Extreme Learning Machine: Towards Tuning-Free Learning

- A Unified Learning Technique for Regression and Multiclass Classification

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Nanyang Technological University, Singapore

Outline

1. Feedforward Neural Networks
   - Single-Hidden Layer Feedforward Networks (SLFNs)
   - Function Approximation of SLFNs
   - Classification Capability of SLFNs
   - Conventional Learning Algorithms of SLFNs
2. Extreme Learning Machine
   - Generalized SLFNs
   - New Learning Theory: Learning Without Iterative Tuning
   - ELM Algorithm
3. ELM, SVM and LS-SVM
4. Online Sequential ELM

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Feedforward Neural Networks with Additive Nodes

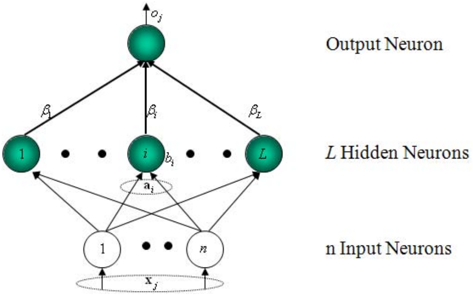

Figure 1: SLFN: additive hidden nodes

Output of hidden nodes

$$G(\mathbf{a}_i, b_i, \mathbf{x}) = g(\mathbf{a}_i \cdot \mathbf{x} + b_i) \qquad (1)$$

$\mathbf{a}_i$: the weight vector connecting the ith hidden node and the input nodes.
b_i: the threshold of the ith hidden node.

Output of SLFNs

$$f_L(\mathbf{x}) = \sum_{i=1}^{L} \beta_i G(\mathbf{a}_i, b_i, \mathbf{x}) \qquad (2)$$

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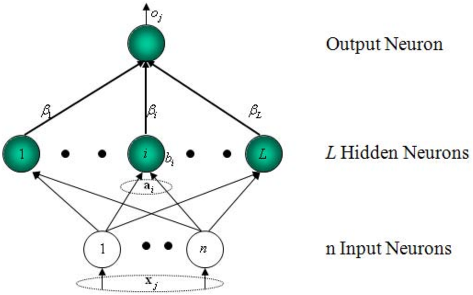

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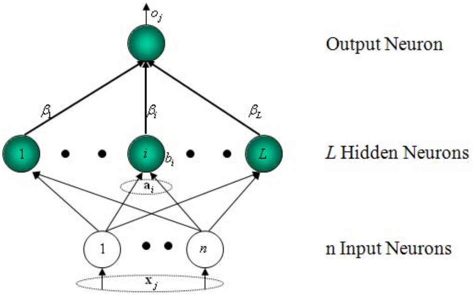

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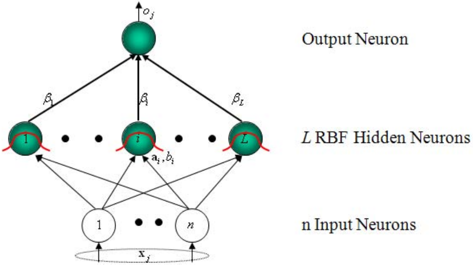

Figure 2: Feedforward Network Architecture: RBF hidden nodes

Output of hidden nodes

$$G(\mathbf{a}_i, b_i, \mathbf{x}) = g\left(b_i \|\mathbf{x} - \mathbf{a}_i\|\right) \qquad (3)$$

$\mathbf{a}_i$: the center of the ith hidden node.
b_i: the impact factor of the ith hidden node.

Output of SLFNs

$$f_L(\mathbf{x}) = \sum_{i=1}^{L} \beta_i G(\mathbf{a}_i, b_i, \mathbf{x}) \qquad (4)$$

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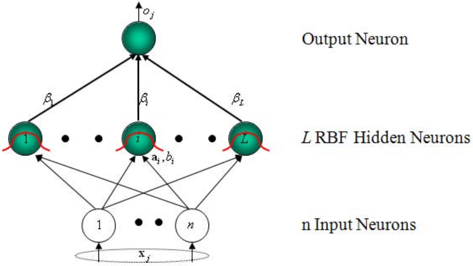

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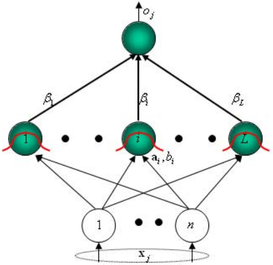

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Function Approximation of Neural Networks

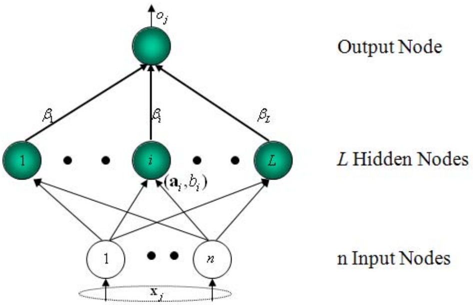


Figure 3: SLFN.

Mathematical Model

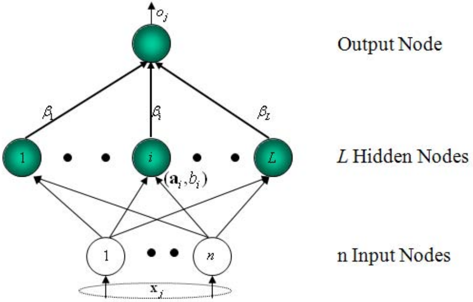
Any continuous target function $f(\mathbf{x})$ can be approximated by SLFNs with adjustable hidden nodes. In other words, given any small positive value ϵ, for SLFNs with enough number of hidden nodes (L) we have

$$\|f_L(\mathbf{x}) - f(\mathbf{x})\| < \epsilon \tag{5}$$

Learning Issue

In real applications, target function f is usually unknown. One wishes that unknown f could be approximated by SLFNs f_L appropriately.

Function Approximation of Neural Networks



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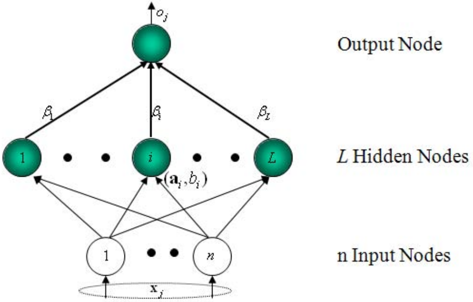

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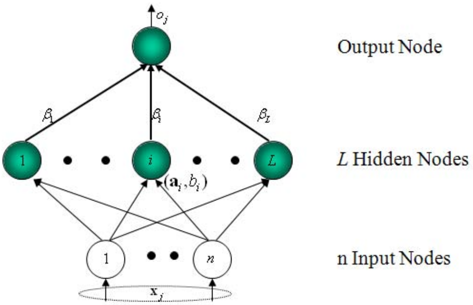

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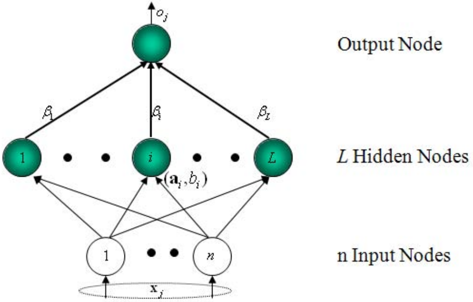

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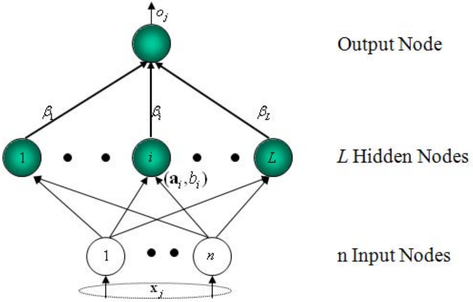
Learning Model

- For N arbitrary distinct samples $(\mathbf{x}_i, \mathbf{t}_i) \in \mathbf{R}^n \times \mathbf{R}^m$, SLFNs with L hidden nodes and activation function $g(x)$ are mathematically modeled as

$$f_L(\mathbf{x}_j) = \mathbf{o}_j, \forall j \tag{6}$$

- Cost function: $E = \sum_{j=1}^{N} \left\| \mathbf{o}_j - \mathbf{t}_j \right\|_2$.
- The target is to minimize the cost function E by adjusting the network parameters: β_i, $\mathbf{a}_i$, b_i.

Function Approximation of Neural Networks



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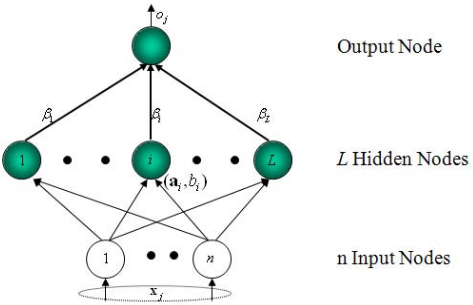

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Classification Capability of SLFNs

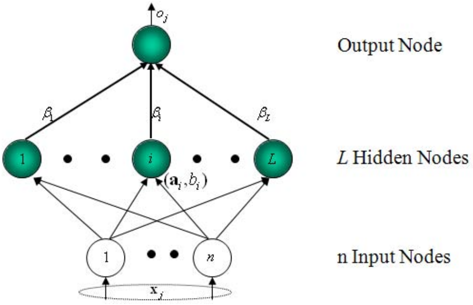

Figure 5: SLFN.

As long as SLFNs can approximate any continuous target function $f(\mathbf{x})$, such SLFNs can differentiate any disjoint regions.

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Learning Algorithms of Neural Networks

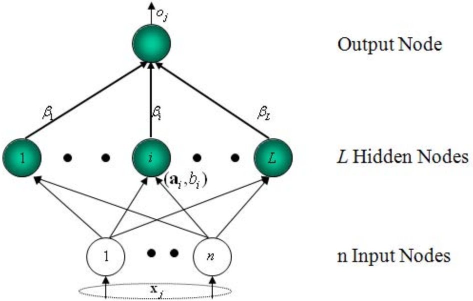

Figure 6: Feedforward Network Architecture.

Learning Methods

- Many learning methods mainly based on gradient-descent/iterative approaches have been developed over the past two decades.
- Back-Propagation (BP) and its variants are most popular.
- Least-square (LS) solution for RBF network, with single impact factor for all hidden nodes.

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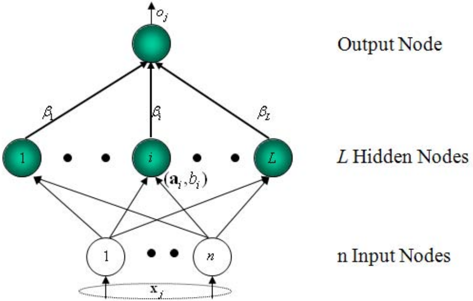

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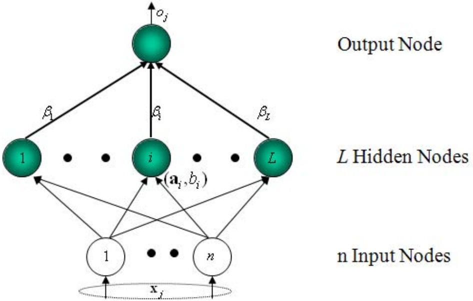

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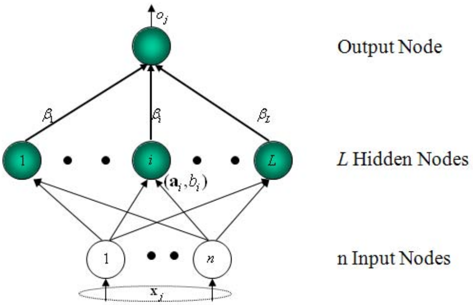

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Support Vector Machine

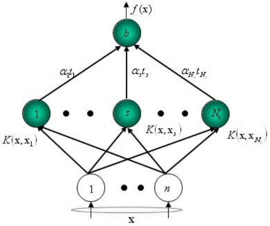

Figure 7: SVM Architecture.

SVM optimization formula:

$$
\begin{aligned}
\text{Minimize: } & L_P = \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{N}\xi_i \\
\text{Subject to: } & t_i(\mathbf{w}\cdot\phi(\mathbf{x}_i)+b) \geq 1-\xi_i, \forall i \\
& \xi_i \geq 0, \forall i
\end{aligned}
\qquad (7)
$$

The decision function of SVM is: $f(\mathbf{x}) = \text{sign}\left(\sum_{s=1}^{N_s}\alpha_s t_s K(\mathbf{x},\mathbf{x}_s)+b\right)$

Q. Liu, et al., "Extreme support vector machine classifier," *LNCS*, vol. 5012, pp. 222-233, 2008.

B. Frénay and M. Verleysen, "Using SVMs with randomised feature spaces: an extreme learning approach,"

ESANN, Bruges, Belgium, pp. 315-320, 28-30 April, 2010.

G.-B. Huang, et al., "Optimization method based extreme learning machine for classification," *Neurocomputing*, vol.

74, pp. 155-163, 2010.

Advantages and Disadvantages

Popularity

- Widely used in various applications: regression, classification, etc.

Limitations

- Usually different learning algorithms used in different SLFNs architectures.
- Some parameters have to be tuned manually.
- Overfitting.
- Local minima.
- Time-consuming.

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Generalized SLFNs

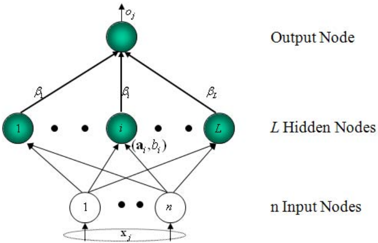

Figure 8: SLFN: any type of piecewise continuous $G(\mathbf{a}_i, b_i, \mathbf{x})$.

General Hidden Layer Mapping

- Output function of SLFNs:
  $f_L(\mathbf{x}) = \sum_{i=1}^{L} \beta_i G(\mathbf{a}_i, b_i, \mathbf{x})$
- The hidden layer output function (hidden layer mapping):
  $h(\mathbf{x}) = [G(\mathbf{a}_1, b_1, \mathbf{x}), \cdots, G(\mathbf{a}_L, b_L, \mathbf{x})]$
- The output functions of hidden nodes can be but are not limited to:
  Sigmoid: $G(\mathbf{a}_i, b_i, \mathbf{x}) = g(\mathbf{a}_i \cdot \mathbf{x} + b_i)$
  RBF: $G(\mathbf{a}_i, b_i, \mathbf{x}) = g\left(b_i \|\mathbf{x} - \mathbf{a}_i\|\right)$

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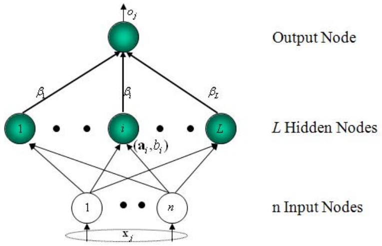

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New Learning Theory: Learning Without Iterative Tuning

New Learning View

- Learning Without Iterative Tuning: Given any nonconstant piecewise continuous function g, if continuous target function $f(\mathbf{x})$ can be approximated by SLFNs with adjustable hidden nodes g then the hidden node parameters of such SLFNs needn't be tuned.
- All these hidden node parameters can be randomly generated without the knowledge of the training data. That is, for any continuous target function f and any randomly generated sequence $\{(\mathbf{a}_i, b_i)_{i=1}^L\}$, $\lim_{L\to\infty} \|f(\mathbf{x}) - f_L(\mathbf{x})\| = \lim_{L\to\infty} \|f(\mathbf{x}) - \sum_{i=1}^L \beta_i G(\mathbf{a}_i, b_i, \mathbf{x})\| = 0$ holds with probability one if β_i is chosen to minimize $\|f(\mathbf{x}) - f_L(\mathbf{x})\|, \forall i$.

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Unified Learning Platform

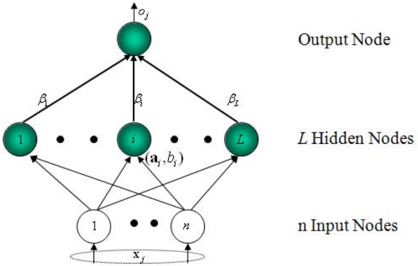

Figure 9: Generalized SLFN: any type of piecewise continuous $G(\mathbf{a}_i, b_i, \mathbf{x})$.

Mathematical Model

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$$\sum_{i=1}^{L} \beta_i G(\mathbf{a}_i, b_i, \mathbf{x}_j) = \mathbf{t}_j, \quad j = 1, \cdots, N \qquad (8)$$

- $(\mathbf{a}_i, b_i)$: hidden node parameters.
- β_i: the weight vector connecting the ith hidden node and the output node.

Unified Learning Platform

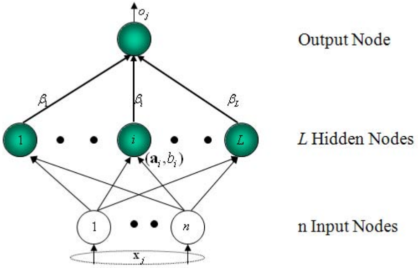

Figure 9: Generalized SLFN: any type of piecewise continuous $G(\mathbf{a}_i, b_i, \mathbf{x})$.

Mathematical Model

- For N arbitrary distinct samples $(\mathbf{x}_i, \mathbf{t}_i) \in \mathbf{R}^n \times \mathbf{R}^m$, SLFNs with L hidden nodes each with output function $G(\mathbf{a}_i, b_i, \mathbf{x})$ are mathematically modeled as

$$\sum_{i=1}^{L} \beta_i G(\mathbf{a}_i, b_i, \mathbf{x}_j) = \mathbf{t}_j, \quad j = 1, \cdots, N \qquad (8)$$

- $(\mathbf{a}_i, b_i)$: hidden node parameters.
- β_i: the weight vector connecting the ith hidden node and the output node.

Extreme Learning Machine (ELM)

Mathematical Model

- $\sum_{i=1}^{L} \beta_i G(\mathbf{a}_i, b_i, \mathbf{x}_j) = \mathbf{t}_j,\ j = 1, \cdots, N$, is equivalent to $\mathbf{H}\beta = \mathbf{T}$, where

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(\mathbf{x}_1) \\ \vdots \\ \mathbf{h}(\mathbf{x}_N) \end{bmatrix} = \begin{bmatrix} G(\mathbf{a}_1, b_1, \mathbf{x}_1) & \cdots & G(\mathbf{a}_L, b_L, \mathbf{x}_1) \\ \vdots & \cdots & \vdots \\ G(\mathbf{a}_1, b_1, \mathbf{x}_N) & \cdots & G(\mathbf{a}_L, b_L, \mathbf{x}_N) \end{bmatrix}_{N \times L} \tag{9}$$

$$\beta = \begin{bmatrix} \beta_1^T \\ \vdots \\ \beta_L^T \end{bmatrix}_{L \times m} \quad \text{and} \quad \mathbf{T} = \begin{bmatrix} \mathbf{t}_1^T \\ \vdots \\ \mathbf{t}_N^T \end{bmatrix}_{N \times m} \tag{10}$$

$\mathbf{H}$ is called the hidden layer output matrix of the neural network; the ith column of $\mathbf{H}$ is the output of the ith hidden node with respect to inputs $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N$.

Outline

Extreme Learning Machine (ELM)

Three-Step Learning Model

Given a training set $\aleph = \{(\mathbf{x}_i, \mathbf{t}_i) | \mathbf{x}_i \in \mathbf{R}^n, \mathbf{t}_i \in \mathbf{R}^m, i = 1, \cdots, N\}$, hidden node output function $G(\mathbf{a}, b, \mathbf{x})$, and the number of hidden nodes L,

1. Assign randomly hidden node parameters $(\mathbf{a}_i, b_i)$, $i = 1, \cdots, L$.
2. Calculate the hidden layer output matrix $\mathbf{H}$.
3. Calculate the output weight β: $\beta = \mathbf{H}^\dagger \mathbf{T}$.

where $\mathbf{H}^\dagger$ is the Moore-Penrose generalized inverse of hidden layer output matrix $\mathbf{H}$.

Source Codes of ELM

http://www.extreme-learning-machines.org

Extreme Learning Machine (ELM)

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ELM Learning Algorithm

Salient Features

- "Simple Math is Enough." ELM is a simple tuning-free three-step algorithm.
- The learning speed of ELM is extremely fast.
- The hidden node parameters $\mathbf{a}_i$ and b_i are not only independent of the training data but also of each other.
- Unlike conventional learning methods which MUST see the training data before generating the hidden node parameters, ELM could generate the hidden node parameters before seeing the training data.
- Unlike traditional gradient-based learning algorithms which only work for differentiable activation functions, ELM works for all bounded nonconstant piecewise continuous activation functions.
- Unlike traditional gradient-based learning algorithms facing several issues like local minima, improper learning rate and overfitting, etc, ELM tends to reach the solutions straightforward without such trivial issues.
- The ELM learning algorithm looks much simpler than many learning algorithms: neural networks and support vector machines.

G.-B. Huang, et al., "Can threshold networks be trained directly?" *IEEE Transactions on Circuits and Systems II*, vol. 53, no. 3, pp. 187-191, 2006.
M.-B. Li, et al., "Fully complex extreme learning machine" *Neurocomputing*, vol. 68, pp. 306-314, 2005.

Output Functions of Generalized SLFNs

Ridge regression theory based ELM

$$\mathbf{f}(\mathbf{x}) = \mathbf{h}(\mathbf{x})\beta = \mathbf{h}(\mathbf{x})\mathbf{H}^T\left(\mathbf{H}\mathbf{H}^T\right)^{-1}\mathbf{T} \Longrightarrow \mathbf{h}(\mathbf{x})\mathbf{H}^T\left(\frac{\mathbf{I}}{C} + \mathbf{H}\mathbf{H}^T\right)^{-1}\mathbf{T}$$

and

$$\mathbf{f}(\mathbf{x}) = \mathbf{h}(\mathbf{x})\beta = \mathbf{h}(\mathbf{x})\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{T} \Longrightarrow \mathbf{h}(\mathbf{x})\left(\frac{\mathbf{I}}{C} + \mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{T}$$

Ridge Regression Theory

A positive value $\frac{\mathbf{I}}{C}$ can be added to the diagonal of $\mathbf{H}^T\mathbf{H}$ or $\mathbf{H}\mathbf{H}^T$ of the Moore-Penrose generalized inverse $\mathbf{H}$ the resultant solution is stabler and tends to have better generalization performance.

A. E. Hoerl and R. W. Kennard, "Ridge regression: Biased estimation for nonorthogonal problems", *Technometrics*, vol. 12, no. 1, pp. 55-67, 1970.

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Output functions of Generalized SLFNs

Valid for both kernel and non-kernel learning

1. Non-kernel based:

$$\mathbf{f}(\mathbf{x}) = \mathbf{h}(\mathbf{x})\mathbf{H}^T \left(\frac{\mathbf{I}}{C} + \mathbf{H}\mathbf{H}^T \right)^{-1} \mathbf{T}$$

   and

$$\mathbf{f}(\mathbf{x}) = \mathbf{h}(\mathbf{x}) \left(\frac{\mathbf{I}}{C} + \mathbf{H}^T\mathbf{H} \right)^{-1} \mathbf{H}^T\mathbf{T}$$

2. Kernel based: (if $\mathbf{h}(\mathbf{x})$ is unknown) $\mathbf{f}(\mathbf{x}) = \begin{bmatrix} K(\mathbf{x}, \mathbf{x}_1) \\ \vdots \\ K(\mathbf{x}, \mathbf{x}_N) \end{bmatrix}^T \left(\frac{\mathbf{I}}{C} + \Omega_{ELM} \right)^{-1} \mathbf{T}$

   where $\Omega_{ELM\,i,j} = \mathbf{h}(\mathbf{x}_i) \cdot \mathbf{h}(\mathbf{x}_j) = K(\mathbf{x}_i, \mathbf{x}_j)$

G.-B. Huang, et al., "Extreme learning machine for regression and multiclass classification", *IEEE Transactions on Systems, Man and Cybernetics - Part B*, vol. 42, no. 2, pp. 513-529, 2012.
K.-A. Toh, "Deterministic Neural Classification", *Neural Computation*, vol. 20, no. 6, pp. 1565-1595, 2008.
G.-B. Huang, et al., "Extreme Learning Machines: A Survey", *International Journal of Machine Leaning and Cybernetics*, pp. 107-122, vol. 2, no. 2, 2011.

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K.-A. Toh, "Deterministic Neural Classification", *Neural Computation*, vol. 20, no. 6, pp. 1565-1595, 2008.
G.-B. Huang, et al., "Extreme Learning Machines: A Survey", *International Journal of Machine Leaning and Cybernetics*, pp. 107-122, vol. 2, no. 2, 2011.

ELM Classification Boundaries

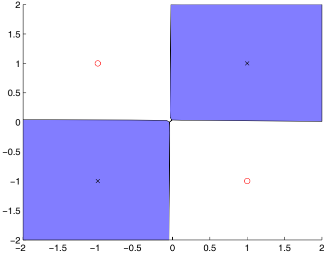


Figure 10: XOR Problem

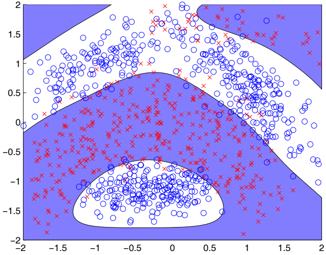


Figure 11: Banana Case

Performance Evaluation of ELM

Datasets	# train	# test	# features	# classes	Random Perm
Letter	13333	6667	16	26	Yes
Shuttle	43500	14500	9	7	No
USPS	7291	2007	256	10	No
MNIST	60,000	10,000	784	10	No

Table 1: Specification of multi-class classification problems

Performance Evaluation of ELM

Datasets	SVM (Gaussian Kernel)			LS-SVM (Gaussian Kernel)			ELM (Sigmoid hidden node)		
	Testing		Training	Testing		Training	Testing		Training
	Rate (%)	Dev (%)	Time (s)	Rate (%)	Dev (%)	Time (s)	Rate (%)	Dev (%)	Time (s)
Letter	92.87	0.26	302.9	93.12	0.27	335.838	93.51	0.15	0.7881
shuttle	*99.74*	*0*	*2864.0*	*99.82*	*0*	**24767.0**	*99.64*	*0.01*	**3.3379**
USPS	*96.51*	*0*	**80.4**	*96.76*	*0*	*59.1357*	*96.28*	*0.28*	**0.6877**

Datasets	SVM (Gaussian Kernel)			LS-SVM (Gaussian Kernel)			ELM (Gaussian Kernel)		
	Testing		Training	Testing		Training	Testing		Training
	Rate (%)	Dev (%)	Time (s)	Rate (%)	Dev (%)	Time (s)	Rate (%)	Dev (%)	Time (s)
Letter	92.87	0.26	302.9	93.12	0.27	335.838	**97.41**	0.13	41.89
shuttle	*99.74*	*0*	*2864.0*	*99.82*	*0*	**24767.0**	*99.91*	*0*	*4029.0*
USPS	*96.51*	*0*	**80.4**	*96.76*	*0*	*59.1357*	**98.9**	*0*	*9.2784*

Table 2: Performance comparison of SVM, LS-SVM and ELM: multi-class datasets.

G.-B. Huang, et al., "Extreme learning machine for regression and multiclass classification", *IEEE Transactions on Systems, Man and Cybernetics - Part B*, vol. 42, no. 2, pp. 513-529, 2012.

Handwritten Characters Recognition

Datasets	SVM (Gaussian Kernel)			Deep Learning			ELM (Gaussian Kernel)		
	Testing		Training	Testing		Training	Testing		Training
	Rate (%)	Dev (%)	Time (s)	Rate (%)	Dev (%)	Time (s)	Rate (%)	Dev (%)	Time (s)
MNIST	*98.6*[a]	-	-	*98.8*[a,b]	-	-	*98.78*[c]	-	-

[a] G. E. Hinton, Science, Vol. 313, July 2006; [b] Deep learning method.
[c] Courtesy to Li Deng, Microsoft Research, Redmond, USA, for running ELM in a computer with large memory.

Table 3: Performance comparison of SVM and ELM in MNIST OCR Applications.

Figure 12: Sample images in MNIST dataset (from www.zjucadcg.cn/dengcai/Data/MNIST/images.html)

Face Recognition

Methods	YALE	ORL
Standard eigenface	76	92.2
Waveletface	83.3	92.5
Curveletface	82.6	94.5
Waveletface + PCA	84	94.5
Waveletface + LDA	84.6	94.7
Waveletface + weighted modular PCA	83.6	95
Curveletface + LDA	83.5	95.6
Waveletface + KAM	84	96.6
Curveletface + PCA	83.9	96.6
Curveletface + PCA + LDA	92	97.7
Curveletface + B2DPCA + ELM	**99.7**	**99.9**

Table 4: Testing accuracy (%) of different methods for YALE and ORL face database.

A. A. Mohammed, et al., "Human face recognition based on multidimensional PCA and extreme learning machine," *Pattern Recognition*, vol. 44, pp. 2588-2597, 2011.

Figure 13: Face samples from YALE.

Face Recognition

Components	Number of hidden nodes						
	35	40	45	50	55	60	Dev
5	92.56	92.92	92.97	92.95	92.62	92.55	0.2047
10	99.80	99.78	99.93	99.77	99.81	99.75	0.0641
15	99.01	99.07	99.01	98.98	99.04	98.94	0.0454
20	99.97	99.95	99.96	99.97	99.89	99.95	0.0299
25	100	100	100	100	100	100	0

Table 5: Average recognition rates(%) for JAFFE database at varying number of hidden nodes: random Sigmoid hidden nodes

Automatic Object Recognition

Dataset	ELM Based	AdaBoost Based	Joint Boosting	Scale-Invariant Learning
Bikes	94.6	93.4	92.5	73.9
Planes	95.3	90.0	90.2	92.7
Cars	99.0	96.0	90.3	97.0
Leaves	98.3	94.2	-	97.8
Faces	97.9	98.0	96.4	-

Table 6: Accuracy comparison (%) of different approaches

R. Minhas, et al., "A fast recognition framework based on extreme learning machine using hybrid object information," *Neurocomputing*, vol. 73, pp. 1831-1839, 2010.

Figure 14: Sample images from CalTech database.

Leukocyte Image Segmentation

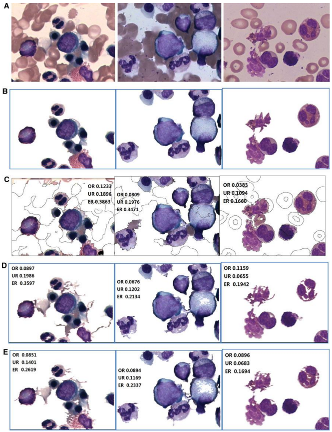


Figure 15: Leukocyte image segmentation

Some examples:

a. Original Leukocyte images from bone marrow smears.
b. Manual segmentation results as ground truth.
c. Segmentation results based on marker-controlled watershed.
d. Segmentation results based on SVM.
e. Segmentation results based on ELM

C. Pan, et al., "Leukocyte image segmentation by visual attention and extreme learning machine," *Neural Computing and Applications*, 2011.

Image Super-Resolution by ELM

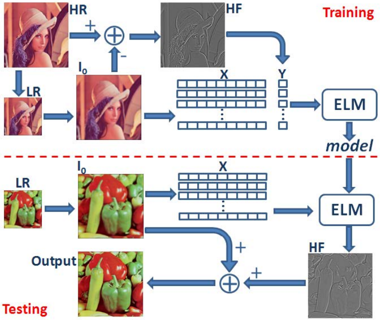


Figure 16: The system diagram of the proposed super-resolution algorithm.

L. An and B. Bhanu, "Image super-resolution by extreme learning machine," *2012 IEEE International Conference on Image Processing*, September 30 - October 3, 2012, Orlando, Florida, USA

Image Super-Resolution by ELM

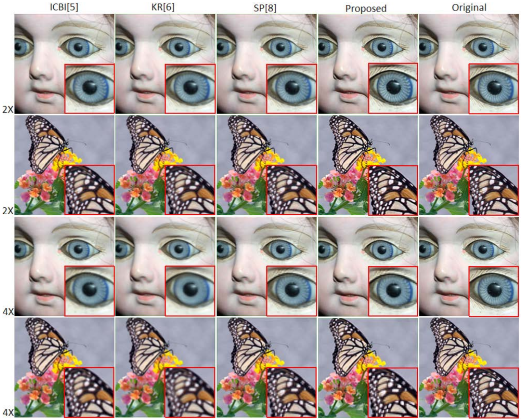
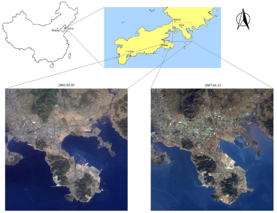


Figure 17: From top to down: super-resolution at 2x and 4x. State-of-the-art methods: iterative curve based interpolation (ICBI), kernel regression based method (KR), compressive sensing based sparse representation method (SR).

Real-Time Remote Satellite Sensing



(a) 2003 images (ba) 2007 images

Figure 18: Location of the Dalian Development Area (DDA), China, and the corresponding SPOT-5 (Satellite for earth observation-5) images from 2003 and 2007.

N.-B. Chang, "Satellite-based multitemporal-change detection in urban environments,"

http://spie.org/x44379.xml?pf=true&ArticleID=x44379, Feb 2011.

Challenging Problems

- Improving land management depends critically on the capacity of (near-)real-time monitoring of land-use/land cover (LULC) change.
- From multitemporal to rapid-change detection, both the resolution of satellite sensors and the computational capacity of the classifiers used for image processing must be well integrated.
- Early remote-sensing image-classification studies employed statistical methods, such as the maximum-likelihood classifier, KNN, and the K-means clustering approach.
- In recent years, methods based on artificial-intelligence and machine-learning techniques have become popular. Approaches based on neural computing, fuzzy logic, evolutionary algorithms, and expert systems are widely used.
- Existing processing techniques using LULC methods are often time-consuming, laborious, and tedious to use, resulting in the unavailability of the results within the designated time window.

Real-Time Remote Satellite Sensing

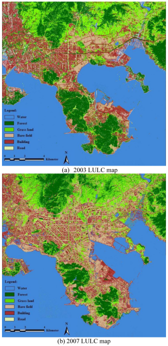


Figure 19: Final classification results produced from (a) the 2003 and (b) 2007 image sets using the PL-ELM.

ELM Solution

- Classifier: Partial Lanczos extreme-learning machine (PL-ELM, Tang and Han, *Neurcomputing*, 2009).
- Texture features and vegetation indices were extracted.
- More features to the image pixels were added and the "normalized differential vegetation index," as well as four commonly used texture features (angular second moment, contrast, correlation, and homogeneity) were introduced. The feature-space dimension for all data points/pixels were expanded to eight.
- The LULC features into six major categories, including water bodies, forests, grasslands, bare fields, buildings, and roads.
- PL-ELM classification approach outperforms five other major algorithms, including the BP, maximum-likelihood, KNN and naive Bayes' algorithms, as well as SVM.
- This case study in the DDA based on images collected in 2003 and 2007 fully supports the monitoring needs and aids in rapid-change detection in terms of both urban expansion and coastal-land reclamation.
- (Near) real-time remotely sensed information can be employed to speed the decision making process for problem resolution.
- ELM based real-time remote sensing can contribute to improved coastal and land management, hazard mitigation, emergency response, and ecosystem-service design.

EEG Based Epileptic Seizure Detection

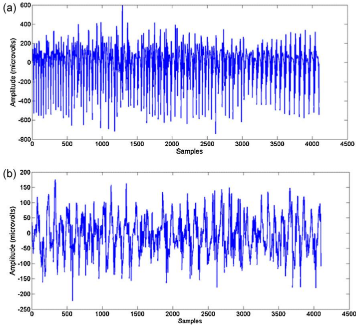


Figure 20: Sample EEG recordings. (a) Ictal EEG (Set S) (b) Interictal EEG (Set F).

Y. Song, et al., "Automatic epileptic seizure detection in EEGs based on optimized sample entropy and extreme learning machine," *Journal of Neuroscience Methods*, vol. 210, pp. 132-146, 2012.

EEG Based Epileptic Seizure Detection

Figure 21: Intracranial electrode placements.

Y. Song, et al., "Automatic epileptic seizure detection in EEGs based on optimized sample entropy and extreme learning machine," *Journal of Neuroscience Methods*, vol. 210, pp. 132-146, 2012.

EEG Based Epileptic Seizure Detection

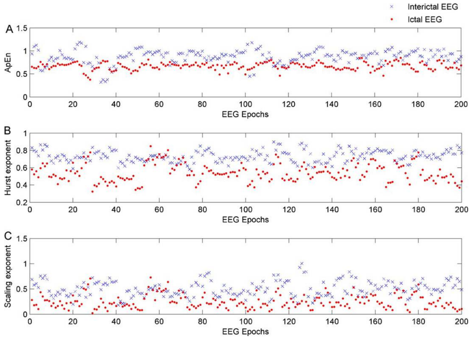


Figure 22: Nonlinear features extracted from EEG signals (approximate entropy (ApEn), Hurst exponent and scaling exponent obtained with detrended fluctuation analysis (DFA)) are employed to characterize interictal and ictal EEGs.

Q. Yuan, et al., "Epileptic EEG classification based on extreme learning machine and nonlinear feature," *Epilepsy Research*, vol. 96, pp. 29-38, 2011.

Epileptic EEG Classification

Classifiers	Sensitivity (%)	Specificity (%)	Accuracy (%)	Train Time (s)	Test Time (s)
ELM	92.50 $\pm$ 2.00	96.00 $\pm$ 2.50	96.00 $\pm$ 0.50	**0.0803**	**0.0135**
BP	91.50 $\pm$ 3.00	94.00 $\pm$ 3.50	95.50 $\pm$ 0.50	1.6363	0.0256
SVM	95.00 $\pm$ 2.00	93.75 $\pm$ 0.25	95.25 $\pm$ 0.25	12.6410	3.6406

Table 7: Nonlinear features extracted from EEG signals (approximate entropy (ApEn), Hurst exponent and scaling exponent obtained with detrended fluctuation analysis (DFA)) are employed to characterize interictal and ictal EEGs. United features of ApEn, Hurst exponent and scaling exponent were used in this work.

Q. Yuan, et al., "Epileptic EEG classification based on extreme learning machine and nonlinear feature," *Epilepsy Research*, vol. 96, pp. 29-38, 2011.

Y. Song, et al., "Automatic epileptic seizure detection in EEGs based on optimized sample entropy and extreme learning machine," *Journal of Neuroscience Methods*, vol. 210, pp. 132-146, 2012.

ELM Based Intrusion Detection

Table 8: 22 Attack Types

Denial of Service (DoS)	Unauthorized Access to Local Root Privileges	Unauthorized Access from a Remote Machine	Probing
Back Neptune Land Teardrop Ping of Death Smurf	Perl Buffer Overflow Load Module Rootkit	FTP Write Guess Password Imap Multihop Phf Spy Warezclient Warezmaster	IP Sweep Nmap Port Sweep Satan

The dataset is from the 1998 DAPRA intrusion detection program. During the evaluation program, an environment was set up in Lincoln Labs to record 9 weeks of raw TCP/IP dump data for a network simulating a typical U.S. air force LAN. Then the LAN was operated under a real environment and blasted with multiple attacks. After that, 7 weeks of raw tcpdump data was processed into millions of connection records. Finally, 41 quantitative and qualitative features were extracted using data mining techniques.

C. Cheng, et al, "Intrusion detection using random features: an extreme learning machine approach," *Proceedings of International Joint Conference on Neural Networks (IJCNN2012)*, June 10 - June 15, 2012, Brisbane, Australia

ELM Based Intrusion Detection

Table 9: Basic Features of Individual TCP Connections

Feature Names	Description	Types
duration	length (number of seconds) of the connection	continuous
protocol_type	type of the protocol	discrete
service	network service on the destination	discrete
src_bytes	number of data bytes from source to destination	continuous
dst_bytes	number of data bytes from destination to source	continuous
flag	normal or error status of the connection	discrete
land	1 if connection is from/to the same host/port, 0 otherwise	discrete
wrong_fragment	number of “wrong” fragments	continuous
urgent	number of urgent packets	continuous

ELM Based Intrusion Detection

Table 10: Content Features Within a Connection Suggested by Domain Knowledge

Feature Names	Description	Types
hot	number of “hot” indicators	continuous
num_failed_logins	number of failed login attempts	continuous
logged_in	1 if successfully logged in, 0 otherwise	discrete
num_compromised	number of “compromised” conditions	continuous
root_shell	1 if root shell is obtained, 0 otherwise	discrete
su_attempted	1 if “su root” command attempted, 0 otherwise	discrete
num_root	number of “root” accesses	continuous
num_file_creations	number of file creation operations	continuous
num_shells	number of shell prompts	continuous
num_access_files	number of operations on access control files	continuous
num_outbound_cmds	number of outbound commands in an ftp session	continuous
is_hot_login	1 if the login belongs to the “hot” list, 0 otherwise	discrete
is_guest_login	1 if the login is a “guest”login, 0 otherwise	discrete

ELM Based Intrusion Detection

Table 11: Traffic Features Computed Using a Two-Second Time Window

Feature Names	Description	Types
count	number of connections to the same host as the current connection in the past two seconds	continuous
serror_rate	% of connections that have "SYN" errors	continuous
rerror_ rate	% of connections that have "REJ" errors	continuous
same_srv_rate	% of connections to the same service	continuous
diff_srv_rate	% of connections to different services	continuous
srv_count	number of connections to the same service as the current connection in the past two seconds	continuous
srv_serror_rate	% of connections that have "SYN" errors	continuous
srv_rerror_rate	% of connections that have "REJ" errors	continuous
srv_diff_host_rate	% of connections to different hosts	continuous

ELM Based Intrusion Detection

Table 12: Binary-Class Performance Comparison Results

Dataset Size	SVM		Basic ELM (Random Sigmoid Nodes)		ELM (Gaussian Kernel)	
Training/Testing	Rate (%)	95% Confidence Interval (%)	Rate (%)	95% Confidence Interval (%)	Rate (%)	95% Confidence Interval (%)
1000/1000	99.15	99.12 - 99.17	99.33	99.15 - 99.51	99.12	99.05 - 99.25
2000/2000	99.43	99.40 - 99.45	99.07	98.90 - 99.24	99.27	99.25 - 98.28
4000/4000	99.77	99.76 - 98.78	99.58	99.50 - 99.66	99.63	99.61 - 99.65

ELM Based Intrusion Detection

Table 13: Multi-Class Performance Comparison Results

Dataset Size	SVM		Basic ELM (Random Sigmoid Nodes)		ELM (Gaussian Kernel)	
Training/Testing	Rate (%)	95% Confidence Interval (%)	Rate (%)	95% Confidence Interval (%)	Rate (%)	95% Confidence Interval (%)
1000/1000	97.58	97.52 - 97.64	96.83	96.40 - 97.23	97.78	97.72 - 97.83
2000/2000	98.31	98.27 - 98.34	97.07	96.77 - 97.37	98.81	98.76 - 98.86
4000/4000	98.69	98.66 - 98.72	97.00	96.68 - 97.32	98.74	98.70 - 98.78

Real-World Very Large Complex Applications

Algorithms	Time (minutes)		Success Rate (%)				# SVs/ nodes
	Training	Testing	Training		Testing		
			Rate	Dev	Rate	Dev	
ELM	1.6148	0.7195	92.35	0.026	90.21	0.024	200
SLFN	12	N/A	82.44	N/A	81.85	N/A	100
SVM	693.6000	347.7833	91.70	N/A	89.90	N/A	31,806

Table 14: Basic ELM: Performance comparison of the ELM, BP and SVM learning algorithms in Forest Type Prediction application. (100, 000 training data and 480,000+ testing data, each data has 53 attributes.)

G.-B. Huang, et al., "Extreme learning machine: theory and applications," *Neurocomputing*, vol. 70, pp. 489-501, 2006.

Artificial Case: Approximation of ‘SinC’ Function

Algorithms	Training Time (seconds)	Training		Testing		# SVs/ nodes
		RMS	Dev	RMS	Dev	
ELM	0.125	0.1148	0.0037	0.0097	0.0028	20
BP	21.26	0.1196	0.0042	0.0159	0.0041	20
SVR	1273.4	0.1149	0.0007	0.0130	0.0012	2499.9

Table 15: Basic ELM: Performance comparison for learning function: SinC (5000 noisy training data and 5000 noise-free testing data) .

Essence of ELM

Key expectations

1. Hidden layer need not be tuned.
2. Hidden layer mapping $\mathbf{h}(\mathbf{x})$ satisfies universal approximation condition.
3. Minimize: $\|\mathbf{H}\beta - \mathbf{T}\|$ and $\|\beta\|$

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Differences between ELM and LS-SVM

ELM (unified for regression, binary/multi-class cases)

1. Non-kernel based:

$$\mathbf{f}(\mathbf{x}) = \mathbf{h}(\mathbf{x})\mathbf{H}^T \left(\frac{\mathbf{I}}{C} + \mathbf{H}\mathbf{H}^T \right)^{-1} \mathbf{T}$$

and

$$\mathbf{f}(\mathbf{x}) = \mathbf{h}(\mathbf{x}) \left(\frac{\mathbf{I}}{C} + \mathbf{H}^T\mathbf{H} \right)^{-1} \mathbf{H}^T\mathbf{T}$$

2. Kernel based: (if $\mathbf{h}(\mathbf{x})$ is unknown)

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} K(\mathbf{x}, \mathbf{x}_1) \\ \vdots \\ K(\mathbf{x}, \mathbf{x}_N) \end{bmatrix}^T \left(\frac{\mathbf{I}}{C} + \mathbf{\Omega}_{ELM} \right)^{-1} \mathbf{T}$$

where $\Omega_{ELM\,i,j} = \mathbf{h}(\mathbf{x}_i) \cdot \mathbf{h}(\mathbf{x}_j) = K(\mathbf{x}_i,\mathbf{x}_j)$

LS-SVM (for binary class case)

$$\begin{bmatrix} \mathbf{0} & \mathbf{T}^T \\ \mathbf{T} & \frac{\mathbf{I}}{C} + \mathbf{\Omega}_{LS-SVM} \end{bmatrix} \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{T}^T \\ \mathbf{T} & \frac{\mathbf{I}}{C} + \mathbf{Z}\mathbf{Z}^T \end{bmatrix} \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \vec{\mathbf{1}} \end{bmatrix}$$

where

$$\mathbf{Z} = \begin{bmatrix} t_1 \phi(\mathbf{x}_1) \\ \vdots \\ t_N \phi(\mathbf{x}_N) \end{bmatrix}$$

$$\mathbf{\Omega}_{LS-SVM} = \mathbf{Z}\mathbf{Z}^T$$

Scalability: ELM vs LS-SVM

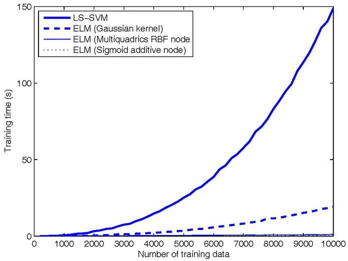

Figure 23: Scalability of different classifiers: Letter dataset.

G.-B. Huang, et al., "Extreme learning machine for regression and multiclass classification", *IEEE Transactions on Systems, Man and Cybernetics - Part B*, vol. 42, no. 2, pp. 513-529, 2012.

Optimization Constraints of ELM and LS-SVM

ELM: Based on Equality Constraint Conditions

ELM optimization formula:

$$\text{Minimize: } L_{P_{ELM}} = \frac{1}{2}\|\beta\|^2 + C\frac{1}{2}\sum_{i=1}^{N}\|\xi_i\|^2 \qquad (11)$$
$$\text{Subject to: } \mathbf{h}(\mathbf{x}_i)\beta = \mathbf{t}_i^T - \xi_i^T, \forall i$$

The corresponding dual optimization problem:

$$\text{minimize: } L_{D_{ELM}} = \frac{1}{2}\|\beta\|^2 + C\frac{1}{2}\sum_{i=1}^{N}\|\xi_i\|^2 - \sum_{i=1}^{N}\sum_{j=1}^{m}\left(\mathbf{h}(\mathbf{x}_i)\beta - \mathbf{t}_i^T + \xi_i^T\right)\alpha_i$$
$$\text{subject to: } \beta = \mathbf{H}^T\alpha, \alpha_i = C\xi_i, \mathbf{h}(\mathbf{x}_i)\beta - \mathbf{t}_i^T + \xi_i^T = 0, \forall i$$

G.-B. Huang, et al., "Extreme learning machine for regression and multiclass classification", *IEEE Transactions on Systems, Man and Cybernetics - Part B*, vol. 42, no. 2, pp. 513-529, 2012.

Optimization Constraints of ELM and LS-SVM

LS-SVM: Based on Equality Constraint Conditions

LS-SVM optimization formula:

$$\text{minimize: } L_{P_{LS-SVM}} = \frac{1}{2}\mathbf{w}\cdot\mathbf{w} + C\frac{1}{2}\sum_{i=1}^{N}\xi_i^2 \tag{12}$$

$$\text{subject to: } t_i(\mathbf{w}\cdot\phi(\mathbf{x}_i)+b) = 1-\xi_i, \forall i$$

The corresponding dual optimization problem:

$$\text{minimize: } L_{D_{LS-SVM}} = \frac{1}{2}\mathbf{w}\cdot\mathbf{w} + C\frac{1}{2}\sum_{i=1}^{N}\xi_i^2 - \sum_{i=1}^{N}\alpha_i\left(t_i\left(\mathbf{w}\cdot\phi(\mathbf{x}_i)+b\right)-1+\xi_i\right)$$

$$\text{subject to: } \mathbf{w} = \sum_{i=1}^{N}\alpha_i t_i\phi(\mathbf{x}_i), \alpha_i = C\xi_i, t_i(\mathbf{w}\cdot\phi(\mathbf{x}_i)+b)-1+\xi_i = 0, \forall i$$

$$\sum_{i=1}^{N}\alpha_i t_i = 0$$

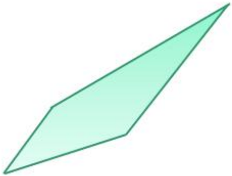

Figure 24: In LS-SVM optimal α_i are found from one hyperplane $\sum_{i=1}^{N}\alpha_i t_i = 0$.

Optimization Constraints of ELM and SVM

ELM variant: Based on Inequality Constraint Conditions

ELM optimization formula:

$$\text{Minimize: } L_P = \frac{1}{2}\|\beta\|^2 + C\sum_{i=1}^{N}\xi_i$$
$$\text{Subject to: } t_i\beta \cdot \mathbf{h}(\mathbf{x}_i) \geq 1 - \xi_i, \forall i \qquad (13)$$
$$\xi_i \geq 0, \forall i$$

The corresponding dual optimization problem:

$$\text{minimize: } L_D = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} t_i t_j \alpha_i \alpha_j \mathbf{h}(\mathbf{x}_i) \cdot \mathbf{h}(\mathbf{x}_j) - \sum_{i=1}^{N}\alpha_i \qquad (14)$$
$$\text{subject to: } 0 \leq \alpha_i \leq C, \forall i$$

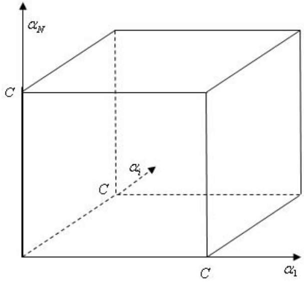

Figure 25: ELM

G.-B. Huang, et al., "Optimization method based extreme learning machine for classification," *Neurocomputing*, vol. 74, pp. 155-163, 2010.

Optimization Constraints of ELM and SVM

SVM Constraint Conditions

SVM optimization formula:

$$\text{Minimize: } L_P = \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{N}\xi_i$$
$$\text{Subject to: } t_i(\mathbf{w}\cdot\phi(\mathbf{x}_i)+b) \geq 1-\xi_i, \forall i$$
$$\xi_i \geq 0, \quad i=1,\cdots,N \tag{15}$$

The corresponding dual optimization problem:

$$\text{minimize: } L_D = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} t_i t_j \alpha_i \alpha_j \phi(\mathbf{x}_i)\cdot\phi(\mathbf{x}_j) - \sum_{i=1}^{N}\alpha_i$$
$$\text{subject to: } 0 \leq \alpha_i \leq C, \forall i \tag{16}$$
$$\sum_{i=1}^{N} t_i\alpha_i = 0$$

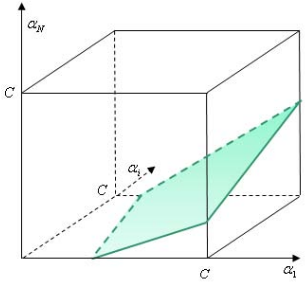

Figure 26: SVM

Optimization Constraints of ELM and SVM

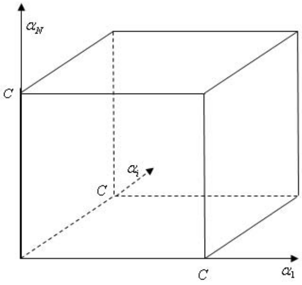

Figure 27: ELM

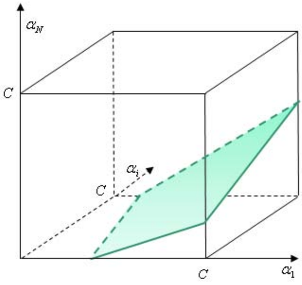

Figure 28: SVM

ELM and SVM have the same dual optimization objective functions, but in ELM optimal α_i are found from the entire cube $[0, C]^N$ while in SVM optimal α_i are found from one hyperplane $\sum_{i=1}^{N} t_i\alpha_i = 0$ within the cube $[0, C]^N$. SVM always provides a suboptimal solution, so does LS-SVM.

Flaws in SVM Theory?

Flaws?

1. SVM is great! Without SVM computational intelligence may not be so successful! Many applications and products may not be so successful either! However ...
2. SVM always searches for the optimal solution in the hyperplane $\sum_{i=1}^{N} \alpha_i t_i = 0$ within the cube $[0, C]^N$ of the SVM feature space.
3. SVMs may apply same application-oriented constraints to irrelevant applications. Given two training datasets $\{(\mathbf{x}_i^{(1)}, t_i^{(1)})\}_{i=1}^N$ and $\{(\mathbf{x}_i^{(2)}, t_i^{(2)})\}_{i=1}^N$ and $\{(\mathbf{x}_i^{(1)}\}_{i=1}^N$ and $\{(\mathbf{x}_i^{(2)}\}_{i=1}^N$ are totally irrelevant/independent, if $[t_1^{(1)}, \cdots, t_N^{(1)}]^T$ is similar or close to $[t_1^{(2)}, \cdots, t_N^{(2)}]^T$ SVM may have similar search areas of the cube $[0, C]^N$ for two different cases.

G.-B. Huang, et al., "Optimization method based extreme learning machine for classification," *Neurocomputing*, vol. 74, pp. 155-163, 2010.

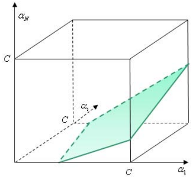

Figure 29: SVM

Reasons

SVM is too "generous" on the feature mappings and kernels, almost condition free except for Mercer's conditions.

1. As the feature mappings and kernels need not satisfy universal approximation condition, b must be present.
2. As b exists, contradictions are caused.
3. LS-SVM inherits such "generosity" from the conventional SVM.

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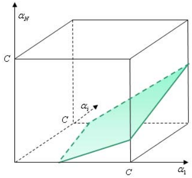

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Online Sequential ELM (OS-ELM) Algorithm

Learning Features

1. The training observations are *sequentially* (one-by-one or chunk-by-chunk with varying or fixed chunk length) presented to the learning algorithm.
2. At any time, only the *newly* arrived single or chunk of observations (instead of the entire past data) are seen and learned.
3. A single or a chunk of training observations is *discarded* as soon as the learning procedure for that particular (single or chunk of) observation(s) is completed.
4. The learning algorithm has *no prior* knowledge as to how many training observations will be presented.

N.-Y. Liang, et al., "A fast and accurate on-line sequential learning algorithm for feedforward networks", *IEEE Transactions on Neural Networks*, vol. 17, no. 6, pp. 1411-1423, 2006.

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OS-ELM Algorithm

Two-Step Learning Model

1. Initialization phase: where batch ELM is used to initialize the learning system.
2. Sequential learning phase: where recursive least square (RLS) method is adopted to update the learning system sequentially.

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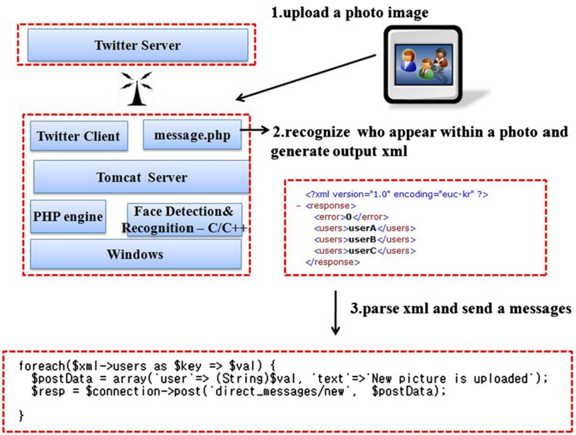
Intelligent Photo Notification System For Twitter Service



K. Choi, et al., "Incremental face recognition for large-scale social network services", *Pattern Recognition*, vol. 45, pp. 2868-2883, 2012.

Intelligent Photo Notification System For Twitter Service

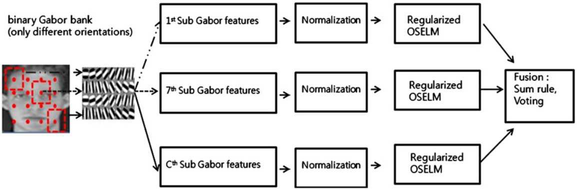


Figure 30: Binary Gabor filter-based OS-ELM (BG-OSELM)

Methods	Baseline		Sequential Subspace			Sequential Classifiers		
Database	PCA	FDA	CCIPCA	IPCA	ILDA	OSELM	BG-OSELM(S)	BG-OSELM(V)
AR	77.0	72.3	55.0	77.3	76.6	80.3	92.0	87.6
EYALE	99.7	96.9	58.5	99.7	100.0	100.0	99.7	99.7
BIOID	98.1	97.3	91.6	97.5	-	98.5	97.4	96.7
ETRI	95.8	95.5	86.9	95.4	-	97.2	97.0	94.2

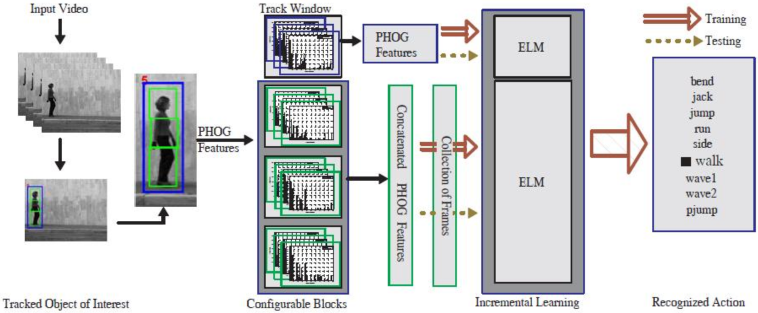
Table 16: Performance comparison of different sequential methods.

Online Sequential Human Action Recognition

Figure 31: Example frames from top row: Weizmann dataset, middle row: KTH dataset, and bottom row: UCF sports dataset

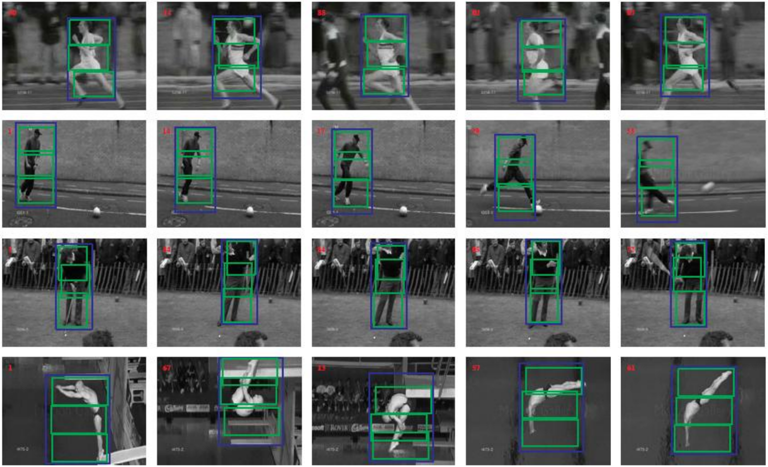
R. Minhas, et al., "Incremental learning in human action recognition based on *Snippets*", *(in press) IEEE Transactions on Circuits and Systems for Video Technology*, 2012.

Online Sequential Human Action Recognition



R. Minhas, et al., "Incremental learning in human action recognition based on *Snippets*", *(in press) IEEE Transactions on Circuits and Systems for Video Technology*, 2012.

Online Sequential Human Action Recognition



Weizmann dataset											
Methods	OS-ELM Based				[32]	[14]	[36]			[11]	
Frames	1/1	3/3	6/6	10/10	1/12	1/9	1/1	7/7	10/10	8/8	20/20
Accuracy	65.2	95.0	99.63	99.9	55.0	93.8	93.5	96.6	99.6	97.05	98.68
KTH dataset											
Methods	OS-ELM Based				[25]	[33]	[43]	[14]	[36]		[12]
Frames	1/1	3/3	6/6	10/10	-	-	-	-	1/1	7/7	20/20
Accuracy	74.4	88.5	92.5	94.4	91.3	90.3	83.9	91.7	88.0	90.9	90.84

Table 17: Classification comparison against different approaches at *snippet*-level.

Weizmann dataset											
Methods	OS-ELM Based				[2]	[32]	[14]	[36]	[41]	[30]	[11]
Frames	1/1	3/3	6/6	10/10	-	-	-	-	-	-	-
Accuracy	100.0	100.0	100.0	100.0	100.0	72.8	98.8	100.0	97.8	99.44	100.0
KTH dataset											
Methods	OS-ELM Based				[14]	[36]	[30]	[21]	[27]	[9]	[44]
Frames	1/1	3/3	6/6	10/10	-	-	-	-	-	-	-
Accuracy	92.8	93.5	95.7	96.1	91.7	92.7	94.83	95.77	97.0	96.7	95.7

Table 18: Classification comparison against different approaches at *sequence*-level.

Summary

- Both G. Hinton and V. Vapnik have made great contributions in neural networks R&D
  - Without Hinton's work on BP in 1982, neural networks might not have revived in 1980's.
  - Without Vapnik's work on SVM in 1995, neural networks might have disappeared although many SVM researchers do not consider SVM a kind of solutions to the traditional neural networks. Without SVM, many applications in pattern recognition, HCI, BCI, computational intelligence and machine learning, etc, may not have appeared.
  - However, both BP and SVM over-emphasize some aspects of learning and overlook the other aspects, and thus, both become incomplete in theory:
    - i) BP gives preference on training but does not consider the stability of the system (consistency of minimum norm of weights in neural networks and matrix theory)
    - ii) SVM confines the research in the maximum margin concept which limits the research in binary classification and does not have direct and efficient solutions to regression and multi-class applications. The consistency between maximum margin, minimum norm of weights in neural networks and matrix theory has been overlooked.
- From learning point of view, ELM theory seems more complete.

Summary

- For generalized SLFNs, learning can be done without iterative tuning.
- ELM is efficient for batch mode learning, sequential learning, incremental learning.
- ELM provides a unified learning model for regression, binary/multi-class classification.
- ELM works with different hidden nodes including random hidden nodes (random features) and kernels.

Summary

- Generally speaking, efficient for regression and classification applications. Existing applications:
  - Biometrics
  - Bioinformatics
  - Image processing (image segmentation, image quality assessment, image super-resolution)
  - Signal processing
  - Human action recognition
  - Disease prediction and eHealthCare
  - Location positioning system
  - Brain computer interface
  - Human computer interface
  - Feature selection
  - Time-series
  - Real-time learning and prediction
  - Security and data privacy
  - Big data analytics
  - Internet of Things
- Potential Influence:
  - Machine learning (resulting in second wave of machine learning and artificial intelligence with the increasing demand in handling big data in different applications?)
  - Matrix theory and optimization theory
  - Functioning artificial "brain" (coming out in 10 years?)
  - Robot and automation
  - Data and knowledge discovery
  - Cognitive and reasoning system

Open Problems

1. As observed in experimental studies, the performance of basic ELM is stable in a wide range of number of hidden nodes. Compared to the BP learning algorithm, the performance of basic ELM is not very sensitive to the number of hidden nodes. However, how to prove it in theory remains open.
2. One of the typical implementations of ELM is to use random nodes in the hidden layer and the hidden layer of SLFNs need not be tuned. It is interesting to see that the generalization performance of ELM turns out to be very stable. How to estimate the oscillation bound of the generalization performance of ELM remains open too.
3. It seems that ELM performs better than other conventional learning algorithms in applications with higher noise. How to prove it in theory is not clear.
4. ELM always has faster learning speed than LS-SVM if the same kernel is used?

Open Problems

5. ELM provides a batch learning kernel solution which is much simpler than other kernel learning algorithms such as LS-SVM. It is known that it may not be straightforward to have an efficient online sequential implementation of SVM and LS-SVM. However, due to the simplicity of ELM, is it possible to implement the online sequential variant of the kernel based ELM?
6. ELM always provides similar or better generalization performance than SVM and LS-SVM if the same kernel is used (if not affected by computing devices' precision)?
7. ELM tends to achieve better performance than SVM and LS-SVM in multiclasses applications, the higher the number of classes is, the larger the difference of their generalization performance will be?
8. Scalability of ELM with kernels in super large applications.
9. Parallel and distributed computing of ELM.
10. ELM will make real-time reasoning feasible?