1. Anika has a bag of green and blue gumballs. She eats 1 green gumball and notices there are now 4 times as many blue gumballs as green gumballs. She eats another 14 blue gumballs and notices there are now 2 times as many green gumballs as blue gumballs. How many gumballs were originally in the bag?

2. Let x and y be positive integers. If x is 150% of y, and $6y$ is $N\%$ of x, what is N?

3. Harry wants to write several positive numbers in a row on a chalkboard. The numbers must follow certain rules:

   - The leftmost number must be 1.
   - The rightmost number must be 24.
   - Every number other than the rightmost must divide the number to its right.
   - No two consecutive numbers in the row are the same.

   What is the maximum possible number of numbers in the row?

4. Worker A can paint a wall in 4 hours, while workers A and B together only take 3 hours to paint the same wall. Assuming each worker paints the wall at a constant rate, how many hours would it take for worker B to paint the wall alone?

5. Simplify $17^2 + (13 + 23) \cdot 17 + 13 \cdot 23$.

6. Joe has a cone-shaped pitcher full of pancake batter with a base of radius 6 inches and a height of 16 inches. Each pancake Joe makes is the exact same size: a cylinder with a radius of 4 inches that is $\frac{1}{2}$ inch thick. Assume that Joe's pancakes don't rise. After making breakfast this morning, Joe notices that the pitcher is now $\frac{1}{3}$ full. How many pancakes did Joe make this morning?

7. How many ways are there to form a 4-letter word from the letters $\{M, U, S, T, A, N, G\}$ with at least one vowel? Note that the vowels are A and U, words cannot contain duplicate letters, and any combination of distinct letters constitutes a word.

8. A palindrome is a positive integer, such as 30303 or 5885, that is the same when read forward or backward. What is the average of all of the 3-digit palindromes?

9. A field of carrots resembles a 4 by 6 rectangular grid of identical square cells, with one carrot planted in each cell. If two rabbits visit the field and randomly select a carrot to eat, one after the other, what is the probability that the two cells no longer containing carrots share a side?

10. The fraction $\frac{25}{28}$ can be written in the form of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$, where a, b, and c are distinct positive integers. What is $a + b + c$?

11. For integers $a \leq b$, define $S(a, b)$ to be the sum of the integers from a to b, inclusive. For example, $S(2, 4) = 2 + 3 + 4 = 9$ and $S(1, 1) = 1$.

    Consider pairs i, j of positive integers between 1 and 5 inclusive, with $i \leq j$. Find the sum of $S(i, j)$ over all 15 such pairs.

12. The diagonals of rectangle $ABCD$ intersect at E and $BE = AB = 6$. Points F and G lie on AC and BD, respectively, such that BF is perpendicular to AC and CG is perpendicular to BD. If BF and CG intersect at H, what is BH?

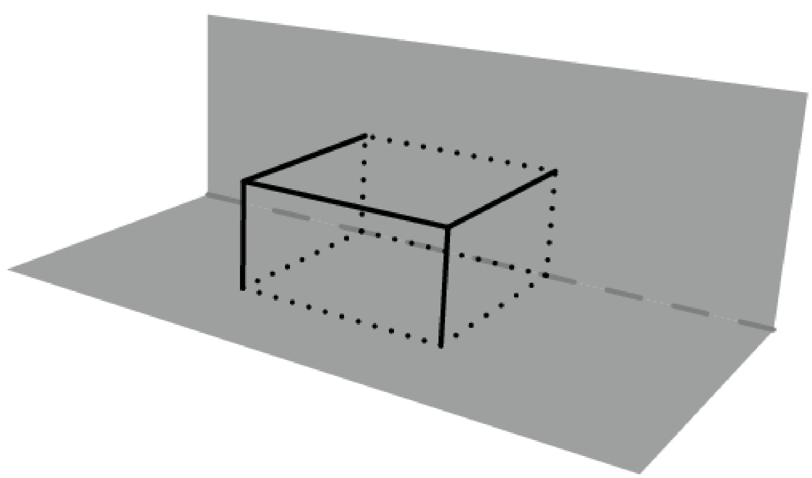
13. The number $717A7B7C$, where A, B, and C are distinct integer digits, is divisible by 12. If x is the maximum possible value of $A + B + C$ and y is the minimum possible value of $A + B + C$, find $x - y$.

14. In trapezoid $ABCD$ with $AB \parallel CD$, E and F are the midpoints of AD and BC, respectively. Given that $CD = 18$, $EF = 12$, and $AE = BF = 5$, what is the area of trapezoid $ABFE$?

15. A race of aliens live on a flat world, with a source of light of negligible size (called the NUS) 100 km above the ground. One day, the NUS is too bright, and the aliens hope to build a bit of shade. They have a 150 m $\times$ 200 m board, and wish to cover an area of 75 km^2 on the ground. Furthermore, while they have the powers of levitation, the aliens can only levitate the board parallel to the ground, as it becomes unstable otherwise. At how many kilometers above the ground do the aliens need to place the board in order to shade the desired area?

16. For any positive integer n, define $f(n)$ as the number of primes less than n. For example, $f(7) = 3$ and $f(8) = 4$. Given that there are 25 primes less than 100 and that the sum of these primes is 1060, what is the value of $f(1) + f(2) + f(3) + \cdots + f(100)$?

17. Bob is making the outline of a wire cage in the shape of a rectangular prism. As shown in the image below, one face of the cage will lie on the ground and one face will lie against the wall. Bob has 9 meters of wiring, which he will use to make the solid black lines below. What is the maximum volume of the cage, in cubic meters, that Bob can achieve?



18. Owen picks letters randomly from the alphabet with replacement, each with equal probability, until he picks two vowels (5 of the 26 letters in the alphabet are vowels). He arranges the letters he picks in the order he picked them to create a word. Find the probability that his word is a palindrome with an even amount of letters.

19. Equilateral triangle ABC with side length 20 is constructed, and a circle is inscribed in it, tangent to lines AB, BC, and AC. A circular coin with radius 1 is then tossed randomly onto the triangle. Given that the coin lands completely within the triangle, what is the probability that at least part of the coin lands outside the circle?

20. Define the function $f(x) = (4x - 6)^{10}$. If $f(x)$ can be represented as $a_{10}x^{10} + a_9x^9 + \cdots + a_1x + a_0$, let $S = a_9 + a_7 + a_5 + a_3 + a_1$. What is the remainder when $|S|$ is divided by 1000?