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Optimization Techniques (MAT-2003)

Lecture-20

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Integer programming

The general integer programming problem is given by

Maximize $z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

subject to $a_{11}\mathrm{x}_1 + a_{12}\mathrm{x}_2 + \cdots + a_{1\mathrm{n}}\mathrm{x}_\mathrm{n} \leq b_1$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$

$$\ldots \ldots \ldots \ldots \ldots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ \ldots + a_{mn}x_n \leq b_m$$

x_i's are non-negative integers

Methods of integer programing problem:

1. Gomory’s Cutting plane method

2. Branch and Bound method (Search Method)

Gomory's Cutting plane method (or) Cutting plane method

Example:

Maximize $\quad Z = x_1 + x_2$

Subject to $\quad 3x_1 + 2x_2 \leq 5$

$$x_2 \leq 2$$

$x_1, x_2 \geq 0$ and are integers.

Solution: First, we have to solve the following LPP by relaxing the integral restrictions either by graphical method or by simplex method.

Standard form of the above LPP is given by

Maximum $Z = x_1 + x_2 + 0 \cdot S_1 + 0 \cdot S_2$

Subjected to $3x_1 + 2x_2 + S_1 = 5$

$x_2 + S_2 = 2$

$x_1, x_2, S_1, S_2 \geq 0.$

	c_j	1	1	0	0		
C_B	Basic Var.	Body of the problem x_1	x_2	s_1	s_2	b	θ
0	s_1	3	2	1	0	5	5/3
0	s_2	0	1	0	1	2	—
	Z_j	0	0	0	0		
	$C_j - Z_J$	1	1	0	0	\$0	

Both variables (x_1 and x_2) are having same most positive value (1), therefore chose x_1 randomly as an outgoing variable.

	c_j	1	1	0	0		
C_B	Basic Var.	Body of the problem				b	θ
		x_1	x_2	s_1	s_2		
1	x_1	1	$2/3$	$1/3$	0	$\frac{5}{3}$	$5/2$
0	s_2	0	(1)	0	1	2	2
	Z_j	1	$2/3$	$1/3$	0	$5/3$	
	$C_j - Z_J$	0	$+1/3$	$-1/3$	0	$\nleq 0$	

	c_j	1	1	0	0	
C_B	Basic Var.	Body of the problem x_1	x_2	s_1	s_2	b
1	x_1	1	0	$1/3$	$-2/3$	$1/3$
1	x_2	0	1	0	1	2
	Z_j	1	1	$1/3$	$1/3$	$7/2$
	$C_j - Z_J$	0	0	$-1/3$	$-1/3$	≤ 0

Optimal Solution obtained. Optimal solution is

$$x_1 = \frac{1}{3}, x_2 = 2, maxz = \frac{7}{2}$$

In the above solution, the values of the basic variables are respectively $x_1 = \frac{1}{3}$ and $x_2 = 2$.

The basic variable x_1 has non-integer solution.

To get the integer solution let us apply Gomory's cutting plane method.

Therefore, the row corresponding to the x_1 will be considered as source row to construct the Gomory's constraint.

Let us consider the x_1 row (first row of the optimal simplex table)

$$x_1 + \frac{1}{3}s_1 - \frac{2}{3}s_2 = \frac{1}{3}$$

Now, separating the integral and fractional parts in each term.

$$(1+0)x_1 + \left(0 + \frac{1}{3}\right)x_2 + \left(-1 + \frac{1}{3}\right)s_2 = \left(0 + \frac{1}{3}\right)$$

Extract the fractional parts from each term we have

$$\frac{1}{3}x_2 + \frac{1}{3}s_2 = \frac{1}{3}$$

The Gomory's constraint can be constructed as

$$G_1 = \sum_{j=1}^{n} f_{ij}y_i - f_i$$

$$G_1 = \frac{1}{3}x_3 + \frac{1}{3}s_2 - \frac{1}{3}$$

$$\Rightarrow -\frac{1}{3}x_3 - \frac{1}{3}s_2 + G = -\frac{1}{3}$$

Add this extra constraint in the final simplex table go for the next iteration.

	c_j	1	1	0	0	0	
C_B	Basic Var.	Body of the problem					b
		x_1	x_2	s_1	s_2	G_1	
1	x_1	1	0	$1/3$	$-2/3$	0	$1/3$
1	x_2	0	1	0	1	0	2
0	G_1	0	0	$-1/3$	$-1/3$	1	$-1/3$
	Z_j	1	1	$1/3$	$-2/3$	0	
	$C_j - Z_J$	0	0	$-1/3$	$+2/3$	0	$\nleq 0$

Since, one of the basis variable having the negative value and $z_j - c_j \geq 0$. We have to apply Dual simplex method.

	c_j	1	1	0	0	0	
C_B	Basic Var.	Body of the problem x_1	x_2	s_1	s_2	σ_1	b
1	x_1	1	0	1/3	-2/3	0	1/3
1	x_2	0	1	0	1	0	2
0	σ_1	0	0	(-1/3)	-1/3	1	-1/3
	Z_j	1	1	1/3	-2/3	0	
	$C_j - Z_J$	0	0	-1/3	2/3	0	≤ 0
	$\frac{C_j - Z_j}{a_{3j}(< 0)}$	—	—	1	1	—	

The in coming variable can be the one with $\boldsymbol{max\{1, 1\} = 1}$. We can chose randomly any of the variable as an in coming variable.

	c_j	1	1	0	0	0	
C_B	Basic Var.	Body of the problem x_1	x_2	s_1	s_2	G_1	b
1	x_1	1	0	0	-1	1	0
1	x_2	0	1	0	1	0	2
0	s_1	0	0	1	1	-3	1
	Z_j	1	1	0	0	1	2
	$C_j - Z_J$	0	0	0	0	1	≤ 0

optimal integer solution obtained.

$\therefore$ The integer solution is $x_1 = 0$, $x_2 = 2$.

max $z = \underline{2}$.

Note:

1) In the optimal non-integer solution, if more than one basic variable are non-integers then we have to chose the variable with highest fractional part and its respective row (constraint) to construct the Gomory's constraint.

2) If more than one variable having the same large fractional value then we can chose any of them.

Note:

If we have more than one fractional part in the basis variables, then we have to choose the maximum fractional part variable and its corresponding row becomes the source row..

If two fractional parts having the same maximum value then we can choose arbitrarily any one of the row as source row.

Example:

Find an optimum integer solution to the following LPP.

$$\text{Maximize} z = x_1 + 2x_2$$

$$\text{Subject to: } 2x_2 \leq 7$$

$$x_1 + x_2 \leq 7$$

$$2x_1 \leq 11$$

$$x_1, x_2 \geq 0 \text{ and } x_1, x_2 \text{ are integers}$$

	$\boldsymbol{C_j}$		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$Z_j - C_J$		

	c_j		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$Z_j - C_J$		

	$\boldsymbol{c_j}$		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$Z_j - C_J$		

	C_j		
C_B	Basic Var.	Body of the problem	b
	Z_j		
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Example:

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$$\text{Maximize} z = x_1 + 2x_2$$

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$$x_1 + x_2 \leq 7$$

$$2x_1 \leq 11$$

$$x_1, x_2 \geq 0 \quad \text{and} \quad x_1, x_2 \quad \text{are integers}$$

Optimal Solution is given by

Max $Z = x_1 + 2x_2 = 1(4) + 2(3) = 10, \; x_1 = 4, \quad x_2 = 3.$