Mustang Math Tournament 2024

Herding Hexes Colt Round

Basic Format

- This round contains 26 problems to be solved in 45 minutes.
- Each problem corresponds to a hexagon on the answer grid (backside).
- A correct answer will grant 2 points each for Problems 1 through 10, 3 points each for Problems 11 through 19, and 4 points each for Problems 20 through 26.
- The score of a hexagonal tile is doubled if it can be connected back to the Free tile through other tiles that contain correct answers.
- **Do not** write below the provided answer blank inside each hexagon (the space is for grading purposes).
- *Feel free to* ***flag*** *down a proctor if you need help* ***deciphering*** *any of the above instructions*

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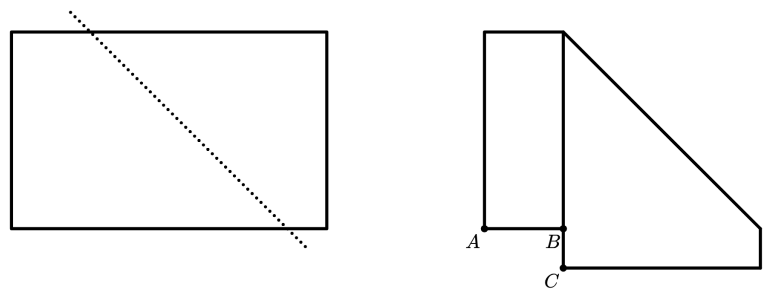
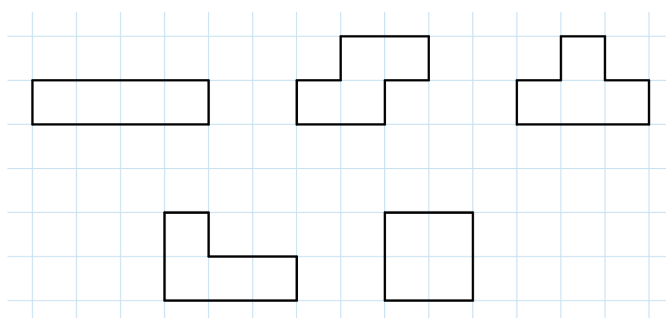
1. A small ice cream truck offers vanilla cones and chocolate cones. A customer can add sprinkles, caramel, both, or neither. How many different cones can be ordered at the truck?

2. A rectangular piece of paper shown on the left is folded along a diagonal line, and the resulting figure is shown on the right. If the original rectangle is 5 inches by 8 inches and the length of AB is 2 inches, and $\angle ABC$ is a right angle, what is the length of BC?



3. Suppose N is a perfect square which has 3 digits. Given that the first and last digits are equal, what is the largest possible value of N?

4. Of the five shapes drawn on the grid below, one of them has a different perimeter from the other four. What is this different perimeter? Each square in the grid has a side length of 1 unit.



5. Susan chooses a random integer between 10 and 99, inclusive. What is the probability that it is a perfect square?

6. Three distinct prime numbers have a sum of 16. What is their product?

7. Suppose x and y are real numbers such that $(x + 2y)^2 = 9$ and $x + 3y = 10$. What is the sum of all possible values of x?

8. Let a, b, and c be integers greater than 1 such that $\gcd(b, c) = \gcd(c, a) = \gcd(a, b) = 1$ and $abc = 34300$. What is the positive difference between the greatest and least numbers among a, b, and c?

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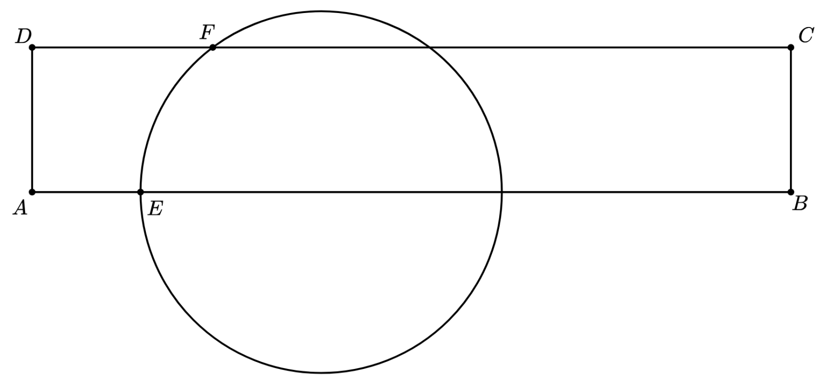
9. The Awesome Intellectual Mustangs Examination is a 10-problem test, and each problem falls into one of four categories: algebra, combinatorics, galloping, and neighing. Among any three consecutive problems, there must be three different categories represented. How many different tests are possible? (Two tests are considered distinct if their sequences of problem categories are distinct.)

10. In the diagram below, $ABCD$ is a rectangle, and E and F lie on a circle whose center is on AB. Given that $AE = 3$, $AD = 4$, and $DF = 6$, what is the radius of the circle?



11. You're playing a game where you need to roll at least n to win. You can either roll two fair 6-sided dice and take the sum of the rolls, or one fair 12-sided die. For what value of n between 1 and 12, inclusive, is the probability you win equal regardless of which option you choose?

12. There is a prime p such that $\frac{10!}{p}$ is a perfect square. What is p? (Note: $10!$ denotes the product of the first 10 positive integers, $1 \times 2 \times 3 \times \cdots \times 10$.)

13. Alice, who was a little sleepy during her 2$^{\text{nd}}$ grade math test, accidentally said $17 \cdot 18 = 2 \cdot 127$. However, Alice, being a clever student, decides to claim that every number was in base b. What value of b must Alice choose so that her multiplication remains valid?

14. Let $m!$ denote the product of the first m positive integers, and say that $0! = 1$. Lenny computes the value of $\frac{(n+1)!}{(n-1)!}$ for each value of n between 1 and 98, inclusive. What is the sum of these 98 values?

15. Carl chooses two positive integers, a and b, with a product of $6^5 = 7776$. He then computes the value, g, of $\gcd(a, b)$. How many different possible values can g take?

16. The ages of Bob's children are all distinct primes less than 20. If the set of distinct remainders when their ages are divided by 5 is the same as the set of distinct remainders when their ages are divided by 6, what is the greatest possible sum of the children's ages?

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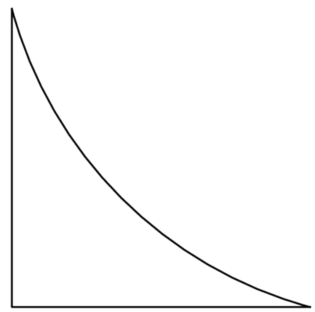
17. Two players, Alice and Bob, play a game. Alice is given three random distinct digits from 1 to 9. Bob is given three random distinct digits from 1 to 9 (not necessarily distinct from Alice's digits). Once both players have their digits, they each arrange their digits in ascending order, forming two 3-digit numbers. Alice wins the game if her number is greater than Bob's. Find the probability Alice wins the game.

18. The diagram below is made up of a $60°$ arc and two congruent sides that meet at a right angle. Given that the shape has an area of $3 + 3\sqrt{3} - 2\pi$, find the length of each congruent side.



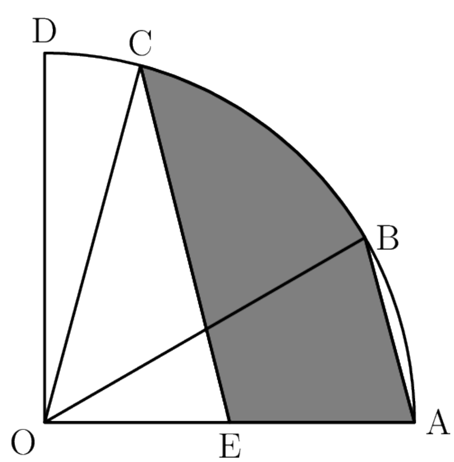
19. Suppose that p, q, r, s, t is an increasing arithmetic sequence such that $p+q+r+s+t = 25$ and $pqrst = 395$. What is the sum of all possible values of pt?

20. Let ABC a triangle with $AB = 5$, $BC = 12$, and $\angle ABC = 90°$. A circle whose center lies on side AC is tangent to sides AB and BC. Find the radius of the circle.

21. How many ways are there to tile a fixed 10×3 rectangle with ten 1×3 rectangular tiles? The rectangular tiles can be rotated.

22. A broken calculator currently displays the number 44. It has two buttons: pressing the first will replace the display number n with $2n - 39$, whereas pressing the second will add 1 to the display number. What is the minimum number of button presses needed to make the calculator display the number 2024?

23. A quarter circle centered at O with radius 2 is shown in the diagram below. If $\angle AOB = 30°$, $\angle BOC = 45°$, and $CE \parallel AB$, find the area of the shaded region.



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24. If $m = 2^{2023}$ and $n = 2^{3202}$, then there exists positive integers x and y such that $mx+ny$ is divisible by 13. Find the minimum value of $x + y$.

25. Let Ω be a circle with diameter AB of length 10. Let C be a point outside Ω, such that segment BC intersects Ω again at D. Suppose that AC and CD have integer lengths and $\angle ABC = 60^\circ$. What is the sum of the possible values of AC?

26. Let A and B be points on a plane that are 1 unit apart. Consider the set of points C on the plane such that $30^\circ < \angle ACB < 60^\circ$. If the area of this set is written as $a\pi + b\sqrt{3}$ for rational numbers a and b, find $a + b$.

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Acceptable Answers

The following rules provide guidelines for acceptable answers in this round. Please note that any specifications provided in a problem will take precedence over these rules. The decisions of MMT coordinators are final.

- Common fractions are defined as a fraction in the form $\pm\frac{a}{b}$ where a and b are natural numbers and $\gcd(a, b) = 1$.
- Ratios and fractional answers should be expressed as common fractions unless otherwise specified.
- Radicals should be simplified. A simplified radical must satisfy:
  - No square factors, fractions, or nested radicals inside a radical
  - No radicals inside the denominator of a fraction
- Answers must be expressed to the exact accuracy called for in the problem (e.g. 25.0 will not be accepted for 25 and 25 will not be accepted for 25.0).
- Do not make approximations for numbers (e.g. 3.14 or $\frac{22}{7}$ for π) unless otherwise specified.
- Units **do not** need to be included but must be correct if included.

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