Mustang Math Tournament 2024

Herding Hexes Foal Round

Basic Format

- This round contains 26 problems to be solved in 45 minutes.
- Each problem corresponds to a hexagon on the answer grid (backside).
- A correct answer will grant 2 points each for Problems 1 through 10, 3 points each for Problems 11 through 19, and 4 points each for Problems 20 through 26.
- The score of a hexagonal tile is doubled if it can be connected back to the Free tile through other tiles that contain correct answers.
- **Do not** write below the provided answer blank inside each hexagon (the space is for grading purposes).
- *Feel free to **flag** down a proctor if you need help **deciphering** any of the above instructions*

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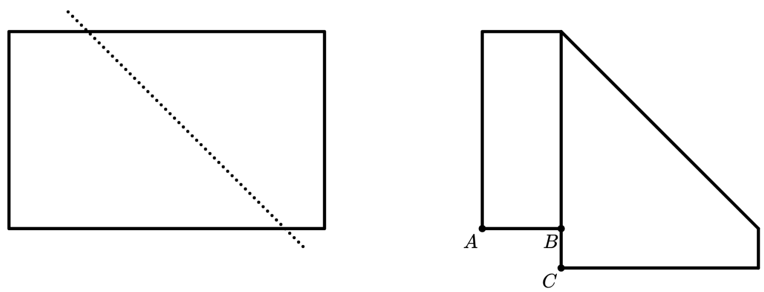
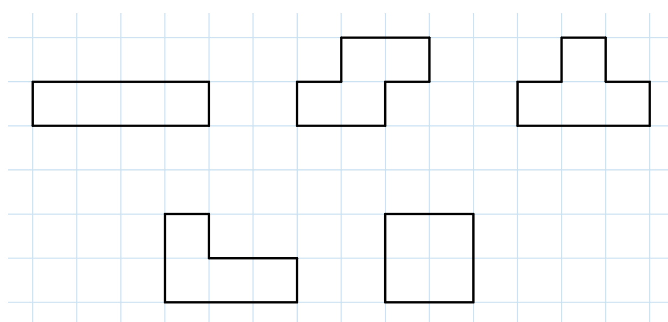
WE WILL NOT GRADE ANY WORK ON THIS PAGE.

1. A rectangular piece of paper shown on the left is folded along a diagonal line, and the resulting figure is shown on the right. If the original rectangle is 5 inches by 8 inches and the length of AB is 2 inches, and $\angle ABC$ is a right angle, what is the length of BC?



2. A small ice cream truck offers vanilla cones and chocolate cones. A customer can add sprinkles, caramel, both, or neither. How many different cones can be ordered at the truck?

3. Of the five shapes drawn on the grid below, one of them has a different perimeter from the other four. What is this different perimeter? Each square in the grid has a side length of 1 unit.



4. Suppose N is a perfect square which has 3 digits. Given that the first and last digits are equal, what is the largest possible value of N?

5. Among a list of 5 distinct integers, the three largest sum to 35. The average of all 5 integers is 10. What is the sum of the two smallest integers in the list?

6. Find the sum of the real numbers x that satisfy the equation $x^2 - 65 = 16$.

7. Susan chooses a random integer between 10 and 99, inclusive. What is the probability that it is a perfect square?

8. Of the numbers 2^{468}, 24^{68}, and 246^8, the one with the most positive divisors has N positive divisors. How many positive divisors does N have?

9. Suppose x and y are real numbers such that $(x + 2y)^2 = 9$ and $x + 3y = 10$. What is the sum of all possible values of x?

10. Three distinct prime numbers have a sum of 16. What is their product?

11. There is a prime p such that $\frac{10!}{p}$ is a perfect square. What is p? (Note: 10! denotes the product of the first 10 positive integers, $1 \times 2 \times 3 \times \cdots \times 10$.)

12. Let A and B be digits such that the five-digit number $A123B$ is a multiple of 5, and the four-digit number $B1A2$ is a multiple of 11. What is the maximum possible value of $A + B$?

13. The Awesome Intellectual Mustangs Examination is a 10-problem test, and each problem falls into one of four categories: algebra, combinatorics, galloping, and neighing. Among any three consecutive problems, there must be three different categories represented. How many different tests are possible? (Two tests are considered distinct if their sequences of problem categories are distinct.)

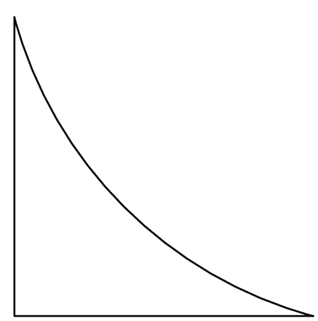
14. Farmer John has n glasses and a bucket filled with 1 gallon of milk. He pours half of his bucket in the first glass, and then half of what remains in his bucket in the second glass, and so on until the nth glass. Once he reaches the last glass, he will cycle back and pour half of his bucket into the first glass again, continuing this process forever.

    Given that the ratio between the amounts in the largest and smallest glasses is $32 : 1$, find the number of glasses.

15. If x, y are positive integers not exceeding 100, compute the maximum possible value of

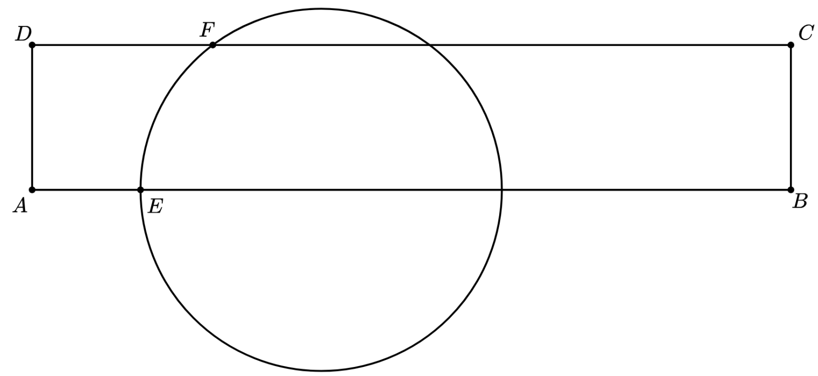
$$\operatorname{lcm}(\gcd(x, y), \operatorname{lcm}(x, y)).$$

16. The diagram below is made up of a 60° arc and two congruent sides that meet at a right angle. Given that the shape has an area of $3 + 3\sqrt{3} - 2\pi$, find the length of each congruent side.



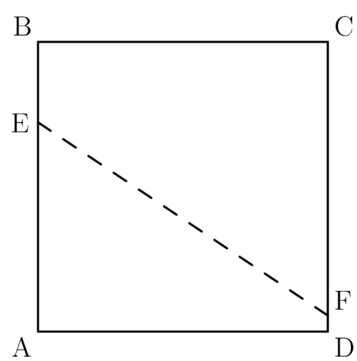
17. You're playing a game where you need to roll at least n to win. You can either roll two fair 6-sided dice and take the sum of the rolls, or one fair 12-sided die. For what value of n between 1 and 12, inclusive, is the probability you win equal regardless of which option you choose?

18. In the diagram below, $ABCD$ is a rectangle, and E and F lie on a circle whose center is on AB. Given that $AE = 3$, $AD = 4$, and $DF = 6$, what is the radius of the circle?



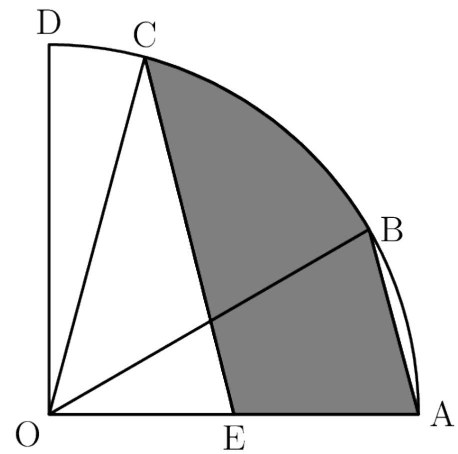
19. Define the polynomial $P(x) = x^3 + 1$. Find the sum of the coefficients of $P(P(P(x)))$.

20. Square $ABCD$ is constructed on the coordinate plane with $A = (0, 0)$ and $B = (0, 3)$. The square is then folded such that A coincides with the point $(2, 3)$. Let the points on the line of folding of the square that intersect AB and CD be E and F, respectively. What is the area of quadrilateral $AEFD$?



21. Carl chooses two positive integers, a and b, with a product of $6^5 = 7776$. He then computes the value, g, of $\gcd(a, b)$. How many different possible values can g take?

22. A quarter circle centered at O with radius 2 is shown in the diagram below. If $\angle AOB = 30°$, $\angle BOC = 45°$, and $CE \parallel AB$, find the area of the shaded region.



23. Let A, B, and C be distinct integers such that A, B, C forms an increasing arithmetic sequence, in that order, and A, C, B forms a geometric sequence, in that order. What is the minimum possible value of C?

24. The ages of Bob's children are all distinct primes less than 20. If the set of distinct remainders when their ages are divided by 5 is the same as the set of distinct remainders when their ages are divided by 6, what is the greatest possible sum of the children's ages?

25. Let $n > k$ be positive integers. Alice is initially tasked with selecting a k-person committee from among n club members. If the committee size were instead to be $k+1$, Alice would have twice as many ways to select the committee as before. If the committee size were instead to be $k-1$, the number of ways to select the committee would be one third the original amount. Find the number of ways for Alice to select the k-person committee.

26. Let $ABCD$ be a square with side length 4. A point P is randomly chosen in the interior of $ABCD$ and a circle ω with radius 1 is drawn with center at P. What is the probability ω intersects the boundary of square $ABCD$ at exactly four points?

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Acceptable Answers

The following rules provide guidelines for acceptable answers in this round. Please note that any specifications provided in a problem will take precedence over these rules. The decisions of MMT coordinators are final.

- Common fractions are defined as a fraction in the form $\pm\frac{a}{b}$ where a and b are natural numbers and $\gcd(a, b) = 1$.
- Ratios and fractional answers should be expressed as common fractions unless otherwise specified.
- Radicals should be simplified. A simplified radical must satisfy:
  - No square factors, fractions, or nested radicals inside a radical
  - No radicals inside the denominator of a fraction
- Answers must be expressed to the exact accuracy called for in the problem (e.g. 25.0 will not be accepted for 25 and 25 will not be accepted for 25.0).
- Do not make approximations for numbers (e.g. 3.14 or $\frac{22}{7}$ for π) unless otherwise specified.
- Units **do not** need to be included but must be correct if included.