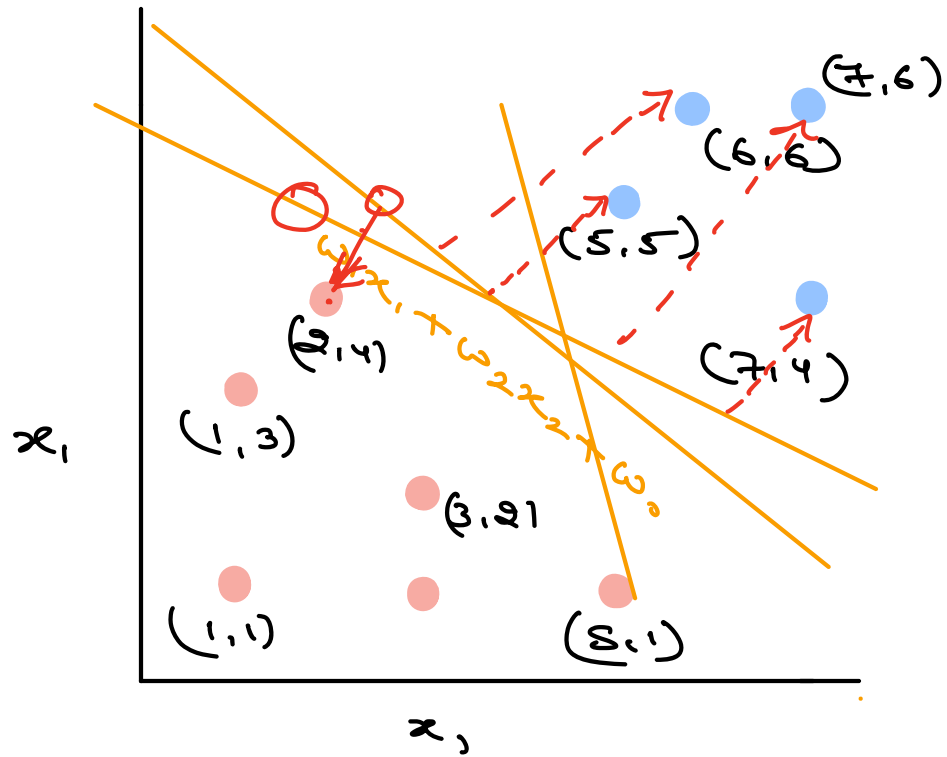


Agenda

- ⇒ Revision
- ⇒ End Goal
- ⇒ Vectors
 - ⇒ intro to Vectors
 - ⇒ Representation of Vectors
 - ⇒ Visualization of Vectors
 - ⇒ Magnitude of Vectors
 - ⇒ Norms: $L1$ and $L2$
 - ⇒ Dot product of Vectors
 - ⇒ Matrix Multiplication
 - ⇒ Angle b/w 2 vectors
 - ⇒ Connection b/w geometry and LA
 - ⇒ Unit Vector
 - ⇒ Vector projection

End Goal

| x_1 | x_2 |
|-------|-------|
| 5 | 5 |
| 2 | 4 |
| 6 | 6 |
| 3 | 2 |
| 7 | 4 |
| 5 | 1 |



w_1 and w_2 and w_3

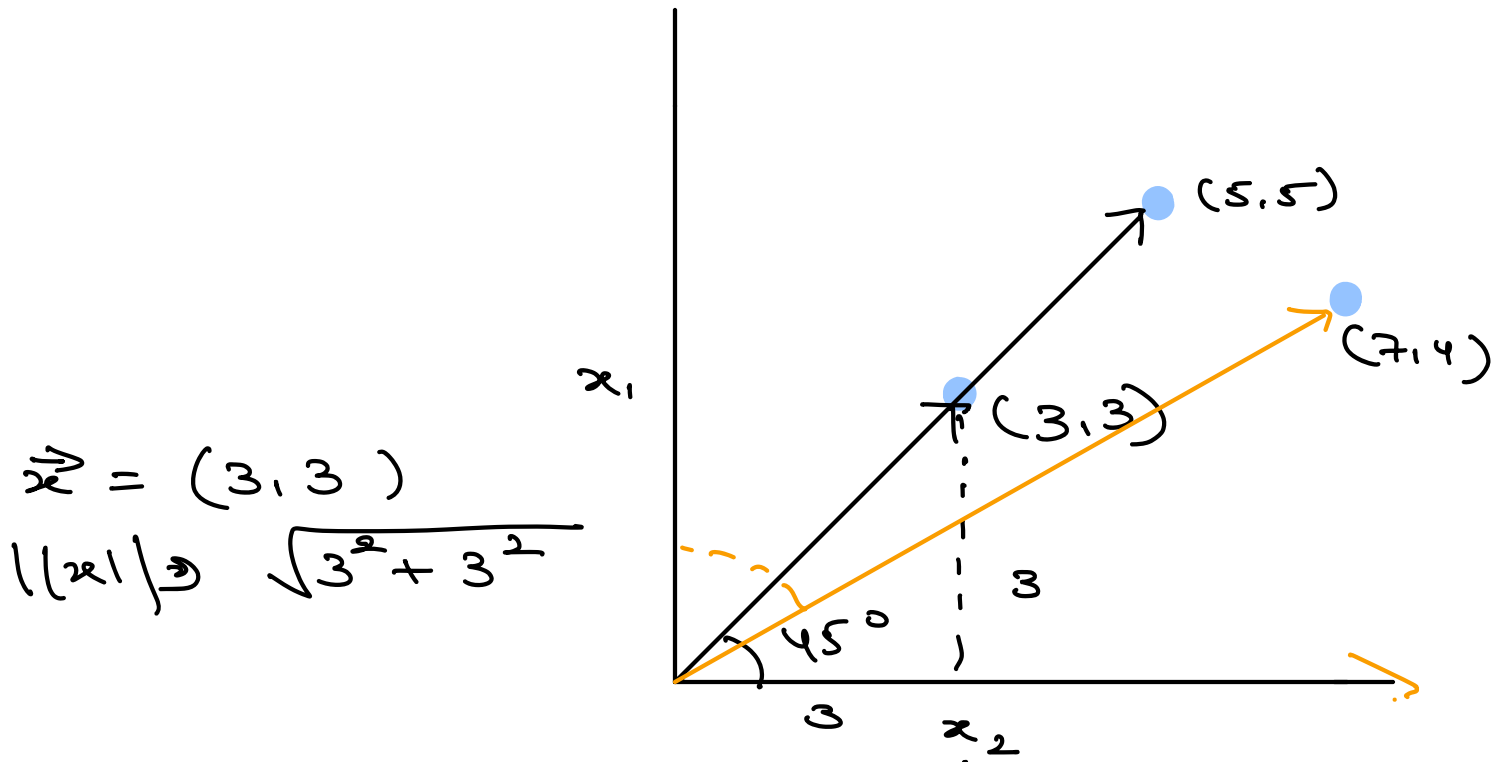
$$w_1, w_2, w_3 \in \mathbb{R}$$

Vectors

→ Magnitude and Direction

$$x = [5, 5] \quad 2D$$

$$x \Rightarrow [4, 5, 6] \quad 3D \Rightarrow \sqrt{4^2 + 5^2 + 6^2}$$



$$\vec{x} = (3, 3)$$

$$||\vec{x}|| \Rightarrow \sqrt{3^2 + 3^2}$$

Norm of Vector

$$||\vec{x}||$$

→ magnitude

$$x = [1, 2]$$

$$x = [1, 2]$$

$$x^T \Rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

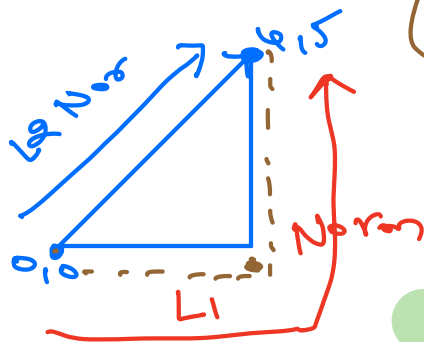
Column Vector

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\vec{x} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

Norm is just another
for distance

L_2 Norm $\Rightarrow \sqrt{x_1^2 + x_2^2 + x_3^2 + \dots + x_n^2}$
(Euclidean distance)



L_1 Norm $\Rightarrow |x_1| + |x_2| + |x_3| + \dots + |x_n|$
(Manhattan distance)

Dot product Vector

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2 \times 1}$$

$$\vec{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{2 \times 1}$$

\Rightarrow Dot product
 \Rightarrow Cross product

$$\vec{x}^T \cdot \vec{y} \quad \text{or} \quad \vec{x}^T \vec{y}^T$$

$$\begin{matrix} n \times b & \cdot & m \times n \\ \hline & b=m & \\ \downarrow & & \\ \text{output } n \times n & & \end{matrix}$$

$$\vec{x}^T \ni \begin{bmatrix} 1 & 2 \end{bmatrix}_{1 \times 2} \quad \vec{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{2 \times 1}$$

$$\vec{x}^T \cdot \vec{y} \ni 1 \times 3 + 2 \times 4 \ni$$

Bank loan approval system

- \ni income \ni 3500
- \ni Cibil score \ni 800
- \ni Current-loans \ni 3

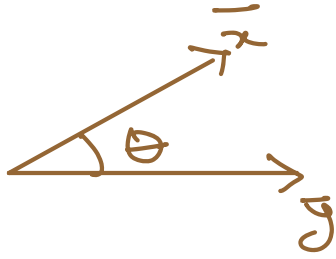
$$w_1 \times \text{income} + w_2 \text{ CbScore} + w_3 \times \text{Call}$$

\geq Target ✓

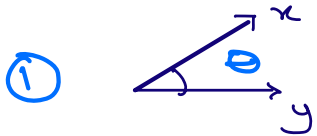
$<$ Target ✗

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}^T \cdot \begin{bmatrix} \text{income} \\ \text{Cibil_Score} \\ \text{Current-loans} \\ \text{no-dep} \end{bmatrix} \ni 100$$

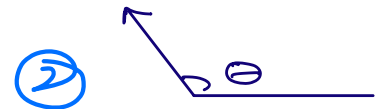
Angle b/w two Vectors



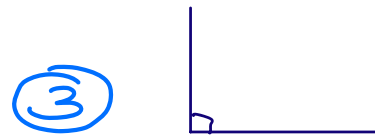
$$\cos \theta = \frac{\vec{x} \cdot \vec{y}}{||\vec{x}|| ||\vec{y}||}$$



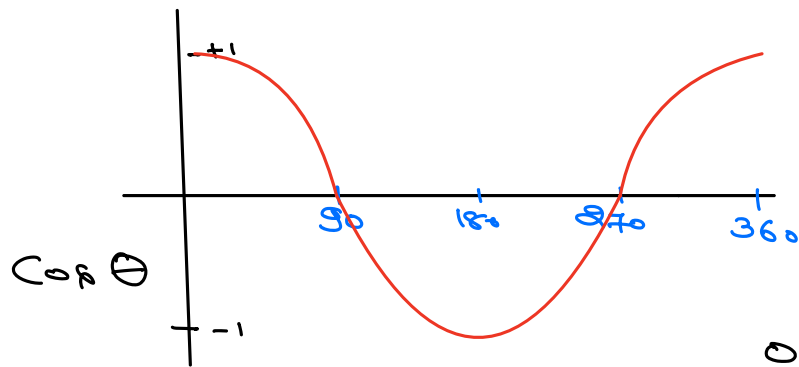
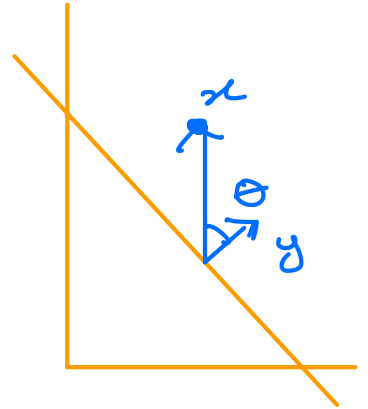
$\theta < 90$ then $\vec{x} \cdot \vec{y} \Rightarrow +ve$



$180 > \theta > 90$ then $\vec{x} \cdot \vec{y} \Rightarrow -ve$



$\theta = 90$ then $\vec{x} \cdot \vec{y} \Rightarrow 0$



$$-1 \leq \cos \theta \leq 1$$

$0 - 90 \Rightarrow +ve$
 $90 - 180 \Rightarrow -ve$

$180 - 270 \Rightarrow -ve$
 $270 - 360 \Rightarrow +ve$

H.W.: Find angle b/w \vec{x} and \vec{y}

$\vec{x} \in [1, 2, 3]$

$\vec{y} \in [10, -2, 3]$

Proof

\vec{w} will be \perp to line $w_1x_1 + w_2x_2 = 0$

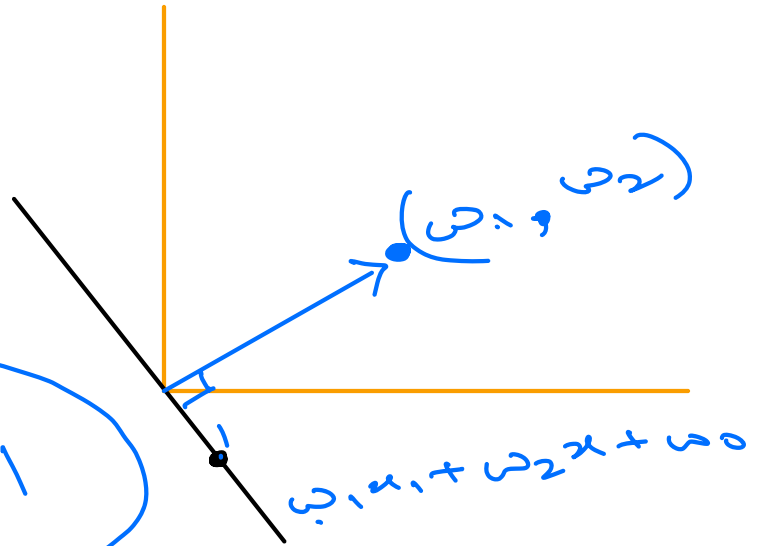
$$w_1x_1 + w_2x_2 = 0$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\cos \theta = \frac{\vec{w}^T \vec{x}}{\|\vec{x}\| \|\vec{w}\|}$$

$$\cos \theta \rightarrow 0$$

$$90^\circ$$



* Unit Vector

It's a vector with magnitude of 1

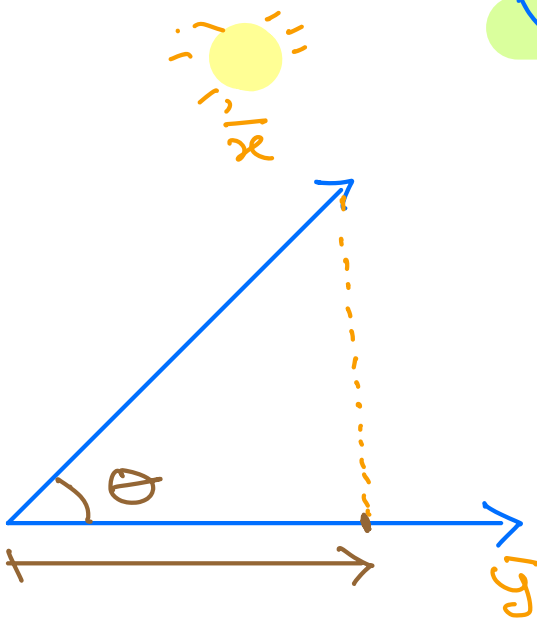
$$\vec{w} = [1, 2, 3]$$

$$\hat{w} = \frac{\vec{w}}{\|\vec{w}\|}$$

$$\rightarrow \frac{1}{\|\vec{w}\|}, \frac{2}{\|\vec{w}\|}, \frac{3}{\|\vec{w}\|}$$

$$\hat{w} = \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$$

Vector projection



P and y will be zero

P lies on y

projection of x on y

Revision and Doubts

$$y = mx + c$$

$$\omega_1 x_1 + \omega_2 x_2 + \omega_0$$

\downarrow \downarrow
 x y

$$\omega_2 x_2 \geq -\omega_1 x_0 - \omega_0$$

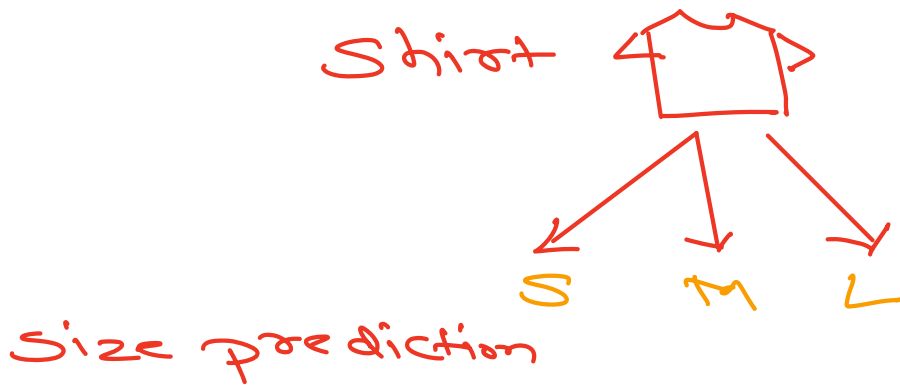
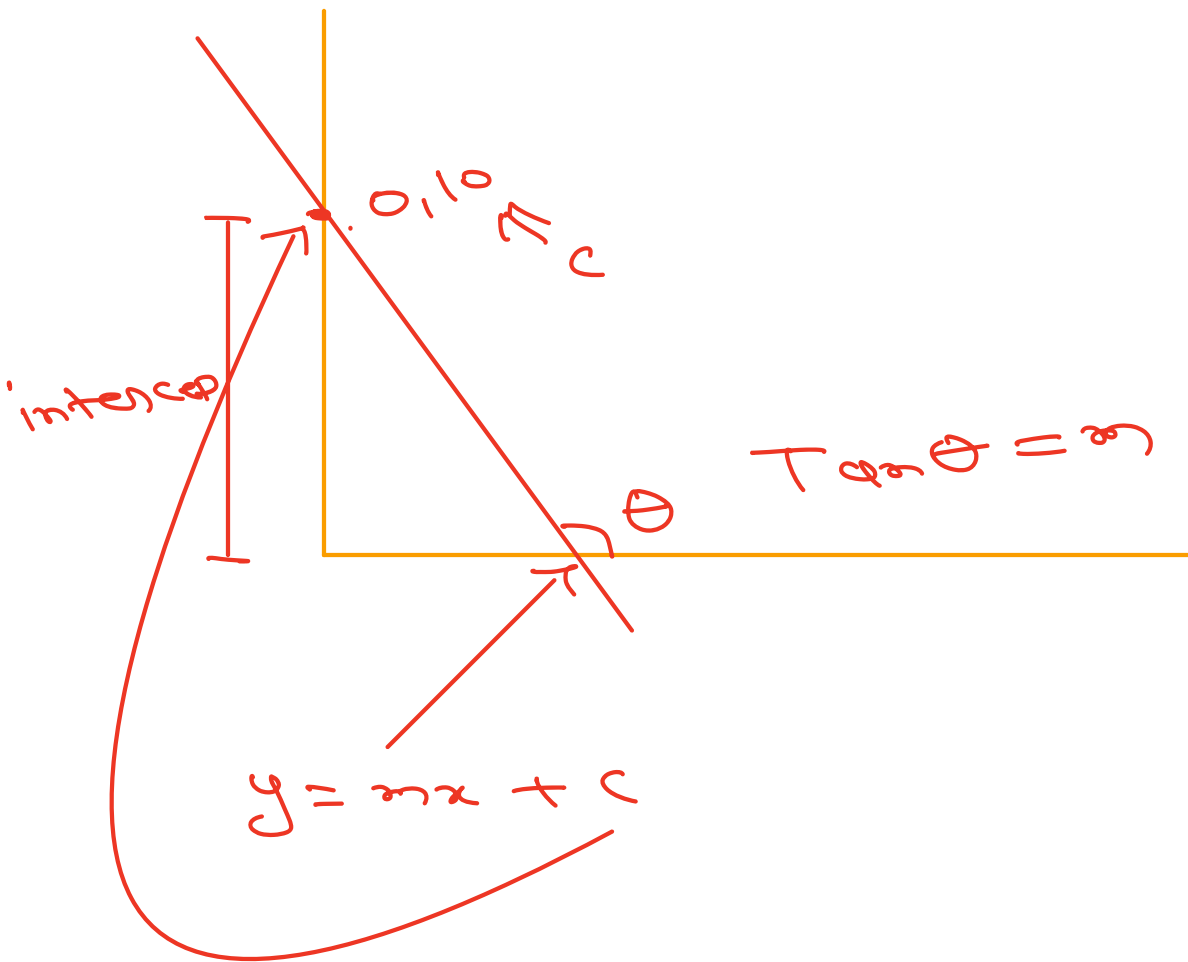
$m \rightarrow \infty$
 \downarrow
Tan 90

$$x_2 \geq -\frac{\omega_1 x_1}{\omega_2} - \frac{\omega_0}{\omega_2}$$

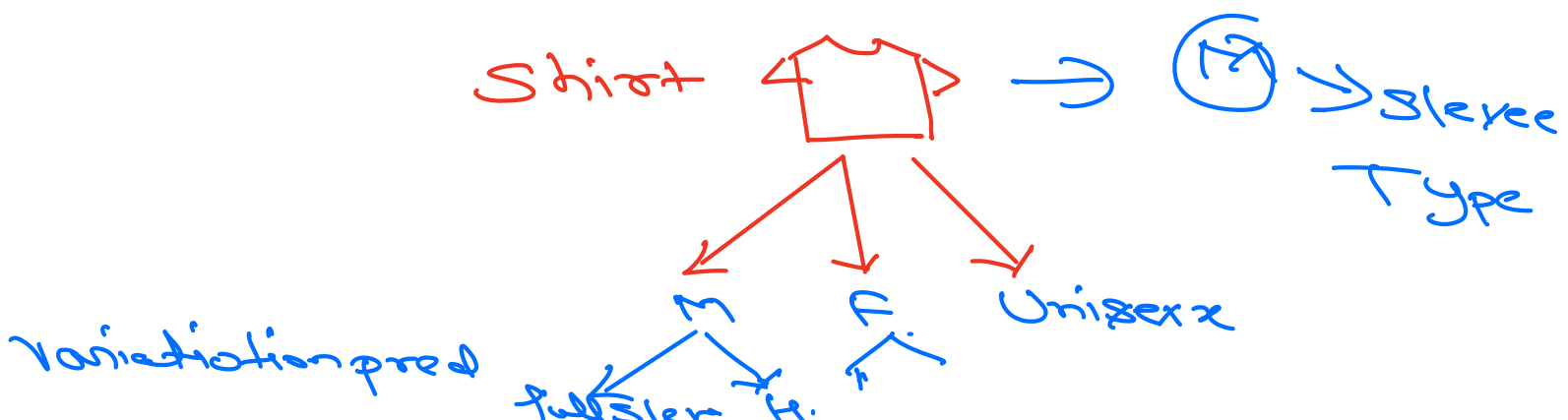
$$y \geq mx + c$$

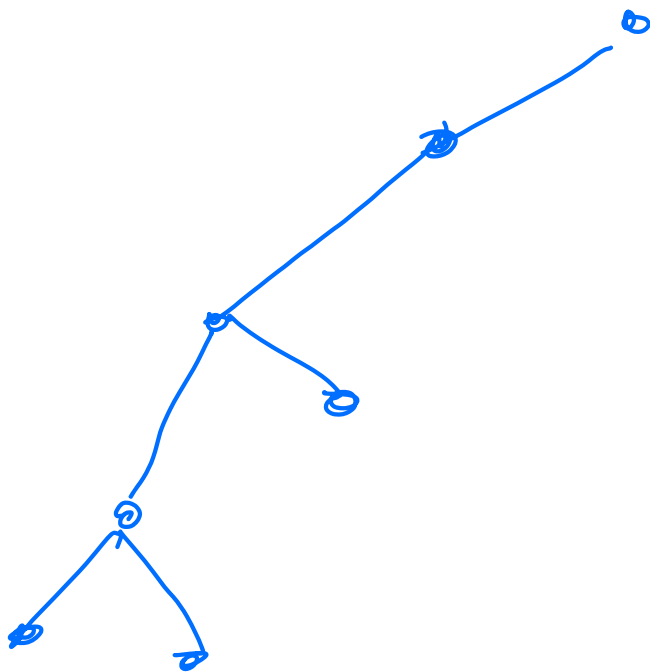
$$\frac{\omega_1}{\omega_2}$$

$$-\frac{\omega_0}{\omega_2}$$

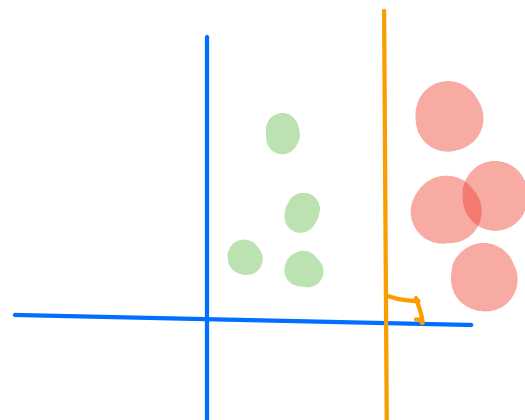


Multi-Class





\Rightarrow Tan \Rightarrow \downarrow
 \Rightarrow Tango \leftarrow



$\Rightarrow \omega_1 x_1 + \omega_2 x_2 + \omega_3 x_3 = 50$
 \downarrow

\Rightarrow

\Rightarrow
 \Rightarrow

| x_1 | x_2 | x_3 | yes |
|-------|-------|-------|-----|
| 10 | 20 | 30 | ✓ |
| 20 | 30 | 10 | ✗ |
| 40 | 50 | 60 | ✓ |

$\Rightarrow \omega^T \cdot x$