Hey Dr., Record the session

Optimization Techniques (MAT-2003)

Lecture-18

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Branch and Bound Method

Example:

Maximize $\quad Z = 2x_1 + 3x_2,$

subject ot $\quad 6x_1 + 5x_2 \leq 25,$

$x_1 + 3x_2 \leq 10,$

x_1, x_2 *non-negative integers.*

Branch and Bound Method

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\end{aligned}$$

First solve the LPP by ignoring the integrality constraints using Graphical method or Simplex method.

Here, we will use Graphical method to solve the LPP.

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Consider the first constraint as equality

$$6x_1 + 5x_2 = 25$$
$$x_1 = 0 \Rightarrow x_2 = 5$$
$$x_2 = 0 \Rightarrow x_1 = \frac{25}{6}$$

$\text{the points are } (0, 5), (\frac{25}{6}, 0)$

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$$x_2 = 0 \Rightarrow x_1 = \frac{25}{6}$$

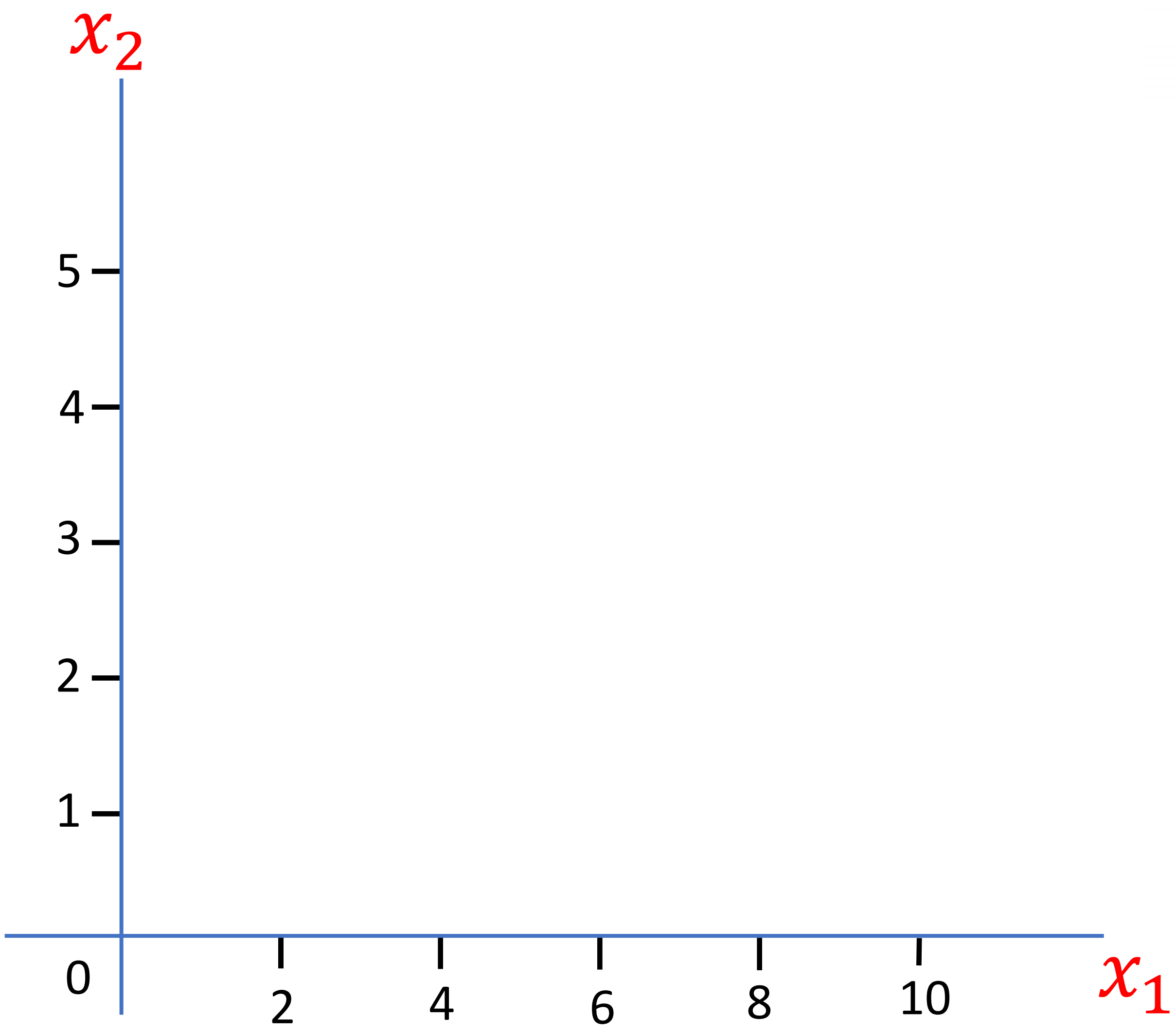
the points are $(0, 5), (\frac{25}{6}, 0)$

Consider the second constraint as equality

$$x_1 + 3x_2 \le 10$$
$$x_1 = 0 \Rightarrow x_2 = 10/3$$
$$x_2 = 0 \Rightarrow x_1 = 10$$

the points are $\left(0, \frac{10}{3}\right), (10,0)$

$$
\begin{aligned}
Z &= 2x_1 + 3x_2, \\
& 6x_1 + 5x_2 \le 25, \\
& x_1 + 3x_2 \le 10, \\
& x_1,\ x_2 \text{ non-negative integers.}
\end{aligned}
$$



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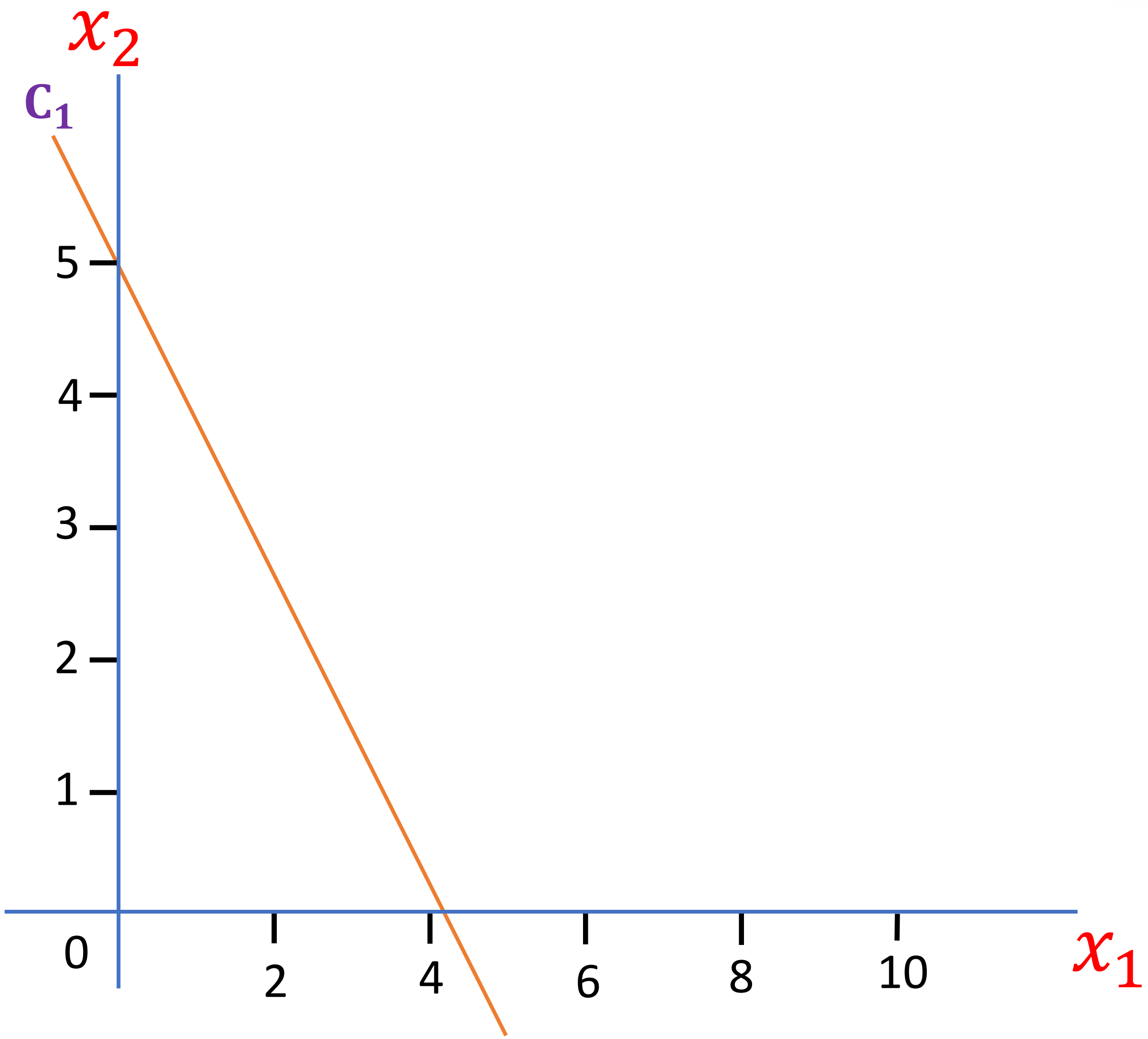

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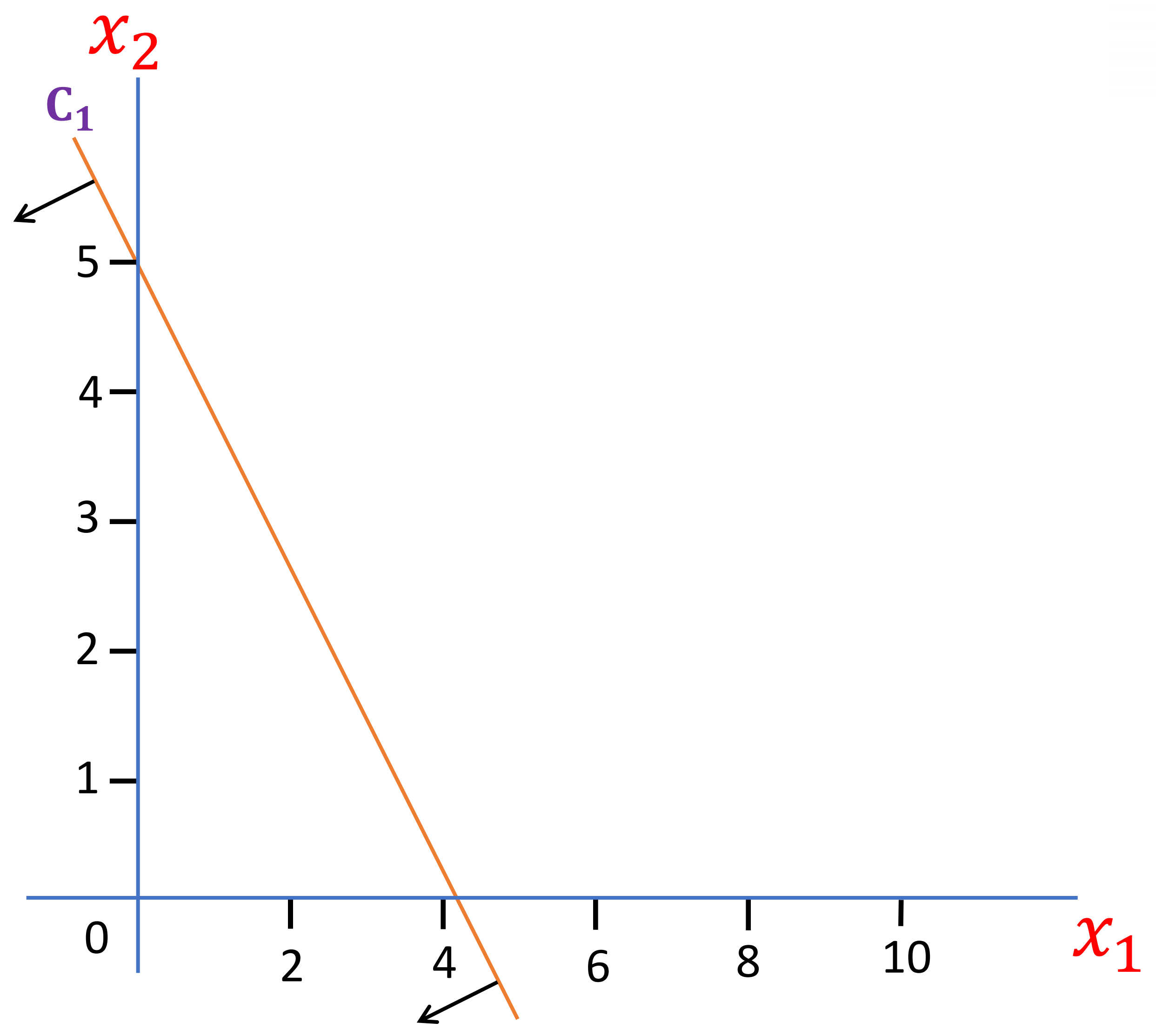

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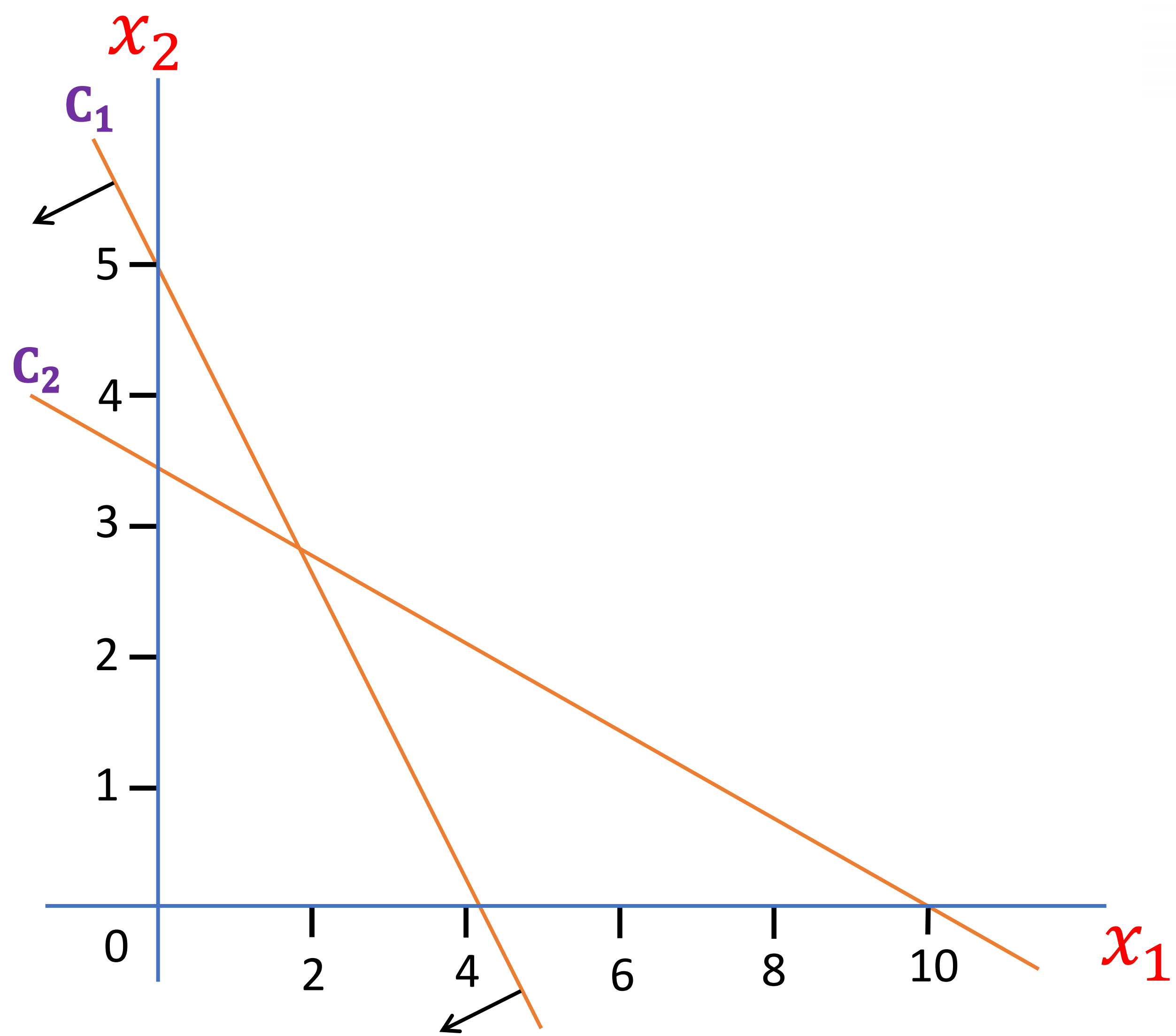

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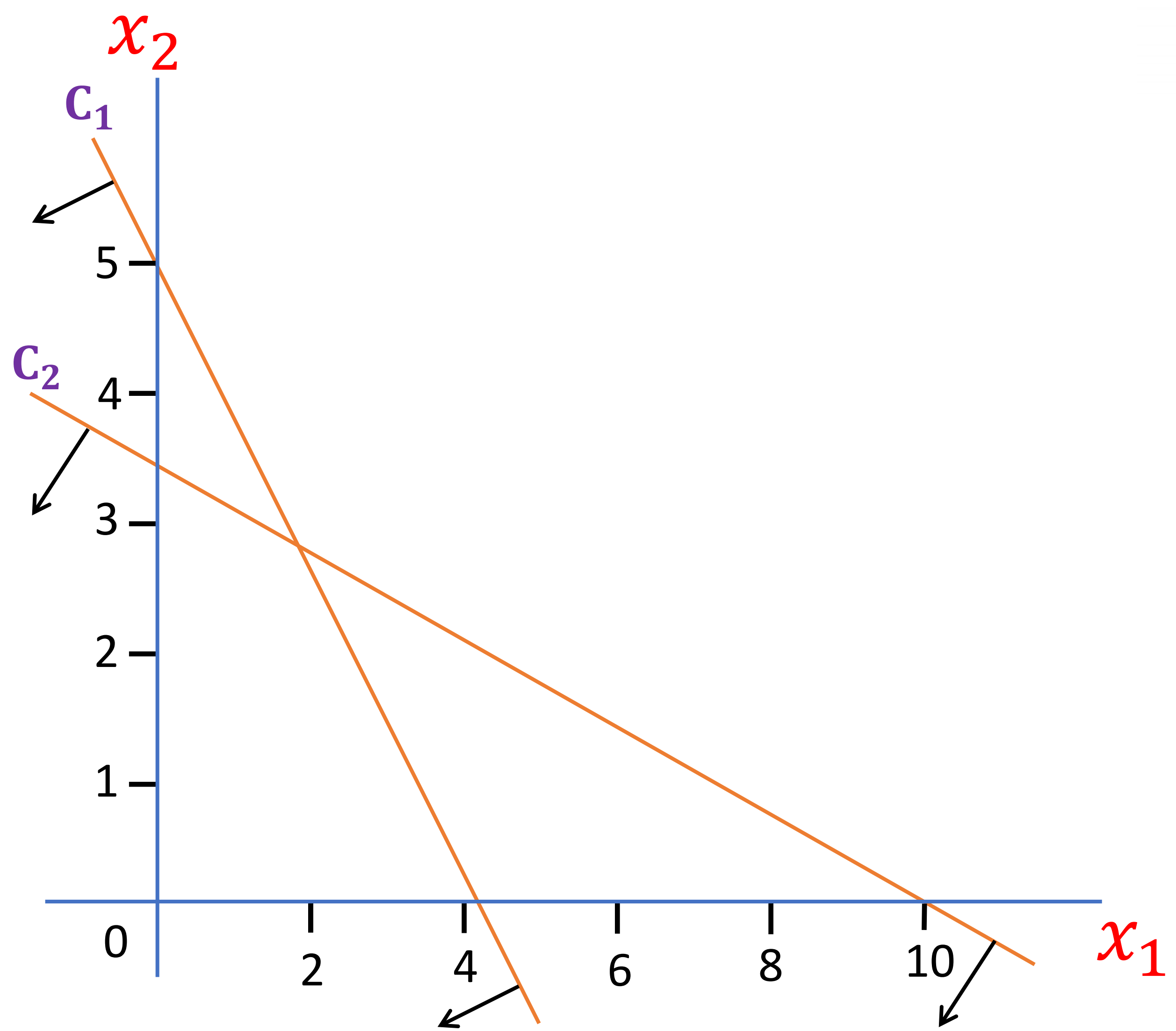

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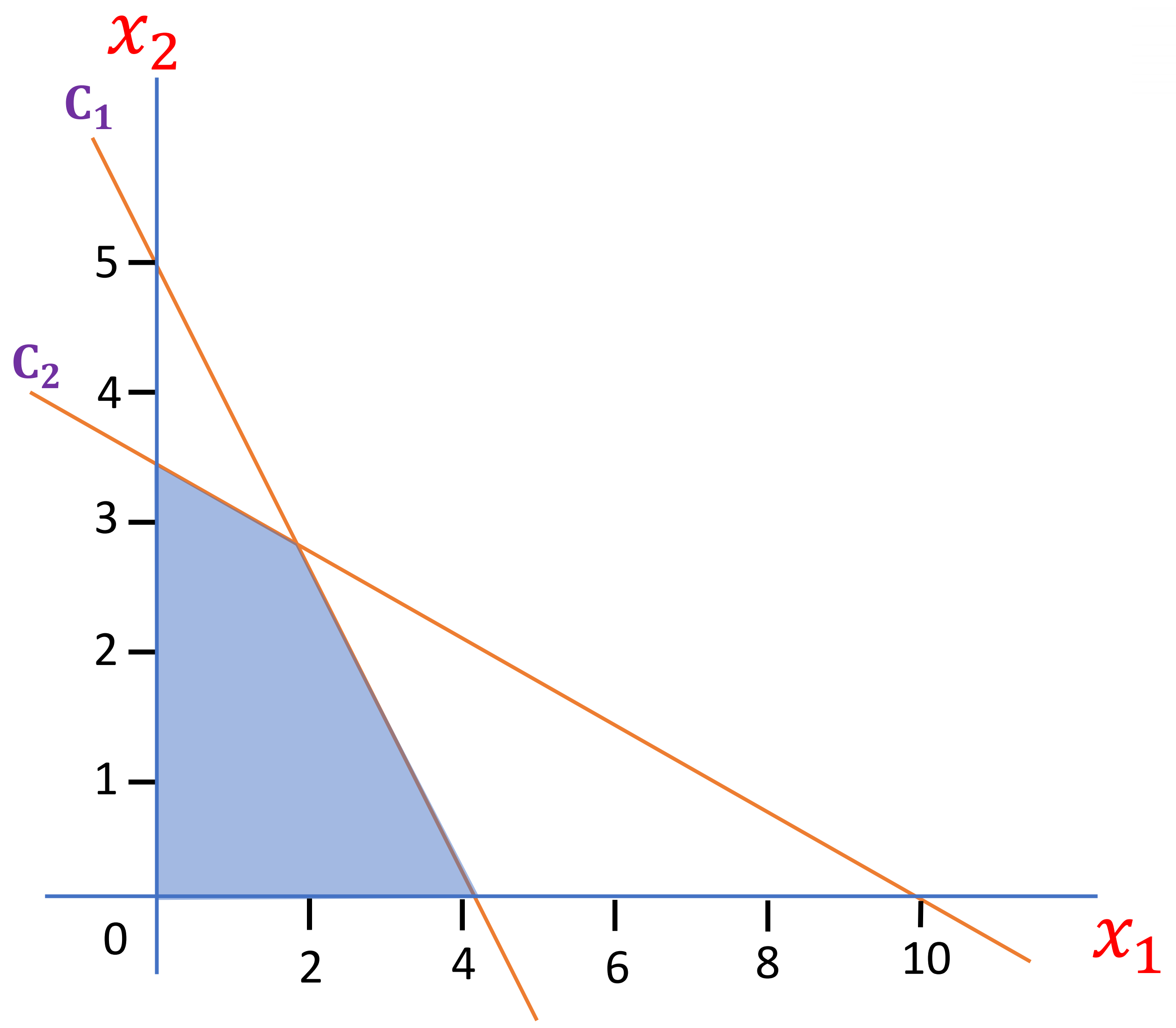

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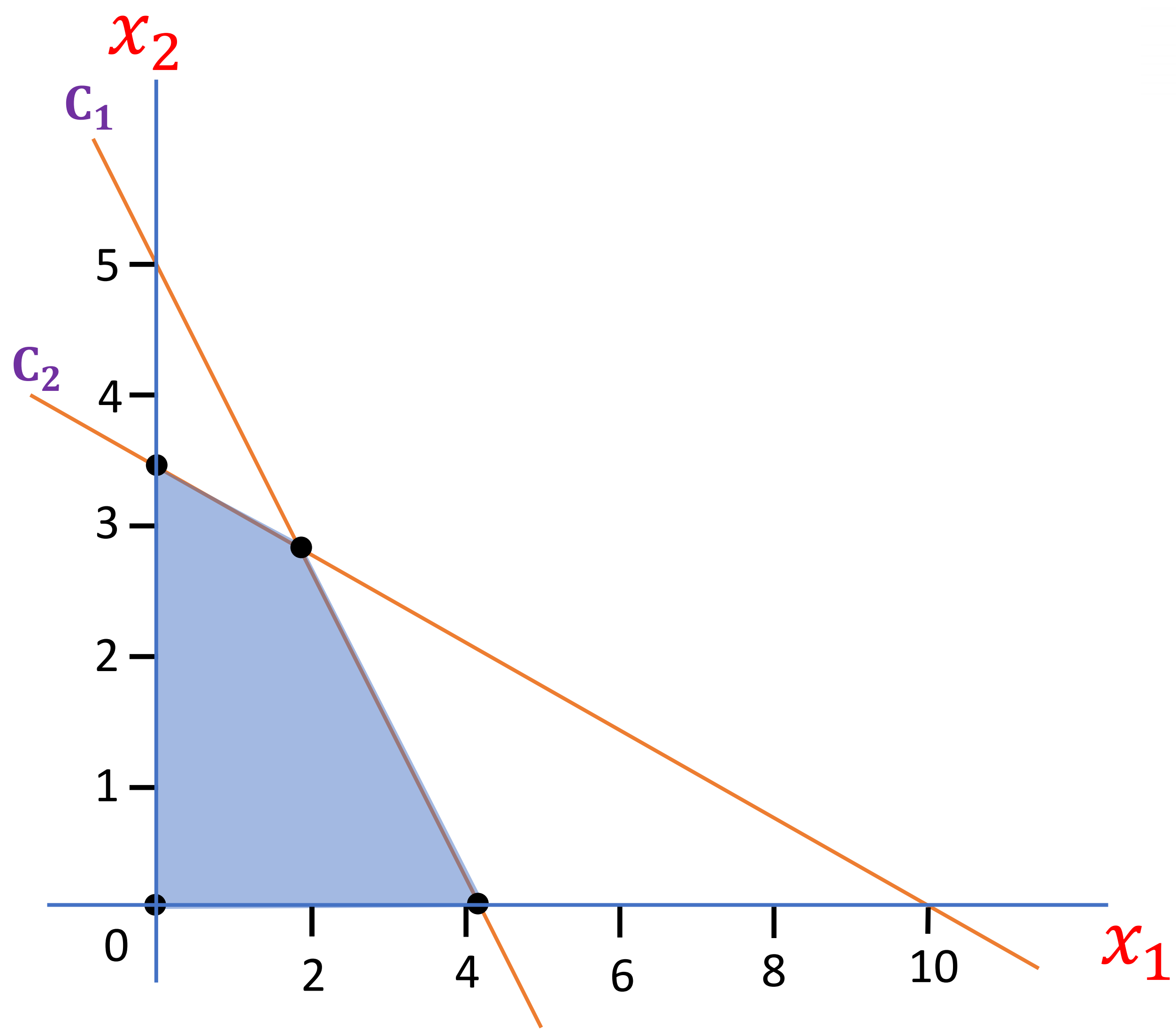

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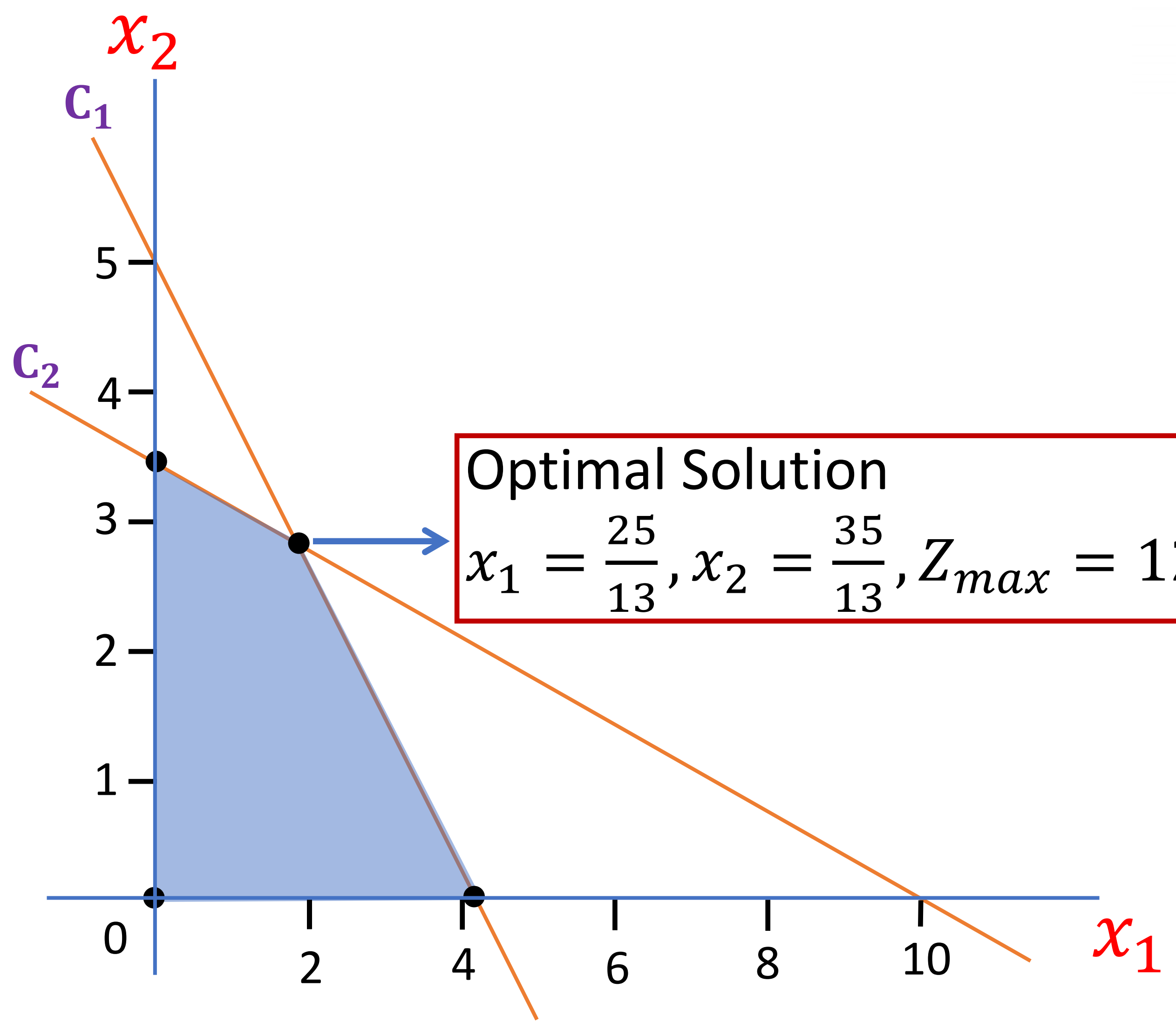

Sub-Problems are:

Sub-Problem 1:

Max $Z = 2x_1 + 3x_2$
$6x_1 + 5x_2 \leq 25$
$x_1 + 3x_2 \leq 10$
$x_2 \leq 2$
x_1, x_2 are non-negative integers

Sub-Problem 2:

Max $Z = 2x_1 + 3x_2$
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$x_1 + 3x_2 \leq 10$
$x_2 \geq 3$
x_1, x_2 are non-negative integers

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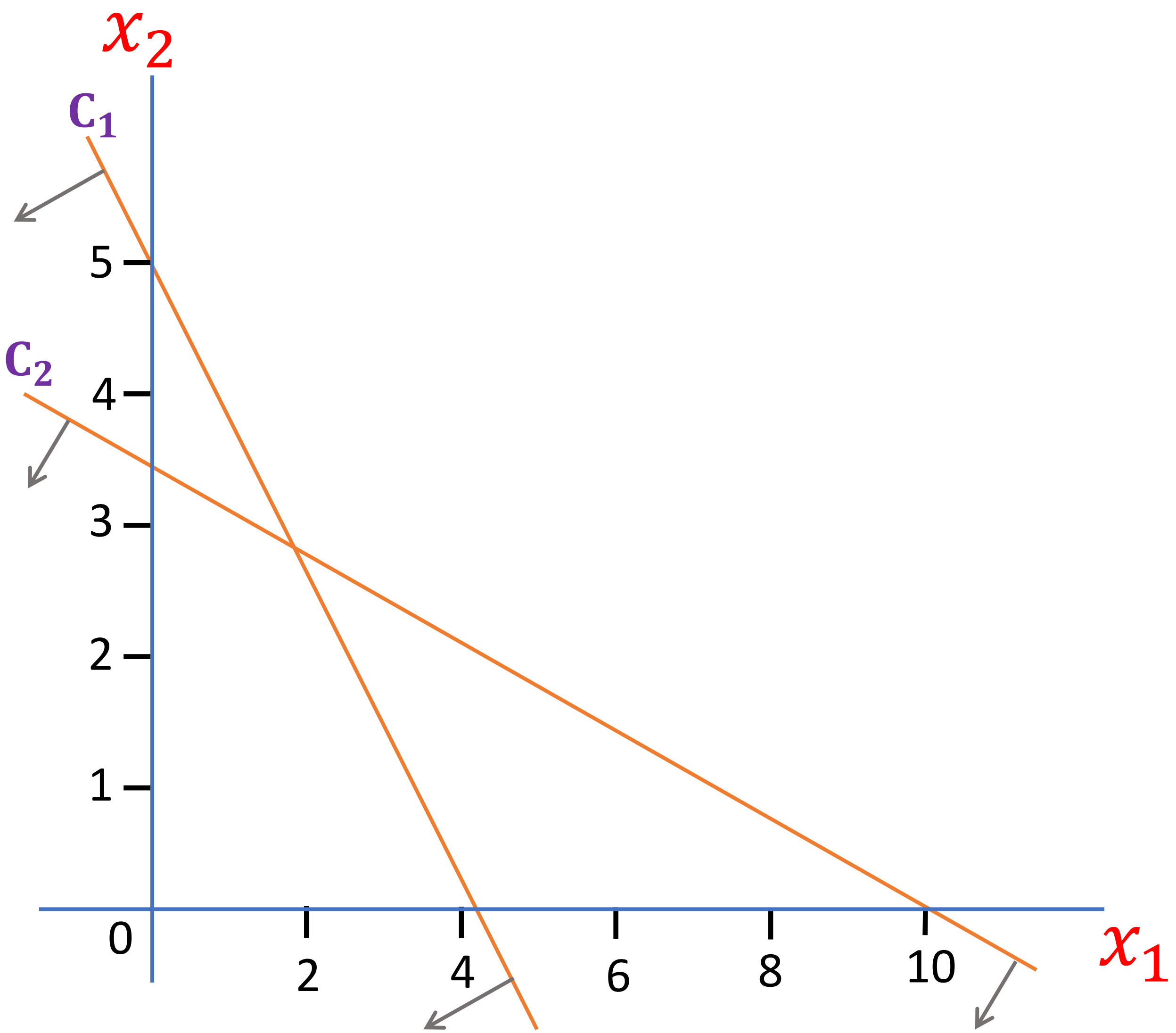

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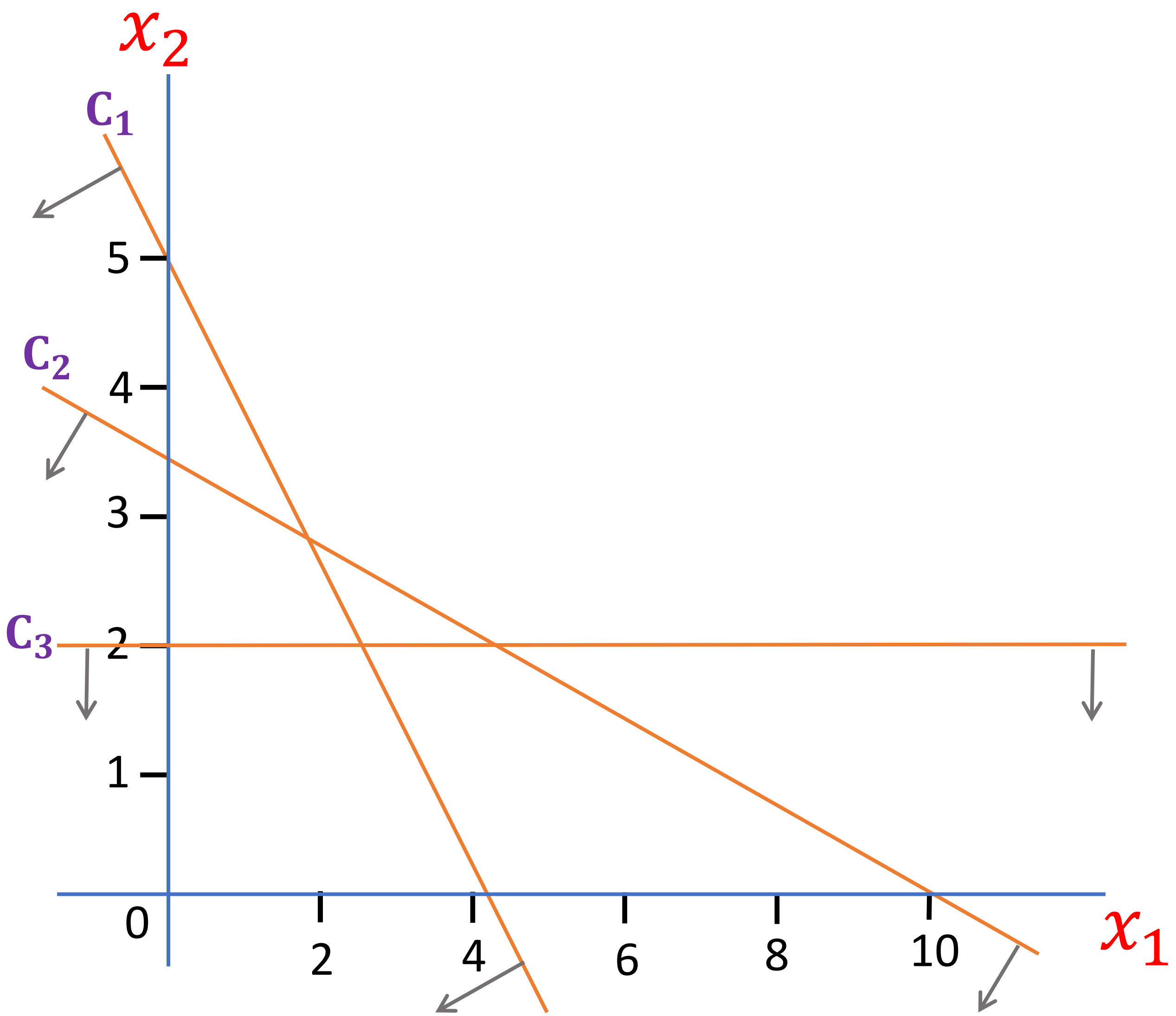

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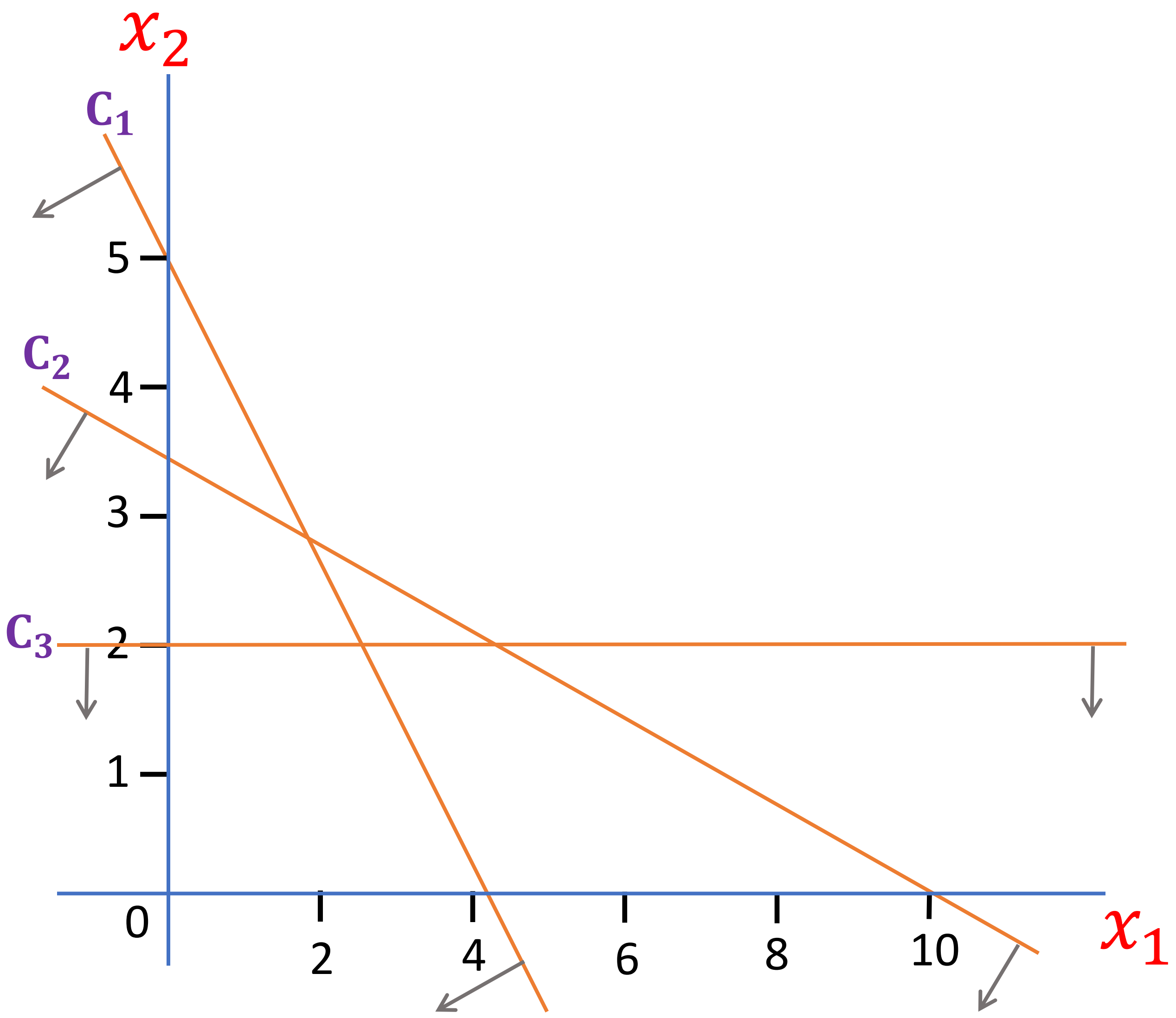
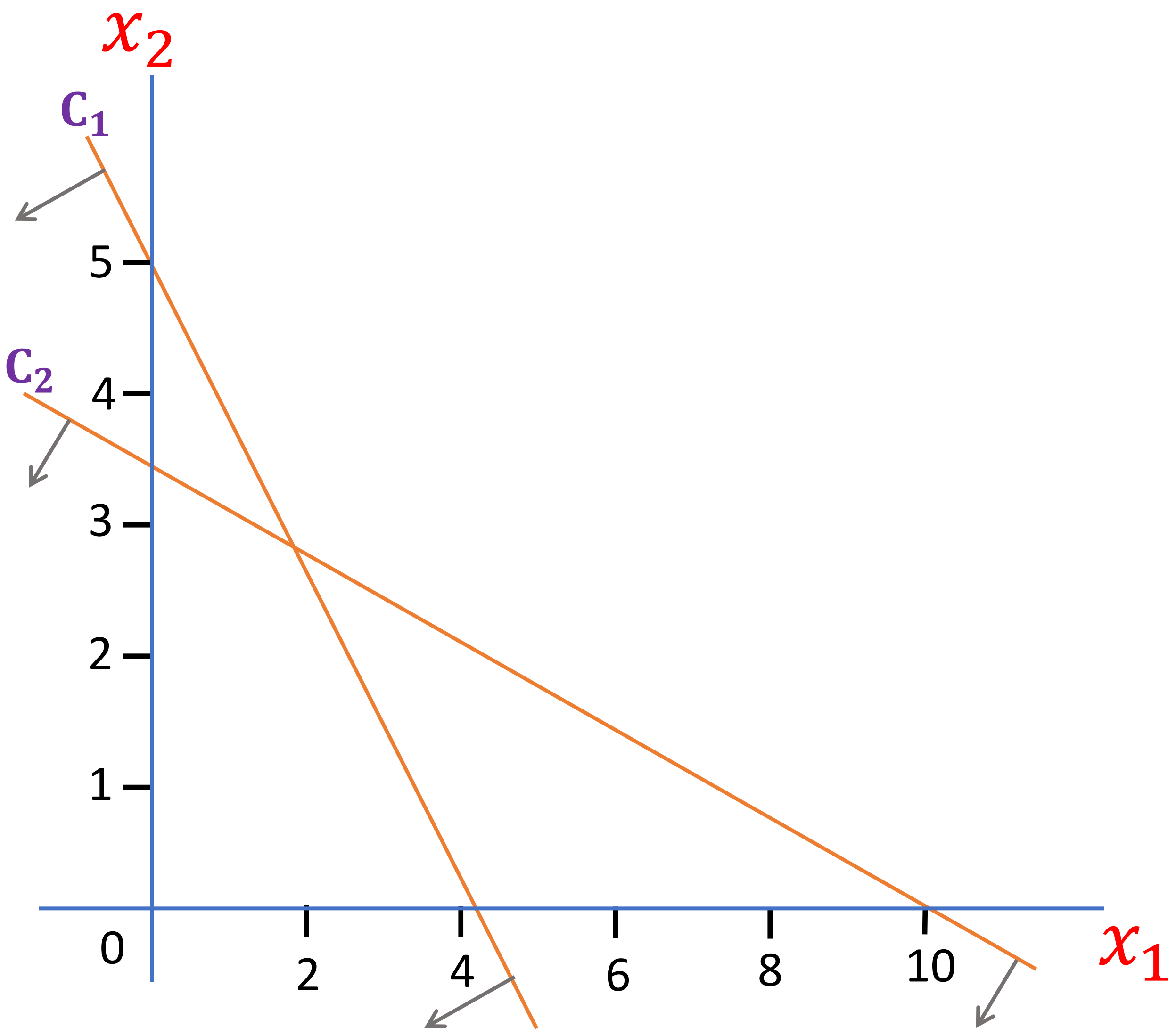


Sub-Problem 2:



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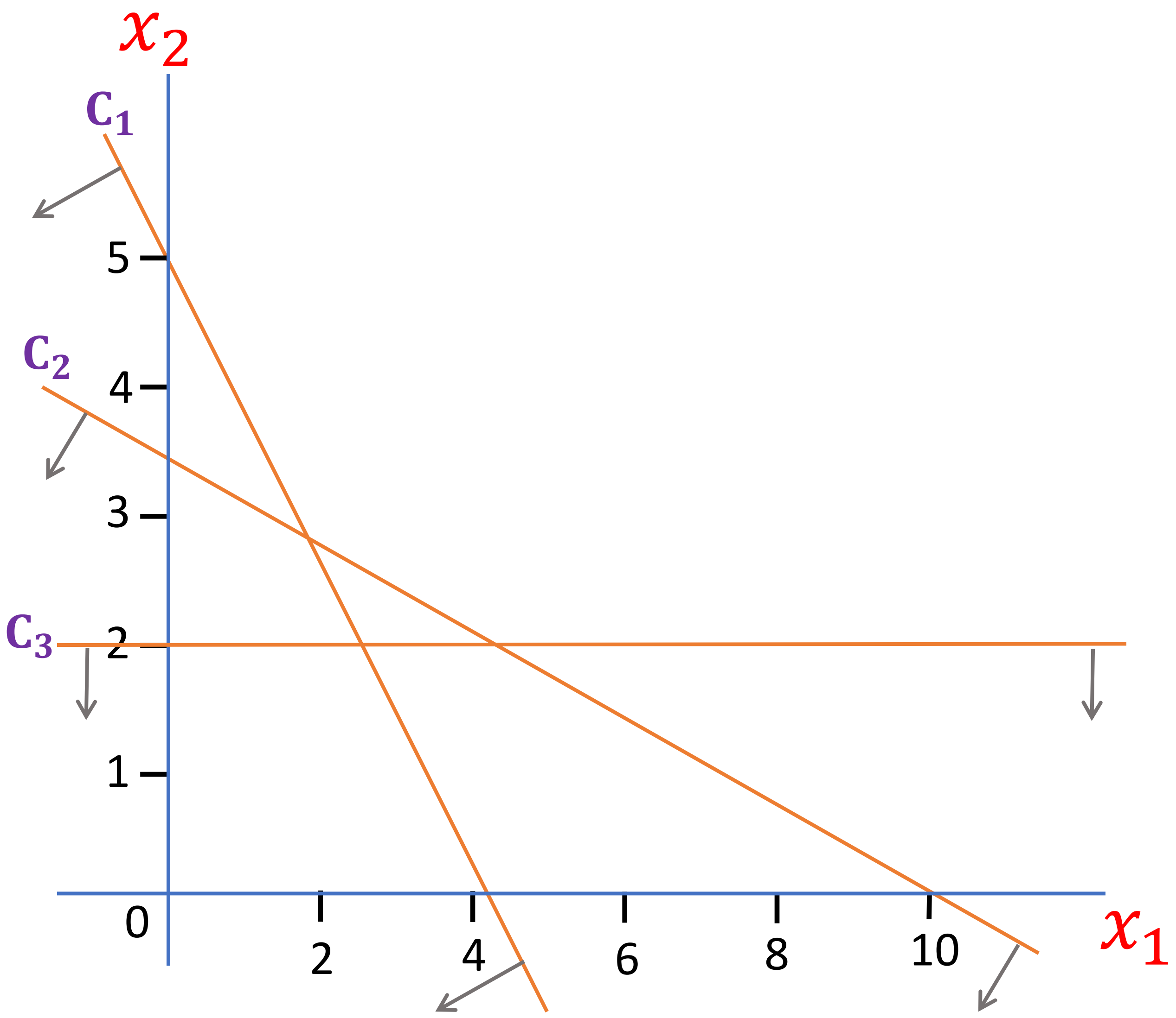

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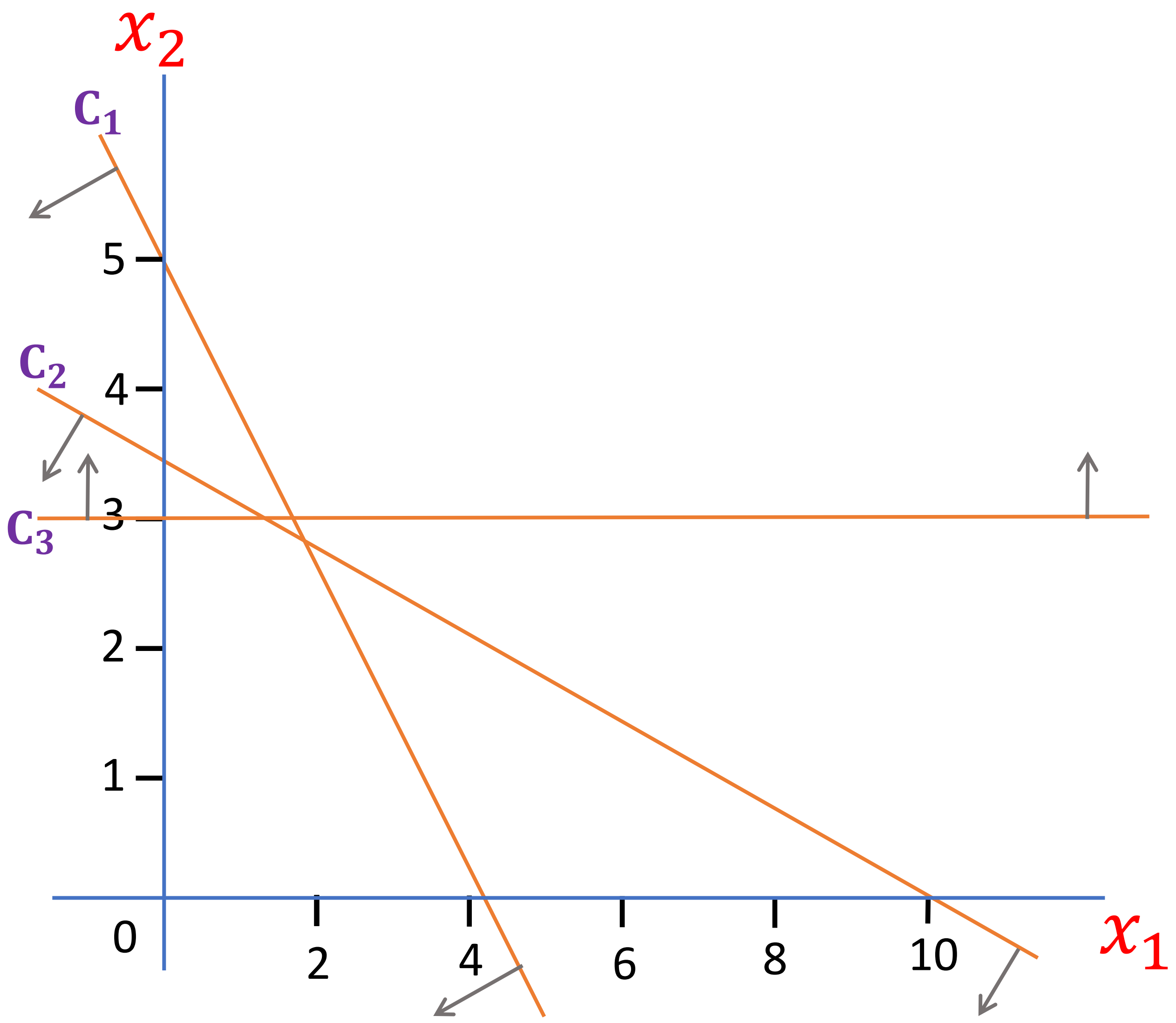

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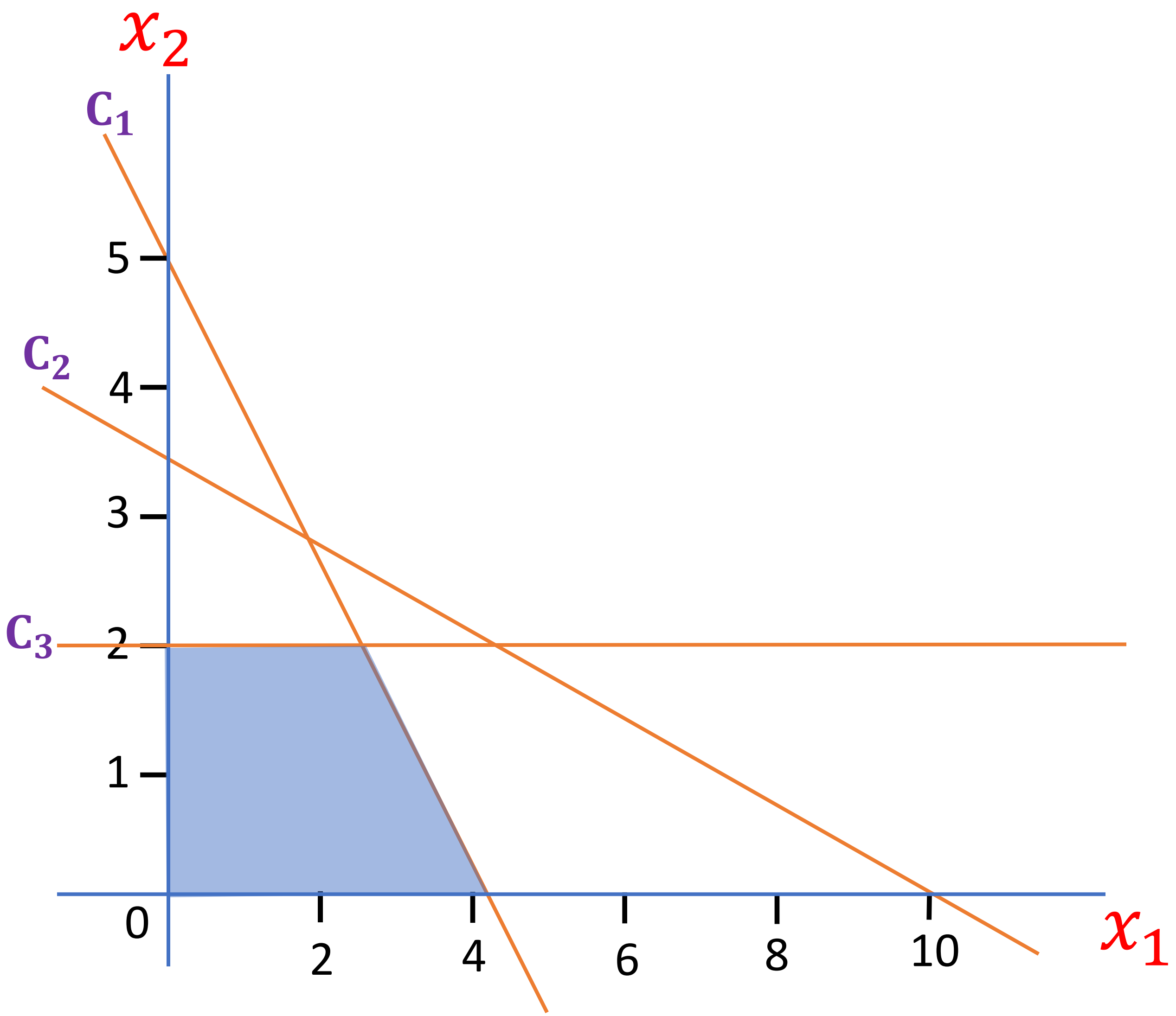

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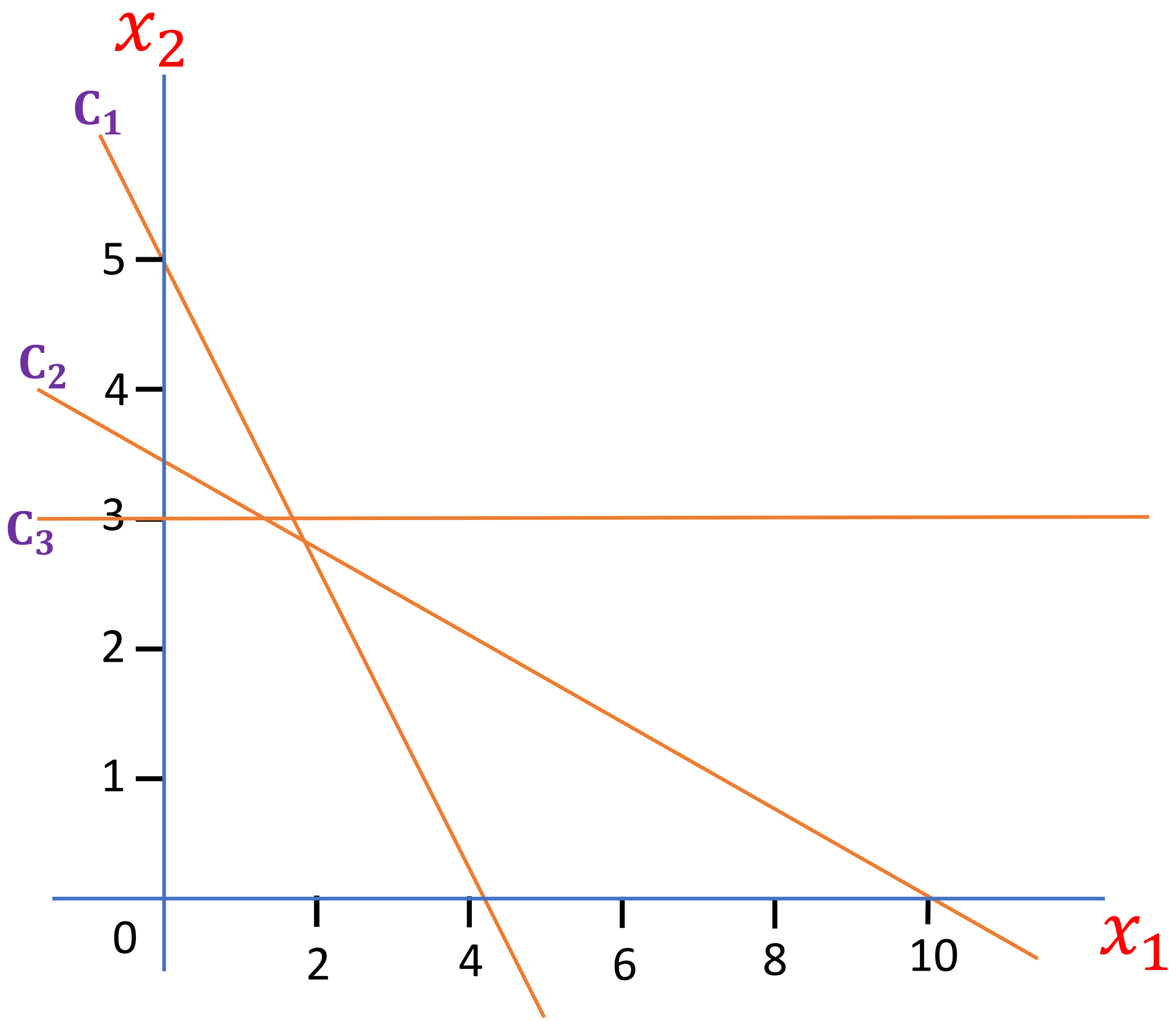

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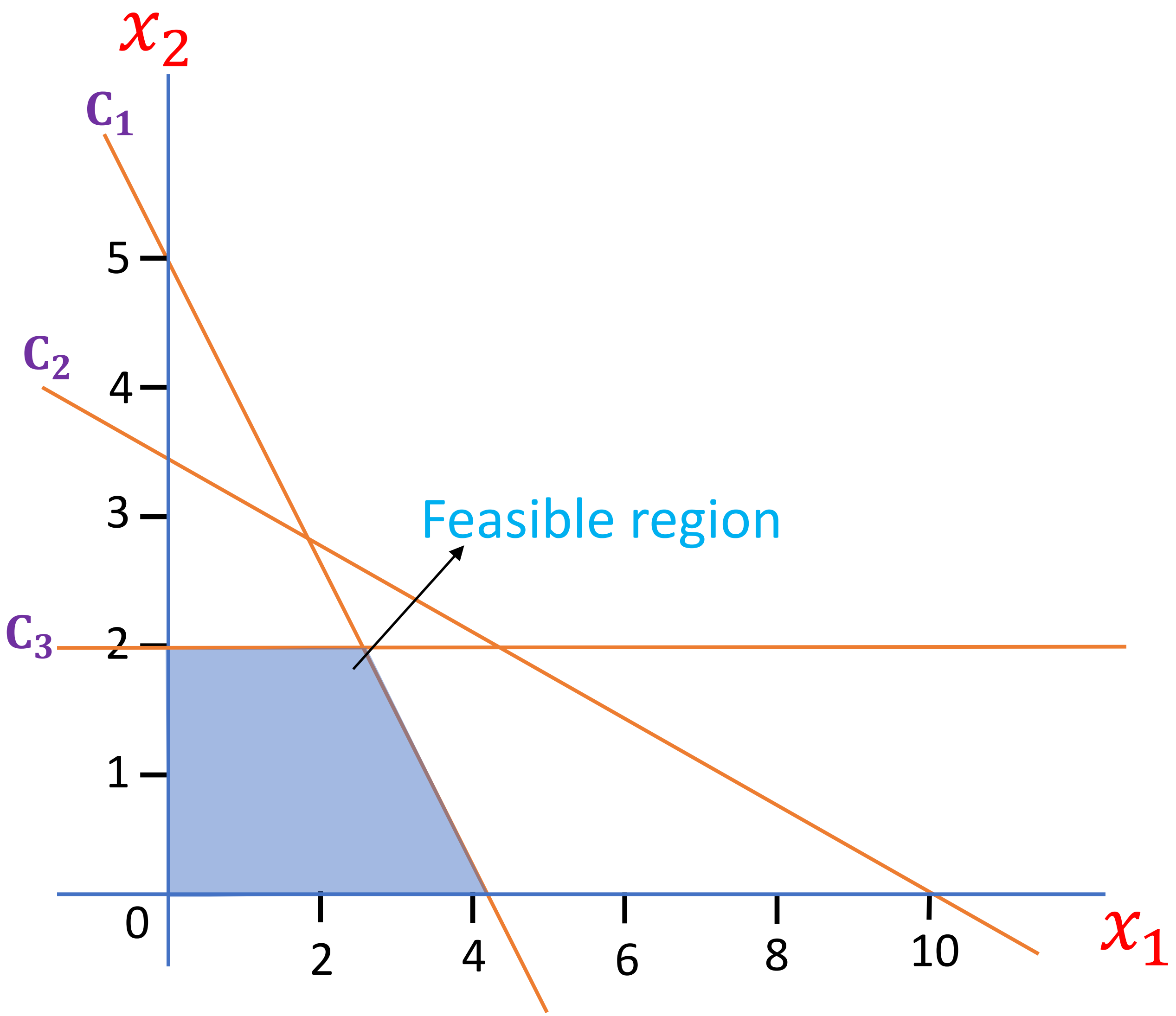

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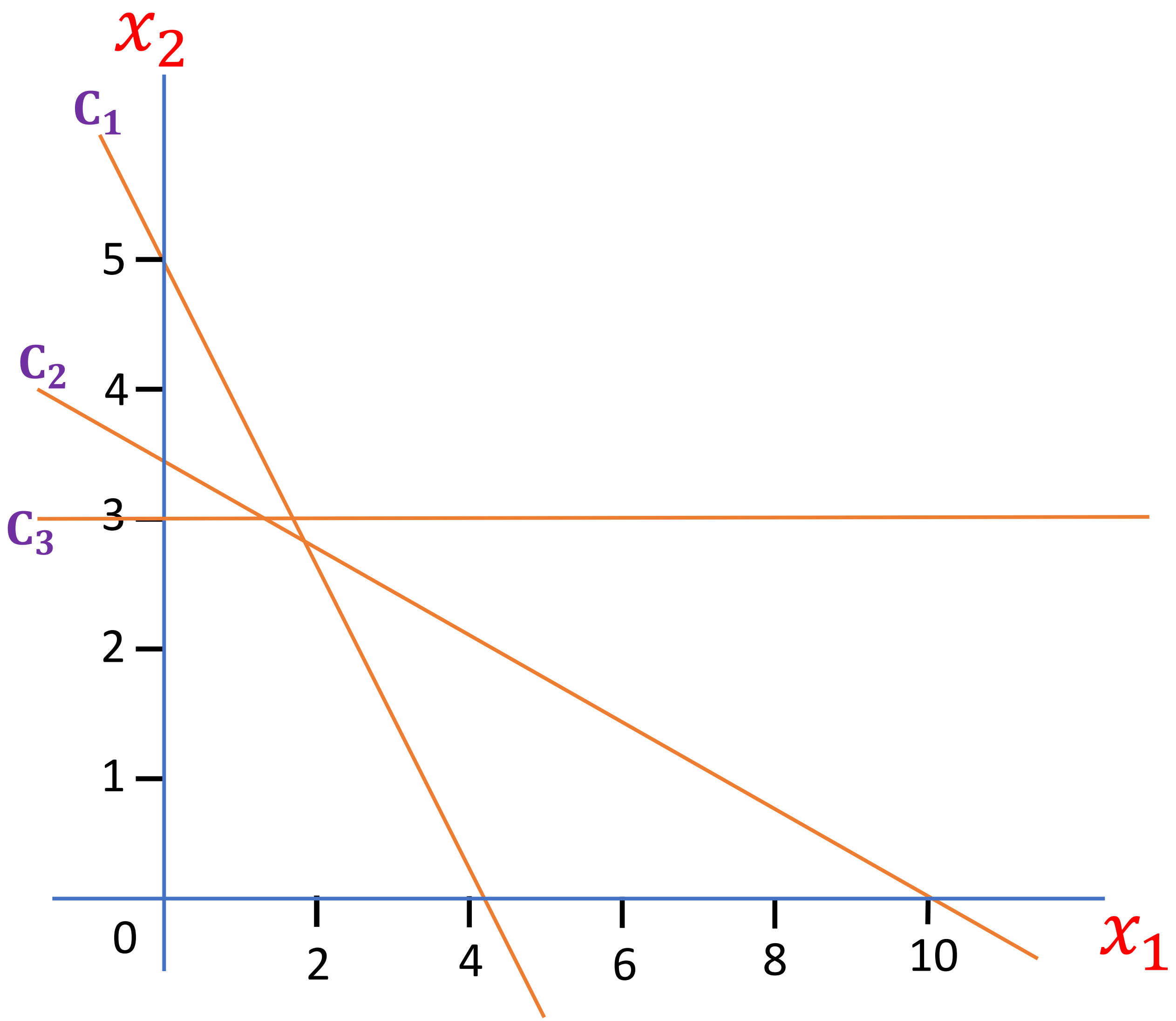

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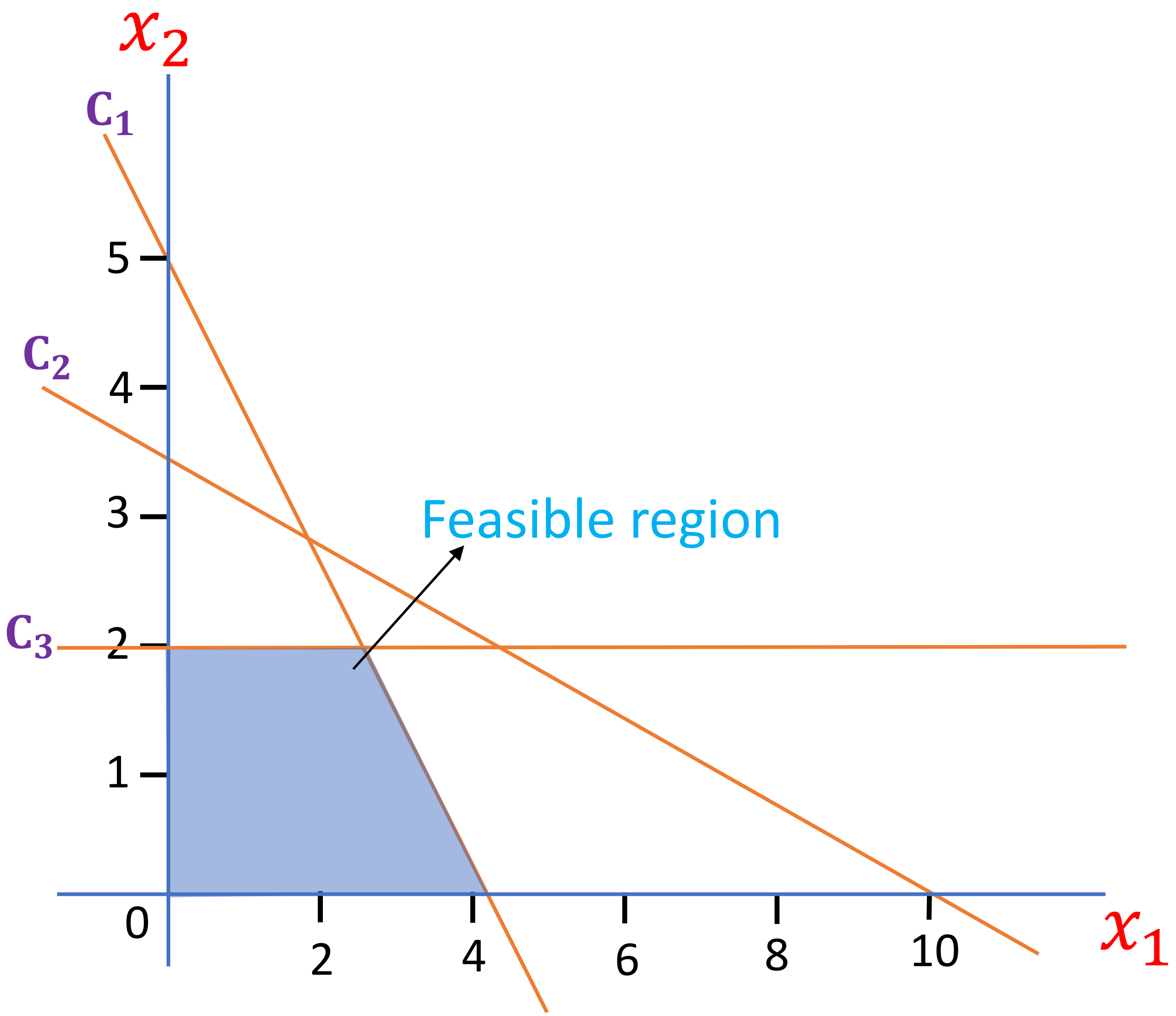

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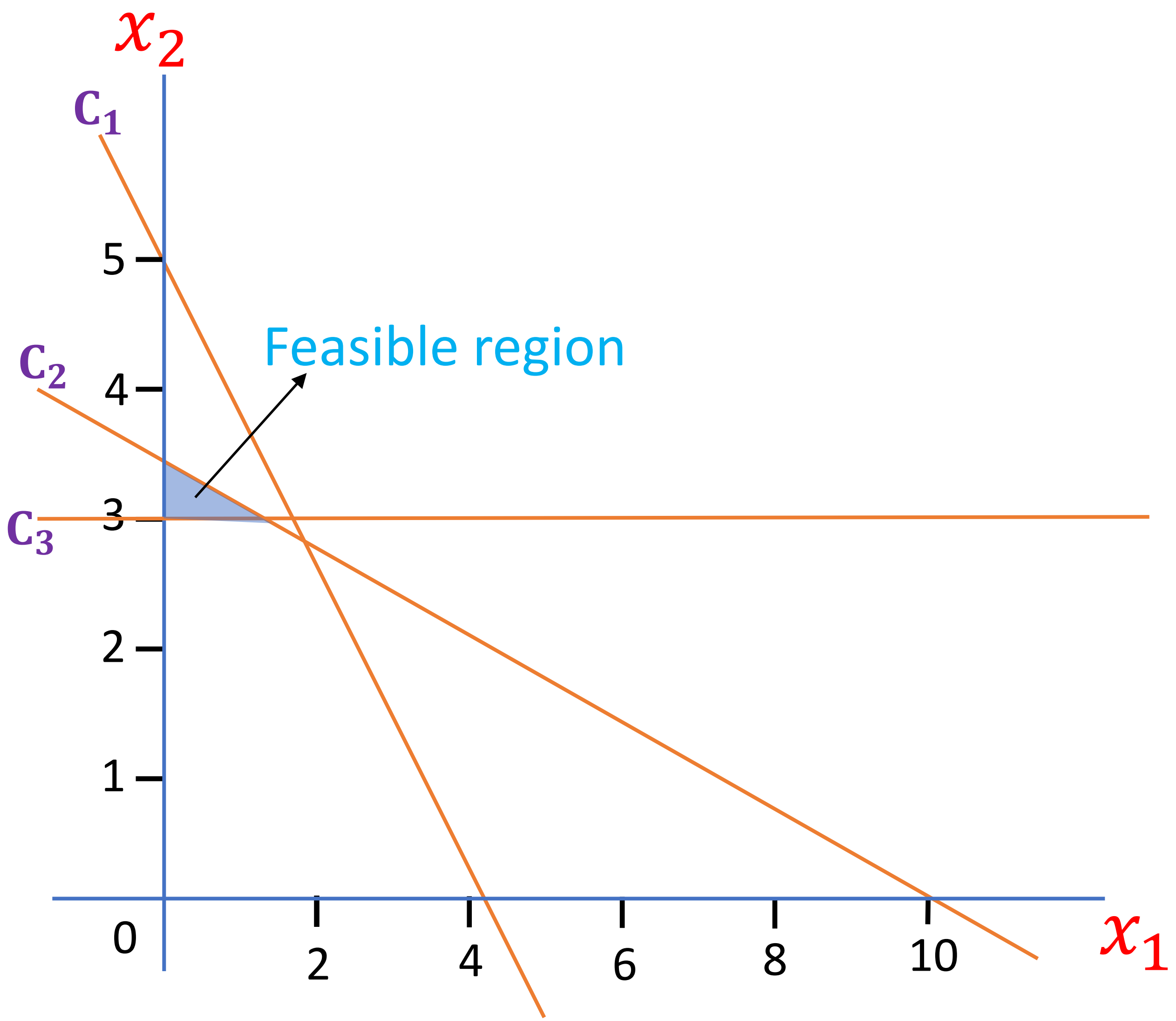

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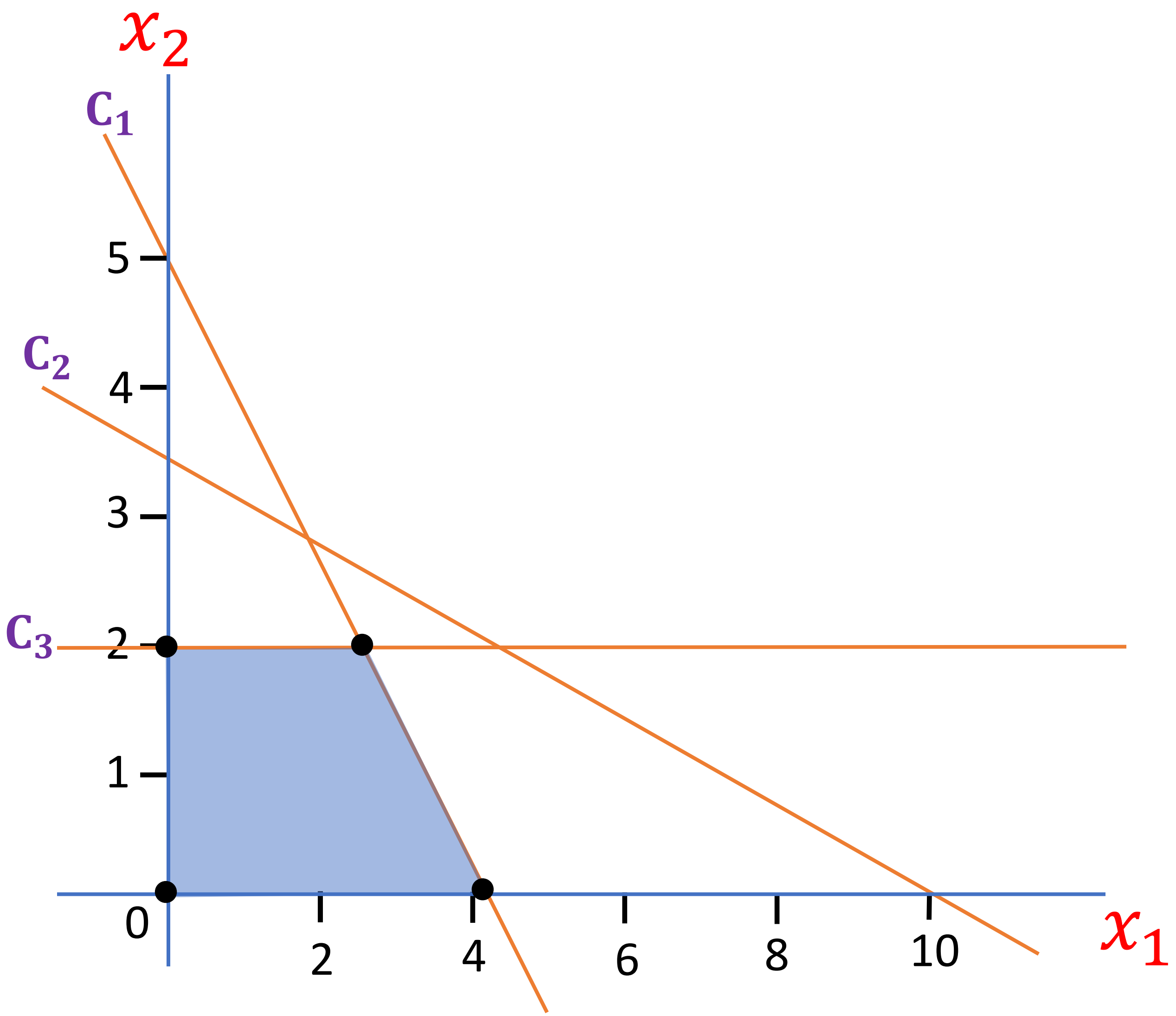

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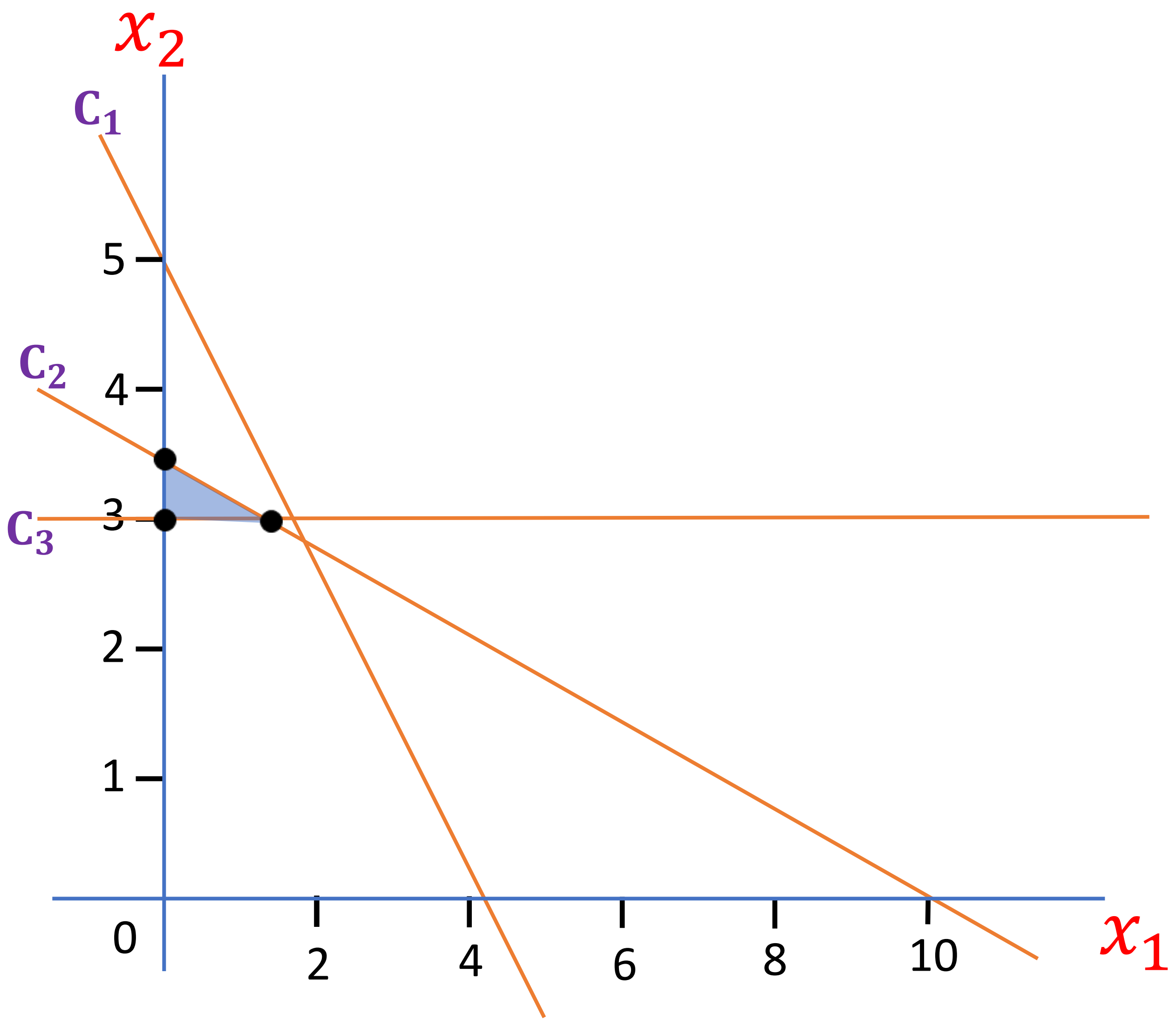

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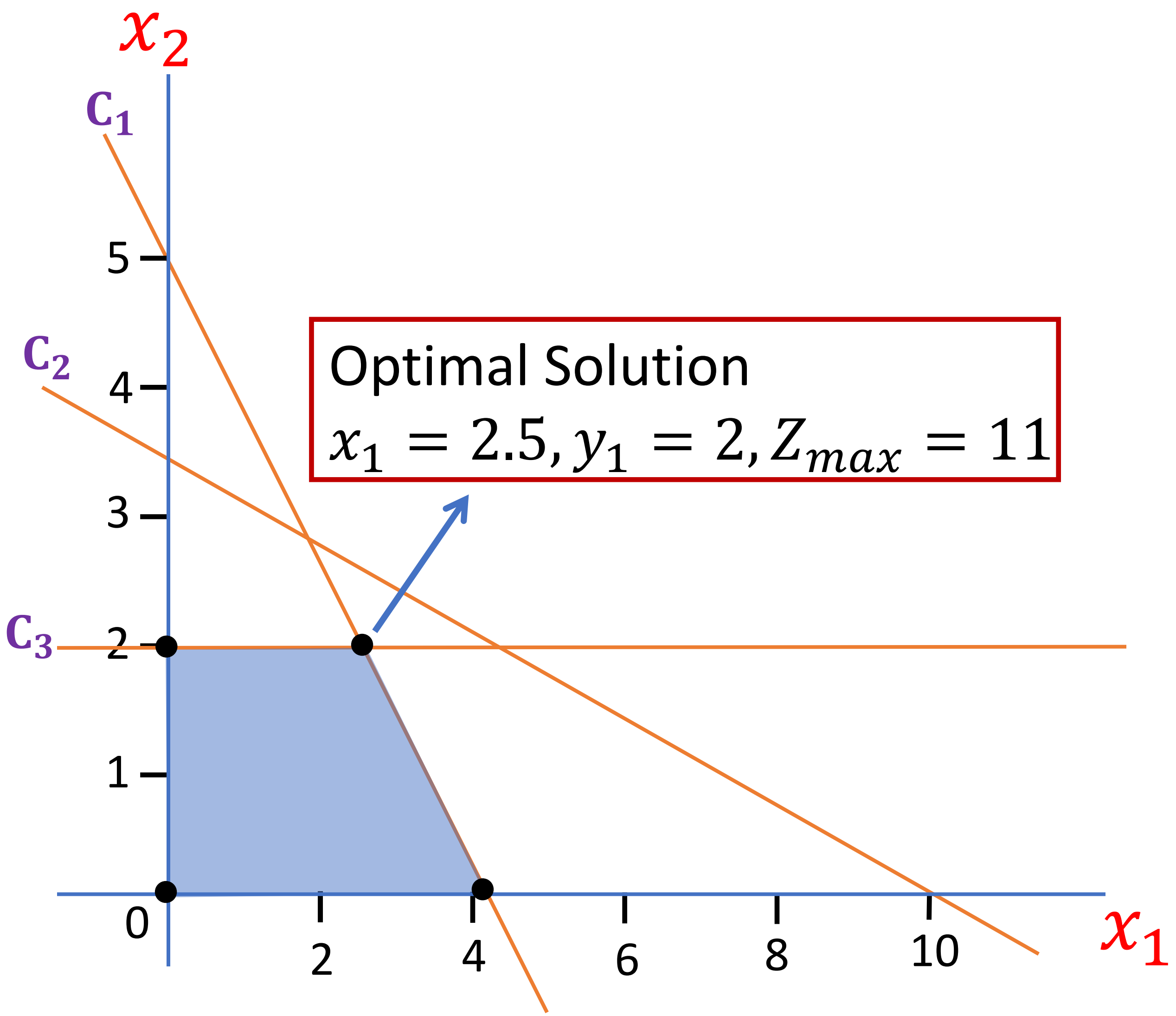

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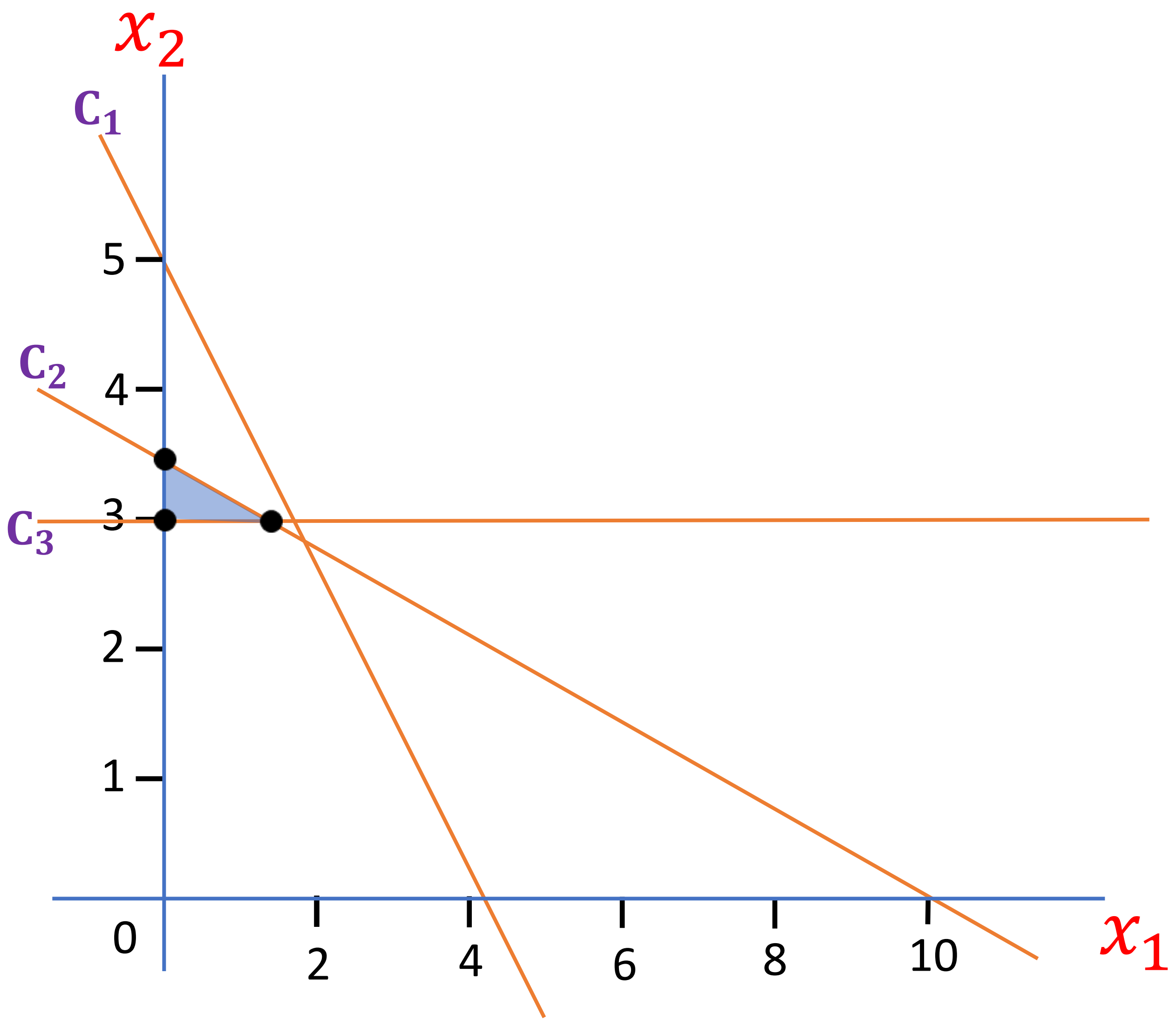

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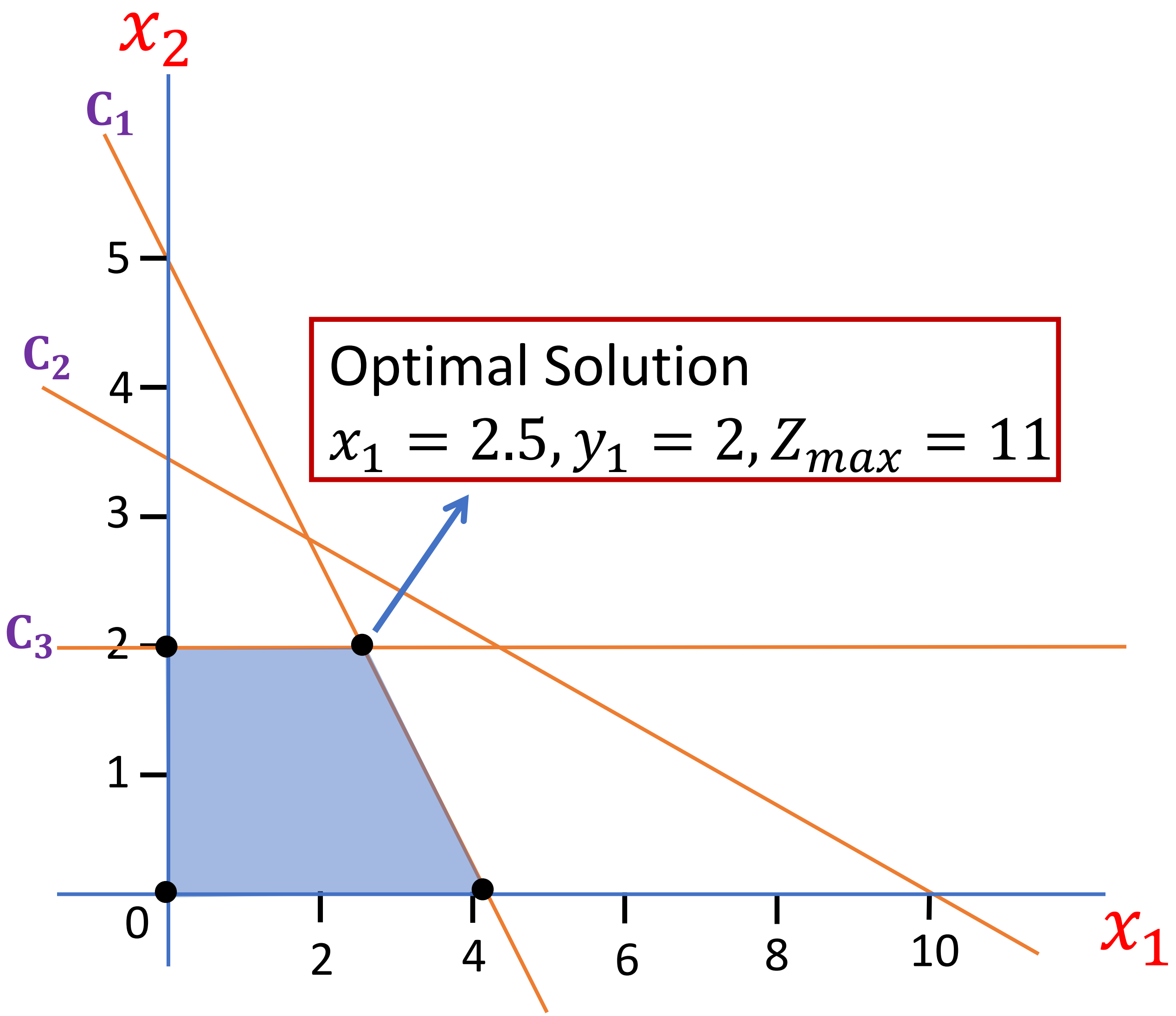

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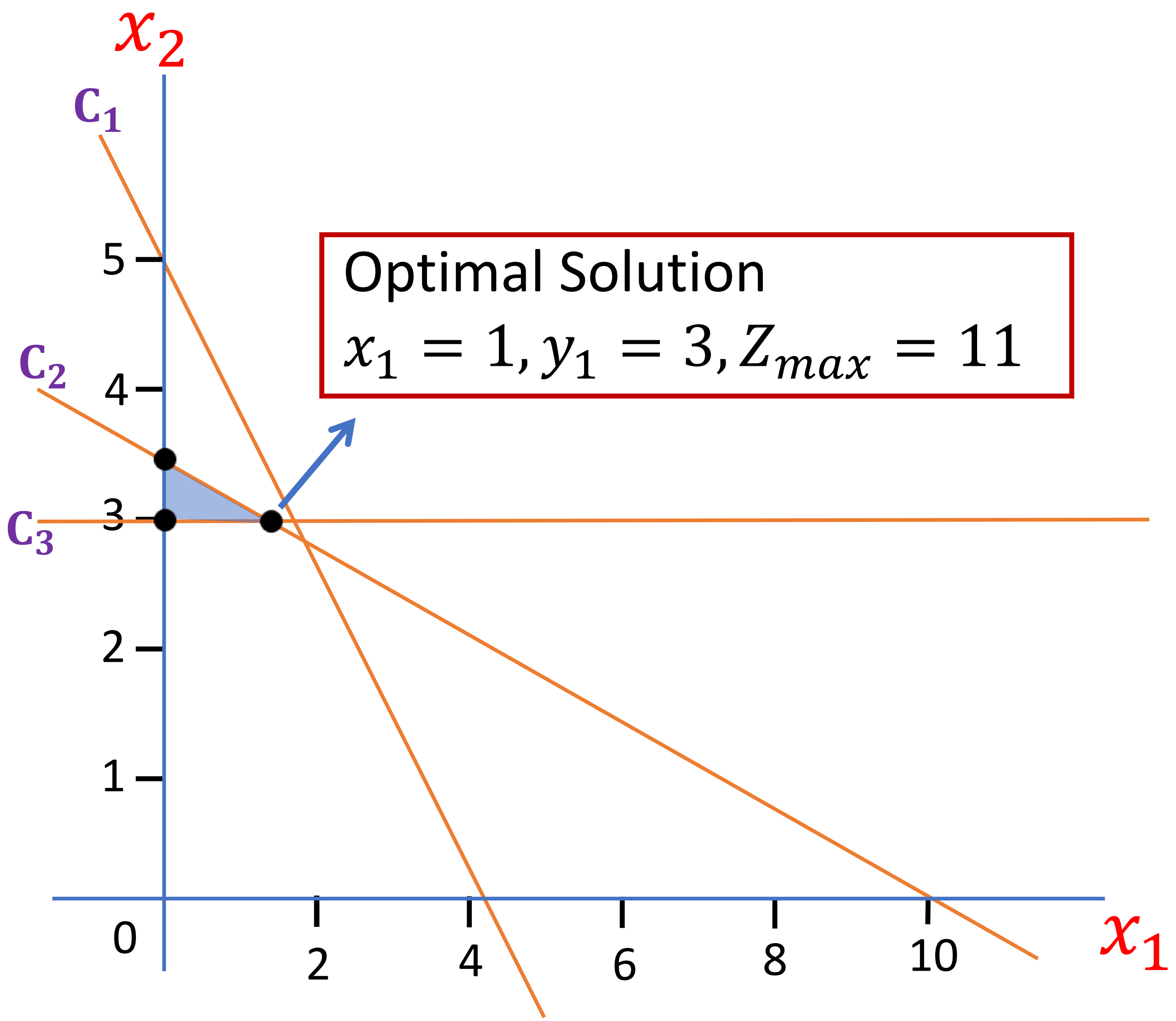

Sub-Problem 1 has non-negative non integer solution. But, Sub-Problem 2 has integer solution. Since Sub-Problem has non integer solution, we have to go for the next step to get the integer solution.

In sub-problem 1, we have the solution $x_1 = 2.5$ and $x_2 = 2$.

The value of the variable x_1 lies between the integers 2 and 3.

Therefore, to get the integer solution we have to take $x_1 \leq 2$ or $x_1 \geq 3$.

The sub-problems corresponds to these constraints are:

Sub-Problem 3:

Max $Z = 2x_1 + 3x_2$

$6x_1 + 5x_2 \leq 25$

$x_1 + 3x_2 \leq 10$

$x_2 \leq 2$

$x_1 \leq 2$

x_1, x_2 are non-negative integers

Sub-Problem 4:

Max $Z = 2x_1 + 3x_2$

$6x_1 + 5x_2 \leq 25$

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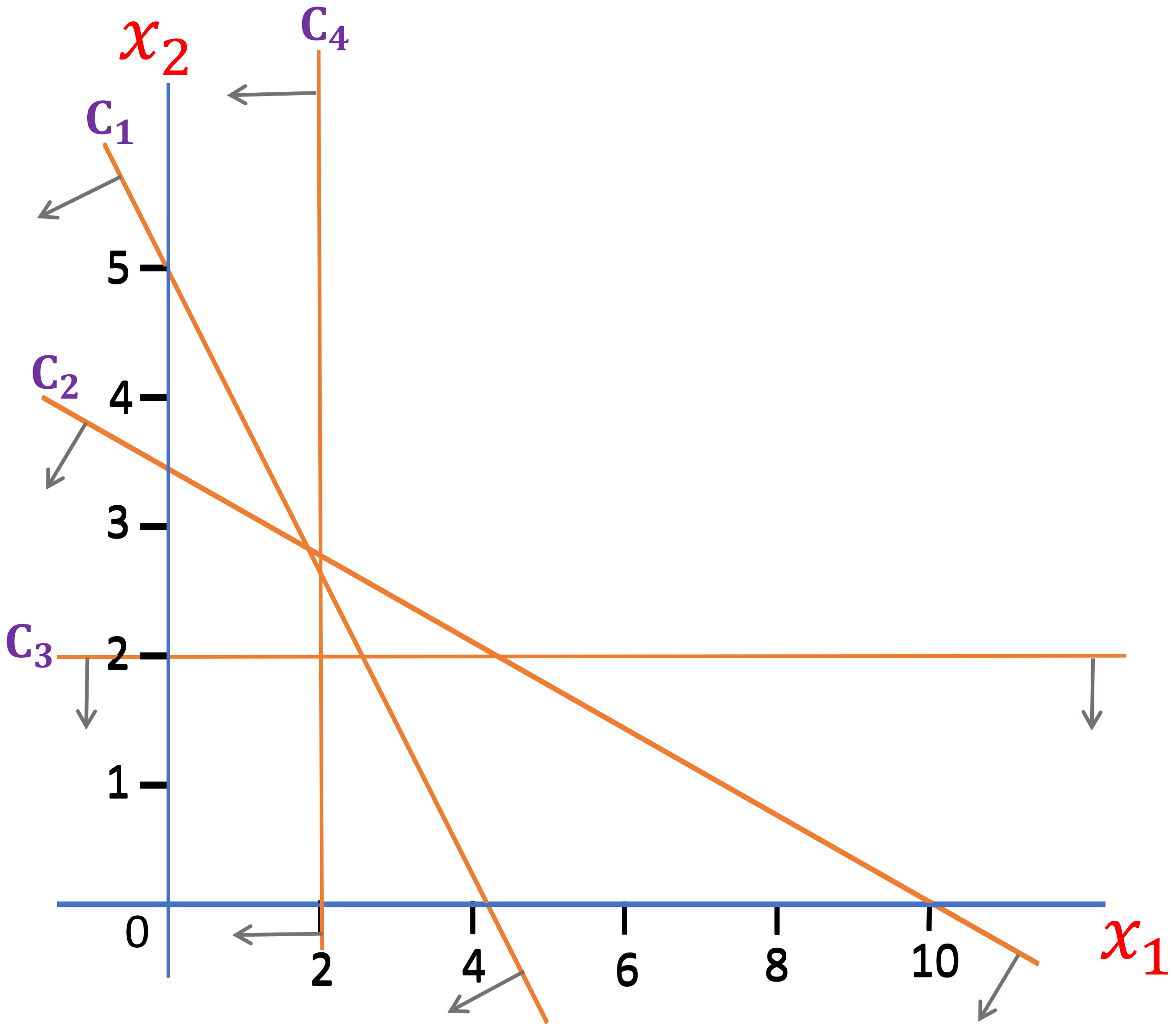
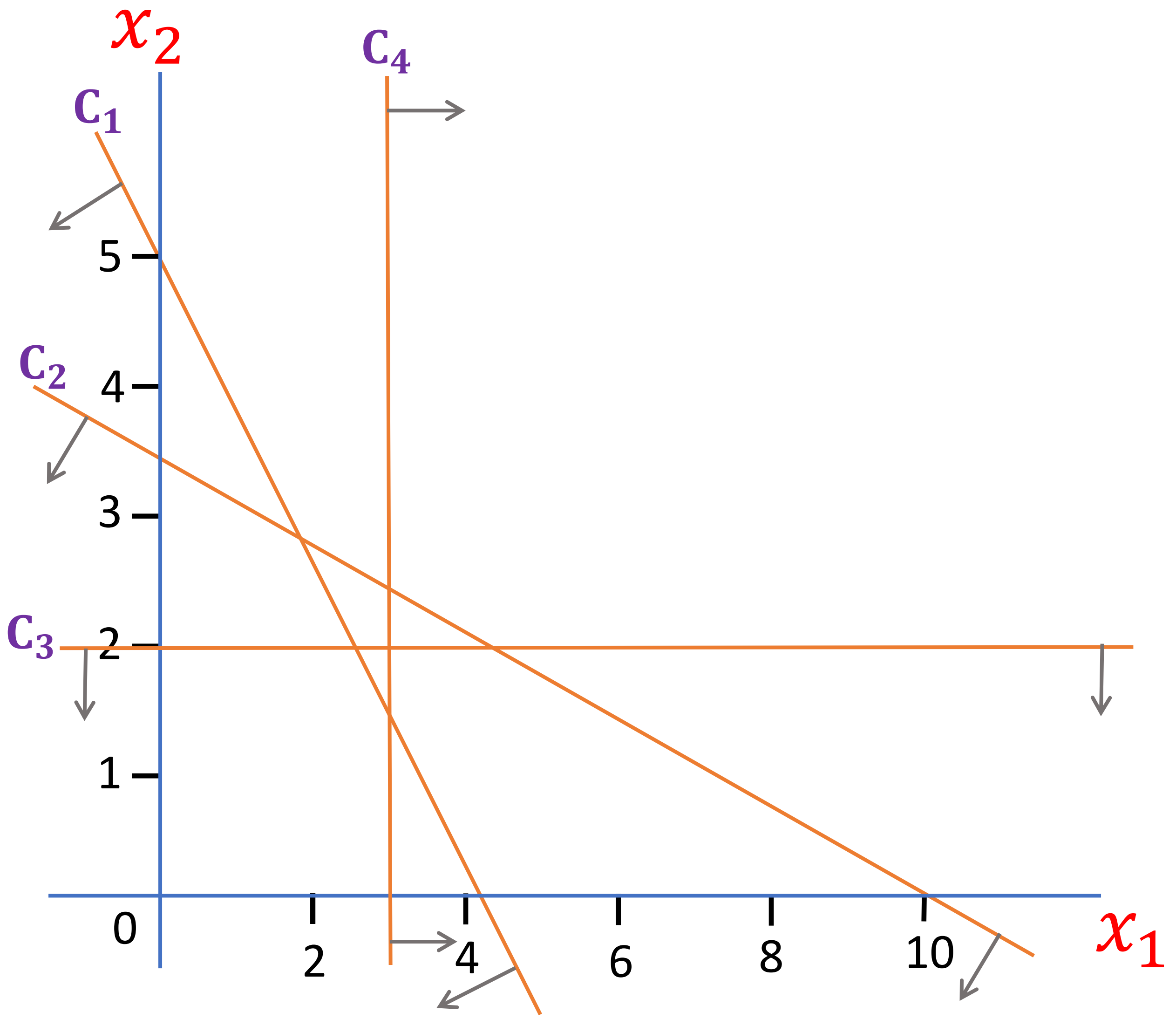


Sub-Problem 4:



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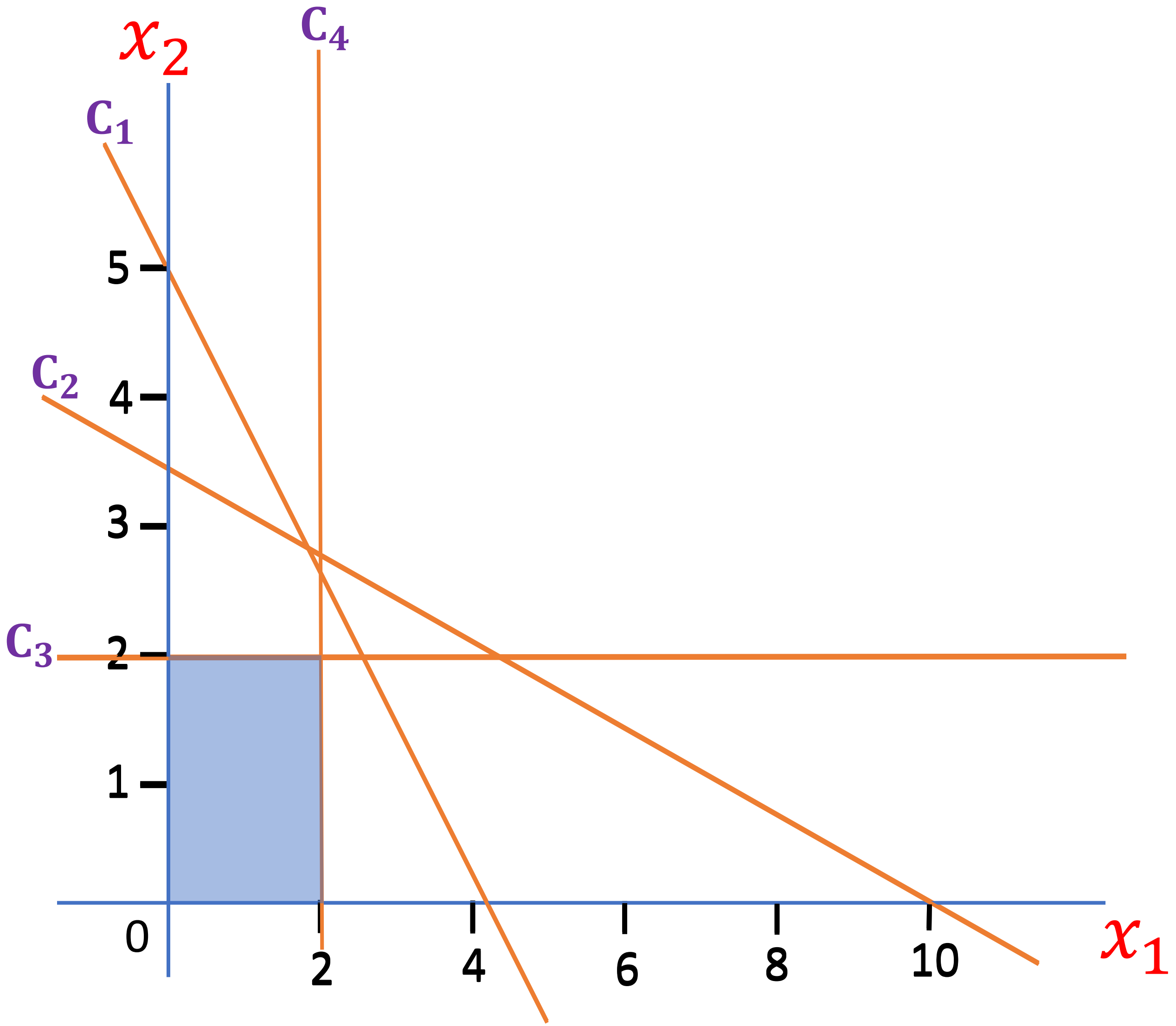

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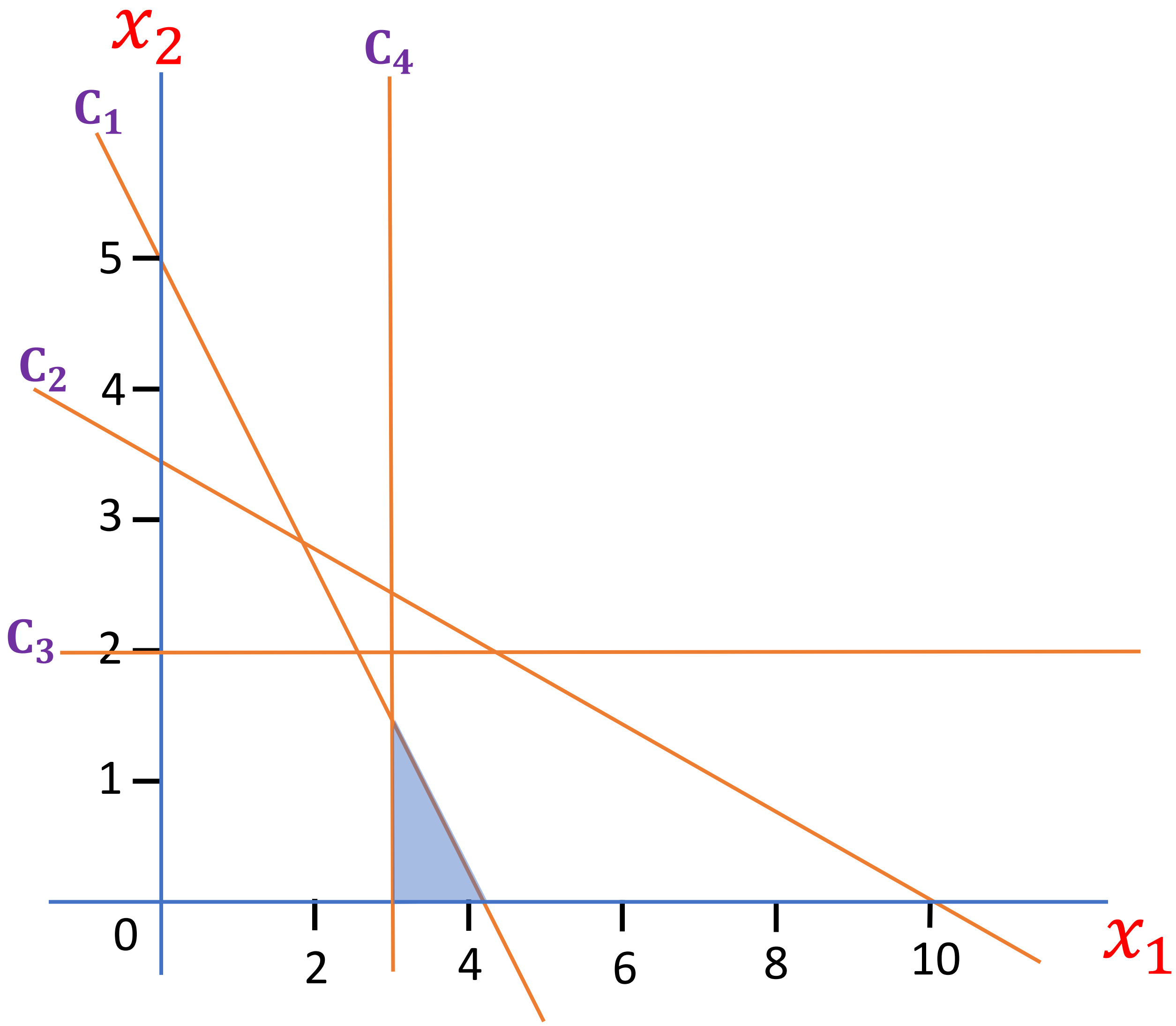

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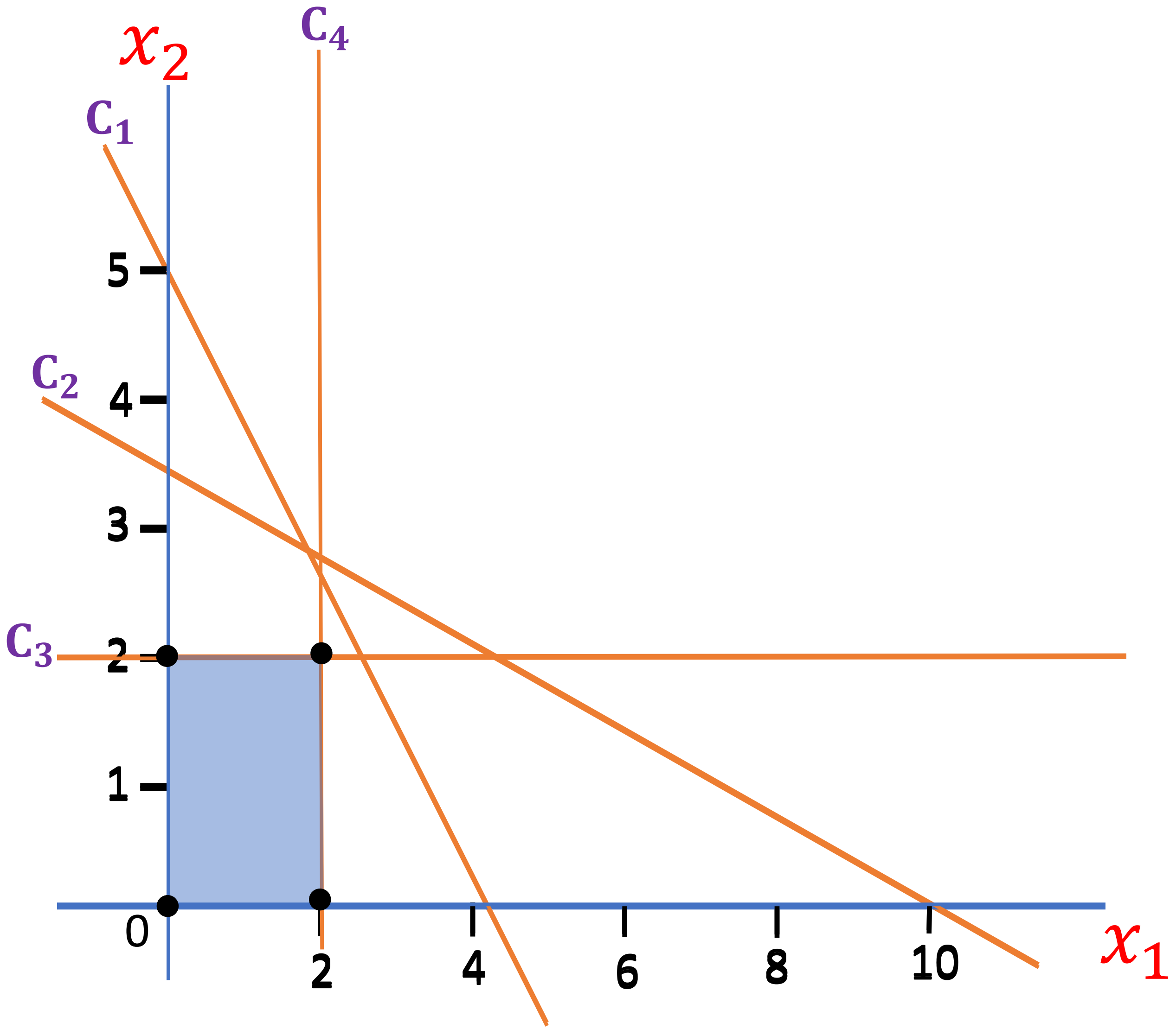

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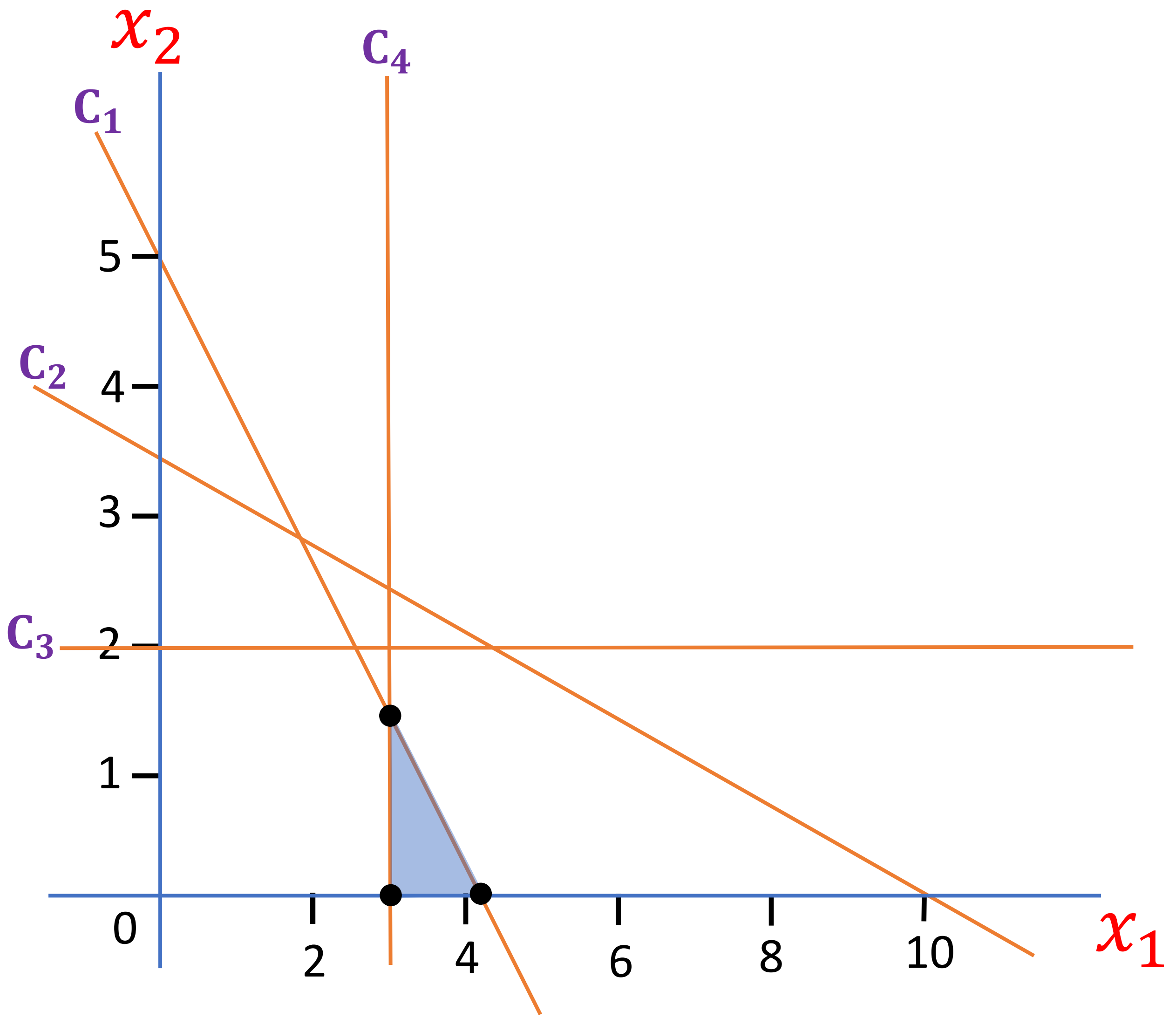

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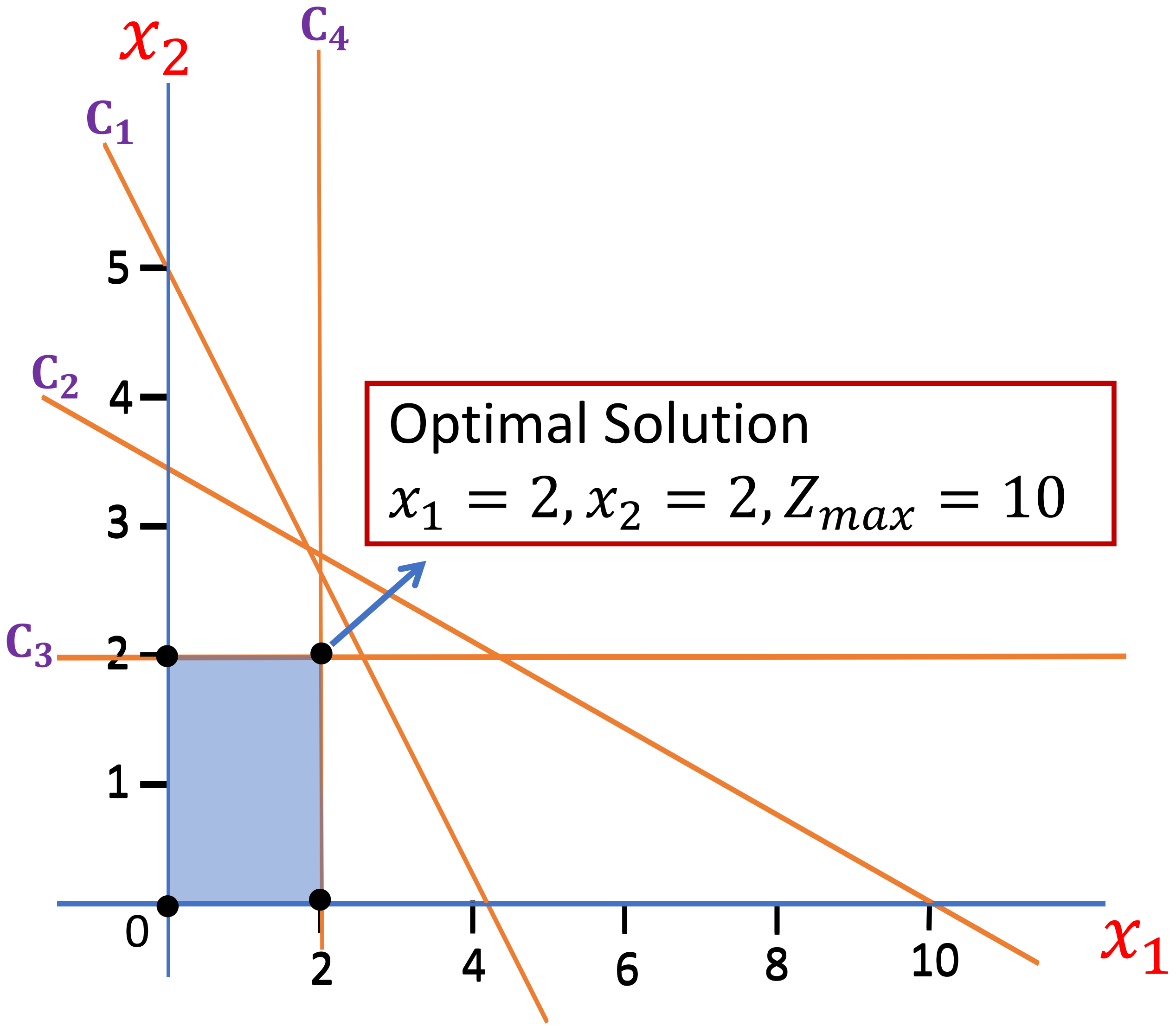

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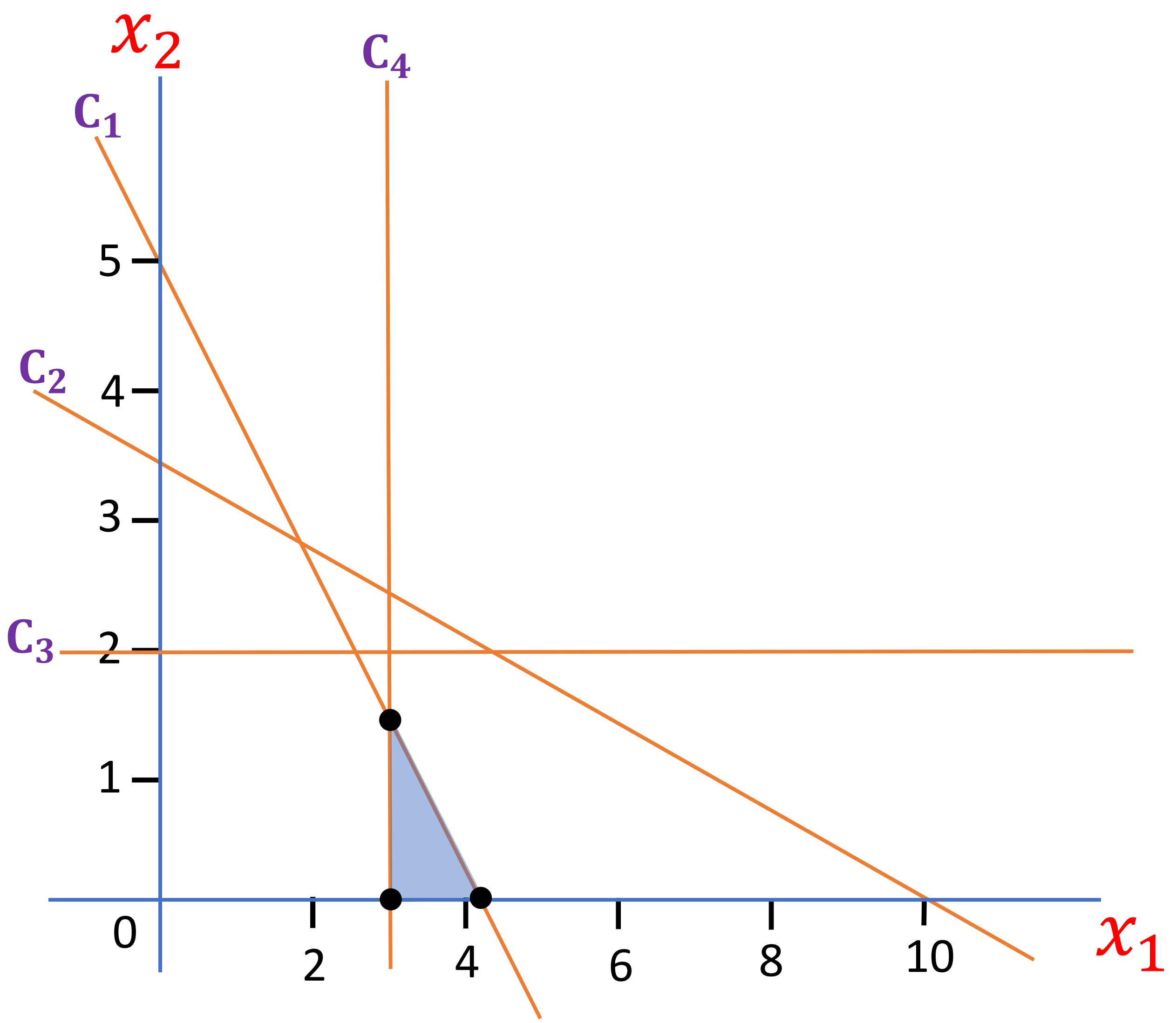

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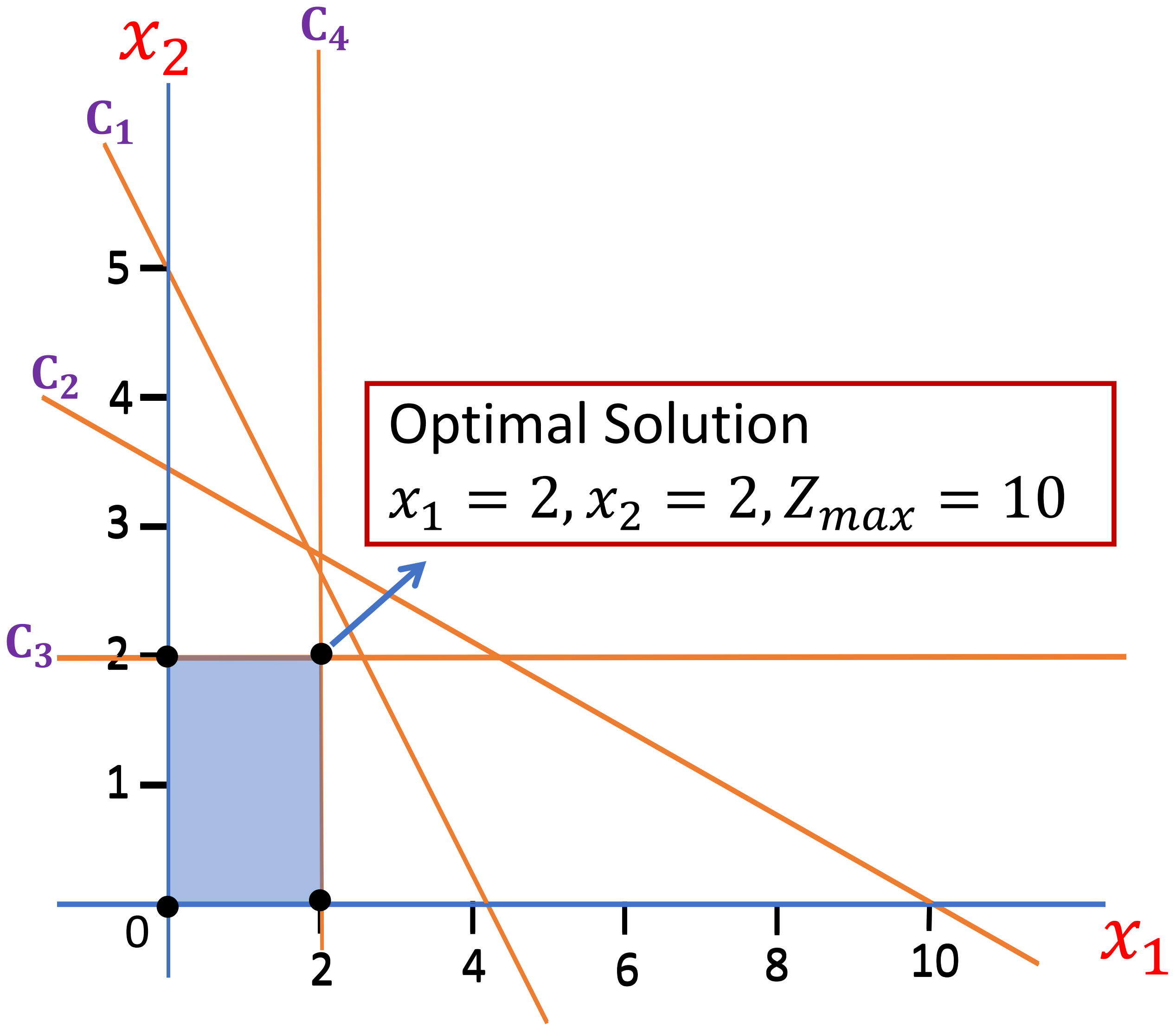

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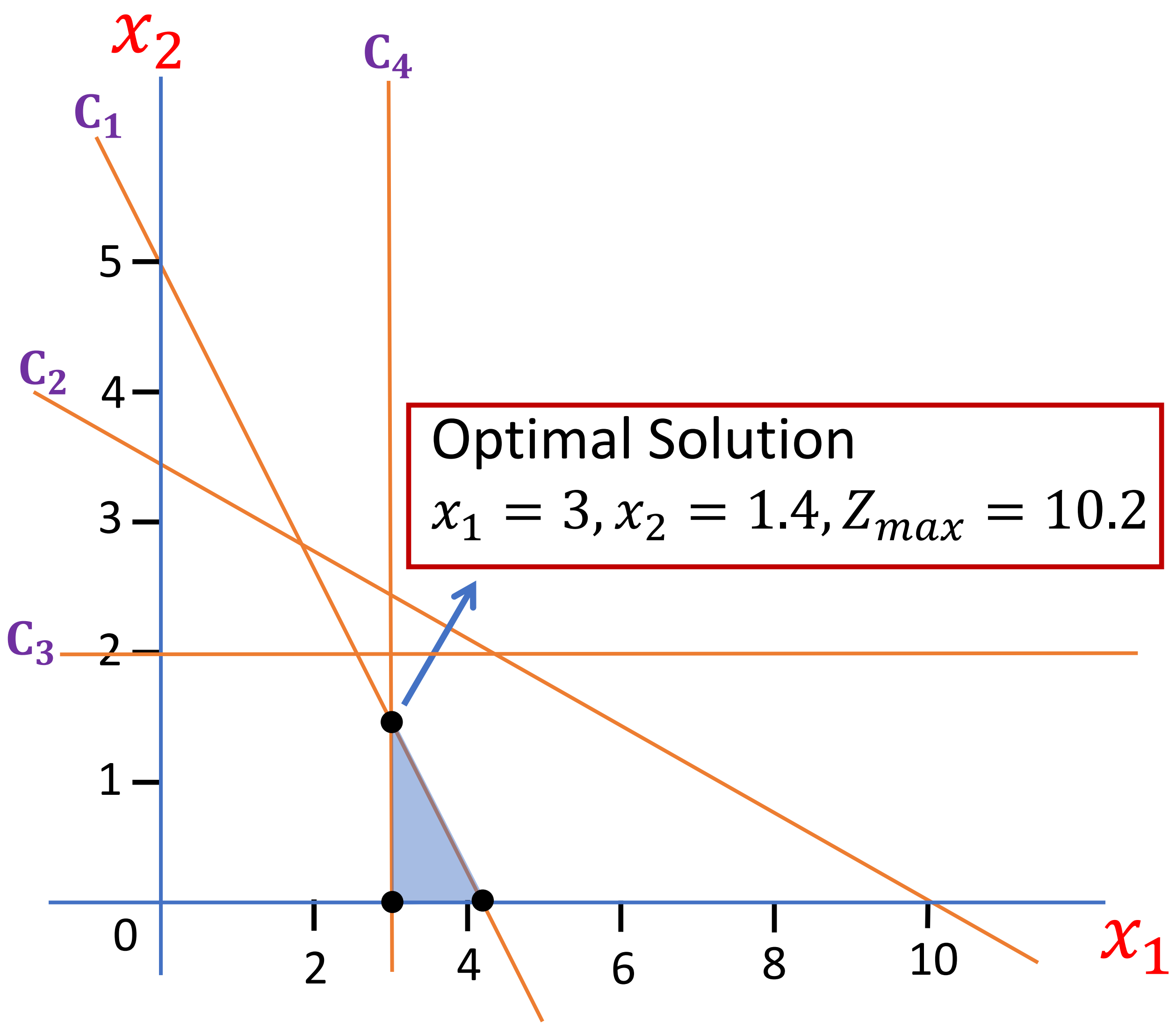

Sub problem 3 is integer solution. However, the objective function value is less than the sub problem 2. Therefore, we can ignore sub problem integer solution.

But the sub problem 4 has non-integer solution. Which implies, we need to branch the subproblem 4.

In sub problem 4 we have the value of x_2 lies between 1 and 2 ($1 \leq x_2 \leq 2$). Therefore,

Sub-Problem 5:

Max $Z = 2x_1 + 3x_2$
$6x_1 + 5x_2 \leq 25$
$x_1 + 3x_2 \leq 10$
$x_2 \leq 2$
$x_1 \geq 3$
$x_2 \leq 1$
x_1, x_2 are non-negative integers.

Sub-Problem 6:

Max $Z = 2x_1 + 3x_2$
$6x_1 + 5x_2 \leq 25$
$x_1 + 3x_2 \leq 10$
$x_2 \leq 2$
$x_1 \geq 3$ $\quad x = 2$
$x_2 \geq 2$
x_1, x_2 are non-negative integers.

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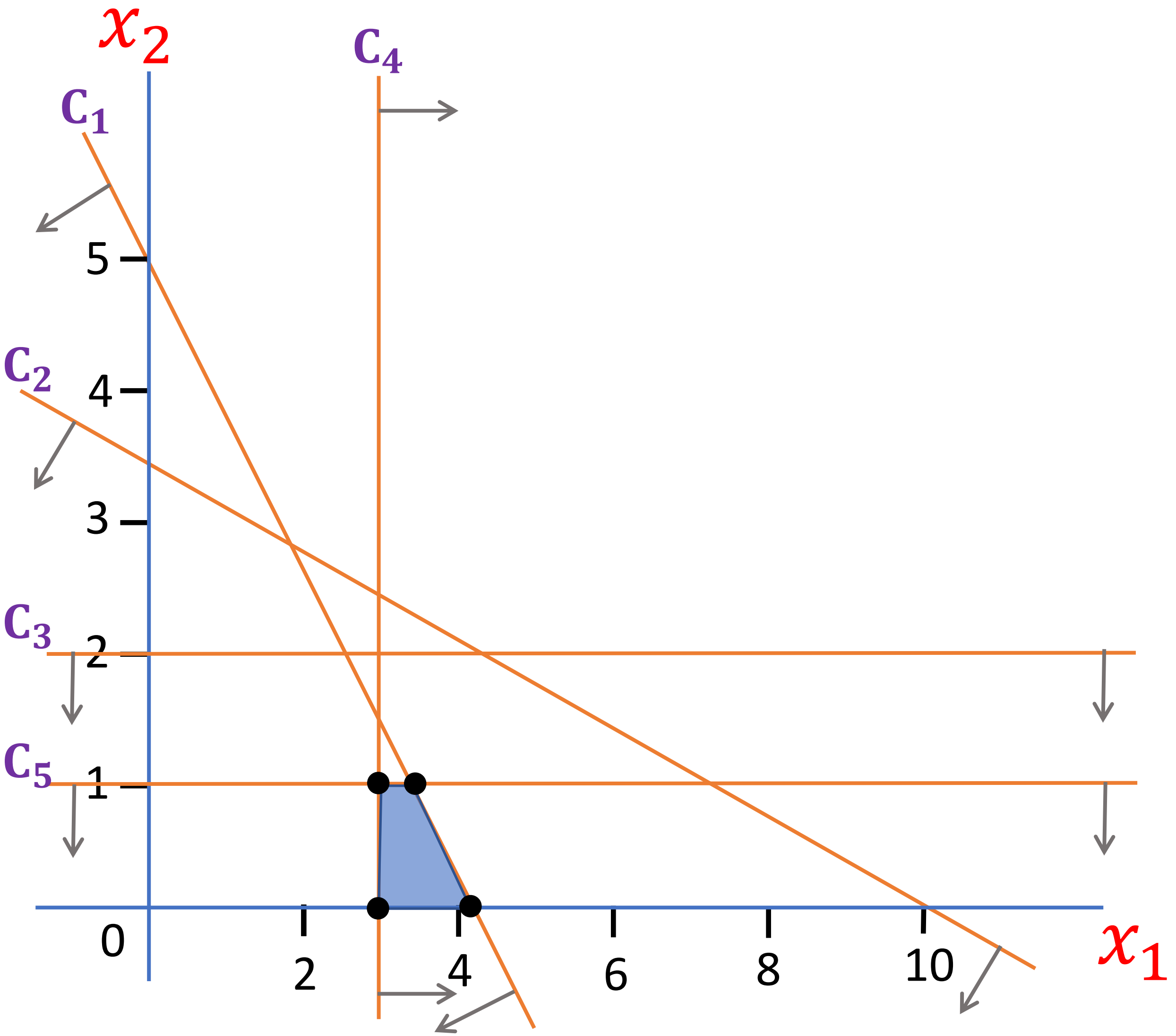

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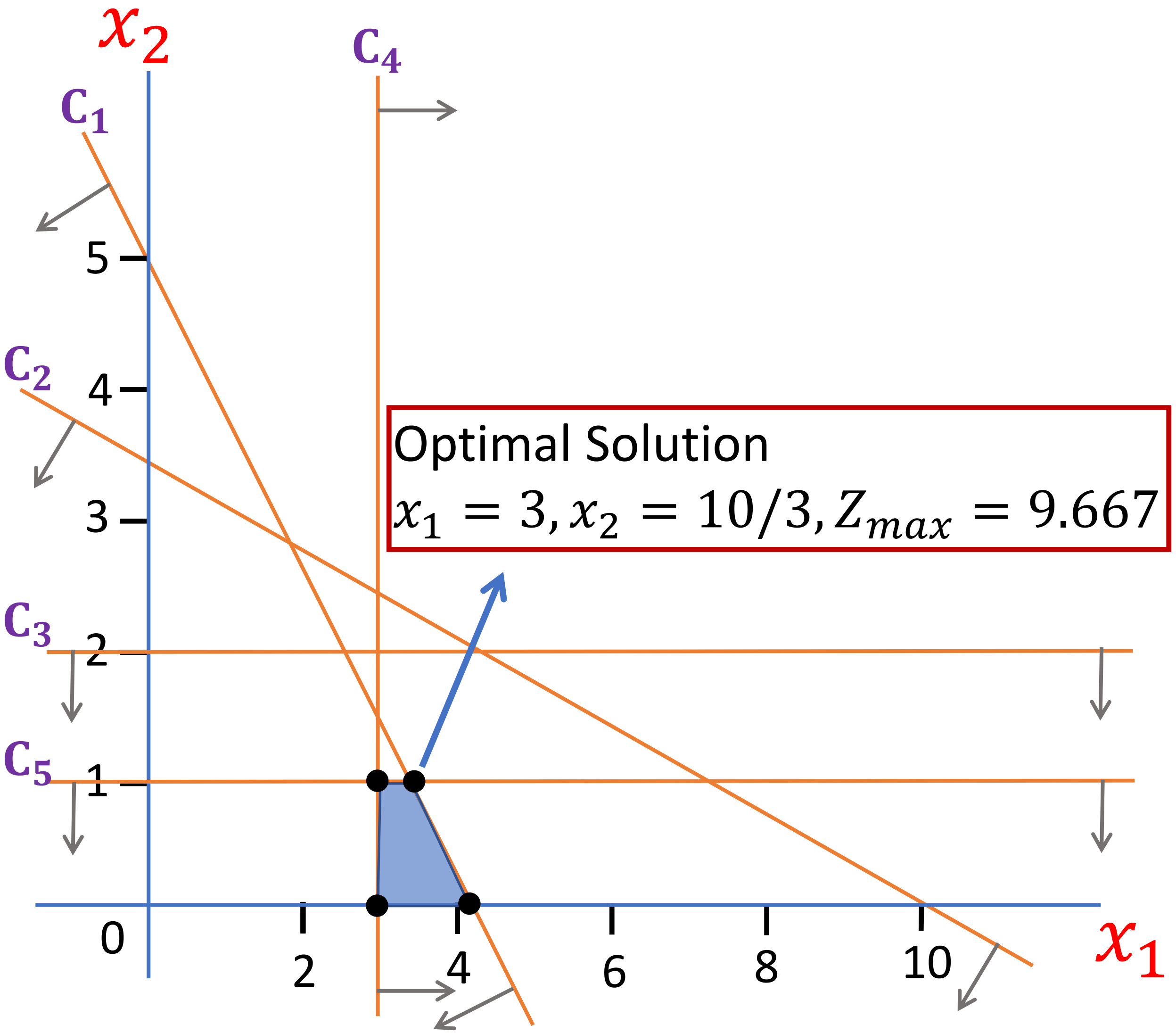
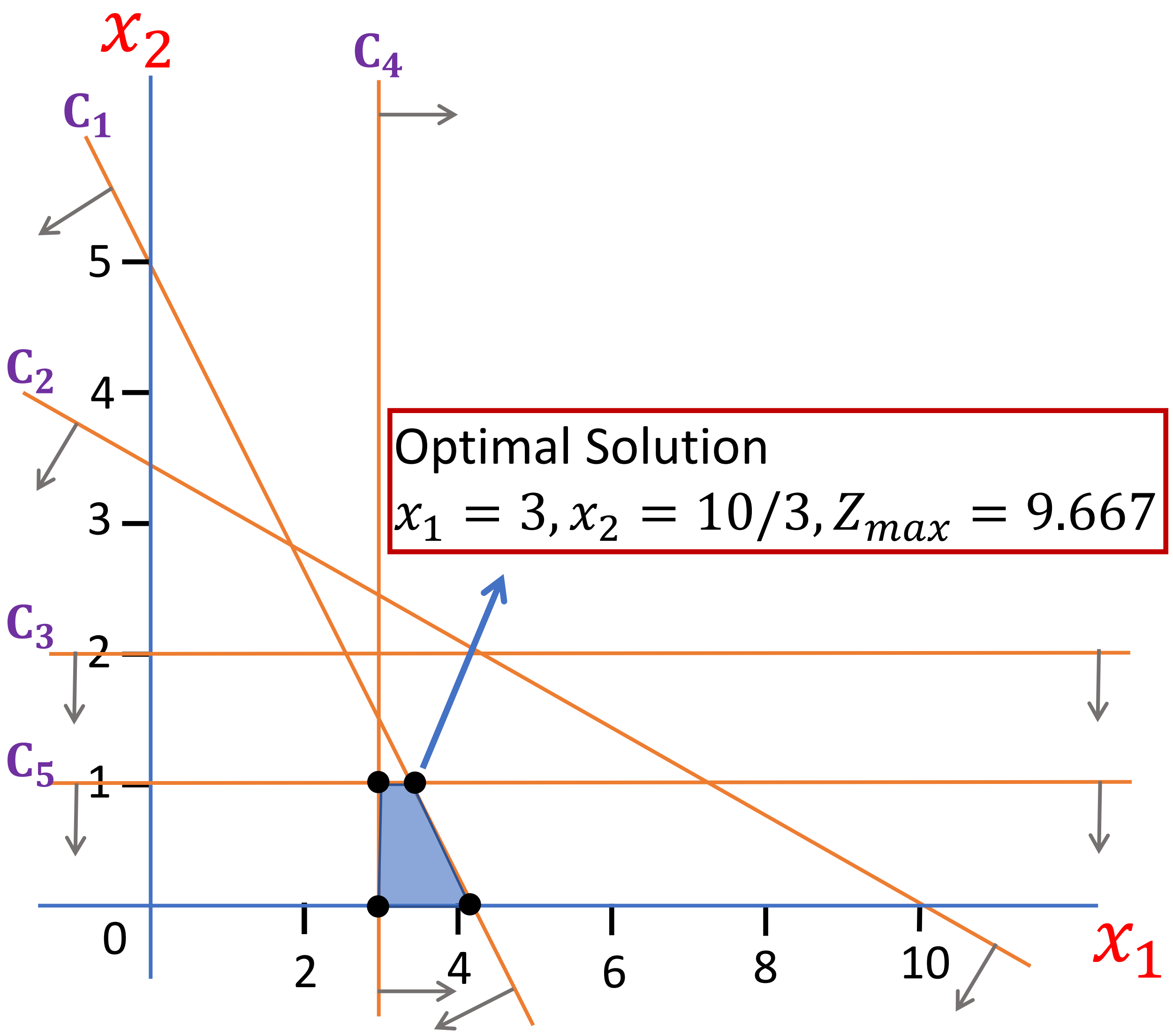
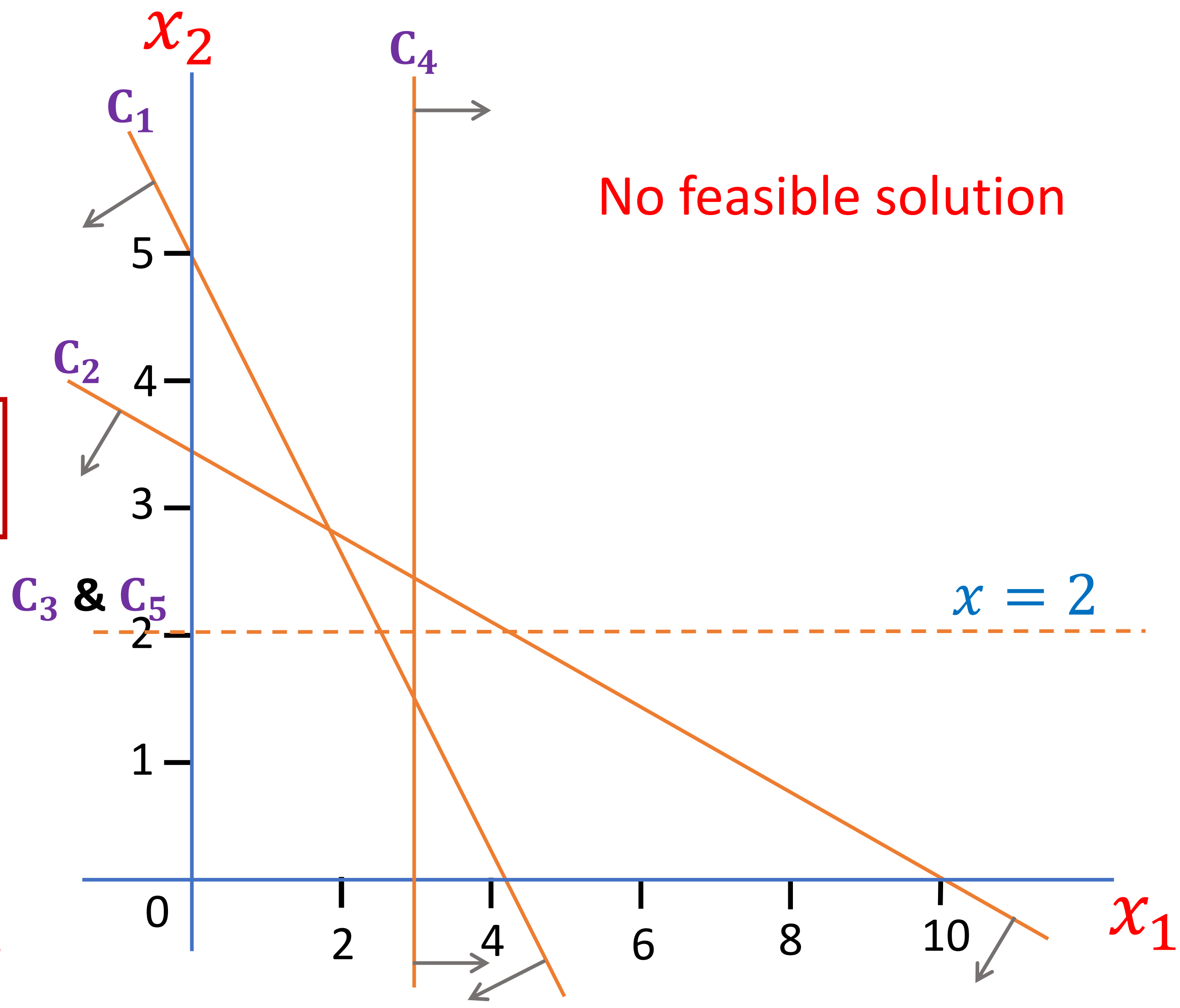


Optimal Solution
$x_1 = 3, x_2 = 10/3, Z_{max} = 9.667$

Sub-Problem 5:



Sub-Problem 6:



In sub problem 5, we didn’t reach a optimal solution again we need branch it. However, the objective function value is decreasing as compared to the previous corresponding subproblems. We can stop our branching and the optimal integer solution is given by sub problem 2.

Therefore optimal integer solution is $x_1 = 1, x_2 = 3, Z_{max} = 11$.

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Max $Z = 2x_1 + 3x_2$
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$x_2 = 2$
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Example:

Branch and Bound Method

Maximize $\quad Z = 2x_1 + 3x_2,$

subject ot $\quad 6x_1 + 5x_2 \leq 25,$

$x_1 + 3x_2 \leq 10,$

x_1, x_2 *non-negative integers.*

Optimal solution is $x_1 = 1, x_2 = 3, Z_{\max} = 11.$

Example:

Solve the following mixed integer problem by the branch and bound technique :

$$\begin{aligned} \text{Maximize} \quad & Z = x_1 + x_2, \\ \text{subject to} \quad & 2x_1 + 5x_2 \le 16, \\ & 6x_1 + 5x_2 \le 30, \\ & x_2 \ge 0, \\ & x_1 \ge 0 \text{ and integer.} \end{aligned}$$

Optimal solution is $x_1 = 4,\ x_2 = \dfrac{6}{5}\ ;\ Z_{\max} = \dfrac{26}{5} = 5.2.$