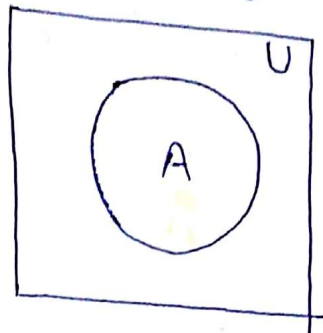
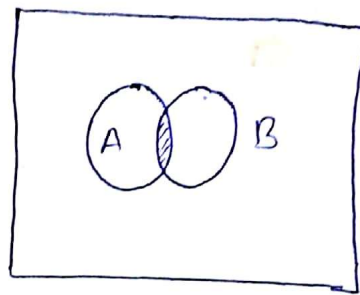
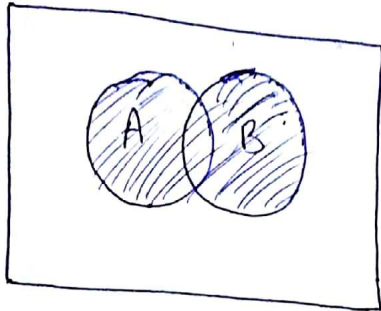


## Venn diagrams:-

A Venn diagram is a schematic representation of a set by set of points. Universal set containing all objects for which the discussion is meaningful. In the Venn diagram the Universal set  $U$  will be denoted by rectangle while the sets within the  $U$  is denoted by circle.

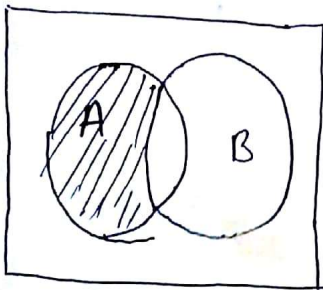


①



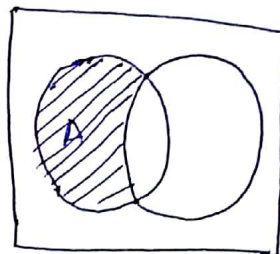
$$A \cup B = \{x : x \in A \text{ or } x \in B\}, \quad A \cap B = \{x : x \in A \text{ and } x \in B\}$$

②



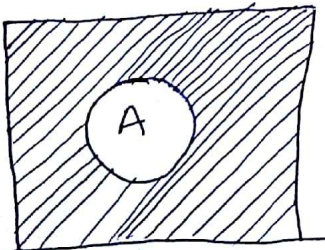
$$A - B$$

③



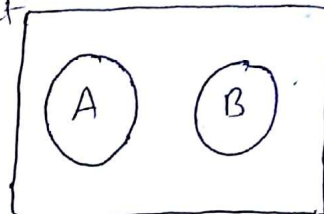
$$A - B$$

③



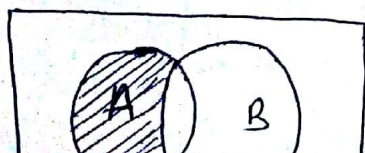
$$A^c \Rightarrow \{x \mid x \notin A\}$$

disjoint set

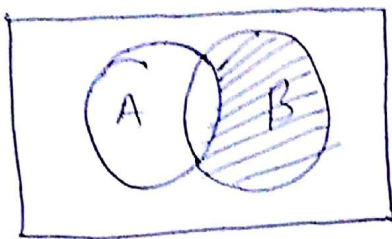


$$A \cap B = \emptyset$$

④

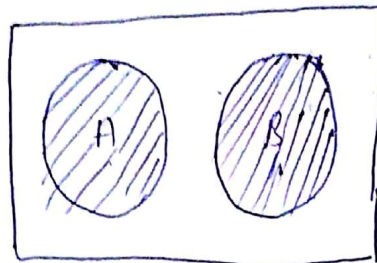


If  $A$  and  $B$  are two sets we define the complement of  $A$  as  $A - B = A \cap B^c$

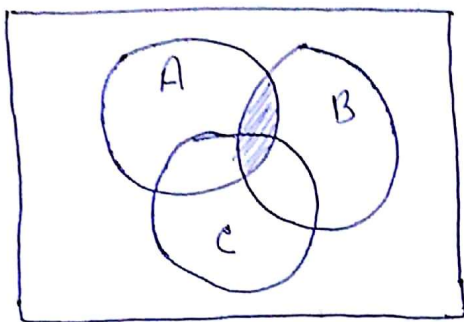


$$B - A = B \cap A^c$$

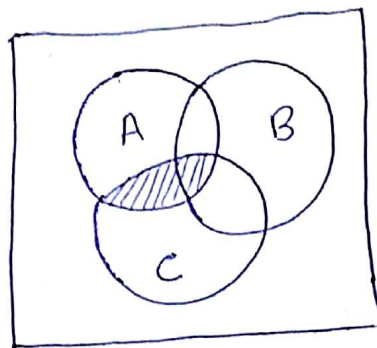
Complement of A  
with respect to B



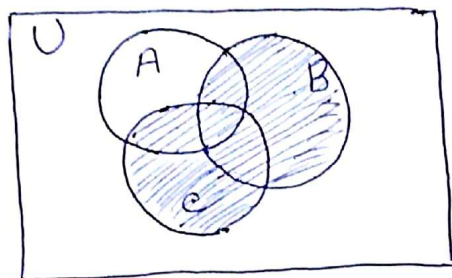
$$A \cup B = \phi$$
  
(disjoint set)



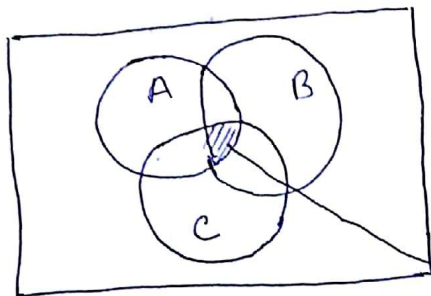
$$A \cap B$$



~~$$A \cup B \cup C$$~~ 
$$A \cap C$$



$$B \cup C$$

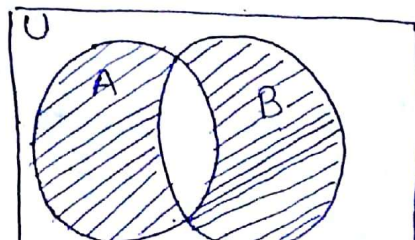


$$A \cap B \cap C$$

If A and B are two sets, we define their symmetric difference as the set of all elements that belong to A or to B but not to both A and B. We denote it by  $A \oplus B$ . Thus

$$A \oplus B = \{ x \mid (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A) \}$$

$$A \oplus B = (A - B) \cup (B - A)$$





Power Set  $P(S) = \{ \{\emptyset\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\} \}$

Q)1 If  $A = \{x : x \in \mathbb{N}, x \text{ is a factor of } 6\}$ .

$B = \{x : x \in \mathbb{N}, x \text{ is a factor of } 8\}$ .

Then find: ①  $A \cup B$  ②  $A \cap B$  ③  $A - B$  ④  $B - A$

Sol:  $A = \{1, 2, 3, 6\}$  &  $B = \{1, 2, 4, 8\}$

$$A \cup B = \{1, 2, 3, 4, 6, 8\}$$

$$A \cap B = \{1, 2\}$$

$$A - B = \{3, 6\}$$

$$B - A = \{4, 8\}$$

Q)2 Find the power set of  $\{4, 6\}$  and  $\{1, 2, 3\}$

Sol: 1) Subset with zero element =  $\{\emptyset\}$

Subset with one element =  $\{4\}, \{6\}$

Subset with two element =  $\{4, 6\}$

Power set  $P(S) = \{ \{\emptyset\}, \{4\}, \{6\}, \{4, 6\} \}$

2) Subset with zero element =  $\{\emptyset\}$

Subset with one element =  $\{1\}, \{2\}, \{3\}$

Subset with two element =  $\{1, 2\}, \{1, 3\}, \{2, 3\}$ ,

Subset with three element =  $\{1, 2, 3\}$ .

Power set  $P(S) = \{ \{\emptyset\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\} \}$ .

(\*) Commutative law

1) Prove that:  $A \cup B = B \cup A$

Proof: LHS  $\Rightarrow x \in A \cup B$   
 $\Rightarrow x \in A$  or  $x \in B$   
 $= x \in B$  or  $x \in A$   
 $= x \in B \cup A$   
 $A \cup B \subseteq B \cup A$  — (1)

RHS  $\Rightarrow x \in B \cup A$   
 $= x \in B$  or  $x \in A$   
 $= x \in A$  or  $x \in B$   
 $= x \in A \cup B$   
 $B \cup A \subseteq A \cup B$  — (2)

From eq<sup>n</sup> (1) & (2)  $A \cup B = B \cup A$

2) Prove that:  $A \cap B = B \cap A$

Proof: LHS  $\Rightarrow x \in A \cap B$   
 $= x \in A$  and  $x \in B$   
or  $= x \in B$  and  $x \in A$   
 $= x \in B \cap A$   
 $A \cap B \subseteq B \cap A$  — (1)

RHS  $\Rightarrow x \in B \cap A$   
 $= x \in B$  and  $x \in A$   
 $= x \in A$  and  $x \in B$   
 $= x \in A \cap B$   
 $B \cap A \subseteq A \cap B$  — (2)

From eq<sup>n</sup> (1) & (2)  $A \cap B = B \cap A$

(\*) Associative law

1) Prove that:  $A \cup (B \cap C) = (A \cup B) \cap C$

Proof: LHS  $= x \in A \cup (B \cap C)$   
 $= x \in A$  or  $x \in (B \cap C)$   
 $= x \in A$  or  $x \in B$  or  $x \in C$   
 $= x \in (A \cup B)$  or  $x \in C$   
 $= x \in (A \cup B) \cap C$   
 $\Rightarrow A \cup (B \cap C) \subseteq (A \cup B) \cap C$  — (1)

RHS  $\Rightarrow x \in (A \cup B) \cap C$   
 $x \in (A \cup B)$  or  $x \in C$   
 $x \in A$  or  $x \in B$  or  $x \in C$   
 $x \in A$  or  $x \in (B \cap C)$   
 $x \in A \cup (B \cap C)$   
 $\Rightarrow (A \cup B) \cap C \subseteq A \cup (B \cap C)$  — (2)



From ① & ②,  $\boxed{A \cup (B \cap C) = (A \cup B) \cap C}$

⑨

A) Prove that:  $A \cap (B \cap C) = (A \cap B) \cap C$

Proof: LHS  $\Rightarrow x \in A \cap (B \cap C)$

$$\begin{aligned} & x \in A \text{ and } x \in (B \cap C) \\ & x \in A \text{ and } x \in B \text{ and } x \in C \\ & x \in (A \cap B) \text{ and } x \in C \\ & x \in (A \cap B) \cap C \\ & A \cap (B \cap C) \subseteq (A \cap B) \cap C \\ & \quad \quad \quad \text{--- ①} \end{aligned}$$

RHS  $\Rightarrow x \in (A \cap B) \cap C$

$$\begin{aligned} & x \in (A \cap B) \text{ and } x \in C \\ & x \in A \text{ and } x \in B \text{ and } x \in C \\ & x \in A \text{ and } x \in (B \cap C) \\ & x \in A \cap (B \cap C) \\ & (A \cap B) \cap C \subseteq A \cap (B \cap C) \\ & \quad \quad \quad \text{--- ②} \end{aligned}$$

From ① & ②,  $\boxed{A \cap (B \cap C) = (A \cap B) \cap C}$

\* Idempotent law

$$A \cup A = A$$

$$A \cap A = A$$

i) Prove that:  $A \cup A = A$

Proof: LHS  $\Rightarrow x \in A \cup A$

$$\begin{aligned} & x \in A \text{ or } x \in A \\ & x \in A \end{aligned}$$

$$\Rightarrow A \cup A \subseteq A \quad \text{--- ①}$$

RHS  $\Rightarrow x \in A$

$$x \in A \text{ or } x \in A$$

$$x \in A \cup A$$

$$A \subseteq A \cup A \quad \text{--- ②}$$

From ① & ②  $\boxed{A \cup A = A}$

2) Prove that:  $A \cap A = A$

Proof: LHS  $\Rightarrow$   $x \in A \cap A$   
 $x \in A$  and  $x \in A$   
 $x \in A$   
 $A \cap A \subseteq A$  — ①

RHS  $\Rightarrow x \in A$   
 $x \in A$  and  $x \in A$   
 $x \in A \cap A$   
 $A \subseteq A \cap A$  — ②

From ① & ②,

$$\boxed{A \cap A = A}$$

\* Identity law

$$A \cup \phi = A$$

$$A \cap U = A$$

1) Prove that:  $A \cup \phi = A$

Proof: LHS  $\Rightarrow x \in A \cup \phi$   
 $x \in A$  or  $x \in \phi$   
 $x \in A$   
 $A \cup \phi \subseteq A$  — ①

RHS  $\Rightarrow x \in A$   
 $x \in A$  or  $x \in \phi$   
 $x \in A \cup \phi$   
 $A \subseteq A \cup \phi$  — ②

From ① & ②,

$$\boxed{A \cup \phi = A}$$

2) Prove that:  $A \cap U = A$

Proof: LHS  $\Rightarrow x \in A \cap U$   
 $x \in A$  and  $x \in U$   
 $x \in A$   
 $A \cap U \subseteq A$  — ①

RHS  $\Rightarrow x \in A$   
 $x \in A$  and  $x \in U$   
 $x \in A \cap U$   
 $A \subseteq A \cap U$  — ②

From ①, & ②,

$$\boxed{A \cap U = A}$$