## 3.2.B Functions on Quantum Computers

The notation  $f: \{0,1\}^n \to \{0,1\}^m$  means the function f takes in a bit string of length n and returns a bit string of length m.

- 1. Apply  $U_f$  to the state  $|\psi\rangle = \frac{1}{\sqrt{3}} \Big( |001\rangle|0\rangle + |010\rangle|0\rangle + |111\rangle|0\rangle \Big)$ , where the function  $f: \{0,1\}^3 \to \{0,1\}$ . Let the first register of qubits be the input to the function and second register be the output register.
- 2. Apply  $U_f$  to the state  $|\psi\rangle = \frac{1}{\sqrt{3}} \left( |001\rangle| \rangle + |010\rangle| \rangle + |111\rangle| \rangle \right)$ , where the function  $f: \{0,1\}^3 \to \{0,1\}$ . Let the first register of qubits be the input to the function and second register be the output register. Recall the phase oracle formula:  $U_f|x\rangle|-\rangle = (-1)^{f(x)}|x\rangle|-\rangle$

## Answers

$$1. \ U_f|\psi\rangle = U_f \frac{1}{\sqrt{3}} \bigg( |001\rangle|0\rangle + |010\rangle|0\rangle + |111\rangle|0\rangle \bigg)$$

$$= \frac{1}{\sqrt{3}} \bigg( U_f|001\rangle|0\rangle + U_f|010\rangle|0\rangle + U_f|111\rangle|0\rangle \bigg)$$

$$= \frac{1}{\sqrt{3}} \bigg( |001\rangle|f(001)\rangle + |010\rangle|f(010)\rangle + |111\rangle|f(111)\rangle \bigg)$$

$$2. \ U_f|\psi\rangle = U_f \frac{1}{\sqrt{3}} \bigg( |001\rangle|-\rangle + |010\rangle|-\rangle + |111\rangle|-\rangle \bigg)$$

$$= \frac{1}{\sqrt{3}} \bigg( U_f|001\rangle|-\rangle + U_f|010\rangle|-\rangle + U_f|111\rangle|-\rangle \bigg)$$

$$= \frac{1}{\sqrt{3}} \bigg( (-1)^{f(001)}|001\rangle|-\rangle + (-1)^{f(010)}|010\rangle|-\rangle + (-1)^{f(111)}|111\rangle|-\rangle \bigg)$$