UNIT 1 : ALGEBRA

1) Permutation and Combinations

Permutation : Total number of arrangements.

Combination : Total number of selections.

Basic Principle of Counting

Sum Rule = "OR" (+)

Product Rule = "AND" (×)

Formula

- Permutation of n different objects taken r at a time is given as,

  $P(n,r)$ or $^nP_r = \frac{n!}{(n-r)!}$, $(r \leq n)$

- If n objects taken at a time in a row is,

  $P(n)$ or $^nP_n = n!$

- In Circle,

  $P(n) = (n-1)!$

- In case of necklace or bracelets,

  $P(n) = \frac{(n-1)!}{2}$

- Permutation of n different objects p is one kind, q second, r third and so on,

  $P = \frac{n!}{p!\,q!\,r!}$

- Number of combinations of n different objects taken r at a time is given by,

  $C(n,r)$ or $^nC_r = \frac{n!}{(n-r)!\,r!}$, $(r \leq n)$

  Also, $^nC_n = 1$

Properties of Combination

- If ${}^nC_r = {}^nC_{r'}$

  Then, either r = r' or r + r' = n

- ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_r$
- ${}^nC_r = {}^nC_{n-r}$

2) Binomial Theorem

For any integer n binomial theorem states that,

$$(a+x)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} x + {}^nC_2\, a^{n-2} x^2 + {}^nC_3\, a^{n-3} x^3 + \dots + {}^nC_n\, x^n$$

General Term is given by,

$$(r+1)^{th} \text{ term} = t_{r+1} = {}^nC_r\, a^{n-r} x^r$$

Middle Term in the expansion of $(a+x)^n$

- If n is even then,

  Middle Term = $t_{n/2+1}$

- If n is odd then,

  Middle Term = $t_{(n+1)/2}$ and $t_{[(n+1)/2]+1}$

NOTE;

$$(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \dots + C_n x^n$$

[where, C_0 , C_1 , C_2 , C_3 , ... , C_n are called binomial coefficients]

Some Important Formulas,

- $C_1 = n$
- $C_2 = \frac{n(n-1)!}{2!}$
- $C_3 = \frac{n(n-1)(n-2)}{3!}$
- $C_4 = \frac{n(n-1)(n-2)(n-3)}{4!}$

- $C_n = 1$

Formula related to expansion of logarithm and exponential series

- $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots..\infty$
- $e^{-x}\ or\ \frac{1}{e^x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots..\infty$
- $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots..\infty$
- $e^{-1}\ or\ \frac{1}{e} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \cdots..\infty$
- $\frac{1}{2}\left(e + \frac{1}{e}\right) = 1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \cdots..\infty$
- $\frac{1}{2}\left(e - \frac{1}{e}\right) = 1 + \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \cdots..\infty$
- $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \cdots..\ \infty$
- $\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \frac{x^6}{6} - \cdots..\ \infty$

Theorems

- Sum of binomial coefficients if 2^n

  $$C_0 + C_1 + C_2 + C_3 + \cdots..+C_n = 2^n$$

- $C_0 + C_2 + C_4 + \cdots..+\text{n}\ terms = C_1 + C_3 + C_5 + \cdots..+\text{n}\ terms = 2^{n-1}$

3) Complex Numbers

z = a + ib = (a,b)

[where, a is real part and b is imaginary part]

$$z = a + ib = r(cos\theta + i.sin\theta) = r.e^{i\theta}$$

Cartesian form	Polar form	Euler's form
↓	↓	↓

Also,

$$z = r(cos\theta + i.sin\theta)$$

$$z = r[\cos(k.360 + \theta) + i.\sin(k.360 + \theta)]$$ (General Polar Form)

$[where, k = 0,1,2,3,4 \dots \dots..n]$

Converting into polar form :

If $x + iy = r(cos\theta + i.sin\theta)$

then,

$$r = \sqrt{x^2 + y^2}$$

And,

$$tan\theta = \frac{y}{x}$$

$$\therefore \theta = \tan^{-1}\frac{y}{x}$$

(-,+) 180 - θ	(+,+) θ = acute angle
(-,-) 270 - θ	(+,-) 360 - θ

Note:

In polar of complex numbers, $z = r(cos\theta + i.sin\theta)$

[where, r is called modulus of complex number]

$i.e. r = |z| = \sqrt{x^2 + y^2}$

θ is amplitude or argument of z.

It is denoted by, $Arg(z)\ or\ Amp(z)$

De-Moivre's Theorem

For any integer n, the De-Moivre's states that,

$$[r(cos\theta + i.sin\theta)]^n = r^n(\cos n\theta + i.\sin n\theta)$$

Properties or Nature of cube roots of unity

1) The square root of one imaginary root of unity is equal to the other.
   If one cube root is ω then the other one is ω^2
2) The sum of three cube roots of unity is equal to zero.
   $i.e. 1+\omega+\omega^2=0$

Formulae:

- $1+\omega+\omega^2=0$
- $\omega^3=1$

4) Sequence and Series

1) **The sum of first n natural numbers, $\mathbf{1+2+3+\cdots+}\boldsymbol{n}$ is,**
   $$\sum n=\frac{n(n+1)}{2}$$
2) **The sum square of first n natural numbers, $\mathbf{1^2+2^2+3^2+\cdots+}\boldsymbol{n^2}$ is**,
   $$\sum n^2=\frac{n(n+1)(2n+1)}{6}$$
3) **The sum cube of first n natural numbers, $\mathbf{1^3+2^3+3^3+\cdots+}\boldsymbol{n^3}$ is,**
   $$\sum n^3=\left[\frac{n(n+1)}{2}\right]^2$$
4) **The sum n terms of Arithmetic Series,**
   $$Sn=\frac{n}{2}[2a+(n-1)d]$$
   $Or,$
   $$Sn=\frac{n}{2}(a+r)$$
5) **The sum n terms of Geometric Series,**
   $$Sn=\frac{a(r^n-1)}{r-1}$$
   $Or,$
   $$Sn=\frac{lr-a}{r-1}$$
6) **$\mathbf{N^{th}}$ term of an Arithmetic Series is,**
   $$tn=a+(n-1)d$$
7) **$\mathbf{N^{th}}$ term of an Geometric Series is,**
   $$tn=a.r^{n-1}$$

Principle of Mathematical Induction

It states that, “If p(n) be the statement such that,

i) p(1) is true.

ii) P(m+1) is true whenever p(m) is true. (m≤n)

Then the statement is true for all n∈N

5) System of Linear Equations (Matrix)

Solution of linear equation by,

i) Matrix Inversion Method
ii) Crammer’s Rule
iii) Row Equivalent matrix method (Gauss-Jordan Method)

Matrix Method or Matrix Inversion Method

Let the linear equation be,

$a_1x + b_1y + c_1z = d_1$ ………. (i)

$a_2x + b_2y + c_2z = d_2$ ………. (ii)

$a_3x + b_3y + c_3z = d_3$ ………. (iii)

The above equations can be written in matrix form as,

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$or, AX = B$

$\therefore X = A^{-1}B \ldots (iv)$

Where,

A = $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

coefficient matrix variable matrix constant matrix

To find A^{-1},

$A^{-1} = \frac{1}{|A|} \times Adjoint\ of\ A$

Then,

From eqn (i),

$X = A^{-1}B$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$

Hence, x=p, y=q and z=r.

NOTE:

$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$ [Basic Rule for Matrix]

Cramer's rule or determinant method

To solve using Cramer's rule,

Coefficient of x	Coefficient of y	Coefficient of z	constant
a	b	c	p
d	e	f	q
g	h	i	r

Then, find

$|d| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}, |d_1| = \begin{vmatrix} p & b & c \\ q & e & f \\ r & h & i \end{vmatrix}, |d_2| = \begin{vmatrix} a & p & c \\ d & q & f \\ g & r & i \end{vmatrix}, |d_3| = \begin{vmatrix} a & b & p \\ d & e & q \\ g & h & r \end{vmatrix}$

Lastly, $x = \frac{d1}{d}, y = \frac{d2}{d}, z = \frac{d3}{d}$

Row-Equivalent Method (Gauss-Jordan Method)

Given equations are:

$a_1x + b_1y + c_1z = d_1$ ………. (i)

$a_2x + b_2y + c_2z = d_2$ ………. (ii)

$a_3x + b_3y + c_3z = d_3$ ………. (iii)

Above equations in augmented matrix form can be written as,

$$\begin{bmatrix} a_1 & b_1 & c_1 & :d_1 \\ a_2 & b_2 & c_2 & :d_2 \\ a_3 & b_3 & c_3 & :d_3 \end{bmatrix} \xRightarrow{Elementary\ row\ operation} \begin{bmatrix} 1 & 0 & 0 & :p \\ 0 & 1 & 0 & :q \\ 0 & 0 & 1 & :r \end{bmatrix}$$

Hence, x=p, y=q and z=r

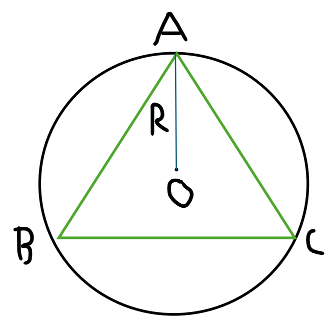
NOTE:

For 2×2 Matrix,

$$\begin{bmatrix} a_1 & b_1 & :c_1 \\ a_2 & b_2 & :c_2 \end{bmatrix} \xRightarrow{Elementary\ row\ operation} \begin{bmatrix} 1 & 0 & :p \\ 0 & 1 & :q \end{bmatrix}$$

UNIT 2 : TRIGONOMETRY

6) Properties of Triangle



Sine Law: In any ΔABC it states that,

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} = 2R$$

Cosine Law: In any ΔABC it states that,

$$cosA = \frac{b^2+c^2-a^2}{2bc}$$

$$cosB = \frac{a^2+c^2-b^2}{2ac}$$

$$cosC = \frac{a^2+b^2-c^2}{2ab}$$

Tangent Law: In any ΔABC it states that,

$$tan\frac{B-C}{2} = \frac{b-c}{b+c}.\cot\frac{A}{2}$$

$$tan\frac{C-A}{2} = \frac{c-a}{c+a}.\cot\frac{B}{2}$$

$$tan\frac{A-B}{2} = \frac{a-b}{a+b}.\cot\frac{C}{2}$$

Projection Law: In any ΔABC it states that,

$$a = b.cosC + c.cosB$$

$$b = a.cosC + c.cosA$$

$$c = a.cosB + b.cosA$$

Half Angle Formula:

- $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$
- $\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$
- $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$
- $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$
- $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$
- $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$
- $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
- $\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$
- $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$
- $\cot\frac{A}{2} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$
- $\cot\frac{B}{2} = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$
- $\cot\frac{C}{2} = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

Other Formula:

We Know,

$$\Delta = Area\ of\ Triangle\ ABC$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

Then,

- $\tan\frac{A}{2} = \frac{\Delta}{s(s-a)}$
- $\tan\frac{B}{2} = \frac{\Delta}{s(s-b)}$
- $\tan\frac{C}{2} = \frac{\Delta}{s(s-c)}$
- $\cot\frac{A}{2} = \frac{s(s-a)}{\Delta}$
- $\cot\frac{B}{2} = \frac{s(s-b)}{\Delta}$
- $\cot\frac{C}{2} = \frac{s(s-c)}{\Delta}$

Also, we know,

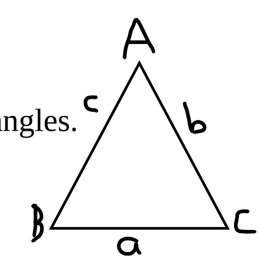
$$Area\ of\ \Delta ABC = \Delta = \frac{1}{2} bc.SinA$$

Then,

- $SinA = \frac{2\Delta}{bc}$
- $SinB = \frac{2\Delta}{ac}$
- $SinC = \frac{2\Delta}{ab}$

7) Solution of Triangles

It means we need to find the unknown sides and unknown angles of given triangles.



RULES:

1) If all three angles of a triangle are given we cannot find length of all three sides but we can find ratio of the sides by using sine law.

   $i.e. a : b : c = sinA : sinB : sinC$

2) If all three sides of a triangle are given we can easily find all three angles by using cosine law.

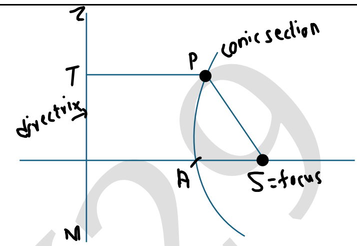
   $cosA = \frac{b^2+c^2-a^2}{2bc}$, $cosB = \frac{a^2+c^2-b^2}{2ac}$

   Then, use $A + B + C = 180°$ to find 'C'.

3) We can use sine law and cosine law to solve the triangle when sides and angles are given.

UNIT 3 : ANALYTICAL GEOMETRY

NOTE:

• **Conic Section :** Locus of a point which moves in a plane in such a way that ratio of its distance from a fixed point to its distance from a fixed straight line is always constant is called conic section.	

The fixed straight line is called directrix and fixed point is called focus and constant is called eccentricity (e).

$$\frac{PS}{PT} = a\ constant$$

There are three cases

- If $e = 1$ then, conic section is called parabola.
- If $e < 1$ then, conic section is called ellipse.
- If $e > 1$ then, conic section is called hyperbola.

8) Circle

1. **Equation of circle having center origin and radius r is,**
   $x^2 + y^2 = r^2$
2. **Equation of circle having center (h,k) and radius r is,**
   $(x-h)^2 + (y-k)^2 = r^2$
3. **Equation of circle in general form is $x^2 + y^2 + 2gx + 2fy + c = 0$ having,**
   $centre\ (h,k) = (-g,-f)$
   $and, radius\ r = \sqrt{g^2 + f^2 - c}$
4. **Equation of circle having end points of a diameter (x_1,y_1) and (x_2,y_2) is,**
   $(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

NOTE;

- If a circle touches x-axis then the y-coordinate of the center is the radius.
  $i.e. r = k$
- If a circle touches y-axis then the x-coordinate of the center is the radius.
  $i.e. r = h$
- If a circle touches both axis then,
  $r = h = k$
- If two circles are said to be concentric then they have same center whatever be their radius.
- Four or more points are said to be concyclic if they lies on same circle.

NOTE:

The general equation of 2nd degree is,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

It represents a circle if,

$$a = 1, b = 1, h = 0$$

Formulas related to tangent and normal

- **The equation of tangent to the circle $x^2 + y^2 = a^2$ at point (x_1, y_1) is**
  $xx_1 + yy_1 = a^2$
- **The equation of normal to the circle $x^2 + y^2 = a^2$ at point (x_1, y_1) is**
  $xy_1 - x_1y = 0$
- **The equation of tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is at point (x_1, y_1) is**
  $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
- **The condition that a line $y = mx + c$ is always tangent to the circle $x^2 + y^2 = a^2$ is,**
  $c = \pm a\sqrt{1 + m^2}$

NOTE:

- A point $P(x_1,y_1)$ lies outside the circle, $x^2 + y^2 + 2gx + 2fy + c = 0$ if,
  $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c > 0$
- A point $P(x_1,y_1)$ lies inside the circle, $x^2 + y^2 + 2gx + 2fy + c = 0$ if,
  $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < 0$

- A point P(x_1,y_1) lies at circumference of the circle, $x^2 + y^2 + 2gx + 2fy + c = 0$ if,
  $$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$$

NOTE:
- Abscissa means x and Ordinate means y

NOTE:
- **The length of tangent from a point (x_1,y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is,**
  $$p = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$
- **If two circles touch externally then,**
  $$r_1 + r_2 = d$$
- **If two circles touch internally then,**
  $r_1 - r_2 = d$ [d= distance between their centres]

NOTE:
- Any line which is parallel to $ax + by + c = 0$ is,
  $$ax + by + k = 0$$
- Any line which is perpendicular to $ax + by + c = 0$ is,
  $$bx - ay + k = 0$$
- A line is tangent to a circle if $r = p$ [r=radius, p=perpendicular distance from a point to line]
- A line is normal to the circle if it passes through center of the circle.

8) Parabola

Parabola : The locus of a point which moves in a plane in such a way that its distance from a fixed point is equal to its distance from a fixed straight line is called parabola.

Vertex of Parabola : The turning point of a parabola is called vertex.

Focus Chord : A chord of the parabola passes through focus is called focus chord.

Latus Rectum : A chord of parabola passes through focus and perpendicular to the axis is called latus rectum.

	Equation of parabola	Vertex	Focus	Equation of axis	Equation of directrix	Length of latus rectum
1	$y^2 = 4ax$	(0,0)	(a,0)	y=0	x=-a	4a
2	$x^2 = 4ay$	(0,0)	(0,a)	x=0	y=-a	4a
3	$(y-k)^2 = 4a(x-h)$	(h,k)	(h+a,k)	y=k	x=h-a	4a
4	$(x-h)^2 = 4a(y-k)$	(h,k)	(h,k+a)	x=h	y=k-a	4a

Equation of parabola $y^2 = 4ax$ in parametric form,

$x = at^2$ and $y = 2at$ [where, t is a parameter]

NOTE:

- The equation of tangent to the parabola $y^2 = 4ax$ at point (x_1,y_1) is,
  $$yy_1 = 2a(x + x_1)$$
- The equation of tangent to the parabola $x^2 = 4ay$ at point (x_1,y_1) is,
  $$xx_1 = 2a(y + y_1)$$
- Any line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$ if,
  $$c = \frac{a}{m}$$
- The equation of normal to the parabola $y^2 = 4ax$ in slope form is,
  $$y = mx - 2am - am^3$$

9) Ellipse

Equation of ellipse	Center	Vertices	Eccentricity	Foci	Major Axis	Minor Axis	Latus Rectum	Eqn of directrix
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a>b)	(0,0)	(±a,0)	$\sqrt{1 - \frac{b^2}{a^2}}$	(±ae,0)	2a	2b	$\frac{2b^2}{a}$	$x = \pm\frac{a}{e}$
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (b>a)	(0,0)	(0,±b)	$\sqrt{1 - \frac{a^2}{b}}$	(0,±be)	2b	2a	$\frac{2a^2}{b}$	$y = \pm\frac{b}{e}$
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (a>b)	(h,k)	(h±a,k)	$\sqrt{1 - \frac{b^2}{a^2}}$	(h±ae,k)	2a	2b	$\frac{2b^2}{a}$	$x = h \pm \frac{a}{e}$
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (b>a)	(h,k)	(h,k±b)	$\sqrt{1 - \frac{a^2}{b^2}}$	(h,k±be)	2b	2a	$\frac{2a^2}{b}$	$y = k \pm \frac{b}{e}$

10) Hyperbola

Equation of hyperbola	Center	Vertices	Eccentricity	Foci	Transverse Axis	Conjugate Axis	Latus Rectum
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	(0,0)	(±a,0)	$\sqrt{1 + \frac{b^2}{a^2}}$	(±ae,0)	2a	2b	$\frac{2b^2}{a}$
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$	(0,0)	(0,±b)	$\sqrt{1 + \frac{a^2}{b^2}}$	(0,±be)	2b	2a	$\frac{2a^2}{b}$
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	(h,k)	(h±a,k)	$\sqrt{1 + \frac{b^2}{a^2}}$	(h±ae,k)	2a	2b	$\frac{2b^2}{a}$
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = -1$	(h,k)	(h,k±b)	$\sqrt{1 + \frac{a^2}{b^2}}$	(h,k±be)	2b	2a	$\frac{2a^2}{b}$

UNIT 4 : PRODUCT OF VECTORS

11) SCALAR PRODUCT AND VECTOR PRODUCT

Dot Product or Scalar Product: Let $\vec{a}$ and $\vec{b}$ be two vectors and θ be the angle between them. Then their dot product is defined as,

$$\vec{a}.\vec{b} = |\overrightarrow{a}|\,|\overrightarrow{b}| cos\theta \qquad [or,\ ab\ cos\theta]$$

Cross Product or Vector Product: Let $\vec{a}$ and $\vec{b}$ be two vectors and θ be the angle between them. Then their cross product is defined as,

$$\vec{a} \times \vec{b} = |\overrightarrow{a}|\,|\overrightarrow{b}|\, sin\theta\ \hat{\eta}$$

[where, $\hat{\eta}$ is called unit vector perpendicular to both a and b]

$$|\vec{a} \times \vec{b}| = |\overrightarrow{a}|\,|\overrightarrow{b}|\, sin\theta \qquad [or,\ ab\ sin\theta]$$

If $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ then,

$$\vec{a}.\vec{b} = (a_1, a_2, a_3).(b_1, b_2, b_3)$$

$$\therefore \vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} a_1 & a_2 & a_3 & a_1 & a_2 \\ b_1 & b_2 & b_3 & b_1 & b_2 \end{pmatrix}$$

$$\therefore \vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - b_1 a_2)$$

NOTE:

- $|\vec{a} \times \vec{b}| = \textit{area of parallelogram}$
- Area of parallelogram when diagonal $\overrightarrow{d_1}$ *and* $\overrightarrow{d_2}$ are given is,

  $$\frac{|\overrightarrow{d_1} \times \overrightarrow{d_2}|}{2}$$

- $\vec{a} \times \vec{b}$ is a vector perpendicular to both $\vec{a}$ *and* $\vec{b}$.

  $i.e. (\vec{a} \times \vec{b}).\vec{a} = 0$

  $and, (\vec{a} \times \vec{b}).\vec{b} = 0$

NOTE:

- Condition that two vectors $\vec{a}$ *and* $\vec{b}$ are perpendicular to each other is,

  $\vec{a} . \vec{b} = 0$

- Condition that two vectors $\vec{a}$ *and* $\vec{b}$ are parallel to each other is,

  $\vec{a} \times \vec{b} = 0$

- The unit vector perpendicular to $\vec{a}$ *and* $\vec{b}$ is,

  $$\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

- Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a} . \vec{b}}{|\vec{b}|}$
- Projection of $\vec{b}$ on $\vec{a}$ = $\frac{\vec{a} . \vec{b}}{|\vec{a}|}$

UNIT 5 : STATSTICS AND PROBABILITY

12) Correlation and Regression

NOTE:

- Correlation coefficient between two variables = r
- Rank correlation coefficient between two variables = R

Interpretation of value of correlation coefficient = r

- If r > 0, then the correlation is positive.
- If r < 0, then the correlation is negative.
- If r = 0, then there is no correlation.
- If r = 1, then the correlation is perfect positive.
- If r = -1, then the correlation is perfect negative.

Formula to find correlation coefficient (r)

- $r = \frac{Cov(x,y)}{\sqrt{(var)_x}\sqrt{(var)_y}}$

  Where, Cov(x,y) = variable of x and y
  $(var)_x$ = variable of X
  $(var)_y$ = variable of Y

- $r = \frac{\sum(X-\bar{X})(Y-\bar{Y})}{n\sigma_x.\sigma_y}$ OR, $\frac{\sum xy}{n\sigma_x.\sigma_y}$

  Where, $x = X - \bar{X}$

  $y = Y - \bar{Y}$

- $r = \frac{\sum(X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum(X-\bar{X})^2}.\sqrt{\sum(Y-\bar{Y})^2}}$ OR, $\frac{\sum xy}{\sqrt{\sum x^2}.\sqrt{y^2}}$

- $r = \frac{n\sum XY - \sum X \sum Y}{\sqrt{n\sum X^2 - (\sum X)^2}.\sqrt{n\sum Y^2 - (\sum Y)^2}}$

Rank Correlation Coefficient = R

- When ranks are given,

  $R = 1 - \frac{6\sum d^2}{n^3 - n}$

  Where, n = no. of observations
  $d = R_1 - R_2$

R_1: Rank of first variable
R_2: Rank of second variable

- When ranks are repeated,

$$R = 1 - 6\left[\frac{\sum d^2 + \frac{m_1(m_1^2 - 1)}{12} + \frac{m_2(m_2^2 - 1)}{12} + \frac{m_3(m_3^2 - 1)}{12}}{n^3 - n}\right]$$

Where, m_1: no. of repetition of one rank
m_2: no. of repetition of 2nd rank
m_3: no. of repetition of 3rd rank
:
:
And so on

Regression Equations and its coefficients

- Regression equation of Y on X is,

$$Y - \bar{Y} = b_{yx}(X - \bar{X})$$

Where,
b_{yx} is regression coefficient of Y on X.

$$\therefore b_{yx} = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

OR,

$$\therefore b_{yx} = \frac{r\sigma_y}{\sigma_x}$$

- Regression equation of X on Y is,

$$X - \bar{X} = b_{xy}(Y - \bar{Y})$$

Where,
b_{xy} is regression coefficient of X on Y.

$$\therefore b_{xy} = \frac{n\sum XY - \sum X \sum Y}{n\sum Y^2 - (\sum Y)^2}$$

OR,

$$\therefore b_{yx} = \frac{r\sigma_x}{\sigma_y}$$

- Relation between regression coefficient and correlation coefficient

$$r = \pm\sqrt{b_{xy} . b_{yx}}$$

i) r is positive if b_{xy} and b_{yx} both are positive.
ii) r is negative if b_{xy} and b_{yx} both are negative.
iii) r is impossible if b_{xy} is negative and b_{yx} is positive and vice-versa.

13) PROBABILITY

The probability of happening an event P(E) is given by,

$$P(E) = \frac{n(E)}{n(S)}$$

Where, n(E) = total no. of favorable cases

n(S) = total no. of cases

NOTE: $0 \leq P(E) \leq 1$

- If P(E) = 0, then event is impossible.
- If P(E) =1, then event is called sure event.

FORMULA:

- The probability of getting anyone of the events A or B is given by,
  $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
- If A and B are mutually exclusive events then, $P(A \cap B) = 0$
  Hence, $P(A \cup B) = P(A) + P(B)$
- If A and B are independent events then,
  $$P(A \cap B) = P(A) \times P(B)$$

Independent Events
Two events are called independent events when occurrence of one event doesn't affect the occurrence of another event.
Example: In the coin tossing experiment, getting head in the first trial and again getting second head in the second trial are independent events.

Dependent Events
Two events are said to be dependent if occurrence of one event affects the another event.
Example: The probability of getting 2 hearts from the deck of 52 cards while taking one after another without replacement are dependent events.

- $P(H_1) = \frac{n(H_1)}{n(S)} = \frac{13}{52}$
- $P(H_2) = \frac{n(H_2)}{n(S)} = \frac{12}{51}$

Conditional Probability

Let A and B be two events then the probability of occurrence of event A when event B has already occurred is called conditional probability. It is denoted by P(A/B).

Mathematically,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Similarly,

$$P(B/A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0$$

- P(A/B): Probability of occurrence of an event A when B has already occurred.
- P(B/A): Probability of occurrence of an event B when A has already occurred.

NOTE:

- $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P\ \overline{(A \cup B)} = P(\bar{A} \cap \bar{B})$

UNIT 6 : CALCULUS

14) Derivatives

Derivatives of hyperbolic function

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{cosech} x = \frac{2}{e^x - e^{-x}}$$

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

CONFUSING FORMULA

- $\cosh^2 x - \sinh^2 x = 1$
- $\cosh^2 x + \sinh^2 x = \cosh 2x$

Derivatives of hyperbolic function

$$\frac{d \sinh x}{dx} = \cosh x \qquad \frac{d \cosh x}{dx} = \sinh x$$

$$\frac{d \tanh x}{dx} = \operatorname{sech}^2 x \qquad \frac{d \coth x}{dx} = -\operatorname{cosech}^2 x$$

$$\frac{d \operatorname{cosech} x}{dx} = -\operatorname{cosech} \text{x}. \coth x \qquad \frac{d \operatorname{sech} x}{dx} = -\operatorname{sech} \text{x}. \tanh x$$

Inverse hyperbolic function

$$\frac{d \sinh^{-1} x}{dx} = \frac{1}{\sqrt{x^2+1}} \qquad \frac{d \cosh^{-1} x}{dx} = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d \tanh^{-1} x}{dx} = \frac{1}{1-x^2} \qquad \frac{d \coth^{-1} x}{dx} = \frac{-1}{x^2-1}$$

$$\frac{d \operatorname{cosech}^{-1} x}{dx} = \frac{-1}{x\sqrt{1+x^2}} \qquad \frac{d \operatorname{sech}^{-1} x}{dx} = \frac{-1}{x\sqrt{1-x^2}}$$

L Hospital's Rule: Let f(x) and g(x) be two differentiable functions.
Then,

$$\lim_{x\to a} \frac{f(x)}{g(x)} \left(\frac{0}{0}\ form\right)$$

$$= \lim_{x\to a} \frac{f'(x)}{g'(x)} \left(\frac{0}{0}\ form\right)$$

$= \frac{f''(a)}{g''(b)}$, [where g''(a)≠0 etc.]

Tangent and Normal

- If y=f(x) be a given curve
  Then,
  Slope of tangent to the curve is,

$$m = \left(\frac{dy}{dx}\right) at\ point\ (x_1, y_1)$$

- If inclination = θ
  Then,
  $$slope(m) = tan\theta$$
- $\frac{dy}{dx} = tan\theta$
- If tangent is parallel to x-axis then,
  $$\frac{dy}{dx} = 0$$
- If tangent is perpendicular to x-axis or parallel to y-axis then,
  $$\frac{dx}{dy} = 0$$
- The equation of tangent to the curve y=f(x) is,
  $y - y_1 = m(x - x_1)$ [where, m is slope of tangent]
- The equation of normal is,
  $$y - y_1 = \frac{-1}{m}(x - x_1)$$
- Angle between two curves,
  $$tan\theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

Rate Measure

If distance travelled (displacement) = s

Then,

$$velocity = \frac{ds}{dt}$$

$$acceleration = \frac{d^2 s}{dt}$$

FORMULA:

- Perimeter or circumference of circle is, $P = 2\pi r$
- Area of circle, $A = \pi r^2$
- CSA of cylinder $= 2\pi rh$
- TSA of cylinder $= 2\pi(r+h)$
- Volume of cylinder $= \pi r^2 h$
- CSA of cone $= \pi rl$
- TSA of cone $= \pi r(r+l)$
- Volume of Cone $= \frac{1}{3}\pi r^2 h$
- Area of sphere $= 4\pi r^2$
- Volume of sphere $= \frac{4}{3}\pi r^3$
- Volume of cube $= l^3$
- Volume of cuboid $= l \times b \times h$

15) Anti-Derivatives

Formula:

- $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$
- $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} log \frac{x-a}{x+a} + c$
- $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} log \frac{a+x}{a-x} + c$
- $\int \frac{dx}{\sqrt{x^2+a^2}} = log(x + \sqrt{x^2+a^2}) + c$
- $\int \frac{dx}{\sqrt{x^2-a^2}} = log(x + \sqrt{x^2-a^2}) + c$
- $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$
- $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} log(x + \sqrt{x^2+a^2}) + c$
- $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} log(x + \sqrt{x^2-a^2}) + c$
- $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

NOTE: $\frac{da^x}{dx} = a^x \log_e a$

Anti-Derivatives of trigonometric functions

- $\int \cosh x \, dx = \sinh x + c$
- $\int \sinh x \, dx = \cosh x + c$
- $\int \operatorname{sech}^2 x \, dx = \tanh x + c$
- $\int \operatorname{sech} x . \tanh x \;\, dx = -\operatorname{sech} x + c$
- $\int \operatorname{coesch} x . coth \, dx = -\operatorname{cosech} x + c$
- $\int \operatorname{cosech}^2 x \, dx = -\coth x + c$
- $\int \sec x \, dx = \log\left(\sec x + \tan x\right) + c \; OR, \log\left[\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right] + c$
- $\int \operatorname{cosec} x \, dx = \log\left(cosec\ x - \cot x\right) + c \; OR, \log\left(\frac{tanx}{2}\right) + c$
- $\int \tan x \, dx = \log \sec x + c \; OR, -\log \cos x + c$
- $\int \cot x \, dx = \log \sin x + c$
- $\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$

Integration by Partial Fraction

Rules for partial fraction:

- $\frac{px+q}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b}$

  [Denominator is product of two linear factors]

- $\frac{px+q}{(x+a)(x^2+b)} = \frac{A}{x+a} + \frac{Bx+C}{x^2+b}$

  [Denominator is product of linear and quadratic functions]

- $\frac{px+q}{(x+a)(x+b)^2} = \frac{A}{x+a} + \frac{B}{x+b} + \frac{C}{(x+b)^2}$

  [Denominator is product of two linear factors]

16) Differential Equations

There are 4 types of differential equations.

1) Variable Separation Form
2) Exact Form

3) Homogeneous Form
4) Linear Form

Differential Equation of a variable separation form

A differential equation of the form, $xdx = ydy$ is called variable separation form.

Where, x = function of x alone or constant
y = function of y alone or constant

NOTE:

To solve the differential equation of a variable separation form, we can integrate directly after separating the variables.

Order and Degree of differential equations

Examples:

- $\frac{d^2y}{dx^2} + 3y = 6$: Order-2 and Degree-1
- $\left(\frac{d^2y}{dx^2}\right)^3 + \frac{3dy}{dx} = 5$: Order-2 and Degree-3

NOTE: Order is the highest order derivative the equation has and the degree is the power of highest order.

Homogeneous Differential Equation

A differential equation of the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ is called homogeneous differential equation.

NOTE:

To solve the homogeneous differential equation, we need to suppose $y = vx$

Then,

$$\frac{dy}{dx} = v + x.\frac{dv}{dx}$$

By substituting the values of y and $\frac{dy}{dx}$, in the given equation, we will get the differential equation of variable separation form in terms of v and x which can be solved easily by the help of variable separation method. Don't forget at last to replace the value of v in terms of y and x .

$i.e. v = \frac{y}{x}$

Differential Equation of Exact Form

A differential equation of the form $p(x, y) + Q(x, y)dy$ is called exact differential equation if their exist $f(x, y)$ such that, $Pdx + Qdy = df$

Linear Differential Equation

A differential equation of the form $\frac{dy}{dx} + Py = Q$ where, P and Q are functions of x alone or constant is called linear differential equation.

NOTE:

To solve the linear differential equation,

$$\frac{dy}{dx} + Py = Q \dots\dots\dots (i)$$

First, we need to find the integrating factor (I.F) which is,

$$I.F = e^{\int Pdx}$$

Then, the given equation (i) is multiplied by Integrating Factor

Then,

Left hand side of the given equation becomes perfect differential.

$$i.e. \frac{dy}{dx} + Py.IF = Q.IF$$

$$or, \frac{d}{dx}(y.IF) = Q.IF$$

$$or, d(y.IF) = Q.IF\ dx$$

Integrating both sides,

$$or, \int d(y.IF) = \int Q.IF\ dx$$

$$\therefore y.IF = \int Q.IF\ dx$$

UNIT 7 : COMPUTATIONAL METHOS

17) Gaussian Elimination

Solution of simultaneous equation of three variables by using Gauss Elimination Method

Given equations are:

$a_1x + b_1y + c_1z = d_1$ ………. (i)

$a_2x + b_2y + c_2z = d_2$ ………. (ii)

$a_3x + b_3y + c_3z = d_3$ ………. (iii)

Converting above equations in the form of,

$$ax + by + cz = d \dots\dots..(1)$$

$$py + qz = r \dots\dots\dots(2)$$

$$wz = e \dots\dots..(3)$$

From equation (3), we get the value of z.

From equation (2), we get the value of y by substituting the value of z.

From equation (1), we get the value of x by substituting the value of y and z.

NOTE:

While solving system of linear equation using Gaussian Elimination method if we get,

0 = Any other number except ‘0’

[Hence, it is impossible]

So, the equation has no solution.

For, 0 = 0 case, it has infinite many solution.

<u>Let, $x = k$</u>

Where, $k \in R$

Then, put <u>$x = k$</u> in equation and solve…..

To check whether the given equations are diagonally dominant or not

Let the given equations be:

$a_1x + b_1y + c_1z = d_1$ ………. (i)

$a_2x + b_2y + c_2z = d_2$ ………. (ii)

$a_3x + b_3y + c_3z = d_3$ ………. (iii)

The system of equation is called diagonally dominant if,

$|a_1| > |b_1| + |c_1|$

$|b_2| > |a_2| + |c_2|$

$|c_3| > |a_3| + |b_3|$

If absolute value of the diagonal coefficient is greater than the sum of the absolute value of other coefficients, then the system of equations are called diagonally dominant. Otherwise, the system of equations are not diagonally dominant.

NOTE:

To solve the system of linear equation by using Gauss-Seidel method, the system of equation must be diagonally dominant. If the system of equation is not diagonally dominant, we should first rearrange them in the form of diagonally dominant then only we can solve.

To solve system of linear equation by Gauss-Siedel method

Let the given equations be:

$a_1x + b_1y + c_1z = d_1$ ………. (i)

$a_2x + b_2y + c_2z = d_2$ ………. (ii)

$a_3x + b_3y + c_3z = d_3$ ………. (iii)

From equation (i), $x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$…… (1)

From equation (ii), $y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$…… (2)

From equation (iii), $z = \frac{1}{c_3}(d_3 - a_3x - b_3y)$…… (3)

Initially, $(x, y, z) = (0,0,0)$

From equation (1), $x = \frac{d_1}{a_1}$

From equation (2), $y = \frac{1}{b_2}\left(d_2 - \frac{a_2d_1}{a_1}\right)$

Again, put the value of x and y in equation (3) to find z.

[The Iteration is continued until two iterations results are similar]

17) Linear Programming Problems

Simplex Method

It is a special method of finding maximum value of a given linear programming problem (LPP).

Example of LPP:

Maximize, $z = 30x + 20y$

Constraints $x + 3y \leq 2$

$2x - y \leq 5$

$x, y \geq 0$

Non-Negative Slacks Variables

A positive variable in the linear programming problem (LPP) is called non-negative slacks variables which converts the given inequality into equation.

In above example, r and s be the non-negative slacks variables then the above inequality converts into equation which is,

$x + 3y + r = 2$

$2x - y + s = 5$

Now, we have the following equation in standard form,

$x + 3y + r + 0.s + 0.z = 2$

$2x - y + 0.r + s + 0.z = 5$

$-30x - 20y + 0.r + 0.s + z = 0$

Then, using simplex method we can solve it.

"MOTIVATE YOURSELF"