

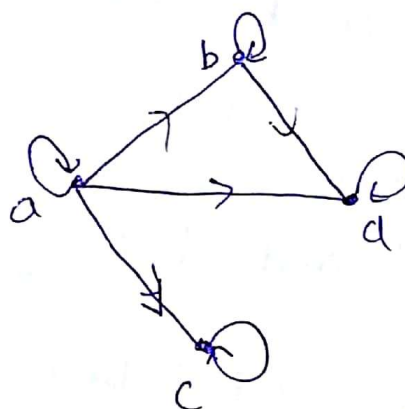
Order of Relation:-

There are two orders of relation

① Partial Order:- A Relation R in a set A is called partial order relation in P iff R is reflexive, anti-symmetric, and transitive. The order pair $\langle A, R \rangle$ is a partially ordered set, or a poset. The relation R is said to be a partial order on A .

Ex.

$$A = \{a, b, c, d\}$$

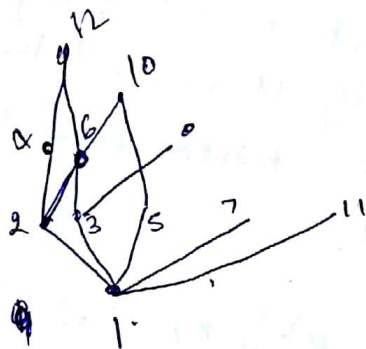


R
 $\langle a, b \rangle$
 $\langle a, c \rangle$
 $\langle b, d \rangle$
 $\langle a, d \rangle$
 $\langle a, a \rangle$
 $\langle b, b \rangle$
 $\langle c, c \rangle$
 $\langle d, d \rangle$

Ex

$$A = \{1, 2, 3, \dots, 12\}$$

aRb if a divides b



this is the poset
 diagram or Hasse diagram

Ex:-

let $X = \{2, 3, 6, 12, 24, 36\}$ and relation \leq be st $x \leq y$ if x divides y . Draw Hasse diagram. ✓

Closure of Relation

Reflexive closure:- If the relation R is not Reflexive, we add necessary elements to the relation to make it reflexive.

ex Set $A = \{a, b, c\}$

$$R = \{(a, b), (a, c), (c, c)\}$$

The relation is not reflexive
To make it reflexive we add link (a, a) and (b, b)

$$R(\tau) = R \cup \Delta_A$$

The reflexive closure of relation R is obtained by union of R and Identity relation Δ_A on the set A .

Symmetric closure:- Let R be a relation on A which is not symmetric and R^{-1} be inverse relation of R on A then symmetric closure R^* is defined as

$$R^* = R \cup R^{-1}$$

Transitive closure:- The relation obtained by adding the least number of order pair to ensure transitivity is called the transitive closure of relation. The transitive closure of R is denoted by R^+ .

$$R^+ = R \cup R^2 \cup R^3 \dots \cup R^m$$

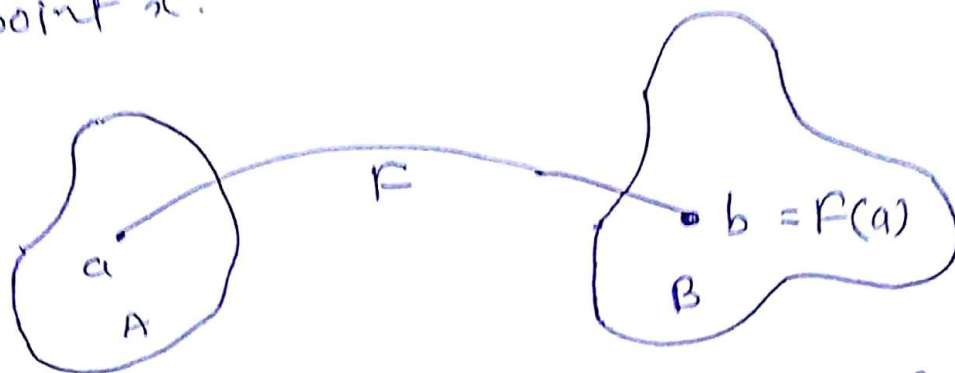
ex Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3), (3, 4)\}$ be a relation on A . Find transitive closure R^+ .

Definition

Function: - If A and B are non-empty set then a rule f , under which to every element x on the set A there corresponds one and only one elements on set B . then the rule f is called the function from A to B . it is denoted by

$$f: A \rightarrow B$$

$f(x)$ is called value of function f at point x .



Example: \rightarrow Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$

and let $f = \{(1, a), (2, a), (3, d), (4, c)\}$

hence we have

$$f(1) = a$$

$$f(2) = a$$

$$f(3) = d$$

$$f(4) = c$$

since each set $f(x)$ is a single value
 f is a function.

Classification of function

1. Real function \rightarrow A function

$f: A \rightarrow B$ is called real valued if the image of every element of A under f is real valued.

if $f(x) \in \mathbb{R} \forall x \in A$ or $y = f(x)$

Called a polynomial function. $\&$

④ Rational functions :-

$$f: A \rightarrow \mathbb{R}; f(x) = \frac{p(x)}{q(x)}$$

$p(x)$ and $q(x)$ are polynomial functions

$$A = \{x: x \in \mathbb{R} \text{ such that } q(x) \neq 0\}$$

⑤ Signum function :-

The function is defined by

$$f(x) = \begin{cases} \frac{|x|}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

$$f(x) = \begin{cases} 1 & \text{when } x > 0 \\ 0 & \text{when } x = 0 \\ -1 & \text{when } x < 0 \end{cases}$$

$$D_f = \mathbb{R} \text{ and } R_f = [-1, 0, 1]$$

Types of function

① Real function

$f: A \rightarrow B$ is called real valued if image of every element of A under f is a real number

if $f(x) \in \mathbb{R}, \forall x \in A$ or $y = f(x)$

~~the $y^2 = 4ax$ and $x^2 = 4ay$ are functions as they contains two dependent variable~~

y is dependent variable and x is independent variable

~~$y^2 = 4ax$~~

② modulus function

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$f(x)$ is called modulus function

$$R_f = \{|x|; x \in \mathbb{R}\} \text{ set of non negative no.}$$

③ Polynomial function -

if $f: \mathbb{R} \rightarrow \mathbb{R}_1$ st

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

where $n \in \mathbb{N}$ and $a_0, a_1, a_2, \dots, a_n \in \mathbb{R}$

① Let $A = \{a_1, a_2, a_3\}$ $B = \{b_1, b_2, b_3\}$, $C = \{c_1, c_2\}$
 $D = \{d_1, d_2, d_3, d_4\}$ Consider the following functions

(a) $f_1 = \{(a_1, b_2), (a_2, b_3), (a_3, b_1)\}$ (b) $f_2 = \{(a_1, d_2), (a_2, d_1), (a_3, d_4)\}$
 $f_3 = \{(b_1, c_1), (b_2, c_2), (b_3, c_1)\}$ (c) $f_4 = \{(d_1, b_1), (d_2, b_2), (d_3, b_1)\}$

Operation on Real valued funⁿ

(i) addition: $(f+g)x = f(x) + g(x)$, $f+g$ will be defined only for those values of x for which both f and g are defined ~~$(f+g)x = f(x) + g(x)$~~

(ii) subtraction: $(f-g)x = f(x) - g(x)$

(iii) multiplication: $(fg)x = f(x) g(x)$

(iv) Division: $\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)}$

Ex) Let $X = \{a, b, c\}$. Define $f: X \rightarrow X$ st
 $f = \{(a, b), (b, a), (c, c)\}$

find f^4

Ex) Let f and g be two funⁿ

$f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{4-x^2}$