Multi Set : A collection of objects that are not necessarily distinct, is called multi set. Number of times an element appears in the multi set is called the multiplicity of that element. Multi set as pair (A, μ) where A is the generic set and μ is the multiplicity.

Eg: if $\{a, b, c, c, a, c\}$ is a multi set

then $\mu(a) = 2$
$\mu(b) = 1$
$\mu(c) = 3$

Q) Consider the multi set $A = \{a, a, a, b, b, d, d, d, d, e\}$

The multiplicity of element b in A $= \mu(a) = 3$.

" " " " d in " $= \mu(d) = 4$

" " " " e " " $= \mu(e) = 1$

" " " " a " " $= \mu(a) = 3$

⊛ Equality of Multi set

If the number of occurence of each element is the same in both the multi set, then multi sets are equal.

Eg: $\{a, b, a, a\} = \{a, a, b, a\}$

But, $\{a, b, a\} \neq \{a, b\}$

Operation on Multi set :-

1) **Intersection of Multi set :** If P and Q are multi set then $P \cap Q$ is defined as the multi set such that for each element $x \in P \cap Q$,

$$\mu(x) = \text{Min} \{ \mu_P(x), \mu_Q(x) \}$$

Eg:
$$P = \{1, 1, 1, 2, 2, 3\}$$
$$Q = \{1, 2, 2, 2, 3, 3\}$$
$$P \cap Q = \{1, 2, 2, 3\}$$

2) **Union of Multi set :** If P and Q are multi set then $P \cup Q$ is defined as the multiset such that each element
$$x \in P \cup Q$$
$$\mu(x) = \text{Max} \{ \mu_P(x), \mu_Q(x) \}$$

Eg:
$$P = \{a, b, b, c\}$$
$$Q = \{b, c, c, d\}$$
$$P \cup Q = \{a, b, b, c, c, d\}$$

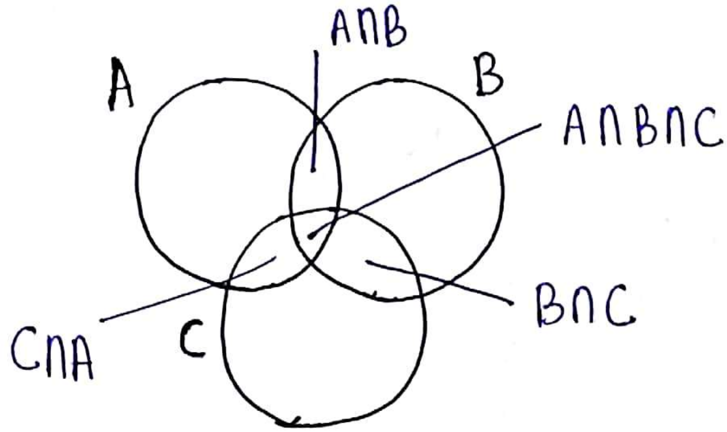
3) **Difference of Multiset :** $x \in P - Q$ [if P & Q are multisets]
$$\mu(x) = \mu_P(x) - \mu_Q(x)$$
$$P = \{a, a, a, a, b, b, c, d\}$$
$$Q = \{a, a, b, d, e\}$$
$$P - Q = \{a, a, b, c\}$$

4) **Sum of Multi set :** $x \in P + Q$
$$\mu(x) = \mu_P(x) + \mu_Q(x)$$

$P = \{a, a, b, b, b, c\}$

$Q = \{a, a, a, b, c, c, d, e\}$

$P + Q = \{a, a, a, a, a, b, b, b, b, c, c, c, d, e$



$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$$

Q) A school has 21 boys in basket ball team, 26 in hockey and 29 in football team. Now if 14 boys play hockey and basket ball, 15 boys play hockey & football, 12 boys play football & basket ball and 8 boys play hockey, football and basket ball all three games. Then what is the total no. of boys playing games.

Solⁿ:- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= 21 + 26 + 29 - 14 - 15 - 12 + 8$

$= \underline{\underline{43}}$ Ans.

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{3, 4, 5, 6, 7, 8, 10\}$

find $(A-B) \cup (B-A)$

Ans:- $A - B = \{2, 12\}$, $B - A = \{3, 5, 7\}$

$(A-B) \cup (B-A) = \{2, 3, 5, 7, 12\}$

Q) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$

$B = \{2, 3, 5, 7\}$, ~~Find~~ ~~AUB &~~ prove

① $(A \cup B)' = A' \cap B'$

② $(A \cap B)' = A' \cup B'$

Soln:- 1) LHS $\Rightarrow$ $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B)' = \{1, 9\}$

RHS $\Rightarrow$ $A' = \{1, 3, 5, 7, 9\}$, $B' = \{1, 4, 6, 8, 9\}$

$A' \cap B' = \{1, 9\}$

$\boxed{LHS = RHS}$ $\Rightarrow (A \cup B)' = A' \cap B'$

2) LHS $\Rightarrow$ $A \cap B = \{2\}$

$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

RHS $\Rightarrow$ $A' = \{1, 3, 5, 7, 9\}$, $B' = \{1, 4, 6, 8, 9\}$

$A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

$\boxed{LHS = RHS}$

$\because (A \cap B)' = A' \cup B'$

Q) Show in Venn Diagram $(A-B)'$