Bxr

(4)

Linear eqⁿ first order PDE
Lagrange's equation

Standard form

$$Pp + Qq = R \quad \text{--- (1)}$$

where P, Q, R are the function of x, y, and z.

$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

Working Rule

(1) Find auxilary eqⁿ for (1)

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \quad \text{--- (2)}$$

(2) Solve auxilary eqⁿ by

1) Grouping method
2) Method of multiplies
3) Combination of (1) and (2)

(3) Suppose $u = a$ and $v = b$ are two solⁿ of (1) which obtained by (2)

where u and v [function (x, y, z)]

(4) complet sol^n of (1)

$$f(u, v) = 0, \text{ or } f(a, b) = 0$$

or $u = \phi(v)$

or $v = \psi(u)$

Note

Grouping Method:-

$$\frac{dx}{P} = \frac{dy}{Q} \Rightarrow \boxed{\frac{dy}{dx} = \frac{Q}{P}}$$

$$\boxed{u(x, y, z) = a}, \quad u(y, z) = b$$

Method of multiplies

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

chose multipliers l, m, n

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{l\,dx + m\,dy + n\,dz}{lP + mQ + nR}$$

$$= \frac{l\,dx + m\,dy + n\,dz}{0}$$

$$l\,dx + m\,dy + n\,dz = 0$$

then integrating

$u = a$

similarly choose another set of multipliers (l_1, m_1, n_1) and if the second solution is $v = c_2$.

Required solution is

$$f(u, v) = 0$$

Solution :-

Solve : $(x^2-yz)p + (y^2-zx)q = z^2-xy$ —— (1)

Here Lagrange's subsidiary eq^n are

$$\frac{dx}{x^2-yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2-xy}$$

$$\frac{dx-dy}{x^2-yz-y^2+zx} = \frac{dy-dz}{y^2-zx-z^2+xy} = \frac{dz-dx}{z^2-xy-x^2+yz}$$

$$\frac{dx-dy}{(x-y)(x+y+z)} = \frac{dy-dz}{(x+y+z)(y-z)} = \frac{dz-dx}{(z-x)(x+y+z)}$$

$$\frac{dx-dy}{(x-y)} = \frac{dy-dz}{(y-z)} = \frac{dz-dx}{(z-x)}$$ —— (2)

Intergrating .

$$\log(x-y) = \log(y-z) + \log c_1$$

$$\log\left(\frac{x-y}{y-z}\right) = \log c_1$$

similar $\quad c_1 = \frac{x-y}{y-z}$

$$\log(y-z) = \log(z-x) + \log c_2$$

$$c_2 = \frac{y-z}{z-x}$$

The required solution is

$$f\left(\frac{x-y}{y-z}, \frac{y-z}{z-x}\right] = 0.$$

2 Solve

$$y q - x p = z$$

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

Sol

$$yq - xp = z$$

$$\frac{dx}{-x} = \frac{dy}{y} = \frac{dz}{z}$$

from ① Part and ② Part

$$-\log x = \log y \cancel{\log z} - \log a$$

$$-\log x = \log y - \log a$$

$$xy = a \quad \text{— ①}$$

$$\log y = \log z + \log b$$

$$\frac{y}{z} = b \quad \text{—— ②}$$

from ① & ②

$$f\left(x, y, \frac{y}{z}\right) = 0$$

Solution :-

H.W

3 $y^2 p - xyq = x(z-2y)$

4 $\frac{y^2 z}{x} p + xzq = y^2$

5 Solve

$$(mz-ny)\frac{\partial z}{\partial x} + (nx-lz)\frac{\partial z}{\partial y} = ly-mx$$

Sol^n

$$(mz-ny)\frac{\partial z}{\partial x} + (nx-lz)\frac{\partial z}{\partial y} = ly-mx$$

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$$

using multipliers (x, y, z), we get

$$= \frac{xdx+ydy+zdz}{x(mz-ny)+y(nx-lz)+z(ly-mx)}$$

$$\frac{xdx+ydy+zdz}{0}$$

$$xdx+ydy+zdz = 0$$

which gives on integration

$$x^2+y^2+z^2=c_1 \quad \text{--- (1)}$$

Again using multiplies l, m, n

we set

each fraction

$$= \frac{l\,dx+m\,dy+n\,dz}{l(mz-ny)+m(nx-lz)+n(ly-mx)}$$

$$= \frac{l\,dx+m\,dy+n\,dz}{0}$$

$$= l\,dx+m\,dy+n\,dz=0$$

integration,

$$lx+my+nz=c_2 \quad \text{--- (2)}$$

from (1) & (2) the required

sol^n is

$$x^2+y^2+z^2=f(lx+my+nz)$$

Ans

6 $(x^2-y^2-z^2)p+2xyq=2xz$ —(1)

Here (1) is Lagrange's of Linear PDE

The auxiliary eqn of (1)

$$\frac{dx}{x^2-y^2-z^2}=\frac{dy}{2xy}=\frac{dz}{2xz} \quad -(2)$$

taking Last two part AE

$$\frac{dx}{2xy}=\frac{dz}{2xz}$$

$$\log y=\log z-\log a$$

$$\boxed{\frac{y}{z}=a}$$

Using multipliers x, y, z

$$\frac{dx}{x^2-y^2-z^2}=\frac{dy}{2xy}=\frac{dz}{2xz}$$

$$=\frac{x\,dx+y\,dy+z\,dz}{x^3-xy^2-xz^2+2xy^2+2xz^2}$$

$$=\frac{x\,dx+y\,dy+z\,dz}{x(x^2+y^2+z^2)} \quad —(3)$$

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Taking last two part

of AE (3)

$$\frac{dz}{2xz} = \frac{x\,dx + y\,dy + z\,dz}{x(x^2+y^2+z^2)}$$

$$\frac{dz}{z} = \frac{2(x\,dx + y\,dy + z\,dz)}{x^2+y^2+z^2}$$

$$\ln z = \log(x^2+y^2+z^2) + \log b$$

$$\boxed{\frac{z}{x^2+y^2+z^2} = b}$$

complete sol^n

$f(a, b) = 0$

$$\boxed{f\left(\frac{y}{z}, \frac{z}{x^2+y^2+z^2}\right) = 0}$$

Ex' H.W

$$(z-y)\,p + (x-z)\,q = y-x$$

The auxilary eq^n of ①

$$\frac{dx}{z-y} = \frac{dy}{x-z} = \frac{dz}{y-z}$$

Taking multipliers 1, 1, 1

$$\frac{dx}{z-y} = \frac{dy}{x-z} = \frac{dz}{y-x} = \frac{dx+dy+dz}{z-y+x-z+y-x}$$

$$= \frac{dx+dy+dz}{0}$$

$dx+dy+dz = 0$

integrating $\boxed{x+y+z=a}$

Taking multiplied x, y, z.

$$\frac{dx}{z-y} = \frac{dy}{x-z} = \frac{dz}{y-x} = \frac{x\,dx+y\,dy+z\,dz}{xz-xy+\cancel{y}x-zy+zy-\cancel{zx}}$$

$$= \frac{x\,dx+y\,dy+z\,dz}{0}$$

$x\,dx + y\,dy + z\,dz = 0$

integrating

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c$$

$x^2+y^2+z^2 = 2c$ It is complete tan

$f(x+y+z,\ x^2+y^2+z^2)=0$

Ques Find the general sol^n of

$$x(z^2-y^2)\frac{\partial z}{\partial x}+y(x^2-z^2)\frac{\partial z}{\partial y}=z(y^2-x^2)$$

Sol^n Given eq^n

$$x(z^2-y^2)\frac{\partial z}{\partial x}+y(\cancel{x}^2-z^2)\frac{\partial z}{\partial y}=z(y^2-x^2) \quad -(1)$$

$$\frac{dx}{x(z^2-y^2)}=\frac{dy}{y(x^2-z^2)}=\frac{dz}{z(y^2-x^2)} \quad -(2)$$

Now multipliers x, y, z we get

$$=\frac{x\,dx+y\,dy+z\,dz}{x^2(z^2-y^2)+y^2(\cancel{x}^2-z^2)+z^2(y^2-x^2)}$$

$$=\frac{x\,dx+y\,dy+z\,dz}{0}$$

$$x\,dx+y\,dy+z\,dz=0$$

$$\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}=c_1$$

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$x^2+y^2+z^2 = 2c_1 = a$

Again use multipliers – $\left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right)$

$$\frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{z^2-y^2+x^2-z^2+y^2-x^2}$$

$$= \frac{\frac{1}{x}dx + \frac{dy}{y} + \frac{dz}{z}}{0}$$

$$\Rightarrow \frac{1}{x}dx + \frac{1}{y}dy + \frac{dz}{z} = 0$$

$$\log x + \ln y + \ln z = \ln c_2$$

$$xyz = c_2 = b$$

complete sol

$$xyz = f(x^2+y^2+z^2).$$