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Optimization Techniques (MAT-2003)

Lecture-26

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Example:

1131. *XYZ* Company has 4 sales managers to be assigned to 5 different sales territories. The weekly wages estimated for each sales manager in different territories in thousands of rupees are shown in the following table :

Sales managers	*Sales territories*				
	I	*II*	*III*	*IV*	*V*
A	75	80	85	70	90
B	91	71	82	75	85
C	78	90	85	80	80
D	65	75	88	85	90

Suggest optimal assignment for the sales manager. Which sales territory will remain unassigned.

Since we are having 5 sales territories and 4 sales managers. We make the transportation problem a balanced one, we are creating a dummy sales manager **E** with all the wages as zeros.

Now you can proceed with the Hungarian method for assignments.

	I	II	III	IV	V
A	75	80	85	70	90
B	91	71	82	75	85
C	78	90	85	80	80
D	65	75	88	85	90
E	0	0	0	0	0

	I	II	III	IV	V
A	5	10	15	$\boxed{0}$	20
B	20	$\boxed{0}$	11	9	14
C	$\boxed{0}$	12	7	2	2
D	~~0~~	10	23	20	25
E	~~0~~	~~0~~	$\boxed{0}$	~~0~~	~~0~~

	I	II	III	IV	V
A	7	10	15	$\boxed{0}$	20
B	22	$\boxed{0}$	11	4	14
C	$\cancel{0}$	10	5	$\cancel{0}$	$\boxed{0}$
D	$\boxed{0}$	8	21	18	23
E	2	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

Now we have 5 assignments which is optimal assignment.

The optimal assignments are:

$$A \to IV, B \to II, C \to V, D \to I, E \to III$$

Min wages =Min Z

=70+71+80+65+0=286 (Thousands)

Maximization in Assignment Problem:

In some cases the pay off elements of the assignment problem may represent revenues or profits instead of costs so that the objective will be maximize the total revenue or profit.

In that case before applying the Hungarian method, first we to convert the maximization problem to minimization problem by subtracting each and every element from the maximum element of the table. After converting into minimization problem now we can apply Hungarian method as usual. Once the optimal assignment is attained, we restore the original values to those cells in which the assignments have been made.

Example:

A student has to select one and only one elective in each semester and the same elective should not be selected in different semesters. Due to various reasons, the expected grades in each subject, if selected in different semesters, vary and they are given below:

Semester	Analysis	Statistics	Graph Theory	Algebra
I	F	E	D	C
II	E	E	C	C
III	C	D	C	A
IV	B	A	H	H

The grade points are: $H = 10$, $A = 9$, $B = 8$, $C = 7$, $D = 6$, $E = 5$ and $F = 4$. How will the student select the electives in order to maximize the total expected points and what will be his maximum expected total points?

$$\begin{bmatrix} 4 & 5 & 6 & 7 \\ 5 & 5 & 7 & 7 \\ 7 & 6 & 7 & 9 \\ 8 & 9 & 10 & 10 \end{bmatrix}$$

Given the maximization problem. We need to convert the maximization problem to the minimization problem as follows.

First we need to pick the maximum value from the entire matrix (which is 10 in our case). Now, we need to subtract every element of the matrix from this maximum value.

The resultant cost matrix becomes:

$$\begin{bmatrix} 6 & 5 & 4 & 3 \\ 5 & 5 & 3 & 3 \\ 3 & 4 & 3 & 1 \\ 2 & 1 & 0 & 0 \end{bmatrix}$$

This is the minimization problem. We can go with the Hungarian method now.

Subtracting respective minimum element of every row from the every element of the respective rows.

$$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 2 & 2 & 0 & 0 \\ 2 & 3 & 2 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix}$$

Subtracting respective minimum element of every column from the every element of the respective columns. We have

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Now, let us make the assignments. Follow the basic rules for making assignments. We have

$$\begin{bmatrix} 2 & 1 & 1 & \boxed{0} \\ \cancel{0} & 1 & \boxed{0} & \cancel{0} \\ \boxed{0} & 2 & 2 & \cancel{0} \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{bmatrix}$$

We have four assignments which is equals to the order of the matrix. We obtained the optimal assignments.

Optimal Assignments are $\rightarrow$ I – Algebra ;
II $\rightarrow$ Graph Theory, III $\rightarrow$ Analysis
IV $\rightarrow$ statistics.

Maximum expected total points are given by. (Initial table)

$$= 7+7+7+9$$

$$= 21+9 = \underline{\underline{30}}$$

In the modification of a plant layout of a factory, four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost of placing of machine i at place j (in hundred rupees) is shown below :

		Location				
		A	B	C	D	E
Machine	M_1	9	11	15	10	11
	M_2	12	9	—	10	9
	M_3	—	11	14	11	7
	M_4	14	8	12	7	8

Find the optimal assignment schedule.

Since it was mentioned in the problem that the machines M2 and M3 are not placed at locations C and A respectively. Therefore, while solving the problem, we need to consider very large cost (say M>>0) at those two places. Therefore the matrix as follows:

	A	B	C	D	E
M1	9	11	15	10	11
M2	12	9	M	10	9
✓ M3	M	11	14	11	7
M4	14	8	12	7	8
Dummy (M5)	0	0	0	0	0

Now Continue with the Hungarian method. We have the assignments as follows.

$$
\begin{matrix}
\boxed{0} & 2 & 6 & 1 & 2 \\
3 & \boxed{0} & M & 1 & \cancel{0} \\
M & 4 & 7 & 4 & \boxed{0} \\
7 & 1 & 5 & \boxed{0} & 1 \\
\cancel{0} & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0}
\end{matrix}
$$

The number of assignments are equal to the order of the cost matrix (5). Therefore, we obtained the optimal assignments.

Optimal assignments for installing the five machines five locations are:

$M1 \rightarrow A, M2 \rightarrow B, M3 \rightarrow E, M4 \rightarrow D, and\ M5 \rightarrow C.$

Minimum cost = Min Z

$= 9 + 9 + 7 + 7 + 0 = 32$ (hundreds RS.)

$= 3200\ (\text{RS.})$