(1)

MATHS

TEST - 1

VARTUL

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(1) Here, $A = 1$, $B = 2x$, $C = 1 - y^2$

$\Rightarrow B^2 - 4AC = 4x^2 - 4(1-y^2) = 4(x^2+y^2-1)$

eqn is elliptical if

$B^2 - 4AC < 0$

$\therefore 4(x^2+y^2-1) < 0$ or $x^2+y^2 < 1$

eqn is Parabolic if,

$B^2 - 4AC = 0$

$\therefore 4(x^2+y^2-1) = 0$ or $x^2+y^2 = 1$

eqn is Parabolic if,

$B^2 - 4AC > 0$

$\therefore 4(x^2+y^2-1) > 0$ or $x^2+y^2 > 1$

(2) Let $u = XY$ — (1)

$$\therefore \frac{\partial u}{\partial y} = \frac{\partial (XY)}{\partial y} = X\frac{dY}{dy} = XY'$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 (XY)}{\partial x^2} = Y\frac{d^2X}{\partial x} = YX''$$

from the given eqn.

$$YX'' = XY' + 2XY$$

$$\frac{X''}{X} = \frac{Y' + 2Y}{Y}$$

$$\frac{X''}{X} = \frac{Y'}{Y} + 2 = k \quad \text{— (2)}$$

1st case :- $\frac{X''}{X} = k$

$$X'' - kX = 0$$

$$\therefore m^2 - k = 0$$

$$m = \pm\sqrt{k}$$

$$C.F. = c_1 e^{\sqrt{k}x} + c_2 e^{-\sqrt{k}x}$$

$$P.I. = 0$$

$$\therefore X = c_1 e^{\sqrt{k}x} + c_2 e^{-\sqrt{k}x} \quad \text{— (3)}$$

Case (ii) $\Rightarrow \frac{Y'}{Y} + 2 = k$

$$\frac{Y'}{Y} = k-2$$

$$\frac{dY}{Y} = (k-2)\,dy$$

Integration both sides

$$\log Y = (k-2)y + \log C_3$$

$$Y = C_3 e^{(k-2)y} \quad \text{--- (4)}$$

$\therefore$ from (1)

$$u(x,y) = \left(C_1 e^{\sqrt{k}x} + C_2 e^{-\sqrt{k}x}\right) C_3 e^{(k-2)y} \quad \text{--- (5)}$$

Applying condition $u(0,y) = 0$ in (5)

$$u(0,y) = 0 = (C_1 + C_2)\, C_3 e^{(k-2)y}$$

$$C_1 + C_2 = 0 \Rightarrow C_2 = -C_1 \quad \text{--- (6)}$$

from (5) most general eqⁿ is

$$u(x,y) = \sum C_1 C_3 \left(e^{\sqrt{k}x} - e^{-\sqrt{k}x}\right) e^{(k-2)y} \quad \text{--- (7)}$$

$$\frac{\partial u}{\partial x} = \sum C_1 C_3 \sqrt{k} \left(e^{\sqrt{k}x} + e^{-\sqrt{k}x}\right) e^{(k-2)y}$$

$$\left(\frac{\partial u}{\partial x}\right)_{x=0} = 1 + e^{-3y} = \sum c_1 c_3 \sqrt{k}\,(2)\, e^{(k-2)y}$$

$$= \sum_{n=1}^{\infty} b_n e^{(k-2)y}$$

Comparing the coff.

Case ① :- $b_1 = 1, \; k-2 = 0$

$$2c_1c_3\sqrt{k} = 1, \; k = 2$$

$$\therefore \; c_1c_3 = \frac{1}{2\sqrt{2}}$$

Case ② :- $b_3 = -1, \; k-2 = -3$

$$2c_1c_3\sqrt{k} = 1, \; k = -1$$

$$c_1c_3 = \frac{1}{2i}$$

$\therefore$ from ⑦

$$u(x,y) = \frac{1}{2\sqrt{2}}\left(e^{\sqrt{2}x} - e^{-\sqrt{2}x}\right) + \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)e^{-3y}$$

$$u(x,y) = \frac{1}{\sqrt{2}}\sinh\sqrt{2}\,x + e^{-3y}\sin x$$