HW06 Coder Bak

I. Unions and Intersections

(a) $(X \cap Y)$ is a subset of X $\Rightarrow$ always countable

(b) Y is a subset of $(X \cup Y)$ $\Rightarrow$ always x

(c). A_i is a subset of $\bigcup_{i \in X} A_i$ $\Rightarrow$ always x

(d) Sometimes:

① $X = \mathbb{N}$. $A_i = \mathbb{R} / \{i\}$ $\bigcap_{i \in X} A_i = \mathbb{R} / \mathbb{N}$ uncountable

② $X = \mathbb{N}$ $A_i = \{(i, x) \mid x \in \mathbb{R}\}$ $\bigcap_{i \in X} A_i = \phi$ Countable

(e) Sometimes:

① $Y = \mathbb{R}$. $B_i = \mathbb{N}$. $\bigcup_{i \in Y} B_i = \mathbb{N}$ countable

② $Y = \mathbb{R}$ $B_i = \mathbb{N} \cup \{i\}$ $\bigcup_{i \in Y} B_i = \mathbb{R}$ uncountable

(f) $\bigcap_{i \in Y} B_i$ is a subset of B_i. $\Rightarrow$ always ✓

II. Count it

(a) finite

(b) $8\mathbb{Z}$, countably infinite.

(c) ?

(d) when given the length, we can calculate the rank in some order eg. apple > apply ...

So we can link them with 1, 2, ...

(e) ?

(f) ?

II. Fixed points

(a) Suppose Find(p) finds the fixed point of program p.

(b)

Consider program F(p):

$$F(p) = \begin{cases} p, & \text{if } p \text{ halts on input } 0 \\ \text{null}, & \text{if } p \text{ doesn't halt on input } 0 \end{cases}$$

if Find(p) can calculate the fixed point of F, then

Find(p) can find all the programs that halts on input 0 ×

(c) if we have Find Fixed(F):

$$\text{Find Fixed}(F) = \begin{cases} n & \text{if } \exists n,\ F(n) = n \\ \text{null} & \text{otherwise.} \end{cases}$$

(skipped)

IV. Unprogrammable Programs

(a).

```
def TestHalt(G, x)
    def F(x):
        G(x)
        return 0
    return P(F, x, 0)
```

$\Rightarrow$ X

(b)

```
def G(x):
    if x==0:
        return 0.
    else:
        return F(x).
```

if F(x) halts on 0, then F.G halt on the same set of inputs

if F(x) doesn't halt on 0, then F.G don't halt on the same ---

$\Rightarrow$. we can define TestHalt(F) = P(F, G) X

to test if F halts on 0.

V. Counting.

(a) 3×2^{18}

(b) $\frac{104!}{2^{52}}$

(c) $\frac{7!}{4!}$ $\frac{7!}{2!\,2!}$

(d) CE $5!$

$\frac{6!}{2}$

(e) 25^8

(f) $\frac{20!}{2^{10} \times 10!}$ $\frac{C_{20}^{10} A_{10}^{10}}{2^{10}}$