Set Theory and Relations Functions

* **Set :-** A set is any well defined collection of distinct or distinguisable elements by a well defined collection means that there exists a rule with the help of which we should be able to say whether any given object belongs to or not to the collection under some specific rule.

The set of students in a class.
The set of months in a year.
$A = \{1, 2, 3\}$.

Generally capital letters A, B, C, X, Y, Z etc are used to denote sets and its elements by lowercase letters a, b, c, x, y, z - - etc.

* **Elements of a set :-**

If $A = \{1, 2, 3\}$ the objects in a set are called elements or members.
The distinct elements means no element is repeated.
The Distinguishable means that given any object or element is either in the set or not in the set. so the elements of a set must be distinct and distinguishable.

The symbol $\in$ (Epsilon) is used to Indicate "belongs to" if 1 is the element of set A then symbolically we write

$$1 \in A$$

The symbol $\notin$ is used to indicate "does not belongs to" if 4 is not an element of set A then symbolically we write -

$$4 \notin A$$

* **Symbols :-**

N = Natural Numbers = $\{1, 2, 3, \ldots\}$
I or Z = set of Integers = $\{\ldots -2, -1, 0, 1, 2, \ldots\}$
Z^+ = set of Positive Integers = $\{0, 1, 2, \ldots\}$
Z^- = set of Negative Integers = $\{\ldots, -2, -1\}$.
R = set of Real Numbers.
Q = Set of Rational Numbers (where $\frac{p}{q}$, $q \neq 0$)
C = set of Complex Numbers.

* Set Representation :-

Set are represented by two forms:-

- Roster or Enumeration or Tabular form.
- Set builder form.

- **Roster form** :- In this method the elements of a set are described by listing or written them within braces seperated by comma (,).

e.g. The set of vowel in English alphabet can be written as-

$V = \{a, e, i, o, u\}$

The set of Natural Number.

$N = \{1, 2, 3, ---\}.$

- **Set builder form** :- In this method the elements of a set are represented by stating a property that uniquely characterised them.

e.g. Let consider the sets

$A = \{1, 2, 3\}$ $B = \{2, 4, 6, 8 ---\}$ $C = \{a, e, i, o, u\}$

$A = \{x : x \in N \ \& \ x < 4\}.$

$B = \{x : x = 2n \ \& \ n \in N\}$

OR

$= \{x : x$ is an even natural number$\}.$

OR

$= \{x : x$ is an even positive integer$\}.$

$C = \{x : x$ is a vowel in English alphabet$\}.$

* Types of Set :-

- **Cardinality of set (OR) Cardinal No. of Set** :-

The number of distinct elements in a set is called cardinal no. of set.

The number of distinct elements in set A is called cardinal no. of set it is represented as -

$O(A)$ (OR) $n(A)$ (OR) $|A|.$

$A = \{1, 2, 3\}$

$\boxed{n(A) = 3}$

- **Finite Set** :- A set is finite if it contains finite no. of

The set of rivers in India.

- Infinite set :- A set which is not finite is called Infinite set.

  e.g. set of Natural Numbers.

- Singelton Set :- A set which contains only 1 element is called singelton set.

  e.g. $A = \{ x : 4 < x < 6 \ \& \ x \in N \}$

  $A = \{ \phi \}$.

- Null set (OR) Empty (OR) Void set :- A set which contain no or zero elements is called Null set.

  This set is denoted by ϕ (OR) $\{ \}$.

  Cardinality of null set is always zero.

- Equality of sets (OR) Equal sets :- Two sets A and B are said to be equal if every element of set A is an element of B and also every element of set B is an element of A. The equality of two sets A and B are denoted by -

  $\boxed{A = B}$.

  symbolically, $A = B$ if and only if $\boxed{x \in A \iff x \in B}$

- Equivalence set :- If the elements of 1 set can be put for one to one maping with the elements of another set then two sets are called equivalence set.

  It is denoted by $\sim$ (OR) $\equiv$ (OR) $\cong$.

  (OR)

  If the elements of sets are same then it is known as equivalent set

* Subset :- Let A and B be <u>two non-empty sets</u>. The set A is subset of B if and only if every element of set A is an element of set B.

  subset is denoted by " $\subseteq$ ".

  symbolically, $A \subseteq B$ iff $x \in A \implies x \in B$.

  e.g. $A = \{1, 2\}$

  $B = \{1, 2, 3\}$.

  Here, $A \subseteq B$

  A is contained in B.

* **Superset** :- It is denoted by "$\supseteq$"
If A is subset of B then, B is superset of A.
$B \supseteq A$.

* **Properties of Subset** :-
  - ϕ is the subset of every set.
  - If $A \subseteq B$ & $B \subseteq A$, then $A = B$.
  - If $A \subseteq B$ & $B \subseteq C$ then $A \subseteq C$. (A is also subset of C).

* **Power set** :- The set of all possible subsets of set A is called Power set of set A. Denoted by "P(A)".

e.g. $A = \{1, 2\}$

$P(A) = \{\{\phi, \{1\}, \{2\}, \{1,2\}\}$.

$A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}; \{1,2\}, \{1,3\}; \{2,3\}, \{1,2,3\}\}$.

NOTE :- If a set has 'n' elements then '2^n' subsets of that set are possible

* Proof :- Let Set A has n elements.
So, no. of subsets taking one element $= {}^nC_1$
No. of subsets taking two element $= {}^nC_2$
No. of subsets taking three element $= {}^nC_3$.
" " " " "
" " " " "
No. of subsets taking n element $= {}^nC_n$.

We know that ϕ is subset of every set.
So, Total no. of subsets

$$= 1 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n.$$
$$= {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n.$$
$$= (1+1)^n$$
$$= (2)^n.$$

Prob. Find P(A) for the following sets.

1) $A = \{a\}$. $P(A) = \{\phi, \{a\}\}$
2) $A = \{\phi, \{\phi\}\}$. $P(A) = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}, \{\{\phi\}\}\}$
3) $A = \{\phi\}$. $P(A) = \{\phi, \{\phi\}\}$
4) $A = \{\{a\}\}$ $P(A) = \{\phi, \{\{a\}\}\}$
5) $A = \phi$ $P(A) = \{\phi\}$

• Union :- Let A and B be two ~~sets~~ non-empty sets the union of A and B is the set of all elements which are either in A or in B or in both A and B. and the union of A and B is denoted by '$A \cup B$'. It is also known as Joint or Logical Sum of A and B.

Symbolically,

$$A \cup B = \{ x : x \in A \text{ or } x \in B \}.$$

e.g. $A = \{1, 2, 3\}$ $\quad B = \{1, 2, 3, 4, 5\}$.

$A \cup B = \{1, 2, 3, 4, 5\}$.

◉ Properties of Union of Set :-

★ Commutative Property :- Union of sets is commutative i.e. $A \cup B = B \cup A$.

Proof :- Let $x \in (A \cup B)$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in (B \cup A)$

So. $(A \cup B) \subseteq (B \cup A)$ —— ①.

Let $x \in (B \cup A)$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in (A \cup B)$

So, $(B \cup A) \subseteq (A \cup B)$ —— ②

From eqn ① & ②

$$\boxed{A \cup B = B \cup A}$$

★ Associative Property :- Union of sets is Associative. i.e $A \cup (B \cup C) = (A \cup B) \cup C$.

Proof :- Let $x \in (A \cup (B \cup C))$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$.

$\Rightarrow x \in (A \cup B)$ or $x \in C$.

$\Rightarrow x \in ((A \cup B) \cup C)$

So, $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ —— ①

Let $x \in ((A \cup B) \cup C)$

$\Rightarrow x \in (A \cup B)$ or $x \in C$.

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ or $x \in B \cup C$

$\Rightarrow x \in A \cup (B \cup C)$

So, $(A \cup B) \cup C \subseteq A \cup (B \cup C)$ —— (2)

From eqn (1) & (2),

$$\boxed{A \cup (B \cup C) = (A \cup B) \cup C}$$

★ Idempotent Property :- Union of sets is idempotent.
i.e. $A \cup A = A$.

Proof :- Let, $x \in A \cup A$

$\Rightarrow x \in A$ or $x \in A$

$\Rightarrow x \in A$.

So, $(A \cup A) \subseteq A$ —— (1)

Let $x \in A$.

$\Rightarrow x \in A$ or $x \in A$.

$\Rightarrow x \in (A \cup A)$

So, $A \subseteq (A \cup A)$ —— (2)

From eqn (1) & (2) $\boxed{A \cup A = A}$

★ Let A and B be two non-empty sets.

$A = \{1, 2, 3\}$

$B = \{1, 2, 4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5\}$.

$\Rightarrow A \subseteq (A \cup B)$

$\Rightarrow B \subseteq (A \cup B)$

★

$A \cup \phi = A$

$A \cup U = U$

★ If $A \subseteq B$ then $A \cup B = B$.

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$.

$\Rightarrow x \in B$ or $x \in B$ ($\because A \subseteq B$ then $x \in A \Rightarrow x \in B$)

$\Rightarrow x \in B$.

So, $A \cup B \subseteq B$ —— (1)

We know that, $B \subseteq A \cup B$. —— (2)

From (1) & (2)

$$\boxed{A \cup B = B}$$

e.g. $A = \{1, 2\}$

$B = \{1, 2, 3\}$

is the set of elements which belongs to both A and B (common to both A and B).
The intersection of A and B is denoted by '$A \cap B$'.
Symbolically,
$$A \cap B = \{x : x \in A \text{ and } x \in B\}.$$

e.g. $A = \{1, 2, 3\}$
$B = \{1, 3, 5\}$.
$A \cap B = \{1, 3\}$.

⊙ Properties of Intersection of Sets :-

→ Commutative Property :- Intersection of sets is commutative.
i.e. $A \cap B = B \cap A$.

Proof :- Let $x \in A \cap B$
$\Rightarrow x \in A$ and $x \in B$.
$\Rightarrow x \in B$ and $x \in A$.
$\Rightarrow x \in (B \cap A)$
So, $A \cap B \subseteq B \cap A$. —— (1)

Let $x \in B \cap A$
$\Rightarrow x \in B$ and $x \in A$
$\Rightarrow x \in A$ and $x \in B$
$\Rightarrow x \in (A \cap B)$
So, $B \cap A \subseteq A \cap B$ —— (2)

From eqn (1) & (2) -
$$\boxed{A \cap B = B \cap A}$$

Associative Property :- Intersection of sets is Associative.
i.e. $A \cap (B \cap C) = (A \cap B) \cap C$.

Proof :- Let $x \in A \cap (B \cap C)$
$\Rightarrow x \in A$ and $x \in B \cap C$
$\Rightarrow x \in A$ and ($x \in B$ and $x \in C$.)
$\Rightarrow x \in A \cap B$ and $x \in C$.
$\Rightarrow x \in (A \cap B) \cap C$.
So, $A \cap (B \cap C) \subseteq (A \cap B) \cap C$. —— (1)

Let $x \in (A \cap B) \cap C$.
$\Rightarrow x \in (A \cap B)$ and $x \in C$.
$\Rightarrow$ ($x \in A$ and $x \in B$) and $x \in C$.
$\Rightarrow x \in A$ and $x \in (B \cap C)$
$\Rightarrow x \in A \cap (B \cap C)$
So, $(A \cap B) \cap C \subseteq A \cap (B \cap C)$ —— (2).

From eqⁿ ① & ② -

$$\boxed{A \cap (B \cap C) = (A \cap B) \cap C}$$

* Idempotent Property :- Intersection of sets is Idempotent ie. $A \cap A = A$.

Proof :- Let $x \in A \cap A$

$\Rightarrow x \in A$ and $x \in A$.

$\Rightarrow x \in A$.

So, $A \cap A \subseteq A$ — ①

Let $x \in A$.

$\Rightarrow x \in A$ and $x \in A$.

$\Rightarrow x \in (A \cap A)$

So, $A \subseteq A \cap A$ — ②

From eqⁿ ① & ②

$$\boxed{A \cap A = A}$$

*

$A = \{1, 2, 3\}$

$B = \{1, 3, 5\}$.

$A \cap B = \{1, 3\}$.

$\Rightarrow A \cap B \subseteq A$

$\Rightarrow A \cap B \subseteq B$.

*

$A \cap \phi = \phi$

$A \cap U = A$.

NOTE: $A \cap B \subseteq A \cup B$.

- Compliment of sets :- Let U be the universal set and A be any subset of universal set the compliment of A is a set containing elements of universal set which do not belong to A.
The compliment of A is denoted by A' (OR) A^c (OR) $\bar{A}$.

e.g. $U = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 4, 6\}$.

$A' = \{2, 3, 5\}$.

Symbolically, $A' = \{x : x \in U \text{ and } x \notin A\}$.

NOTE: $x \in A^c$ (OR) $x \notin A$ are same.

difference of

those elements which belongs to A but does not belongs to B.
It is denoted by 'A − B' (or) 'A / B'.
Symbolically,

$$A - B = \{ x : x \in A \text{ and } x \notin B \}$$

e.g. $A = \{1, 2, 3\}$
$B = \{2, 3, 4, 5\}$.

$A - B = \{1\}$
$B - A = \{4, 5\}$.

<u>NOTE</u>:- $A - B = A - (A \cap B)$
$B - A = B - (A \cap B)$

$\boxed{A - B \neq B - A}$

* **Symmetric Difference of sets** :- Let A and B be two sets then symmetric difference of A and B is a set containing all the elements that belong to A or B but not both.
It is denoted by '$A \oplus B$'.

e.g. $A = \{1, 2, 3, 4, 5\}$
$B = \{4, 5, 6\}$

$A \oplus B = \{1, 2, 3, 6\}$.

$A \oplus B = (A \cup B) - (A \cap B)$
$A \cup B = \{1, 2, 3, 4, 5, 6\}$
$A \cap B = \{4, 5\}$

$A \oplus B = (A \cup B) - (A \cap B) = \{1, 2, 3, 6\}$.

* **Disjoint Sets** :- Let A and B be two sets then, if there is no common element b/w A and B then, they are said to be disjoint sets.

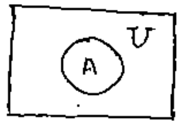
e.g. $A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$.

If $A \cap B = \Phi$ Then A and B are disjoint sets.

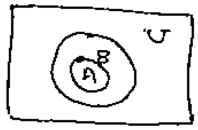
* **Proper Subset** :- Let A and B be two non-empty sets and $A \subseteq B$ if there is atleast one element in B which does not belong to A then, A is called proper subset of B and it is denoted by '$\subset$' ($A \subset B$)

* **Venn Diagram** :- Venn Diagram is a pictorial representation of sets which are used to show relationships ~~b/w~~ among sets.
The universal set is represented by interior of a rectangle and its subsets are represented by circular areas within the rectangle.

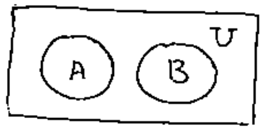
$A \subseteq U$



$A \subseteq B$

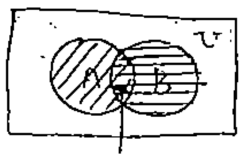


Disjoint sets

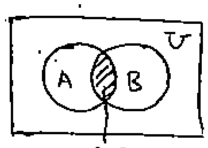


$A \cap B \neq \phi$

$A \cup B$
$A \cap B$.

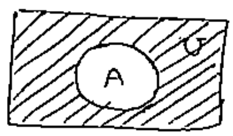


$A \cap B$



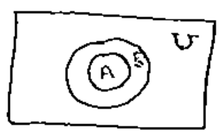
$A \cap B$

A^c



$A - B$

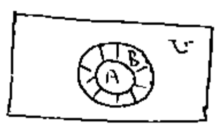
$A \subseteq B$



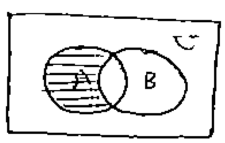
No shaded area means
So, $A - B = \phi$.

$B - A$.

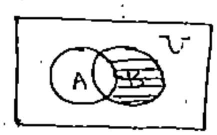
$A \subseteq B$



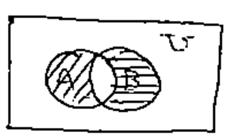
$A - B$



$B - A$



$A \oplus B$

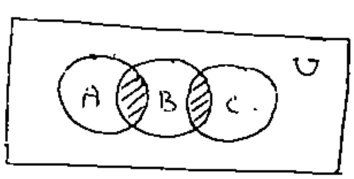


<u>NOTE:</u>

$$\boxed{A \oplus B = (A - B) \cup (B - A)}$$

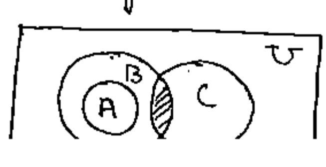
Q. Draw a venn Diagram of sets A, B & C where -
A and B have common elements; B and C have common elements but A and C are disjoint.

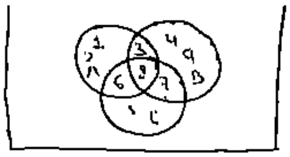
<u>Soln</u>



Q $A \subseteq B$, set A and C are disjoint but B and C have elements in common.

<u>soln</u>.





1) $A - (B \cap C)$
2) $(A \cup B) - C$
3) $B \oplus C$
4) $A - (B - C)$
5) $A - (B \oplus C)$
6) $A \cap (B \oplus C)$

$A = \{1, 2, 6, 3, 8$

$B = \{3, 4, 7, 8, 9$

$C = \{5, 6, 7, 8\}$.

$B \cap C = \{7, 8\}$

$A \cup B = \{1, 2, 3, 4, 6$

$B \oplus C = (B \cup C) - ($

or $(B - C) \cup ($

$B - C = \{3, 4, 9\}$

1) $A - (B \cap C) = \{1, 2, 3, 8, 6\}$

2) $(A \cup B) - C = \{1, 2, 3, 4, 9\}$

3) $(B \oplus C) = \{6, 3, 4, 5, 9\}$

4) $A - (B - C) = \{1, 2, 6, 8\}$.

5) $A - (B \oplus C) = \{1, 2, 8\}$.

6) $A \cap (B \oplus C) = \{6, 3\}$.

* Algebra of Sets :-

- Idempotent Law :- 1) $A \cup A = A$
  2) $A \cap A = A$.

- Commutative Law :- 1) $A \cup B = B \cup A$
  2) $A \cap B = B \cap A$.

- Associative Law :- 1) $A \cup (B \cup C) = (A \cup B) \cup C$.
  2) $A \cap (B \cap C) = (A \cap B) \cap C$.

- Identity Law :- 1) $A \cup \phi = A$
  2) $A \cap U = A$.

- Bound Law :- 1) $A \cup U = U$ ([illegible])
  2) $A \cap \phi = \phi$. ([illegible]).

- Compliment Law :- 1) $U^c = \phi$
  2) $\phi^c = U$
  3) $A \cup A^c = U$
  4) $A \cap A^c = \phi$.

- Involution Law :- $(A^c)^c = A$.

- Distributive Law :-

Q Prove that union of sets is distributive over intersect of sets and vice-versa.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Prove :- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Proof :- Let $x \in (A \cup (B \cap C))$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A$ or $x \in B$ and $x \in C$.

$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$

$\Rightarrow (x \in A \cup B) \text{ and } (x \in A \cup C).$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$ — ①

So, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Let, $x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow x \in (A \cup B) \text{ and } x \in (A \cup C)$

$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$

$\Rightarrow x \in A \text{ or } x \in B \text{ and } x \in C.$

$\Rightarrow x \in A \text{ or } x \in B \cap C.$

$\Rightarrow x \in A \cup (B \cap C)$

So, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ — ②

from eqⁿ ① & ②

$\Rightarrow \boxed{A \cup (B \cap C) = (A \cup B) \cap (A \cup C).}$

Prove $\quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof: Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A \text{ and } x \in (B \cup C)$

$\Rightarrow x \in A \text{ and } x \in B \text{ or } x \in C.$

$\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$

$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C).$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

So, $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ — ①

Let, $(A \cap B) \cup (A \cap C) \ni x.$

$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$

$\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$

$\Rightarrow x \in A \text{ and } (x \in B \text{ or } x \in C)$

$\Rightarrow x \in A \text{ and } x \in (B \cup C)$

$\Rightarrow x \in A \cap (B \cup C)$

So, $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ — ②

From eqⁿ ① & ②

$\Rightarrow \boxed{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

Demorgan's Law :- 1) $(A \cup B)^c = A^c \cap B^c$

2) $(A \cap B)^c = A^c \cup B^c$.

<u>Prove</u>:- $(A \cup B)^c = A^c \cap B^c$.

<u>Proof</u>:- Let $x \in (A \cup B)^c$.

$\{ x \in A^c$
$x \notin A \}$.

{ In negativity for union we use [illegible] & for [illegible] we [illegible]

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ **and** $x \notin B$.

$\Rightarrow x \in A^c$ and $x \in B^c$.

$\Rightarrow x \in (A^c \cap B^c)$

So, $(A \cup B)^c \subseteq (A^c \cap B^c)$ —— 1.

Let, $x \in A^c \cap B^c$

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \in (A \cup B)^c$.

So, $(A^c \cap B^c) \subseteq (A \cup B)^c$ —— 2

From eqn 1 & 2 -

$\Rightarrow \boxed{(A \cup B)^c = (A^c \cap B^c)}$

<u>Prove</u>: $(A \cap B)^c = A^c \cup B^c$.

<u>Proof</u>: Let $x \in (A \cap B)^c$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ and $x \notin B$.

$\Rightarrow x \in A^c$ or $x \in B^c$.

$\Rightarrow x \in A^c \cup B^c$

So, $(A \cap B)^c \subseteq (A^c \cup B^c)$ —— 1

Let, $x \in A^c \cup B^c$.

$\Rightarrow x \in A^c$ or $x \in B^c$.

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \in (A \cap B)^c$

So, $(A^c \cup B^c) \subseteq (A \cap B)^c$ —— 2

From eqn 1 & 2

$\Rightarrow \boxed{(A \cap B)^c = A^c \cup B^c}$

Q Show that $(A-B)-C = A-(B\cup C)$.

Let $x \in (A-B)-C$

$\Rightarrow x \in (A-B)$ and $x \notin C$.

$\Rightarrow (x \in A$ and $x \notin B)$ and $x \notin C$.

$\Rightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow x \in A$ and $(x \notin (B\cup C))$

$\Rightarrow x \in A-(B\cup C)$

So, $(A-B)-C \subseteq A-(B\cup C)$ —— (1)

Let, $x \in A-(B\cup C)$

$\Rightarrow x \in A$ and $x \notin (B\cup C)$

$\Rightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow (x \in A$ and $x \notin B)$ and $x \notin C$

$\Rightarrow x \in A-B$ and $x \notin C$

$\Rightarrow x \in (A-B)-C$.

So, $A-(B\cup C) \subseteq (A-B)-C$. —— (2)

from eqn (1) & (2)

$\Rightarrow \boxed{(A-B)-C = A-(B\cup C)}$

Q Prove that $(A-B)\cap(B-A) = \phi$.

Let, $x \in (A-B)\cap(B-A)$

$\Rightarrow x \in (A-B)$ and $x \in (B-A)$.

$\Rightarrow (x \in A$ and $x \notin B)$ and $(x \in B$ and $x \notin A)$

$\Rightarrow (x \in A$ and $x \notin A)$ and $(x \in B$ and $x \notin B)$.

$\Rightarrow x \in \phi$ and $x \in \phi$.

$\Rightarrow x \in \phi$.

So, $(A-B)\cap(B-A) \subseteq \phi$. —— (1)

~~Let, x~~ We know that ϕ is subset of every set

$\phi \subseteq (A-B)\cap(B-A)$ —— (2)

From (1) & (2)

$\Rightarrow \boxed{(A-B)\cap(B-A) = \phi.}$

Q Prove that $A-(A\cap B) = (A-B)$

Let, $x \in A-(A\cap B)$

$\Rightarrow x \in A$ and $x \notin (A\cap B)$

$\Rightarrow x \in A$ and $(x \notin A$ or $x \notin B)$

$\Rightarrow (x \in A$ and $x \notin A)$ or $(x \in A$ and $x \notin B)$

$\Rightarrow x \in \phi$ or $x \in (A-B)$

$\Rightarrow x \in \phi \cup (A-B)$

$\Rightarrow x \in (A-B)$

So, $A-(A\cap B) \subseteq (A-B)$ —— (1)

$\Rightarrow$ ~~$x \in A$ and $x \notin B$~~

$\Rightarrow x \in \phi \cup (A - B)$

$\Rightarrow x \in \phi$ or $x \in (A - B)$.

$\Rightarrow x \in \phi$ or $x \in A$ and $x \notin B$.

$\Rightarrow (x \in A$ and $x \notin A)$ or $(x \in A$ and $x \notin B)$.

$\Rightarrow x \in A$ and $(x \notin A$ or $x \notin B)$

$\Rightarrow x \in A$ and $x \notin (A \cap B)$

$\Rightarrow x \in A - (A \cap B)$.

So, $(A - B) \subseteq A - (A \cap B)$ —— ②

$\Rightarrow \boxed{A - (A \cap B) = A - B}$ From ① & ②.

Q If A and B are two sets then $(A \cap B) \cup (A \cap \sim B)$ and $A \cap (\sim A \cup B)$ are equal to ?

$\sim B$ = compliment
$\sim(A \cup B) = (A \cup B)'$

$(A \cap B) \cup (A \cap \sim B)$.

$\Rightarrow A \cap (B \cup \sim B)$ [By Distributive Law].

$\Rightarrow A \cap U$ [By compliment Law].

$\Rightarrow A$ [By Identity Law].

$A \cap (\sim A \cup B)$

$\Rightarrow (A \cap \sim A) \cup (A \cap B)$ [Distributive Law]

$\Rightarrow \phi \cup (A \cap B)$ [By compliment Law]

$\Rightarrow A \cap B$ [By Identity Law].

Q If A and B are two subsets of universal set then prove the following -

1) $A - B = B - A$ if & only if $A = B$.

2) $A - B = A$ iff $A \cap B = \phi$.

1) (i) If $A = B$ then $A - B = B - A$.

Let $x \in A - B$

$\Rightarrow x \in A$ and $x \notin B$.

$\Rightarrow x \in B$ and $x \notin A$.

$\Rightarrow x \in (B - A)$

So, $(A - B) \subseteq B - A$ —— ①.

Let $x \in (B - A)$

$\Rightarrow x \in B$ and $x \notin A$.

$\Rightarrow x \in A$ and $x \notin B$.

$\Rightarrow x \in (A - B)$

So, $(B - A) \subseteq (A - B)$ —— ②

From ① & ②.

$\Rightarrow \boxed{(A - B) = (B - A)}$

(1) If $(A-B) = (B-A)$ then $A = B$.

$x \in (B-A)$

$\Rightarrow x \in B$ and $x \notin A$. — ①

$x \in (A-B)$

$\Rightarrow x \in A$ and $x \notin B$. — ②

Eqⁿ ① & ② can be equal only when $A = B$.

(2) (i) ~~If $A \cap B = \phi$ then~~ If $A - B = A$ then $A \cap B = \phi$

~~$x \in A - B$.~~

~~$\Rightarrow x \in A$ and $x \notin B$.~~

Let, $A \cap B \neq \phi$.

$\Rightarrow x \in (A \cap B)$

$\Rightarrow x \in (A-B) \cap B$. $\quad \because [A - B = A]$

$\Rightarrow x \in A-B$ and $x \in B$.

$\Rightarrow (x \in A$ and $x \notin B)$, and $x \in B$.

$\Rightarrow x \in A$ and $(x \notin B$ and $x \in B)$

$\Rightarrow x \in A$ and $x \notin \phi$.

$\Rightarrow x \in A \cap \phi$.

$\Rightarrow x \in \phi$.

So, $A \cap B \subseteq \phi$. — ① But this contradicts our assumption $A \cap B = \phi$.

But, we know that ϕ is subset of every set — ②

$\phi \subseteq A \cap B$. — ②

from eqⁿ ① & ②

$\boxed{A \cap B = \phi}$

(ii) If $A \cap B = \phi$ then $A - B = A$

Let, $x \in A - B$.

$\Rightarrow x \in A$ and $x \notin B$.

$\Rightarrow x \in A$ $\quad (\because A \cap B = \phi)$.

So, $A - B \subseteq A$ — ①

Let, $x \in A$

$\Rightarrow x \in A$ and $x \notin B$. $\quad [\because A \cap B = \phi]$

$\Rightarrow x \in (A-B)$

So, $A \subseteq A - B$ — ②

From ① & ②

$\boxed{A - B = A}$

(OR)

If $(A \cap B) = \phi$ then $A - B = A$

Let, $x \in (A-B)$

$\Rightarrow x \in (A-B) \cup \phi$

$\Rightarrow x \in (A-B)$ or $x \in (A \cap B)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \in B)$

$\Rightarrow (x \in A$ and $x \in B')$ or $(x \in A$ and $x \in B)$.

$\Rightarrow x \in A$ and $(x \in B'$ ~~and~~ or $x \in B)$

$\Rightarrow x \in A$ and $x \in (B \cup B')$

$\Rightarrow x \in A$ and $x \in U$

$\Rightarrow x \in (A \cap U)$

$\Rightarrow x \in A$

So, $A - B \subseteq A$ —— ①.

* Cartesian Product :-

$A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$.

e.g. ordered pair . $A = \{1, 2, 3\}$

$B = \{a, b\}$.

$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$.

$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

Ordered Pair :- It is a pair of objects formed by using the two components in specified order

In the ordered pair (x, y), x is the first component and y is the second component.

Consider two sets A and B, then cartesian product of A and B is denoted by '$A \times B$' and it is the set of all possible ordered pairs (x, y) with $x \in A$ and $y \in B$.

Symbolically,

$A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$.

<u>NOTE</u>: $A \times B \neq B \times A$.

* Multiset :-

Multisets are sets where an element appea more than ones.

e.g. $A = \{1, 1, 1, 2, 2, 3\}$.

multiset A can also be written as-

$A = \{3 \cdot 1, 2 \cdot 2, 1 \cdot 3\}$.

Multiplicity :- The multiplicity of an element in a multiset is defined to be the no. of times an element appears in the multiset.

3 is the multiplicity of 1.

2 " " " " 2.

1 " " " " 3.

NOTE: [illegible] is a special type of multiset in which the multiplicity of every element is one or zero

- Cardinality of Multiset :- Cardinality of Multiset is equal to the cardinality of corresponding set

(OR)

Cardinality of Multiset is defined as the cardinality of its corresponding set. Assuming that all the elements are distinct in the ~~set~~ multiset.

So cardinality of multiset A = Cardinality of corresponding set A.

e.g. $A = \{1, 1, 1, 2, 2, 3\}$

$A = \{1, 2, 3\}$.

Cardinality of Multiset $A = 3$.

- Operation in Multisets :-

Let, $A = \{3\cdot a,\ 2\cdot b,\ 1\cdot c\}$.

$B = \{2\cdot a,\ 1\cdot b,\ 1\cdot d\}$.

Union of multisets ($\cup$) :- $(A \cup B) = \{3\cdot a, 2\cdot b, 1\cdot c, 1\cdot d\}$

Intersection of Multisets ($\cap$) :- $A \cap B = \{2\cdot a,\ 1\cdot b\}$

Difference ($-$) :- $A - B = \{1\cdot a,\ 1\cdot b,\ 1\cdot c\}$

Sum ($+$) :- $B - A = \{1\cdot d\}$.

$A + B = \{5\cdot a,\ 3\cdot b,\ 1\cdot c,\ 1\cdot d\}$.

Union :- $A \cup B$ is the multiset where the multiplicity of an element is maximum of its multiplicities in A and B.

eg $A \cup B = \{3\cdot a,\ 2\cdot b,\ 1\cdot c,\ 1\cdot d\}$.

Intersection :- $A \cap B$ is the multiset where the multiplicity of an common element is minimum of its multiplicities in A and B.

eg $A \cap B = \{2\cdot a,\ 1\cdot b\}$.

Difference :- A-B is the multiset where the multiplicity of an element is equal to multiplicity of element in A minus multiplicity of element in B, if the difference is positive but it is equal to zero if the difference is zero and negative.

e.g $A - B = \{1\cdot a,\ 1\cdot b,\ 1\cdot c\}$

$B - A = \{1\cdot d\}$.

Sum :- A+B is the multiset where multiplicity of an element is equal to the sum of multiplicities of an element in both multisets A and B.

Eg $A + B = \{5\cdot a,\ 3\cdot b,\ 1\cdot c,\ 1\cdot d\}$.

$Q = \{3\cdot a,\ 3\cdot b,\ 2\cdot d\}$.

find 1) $P \cup Q$, 2) $P \cap Q$, 3) $P - Q$, 4) $Q - P$, 5) $P +$

1) $P \cup Q = \{4\cdot a,\ 3\cdot b,\ 1\cdot c,\ 2\cdot d\}$.

2) $P \cap Q = \{3\cdot a,\ 3\cdot b\}$

3) $P - Q = \{1\cdot a,\ 1\cdot c\}$.

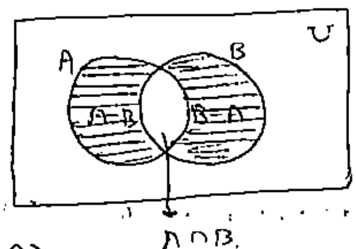
4) $Q - P = \{2\cdot d\}$.

5) $P + Q = \{7\cdot a,\ 6\cdot b,\ 1\cdot c,\ 2\cdot d\}$.

* Set Inclusion Exclusion Principle :-

(OR)

Counting Principle :-



$n(A) = n(A-B) + n(A\cap B)$ — ①

$n(B) = n(B-A) + n(A\cap B)$ — ②

$n(A\cup B) = n(A-B) + n(B-A) + n(A\cap B)$ — ③

Put the values of $n(A-B)$ & $n(B-A)$ from eqn ① & ② in eqn ③ :-

$n(A\cup B) = n(A) - n(A\cap B) + n(B) - \cancel{n(A\cap B)} + \cancel{n(A\cap B)}$

$$\boxed{n(A\cup B) = n(A) + n(B) - n(A\cap B)}$$

Corollary :-

$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(A\cap C) - n(B\cap C)$
$+ n(A\cap B\cap C)$

Proof: Let $B\cup C = D$.

$n(A\cup D) = n(A) + n(D) - n(A\cap D)$
$= n(A) + n(B\cup C) - n[A\cap(B\cup C)]$
$= n(A) + n(B) + n(C) - n(B\cap C) - n[(A\cap B)\cup(A\cap C)]$
$= n(A) + n(B) + n(C) - n(B\cap C) - n(A\cap B) - n(A\cap C)$
$+ n[(A\cap B)\cap(A\cap C)]$
$= n(A) + n(B) + n(C) - n(B\cap C) - n(A\cap B) - n(A\cap C)$
$+ n(A\cap B\cap C)$.

Q 40 lecturers were interviewed for a job, 25 were mathematicians, 28 were physicist and 7 were niether. How many lecturers were matematicians and physicists.

→ Let, M be the set of mathematicians and
P be the set of Physicists.

Given :- $n(M) = 25$
$n(P) = 28$.

Find :- $n(M \cap P) = ?$

Given :- $n(M \cup P) = 40 - 7 = 33$

$n(M \cup P) = n(M) + n(P) - n(M \cap P)$ {By set incl. ex. principle}

$n(M \cap P) = n(M) + n(P) - n(M \cup P)$
$= 25 + 28 - 33$
$= 53 - 33$

$\boxed{n(M \cap P) = 20}$

So, 20 lectureres were both mathematicians and physicists.

Q In a survey of 600 TV viewers given the following information.
1) 385 watch cricket matches.
2) 295 watch hockey matches.
3) 215 watch football matches.
4) 145 watch cricket & football matches both.
5) 170 watch cricket & hockey matches both.
6) 150 watch hockey & football matches both.
7) 150 does not watch any of the three games.

Find 1) How many people watch all three kind of matches.
2) How many people watch exactly one sport.

→ Let, C be the set of cricket viewers.
H be the set of Hockey viewers.
F be the set of football viewers.

Given :- $n(C) = 385$
$n(H) = 295$
$n(F) = 215$
$n(C \cap F) = 145$
$n(C \cap H) = 170$
$n(H \cap F) = 150$.
$n(C \cup H \cup F) = 600 - 150 = 450$

Now, $n(C \cup H \cup F) = n(C) + n(H) + n(F) - n(C \cap H) - n(C \cap F) - n(H \cap F) + n(C \cap H \cap F)$.

$450 = 385 + 295 + 215 - 170 - 145 - 150 + n(C \cap H \cap F)$
$450 = 895 - 465 + n(C \cap H \cap F)$
$450 = 430 + n(C \cap H \cap F)$

$\boxed{n(C \cap H \cap F) = 20}$ (By set Inc. Exc. Principle).