Reg. No.															

B.Tech. DEGREE EXAMINATION, MAY 2022

Third & Fourth Semester

18MAB203T – PROBABILITY AND STOCHASTIC PROCESSES

(For the candidates admitted from the academic year 2018-2019 to 2019-2020)

(Statistical tables are to be provided)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40^{th} minute.

(ii) **Part - B** should be answered in answer booklet.

Time: 2½ Hours Max. Marks: 75

PART – A (25 × 1 = 25 Marks)

Answer **ALL** Questions

	Marks	BL	CO	PO
1. A coin is tossed twice and X denotes the number of heads. The mean of X is (A) 1 (B) 0 (C) 2 (D) –1	1	2	1	2
2. If F is the cumulative distribution function of a continuous random variable X and if $a<b$, then $p(a<X\le b)$ is (A) F(b)+F(a) (B) aF(b)–bF(a) (C) aF(b)+bF(a) (D) F(b)–F(a)	1	1	1	1
3. The mean of a binomial distribution is 20 and standard deviation is 4. Find the parameter n (A) 100 (B) 200 (C) 50 (D) 25	1	2	1	2
4. The characteristic function of the exponential distribution is (A) $\frac{\lambda}{\lambda-i\omega}$ (B) $\frac{\lambda}{\lambda+i\omega}$ (C) $\frac{1}{\lambda-i\omega}$ (D) $\frac{1}{\lambda+i\omega}$	1	1	1	1
5. A discrete random variable X has the following probability distribution. Find $p(-2<X<2)$	1	2	1	2

x:	–2	–1	0	1	2	3
$p(x)$:	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{3}{10}$

(A) 0.1 (B) 0.4
(C) 0.3 (D) 0.5

6. If $F(x,y)$ is the joint cumulative distribution function, then $F(\infty,\infty)=$ (A) 0 (B) ∞ (C) 1 (D) $-\infty$	1	1	2	1

7. If U and V are two independent variables, then COV(U,V) is 1 1 2 1

(A) 0 (B) 1
(C) −1 (D) ∞

8. From the following joint probability distribution of X and Y find P(X=0) 1 2 2 2

Y \ X	0	1	2
0	3/28	9/28	3/28
1	3/14	3/14	0
2	1/28	0	0

(A) 3/28 (B) 7/14
(C) 5/28 (D) 5/14

9. The conditional probability density function of Y given X is 1 1 2 1

(A) $\dfrac{f(x,y)}{f(x)}$ (B) $\dfrac{f(x,y)}{f(y)}$
(C) $f(x,y).f(x)$ (D) $f(x,y).f(y)$

10. If X and Y have joint probability density function $f(x,y)=\frac{2}{3}(2x+y); 0<x<1, 0<y<1$, then $f(x)=$ 1 2 2 2

(A) $\dfrac{1+2y}{2}\left(\dfrac{2}{3}\right)$ (B) $\dfrac{2}{3}\left(\dfrac{4x+1}{2}\right)$
(C) $\dfrac{1+x}{2}$ (D) $\dfrac{1+y}{2}$

11. Let X follows an exponential distribution with parameter λ. Using Markov's inequality, an upper bound for $P(X\geq a)\leq$ 1 1 3 1

(A) $\dfrac{1}{\lambda a}$ (B) $\dfrac{\lambda}{a}$
(C) $\dfrac{1}{a}$ (D) λ

12. If $E(X)=\mu$ and $Var(X)=\sigma^2$, then for $a>0$, $P(X\leq\mu-a)\leq$ 1 1 3 1

(A) $\dfrac{\sigma^2}{\sigma^2-a^2}$ (B) $\dfrac{\sigma^2}{\sigma^2+a^2}$
(C) $\dfrac{\sigma}{\sigma+a}$ (D) $\dfrac{\sigma}{\sigma-a}$

13. Cauchy-Schwartz inequality states that for any two random variables, 1 1 3 1

(A) $E(XY)\leq E(X)E(Y)$ (B) $E(XY)^2\leq E(X)E(Y)$
(C) $E(XY)^2\leq E\left(X^2\right)E\left(Y^2\right)$ (D) $E(XY)^2\leq E(XY)$

14. If $Var(X)=0$, then $P(X=\mu)$ 1 1 3 1

(A) 0
(B) 1
(C) 2
(D) 3

15. Chernoff's inequality gives the ________ bounds compared to Markov's inequality and Tchebycheff's inequality. 1 1 3 1

(A) Weakest
(B) Strongest
(C) Same
(D) Approximate

16. $R_{XY}(-\tau)=$ 1 1 4 1

(A) $-R_{XY}(\tau)$
(B) $R_{XY}(\tau)$
(C) $R_{YX}(\tau)$
(D) $-R_{XY}(-\tau)$

17. If the random processes {X(t)} and {Y(t)} are jointly wide-sense stationary and independent, then $R_{XY}(\tau)$

(A) $E\{X^2(t)\}\cdot E\{Y^2(t)\}$
(B) $E\{X(t)\}\cdot E\{Y(t)\}$
(C) 0
(D) 1

18. The average power of the random process {X(t)} is defined by 1 1 4 1

(A) $R_{XX}(\tau)$
(B) $R_{XX}(0)$
(C) $R_{XX}(-\tau)$
(D) $S_{XX}(0)$

19. If a stationary process has autocorrelation function given by $R(\tau)=2+4e^{-2|\tau|}$, then the mean square value is 1 2 4 2

(A) 2
(B) 4
(C) 6
(D) 8

20. A random process is defined by X(t)=A, where A is a continuous random variable with probability density function $f(x)=1, 0<a<1$. The mean of the process X(t) is 1 2 4 2

(A) 0
(B) 1
(C) 2
(D) 1/2

21. The power spectral density of a random signal with autocorrelation function $e^{-\lambda|\tau|}$ is 1 2 5 2

(A) $\dfrac{\lambda}{\lambda^2+\omega^2}$
(B) $\dfrac{\omega}{\lambda^2+\omega^2}$
(C) $\dfrac{2\lambda}{\lambda^2+\omega^2}$
(D) $\dfrac{2\omega}{\lambda^2+\omega^2}$

22. Unit impulse response for a causal system {h(t)} is zero when 1 1 5 1

(A) t>0
(B) t=0
(C) t<0
(D) Always

23. The power spectral density satisfies the ________ condition if X(t) is real 1 1 5 1

(A) $\delta_{XX}(\omega)=-\delta_{XX}(-\omega)$
(B) $\delta_{XX}(\omega)=\delta_{XX}(-\omega)$
(C) $\delta_{XX}(\omega)=\infty \; at \; \omega=0$
(D) $\delta_{XX}(\omega)=\delta_{XX}(\omega^2)$

24. If $\{X(t)\}$ and $\{Y(t)\}$ are orthogonal, then ______ 1 1 5 1

(A) $\delta_{XY}(\omega)=0 \quad \delta_{YX}(\omega)=1$ (B) $\delta_{XY}(\omega)=0 \quad \delta_{YX}(\omega)=0$

(C) $\delta_{XY}(\omega)=1 \quad \delta_{YX}(\omega)=1$ (D) $\delta_{XY}(\omega)=1 \quad \delta_{YX}(\omega)=0$

25. A random process $\{X(t)\}$ is applied to a linear system with impulse response $h(t)=e^{-2t};t\geq 0$. The power transfer function of the system is 1 2 5 2

(A) $\dfrac{1}{2+i\omega}$ (B) $\dfrac{4}{4+\omega^2}$

(C) $\dfrac{1}{4+\omega^2}$ (D) $\dfrac{2}{2+i\omega}$

PART – B ($5 \times 10 = 50$ Marks)

Answer ALL Questions

Marks BL CO PO

26. a. The discrete random variable X has the probability distribution given by 10 3 1 1,2

x	0	1	2	3	4
$p(x)$	k	$3k$	$5k$	$7k$	$9k$

Compute,

(i) The value of k

(iii) Cumulative distribution function

(iv) Variance and

(v) $P(0<X<3/X>1)$

(OR)

b. In a normal distribution, 30% of the items are under 45 and 80% of the items are over 60. Compute the mean and standard deviation of the distribution. 10 4 1 1,2

27. a.i. If the joint probability distribution function of (X,Y) is given by 5 3 2 1,2

$$f(x,y)=\begin{cases} x+y; 0\leq x\leq 1 \\ \qquad 0\leq y\leq 1 \\ 0 \quad ;otherwise \end{cases}$$

Compute the probability density function of U=XY.

ii. The joint probability distribution of (X,Y) is given by 5 3 2 1,2

X	0	1	2
0	0.1	0.04	0.06
1	0.2	0.08	0.12
2	0.2	0.08	0.12

Examine if X and Y are independent.

(OR)

b. Let X and Y be random variables having the following joint probability distribution. Compute the correlation coefficient between X and Y. 10 4 2 1,2

X	Y		
	0	1	2
0	1/16	2/16	1/16
1	2/16	4/16	2/16
2	1/16	2/16	1/16

28. a. An unbiased coin is tossed 100 times. Show that the probability that the number of heads will be between 30 and 70 is greater than 0.9375. **(OR)** b. In a particular circuit 20 resistors are connected in series. The mean and variance of the resistance of each resistor is 5 and 0.2 respectively. Using central limit theorem, compute the probability that the total resistance of the circuit will exceed 98, assuming independence.	10 10	3 4	3 3	1,2 1,2
29. a. If $X(t)=5\cos(10t+\theta)$ and $Y(t)=20\sin(10t+\theta)$ where θ is a random variable uniformly distributed in (0,2π), show that the processes {X(t)} and {Y(t)} are jointly wide-sense stationary. **(OR)** b. Consider two random processes $X(t)=3\cos(\omega t+\theta)$ *and* $Y(t)=2\cos(\omega t+\theta-\pi/2)$, where θ is a random variable uniformly distributed in (0,2π). Show that $\lvert R_{xy}(0)\rvert\le\sqrt{R_{xx}(0)R_{yy}(0)}$.	10 10	3 4	4 4	1,2 1,2
30. a. A wide-sense stationary random process X(t) has power spectral density $\delta_{XX}(\omega)=\dfrac{\omega^2}{\omega^4+10\omega^2+9}$ compute auto correlation function and mean square value of the process. **(OR)** b. A wide-sense stationary process X(t) is the input to a linear system with impulse response $h(t)=2e^{-t};t\ge 0$ *if* $R_{XX}(\tau)=e^{-2\lvert\tau\rvert}$ Compute the power spectral density function of the output process Y(t).	10 10	3 4	5 5	1,2 1,2

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