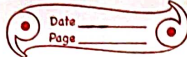


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Parameter Estimation



Ans: A sample size  $n$  is taken as  $x_1, x_2, \dots, x_n$

normal distribution density is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

density of  $x_i$  is:

$$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} \quad i=1, 2, \dots, n$$

Joint density of  $(x_1, x_2, \dots, x_n)$ :

$$L(x_1, x_2, \dots, x_n) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

Take natural log both side.

$$\ln L(x_1, x_2, \dots, x_n) = -n \ln(\sqrt{2\pi}\sigma) - \frac{1}{2\sigma^2} \sum_{i=1}^n (x_i - \mu)^2$$

Maximum likelihood estimation for  $\mu$ :

$$\frac{\partial}{\partial \mu} \ln L(\mu, \sigma^2 | x_1, x_2, \dots, x_n) = \frac{1}{\sigma^2} \sum_{i=1}^n (x_i - \mu) = 0$$

$$\sum_{i=1}^n (x_i - n\mu) = 0$$

$$\mu = \frac{1}{n} \sum_{i=1}^n x_i = \bar{x}$$

$$\boxed{\mu = \bar{x}}$$

for  $\sigma^2$

$$\frac{\partial}{\partial \sigma^2} \ln L(\mu, \sigma^2 | x_1, \dots, x_n) = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^n (x_i - \mu)^2 = 0$$

$$-n + \frac{1}{\sigma^2} \sum_{i=1}^n (x_i - \mu)^2 = 0$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2 = s^2$$

$$\boxed{\sigma^2 = s^2}$$

$\therefore$  MLE for  $\mu = \bar{x}$  (mean)

MLE for  $\sigma^2 = s^2$  (variance)

Ans:  $L(\theta | x_1, x_2, \dots, x_n) = \prod_{i=1}^n \left[ \binom{n}{x_i} \theta^{x_i} (1-\theta)^{n-x_i} \right]$

MLE for  $\theta$ :

$$\frac{\partial}{\partial \theta} \ln L(\theta | x_1, x_2, \dots, x_n) = \sum_{i=1}^n \left[ \frac{x_i - n\theta}{\theta(1-\theta)} \right] = 0$$

$$\sum_{i=1}^n \left[ \frac{x_i(1-\theta) - \theta(n-x_i)}{\theta(1-\theta)} \right] = 0$$

$$\Rightarrow \hat{\theta} = \frac{\sum_{i=1}^n x_i}{n + \sum_{i=1}^n x_i}$$

$$\text{let } \sum_{i=1}^n x_i = S$$

$$\therefore \hat{\theta} = \frac{S}{n+S}$$