J-Electromagnetic Theory, Quantum Mechanics, Waves and Optics

Module 3- Lecture-III

de-Broglie hypothesis for matter waves, Physical significance of wave function

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Topics to be Discussed:-

de-Broglie hypothesis for matter waves

Physical significance of wave function

de-Broglie Concept of Dual Nature

- The universe is made of Radiation (light) and matter (particles). The light exhibits the dual nature (i.e.,) it can behaves both as a wave (Interference, diffraction phenomenon) and as a particle (Compton effect, photo-electric effect etc).
- Since the nature loves symmetry, in 1924 Louis de-Broglie suggested that an electron (or) any other material particle must exhibit wave like properties in addition to particle nature.
- The waves associated with a material particle are called as Matter waves.

4.2.2 de-Broglie Wavelength

From the theory of light, considering a photon as a particle, the total energy of the photon is given by $E = mc^2$. (1)

where $m \rightarrow$ Mass of the particle

$c \rightarrow$ Velocity of light

Considering the photon as a wave, the total energy is given by $E = h\nu$ (2)

where $h \rightarrow$ Planck's constant

$\nu \rightarrow$ Frequency of radiation

From equation (1) and (2) we can write $E = mc^2 = h\nu$ (3)

We know momentum = mass × velocity

$$p = mc$$

$\therefore$ Equation (3) becomes $h\nu = pc$

$$p = \frac{h\nu}{c}$$

Since $\dfrac{c}{v} = \lambda$ we can write $p = \dfrac{h}{\lambda}$ (4)

de-Broglie suggested the equation (4) can be applied both for photons and material particles. If m is the mass of the particle and v is the velocity of the particle, then

Momentum $p = mv$.

$\therefore$ de-Broglie wavelength $\lambda = \dfrac{h}{mv}$ (5)

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Other forms of de-Broglie Wavelength

i) de-Broglie wavelength in terms of Energy

We know kinetic energy $E = \frac{1}{2} mv^2$

Multiplying by 'm' on both sides we get

$$Em = \frac{1}{2} m^2 v^2$$

$$\text{(or)} \quad m^2 v^2 = 2Em$$

$$mv = \sqrt{2Em}$$

$$\therefore \text{ de-Broglie wavelength } \lambda = \frac{h}{\sqrt{2mE}} \qquad (6)$$

ii) de-Broglie Wavelength in terms of voltage

If a charged particle of change '*e*' is accelerated through a potential difference '*V*'

Then the kinetic energy of the particle $=\frac{1}{2}mv^2$ (7)

Also we know energy = eV (8)

Equating equations (7) and (8) we get

$$=\frac{1}{2}mv^2 = \text{eV}$$

Multiplying by '*m*' on both sides we get

$$m^2v^2 = 2meV$$

(or) $$mv = \sqrt{2meV} \quad (9)$$

Substituting equation (18) in (14), we get

$$\text{de-Broglie wavelength } \lambda = \frac{h}{\sqrt{2meV}} \quad (10)$$

iii) de-Broglie wavelength in terms of Temperature

When a particle like neutron is in thermal equilibrium at temperature *T*, then they possess Maxwell distribution of velocities.

$$\therefore \text{ Their kinetic energy } E_k = \frac{1}{2} m v_{rms}^2 \qquad (11)$$

Where v_{rms} is the Root mean square velocity of the particle.

$$\text{Also, we know Energy } = \frac{3}{2} K_B T \qquad (12)$$

Where K_B is the Boltzmann constant.

$\therefore$ Equating equations (11) and (12) we get

$$\frac{1}{2} m v^2 = \frac{3}{2} K_B T$$

(or) $$m^2 v^2 = 3 m K_B T$$

$$m v = \sqrt{3 m K_B T}$$

$$\therefore \text{ de-Broglie wavelength } \lambda = \frac{h}{mv} = \frac{h}{\sqrt{3 m K_B T}} \qquad (13)$$

Physical Significance of the wave function (ψ)

1. It relates the particle and ware nature of matter statistically.
2. It is a complex quantity and hence we cannot measure it.
3. It must be single valued and continuous everywhere.
4. The probability density is given by square of its magnitude

$$P = |\psi|^2 = \psi\psi^*$$

where ψ^* *is the complex conjugate of* ψ.

5. *The probability of finding a particle in a volume dτ* is *given by,* $P = |\psi|^2 \, d\tau$

6. Further if the particle is certainly to be found somewhere is space then the probability value is equal to one.

$$\iiint |\psi|^2 \, d\tau = 1$$

$$\iiint \psi\psi^* \, d\tau = 1$$

A wave function satisfying this condition is called normalized wave function and this condition is called normalization condition