Mustang Math Tournament 2024

Relay Rodeo Foal Round

Basic Format

- This round contains 21 problems to be solved in 45 minutes.
- The problems are divided into four suits ($\diamondsuit$, $\heartsuit$, $\clubsuit$, $\spadesuit$) with five problems each, plus one Joker problem.
- Some of the problems refer to the answer to earlier problems in the same suit. For example, problem [$\diamondsuit$2] may begin with, "Let N be the answer to [$\diamondsuit$1]." So, [$\diamondsuit$2] can only be attempted after [$\diamondsuit$1] has been answered. In a similar manner, the Joker problem refers to the answer to the last problem in every suit.
- The first four problems in each suit are worth 2 points each, and the last problem in each suit is worth 3 points. The Joker problem is worth 6 points.
- **Do not** write below the provided answer blank inside each space on the answer sheet (the space is for grading purposes).
- *Be sure to **dot** your i's and **dash** your t's (i.e. carefully check your work) to submit each set with no re**morse**.*

Diamonds $\diamondsuit$

[$\diamondsuit$1]. Charlie the Colt is standing in line to get ice cream from the ice cream truck. There are 4 colts in front of Charlie and 5 colts behind Charlie. How many colts are in the line, in total?

[$\diamondsuit$2]. Let N be the answer to [$\diamondsuit$1]. The Martian alphabet contains N letters. Half of them are vowels, and the other half of them are consonants. A Martian syllable is any string of two letters that has exactly one vowel. How many different Martian syllables are there?

[$\diamondsuit$3]. Let N be the answer to [$\diamondsuit$1]. A rectangle has integer side lengths and a perimeter of $N-4$. What is the area of the rectangle?

[$\diamondsuit$4]. Let N be the answer to [$\diamondsuit$1]. Sally evaluates the two expressions 2^N+N^3 and 3^N+N^2, and writes both results on the chalkboard. What is the smaller of the two numbers written?

[$\diamondsuit$5]. Let N be the answer to [$\diamondsuit$1]. Suppose m is a positive integer that has exactly N positive divisors, one of which is 20. What is m?

Clubs $\clubsuit$

[$\clubsuit$1]. Anna chooses three positive integers that have a product of 36. Her friend, Yuuki, chooses a different set of three positive integers that also have a product of 36. Anna then points out that the sum of her numbers is equal to the sum of Yuuki's numbers. What was that sum?

[$\clubsuit$2]. Rectangle $ABCD$ with side length $AB = 8$ is inscribed in a circle with diameter 17. What is the length of $\overline{BC}$?

[$\clubsuit$3]. Let N be the answer to [$\clubsuit$1]. Jane, Kate, and Harold have a total of 167 berries. Kate has N more than 3 times of Jane's berries. Harold has 8 more than 2 times of Kate's berries. Kate gives Harold 14 berries. If Harold now has n berries in total, what is the value of $\frac{n}{5}$?

[$\clubsuit$4]. Let N be the answer to [$\clubsuit$2]. Two distinct integers from 1 to N, inclusive, are chosen at random. The probability that both integers are prime and their sum is also prime can be expressed as a fully simplified fraction $\frac{m}{n}$. Find $m+n$.

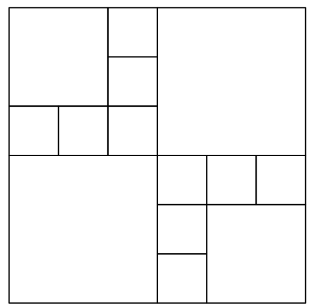
[$\clubsuit$5]. Let m and n be the answers to [$\clubsuit$3] and [$\clubsuit$4], respectively. Let ABC be an isosceles triangle with $\angle A = (n-m)^\circ$ and $\angle B = \angle C$. Suppose that D lies on the segment AB such that $\triangle ACD$ is isosceles. What is the measure of $\angle BCD$, in degrees?

Hearts ♡

[♡1]. There are 7 balls placed in a bag, each with a different color. When 3 balls are drawn from the bag with replacement, the probability that all three of them have the same color can be expressed as $\frac{1}{a}$. What is a?

[♡2]. Let N be the answer to [♡1]. N is a perfect square that has the property that the digits of N are positive perfect squares. What is the smallest perfect square greater than N that has the same property?

[♡3]. Let N be the answer to [♡2]. A large square with area N is cut into many smaller squares, as shown below. What is the perimeter of the smallest square?



[♡4]. Let N be the answer to [♡3]. A fair coin is flipped N times. What is the probability that tails is flipped exactly once?

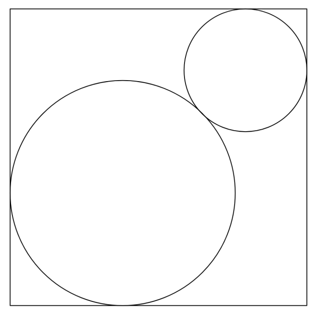
[♡5]. Let N be the answer to [♡4]. A positive integer a has exactly d positive divisors, and exactly $d^2N + 1$ of these divisors are composite numbers. What is the value of d?

Spades ♠

[♠1]. Let X be the units digit of $2025^{2024^{2023^{2022}}}$ and Y be the units digit of $2024^{2023^{2022^{2021}}}$. What is $10X+Y$? Note that exponents are computed from right to left, so 2^{2^3} equals 2^8, not 4^3, for example.

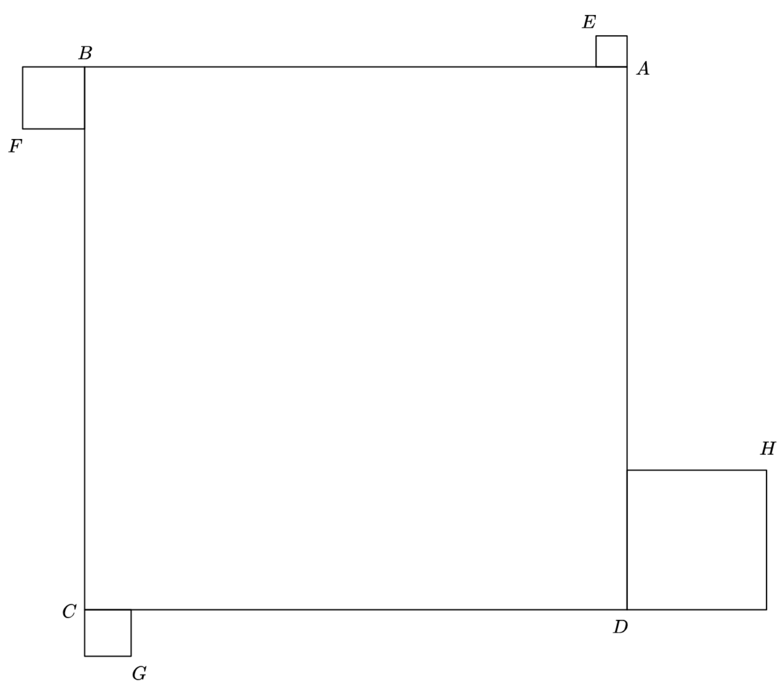
[♠2]. Cathy takes two distinct numbers from the list $1, 2, 3, 4, 10, 20, 30, 40$ and adds them. How many different sums are possible?

[♠3]. Two circles are placed inside of a square so that each is tangent to the other and to two sides of the square, as shown. Given that the circles have radii of 1.5 and 2.5 and that the square has an area of A, what is the greatest integer less than A?



[♠4]. The graph of the equation $|x|+|y|+|x+20|+|y+24|=45$ encloses a region in the plane. What is the area of this region?

[♠5]. Let $ABCD$ be a square of side length 10000. Let a, b, c, and d be the answers to [♠1], [♠2], [♠3], and [♠4], respectively. Squares with diagonals $\overline{AE}$, $\overline{BF}$, $\overline{CG}$, and $\overline{DH}$ and side lengths of a, b, c, and d, respectively, are constructed outside $ABCD$, as shown in the diagram below. What is the area of $EFGH$?



JOKER

[J1]. Let a, b, and c be the answers to [$\diamondsuit$5], [$\clubsuit$5], and [$\heartsuit$5], respectively. Let d be the sum of the digits of the answer to [$\spadesuit$5]. What is the greatest integer that cannot be written as $pa + qb + rc + sd$ for nonnegative integers p, q, r, and s?

Acceptable Answers

The following rules provide guidelines for acceptable answers in this round. Please note that any specifications provided in a problem will take precedence over these rules. The decisions of MMT coordinators are final.

- Common fractions are defined as a fraction in the form $\pm\frac{a}{b}$ where a and b are natural numbers and $\gcd(a, b) = 1$.
- Ratios and fractional answers should be expressed as common fractions unless otherwise specified.
- Radicals should be simplified. A simplified radical must satisfy:
  - No square factors, fractions, or nested radicals inside a radical
  - No radicals inside the denominator of a fraction
- Answers must be expressed to the exact accuracy called for in the problem (e.g. 25.0 will not be accepted for 25 and 25 will not be accepted for 25.0).
- Do not make approximations for numbers (e.g. 3.14 or $\frac{22}{7}$ for π) unless otherwise specified.
- Units **do not** need to be included but must be correct if included.

