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Optimization Techniques (MAT-2003)

Lecture-21

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Vogel’s approximation method:

Example:

Distribution centres

		1	2	3	4	*Supply*
	1	2	3	11	7	6
Plants	2	1	0	6	1	1
	3	5	8	15	9	10
Requirement		7	5	3	2	

First, we have to calculate the row and column penalties. i.e., for each row and column we have to calculate the difference between the second lowest element to the first lowest elements.

					Supply	Row Penalties
	2	3	11	7	6	1
	1	0	6	1 $\boxed{1}$	~~1~~ 0	~~1~~ 0
	5	8	15	9	10	3
Demand	7	5	3	~~2~~ 1		
Column Penalties	1	3	5	6		

Here, the maximum penalty is 6 which is in fourth column. Now in the fourth column, the least cost is 1 (at (2,4) location) where we have to make our first allocation (minimum of 6, 1 is 1) and cross out the second row.

Now, we left with

					Supply	Row Penalties
	2	3 $\boxed{5}$	11	7	~~6~~ 1	1
	5	8	15	9	10	3
Demand	7	~~8~~ 0	3	1		
Column Penalties	2	5	4	2		

Again, calculate the row and column penalties for the leftover matrix. The maximum penalty is 5 corresponding to second column of the matrix. In the second column, least cost is 3 where we have to make our second allocation (minimum of 5, 6 and we have to subtract this minimum value from the other value) and cross out the respective column.

				Supply	Row Penalties
	2 1	11	7	~~1~~0	5
	5	15	9	10	4
Demand	~~7~~6	3	1		
Column Penalties	3	4	2		

Calculate the row and column penalties for the leftover matrix. The maximum penalty is 5 corresponding to first row of the matrix. In the first row, least cost is 2 where we have to make our third allocation (minimum of 1, 7 and we have to subtract this minimum value from the other value) and cross out the respective row.

				Supply	Row Penalties
	5 $\boxed{6}$	15 $\boxed{3}$	9 $\boxed{1}$	~~10~~ ~~4~~ ~~3~~ 0	(4)
Demand	~~6~~ 0	~~3~~ 0	~~1~~ 0		
Column Penalties	0	0	0		

For the leftover matrix, calculate the row and column penalties. Out of which 4 is the maximum penalty corresponds to third row of the original matrix. In that row we have least cost 5 where we have to make our fourth allocation and cancel respective column. Here, we have only one row. Therefore, no need to find further penalties. Directly, we can make our fifth allocation at the least cost i.e., 9 and cross out the respective column.

(here, we left only one row. Therefore directly we can allocate at the minimum cost of the row without go for penalties. Similarly we can do whenever we left with one column.)

Finally, we left with one cell where we can make our last (sixth) allocation.

Allocation matrix is given by:

2 $\boxed{1}$	3 $\boxed{5}$	11	7	6
1	0	6	1 $\boxed{1}$	1
5 $\boxed{6}$	8	15 $\boxed{3}$	9 $\boxed{1}$	10
7	5	3	2	

Total cost $= 2 \times 1 + 3 \times 5 + 1 \times 1 + 5 \times 6 + 15 \times 3 + 9 \times 1 = 102.$

Procedure for Vogel's Approximation Method:

The Vogel's Approximation Method takes into account not only the least cost c_{ij} but also the costs just exceed c_{ij}. The steps of the method are given as follows:

Step 1: For each row of the transportation table identify the smallest and the next –to –smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly, compute the differences for each column.

Step 2: Identify the row or column with the largest difference among all the rows and columns. If tie occurs, use any arbitrary (row or column) for tie breaking choice. Let the greatest difference correspond to i^{th} row and let c_{ij} be the smallest cost in the i^{th} row. Allocate the maximum feasible amount $x_{ij} = min.(a_i, b_j)$ in the $(i, j)^{th}$ cell and cross off either the i^{th} row or the j^{th} column in the usual manner.

Step 3: Recompute the column and row differences for the reduced transportation table and go to step 2. Repeat the procedure until all the rim requirements are satisfied.

Note: Among all the methods discussed for finding the initial basic feasible solution Vogel's approximation method is the best one and gives the most approximated solution near to the optimal solution.

Example:

Find the initial basic feasible solution to the following transportation problem. Given the cost matrix:

$$\begin{array}{ccc}
 & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} & \text{Supply} \\
\begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} & \begin{bmatrix} 20 & 25 & 28 & 31 \\ 32 & 28 & 32 & 41 \\ 18 & 35 & 24 & 32 \end{bmatrix} & \begin{array}{c} 200 \\ 180 \\ 110 \end{array} \\
\text{Demand:} & \begin{array}{cccc} 150 & 40 & 180 & 170 \end{array} &
\end{array}$$