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Optimization Techniques (MAT-2003)

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Definitions:

1. Feasible Solution: A feasible solution to a transportation problem is a set of non-negative allocations, x_{ij} if that satisfies the rim (row and column) restrictions.

2. Basic Feasible Solution: A feasible solution to a transportation problem is said to be a basic feasible solution if it contains no more than $m + n - 1$ non-negative allocations, where m is the number of rows and n is the number of columns of the transportation problem.

3. Optimal Solution: A feasible solution (not necessarily basic) that minimizes (maximizes) the transportation cost (profit) is called an optimal solution.

4. Non-degenerate Basic Feasible Solution: A basic feasible solution to a $(m \times n)$ transportation problem is said to be non-degenerate if,

(a) the total number of non-negative allocations is exactly $m + n - 1$ (*i.e.*, number of independent constraint equations), and

(b) these $m + n - 1$ allocations are in independent positions.

5. Degenerate Basic Feasible Solution: A basic feasible solution in which the total number of non-negative allocations is less than $m + n - 1$ is called degenerate basic feasible solution.

Optimality Test:

Here, we will verify the obtained feasible solution is optimal or not. Optimality test can only be performed on the feasible solution in which

(a) number of allocations is $m + n - 1$(Non-degenrate), where m is the number of rows and n is the number of columns.

(b) these $(m + n - 1)$ allocations should be in independent positions. i.e., it is impossible to increase or decrease any allocation without either changing the position of the allocations or violating the row and column restrictions.

A simple rule for allocations to be in independent positions is that it is impossible to travel from any allocation(thereby forming a closed loop), back to itself by a series of horizontal and vertical jumps from one occupied cell to another, without a direct reversal of route.

Example:

2	3	11	7	6
1	0	6	1	1
5	8	15	9	10
7	5	3	2	

Now the **procedure for the optimality test** as follows:

Step 1: Determining the net evaluations for non-basic variables (unoccupied cells).

Step 2: Choosing that net evaluation which may improve the current basic feasible solution.

Step 3: Determining the current occupied cell which leaves the basis (i.e., becomes unoccupied) and repeating (1) through (3) until an optimum solution attained.

Optimality test:

We have to find the values of u_i and v_j (Dual variables) such that $u_i + v_j = c_{ij}$ with respect to the c_{ij}'s corresponding to the allocated cells (basic cells).

					u_i
	2 (1)	3 (5)	11	7	0
	1	0	6	1 (1)	-5
	5 (6)	8	15 (3)	9 (1)	3
v_j	2	3	12	6	

Now, we have to calculate the net-evaluations $(c_{ij} - z_{ij})$ for all un-allocated cells (non-basic cells).

Where, $c_{ij} - z_{ij} = c_{ij} - (u_i + v_j)$.

cell $(1,3) \rightarrow c_{13} - z_{13} = c_{13} - (u_1 + v_3) -= 11 - (12 + 0) = -1 \ngeq 0$

$(1,4) \rightarrow c_{14} - z_{14} = \mathrm{c}_{14} - (u_1 + \mathrm{v}_4) = 7 - (6 + 0) = 1 \geq 0$

$(2,1) \rightarrow c_{21} - z_{21} = c_{21} - (u_2 + v_1) = 1 - (2 - 5) = 4 \geq 0$

$(2,2) \rightarrow c_{22} - z_{22} = c_{22} - (u_2 + v_2) = 0 - (-5 + 3) = 2 \geq 0$

$(2,3) \rightarrow c_{23} - z_{23} = c_{23} - (u_2 + v_3) = 6 - (-5 + 12) = -1 \ngeq 0$

$(3,2) \rightarrow c_{32} - z_{32} = c_{32} - (u_3 + v_2) = 8 - (3 + 3) = 2 \geq 0$

For the cells (1,3) and (2,3) have positive net evaluations. Therefore, not satisfying the optimality test. We have to improve the solution to obtain the optimal solution.

Modified Distribution (MODI) Method (or) $u - v$ Method

Example:

Find the optimal solution of the transportation problem. If the initial basic feasible solution using VAM method is given as follows.

4 (20)	6 (20)	8	8	40
6	8	6 (50)	7 (10)	60
5	7 (10)	6	8 (40)	50
20	30	50	50	

Initial basic feasible solution using VAM method is given by

4 (20)	6 (20)	8	8	40
6	8	6 (50)	7 (10)	60
5	7 (10)	6	8 (40)	50
20	30	50	50	

Here, the number of allocations are $= 6 = m + n - 1 = 3 + 4 - 1.$

The transportation problem is a non-degenerated one.

Now, we need to check whether this allocation is a optimal allocation or not. Therefore, we have to verify the optimality test.

					u_i
	4 $\boxed{20}$	6 $\boxed{20}$	8	8	0 → starts with '0' here.
	6	8	6 $\boxed{50}$	7 $\boxed{10}$	0
	5	7 $\boxed{10}$	6	8 $\boxed{40}$	1
v_j	4	6	6	7	

We have find u_i & v_i values with respect to the c_{ij}'s of the allocated cells such that $u_i + v_j = c_{ij}$.

Now we need to calculate the net evaluations for the un-allocated cells (non-basic cells).

Now, we have to calculate the net-evaluations $(c_{ij} - z_{ij})$ for all un-allocated cells (non-basic cells).

Where, $c_{ij} - z_{ij} = c_{ij} - (u_i + v_j)$.

cell $(1,3) \rightarrow c_{13} - z_{13} = c_{13} - (u_1 + v_3) = 8 - (0 + 6) = 2 \geq 0$

$(1,4) \rightarrow c_{14} - z_{14} = \mathrm{c}_{14} - (u_1 + \mathrm{v}_4) = 8 - (7 + 0) = 1 \geq 0$

$(2,1) \rightarrow c_{21} - z_{21} = c_{21} - (u_2 + v_1) = 6 - (4 + 0) = 2 \geq 0$

$(2,2) \rightarrow c_{22} - z_{22} = c_{22} - (u_2 + v_2) = 8 - (6 + 0) = 2 \geq 0$

$(3,1) \rightarrow c_{31} - z_{31} = c_{31} - (u_3 + v_1) = 5 - (4 + 1) = 0 \geq 0$

$(3,3) \rightarrow c_{33} - z_{33} = c_{33} - (u_3 + v_3) = 6 - (6 + 1) = -1 \ngeq 0$

For the cell (3,3), the net evaluation is less than zero ($\ngeq$) which fails to satisfy the optimality test. Therefore, we need to improve the solution.

We have to apply the MODI method to obtain the optimal solution.

Now, we need to make a new allocation $(+\theta)$ at cell which is having negative net evaluation ($(3,3)$). To do that we need to form a closed loop starting with cell $(3,3)$ jumps from vertical and horizontal jumps from one occupied cell to other.

4 20	6 20	8	8
6	8	6 $-\theta$ 50	7 $+\theta$ 10
5	7 10	6 $+\theta$	8 $-\theta$ 40

From the cells $(2,3)$ and $(3,4)$ (from $-\theta$ values in the loop) the minimum allocation is 40. which we have to add at the $+\theta$ values in the loop.

4 $\boxed{20}$	6 $\boxed{20}$	8	8
6	8	6 $\boxed{10}$	7 $\boxed{50}$
5	7 $\boxed{10}$	6 $\boxed{40}$	8

Now, again we need to perform the optimality test to verify these allocation is optimal or not.

We need to calculate the u_i & v_j values.

					u_i
	4 $\boxed{20}$	6 $\boxed{20}$	8	8	0
	6	8	6 $\boxed{10}$	7 $\boxed{50}$	1
	5	7 $\boxed{10}$	6 $\boxed{40}$	8	1
v_j	4	6	5	6	

Now, we have to calculate the net-evaluations $(c_{ij} - z_{ij})$ for all un-allocated cells (non-basic cells).

Where, $c_{ij} - z_{ij} = c_{ij} - (u_i + v_j)$.

cell $(1,3) \rightarrow c_{13} - z_{13} = c_{13} - (u_1 + v_3) = 8 - (5 + 0) = 3 \geq 0$

$(1,4) \rightarrow c_{14} - z_{14} = \mathrm{c}_{14} - (u_1 + \mathrm{v}_4) = 8 - (6 + 0) = 2 \geq 0$

$(2,1) \rightarrow c_{21} - z_{21} = c_{21} - (u_2 + v_1) = 6 - (4 + 1) = 1 \geq 0$

$(2,2) \rightarrow c_{22} - z_{22} = c_{22} - (u_2 + v_2) = 8 - (6 + 1) = 1 \geq 0$

$(3,1) \rightarrow c_{31} - z_{31} = c_{31} - (u_3 + v_1) = 5 - (4 + 1) = 0 \geq 0$

$(3,4) \rightarrow c_{34} - z_{34} = c_{34} - (u_3 + v_4) = 8 - (6 + 1) = 1 \geq 0$

Here, all the net evaluations are ≥ 0. Therefore, we obtained the optimal allocation.

The optimal (minimum cost) is $= 4 \times 20 + 6 \times 20 + 6 \times 10 + 7 \times 50 + 7 \times 10 + 6 \times 40 = 920$.

Example:

Find the optimal solution of the transportation problem using MODI (or) u-v method.

	D_1	D_2	D_3	D_4	**Supply**
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	3	3	2	2	