Principles of Mathematical Induction

Let $P(n)$ be a statement involving natural no. n. To prove that $P(n)$ is true for all natural number we proceed as follows:

1. the statement is true for $n = 1$ i.e. $P(1)$ is true
2. if the statement is true for $n = k$ (where k is the some possible integer) then the statement is also true for $n = k+1$, truth of $p(k)$ implies the truth of $p(k+1)$ then $P(n)$ is true for all natural number n.

Ques $1+2+3+ \dots n = \frac{n(n+1)}{2}$

for $n = 1$

LHS $= n = 1$

RHS $= \frac{n(n+1)}{2} = \frac{2}{2} = 1$

LHS = RHS

$\Rightarrow$ $P(1)$ is true

Let $P(n)$ is true for $n = k$

LHS = RHS

$$\boxed{1 + 2 + 3 + \dots k = k(k+1)/2} \quad (1)$$

For $n = k+1$

LHS $1+2+3 \dots k + k+1 = \frac{k(k+1)}{2} + k+1$

$= \frac{1}{2}[k(k+1) + 2k + 2]$

$= \frac{1}{2}[k^2 + 3k + 2]$

$$P(k+1) = \frac{1}{2}(k+2)(k+1)$$

RHS $\Rightarrow$ $$P(k+1) = \frac{(k+1)(k+2)}{2}$$

LHS = RHS

$\because$ $P(n)$ is true $\forall\, n \in N$

<u>Ques</u> $$1^2+2^2+3^2+ \dots n^2 = \frac{n(2n+1)(n+1)}{6}$$

for $n=1$

LHS $P(1) = n^2 = 1$

RHS $$P(1) = \frac{1(2(1)+1)(1+1)}{6} = 1$$

LHS = RHS

so, $P(1)$ is true

Let $(P(n))$ is true for $n=k$

$\Rightarrow$ LHS = RHS

$$\boxed{1^2+2^2+3^2 \dots k^2 = \frac{k(2k+1)(k+1)}{6}} \quad -(1)$$

for $n=k+1$

LHS $\Rightarrow$

$1^2+2^2+3^2 \dots k^2+(k+1)^2$ from (1)

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

$$\Rightarrow \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{k^2+k+k+1}{(k+1)(k+2)}$$

$$= \frac{k(k+1)+k+1}{(k+1)(k+2)}$$

$$= \frac{\cancel{(k+1)}(k+1)}{\cancel{(k+1)}(k+2)}$$

$$RHS = \frac{k+1}{k+1+1} = \frac{(k+1)}{(k+2)}$$

LHS = RHS

$\because$ P(n) is true $\forall\, n \in N$

Ques $1.2.3 + 2.3.4 + 3.4.5 + \dots\dots$

$+ n(n+1)(n+2) = \frac{1}{4} n(n+1)(n+2)(n+3)$

$\forall\, n \in N$

for $n=1$

$LHS \Rightarrow n(n+1)(n+2) = 1(1+1)(1+2)$

$= 2(3) = 6$

$RHS \Rightarrow \frac{1}{4}(1)(1+1)(1+2)(1+3) = 6$

LHS = RHS

$\Rightarrow$ P(1) is true

Let P(n) is true for $n=k$

$\Rightarrow$ LHS = RHS

$$\boxed{1.2.3 + 2.3.4 + 3.4.5 + \dots + k(k+1)(k+2) = \frac{k(k+1)(k+2)(k+3)}{4}}$$

for $n = k+1$

$$LHS = 1.2.3 + 2.3.4 + \cdots k(k+1)(k+2) = (k+1)(k+2)(k+3)$$

from ①

$$LHS = \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3)$$

$$= \frac{k(k+1)(k+2)(k+3) + 4(k+1)(k+2)(k+3)}{4}$$

$$= \frac{(k+1)(k+2)(k+3)[k+4]}{4}$$

$$= \frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

$$RHS = \frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

LHS = RHS

$\therefore p(n)$ is true $\forall n \in N$

Ques $(n+1)^2 + (n+2)^2 + (n+3)^2 + \cdots (2n)^2 = \frac{n(2n+1)(7n+1)}{6}$

for $n=1$

$LHS = (2n)^2 = 4$

$RHS = \frac{n(2n+1)(7n+1)}{6} = 4$

$\Rightarrow p(1)$ is true