find the PDE by Eliminating arbitrary function

$f(x+y+z,\ x^2+y^2+z^2)=0$

$f(x+y+z,\ x^2+y^2+z^2)=0$ ——— (i)

Diff. partially w.r.t. x & y resp. we get

$f'(x+y+z,\ x^2+y^2+z^2)$.

Q:. Find PDE by eliming arbitray fun^n $f(x+y+z, x^2+y^2+z^2)$

Sol^n

$$\left.\begin{aligned} f(x+y+z,\ x^2+y^2+z^2) &= 0 \\ f(u,\ v) &= 0 \end{aligned}\right\} \text{— (i)}$$

where $u=x+y+z;\ v=x^2+y^2+z^2$ - - (ii)

Diff. w.r.t. x, partially (i), we get -

$$\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x}+p\frac{\partial u}{\partial z}\right)+\frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial x}+p\frac{\partial v}{\partial z}\right)=0 \text{ - - (iii)}$$

From (ii), we get: $\frac{\partial u}{\partial x}=1,\ \frac{\partial u}{\partial z}=1,\ \frac{\partial v}{\partial x}=2x,$

$$\frac{\partial v}{\partial y}=2y,\ \frac{\partial v}{\partial z}=2z,\ \frac{\partial u}{\partial y}=1$$

$\cancel{\frac{\partial v}{\partial y}=2y}$

Putting these values in eq^n (iii), we get

$$\frac{\partial f}{\partial u}(1+p)+\frac{\partial f}{\partial v}(2x+2pz)=0$$

or $\frac{\partial f}{\partial u}(1+p)+2\frac{\partial f}{\partial v}(x+pz)=0$ - - (iv)

or $\frac{\partial f}{\partial u}(1+p)=-2\frac{\partial f}{\partial v}(x+pz)$

or $\frac{(\partial f/\partial u)}{(\partial f/\partial v)}=\frac{-2(x+pz)}{(1+p)}$

$$\Rightarrow \boxed{\frac{(\partial f/\partial u)}{(\partial f/\partial v)} = \frac{-2(x+pz)}{(1+p)}} \quad \text{--- (v)}$$

Again Diff (i) w.r.t. 'y' partially, we get

$$\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial y} + q\frac{\partial u}{\partial z}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y} + q\frac{\partial v}{\partial z}\right) = 0$$

Putting all the values of derivali

$$\frac{\partial f}{\partial u}(1+q) + 2\frac{\partial f}{\partial v}(y+zq) = 0$$

$$\Rightarrow \boxed{\frac{\left(\frac{\partial f}{\partial u}\right)}{\left(\frac{\partial f}{\partial v}\right)} = \frac{-2(y+zq)}{(1+q)}} \quad \text{--- (vi)}$$

From (v) & (vi), we get equating them

$$\frac{-\cancel{2}(x+pz)}{(1+p)} = \frac{-\cancel{2}(y+zq)}{(1+q)}$$

$$\Rightarrow (1+q)(x+pz) = (1+p)(y+zq)$$

$$\Rightarrow x + pz + qx + \cancel{qpz} - y - zq - py - \cancel{pzq}$$

$$\Rightarrow (x-y) + (z-y)p + (x-z)q = 0$$

$$\Rightarrow \boxed{(y-z)p + (z-x)q = (x-y)} \quad \text{Ans}$$

$$\Rightarrow \boxed{\frac{(\partial f/\partial u)}{(\partial f/\partial v)} = \frac{-2(x+pz)}{(1+p)}} \quad \text{--- (v)}$$

Again Diff (i) w.r.t. 'y' partially, we get

$$\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial y} + q\frac{\partial u}{\partial z}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y} + q\frac{\partial v}{\partial z}\right) = 0$$

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From (v) & (vi), we get (equating them)

$$\frac{-\cancel{2}(x+pz)}{(1+p)} = \frac{-\cancel{2}(y+zq)}{(1+q)}$$

$$\Rightarrow (1+q)(x+pz) = (1+p)(y+zq)$$

$$\Rightarrow x + pz + qx + \cancel{qpz} - y - zq - py - \cancel{pzq} =$$

$$\Rightarrow (x-y) + (z-y)p + (x-z)q = 0$$

$$\Rightarrow \boxed{(y-z)p + (z-x)q = (x-y)} \text{ Ans}$$