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Parameter Estimation

Ans1 A sample size n is taken as $x_1, x_2 \ldots x_n$

normal distribution/density is:

$$f(x) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

density of x_i is:

$$\frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}} \quad i = 1, 2 \ldots n$$

Joint density of $(x_1, x_2 \ldots x_n)$:

$$L(x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

Take natural log both sides:

$$\ln(L(x_1, x_2 \ldots x_n)) = -\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Maximum likelihood estimation for θ_1:

$$\frac{\partial}{\partial\theta_1}\ln(\theta_1, \theta_2 | x_1 x_2 \ldots x_n) = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i - \theta_1) = 0$$

$$\sum_{i=1}^{n}(x_i - n\theta_1) = 0$$

$$\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x}$$

$$\boxed{\theta_1 = \bar{x}}$$

for θ_2

$$\frac{\partial}{\partial\theta_2}\ln(\theta_1, \theta_2 | x_1 \ldots x_n) = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2}\sum_{i=1}^{n}(x_i-\theta_1)^2 = 0$$

$$-n + \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2 = 0$$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2 = S^2$$

$$\boxed{\theta_2 = S^2}$$

$\therefore$ MLE for $\theta_1 = \bar{x}$ (mean)

MLE for $\theta_2 = S^2$ (variance)

Ans2 $L(\theta | x_1, x_2 \ldots x_n) = \sum_{i=1}^{n} \left[\ln\binom{m}{x_i} + x_i \ln\theta + (m - x_i)\ln(1-\theta)\right]$

MLE for θ:

$$\frac{\partial}{\partial\theta}\ln(\theta | x_1, x_2 \ldots x_n) = \sum_{i=1}^{n}\left[\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta}\right] = 0$$

$$\sum_{i=1}^{n}\left[\frac{x(1-\theta) - \theta(m - x_i)}{\theta(1-\theta)}\right] = 0$$

$$\Rightarrow \hat{\theta} = \sum_{i=1}^{n} x_i \Big/ \left(mn + \sum_{i=1}^{n} x_i\right)$$

let $\sum_{i=1}^{n} x_i = S$

$$\therefore \boxed{\hat{\theta} = \frac{S}{mn + S}}$$