HW 1 CoderBak

I Logical Equivalence ?

(a) if $\forall x\,(P(x) \wedge Q(x))$ is true, then we have $\forall x.\ P(x)$ and $\forall x.\ Q(x)$
✓
if $\forall x\,(P(x) \wedge Q(x))$ is false, then $\exists x,\ (\neg P(x) \vee \neg Q(x))$ is true

so there exists some x, $P(x)$ is false or $Q(x)$ is false.

✗ (b) example: $x \in \mathbb{N}_+$, $P(x) =$ x is a prime number $Q(x) =$ x is not a prime number

(c) if $\exists x\,(P(x) \vee Q(x))$ is true, then there exists some x, one of $P(x)$ and $Q(x)$
✓ is true, so $\exists x\, P(x) \vee \exists x\, Q(x)$ is true.

if $\exists x\,(P(x) \vee Q(x))$ is false, then $\forall x, (\neg P(x) \wedge \neg Q(x))$ is true, thus

$\forall x\, \neg P(x) \wedge \forall x\, \neg Q(x)$, so $\exists x\, P(x) \vee \exists x\, Q(x)$ is false.

(d) the same example ~~is~~ in (b)
✗

II. Prove or disprove

(a) n is odd $\Rightarrow \exists k \in \mathbb{N},\ n = 2k \Rightarrow n^2+4n = 4k^2+8k = 2(2k^2+4k)$ is odd. ✓

(b) ~~example: a=12 b~~

Pf. $(a+b \leq 15) \Rightarrow (a \leq 11 \vee b \leq 4)$

$\Leftrightarrow$ $(a > 11 \wedge b > 4) \Rightarrow (a+b > 15)$ obvious ! ✓

(c) Pf. $(r^2 \notin \mathbb{Q}) \Rightarrow (r \notin \mathbb{Q})$

$\Leftrightarrow$ $(r \in \mathbb{Q}) \Rightarrow (r^2 \in \mathbb{Q})$ By setting $r = p/q$ $r^2 = p^2/q^2$ ✓

(d). example: $n=5$ ✗

(e) Pf. $(x \notin \mathbb{Q} \wedge y \in \mathbb{Q}\setminus 0) \Rightarrow (xy \notin \mathbb{Q})$ $\Leftrightarrow$ if $xy \in \mathbb{Q}$, $xy = p/q$ $y = r/s \Rightarrow x = ps/qr \in \mathbb{Q}$ ✓

III. Twin primes

(a) if there exists a prime p, such that $\forall k \in \mathbb{Z}$, $p \neq 3k+1 \wedge p \neq 3k-1$

We claim that $3 \mid p$. otherwise $p = 3l+2$ or $3l+1$, which is impossible

However, $3 \mid p$ contradicts with the fact that p is a prime > 3.

(b) We want to prove that: the only $p \in \mathbb{P}$ (prime) satisfying

$p-2$. p. $p+2$ are all primes is 5

$p-2 \geq 2$, so we assume that $p \geq 5$.

from previous result, $p = 3k+1$ or $p = 3k-1$

if $p = 3k+1$: $p+2 = 3(k+1)$ ~~if~~ is not a prime

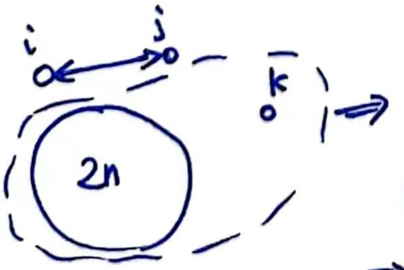
if $p = 3k-1$: $p-2 = 3(k-1)$ is a prime iff $k=2$, ~~eg~~ i.e. $p=5$

IV. Airport

consider the basic case: $n=1$

induction: adding 2 points at a time.

$2n+3$ points. We select 2 that has minimum distance, $i,j = \underset{i,j \in V}{\operatorname{argmin}}\, d_{ij}$



$\Rightarrow$ these $2n+1$ points "isolates" point k

both i,j don't connect k

$\Rightarrow$ k is isolated. #

V. A coin game

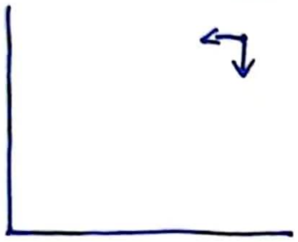
set $f(n)$ to be the result of this game starting from n.

by strong induction

$$f(x)+f(n-x)=\frac{x(x-1)}{2}+\frac{(n-x)(n-x-1)}{2}+x(n-x)$$

$$=\frac{x^2-x+n^2-2nx+x^2-n+x+2nx-2x^2}{2}=\frac{n^2-n}{2} \quad \#$$

VI Grid induction



$i+j$ always decreases by one #

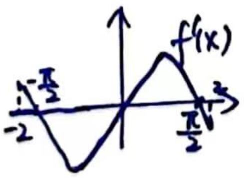
VII. Calculus review

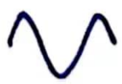
(a) $\int_0^\infty \sin t\, e^{-t}\, dt = I_1$ $\quad \int_0^\infty \cos t\, e^{-t}\, dt = I_2$ $\quad \Rightarrow I_1 = \frac{1}{2}$

(b) $f(x)=\int_0^{x^2} t\cos(\sqrt{t})\, dt$

set $\sqrt{t}=u$ $\quad x>0:\ f(x)=\int_0^x 2u^3\cos u\, du$

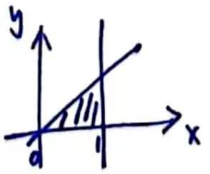
$f'(x)=2x^3\cos x$





local maxima: $x=\pm\frac{\pi}{2}$ $\quad$ local minima: $x=0$

(c)



$$\iint_R 2x+y\, dx\, dy = \int_0^1 dx \int_0^x 2x+y\, dy = \frac{5}{6}$$