KAS 302/402 : Mathematics-IV

Unit- 1

Partial Differential Equations

Part 1 :- ☐ Origin of Partial differential Equation

☐ Linear and Non linear Partial differential equation (PDE) of First order

☐ Lagrange's Equations.

Introduction to Partial differential Eqn

Partial Differential Equations:- (PDE)

1. **Definition :-**

Those Eqn which have partial derivative, derivative of Dependent variable w.r.t. Independent variable (two or more) are called partial differential equation (PDE)

For example

If $z = f(x, y)$

z ↓ Dependent variable

x, y ↓ Independent variable

$$\Rightarrow \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x \partial y} + Py = Q$$

is a PDE of order 2nd, where P and Q may be consider function of x, and y.

Notations :-

$$\frac{\partial z}{\partial x} = p, \quad \frac{\partial z}{\partial y} = q$$

$$\frac{\partial^2 z}{\partial x^2} = r, \quad \frac{\partial^2 z}{\partial x \partial y} = s$$

$$\frac{\partial^2 z}{\partial y^2} = t.$$

2. Formation of PDE :→

a) By Eliminating arbitrary ~~ta~~ Constant

(b) By Eliminating arbitrary function

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Solve Problem

(A) By Elimination of arbitary Constants

Problem. 1 :- $(x-a)^2 + (y-b)^2 = (c-z)^2$
where a, b, c are arbitary constants

Problem. 2 :- $z = (x+a)(y+b)$

Problem. 3 :- $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
where a, b are arbitary constant

Problem. 4 :- Find the PDE of all sphere whose centre be on z axis and given eqn $x^2 + y^2 + (z-a)^2 = b^2$, where a, b are constants.

Solve Problem. 1

Give $(x-a)^2 + (y-b)^2 = c - z^2$ —— (1)

Paritially differente (1) w.r.to x and y respectively

$$2(x-a)\frac{\partial}{\partial x}(x-a) + 0 = 0 - 2z\frac{\partial z}{\partial x}$$

$$2(x-a) = -2z\frac{\partial z}{\partial x}$$

$$(x-a) = -z\frac{\partial z}{\partial x}$$

$$(x-a) = -zp \quad \text{—— (2)}$$

Again

$$0 + 2(y-b)\frac{\partial}{\partial y}(y-b) = 0 - 2z\frac{\partial z}{\partial y}$$

$$2(y-b) = -2z\frac{\partial z}{\partial y}$$

$$(y-b) = -zq \quad \text{—— (3)}$$

from 1, 2 and 3

$$(-zp)^2 + (-zq)^2 = c - z^2$$

$$(p^2 + q^2 + 1)z^2 = c$$ Required PDE

Solve Problem. 2

Given $z = (x+a)(y+b)$ —— (1)

Partially differenting (1) w.r.to x and y

$$\frac{\partial z}{\partial x} = (y+b)\frac{\partial}{\partial x}(x+a)$$

$$\frac{\partial z}{\partial x} = y+b$$

$$p = y+b \text{ —— } (2)$$

$$\frac{\partial z}{\partial y} = (x+a)\frac{\partial}{\partial y}(y+b)$$

$$q = x+a \text{ —— } (3)$$

from (1), (2) and (3)

$$z = pq$$

Required PDE.

Solve Problem. 3

Given $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ —— (1)

Partially differenting (1) w.r.to x and y

$$2\frac{\partial z}{\partial x} = \frac{2x}{a2} + 0$$

$$\frac{1}{x}\frac{\partial z}{\partial x} = \frac{1}{a2} \quad \text{---} (2)$$

$$\frac{1}{y}\frac{\partial z}{\partial y} = \frac{1}{b^2} \quad \text{---} (3)$$

from (1), (2) and (3)

$$2z = x^2\left(\frac{1}{x}\frac{\partial z}{\partial x}\right) + y^2\left(\frac{1}{y}\frac{\partial z}{\partial y}\right)$$

$$2z = xp + yq$$

Required PDE.

Solve Problem. 4

Given $x^2 + y^2 + (z-a)^2 = b^2$ — (1)

Partially differenting (1) w.r.to x and y,

$$2x + 2(z-a)\frac{\partial z}{\partial x} = 0$$

$$x + (z-a)\frac{\partial z}{\partial x} = 0$$

$$x + (z-a)p = 0 \quad \text{——— } (2)$$

Again

$$2y + 2(z-a)\frac{\partial z}{\partial y} = 0$$

$$y + (z-a)q = 0 \quad \text{——— } (3)$$

from (2) $(z-a) = -\left(\frac{x}{p}\right)$ Putting this value in eqn (3) we get

$$y - \frac{x}{p}q = 0$$

$$yp - xq = 0$$

or $$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$$

This is required PDE

B. Solve problem

→ By Elimination of arbitrary function

Problem 1:- $z = f(x+iy) + F(x-iy)$

Problem 2: $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$

Problem 3:- From the PDE from

$$z = f(x^2 - y^2)$$

Problem 4:- $z = f(x^2 + y^2)$

Problem 5:- $F(x^2 + y + z,\ x^2 + y^2 + z) = 0$

Solve Problem.3 :-

$$z = f(x^2 - y^2)$$

Differate w.r. to x and y

$$p = \frac{\partial z}{\partial x} = f'(x^2 - y^2) \times 2x$$

$$p = f'(x^2 - y^2) \times 2x \quad \text{---} (2)$$

$$q = \frac{\partial z}{\partial y} = f'(x^2 - y^2)(-2y)$$

$$q = f'(x^2 - y^2)(-2y) \quad \text{---} (3)$$

from (2) and (3)

$$\frac{p}{q} = \frac{2x\, f'(x^2 - y^2)}{-2y\, f'(x^2 - y^2)}$$

$$\frac{p}{q} = -\frac{x}{y}$$

$$yp + qx = 0$$

Required PDE.

Solve problem 2 :→

Given $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$ — ①

Partially differentiate w.r.to x and y

$$\frac{\partial z}{\partial x} = 0 + 2f'\left(\frac{1}{x} + \log y\right)\frac{\partial}{\partial x}\left(\frac{1}{x} + \log y\right)$$

$$\frac{\partial z}{\partial x} = 2f'\left(\frac{1}{x} + \log y\right)\left(-\frac{1}{x^2}\right) \quad \text{--- (2)}$$

$$\frac{\partial z}{\partial y} = 2y + 2f'\left(\frac{1}{x} + \log y\right)\frac{\partial}{\partial y}\left(\frac{1}{x} + \log y\right)$$

$$= 2y + 2f'\left(\frac{1}{x} + \log y\right)\left(0 + \frac{1}{y}\right)$$

$$\frac{\partial z}{\partial y} = 2y + 2\left(-\frac{x^2}{2}\frac{\partial z}{\partial x}\right)\cdot\frac{1}{y}$$

[By Eqn (2)

$$\frac{\partial z}{\partial y} = 2y - \frac{x^2}{y}\frac{\partial z}{\partial x}$$

$$q = 2y - \frac{x^2}{y}p$$

$$\boxed{qy = 2y^2 - px^2}$$

Required PDE.

Remark

If NO. of arbitrary constant is Equal to NO. of independent variables than order of PDE is 1.

And if No. of arbitrary constant greater than no. of independent variables than order of PDE is greater 1.

No of arbitrary function is Equal to order PDE.

Home Assingments

To solve Problems → 1, 4, 5,