Relation :-

Let A and B be non empty set then any subset R of the Cartesian product $A \times B$ is called a Relation from A to B and is denoted by R. Thus R is a Relation from A to B $\Rightarrow R \subseteq A \times B$.

$$R = \{(x, y) : x \in A,\ y \in B \text{ and } xRy\}$$

Ex :- Let $A = \{1, 2, 5\}$ and $B = \{2, 4\}$ be two given set. Now suppose a relation from the set A to B is expressed by statement is less than.

$$A \times B = \{(1,2)\ (1,4), (2,2)\ (2,4), (5,2), (5,4)\}$$

the subset $A \times B$ whose elements are related in the relation R is given by

$$R = \{(1,2), (1,4), (2,4)\}$$

$$R \subset A \times B$$

Operation on Relation :-

① Complement of Relation :-

Consider a Relation R from set A to B, the complement of relation R denoted by $\bar{R}$ or R' is a relation from A to B such that

$$\bar{R} = \{(a,b) : (a,b) \notin R\}$$

Ex :- Let R be relation from X to Y where $X = \{1, 2, 3\}$ and $Y = \{8, 9\}$ and $R = \{(1,8), (2,8), (1,9), (3,9)\}$ find the complement of relation R.

Sol^n we first find $X \times Y = \{(1,8), (1,9)$ ...
$(2,8), (2,9), (3,8), (3,9)\}$

then Complement relation $\bar{R}$ of R' w.r.t X,

$$\bar{R} = \{(2,9), (3,8)\}$$

$\boxed{\text{Inverse Relation:}}$ → let R be relation from a set A to B. the inverse of Relation R^{-1} is a relation from B to A such that $b R^{-1} a$

$$R^{-1} = \{(b,a) : (a,b) \in R\}$$

Thus, to find R^{-1} we write in reverse order all order pair belonging to R

Ex. let $A = \{1,2,3\}$ and relation (R) is $\leq$ on A. Determine its inverse

$R = \{(1,2)\ (1,3)\ (1,1)\ (2,2)\ (2,3), (3,3)\}$

$R^{-1} = \{(2,1)\ (3,1)\ (1,1)\ (2,2)\ (3,2)\ (3,3)\}$

$R \cup S = \{(x,y) : x R y \text{ or } x S y\}$

$R \cap S = \{(x,y) : x R y \text{ or } x S y\}$

Properties of Relation:-

(I) **Reflexive Relation:-** A Relation R on a set A is reflexive if a R a for every $a \in A$, that is

$(a, a) \in R \quad \forall\, a \in A$

Ex:- If $R_1 = \{(1,1)\ (1,2)$

(II) **Irreflexive Relation:-** A relation R on a set A is irreflexive if, for every $a \in A$, $(a,a) \notin R$

the relation $R_1 = \{(1,2), (1,3), (2,1), (2,3)\}$ on $A = \{1,2,3\}$ is irreflexive since $(x,x) \notin R_1$ for every $x \in R_1$ (a set of real numbers)

(III) **non ~~reflexive~~:-**

(IV) **Symmetric:-** A relation R on a set A is symmetric if whenever $(a,b) \in R$ then $(b,a) \in R$ i.e if $aRb \Rightarrow bRa$

(a) $R_1 = \{(1,1), (1,2), (1,3), (2,2), (2,1), (3,1)\}$ on $A = \{1,2,3\}$ is a symmetric relation.

(b)

Transitivity relation :-

a Relation R on a set A is transitivity if whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ i.e

$$aRb \text{ and } bRc \Rightarrow aRc$$

Property	Meaning
1. Reflexive	$(a,a) \in R$ i.e aRa for all $a \in A$
2. Irreflexive	$(a,a) \notin R$ i.e $a \not R a$ for all $a \in A$
3. Symmetric	$(a,b) \in R \Rightarrow (b,a) \in R$ i.e $aRb \Rightarrow bRa$ for all $a, b \in A$
4. Antisymmetric	$(a,b) \in R \cap (b,a) \in R \Rightarrow a = b$ i.e aRb and $bRa \Rightarrow a = b$ for all $a, b \in A$
5. Asymmetric	$(a,b) \in R \Rightarrow (b,a) \notin R$ i.e $aRb \Rightarrow b \not R a$ for all $a, b \in A$
6. Transitivity	$(a,b) \in R \cap (b,c) \in R \Rightarrow (a,c) \in R$ i.e aRb and $bRc \Rightarrow aRc$ for all $a, b, c \in A$

Equivalance Relation :-

A Relation R in a set A is said to be equivalance relation if R is reflexive, symmetric, transitive.

Ex Set $A = \{$ set of line in a plan $\}$ and R is a relation

$R = \{(l_1, l_2) : l_1$ is parallel to $l_2\}$

Reflexive : for each $l \in A$, $(l, l) \in R$

Symmetric : for every $(l_1, l_2) \in R \Rightarrow (l_2, l_1) \in R$ where $l_1, l_2 \in A$

Transitive : if $(l_1, l_2) \in R$ and $(l_2, l_3) \in R$

$\Rightarrow (l_1, l_3) \in R$ where $l_1, l_2, l_3 \in A$

Since R is Reflexive, Symmetric and transitive, so Relation R is an equivalance Relation.

Composite Relation: — Let A, B, C be sets. Let R be a relation from A to B, and let S be a relation from B to C. The composite of R and S, denoted by $S \circ R$ is the relation from A to C given by

$$S \circ R = \{(a,c) \in A \times C : (\exists b \in B)\,[(a,b) \in R \text{ and } (b,c) \in S]\}$$

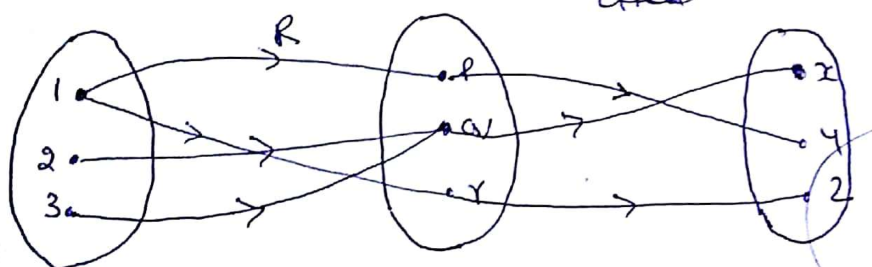
Let R be a relation on the set A.

Then $R \circ R$ is composition of relation ~~itself is denoted~~ ~~on the set A~~ with its self

$$R \circ R = R^2$$

$$R^3 = R^2 \circ R = R \circ R \circ R$$

Ex:- let $A = \{1, 2, 3\}$, $B = \{p, q, r\}$ and $C = \{x, y, z\}$



And let $R = \{(1,p), (1,r)\, (2,q)\, (3,q)\}$

and $S = \{(p,y), (q,x), (r,z)\}$

Compute $R \circ S =$

~~Theorem~~ **Theorem:-** Let R be a relation from the set A to the set B and S be a relation from the set B to set C then

$$(S \circ R)^{-1} = R^{-1} \circ S^{-1}$$

Proof:- let $(c, a) \in (S o R)^{-1}$

$\Rightarrow (c, a) \in (S o R)^{-1}$

$\Rightarrow (a, c) \in (S o R) \ \forall a \in A, c \in C$

$\therefore$ there exist an element $b \in B$ with $(a, b) \in R$ and $(b, c) \in S$

$(a, b) \in R$ and $(b, c) \in S \Rightarrow (b, a) \in R^{-1}$ and $(c, b) \in S^{-1}$

$\Rightarrow (c, b) \in S^{-1}$ and $(b, a) \in R^{-1}$

$\Rightarrow (c, a) \in R^{-1} o S^{-1}$

$(c, a) \in (S o R)^{-1}$

$\Rightarrow (c, a) \in R^{-1} o S^{-1}$

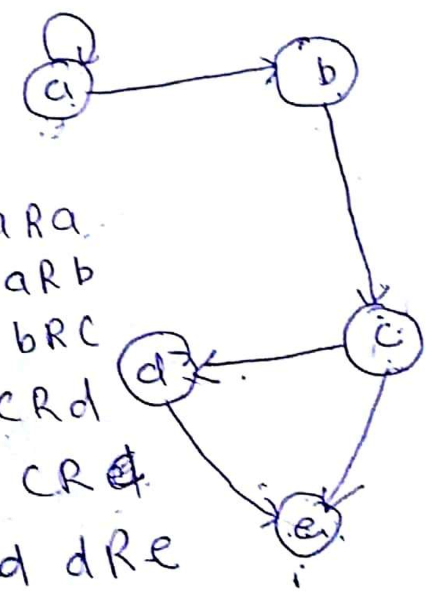
$\Rightarrow (S o R)^{-1} = R^{-1} o S^{-1}$

Example:- let $A = \{a, b, c, d, e\}$

$A^{+} = R \cup R^2 \cup R^3 \cup R^4 \ldots$

$R = \{(a,a) (a, b) (b,c) (c, e), (c,d) (d,e)\}$

(Sol^n)



$aR^2a =$ aRa and aRa

$aR^2b =$ aRa and aRb

$aR^2c =$ aRb and bRc

$bR^2d =$ bRc and cRd

$bR^2e =$ bRc and cRe

$cR^2e =$ cRd and dRe

$aR^3 =$ aRa aRa aRc

$aR^2b =$ aRa aRa aRb

$aR^3c =$ aRa aRb bRc

$bR^3e =$

$R^2 = \{(a,a) (a, b), (a,c), (b, e), (b, d), (c,e)\}$

Q// given : $A = \{1, 2, 3, 4\}$. Consider the following relation in A.

$R = \{(1,1), (2,2)\ (2,3), (3,2)\ (4,2), (4,4)\}$

Soln :-

① R is not reflexive because $3 \in A$ but $3 \not R 3$, i.e $(3,3) \notin R$

(ii) R is not Symmetric $4 R 2$ but $2 \not R 4$, i.e $(4,2) \in R$ but $(2,4) \notin R$

(iii) R is not tansitivity because $4 R 2$ and $2 R 3$ but $4 \not R 3$, i.e $(4,2) \in R$ and $(2,3) \in R$

$R = \{(1,2), (2,3), (1,1), (2,2)\}$ on the same set A is anti Symmetric but it is not reflexive because $(3,3)$ is missing.