Optimization Techniques (MAT-2003)

Dr. Yada Nandukumar
Department of Mathematics
School of Advanced Sciences (SAS)
VIT-AP University

Duality in Linear Programming

For every Linear programming problem there is a related unique L. P. problem involving the same data which also describes the original problem. The given original problem is called **primal problem** and while the associated problem is called **dual problem**.

Importance of Duality:

- It simplifies the L. P. Problem
- Useful for management to compare alternate course of action and their relative values.
- Calculation of Dual checks the accuracy of the primal solution.
- It has applications in economics and Physics and other fields. In economics it is used in the formation of input and out put systems. In physics it is used in the series circuit and parallel circuit theory.
- Duality is used to solve the L. P. problems in which initial condition is infeasible.

If the general L. P. problem is in Canonical form

$$\begin{aligned}
\text{Max } Z = \; & c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n, \\
& a_{11}\,x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}\,x_n \leq b_1, \\
& a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n \leq b_2, \\
& \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
& \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
& a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n \leq b_m, \\
& x_1, x_2, x_3, \dots, x_n, \text{ all} \geq 0.
\end{aligned}$$

If the general L. P. problem is in Canonical form

$$
\begin{aligned}
\text{Max } Z = \; & c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n, \\
& a_{11}\,x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}\,x_n \le b_1, \\
& a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n \le b_2, \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
& a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n \le b_m, \\
& x_1, x_2, x_3, \ldots, x_n, \text{ all} \ge 0.
\end{aligned}
$$

If the general L. P. problem is in Canonical form

$$
\begin{aligned}
\text{Max } Z = \ & c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n, \\
& a_{11}\, x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}\, x_n \le b_1, \\
& a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n \le b_2, \\
& \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
& \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
& a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n \le b_m, \\
& x_1, x_2, x_3, \dots, x_n, \text{ all} \ge 0.
\end{aligned}
$$

The associated **Dual** problem can be written as

$$
\begin{aligned}
\text{Min } W = \ & b_1y_1 + b_2y_2 + b_3y_3 + \dots + b_my_m, \\
& a_{11}y_1 + a_{21}y_2 + a_{31}y_3 + \dots + a_{m1}y_m \ge c_1, \\
& a_{12}y_1 + a_{22}y_2 + a_{32}y_3 + \dots + a_{m2}y_m \ge c_2, \\
& \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
& \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
& a_{1n}y_1 + a_{2n}y_2 + a_{3n}y_3 + \dots + a_{mn}\, y_m \ge c_n, \\
& y_1, y_2, y_3, \dots, y_m, \text{ all} \ge 0.
\end{aligned}
$$

	Primal	*Dual*
(*i*)	Maximization problem	Minimization problem
(*ii*)	n variables and m constraints	m variables and n constraints
(*iii*)	$\leq$ type constraints	$\geq$ type constraints
(*iv*)	Objective function coefficients	R.H.S. constants of the constraints
(*v*)	R.H.S. constants of the constraints	Objective function coefficients
(*vi*)	= type ith constraint	unrestricted ith variable
(*vii*)	Variable x_j unrestricted	jth constraint = type

Example

Find the Dual to the Primal problem given by

Maximize $Z = 3x_1 + 5x_2,$

Subjected to

$$2x_1 + 6x_2 \le 50,$$
$$3x_1 + 2x_2 \le 35,$$
$$5x_1 - 3x_2 \le 10,$$
$$x_2 \le 20,$$
$$x_1 \ge 0,\ x_2 \ge 0.$$

The given problem is Primal Problem

Example

Find the Dual to the Primal problem given by

Maximize $Z = 3x_1 + 5x_2,$

Subjected to $2x_1 + 6x_2 \leq 50,$

$3x_1 + 2x_2 \leq 35,$

$5x_1 - 3x_2 \leq 10,$

$x_2 \leq 20,$

$x_1 \geq 0,\ x_2 \geq 0.$

The given problem is Primal Problem

Dual Problem:

Min $Z = 50y_1 + 35y_2 + 10y_3 + 20y_4$

Sub to $2y_1 + 3y_2 + 5y_3 \geq 3$

$6y_1 + 2y_2 - 3y_3 + y_4 \geq 5$

$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0,\ y_4 \geq 0$

Example

Construct the dual of the problem

$$
\begin{aligned}
&\textit{minimize} && Z = 3x_1 - 2x_2 + 4x_3,\\
&\textit{subject to the constraints} && 3x_1 + 5x_2 + 4x_3 \geq 7,\\
& && 6x_1 + x_2 + 3x_3 \geq 4,\\
& && 7x_1 - 2x_2 - x_3 \leq 10,\\
& && x_1 - 2x_2 + 5x_3 \geq 3,\\
& && 4x_1 + 7x_2 - 2x_3 \geq 2,\\
& && x_1, x_2, x_3 \geq 0.
\end{aligned}
$$

Example

Construct the dual of the problem

$$\text{minimize} \quad Z = 3x_1 - 2x_2 + 4x_3,$$

subject to the constraints

$$\begin{aligned}
3x_1 + 5x_2 + 4x_3 &\geq 7,\\
6x_1 + x_2 + 3x_3 &\geq 4,\\
7x_1 - 2x_2 - x_3 &\leq 10,\\
x_1 - 2x_2 + 5x_3 &\geq 3,\\
4x_1 + 7x_2 - 2x_3 &\geq 2,\\
x_1, x_2, x_3 &\geq 0.
\end{aligned}$$

Convert the third constraint to $\geq$ type.

$$-7x_1 + 2x_2 + x_3 \geq -10$$

Then the problem becomes Primal.

Example

Construct the dual of the problem

$$\text{minimize} \quad Z = 3x_1 - 2x_2 + 4x_3,$$

$$\text{subject to the constraints} \quad 3x_1 + 5x_2 + 4x_3 \geq 7,$$
$$6x_1 + x_2 + 3x_3 \geq 4,$$
$$7x_1 - 2x_2 - x_3 \leq 10,$$
$$x_1 - 2x_2 + 5x_3 \geq 3,$$
$$4x_1 + 7x_2 - 2x_3 \geq 2,$$
$$x_1, x_2, x_3 \geq 0.$$

Convert the third constraint to ≥ type.

$$-7x_1 + 2x_2 + x_3 \geq -10$$

Then the problem becomes Primal.

Dual Problem:

$$\text{Max } Z = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5$$

$$\text{Sub to } 3y_1 + 6y_2 + 7y_3 + y_4 + 4y_5 \leq 3$$
$$5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2$$
$$4y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \leq 4$$

$$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0, y_5 \geq 0$$

Example

Construct the dual of the problem

$$\begin{aligned} &\textit{maximize} && Z = 3x_1 + 17x_2 + 9x_3, \\ &\textit{subject to} && x_1 - x_2 + x_3 \geq 3, \\ & && -3x_1 + 2x_3 \leq 1, \\ & && x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0. \end{aligned}$$

Example

Construct the dual of the problem

$$\begin{aligned}
&\text{maximize} && Z = 3x_1 + 10x_2 + 2x_3,\\
&\text{subject to} && 2x_1 + 3x_2 + 2x_3 \le 7,\\
& && 3x_1 - 2x_2 + 4x_3 = 3,\\
& && x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0.
\end{aligned}$$

Example

Construct the dual of the problem

$$\begin{aligned} \text{maximize} \quad & Z = 3x_1 + 10x_2 + 2x_3, \\ \text{subject to} \quad & 2x_1 + 3x_2 + 2x_3 \leq 7, \\ & 3x_1 - 2x_2 + 4x_3 = 3, \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0. \end{aligned}$$

Primal:

Max $Z = 3x_1 + 10x_2 + 2x_3$,

Subjected to $2x_1 + 3x_2 + 2x_3 \leq 7$,

$3x_1 - 2x_2 + 4x_3 \leq 3$,

$-3x_1 + 2x_2 - 4x_3 \leq -3$,

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

Example

Construct the dual of the problem

$$\begin{aligned} &\text{maximize} && Z = 3x_1 + 10x_2 + 2x_3, \\ &\text{subject to} && 2x_1 + 3x_2 + 2x_3 \leq 7, \\ &&& 3x_1 - 2x_2 + 4x_3 = 3, \\ &&& x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0. \end{aligned}$$

Primal:

$$\begin{aligned} &\text{Max } Z = 3x_1 + 10x_2 + 2x_3, \\ &\text{Subjected to} \quad 2x_1 + 3x_2 + 2x_3 \leq 7, \\ &\qquad 3x_1 - 2x_2 + 4x_3 \leq 3, \\ &\qquad -3x_1 + 2x_2 - 4x_3 \leq -3, \\ &\qquad x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0. \end{aligned}$$

Dual Problem:

$$\text{Min } Z = 7y_1 + 3y_2$$

$$\begin{aligned} &\text{Subjected to} \quad 2y_1 + 3y_2 \geq 3 \\ &\qquad 3y_1 - 2y_2 \geq 10 \\ &\qquad 2y_1 + 4y_2 \geq 2 \end{aligned}$$

$y_1 \geq 0$, y_2 is unrestricted.

Example

Construct the dual of the problem

$$\begin{array}{lll} \text{minimize} & Z = & x_2 + 3x_3, \\ \text{subject to} & & 2x_1 + x_2 \leq 3, \\ & & x_1 + 2x_2 + 6x_3 \geq 5, \\ & & -x_1 + x_2 + 2x_3 = 2, \\ & & x_1, x_2, x_3 \geq 0. \end{array}$$

Example

Minimize $Z = 3x_1 - 2x_2 + x_3,$

Subjected to
$$2x_1 - 3x_2 + x_3 \leq 5,$$
$$4x_1 - 2x_2 \geq 9,$$
$$-8x_1 + 4x_2 + 3x_3 = 8,$$
$$x_1, x_2 \geq 0, \; x_3 \text{ is unrestricted.}$$

Example

Minimize $Z = 3x_1 - 2x_2 + x_3,$

Subjected to $2x_1 - 3x_2 + x_3 \leq 5,$

$4x_1 - 2x_2 \geq 9,$

$-8x_1 + 4x_2 + 3x_3 = 8,$

$x_1, x_2 \geq 0,\ x_3$ *is unrestricted.*

Primal:

Minimize $Z = 3x_1 - 2x_2 + x_3,$

Subjected to $-2x_1 + 3x_2 - x_3 \geq -5,$

$$4x_1 - 2x_2 \geq 9,$$

$$-8x_1 + 4x_2 + 3x_3 \geq 8$$

$$8x_1 - 4x_2 - 3x_3 \geq -8$$

$x_1, x_2 \geq 0, x_3$ is unrestricted

Example

Minimize $Z = 3x_1 - 2x_2 + x_3,$

Subjected to $2x_1 - 3x_2 + x_3 \le 5,$

$4x_1 - 2x_2 \ge 9,$

$-8x_1 + 4x_2 + 3x_3 = 8,$

$x_1, x_2 \ge 0,\ x_3$ *is unrestricted.*

Primal:

Minimize $Z = 3x_1 - 2x_2 + x_3,$

Subjected to

$$-2x_1 + 3x_2 - x_3 \ge -5,$$
$$4x_1 - 2x_2 \ge 9,$$
$$-8x_1 + 4x_2 + 3x_3 \ge 8$$
$$8x_1 - 4x_2 - 3x_3 \ge -8$$
$$x_1, x_2 \ge 0, x_3 \text{ is unrestricted}$$

Dual:

Maximize $Z = -5y_1 + 9y_2 + 8y_3$

Subjected to

$$-2y_1 + 4y_2 - 8y_3 \le 3$$
$$3y_1 - 2y_2 + 4y_3 \le -2$$
$$-y_1 + 3y_3 = 1$$
$$y_1, y_2 \ge 0, y_3\ is\ unrestricted.$$

Properties:

1) The Dual of the Dual is Primal.
2) If the primal problem involves n variables and m constraints, the dual involves n constraints and m variables.
3) The value of the objective function Z for any feasible solution of the primal is $\leq$ the value of the objective function W for any feasible solution of the dual.
4) If primal has finite optimal solution, then the dual has finite optimal solution (vice-versa).
5) An optimal solution to the dual exists only when the primal has an optimal solution (vice-versa).

7. The solution of the dual problem can be obtained from the solution of the primal problem (vice-versa).

8. If one of primal/dual has unbounded solution then the other dual/primal do not have feasible solution.

Example

A feed mixing operation can be described in terms of the two activities. The required mixture must contain four kinds of ingredients w, x, y and z. Two basic feeds A and B, which contain the required ingredients are available in the market. One kg. of A contains 0.1 kg. of w, 0. 1 kg. of y and 0.2 kg. of z. Likewise, one kg. of feed B contains 0.1 kg. of x, 0.2 kg. of y and 0.1 kg. of z. The daily per head requirement is of at least 0.4 kg. of w, 0.6 kg. of x, 2 kg. of y and 1.8 kg. of z. Feed A can be bought for £ 0.07 per kg. and B can be bought for £ 0.05 per kg. The availabilities, requirements and costs are summarized in the table below.

Ingredient	Feed A (kg.)	Feed B (kg.)	Requirement (kg.)
w	0.1	0.0	0.4
x	0.0	0.1	0.6
y	0.1	0.2	2.0
z	0.2	0.1	1.8
Cost	£0.07	£0.05	

Determine the quantities of feeds A and B in the mixture so that the total cost is minimum.

	C_j			
C_B	Basic Var.	Body of the problem	b	θ
	Z_j			
	$Z_j - C_J$			

Dual Case

	C_j			
C_B	Basic Var.	Body of the problem	b	θ
	Z_j			
	$Z_j - C_J$			

	c_j			
C_B	Basic Var.	Body of the problem	b	θ
	Z_j			
	$Z_j - C_J$			

	$\boldsymbol{C_j}$			
C_B	Basic Var.	Body of the problem	b	θ
	Z_j			
	$Z_j - C_J$			

Example

A Patient in a hospital is required to have at least 84 units of drug A and 120 units of drug B each day. Each gram of substance M contains 10 units of drug A and 8 units of drug B, and each gram of substance N contains 2 units of drug A and 4 units of drug B. Now suppose that both M and N contain an undesirable drug C, 3 units per gram in M and 1 unit per gram in N. How many grams of substances M and N should be mixed to meet the minimum daily requirements at the same time minimize the intake of drug C? How many units of the undesirable drug C will be in this mixture by using Duality?

	C_j			
C_B	Basic Var.	Body of the problem	b	θ
	Z_j			
	$Z_j - C_J$			

	$\boldsymbol{C_j}$			
C_B	Basic Var.	Body of the problem	b	θ
	Z_j			
	$Z_j - C_J$			

	C_j			
C_B	Basic Var.	Body of the problem	b	θ
	Z_j			
	$Z_j - C_J$			

Example

Dual Simplex Method

Solve by dual simplex method the following problem :

$$\begin{aligned}
&\text{Minimize} && Z = 2x_1 + 2x_2 + 4x_3,\\
&\text{subject to} && 2x_1 + 3x_2 + 5x_3 \geq 2,\\
& && 3x_1 + x_2 + 7x_3 \leq 3\\
& && x_1 + 4x_2 + 6x_3 \leq 5,\\
& && x_1, x_2, x_3 \geq 0.
\end{aligned}$$

Solution:

Convert the object function to the maximization type.

Maximize $(-z) = -$ minimize $z = -2x_1 - 2x_2 - 4x_3$. Let us consider Maximize $(-z)$=Maximize G. Also, except the second constraint rest of the problem is in canonical form. Therefore, convert the second constraint to $\leq$ type by multiplying $-$ on both sides. The second constraint becomes

$$-2x_1 - 3x_2 - 5x_3 \leq -2.$$

Now, convert the problem into standard form, we have

$$\begin{aligned}
\text{maximize} \quad & G = -2x_1 - 2x_2 - 4x_3 + 0s_1 + 0s_2 + 0s_3, \\
\text{subject to} \quad & -2x_1 - 3x_2 - 5x_3 + s_1 = -2, \\
& 3x_1 + x_2 + 7x_3 + s_2 = 3, \\
& x_1 + 4x_2 + 6x_3 + s_3 = 5, \\
& x_1, x_2, x_3, s_1, s_2, s_3, \text{ all} \geq 0.
\end{aligned}$$

$m + n = 6$ and m $= 3$.

$6 - 3 = 3$ variables we have take zeros.

Putting $x_1 = x_2 = x_3 = 0$, the initial basic solution is $s_1 = -2$, $s_2 = 3$, $s_3 = 5$. Since s_1 is negative, solution is infeasible. The above information is expressed in table 6.21, called starting dual simplex table.

	c_j	-2	-2	-4	0	0	0	
C_B	Basic Var. (X_B)	x_1	x_2	x_3	s_1	s_2	s_3	b
0	s_1	-2	(-3)	-5	1	0	0	-2
0	s_2	3	1	7	0	1	0	3
0	s_3	1	4	6	0	0	1	5
	z_j	0	0	0	0	0	0	0
	$c_j - z_J$	-2	-2	-4	0	0	0	≤ 0
	$\dfrac{c_j - z_J}{a_{1j}(< 0)}$	1	$\dfrac{2}{3}$	$\dfrac{4}{5}$	—	—	—	

Infeasible and most negative

Optimality satisfies

Minimum ratio

	c_j	-2	-2	-4	0	0	0	
C_B	Basic Var.	x_1	x_2	x_3	s_1	s_2	s_3	b
-2	x_2	$2/3$	1	$5/3$	$-1/3$	0	0	$2/3$
0	s_2	$7/3$	0	$16/3$	$1/3$	1	0	$7/3$
0	s_3	$-5/3$	0	$-2/3$	$4/3$	0	1	$7/3$
	Z_j	$-4/3$	-2	$-10/3$	$2/3$	0	0	$-4/3$
	$C_j - Z_J$	$-2/3$	0	$-2/3$	$-2/3$	0	0	≤ 0

Feasible

Optimality satisfies

We obtained the optimal feasible solution. The optimal solution is

$x_2 = \frac{2}{3}, s_2 = \frac{7}{3}, S_3 = \frac{7}{3}, x_1 = 0, x_3 = 0, s_1 = 0$ and

Maximize G= Maximize (-Z)= $-4/3$ $\Rightarrow$ Minimize $Z = 4/3$

Algorithm for Dual Simplex Method:

This method will be applicable to those LP problems which start with infeasible solution but having optimum solution. The procedure as follows:

Step 1: (a) Convert the objective function of Minimization type to Maximization type.

(b) If the constraints are of $\geq$ type convert them to $\leq$ by multiplying negative sign.

(c) Convert the problem to standard form by adding slack variables.

(d) Now, write the initial basic solution.

Step 2: (a) If the current basic solution is feasible, use simplex method to solve.

(b) If the current basic solution is infeasible, i.e., at least one value of the basic variable is < 0 then go to next step.

Step 3: Check whether the solution is optimal.

(a) If the solution is not optimum, add an artificial constraint in such a way that the condition of the optimality satisfied.

(b) If the solution is optimum, go to next step.

Step 4: Select the basic variable having the most negative value. This basic variable becomes the leaving variable and the row corresponding to it becomes the key row.

Step 5: Obtain the ratios of the net evaluations $(C_j - Z_j)$ to the corresponding coefficients in the key row. Ignore the ratios associated with positive and zero denominators. The entering vector is the one with the smallest absolute value (or maximum ratio) of the ratios. Column corresponding to the entering vector becomes the key column.

Step 6: Reduce the leading element into unity and all other entries of the key column to zero by elementary row operations.

Step 7: Go to Step 2 and repeat the procedure until an optimum basic feasible solution is attained.

Example

Use dual simplex method to

$$\begin{aligned}
&\textit{maximize} && Z = -3x_1 - 2x_2,\\
&\textit{subject to} && x_1 + x_2 \geq 1,\\
& && x_1 + x_2 \leq 7,\\
& && x_1 + 2x_2 \geq 10,\\
& && x_2 \leq 3,\\
& && x_1, x_2, \geq 0.
\end{aligned}$$

Example

Use dual simplex method to

$$
\begin{aligned}
&\textit{maximize} && Z = -3x_1 - 2x_2, \\
&\textit{subject to} && x_1 + x_2 \geq 1, \\
& && x_1 + x_2 \leq 7, \\
& && x_1 + 2x_2 \geq 10, \\
& && x_2 \leq 3, \\
& && x_1, x_2, \geq 0.
\end{aligned}
$$

$$x_1 = 4, x_2 = 3, and\ Z_{max} = -18$$

	$\boldsymbol{c_j}$		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$C_j - Z_J$		

	$\boldsymbol{c_j}$		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$C_j - Z_J$		

	c_j		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$C_j - Z_J$		

Example

Use dual simplex method to solve the L.P.P. :

$$\begin{aligned} &\text{Minimize} && Z = x_1 + x_2, \\ &\text{subject to} && 2x_1 + x_2 \geq 2, \\ &&& -x_1 - x_2 \geq 1, \\ &&& x_1, x_2 \geq 0. \end{aligned}$$

Example

Solve the following L.P.P. by dual simplex method.

Maximize $Z = 2x_3$

Subjected to $-x_1 + 2x_2 - 2x_3 \geq 8,$

$-x_1 + x_2 + x_3 \leq 4,$

$2\mathrm{x}_1 - x_2 + 4x_3 \leq 10;$

$x_1, x_2, x_3 \geq 0.$

	c_j		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$C_j - Z_J$		

	$\boldsymbol{c_j}$		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$C_j - Z_J$		

	c_j		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$C_j - Z_J$		

	c_j		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$C_j - Z_J$		

	$\boldsymbol{c_j}$		
C_B	Basic Var.	Body of the problem	b
	Z_j		
	$C_j - Z_J$		

Example

$Maximize\ z = 2x_1 + x_2 + x_3$

Subjected to $2x_1 + 3x_2 - 5x_3 = 4,$

$$-x_1 + 9x_2 - x_3 \geq 3,$$

$$4x_1 + 6x_2 + 3x_3 \leq 8$$

$$x_1, x_2, x_3 \geq 0$$