

AI by Hand



Volume 1

Basic x 5
Advanced x 20

Prof. Tom Yeh

2024

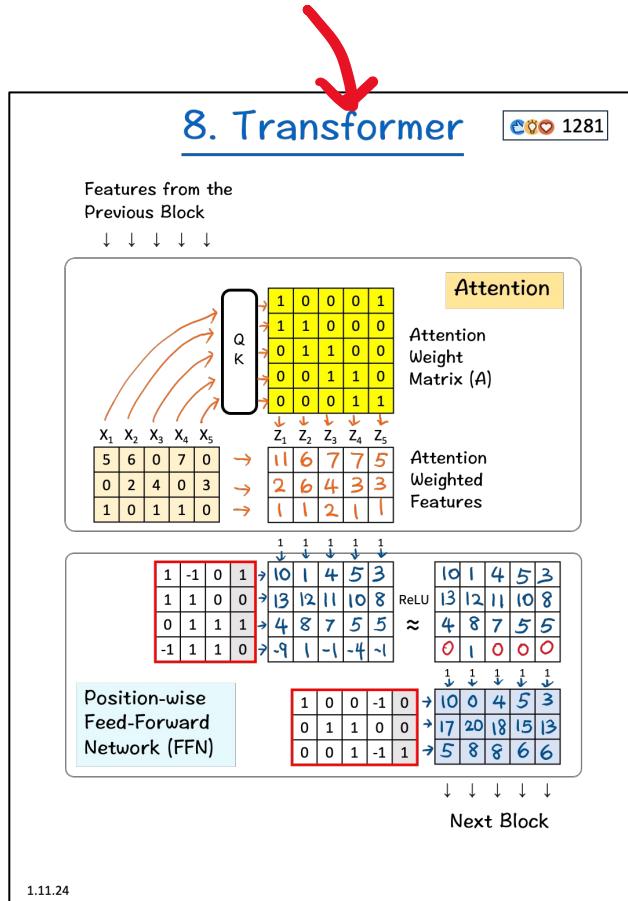
Basic

- I. One Node
- II. Four Nodes
- III. One Hidden Layer
- IV. Three Inputs
- V. Seven Layers

Advanced

- 1. Mixture of Experts (MOEs)
- 2. Recurrent Neural Network (RNN)
- 3. Mamba
- 4. Matrix Multiplication
- 5. LLM Sampling
- 6. MLP in PyTorch
- 7. Backpropagation
- 8. Transformer
- 9. Batch Normalization
- 10. Generative Adversarial Network (GAN)
- 11. Self Attention
- 12. Dropout
- 13. Autoencoder
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- 15. CLIP
- 16. Residual Network (ResNet)
- 17. Graph Convolution Network (GCN)
- 18. SORA's Diffusion Transformer (DiT)
- 19. Gemini 1.5's Switch Transformer
- 20. Reinforcement Learning with Human Feedback (RLHF)

Link to my original LinkedIn post
with animation and explanation



Date originally posted

I. One Node

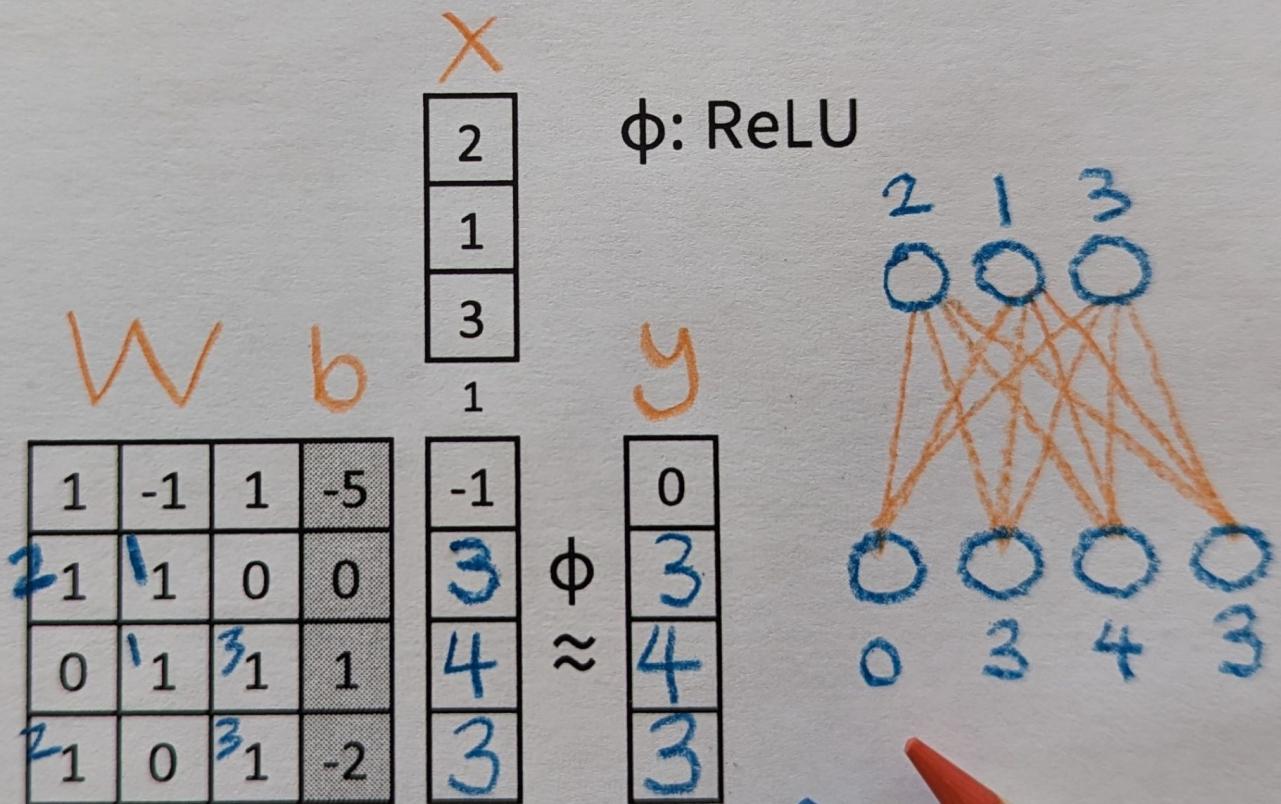
$$\begin{array}{c} \boxed{2} \\ \boxed{1} \\ \boxed{3} \end{array} \times \quad \text{ReLU}$$

w b 1 ϕ \approx 0

$$\begin{array}{|c|c|c|c|} \hline 1 & -1 & 1 & -5 \\ \hline \end{array}$$
$$\begin{array}{|c|} \hline -1 \\ \hline \end{array}$$

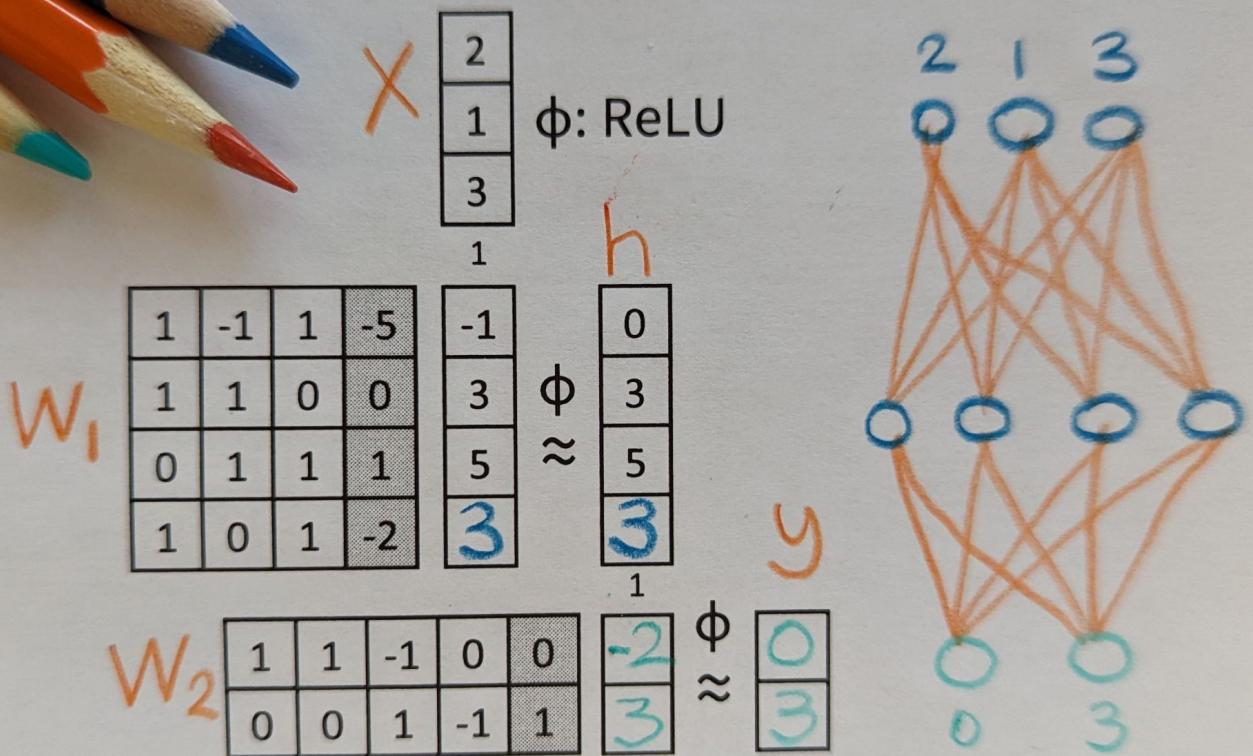
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II. Four Nodes



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III. Hidden Layer



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IV. Three Inputs

2	1	0
1	1	1
3	0	1

1 1 1

ϕ : ReLU

1	-1	1	-5
1	1	0	0
0	1	1	1
1	0	1	-2

-1	-5	-5
3	2	1
5	2	3
3	-1	-1

$\phi \approx$

0	0	0
3	2	1
5	2	3
3	0	0

1 1 1

1	1	-1	0	0
0	0	1	-1	1

-2	0	-2
3	3	4

$\phi \approx$

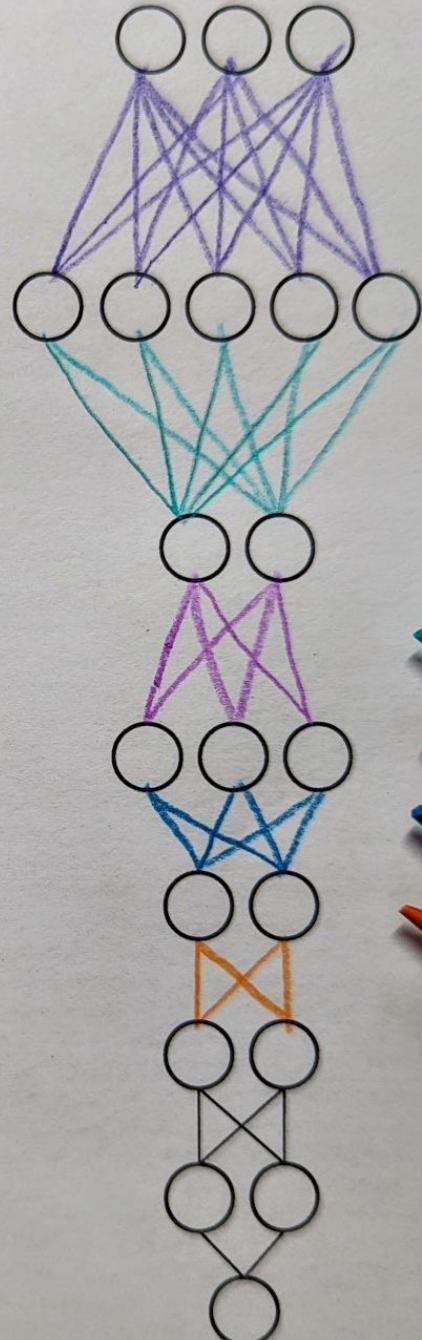
0	0	0
3	3	4



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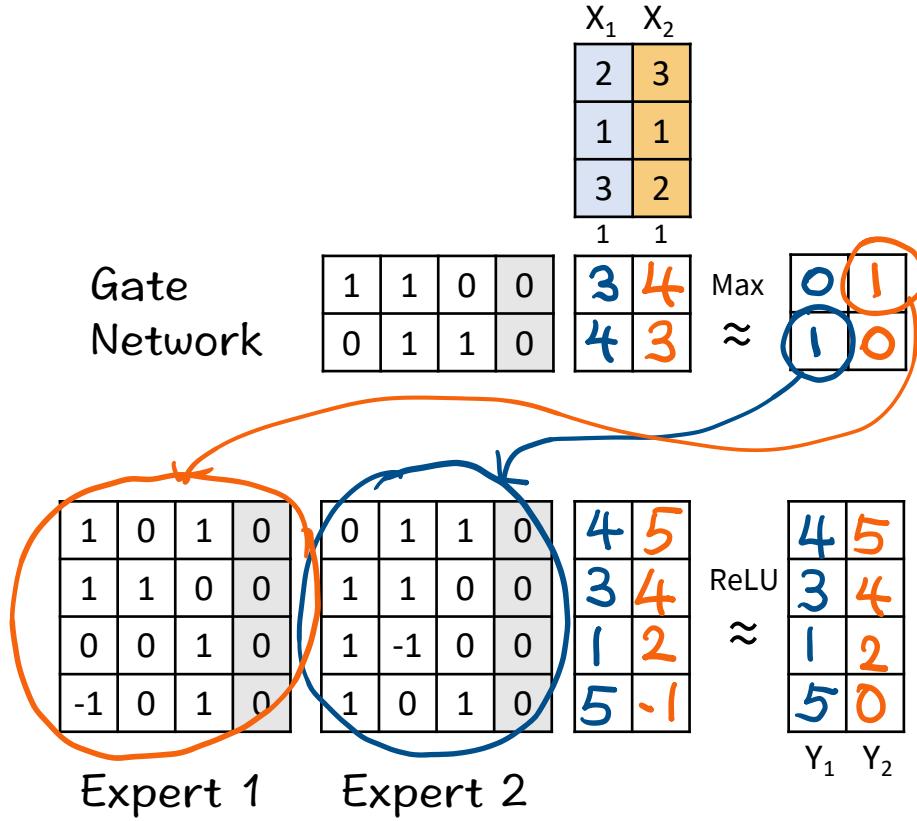
V. Seven Layers

					3 5
					4 4
					5 3
	0 0 1				5 3
w_1	0 1 0				4 4
	1 0 0				3 5
	1 1 0				7 9
	0 1 1				9 7
w_2	1 1 -1 0 0				6 2
	0 0 1 1 -1				1 7
w_3	1 1				7 9
	1 -1				5 -5
	1 2				8 16
w_4	1 -1 0				2 9
	0 -1 1				3 16
w_5	0 1				3 21
	1 0				2 9
w_6	1 -1				1 12
	1 1				5 30
w_7	1 -1		-4	-18	



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1. Mixture of Experts



2. Recurrent Neural Network (RNN)



Input Sequence

X	3	4	5	6
---	---	---	---	---

Parameters

$$A \begin{array}{|c|c|} \hline 1 & -1 \\ \hline 1 & 1 \\ \hline \end{array} \quad B \begin{array}{|c|c|} \hline 1 \\ \hline 2 \\ \hline \end{array} \quad C \begin{array}{|c|c|} \hline -1 & 1 \\ \hline \end{array}$$

Activation Function

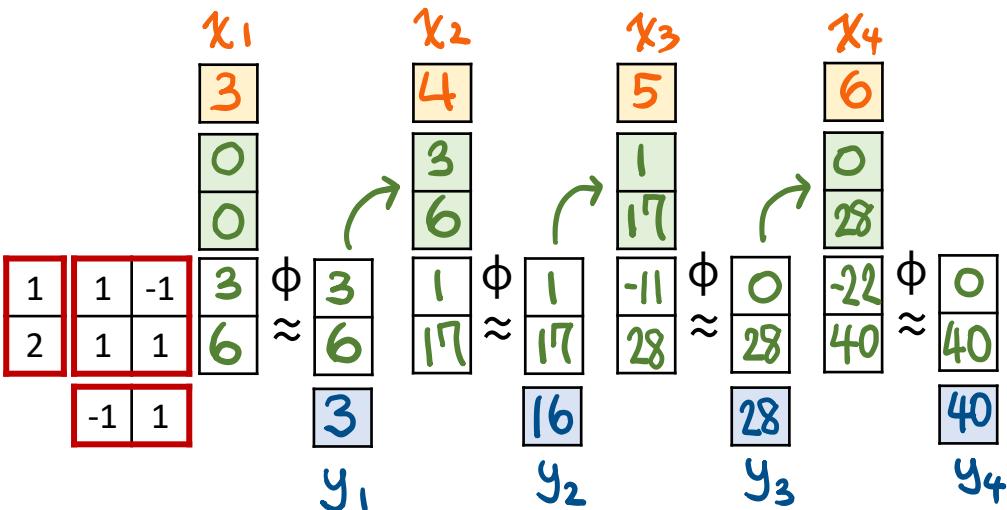
ϕ : ReLU

Hidden States

H_0	0	
	0	

Output Sequence

y				
---	--	--	--	--

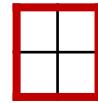


3. Mamba's S6 Model

Input Sequence

3	4	5	6
---	---	---	---

Parameters

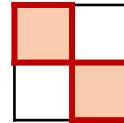


Output Sequence

-1	2	-3	24	1
----	---	----	----	---

Selective

Structured



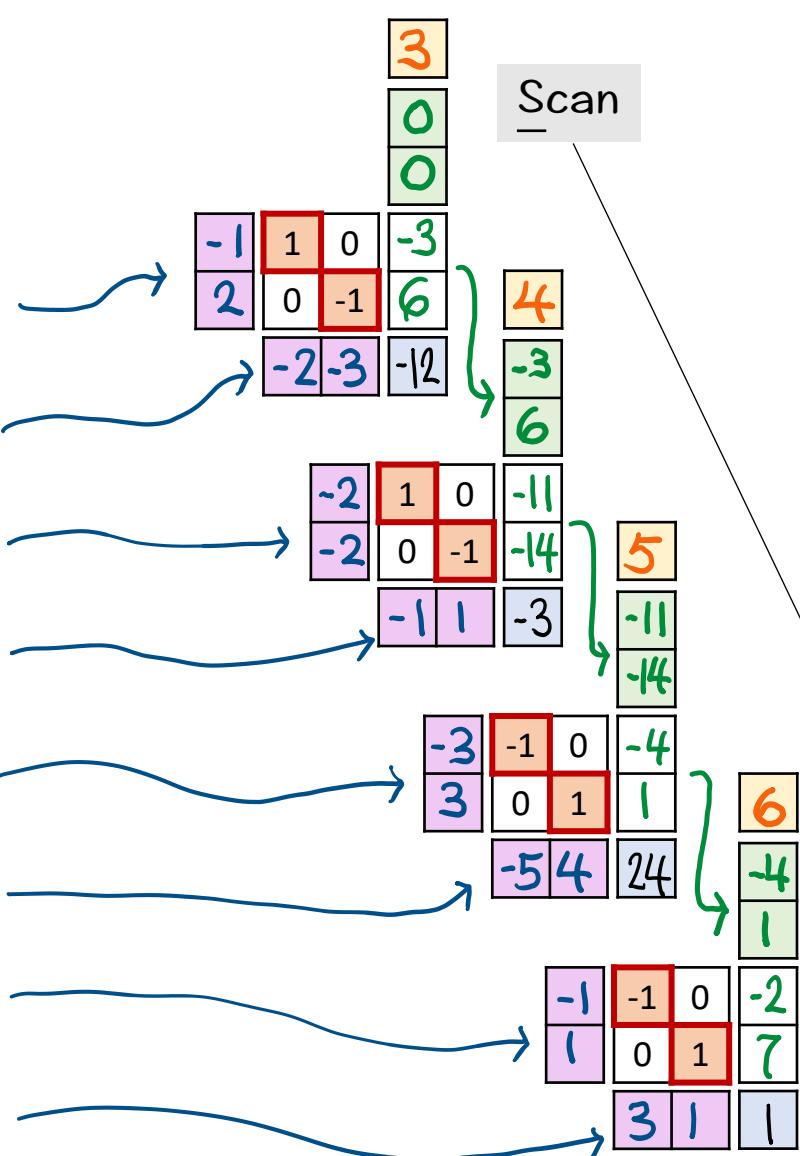
State-Space



1	-1	0	0
0	-1	0	1
1	0	-1	0
1	0	0	-1
1	0	-1	0
0	1	0	-1
1	-1	0	0
0	0	-1	1
-1	0	0	0
1	0	0	0
0	0	-1	0
0	1	0	0
1	-1	0	0
0	0	-1	1
1	0	0	0
0	-1	1	0

3
4
5
6

-1
2
-2
-3
-2
-2
-1
1
-3
3
-5
4
-1
1
3
1



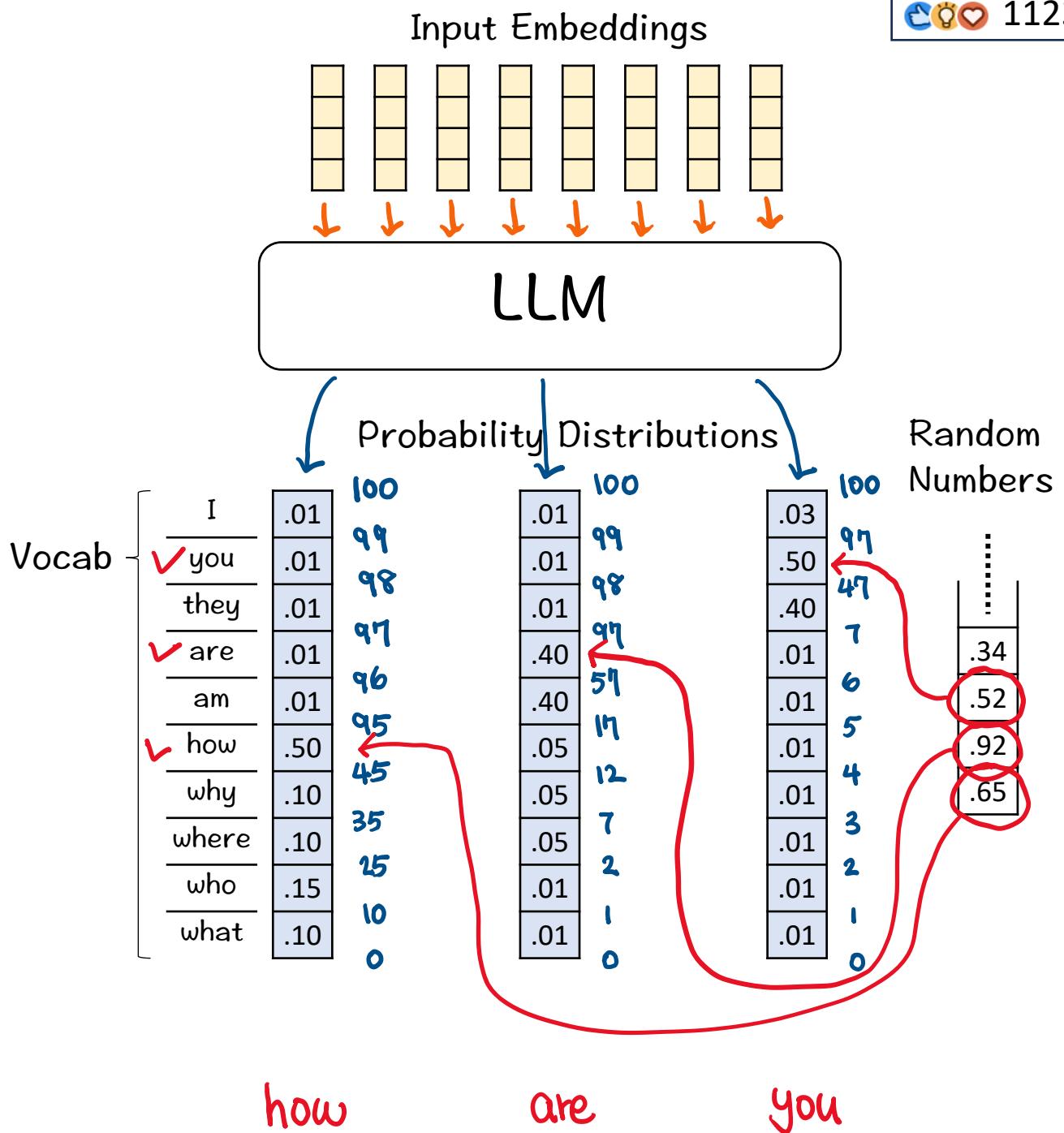
4. Matrix Multiplication

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 5 & 2 \\ 2 & 4 & 2 \end{bmatrix} = ?$$

1	5	2
2	4	2

1	1	-1
3	9	4
1	-1	0

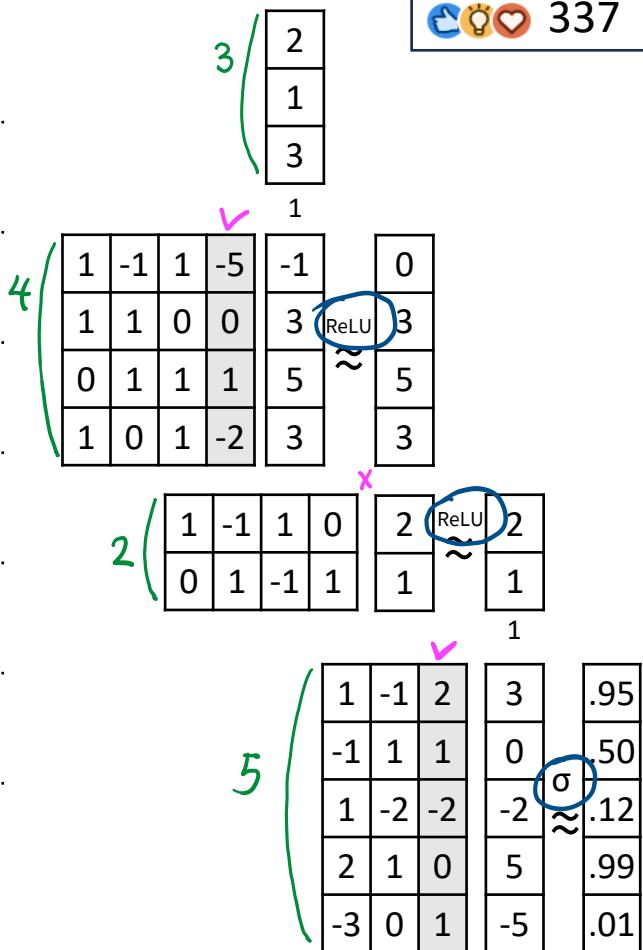
5. How does an LLM sample a sentence?



6. Multi Layer Perceptron in pytorch



```
1 mlp_model = nn.Sequential(  
.....  
2     nn.Linear( 3, 4, bias = T ),  
.....  
3     nn.ReLU(),  
.....  
4     nn.Linear( 4, 2, bias = F ),  
.....  
5     nn.ReLU(),  
.....  
6     nn.Linear( 2, 5, bias = T ),  
.....  
7     nn.Sigmoid()  
.....  
8 )
```



Hints:

Linear Layer: { Identity | Linear | Bilinear }

Activation Function: { ReLU | Tanh | Sigmoid }

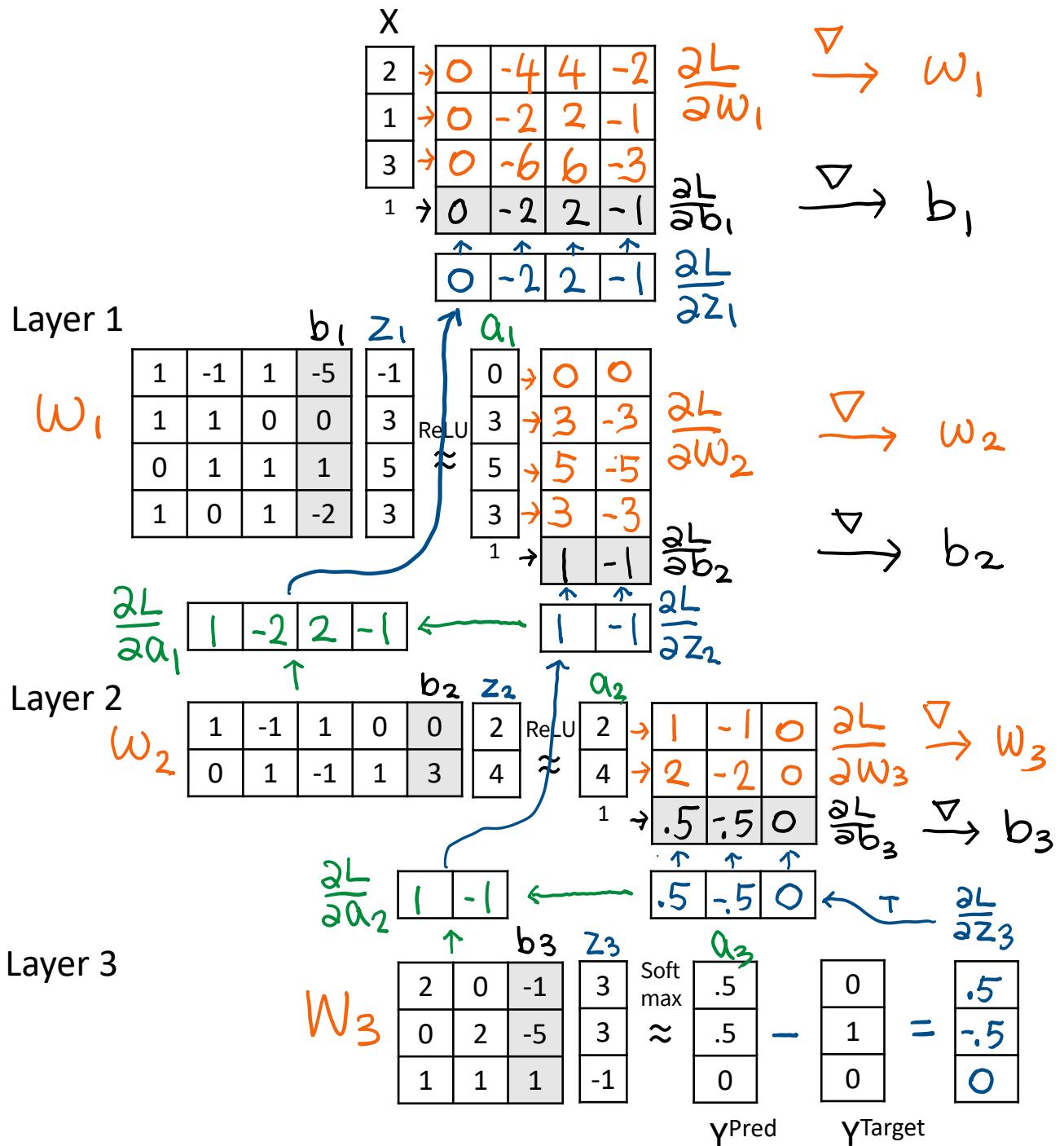
in_features: { int }

out_features: { int }

bias: { T | F }

7. Backpropagation

1260

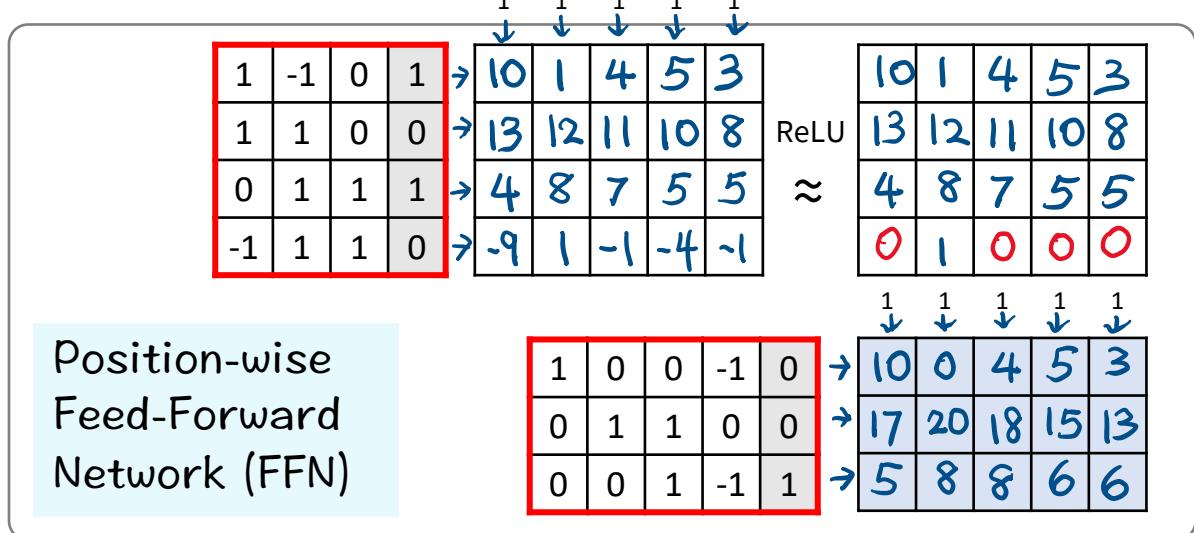
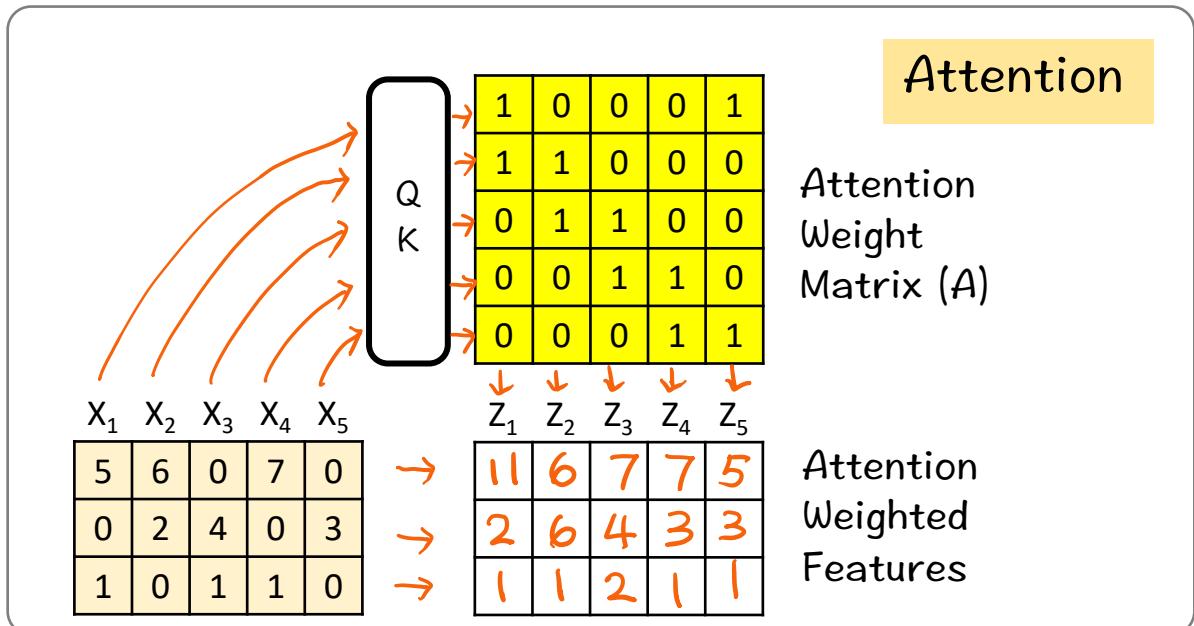


L : Cross-Entropy Loss

8. Transformer

Features from the
Previous Block

↓ ↓ ↓ ↓ ↓



↓ ↓ ↓ ↓ ↓

Next Block

9. Batch Normalization

Mini-batch: $x_1 \ x_2 \ x_3 \ x_4$

1	0	3	0
0	3	1	1
2	1	0	2

Linear Layer

1	0	1	0	→	3	1	3	2
1	1	0	-1	→	0	2	3	0
0	2	-1	0	→	-2	5	2	0

ReLU
≈

3	1	3	2
0	2	3	0
0	5	2	0

Σ	μ	σ^2	σ
9	2	1	1
5	1	1	1
7	2	4	2

Normalize

$$\begin{array}{r} \mu \\ - \\ \hline 2 \\ 1 \\ 2 \end{array}$$

Sum (Σ)
Mean (μ)
Variance (σ^2)
Std Dev (σ)

1	-1	1	0
-1	1	2	-1
-2	3	0	-2

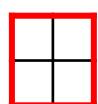
$$\begin{array}{r} \sigma \\ \div \\ \hline 1 \\ 1 \\ 2 \end{array}$$

1	-1	1	0
-1	1	2	-1
-1	1	0	-1

Scale & Shift

2	0	0	0	→	2	-2	2	0
0	3	0	0	→	-3	3	6	-3
0	0	-1	1	→	2	0	1	2

Trainable Parameters



Next Layer

10. Generative Adversarial Network (GAN)



Generator

Noise: N ₁ N ₂ N ₃ N ₄			
1	1	0	1
1	0	1	-1

1 1 0	→	2 1 1 0
0 1 2	→	3 2 3 1
-1 1 0	→	0 1 1 0

[≈ ReLU]

Fake: F₁ F₂ F₃ F₄

-1 1 0 0	→	1 1 2 1
1 0 1 0	→	2 1 2 0
0 1 1 0	→	3 2 4 1
0 0 1 1	→	1 1 2 1

[≈ ReLU]

Real:

X ₁	X ₂	X ₃	X ₄
2	3	3	4
1	1	1	1
2	3	4	3
1	1	1	1

↓ ↓ ↓ ↓

Discriminator

1 0 0 -1 0	→	0 0 0 0
0 1 1 0 0	→	5 3 6 1
0 0 1 -1 1	→	3 2 3 1

[≈ ReLU]

1 1 -1 -1	→ Z	1 0 2 -1
-----------------	-----	----------------

[≈ σ]

1	2	2	3
3	4	5	4
2	3	4	3

↓ ↓ ↓ ↓

1 | 2 | 2 | 3

[≈ σ]

Predictions: Y .7 .5 .9 .3

.7 .9 .9 .1

Training the Discriminator

Targets: - Y_D 0 | 0 | 0 | 0

1 | 1 | 1 | 1

Loss Gradients: $\frac{\partial L_D}{\partial Z}$.7 .5 .9 .3 - .3 -.1 -.1 0

Training the Generator

Targets: - Y_G 1 | 1 | 1 | 1

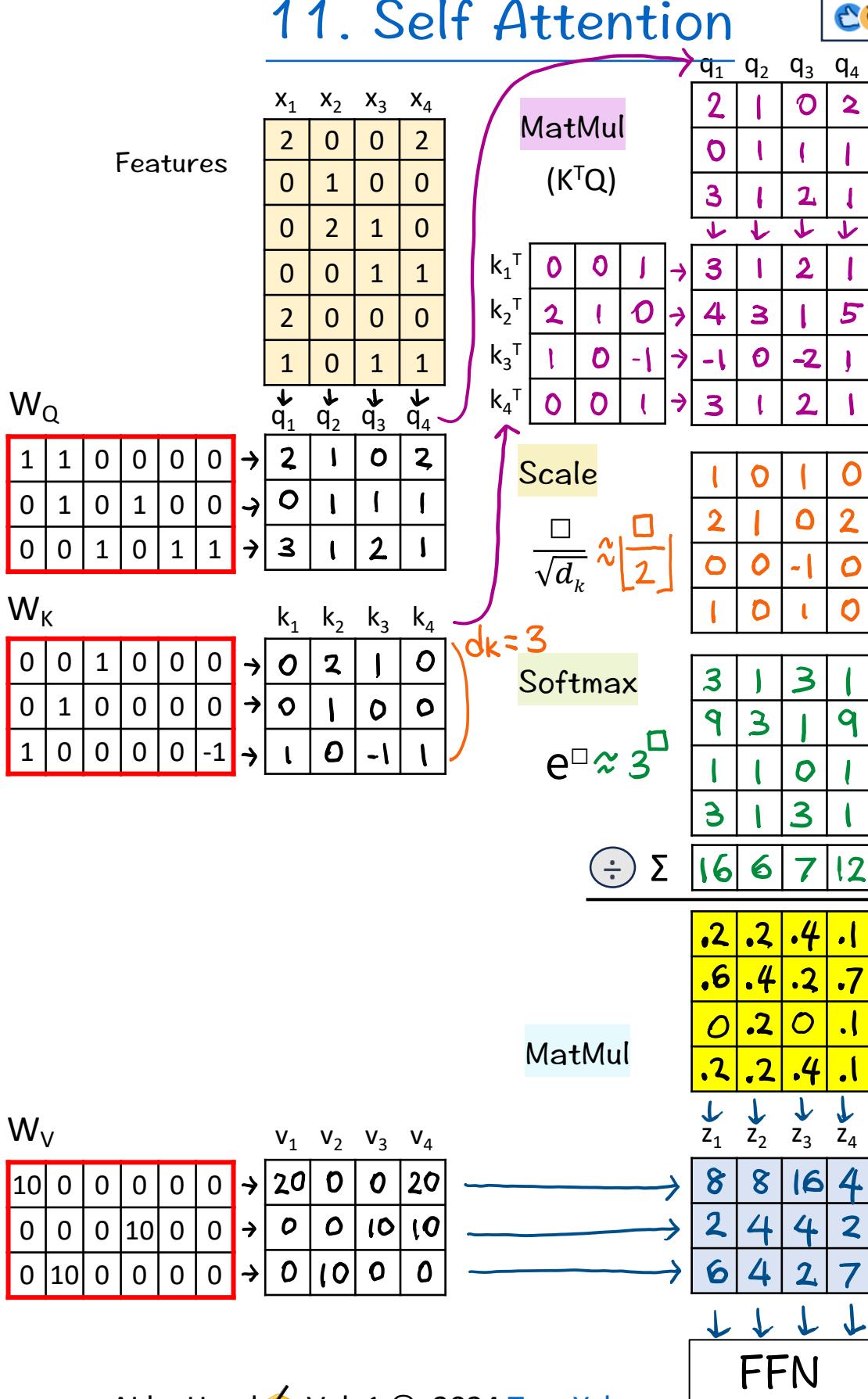
1 | 1 | 1 | 1

Loss Gradients: $\frac{\partial L_G}{\partial Z}$ -.3 -.5 -.1 -.7

11. Self Attention



1376



12. Dropout



557

Random Sequence

.61 >.5

.39
.75
40
.65
.42
.23

Linear

Training Data:

X_1	X_2
3	5
4	1

1 1

1	0	0
1	1	0
0	1	1
1	-1	0

[≈ ReLU]

3	5
7	6

5 2

4

Dropout
($p=0.5$)

$$\frac{1}{1-p} = 2$$

2	0	0	0
0	0	0	0
0	0	2	0
0	0	0	0

6	10
0	0

10 4

0 0

Linear

1	0	0	1	0
0	1	1	0	0
1	0	-1	-1	1

[≈ ReLU]

6	10
10	4

4 6

Dropout
($p=0.33$)

$$\frac{1}{1-p} = 1.5$$

1.5	0	0
0	1.5	0
0	0	0

9	15
15	6

0 0

1 1

Linear

1	-1	0	0
0	1	-1	-2

-6	9
13	4

Training



-4	7
10	5

-2	2
3	-1

Targets

Y'

$\times 2$

-4	4
6	-2

MSE Loss Gradients

$$\frac{\partial L}{\partial Y}$$

Inference

Unseen Data:

3	3
2	1

1 1

1	0	0	0
1	1	1	1
-1	1	1	1
1	-1	0	0

[≈ ReLU]

3	3
6	5

0 X

1 2

1 1

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

3	3
6	5

0 0

1 2

1	0	1	1	0
1	1	1	0	0
1	0	-1	0	1

[≈ ReLU]

4	5
9	8

4 4

1 1

1	0	0
0	1	0
0	0	1

4	5
9	8

4 4

1 1

1	1	0	0
0	1	-1	-1

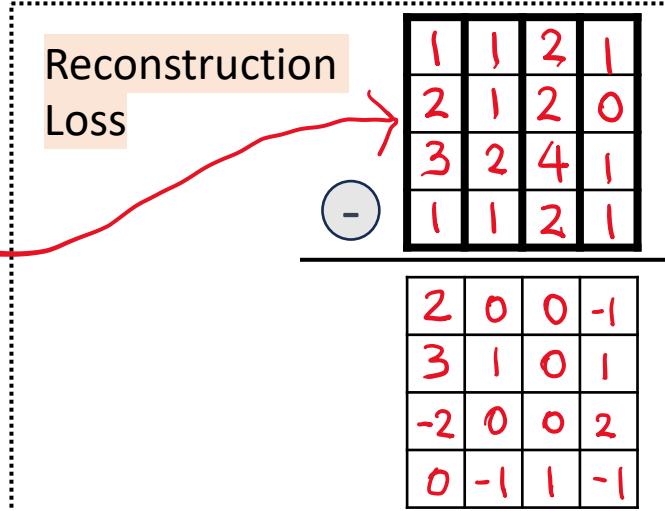
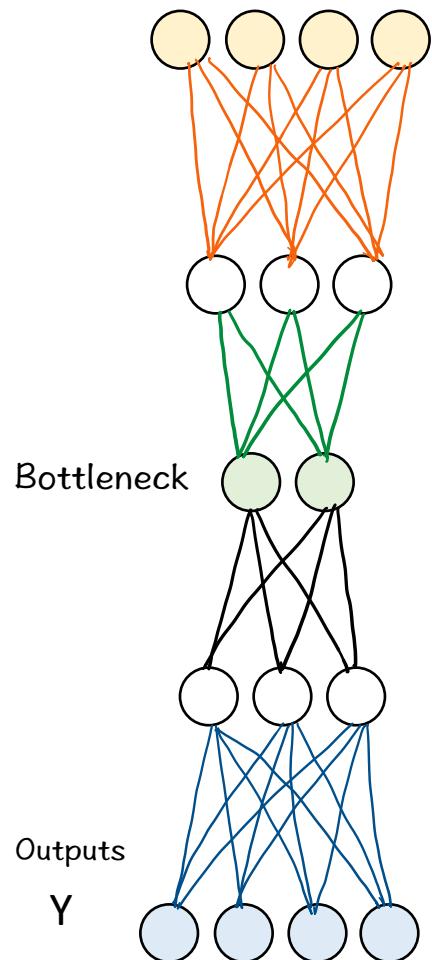
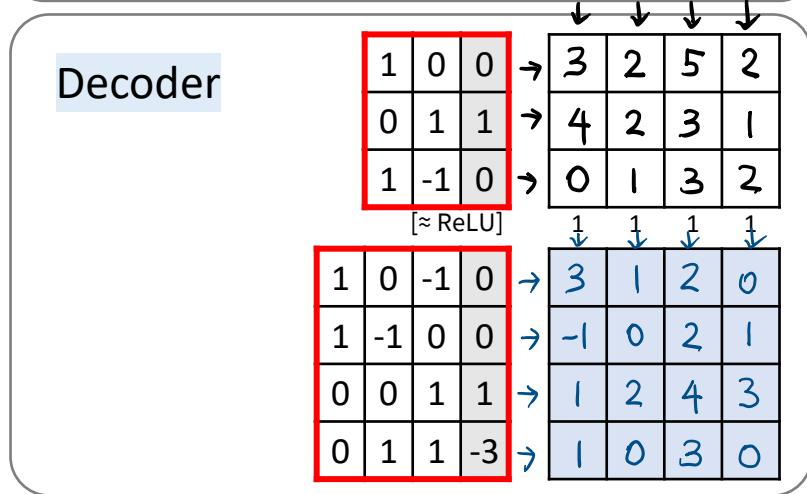
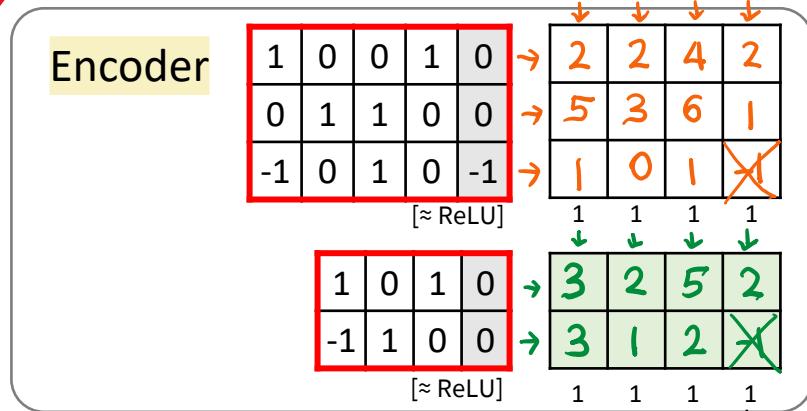
13	13
4	3

4 3

1 1

13. Autoencoder

849



14. Vector Database

 2224

Query

Data [how are you] [who are you] [who am I]

[am I you]

Word Embeddings

a	an	the	how	why	who	what	are	is	am	be	was	you	we	I	they	she	he	she	me	him	her
0	-1	0	1	0	1	0	0	-1	1	0	0	0	3	1	0	-1	0	0	0	-1	0
2	0	2	0	0	0	-1	1	0	0	0	2	1	0	2	0	2	0	0	2	0	0
-1	0	-1	1	2	0	0	1	0	1	-1	0	0	-1	0	3	0	0	-1	0	2	-1
0	1	0	0	1	0	1	0	1	0	1	-2	0	0	0	1	0	1	0	1	0	1

Text Embeddings

1 0 0 0 1 1 1 1 0 0 0 0	1 0 0 0 1 1 0 1 0 0 0 0	1 1 1 0 0 2 0 1 0 0 0 0	1 1 0 0 2 1 1 0 0 0 0 0
1 1 1 1 1 1 1 1			

Encoder

Linear &
ReLU

1	1	0	0	0
0	1	0	1	0
1	0	1	0	-1
1	-1	0	0	0

1 1 1 0 1 1 0 0 2 0 1 0
1 1 3 0 0 2 0 1 0 0 1 0

1 3 1 0 2 1 1 0 0 1 0 0
1 3 1 0 2 1 1 0 0 1 0 0

Mean Pooling

$$\Sigma / 3$$

3/3
2/3
1/3
1/3

3/3
2/3
0

5/3
2/3
1/3
1/3

5/3
3/3
1/3
1/3

Indexing

Projection

1	1	0	0
0	0	1	1

5/3
2/3

5/3
0

7/3
2/3

Vector Storage

8/3
2/3

Retrieval

Dot Products

44/9

40/9

60/9

8/3 2/3 T

Nearest Neighbor (argmax)

✓

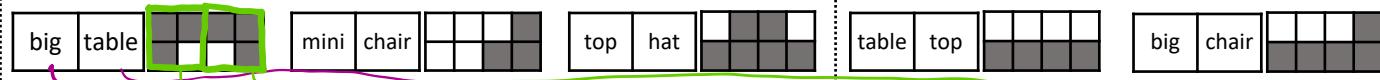
who am I

15. CLIP

885

400 millions more ...

mini batch of text-image pairs



word2vec

	mini	big	top	hat	chair	table
0	1	1	0	1	0	0
1	0	0	0	1	1	1
2	1	0	1	0	1	1

Word
Embeddings

1	0	0	1	0
0	1	1	1	0
1	1	0	0	1

Text Encoder

1	0	1	0	→	2	1	0	1	1
0	3	0	-2	→	2	1	1	1	-2
1	1	0	1	→	2	2	2	3	2

[Mean Pooling]
(round)

2	1	1
1	1	3
2	0	2

[Projection]

1	1	0	-2
0	1	1	-2

1	0	-1
1	2	0

T₁ T₂ T₃

1	0	-1
1	2	0

[Softmax]

$$e^{\square} \approx 3^{\square}$$

$$\frac{\Sigma}{\Sigma}$$

Shared Embedding Space

Similarity
Image → Text

Target

Cross Entropy
Loss Gradients

Image → Text

-1	.1	0
.75	.25	0
.7	.3	-1

Text → Image

-8	.1	0
.1	-.9	0
.7	.8	-1

2	0
1	0
2	1

2	0	-2
3	1	0
27	9	0

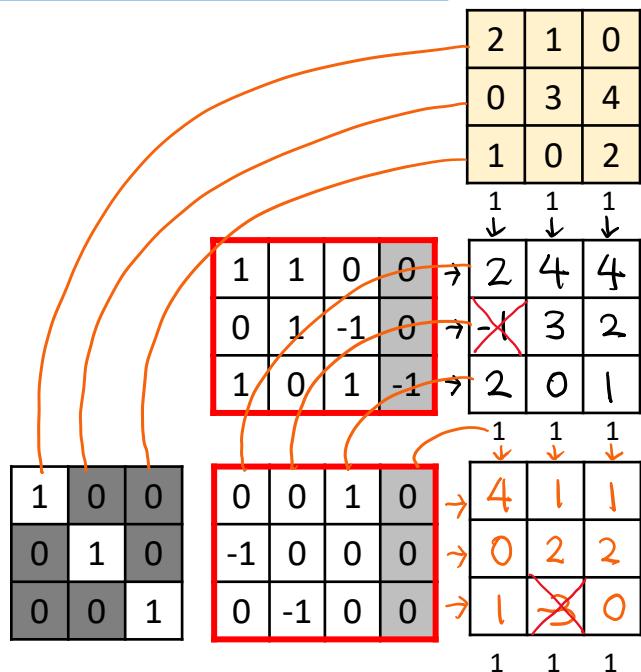
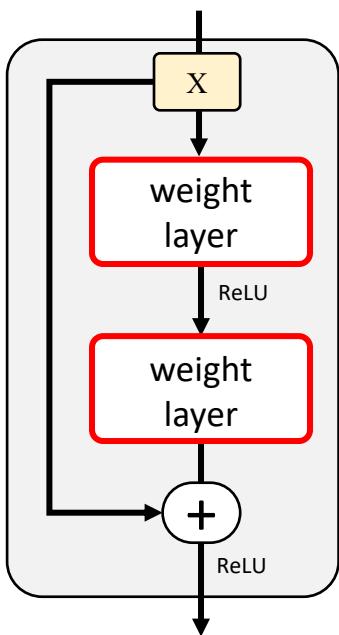
.9	.1	0
.75	.25	0
.7	.3	0

1	0	0
0	1	0
0	0	1

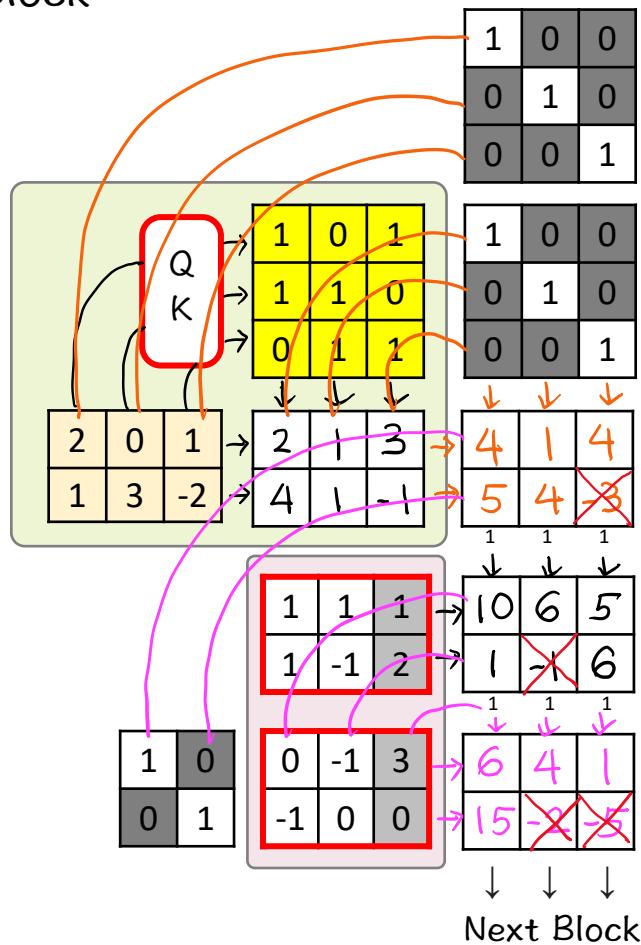
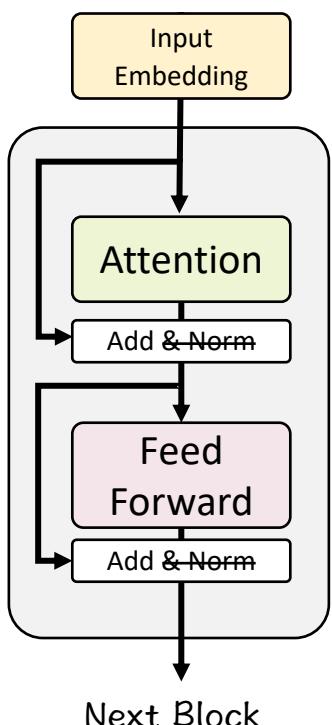
Similarity
Text → Image

.2	.1	0
.1	.1	0
.7	.8	0

16. Residual Network

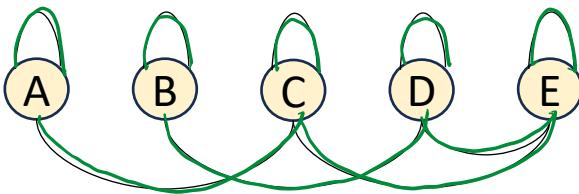


Transformer's Encoder Block



17. Graph Convolutional Network

Graph Data



573

Graph
Convolutional
Network

	A	B	C	D	E
A	2	0	1	0	1
B	1	1	0	0	0
C	0	0	-1	1	1
D	0	3	0	1	0
E					

1	1	0	0	0
0	1	0	-1	0
1	0	0	1	-1

[ReLU]

1	1	0	0	0
0	1	1	0	1
1	2	0	0	0

Messages

	A	B	C	D	E
A	1	1	1		
B	1	1	1	1	
C	1	1	1	1	1
D	1	1	1	1	1
E			1	1	1

1	1	1	1	1
0	1	3	1	3
1	2	1	2	0

Messages

	A	B	C	D	E
A	1	1	1	1	
B	1	1	1	1	
C	1	1	1	1	1
D	1	1	1	1	1
E			1	1	1

Fully
Connected
Network

1	0	0	-2	
0	1	0	-2	
0	0	1	-5	
1	-1	0	0	
1	0	-1	0	

[ReLU]

1	1	1	1	1	-9
---	---	---	---	---	----

σ

0	4	0	4	2
3	-2	3	-2	1
2	-4	3	-1	1
-3	6	-3	6	1
-7	3	8	2	4

1	1	1	1	1
-4	4	5	3	0
0	1	1	1	0.5

18. SORA's Diffusion Transformer

 2118

Training Video

1 0	2 0	0 1	0 1
0 1	1 0	3 0	4 0

Spacetime
Patches
(Pixels)

1 0 0 1
2 0 1 0
0 1 3 0
0 1 4 0

Visual Encoder

1 0 -1 0 0
0 1 0 1 1

[ReLU]

Sampled
Noise



0 2 1 -1
-1 0 -2 1

Noised
Latent

1 2 1 0
2 2 4 2

Predicted
Noise



0 -2 2 2
1 -1 -1 -2

Noise-free
Latent

1 4 -1 -2
1 3 5 4

Visual Decoder

1 0 1
0 1 0
1 1 0
-1 1 0

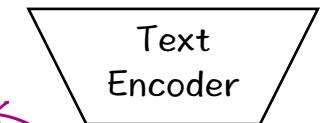
[ReLU]

Diffusion

Step $t = 3$

1
1

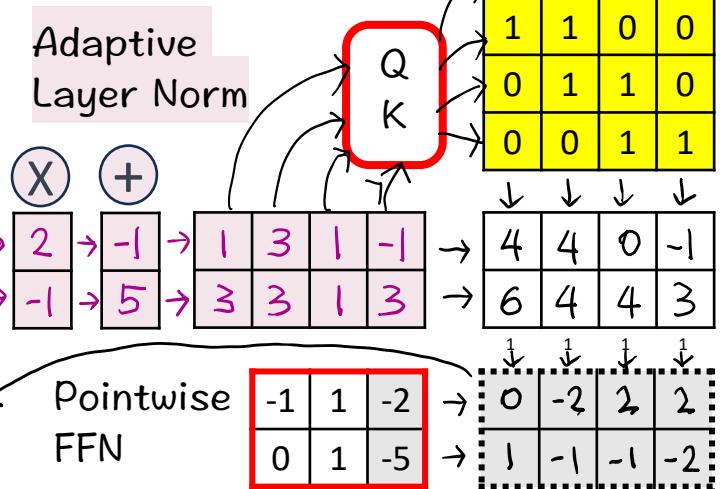
Prompt
“sora is sky”



0
1
-1

1 0 0 1 0 0
0 1 0 -1 1 0
1 1 0 0 1 0
0 2 0 1 0 2

Self-Attention



-1 1 -2
0 1 -5

Pointwise
FFN

0 -2 2 2
1 -1 -1 -2

Train

Sampled
Noise



0 2 1 -1
-1 0 -2 1

MSE Loss
Gradients

0 -4 1 3
2 0 1 -3

Generated Video

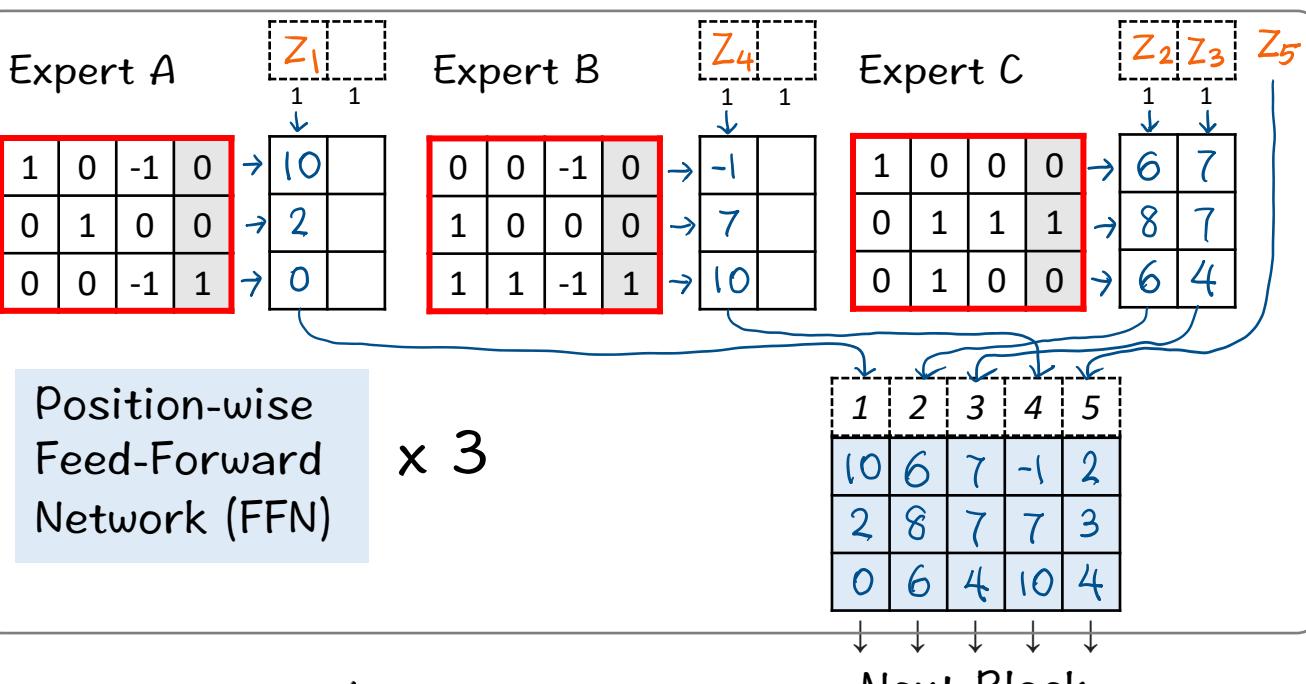
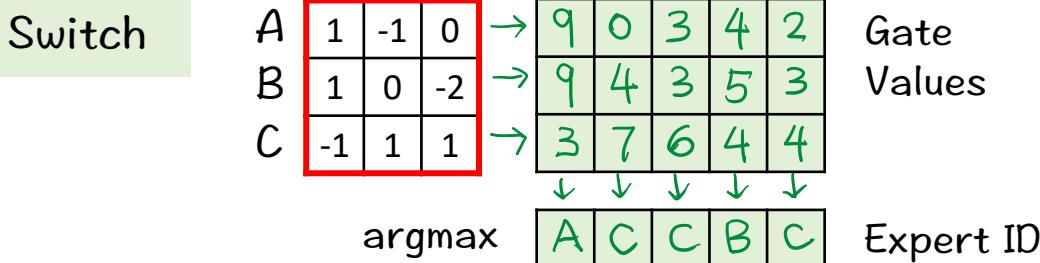
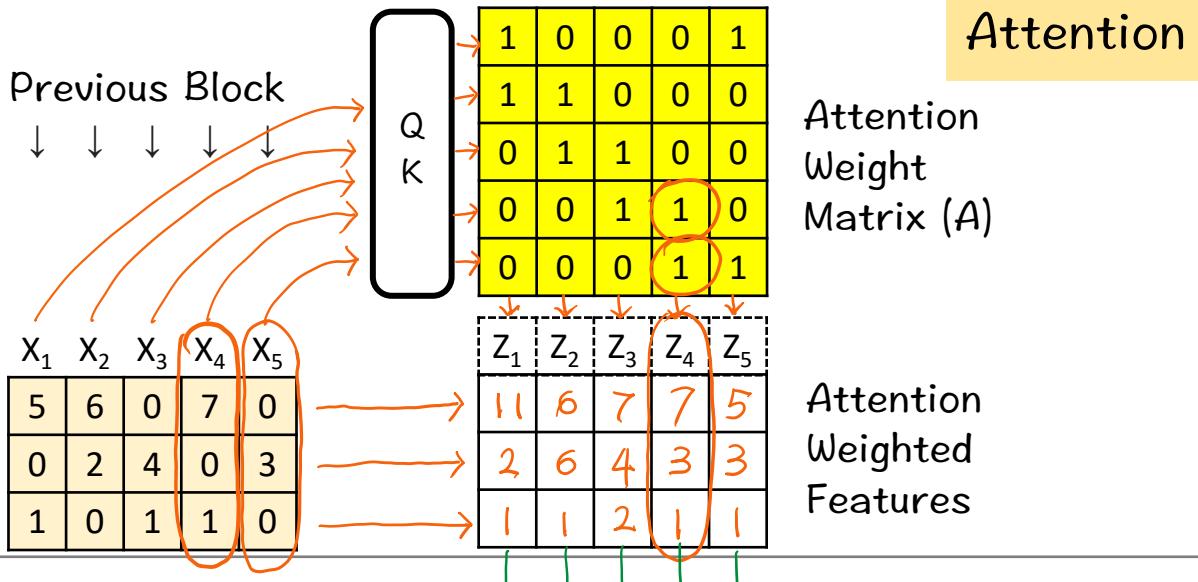
2 5	1 3	2 7	0 0
0 0	5 4	4 2	6 6

19. Switch Transformer



576

(Gemini 1.5's Sparse Mixture of Experts)



20. Reinforcement Learning with Human Feedback (RLHF)

