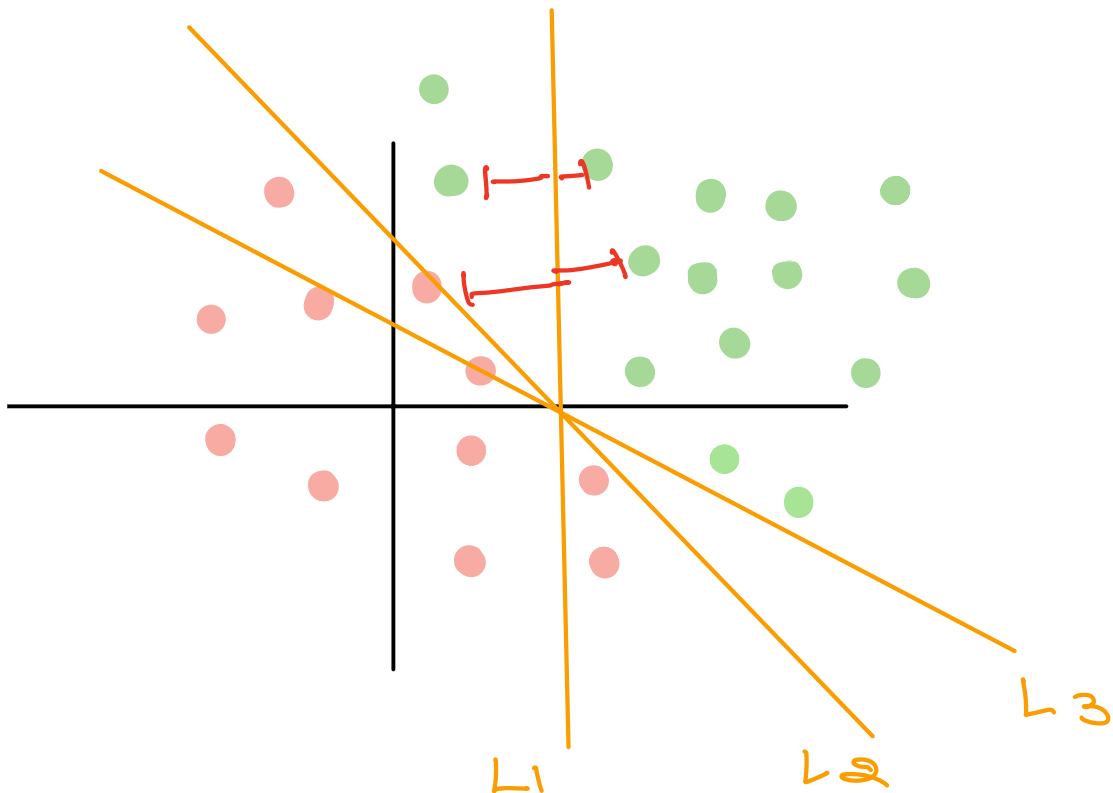


Agenda

- 1) Basic intuition of Classifier
- 2) Searching Algorithm: Grid Search
- 3) Optimization problem: Topics
- 4) Classification problem: Maths
- 5) Relationship between gain function and distance
- 6) Function: Domain and Range
- 7) Limits and Continuity
- 8) Homework: Some important $f(x)$'s

Basic intuition of a Classifier



Metric :

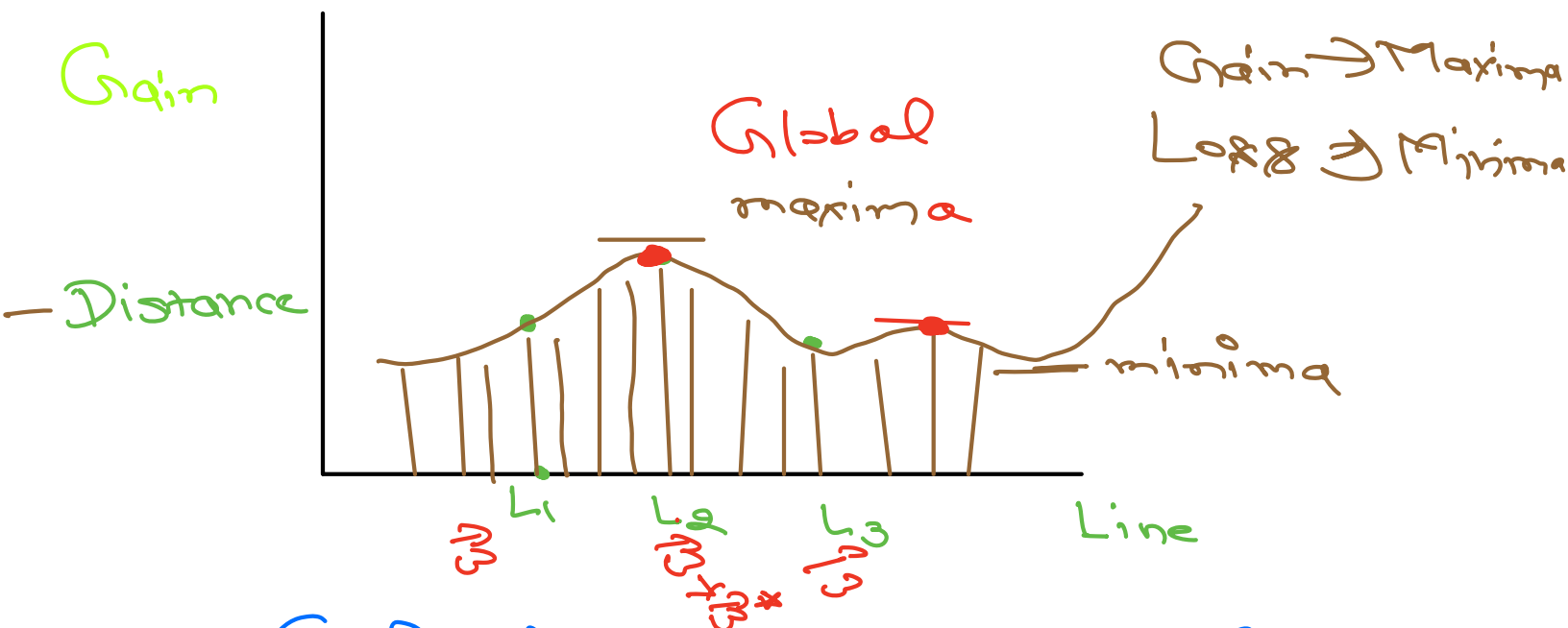
$$L_1 = 100\%$$

$$L_2 \Rightarrow 100\%$$

$$L_3 \Rightarrow 91\%$$

① Accuracy

② Distance



Calculus \Rightarrow Derivatives?

$$10^3 \times 10^3 \times 10^3 = 10^9 \text{ times}$$

Grid Search

$$\begin{cases} \omega_1 \Rightarrow -10, 10 \Rightarrow 10^3 \\ \omega_2 \Rightarrow -10, 10 \Rightarrow 10^3 \\ \omega_3 \Rightarrow -10, 10 \Rightarrow 10^3 \end{cases}$$

$$G \Rightarrow \sum_{i=1}^n \left(\frac{\vec{\omega}^T \vec{x}_i + \omega_0}{\|\vec{\omega}\|} \right) \times y_i$$



maximum

$$10^9 \times 10^{-6} \text{ seconds} \Rightarrow 1000 \text{ sec}$$

Optimize the values of $\omega_1, \omega_2, \omega_3$ to get the maximum G .

Optimization

Gradient Descent ←

(It Help us find the best (approximate) value of parameter for any function)

Minima and Maxima

Calculus (Multi Variate)

→ Single Variate

Slope, Tangents and Derivatives

→ Limit, Continuity and Differentiability

→ Functions

Defining Classification problem Mathematically

Given Dataset

$$D = \left\{ (x_i, y_i) : x_i \in \mathbb{R}^d, y_i \in \{1, 2, 3\} \right\}_{i=1}^n$$

↗ feature vector ↗ labels

Goal: Find a function $f(x)$ such that

$$f(\vec{x}) = y_i \quad y_i' = y_i$$

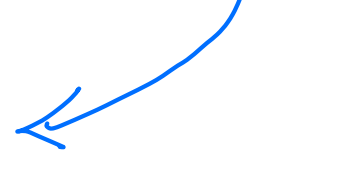
where x is new vector and y_i is pred

$$G(D, \vec{w}, w_0)$$

$$G \Rightarrow \sum_i \left(\frac{\vec{w} \cdot x_i + w_0}{\|\vec{w}\|} \right) \times y_i$$

$$G \Rightarrow \sum_i g$$

$$G \Rightarrow \sum_i g(x_i, y_i, \vec{w}, w_0) / n$$



Distance Function

$$g(x_i, y_i, \vec{w}, w_0) \Rightarrow \left(\frac{\vec{w} \cdot x_i + w_0}{\|\vec{w}\|} \right) \times y_i$$

problem y_i or $y_i \leftarrow$ Not Unit Vector

$$\vec{w}^*, w_0^* = \underset{\vec{w}, w_0}{\operatorname{argmax}} G(D, \vec{w}, w_0)$$

$\vec{w}^*, w_0^* \rightarrow$ optimal value of \vec{w} and w_0 .

Question: Relationship between Gain function and Distance

$$\textcircled{1} G(D, \vec{w}, w_0) = \sum_{i=1}^n g(x_i, y_i, \vec{w}, w_0)$$

$$\textcircled{2} G(D, \vec{w}, w_0) = \prod_{i=1}^n g(x_i, y_i, \vec{w}, w_0)$$

Σ summation
✓

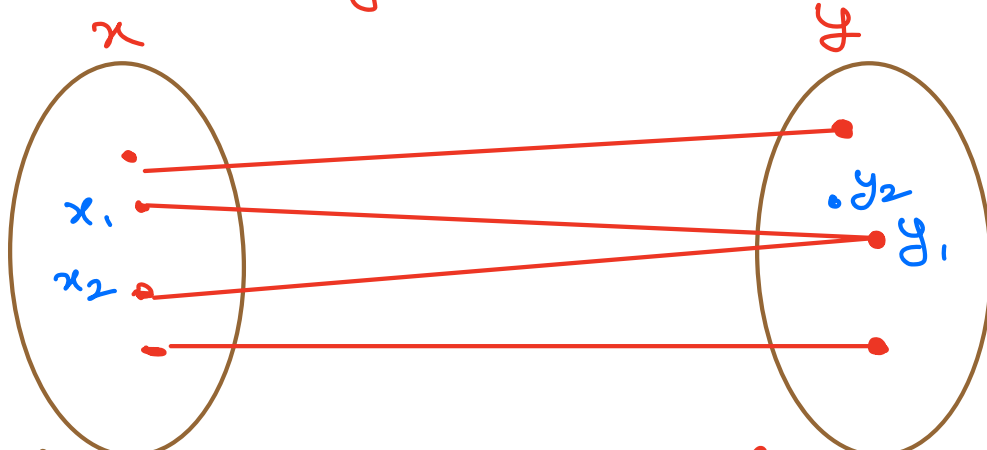
Π multiplication
/ 0 X

Functions

$$f(x) = y$$

mapping between input and output

$$f(x) = x^2$$



This is a valid $f(x)$

(All possible input x can take)
Domain

(All possible output)
Range

Ex: $f(x) = x^2$

Domain

$(-\infty, \infty)$

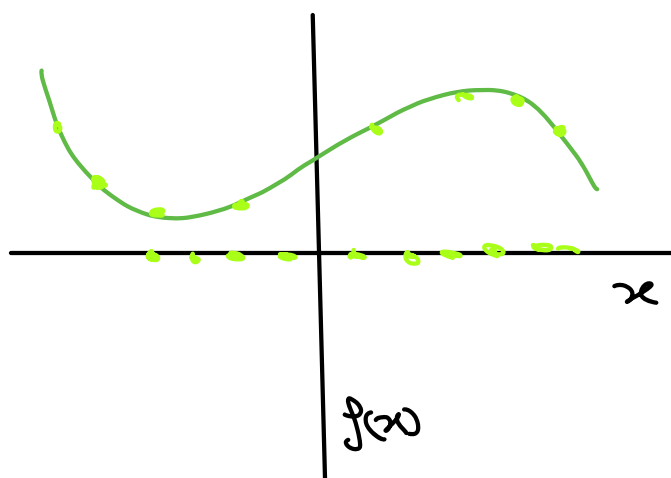
$x \in \mathbb{R}$

Range

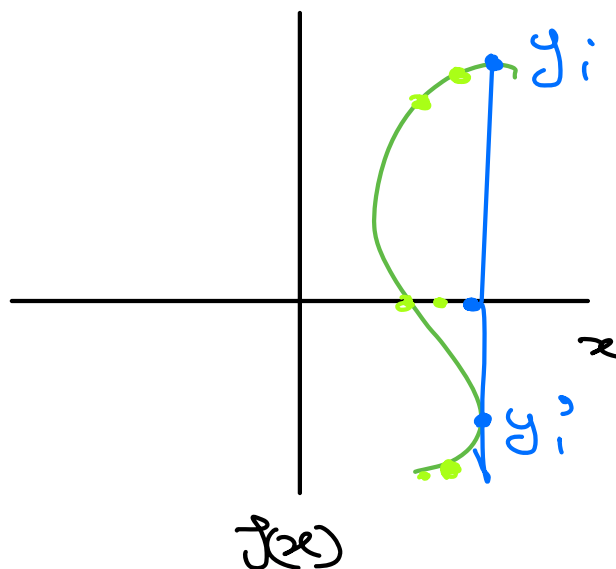
$[0, \infty)$

Q:

①



②

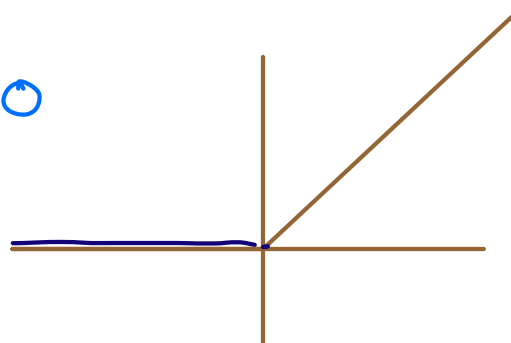


Valid Function

for one value of x you can get one and only one y

$f(x) = x \quad \begin{cases} x > 0 \\ 0 \end{cases}$

(Relu)



Limits and Continuity

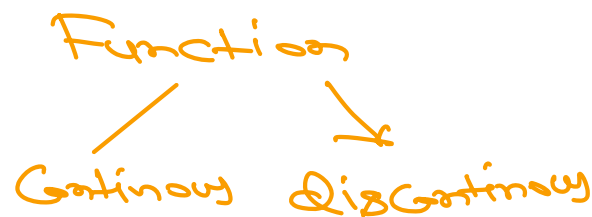
L.H.L

$$\lim_{x \rightarrow a^-} f(x)$$

(x is approaching a from Left hand Side)

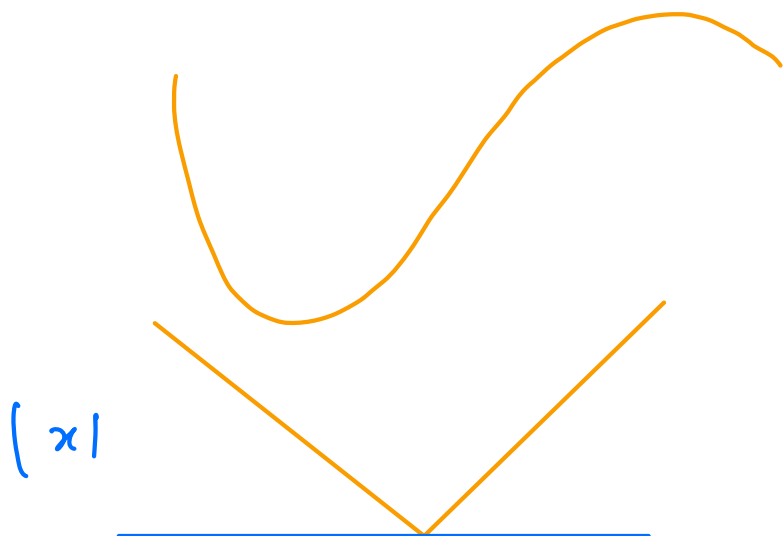
R.H.L

$$\lim_{x \rightarrow a^+} f(x)$$



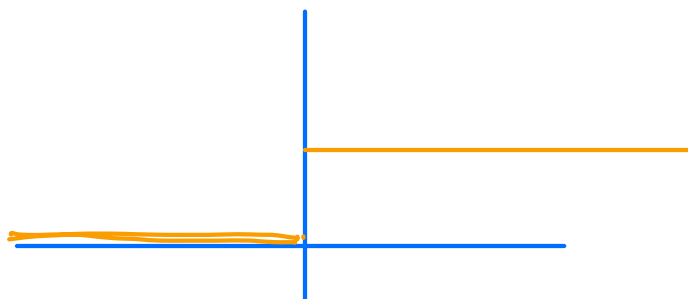
$$\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = f(x)_{x=a}$$

($f(x)$ is Continuous at $x=a$)



Step function

$$f(x) \ni \begin{cases} 1; & x \geq 0 \\ 0; & x < 0 \end{cases}$$

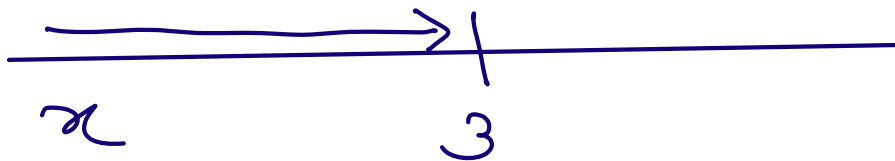


$$y = x^2 \Rightarrow L.H.L = R.H.L \\ \Rightarrow f(x)_3$$

$$a \rightarrow 3$$

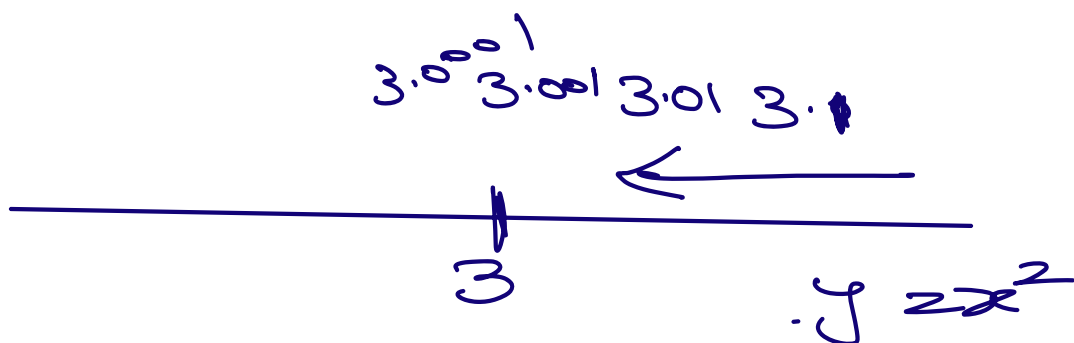
$$\lim_{x \rightarrow 3^-} f(x) = 9$$

$$2.9, 2.99, 2.999, 2.9999$$

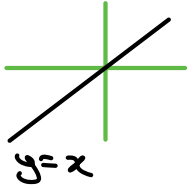


$$y \rightarrow 9$$

$$\lim_{x \rightarrow 3^+} f(x) = 9$$



H.W

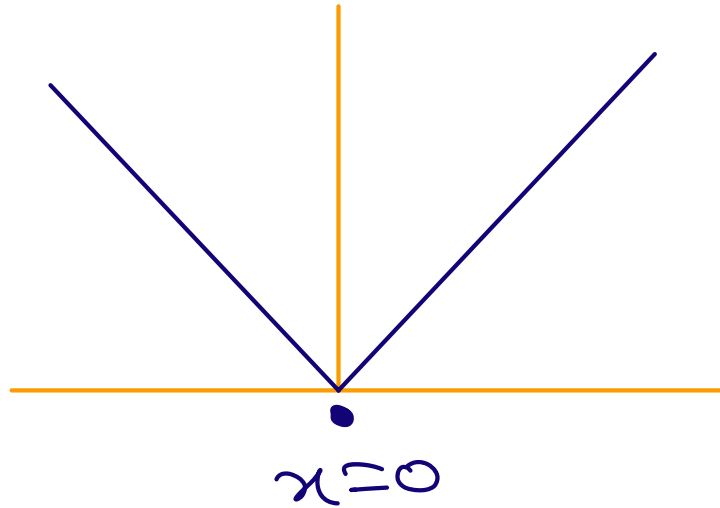
$f(x)$	Domain	Range	Continuous	Plot
① $y = x$	$(-\infty, \infty)$	$(-\infty, \infty)$	✓	
② $y = \frac{1}{x}$				
③ $y = e^x$				
④ $y = x $				
⑤ $y = \log(x)$				
⑥ $y = \frac{1}{1 + e^{-x}}$				
⑦ $\sin \theta$				

⑧ $y = \cos \theta$

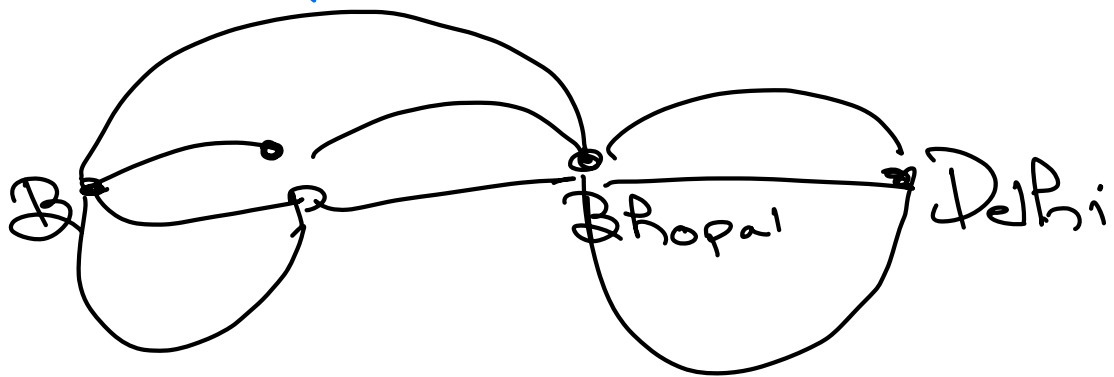
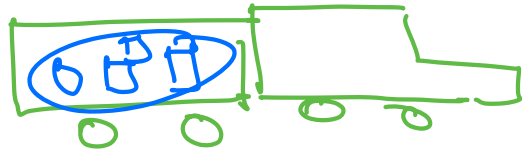
⑨ $y = \tan \theta$

Check Continuity at $x=0$

H.W



Delivery Intro

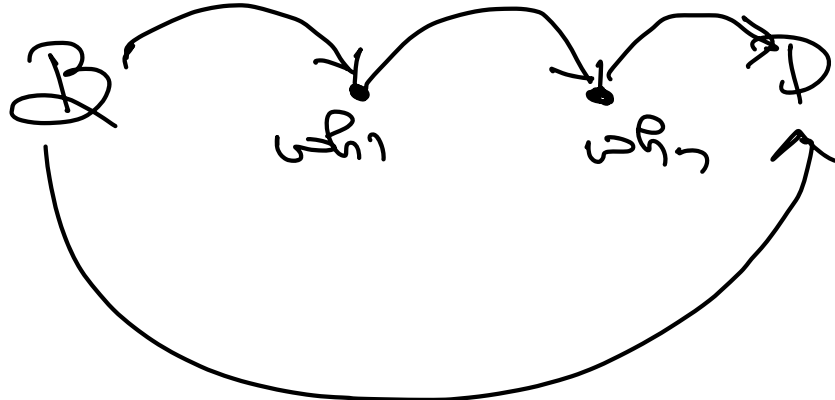


OSRM \Rightarrow Source Destination
 \rightarrow suggest most optimal path
 \rightarrow Time \leftarrow

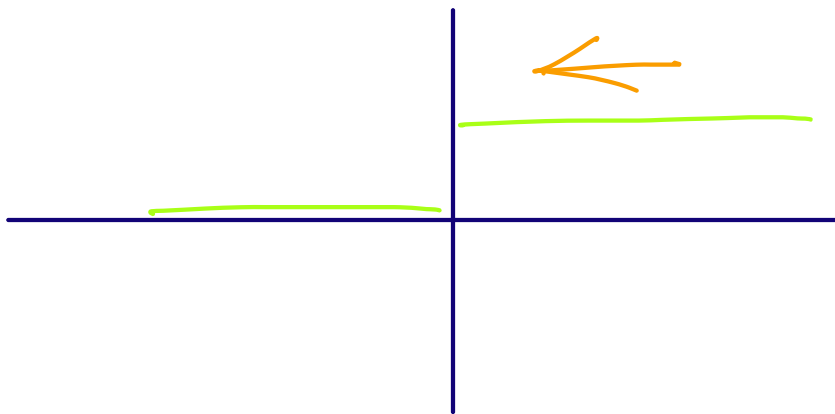
Actual Time

Actual Distance

Actual Time vs OSRM Time



Ex: Discontin



$$f(x) \equiv \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

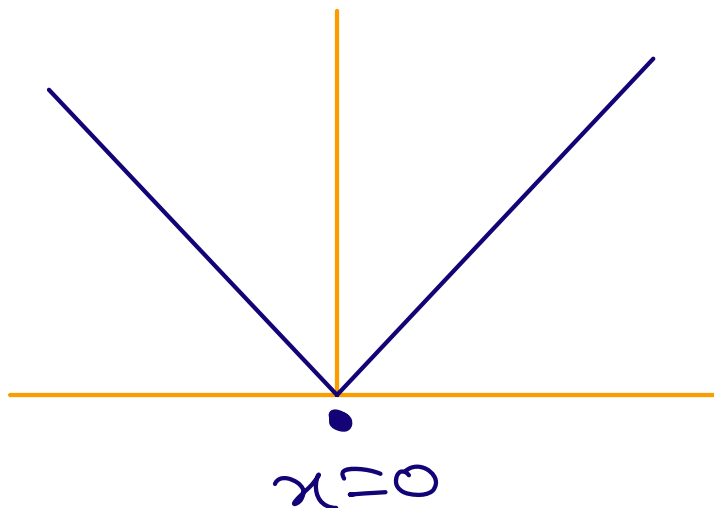
$$\text{L.H.L. } f(x) \equiv 0 \\ x \rightarrow 0^-$$

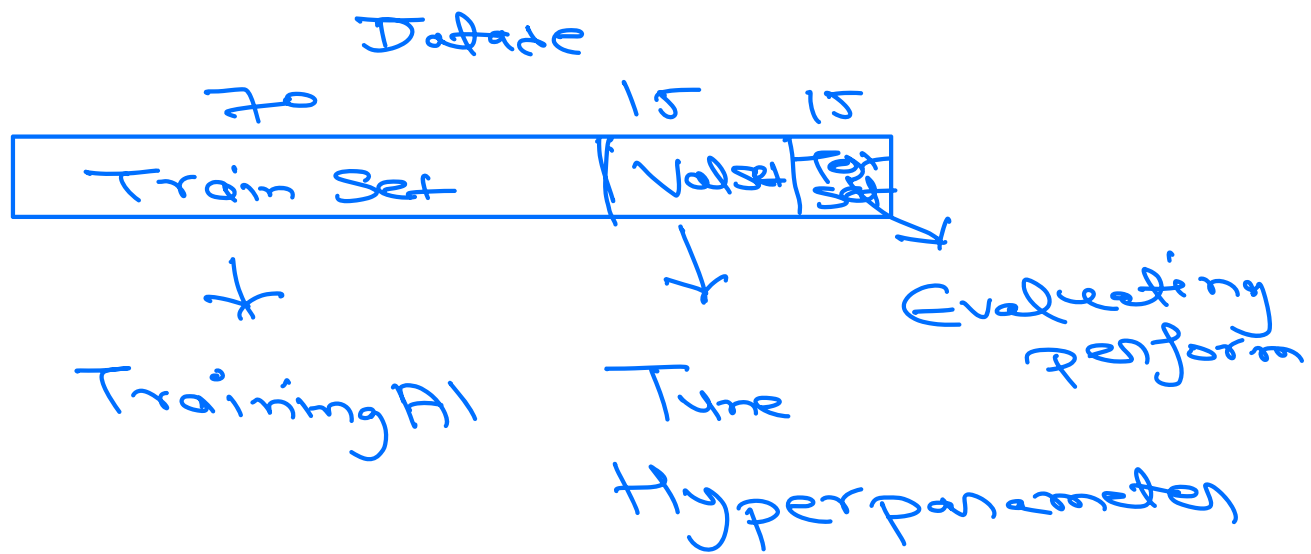
$$\text{R.H.L. } f(x) \equiv 1 \quad \begin{matrix} 0.1, 0.01 \\ 0.001 \\ 0.0001 \end{matrix} \\ x \rightarrow 0^+$$

$$f(x) \Big|_{x=0} \equiv 1$$

$$\text{L.H.L.} \neq \text{R.H.L.} = f(x)$$

H. 63





$x = 4, 3, 1$

a) $(1, 2, -1)$
 b) $(3, 1, 5)$

$$\vec{x} \cdot \hat{a} = \frac{\vec{x} \cdot \vec{a}}{\|\vec{a}\|}$$

$$\vec{x} \cdot \hat{b} = \frac{\vec{x} \cdot \vec{b}}{\|\vec{b}\|}$$

