ASSIGNMENT-II

(2 Marks Questions for Section–A)

1. Classify the differential equation: $\frac{\partial^2 z}{\partial x^2}+x^2\frac{\partial^2 z}{\partial y^2}=0$. (M.T.U. 2011)

2. Classify the partial differential equation: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{c^2}\frac{\partial u}{\partial t}$.

3. Classify the partial differential equation: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$. Also explain your answer. (A.K.T.U. 2016) (U.P.T.U. 2014)

4. Classify: $f_{xx}+2f_{xy}+4f_{yy}=0$. [G.B.T.U. (SUM) 2010]

5. Classify: $\frac{\partial^2 z}{\partial x^2}=\frac{\partial^2 z}{\partial y^2}$.

6. Classify the partial differential equation: $\frac{\partial^2 u}{\partial x^2}+3\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}=0$. (U.P.T.U. 2015)

7. Explain briefly the method of separation of variables in solving a given partial differential equation. (M.T.U. 2012)

8. Write down the two-dimensional wave equation. (A..K.T.U. 2017)

9. Mention two applications of partial differential equations in engineering. [G.B.T.U. (AG) 2012]

10. Name the following equations: [G.B.T.U. (AG) 2012]

 (i) $\frac{\partial^2 u}{\partial t^2}=c\frac{\partial^2 u}{\partial x^2}$ (ii) $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$.

11. Apply the method of separation of variables to find the most appropriate solution of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{c^2}\frac{\partial u}{\partial t}$. [G.B.T.U. (A.G.) 2012]

12. What does the two-dimensional wave equation represent?

13. Write down the partial differential equation for one-dimensional wave equation.

14. Solve: $4u_x+u_y=3u;\ u(0,y)=e^{-5y}$. (A.K.T.U. 2015, 2017)

15. Solve: $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ in the steady state. [G.B.T.U. (A.G.) 2011]

16. Classify: $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$

17. Name the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

18. Write down the equation of steady state heat conduction in the rectangular plate.

19. Write down the two-dimensional steady state heat flow equation in polar coordinates.

20. Solve: $3u_x + 2u_y = 0$ where $u_x = \frac{\partial u}{\partial x}$, $u_y = \frac{\partial u}{\partial y}$.

21. Classify the following differential equation in the first quadrant:

$$y^2 u_{xx} - x^2 u_{yy} = 0$$ (G.B.T.U. 2013)

22. Solve $\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t}$ using method of separation of variables. (G.B.T.U. 2013)

23. Write the boundary conditions and initial conditions for the displacement of a finite string of length L that is fixed at both ends and is released from rest with an initial displacement $f(x)$. (G.B.T.U. 2013)

24. (*i*) Write telegraph equations. (U.P.T.U. 2014)
(*ii*) Write two-dimensional heat equation. (A.K.T.U. 2016)

25. (*i*) Characterize the following partial differential equation into elliptic, parabolic and hyperbolic equations: $a\frac{\partial^2 z}{\partial x^2} + 2h\frac{\partial^2 z}{\partial x \partial y} + b\frac{\partial^2 z}{\partial y^2} + 2f\frac{\partial z}{\partial x} + 2g\frac{\partial z}{\partial x} + cz = f(x, y)$ where a, b, c, h, f, g are constants. (M.T.U. 2013)
(*ii*) Specify with suitable example, the classification of partial differential (PDE) for elliptic, parabolic and hyperbolic differential equations. (A.K.T.U. 2017)

26. Find the condition for which the following partial differential equation is parabolic:

$$yu_{xx} + (x + y)u_{xy} + xu_{yy} = 0$$ (U.P.T.U. 2013)

27. Classify the partial differential equation:

$$2\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} - 3\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0.$$ (U.P.T.U. 2015)

28. Classify the following partial differential equation along the line $y = x$:

$$yu_{xx} + (x + y)u_{xy} + xu_{yy} = 0.$$ (U.P.T.U. 2014)

29. (*i*) Find the steady state temperature distribution in a rod of length L when its one end is kept at 0°C and the other end is kept at 100°C. (U.P.T.U. 2013)
(*ii*) Find the steady state temperature distribution in a plate of length of 20 whose ends are kept at 40°C and 100°C respectively. (U.P.T.U. 2015)

30. (*i*) Find the steady state temperature distribution in a rod of 2 m whose ends are kept at 30°C and 70°C respectively. (U.P.T.U. 2015)
(*ii*) Find the steady state temperature distribution in a rod of length 20 cm, whose ends are kept at 0°C and 60°C. (U.P.T.U. 2014)

Answers

1. Elliptic
2. Parabolic
3. Elliptic
4. Elliptic
5. Hyperbolic
6. Hyperbolic

8. $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

10. (*i*) One-dimensional wave equation (*ii*) One-dimensional heat equation

11. $u(x, y, t) = (c_1 \cos k_1 x + c_2 \sin k_1 x)(c_3 \cos k_2 y + c_4 \sin k_2 y)\, c_5 e^{-c^2 k^2 t}$

12. Vibrations of a tightly stretched membrane

13. $\dfrac{\partial y}{\partial t} = c^2 \dfrac{\partial^2 y}{\partial x^2}$

14. $u(x, y) = e^{2x - 5y}$

15. $u = c_1 x + c_2$

16. Parabolic

17. Laplace equation in two dimensions

18. $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$

19. $r^2 \dfrac{\partial^2 u}{\partial r^2} + r \dfrac{\partial u}{\partial r} + \dfrac{\partial^2 u}{\partial \theta^2} = 0$

20. $u(x, y) = c\, e^{\frac{k}{6}(2x - 3y)}$

21. hyperbolic

22. $u(x, t) = c_1 c_2\, e^{-p^2(3x + t)}$

23. $y(0, t) = 0 = y(L, t)\}$ boundary conditions

$$\left.\begin{array}{l} \left(\dfrac{\partial y}{\partial t}\right)_{t=0} = 0 \\ y(x, 0) = f(x) \end{array}\right\} \text{initial conditions}$$

24. (i) $\dfrac{\partial^2 V}{\partial x^2} = RC \dfrac{\partial V}{\partial t}$ and $\dfrac{\partial^2 I}{\partial x^2} = RC \dfrac{\partial I}{\partial t}$ (ii) $\dfrac{\partial u}{\partial t} = c^2 \left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right)$

25. $h^2 < ab \rightarrow$ elliptic
$h^2 = ab \rightarrow$ parabolic
$h^2 > ab \rightarrow$ hyperbolic

26. $y = x$

27. hyperbolic

28. parabolic

29. (i) $u(x, 0) = \dfrac{100}{L} x$ (ii) $u(x, 0) = 40 + 3x$

30. (i) $u(x, 0) = 30 + 20x$ (ii) $u(x, 0) = 3x$