1. Simplify $17^2 + (13 + 23) \cdot 17 + 13 \cdot 23$.

2. Let x and y be positive integers. If x is 150% of y, and $6y$ is N% of x, what is N?

3. Harry wants to write several positive numbers in a row on a chalkboard. The numbers must follow certain rules:

   - The leftmost number must be 1.
   - The rightmost number must be 24.
   - Every number other than the rightmost must divide the number to its right.
   - No two consecutive numbers in the row are the same.

   What is the maximum possible number of numbers in the row?

4. Joe has a cone-shaped pitcher full of pancake batter with a base of radius 6 inches and a height of 16 inches. Each pancake Joe makes is the exact same size: a cylinder with a radius of 4 inches that is $\frac{1}{2}$ inch thick. Assume that Joe's pancakes don't rise. After making breakfast this morning, Joe notices that the pitcher is now $\frac{1}{3}$ full. How many pancakes did Joe make this morning?

5. For integers $a \leq b$, define $S(a, b)$ to be the sum of the integers from a to b, inclusive. For example, $S(2, 4) = 2 + 3 + 4 = 9$ and $S(1, 1) = 1$.

   Consider pairs i, j of positive integers between 1 and 5 inclusive, with $i \leq j$. Find the sum of $S(i, j)$ over all 15 such pairs.

6. Distinct digits A, B, and C are chosen such that the six-digit number $7A7B7C$ is divisible by 12. What is the largest possible value of C?

7. Four consecutive positive integers have a product of 93024. What is their sum?

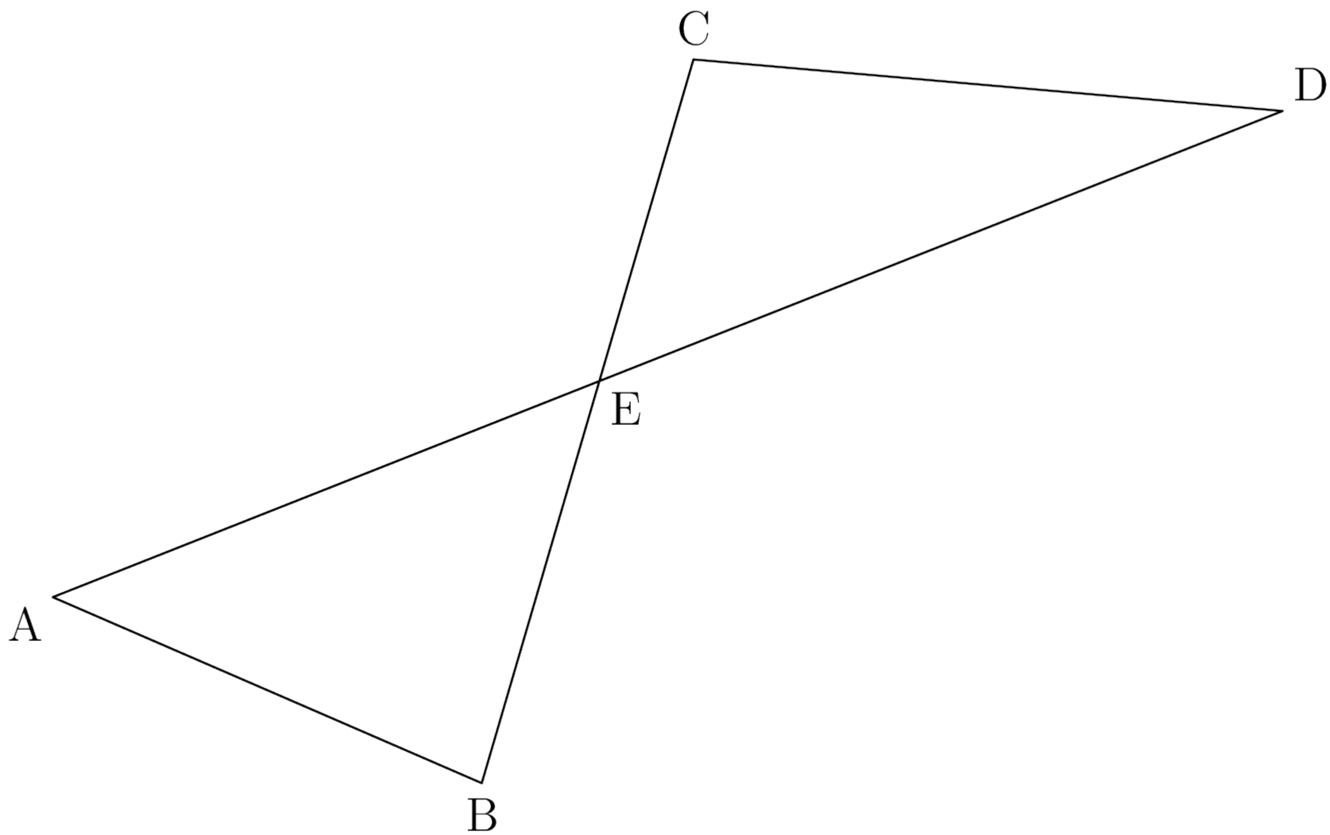
8. The fraction $\frac{25}{28}$ can be written in the form of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$, where a, b, and c are distinct positive integers. What is $a + b + c$?

9. A race of aliens live on a flat world, with a source of light of negligible size (called the NUS) 100 km above the ground. One day, the NUS is too bright, and the aliens hope to build a bit of shade. They have a 150 m $\times$ 200 m board, and wish to cover an area of 75 km^2 on the ground. Furthermore, while they have the powers of levitation, the aliens can only levitate the board parallel to the ground, as it becomes unstable otherwise. At how many kilometers above the ground do the aliens need to place the board in order to shade the desired area?

10. For any real number z, let $\lfloor z \rfloor$ denote the greatest integer that is less than or equal to z. Find the area of the region formed by all points (x, y) on the coordinate plane satisfying $x \geq 0$, $y \geq 0$, and $\left\lfloor \frac{x}{3} \right\rfloor + \left\lfloor \frac{y}{3} \right\rfloor = 9$.

11. In the diagram below, E lies on both $\overline{AD}$ and $\overline{BC}$. Given that $\angle DAC = \angle ADB$ and $[ABE] = 2[CDE] - 23$, find $[CDE]$. (Note that $[ABE]$ and $[CDE]$ refer to the areas

of triangles $\triangle ABE$ and $\triangle CDE$, respectively.)



12. Let $n!$ denote the product of the integers between 1 and n, inclusive. A positive integer divisor d of $41!$ is chosen at random. What is the probability that d is divisible by 8, but is not divisible by 16?

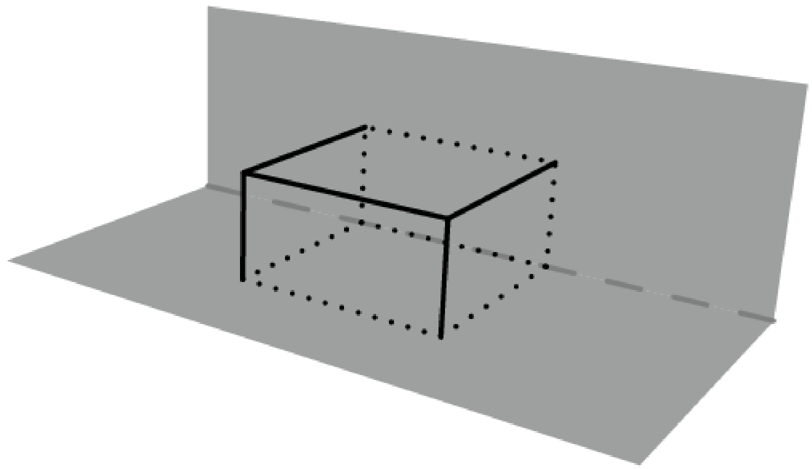
13. Owen picks letters randomly from the alphabet with replacement, each with equal probability, until he picks two vowels (5 of the 26 letters in the alphabet are vowels). He arranges the letters he picks in the order he picked them to create a word. Find the probability that his word is a palindrome with an even amount of letters.

14. Let $\triangle ABC$ be a right triangle with $AB = 3$, $BC = 4$, and $CA = 5$. Let D and E be points on sides BC and AB, respectively, such that AD bisects $\angle A$ and CE bisects $\angle C$. Define F to be the intersection point of AD and CE, and let O be the circumcenter of $\triangle DEF$. Find BO.

15. Find the greatest positive integer n for which
$$2^{40} + 2^{32} + 2^{22} + 2^{21} + 2^{12} + 1$$
is divisible by 5^n.

16. Bob is making the outline of a wire cage in the shape of a rectangular prism. As shown in the image below, one face of the cage will lie on the ground and one face will lie

against the wall. Bob has 9 meters of wiring, which he will use to make the solid black lines below. What is the maximum volume of the cage, in cubic meters, that Bob can achieve?



17. A sequence of integers is defined recursively by $a_0 = 0$ and $a_{n+1} = 17a_n + 1$ for all $n \geq 0$. Find the remainder when a_{200} is divided by 2023.

18. Define the function $f(x) = (4x - 6)^{10}$. If $f(x)$ can be represented as $a_{10}x^{10} + a_9x^9 + \cdots + a_1x + a_0$, let $S = a_9 + a_7 + a_5 + a_3 + a_1$. What is the remainder when $|S|$ is divided by 1000?

19. Two circles $\odot O_1$ and $\odot O_2$ are drawn such that each circle passes through the center of the other circle. $\odot O_1$ and $\odot O_2$ intersect at points A and B. Points P and Q are chosen independently and uniformly at random on $\odot O_1$ and $\odot O_2$, respectively. What is the probability that line segment $\overline{PQ}$ intersects line segment $\overline{AB}$?

20. The equation $x^4 + ax^3 + 123x^2 - 123x + b = 0$ has 4 distinct real solutions. Given that the sum of two of the solutions is 10 and that the product of the other two solutions is 20, find $a + b$.