Set Theory

* **Set** : A well defined collection of distinct objects are called a set.

Eg: 1) A set of all planets in our solar system.
2) A set of all positive numbers.
3) A set of all the states in India.
4) A set of all the lower case letters in the alphabet.

* Representation of a Set

1) **Roster or Tabular form** :- The set is represented by listing all the elements comprising it. The elements are enclosed with braces and separated by commas.

Eg: 1) Set of vowels in english alphabets.

$A = \{a, e, i, o, u\}$

2) Set of odd numbers less than 10.

$A = \{1, 3, 5, 7, 9\}$

2) **Set Builder Notation** :- The set is defined by specifying a property that elements of set have in common.

Eg: 1) $A = \{x : P(x)\}$

2) The set $\{a, e, i, o, u\}$ is written as,

$A = \{x : x \text{ is a vowel in english alphabet}\}$

3) For, $A = \{0, 2, 4, 6, 8\}$

$A = \{x : 1 \leq x < 10 \text{ and } x \% 2 = 0\}$

4) For, $A = \{1, 3, 5, 7, 9\}$

$A = \{x : 1 \leq x < 10 \text{ and } x \% 2 \neq 0\}$

Some Important Sets :-

N = The set of all natural numbers $= \{1, 2, 3 - - -\}$

Z = The set of all integers = $\{ \cdots -3, -2, -1, 0, 1, 2, 3 \cdots \}$
Z^+ = The set of all positive integers = $\{1, 2, 3 \cdots\}$
Q = The set of all rational numbers.
R = The set of all real numbers.

⊛ Cardinality of Set :- Cardinality of a set S, denoted by $|S|$, is the number of elements of the set, the number is also referred as the cardinal number. If a set has an infinite number of elements its cardinality is ∞.

$|X| = |Y| \Rightarrow$ Bijective function
$|X| \leq |Y| \Rightarrow$ Injective function
$|X| < |Y| \Rightarrow$ Injective function

⊛ Types of Set :-

1) **Finite Set :-** A set which contains a definite number of element is called a finite set.
Eg :- $A = \{x : x \in N \text{ and } 60 < x < 70\}$.

2) **Infinite Set :-** A set which contains infinite number of elements is called an infinite set.
Eg :- $A = \{x : x \in N \text{ and } x > 10\}$.

3) **Subset :-** A set X is a subset of set Y written as $X \subseteq Y$ [X is a subset of Y] if every element of X is an element of set Y.
Let $X = \{1, 2, 3, 4, 5, 6,\}$ & $Y = \{1, 2\}$
$\Rightarrow$ Set Y is a subset of set X $[Y \subseteq X]$

Proper Subset : The term proper subset can be defined as "subset of but not equal to".
A set X is a proper subset of set Y, is written as $X \subset Y$ if every element of X is an element of set Y and $|X| < |Y|$.

Eg:- Let $X = \{1, 2, 3, 4, 5, 6\}$
$Y = \{1, 2\}$
$Y \subseteq X$
$Y \subset X$

5) **Universal Set** :- It is a collection of all elements in a particular context or application.
All the sets in that context or application are essentially subset of the universal set. Universal sets are represented as U.

Eg: U as the set of all animals on the earth.
In this case, set of all mammals is a subset of U, set of all fishes is a subset of U.

6) **Singleton or Unit Set** :- Unit Set contains only one element. A singleton set is denoted by $\{S\}$.

Eg: $S = \{x : x \in N,\ 7 < x < 9\}$
$= \{8\}$

7) **Equal Set** :- If two sets contain the same elements they are said to be equal.

Eg: $A = \{1, 2, 6\}$ and $B = \{6, 1, 2\}$
A & B are equal sets.

(8) **Equivalent Set** :- If the cardinalities of two sets are same, they are called equivalent set.

Eg : $A = \{1, 2, 4\}$, $B = \{5, 6, 7\}$

$$|A| = |B| = 3$$

$\because$ They are equivalent sets.

(9) **Overlapping Set** : Two sets that have atleast one common element are called overlapping sets.

Eg :- $A = \{1, 2, 6\}$, $B = \{6, 12, 4/6\}$

There is a common element 6. Hence these sets are overlapping sets.

(10) **Disjoint Set** :- Two sets A and B are called disjoint sets if they don't have even one element in common. Therefore disjoint sets have following property,

$$n(A \cap B) = \phi$$

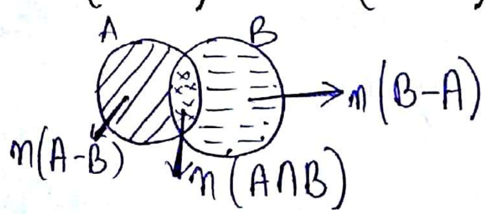
$$n(A \cup B) = n(A) + n(B)$$

Eg: Let $A = \{1, 2, 6\}$ & $B = \{7, 9, 14\}$

There is not a single common element, $\because$ these sets are disjoint sets.

Generally :-

* $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
* $n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$



are $n(A) = n(A-B) + n(A \cap B)$ ⑤
$n(B) = n(B-A) + n(A \cap B)$

Venn Diagram

1) **Set Union** :- The union of sets A and B (denoted by $A \cup B$) is the set of all elements which are in A or in B or in both A and B.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$
$$A = \{10, 11, 12, 13\}$$
$$B = \{13, 14, 15\}$$
$$A \cup B = \{10, 11, 12, 13, 14, 15\}.$$

2) **Set Intersection** :- The intersection of sets A & B (denoted by $A \cap B$) is the set of elements which are in both A and B. Hence,

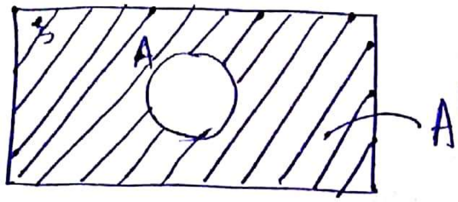
$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$
$$A = \{10, 11, 12, 13\}$$
$$B = \{13, 14, 15\}$$
$$A \cap B = \{13\}$$

3) **Set Difference / Relative Complement** :-

If $A = \{10, 11, 12, 13\}$
$B = \{13, 14, 15\}$
$A - B = \{10, 11, 12\}$
$B - A = \{14, 15\}$
$\because B - A = \{x : x \in B \text{ and } x \notin A\}$
$A - B = \{x : x \in A \text{ and } x \notin B\}$

4) **Complement of a set** :- The complement of a set A (denoted by A') is the set of elements which are not in set A. Hence,

$$A' = \{x : x \notin A\}$$



5) **Cartesian Product** : The cartesian product of n number of sets $A_1, A_2 --- A_n$ denoted as $A_1 \times A_2 \times ---- \times A_n$, defined as all possible order pair $(X_1, X_2, ----, X_n)$, where $X_1 \in A_1, X_2 \in A_2 ---- X_n \in A_n$.

eg: $A = \{1, 2\}$

$B = \{a, b\}$

$A \times B = \{(1,a), (1,b), (2,a), (2,b)\}$

$B \times A = \{(a,1), (a,2), (b,1), (b,2)\}$

6) **Power Set** : Power set of a set is the set of all possible subsets of S including the empty set. The cardianality of a power set of cardianality n is 2^n. Power set is denoted as P(S).

eg: $S = \{a, b, c, d\}$

Subset with zero element $= \{\phi\}$

Subset with one element $= \{a\}, \{b\}, \{c\}, \{d\}$.

Subset with two elements $= \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}$.

Subset with three elements $= \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, \{a,c,d\}$

Subset with four elements $= \{a, b, c, d\}$.