HW 02 CoderBak

I. Universal Preference

(a) Consider the procedure of matching.

① ~~Every~~ 1st morning, every company send offer to C1

② ~~Every~~ 1st afternoon, C1 responses "maybe" to J1, and rejects $J_2 \sim J_n$

⋮

Eventually, we have $C1 \sim J1$ ⋯ $C_k \sim J_k$ ⋯ $C_n \sim J_n$

(b) it's the same as (a)

II. Pairing Up

① consider the simplest case:

$J_1: C_1 > C_2$ $\quad C_1: J_2 > J_1$

$J_2: C_2 > C_1$ $\quad C_2: J_1 > J_2$

stable matching: $\{J_1 \sim C_1, J_2 \sim C_2\}$ $\quad \{J_1 \sim C_2, J_2 \sim C_1\}$

② Induction:

$J_1 \sim J_{2k}$ $\quad C_1 \sim C_{2k}$ $\Rightarrow$

$J_1: \boxed{\quad} > C_{2k+1} > C_{2k+2}$ $\quad C_1: \boxed{\quad} > J_{2k+1} > J_{2k+2}$

$\vdots$

$J_{2k}: \boxed{\quad} > C_{2k+1} > C_{2k+2}$ $\quad C_{2k}: \boxed{\quad} > J_{2k+1} > J_{2k+2}$

inner pair ←

$J_{2k+1}: C_{2k+1} > C_{2k+2} > \cdots$ $\quad C_{2k+1}: J_{2k+2} > J_{2k+1} > \cdots$

$J_{2k+2}: C_{2k+2} > C_{2k+1} > \cdots$ $\quad C_{2k+2}: J_{2k+1} > J_{2k+2} > \cdots$

III. Upper Bound

(b) We construct the following example:

Jobs $1 \sim 4$ Candidates $A \sim D$

1: $A > B > C > D$

2: $A > C > B > D$

3: $B > C > A > D$

4: $C > A > B > D$

A: $3 > 4 > 1 > 2$

B: $4 > 2 > 1 > 3$

C: $1 > 3 > 2 > 4$

D: $1 > 2 > 3 > 4$

stable matching:

A	B	C	D
3	4	1	2

Intuition:

One rejection each time

(a) Jobs: $J_1 \sim J_n$ Candidates: $C_1 \sim C_n$

J_1: $a_{11} > a_{12} > \cdots > a_{1n}$

J_2: $a_{21} > a_{22} > \cdots > a_{2n}$

$\vdots$

J_n: $a_{n1} > a_{n2} > \cdots > a_{nn}$

n^2

After $(n-1)^2$ days,

at least $(n-1)^2$ rejected offers

Consider the final stable pairs

rejects

J_1: $\boxed{a_{11} > a_{12}}$ $\boxed{a_{13}}$ $\cdots$ a_{1n}

J_2: $\boxed{a_{21}}$ $\boxed{a_{22}}$ a_{23} $\cdots$ a_{2n}

J_3: $\boxed{a_{31} \quad a_{32} \quad a_{33} \quad \cdots}$ $\boxed{a_{3n}}$

$\vdots$ $\boxed{}$

J_n $\boxed{a_{n1}}$ $\boxed{a_{n2}}$ a_{n3} $\cdots$ a_{nn}

$J_k \sim a_{ki_k}$ $\{a_{ki_k}\} = \{C_k\}$.

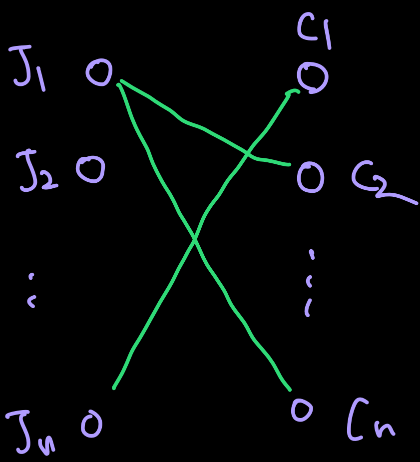
Stable: eg. $J_1 \sim a_{13}$. then J_1 is not the top on a_{11}. a_{12}'s list.

we are actually having $(n-1)^2$ constraints

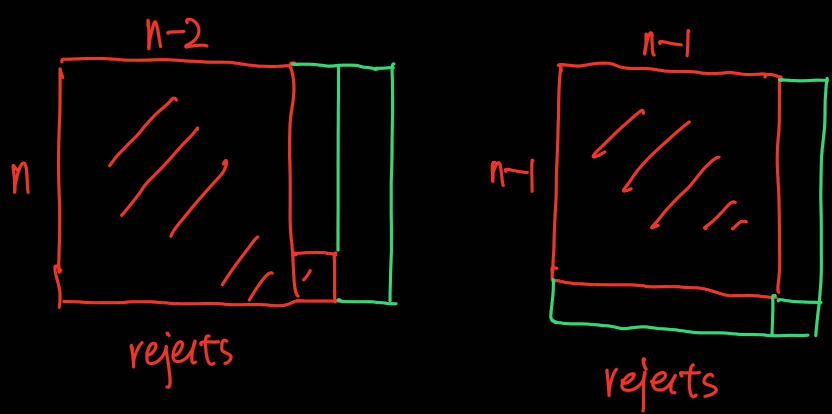
(J_u is not the top on p. q. r. $\cdots$)

Connected: may be favourite.

at most $2n-1$ edges.

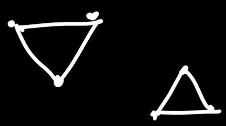


Intuition:



IV. Build-up Error?

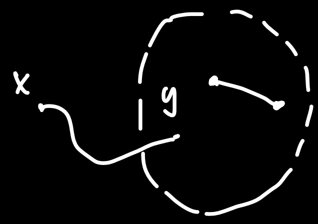
Example:



Reason: it's not always possible to find x–y,

when erasing x–y, y has degree ≥ 1.

e.g.



V. Proofs in Graphs

(a). n cities. $X \to Y$ or $Y \to X$.

Prove that there existed a city which was reachable from every other city by traveling through ≤ 2 roads

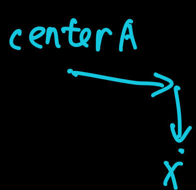
Opposite: prove that exist a city (center)

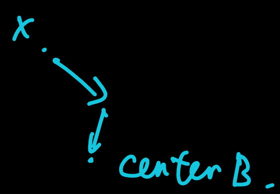
(By reversing all the paths)

Claim: $\exists$ center A (start)

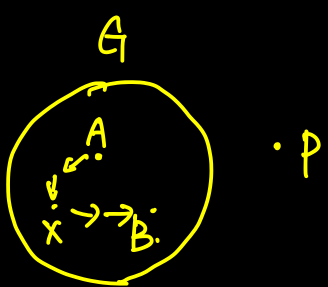
$\Updownarrow$

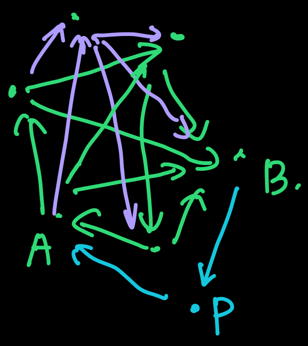
$\exists$ center B (end)





Induction:

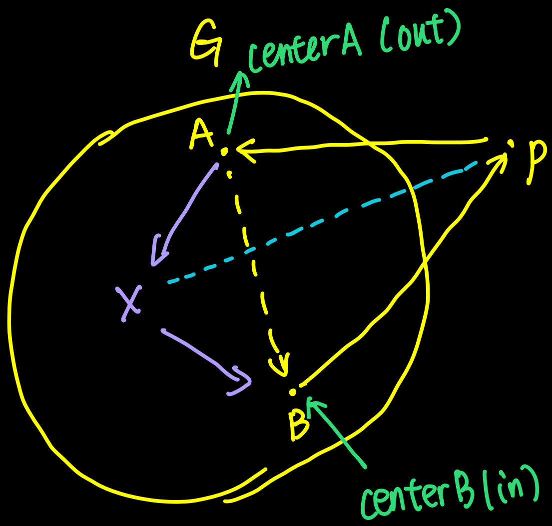




① $A = B$ $\quad A \to P$ or $A \leftarrow P$ ✓

② $A \neq B$ $\quad$ if $A \to P$ ✓ $\quad$ if $B \leftarrow P$ ✓

Otherwise:



Consider how A goes to B.

i) $A \to B$: we have done.

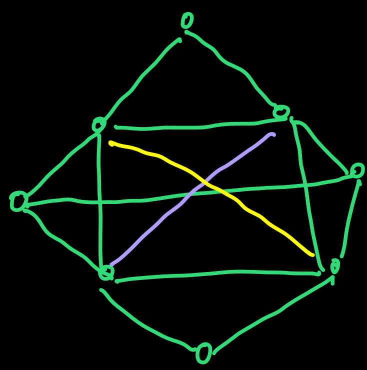
ii) $A \to X \to B$:

no matter $P \to X$ / $P \leftarrow X$

one of the centers is still A / B #

(b) Consider a connected graph G, n vertices, with 2m odd degree. Prove that there are m walks that together cover edges of G.

example:

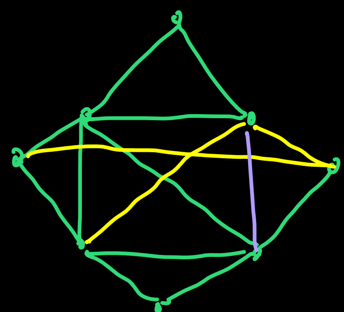


Consider the fact that

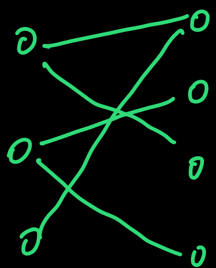
if only 2 vertices have odd degree, than an Euler trail exists.

Randomly select two with odd degree.

Delete the Euler trail from graph and continue.



(c) Prove: G is bipartite iff. it has no tours of odd length



bipartite $\Rightarrow$ no odd tour is obvious. (coloring)

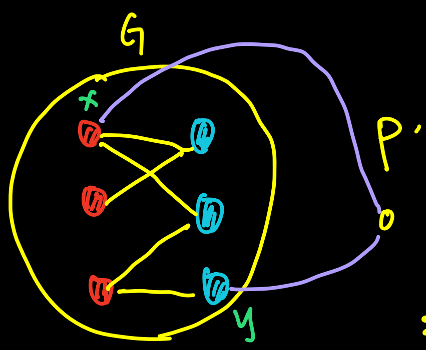
Consider "$\Leftarrow$": (note: assume G is connected)

Induction: if no tours of odd length in $G+P$

then G itself has no tours of odd length. $\Rightarrow$ G is bipartite.

$\Rightarrow$ G can be separated into r / b

(one of them may be ϕ).



We claim that P can be categorized into r/b.

Otherwise $\exists x, y$. $x - P - y$ $\times$ #