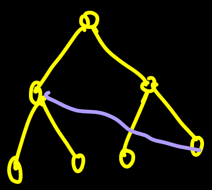
HW03 CoderBak.

I. Short Tree Proofs (undirected)

1. Prove that every connected component in an acyclic graph is a tree

if a connected, undirected, acyclic graph is not a tree

we consider its spanning tree.

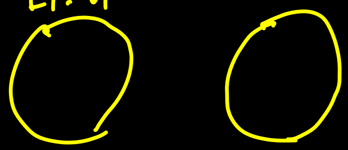


the extra edge leads to a cycle #

2. Suppose G has k connected components.

Prove that if G is acyclic, $|E| = |V| - k$.

$E_1 . V_1$



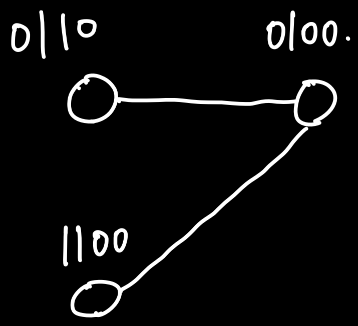
$\Rightarrow \sum e_i = |E| = \sum V_i - k = |V| - k$

$e_1 = V_1 - 1$ $e_2 = V_2 - 2$

3. Prove that a graph with $|V|$ edges contains a cycle.

Otherwise divide it into k parts. $|V| - k = |V|$ X

II. Touring Hypercube

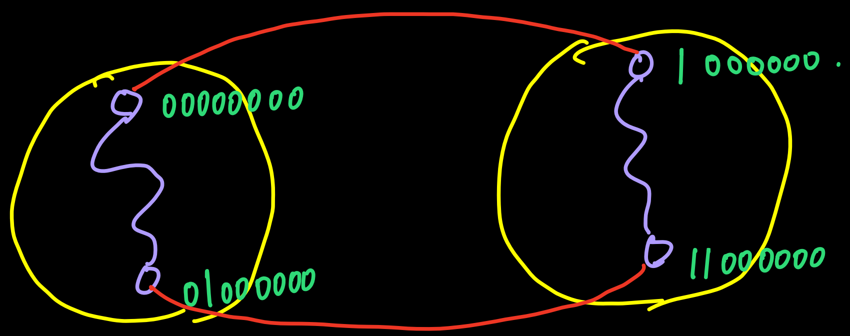


① exist Eulerian tour $\Leftrightarrow$

every vertex has even degree $\Leftrightarrow$

n is even

② Induction. n digits. 00..0 ~ 10..0



III. Planarity and Graph Complements.

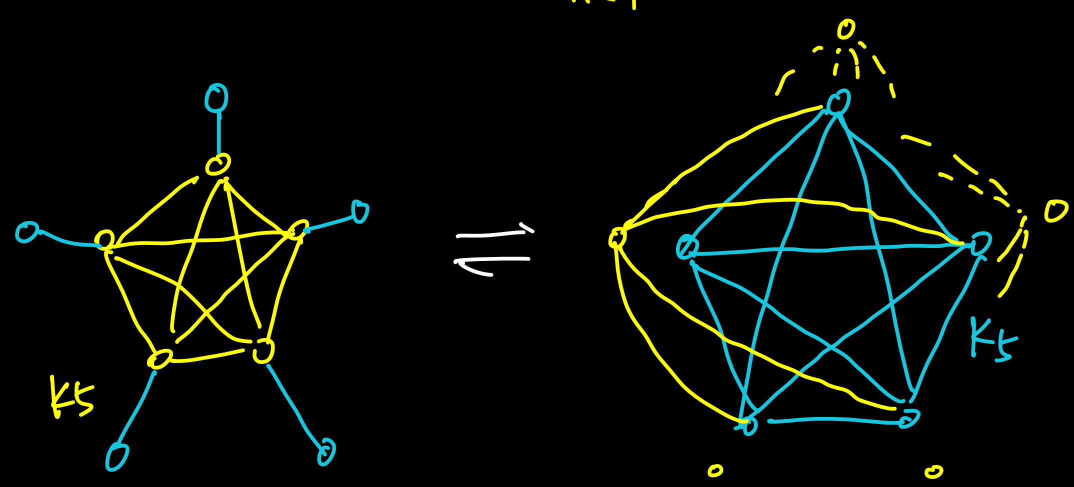
(a). $\frac{n(n-1)}{2} - e$.

(b) if G and $\bar{G}$ are all planar:

$e \le 3n-6$ $\Rightarrow$ $n(n-1) \le 4(3n-6)$

$\frac{n(n-1)}{2} - e \le 3n-6$ $\Rightarrow$ $n \le 10$

(c)



IV. Modular Practice.

(a) $9x+5 \equiv 7 \pmod{13}$.

$9x \equiv 2 \pmod{13}$.

$9 \cdot 3x \equiv 6 \pmod{13} \Rightarrow x \equiv 6 \pmod{13}$

(b) $3x+12 \equiv 0 \pmod 3$

$\not\equiv 1 \pmod 3$

(c) $\begin{cases} 5x+4y \equiv 0 \pmod 7 \\ 2x+y \equiv 4 \pmod 7 \Rightarrow x+4y \equiv 2 \pmod 7 \end{cases}$

$\Rightarrow 4x \equiv 5 \pmod 7 \Rightarrow x \equiv 3 \pmod 7 \Rightarrow y \equiv 5 \pmod 7$

(d) $x \equiv 1 \pmod{12}$

(e) $(7^{10})^6 \cdot 49 \equiv 5 \equiv x \pmod{11}$

V. Modular Arithmetic

(a) $n-1$

(b) $x \equiv 2 \pmod{17}$

(c) True

(d) $x \equiv 49 \pmod m$

VI. Wilson Theorem (well-known. skipped).