**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans : The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager have to complete work in 50 mins.

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|  | P(X > 50) = 1 – P(X ≤ 50). |
|  | Z = (X - *μ* )/ *σ* = (X - 45)/8.0 |
|  | Thus the question can be answered by using the normal table to find |
|  | P(X ≤ 50) = P(Z ≤ (50 - 45)/8.0) = P(Z ≤ 0.625)=73.4% |
|  | Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676 |

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans : The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6.

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|  | a)Probabilty of employees greater than age of 44= P(X>44) |
|  | P(X > 44) = 1 - P(X ≤ 44). |
|  | Z = (X - )/ = (X - 38)/6 |
|  | Thus the question can be answered by using the normal table to find |
|  | P(X ≤ 44) = P(Z ≤ (44 - 38)/6) = P(Z ≤ 1)=84.1345% |
|  | Probabilty that the employee will be greater than age of 44 = 100-84.1345=15.86% |
|  | So the probability of number of employees between 38-44 years of age = P(X<44)-0.5=84.1345-0.5= 34.1345% |
|  | Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is False. |
|  | b) Probabilty of employees less than age of 30 = P(X<30). |
|  | Z = (X - )/ = (30 - 38)/6 |
|  | Thus the question can be answered by using the normal table to find |
|  | P(X ≤ 30) = P(Z ≤ (30 - 38)/6) = P(Z ≤ -1.333)=9.12% |
|  | So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48( or 36 employees). |
|  | Therefore the statement B of the question is also TRUE. |

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans : As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) . Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ). Therefore in the question 2X1~ N(2 u,4 σ^2) and X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

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1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: p(a<x<b) = 0.99 ,m ean =100,standardDeviation = 20

From the above details,we have to excluded area of .005 in each of the left and right tails.

Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

          Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

          Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

      a = -(20\*2.576) + 100= 48.5

b = (20\*2.576)+100= 151.5

Two values symmetric about mean for the given standard normal distribution

Are D[48.5,151.5]

Or else we can use code

from scipy import stats

stats**.** norm**.** interval(0.99,100,20)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 3) and Profit2 ~ N(7, 4) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Ans: Mean profit from two different divisions of a company = Mean1 + Mean2

Mean = 5+7

Mean Profit is 12\*45 = 540rs.

Sd = sqrt((9)+(16))

Standard Deviation is sd\*45 = 225rs.

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans : stats.norm.interval(0.95,540,225)

Range(99.0081,980.9918)

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans : To compute 5th percentile we use the formula x = μ+zσ

x = 540 + (-1.645)\*(225) = 170rs.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: stats.norm.cdf(0,5,3) = 0.0477