① HOME WORK

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①

when $x=0$,

$$\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x \quad -①$$

$$[u = X.T.] \quad -②$$

$$\boxed{\frac{\partial u}{\partial x} = \frac{\partial x}{\partial x} T} \quad ④ \quad , \quad \boxed{\frac{\partial u}{\partial t} = X\frac{\partial T}{\partial t}} \quad -③$$

$$\left(\frac{\partial X}{\partial x}\right)T\left(X\frac{\partial T}{\partial t}\right) = e^{-t}\cos x$$

$$\left(T\frac{\partial X}{\partial x}\right)\left(X\frac{\partial T}{\partial t}\right) = e^{-t}\cos x \quad -⑤$$

from ① put these values

$$\frac{\partial T}{\partial t}\cdot\frac{\partial X}{\partial x} = e^{-t}\cos x \quad -⑤$$

seperating values, we get

$$e^{t}\frac{dT}{dt} = \frac{\cos x}{\partial X/\partial x} = -p^2 \text{ (say)}$$

$$e^{t}\frac{dT}{dt} = -p^2$$

$$\int dT = \int -p^2 e^{-t}\,dt$$

$$T = -p^2\frac{e^{-t}}{-1}dt + C_1 \quad -⑥$$

$$\frac{\cos x}{\frac{dx}{X}} = -p^2 = \frac{dx}{dx} = \frac{-1}{p^2}\cos x$$

$$\int dX = \int -\frac{1}{p^2}\cos x\,dx$$

$$X = -\frac{1}{p^2}\sin x + C_2 \quad \text{--- (7)}$$

Putting the values of X & T on eqⁿ -- (2)

$$u = XT = \left(\frac{-1}{p^2}\sin x + C_2\right)\left(p^2 e^{-t} + C_1\right) \text{ --- (8)}$$

from (8), diff. w.r.t t, we get

$$\frac{\partial u}{\partial t} = \left(\frac{-1}{p^2}\sin t + C_2\right)\left(-p^2 e^{-t} + C_1\right) \text{ --- (9)}$$

given $\frac{\partial u}{\partial t} = 0$, when $x = 0$ in (9), we get

$$0 = C_2(-p^2 e^{-t}), \boxed{C_2 = 0}$$

Substituting C_1 & C_2 in (8)

$$u = -\frac{1}{p^2}\sin x\,(p^2 e^{-t} - p^2) \text{ or } \left[u = (1 - e^{-t})\sin x\right]$$

(2) $A = x^2,\ B = 3,\ C = x$

$$3^2 - 4AC = 3^2 - 4x^2 \cdot x = 9 - 4x^3$$

The eqⁿ is elliptic if $9 - 4x^3 < 0$

" " " parabollic " " $= 0$

" " " hyperbolic " " > 0

" " "

3) The given differential eqn is

$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x} = 0$$

$A = 1,\ B = 0,\ C = -c^2$

$B^2 - 4AC = 0^2 - 4(1)(-c^2)$

$= 4c^2$

which is always greater than 0

hence $B^2 - 4AC > 0$

So, the given eqn is hyperbolic