Hey Dr., Record the session

Optimization Techniques (MAT-2003)

Lecture-28

Dr. Yada Nandukumar
Department of Mathematics
School of Advanced Sciences (SAS)
VIT-AP University

Result: Let (a_{ij}) be the $m \times n$ payoff matrix for a two-person zero sum game. If $\underline{v}$ denotes the maximin value and $\overline{v}$ the minimax value of the game, then $\overline{v} \geq \underline{v}$. That $is,$ $\min\limits_{1 \leq j \leq n} \left[\max\limits_{1 \leq i \leq m} \{a_{ij}\} \right] \geq \max\limits_{1 \leq i \leq m} \left[\min\limits_{1 \leq j \leq n} \{a_{ij}\} \right].$

Example:

Determine the range of value of p and q that will make the payoff element a_{22}, a saddle point for the game whose pay-off matrix (a_{ij}) is given below :

$$\begin{array}{cc} & \text{Player } B \\ \text{Player } A & \begin{bmatrix} 2 & 4 & 5 \\ 10 & 7 & q \\ 4 & p & 8 \end{bmatrix} \end{array}$$

Example:

Determine the range of value of p and q that will make the payoff element a_{22}, a saddle point for the game whose pay-off matrix (a_{ij}) is given below :

$$\begin{array}{cc} & \text{Player } B \\ \text{Player } A & \begin{bmatrix} 2 & 4 & 5 \\ 10 & 7 & q \\ 4 & p & 8 \end{bmatrix} \end{array}$$

				min
	2	4	5	2
	10	7	q	7
	4	p	8	4
max	10	7	8	

minimax = 7

maxmin = 7

The values of p and q which makes the payoff element a_{22}, a saddle point are $p < 7$ and $q > 7$

Games without saddle points- Mixed strategies:

If the game has no saddle point, then the game is said to have mixed strategies.

A game without saddle point can be solved by various selection methods i.e., Dominance property, graphical method, simplex method etc.

We call these games as probabilistic mixed strategy games without saddle point.

Procedure to determine mixed strategies:

Consider a 2×2 payoff matrix with respect to player A which has no saddle point

	1	2
1	a	b
2	c	d

Step 1: Find the absolute value of $a - b$ (i.e., $|a - b|$) and write it against row 2.

Step 2: Find the absolute value of $c - d$ (i.e., $|c - d|$) and write it against row 1.

Step 3: Find the absolute value of $a - c$ (i.e., $|a - c|$) and write it against column 2.

Step 4: Find the absolute value of $b - d$ (i.e., $|b - d|$) and write it against column 1 The results of the above steps are summarized . The absolute values are called a oddments

Step 5: Compute the probabilities of selection of the alternative of player A (p_1 and p_2) and that of Player B (q_1 and q_2).

$$p_1 = \frac{|c-d|}{|a-b|+|c-d|}, \quad p_2 = \frac{|a-b|}{|a-b|+|c-d|}$$
$$q_1 = \frac{|b-d|}{|a-c|+|b-d|}, \quad q_2 = \frac{|a-c|}{|a-c|+|b-d|}$$

Step 6: The value of the game can be computed using the following formula

$$V = \frac{a|c-d|+c|a-b|}{|a-b|+|c-d|}$$

Example:

Consider the following payoff matrix and solve it optimally

$$\begin{array}{cc} & Y \\ X & \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \end{array}$$

Example:

Consider the following payoff matrix and solve it optimally

$$
\begin{array}{cc}
 & Y \\
X & \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}
\end{array}
$$

Solution:

			min
	4	1	1
	2	3	2
max.	4	3	

minimax = 3
maximin = 2

$\neq$.

The game has no saddle point

i.e, The Game has no pure strategy.

$\therefore$ The player have to choose mixed strategies.

Let X_1 and X_2 are the strategies of the player X and let Y_1 and Y_2 are the strategies of the player Y.

Let $p_1, and\ p_2$ are the probabilities of the player X using the strategies X_1 and X_2.

Let $q_1, and\ q_2$ are the probabilities of the player Y using the strategies Y_1 and Y_2.

$$\begin{array}{cc|l} 4_a & 1_b & |2-3| = 1 \\ 2_c & 3_d & |4-1| = 3. \\ \hline |1-3| & |4-2| & \\ = 2 & = 2 & \end{array}$$

$$p_1 = \frac{|c-d|}{|a-b| + |c-d|} = \frac{1}{1+3} = 1/4.$$

$$p_2 = \frac{|a-b|}{|a-b| + |c-d|} = \frac{3}{1+3} = 3/4.$$

$$q_1 = \frac{|b-d|}{|a-c| + |b-d|} = \frac{2}{4} = 1/2.$$

$$q_2 = \frac{|a-c|}{|a-c| + |b-d|} = \frac{2}{2+2} = 1/2.$$

Value of ~~the~~ the game $(V) = \dfrac{a|c-d| + c|a-b|}{|a-b| + |c-d|}$

$$= \frac{4(1) + 2(4-1)}{1+3}$$

$$= \frac{10}{4} = \frac{5}{2}.$$

Example:

Solve the following game and determine the value of the game

	B_1	B_2
A_1	9	7
A_2	5	11