UNIVERSAL AI UNIVERSITY, KARJAT

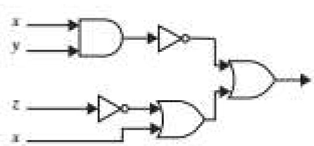
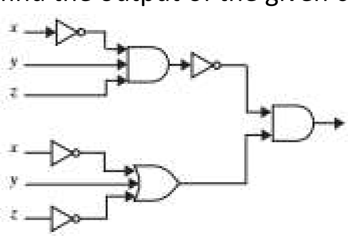
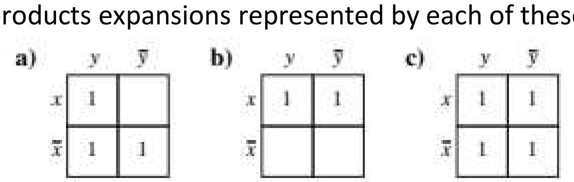
ASSIGNMENT-I- AUG 2024

Program Name:	**B.Tech (AI & ML)**	Semester:	**III Year: 2024-2025 Date:09.09.2024**
Course Code:	**QTM5.02001**	Course Name:	**Discrete Mathematics**
Max. Marks:	**50**	Total Questions: **23**	Duration: **7 Days**

Instructions -

		Marks	*BL*	CO	PO
1	Let p and q be the propositions p: You drive over 65 miles per hour. q: You get a speeding ticket. Write these propositions using p and q and logical connectives (including negations). **a)** You do not drive over 65 miles per hour. **b)** You drive over 65 miles per hour, but you do not get a speeding ticket. **c)** You will get a speeding ticket if you drive over 65 miles per hour. **d)** If you do not drive over 65 miles per hour, then you will not get a speeding ticket. **e)** Driving over 65 miles per hour is sufficient for getting a speeding ticket. **f)** You get a speeding ticket, but you do not drive over 65 miles per hour. **g)** Whenever you get a speeding ticket, you are driving over 65 miles per hour.	3			
2	Write each of these statements in the form "if p, then q" in English. [*Hint:* Refer to the list of common ways to express conditional statements.] **a)** It snows whenever the wind blows from the northeast. **b)** The apple trees will bloom if it stays warm for a week. **c)** That the Pistons win the championship implies that they beat the Lakers. **d)** It is necessary to walk 8 miles to get to the top of Long's Peak. **e)** To get tenure as a professor, it is sufficient to be world famous. **f)** If you drive more than 400 miles, you will need to buy gasoline. **g)** Your guarantee is good only if you bought your CD player less than 90 days ago. **h)** Jan will go swimming unless the water is too cold.	3			
3	Construct a truth table for each of these compound propositions. **a)** $(p \vee q) \rightarrow (p \oplus q)$ **b)** $(p \oplus q) \rightarrow (p \wedge q)$ **c)** $(p \vee q) \oplus (p \wedge q)$ **d)** $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$ **e)** $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$ **f)** $(p \oplus q) \rightarrow (p \oplus \neg q)$	2			
4	Express these system specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives (including negations). **a)** The message is scanned for viruses whenever the message was sent from an unknown system. **b)** The message was sent from an unknown system but it was not scanned for viruses. **c)** It is necessary to scan the message for viruses whenever it was sent from an unknown system. **d)** When a message is not sent from an unknown system it is not scanned for viruses.	3			

5	Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.	2			
6	Determine whether each of these compound propositions is satisfiable. **a)** $(p \lor \neg q) \land (\neg p \lor q) \land (\neg p \lor \neg q)$ **b)** $(p \to q) \land (p \to \neg q) \land (\neg p \to q) \land (\neg p \to \neg q)$ **c)** $(p \leftrightarrow q) \land (\neg p \leftrightarrow q)$	2			
7	Let $Q(x, y)$ denote the statement "x is the capital of y." What are these truth values? a) Q(Denver, Colorado) *b)* Q(Detroit, Michigan) c) Q(Massachusetts, Boston) d) Q(New York, New York)	2			
8	Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class. **a)** A student in your class has a cat, a dog, and a ferret. **b)** All students in your class have a cat, a dog, or a ferret. **c)** Some student in your class has a cat and a ferret, but not a dog. **d)** No student in your class has a cat, a dog, and a ferret. **e)** For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.	3			
9	Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables. **a)** A student in your school has lived in Vietnam. **b)** There is a student in your school who cannot speak Hindi. **c)** A student in your school knows Java, Prolog, and C++. **d)** Everyone in your class enjoys Thai food. **e)** Someone in your class does not play hockey.	2			
10	Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a professor," "x is ignorant," and "x is vain," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$, where the domain consists of all people. **a)** No professors are ignorant. **b)** All ignorant people are vain. **c)** No professors are vain. **d)** Does (c) follow from (a) and (b)?	2			
11	Translate these statements into English, where the domain for each variable consists of all real numbers. **a)** $\forall x \exists y (x < y)$ **b)** $\forall x \forall y (((x \geq 0) \land (y \geq 0)) \to (xy \geq 0))$ **c)** $\forall x \forall y \exists z (xy = z)$	2			
12	Determine the truth value of each of these statements if the domain for all variables consists of all integers. **a)** $\forall n \exists m (n2 < m)$ **b)** $\exists n \forall m (n < m2)$ **c)** $\forall n \exists m (n + m = 0)$ **d)** $\exists n \forall m (nm = m)$ **e)** $\exists n \exists m (n2 + m2 = 5)$ **f)** $\exists n \exists m (n2 + m2 = 6)$ **g)** $\exists n \exists m (n + m = 4 \land n - m = 1)$ **h)** $\exists n \exists m (n + m = 4 \land n - m = 2)$ **i)** $\forall n \forall m \exists p (p = (m + n)/2)$	2			

13	What rule of inference is used in each of these arguments? **a)** Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major. **b)** Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major. **c)** If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed. **d)** If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today. **e)** If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore,	2			
14	Use a table to express the values of each of these Boolean functions. **a)** *F (x, y, z) = xy* **b)** *F (x, y, z) = x + yz* **c)** *F (x, y, z) = xy + (xyz)* **d)** *F (x, y, z) = x(yz + y z)*	2			
15	Show that *xy + yz + xz = xy + yz + xz*	2			
16	Find the sum-of-products expansions of these Boolean functions. **a)** F (x, y, z) = x + y + z **b)** F (x, y, z) = (x + z) y **c)** F (x, y, z) = x **d)** F (x, y, z) = x y	2			
17	find the output of the given circuit. 	2			
18	find the output of the given circuit. 	2			
19	Construct a circuit for a half subtractor using AND gates, OR gates, and inverters. A **half subtractor** has two bits as input and produces as output a difference bit and a borrow.	2			
20	Construct a circuit for a full subtractor using AND gates, OR gates, and inverters. A **full subtractor** has two bits and a borrow as input, and produces as output a difference bit and a borrow.	2			
21	Find the sum-of-products expansions represented by each of these K-maps. 	2			

22	Use a K-map to find a minimal expansion as a Boolean sum of Boolean products of each of these functions of the Boolean variables *x* and *y*. **a)** xy + x y **b)** xy + xy **c)** xy + xy + xy + x	**2**			
23	Use the Quine–McCluskey method to simplify the sum-of-products expansions. **a)** wxyz + wxyz + wxyz + wxyz + w xyz + w x yz **b)** wxyz + wxyz + wxyz + wx yz + wx y z + wxyz + w xyz + w x yz **c)** wxyz + wxyz + wxyz + wxyz + wxyz + wxyz + w xyz + w xyz + w x yz	**2**			
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