

1. Anika has a bag of green and blue gumballs. She eats 1 green gumball and notices there are now 4 times as many blue gumballs as green gumballs. She eats another 14 blue gumballs and notices there are now 2 times as many green gumballs as blue gumballs. How many gumballs were originally in the bag?
2. Simplify $17^2 + (13 + 23) \cdot 17 + 13 \cdot 23$.
3. Let x and y be positive integers. If x is 150% of y , and $6y$ is $N\%$ of x , what is N ?
4. Harry wants to write several positive numbers in a row on a chalkboard. The numbers must follow certain rules:
 - The leftmost number must be 1.
 - The rightmost number must be 24.
 - Every number other than the rightmost must divide the number to its right.
 - No two consecutive numbers in the row are the same.

What is the maximum possible number of numbers in the row?

5. Joe has a cone-shaped pitcher full of pancake batter with a base of radius 6 inches and a height of 16 inches. Each pancake Joe makes is the exact same size: a cylinder with a radius of 4 inches that is $\frac{1}{2}$ inch thick. Assume that Joe's pancakes don't rise. After making breakfast this morning, Joe notices that the pitcher is now $\frac{1}{3}$ full. How many pancakes did Joe make this morning?
6. How many ways are there to form a 4-letter word from the letters $\{M, U, S, T, A, N, G\}$ with at least one vowel? Note that the vowels are A and U , words cannot contain duplicate letters, and any combination of distinct letters constitutes a word.
7. The Mustangs and the Colts are two of the 20 soccer teams in the Horsey Soccer League. The league is randomly split into two divisions with 10 teams in each. Find the probability that the Mustangs and the Colts are in different divisions.
8. For integers $a \leq b$, define $S(a, b)$ to be the sum of the integers from a to b , inclusive. For example, $S(2, 4) = 2 + 3 + 4 = 9$ and $S(1, 1) = 1$.

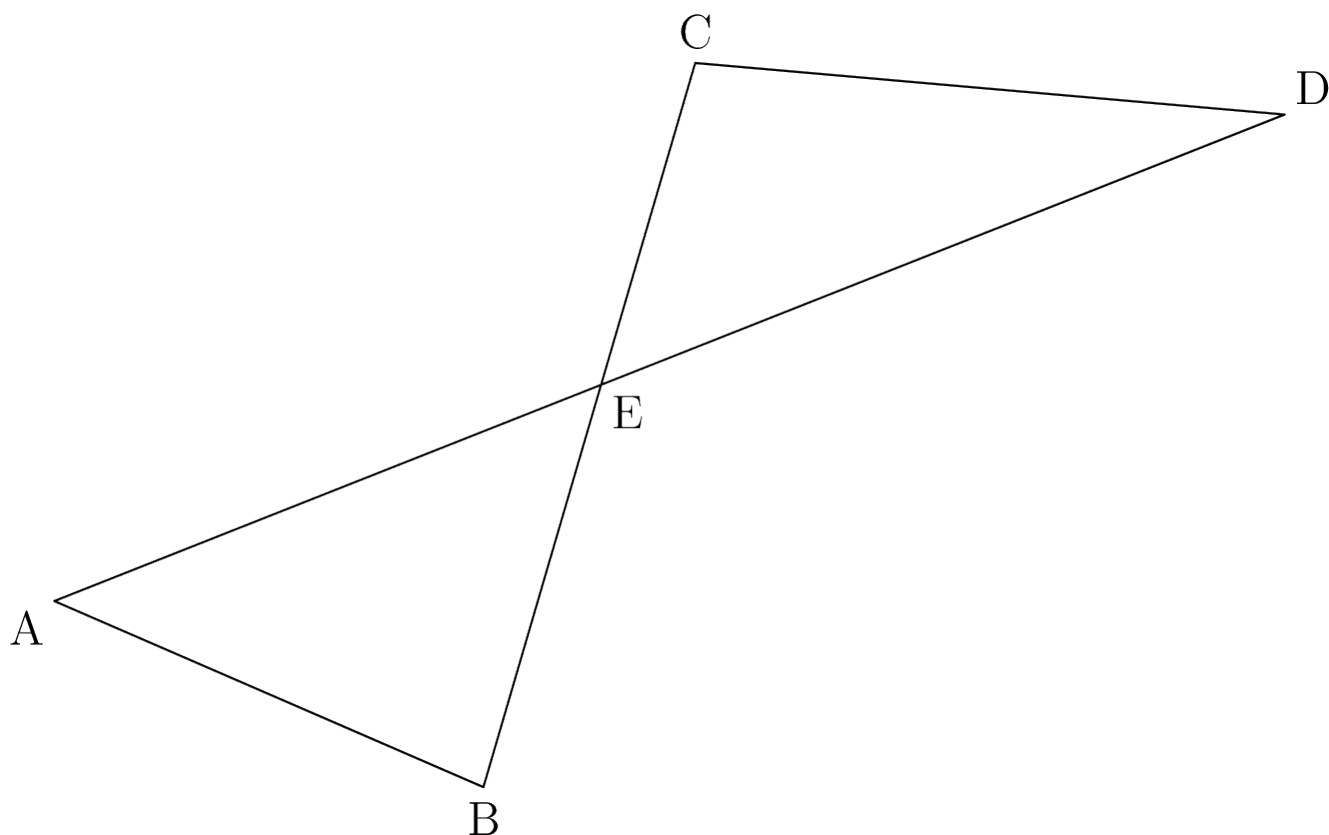
Consider pairs i, j of positive integers between 1 and 5 inclusive, with $i \leq j$. Find the sum of $S(i, j)$ over all 15 such pairs.
9. The number $717A7B7C$, where A , B , and C are distinct integer digits, is divisible by 12. If x is the maximum possible value of $A + B + C$ and y is the minimum possible value of $A + B + C$, find $x - y$.
10. A race of aliens live on a flat world, with a source of light of negligible size (called the NUS) 100 km above the ground. One day, the NUS is too bright, and the aliens hope to build a bit of shade. They have a $150 \text{ m} \times 200 \text{ m}$ board, and wish to cover an area of 75 km^2 on the ground. Furthermore, while they have the powers of levitation, the aliens can only levitate the board parallel to the ground, as it becomes unstable otherwise. At

how many kilometers above the ground do the aliens need to place the board in order to shade the desired area?

11. The fraction $\frac{25}{28}$ can be written in the form of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$, where a , b , and c are distinct positive integers. What is $a + b + c$?
12. Find the value of x that satisfies the equation

$$\sqrt{2 + \sqrt{x + \sqrt{2 + \sqrt{x + \dots}}}} = \frac{15}{2 + \frac{15}{2 + \frac{15}{2 + \dots}}}.$$

13. Four consecutive positive integers have a product of 93024. What is their sum?
14. Owen picks letters randomly from the alphabet with replacement, each with equal probability, until he picks two vowels (5 of the 26 letters in the alphabet are vowels). He arranges the letters he picks in the order he picked them to create a word. Find the probability that his word is a palindrome with an even amount of letters.
15. Let $n!$ denote the product of the integers between 1 and n , inclusive. A positive integer divisor d of $41!$ is chosen at random. What is the probability that d is divisible by 8, but is not divisible by 16?
16. For any real number z , let $\lfloor z \rfloor$ denote the greatest integer that is less than or equal to z . Find the area of the region formed by all points (x, y) on the coordinate plane satisfying $x \geq 0$, $y \geq 0$, and $\lfloor \frac{x}{3} \rfloor + \lfloor \frac{y}{3} \rfloor = 9$.
17. In the diagram below, E lies on both \overline{AD} and \overline{BC} . Given that $\angle DAC = \angle ADB$ and $[ABE] = 2[CDE] - 23$, find $[CDE]$. (Note that $[ABE]$ and $[CDE]$ refer to the areas of triangles $\triangle ABE$ and $\triangle CDE$, respectively.)



18. Define the function $f(x) = (4x - 6)^{10}$. If $f(x)$ can be represented as $a_{10}x^{10} + a_9x^9 + \cdots + a_1x + a_0$, let $S = a_9 + a_7 + a_5 + a_3 + a_1$. What is the remainder when $|S|$ is divided by 1000?
19. A sequence of integers is defined recursively by $a_0 = 0$ and $a_{n+1} = 17a_n + 1$ for all $n \geq 0$. Find the remainder when a_{200} is divided by 2023.
20. Two circles $\odot O_1$ and $\odot O_2$ are drawn such that each circle passes through the center of the other circle. $\odot O_1$ and $\odot O_2$ intersect at points A and B . Points P and Q are chosen independently and uniformly at random on $\odot O_1$ and $\odot O_2$, respectively. What is the probability that line segment \overline{PQ} intersects line segment \overline{AB} ?