De Morgan's Law

$$(A \cup B)' = A' \cap B'$$
$$(A \cap B)' = A' \cup B'$$

1) Prove that: $(A \cup B)' = A' \cap B'$

__Proof__: LHS $\Rightarrow x \in (A \cup B)'$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$x \in (A' \cap B')$

$(A \cup B)' \subseteq A' \cap B'$ —— (1)

RHS $\Rightarrow x \in A' \cap B'$

$x \in A'$ and $x \in B'$

$x \notin A$ or $x \notin B$

$x \notin (A \cup B)$

$x \in (A \cup B)'$

$A' \cap B' \subseteq (A \cup B)'$ —— (2)

From (1) & (2),

$$\boxed{(A \cup B)' = A' \cap B'}$$

2) Prove that: $(A \cap B)' = A' \cup B'$

Proof: LHS ⇒ $x \in (A \cap B)'$

$x \notin A \cap B$

$x \notin A$ and $x \notin B$

$x \in A'$ or $x \in B'$

$x \in A' \cup B'$

$(A \cap B)' \subseteq A' \cup B'$ — ①

RHS ⇒ $x \in A' \cup B'$

$x \in A'$ or $x \in B'$

$x \notin A$ and $x \notin B$

$x \notin A \cap B$

$x \in (A \cap B)'$

$\cancel{(A \cap B)' \subseteq A' \cup}$

$A' \cup B' \subseteq (A \cap B)'$ — ②

From ① & ②, $\boxed{(A \cap B)' = A' \cup B'}$

⊛ Distributive law

1) Prove that: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof: LHS ⇒ $x \in A \cup (B \cap C)$

$x \in A$ or $x \in (B \cap C)$

$x \in A$ or $[x \in B$ and $x \in C]$

$x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$x \in (A \cup B)$ and $x \in (A \cup C)$

$x \in (A \cup B) \cap (A \cup C)$

$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ — ①

RHS ⇒ $x \in (A \cup B) \cap (A \cup C)$

$x \in (A \cup B)$ and $x \in (A \cup C)$

$(x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$x \in A$ or $(x \in B$ and $x \in C)$

$x \in A$ or $(x \in (B \cap C))$

$x \in A \cup (B \cap C)$

$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ — ②

From ① & ②, $\boxed{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$

2) Prove that: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof: LHS $\Rightarrow$ $x \in A \cap (B \cup C)$

$x \in A$ and $x \in (B \cup C)$

$x \in A$ and $[x \in B$ or $x \in C]$

$x \in A$ and $x \in B$ or $x \in A$ and $x \in C$

$x \in (A \cap B)$ or $x \in (A \cap C)$

$x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ — ①

RHS $\Rightarrow$ $x \in (A \cap B) \cup (A \cap C)$

$x \in (A \cap B)$ or $x \in (A \cap C)$

$x \in A$ and $x \in B$ or $x \in A$ and $x \in C$

$x \in A$ and $[x \in B$ or $x \in C]$

$x \in A$ and $x \in (B \cup C)$

$x \in A \cap (B \cup C)$

$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ — ②

From ① & ②, $\boxed{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$