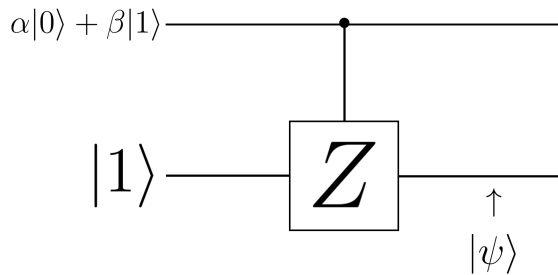


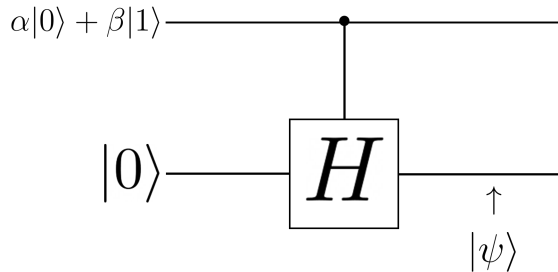
2.6 Phase Kickback

1. We know that $|1\rangle$ is an eigenvector of the Z-gate since $Z|1\rangle = -|1\rangle$. Consider the circuit below,



Prove that phase kickback occurs by showing a relative phase of -1 is applied to the control qubit

2. Does phase kickback occur in the following circuit. Why/why not?



Answers

$$\begin{aligned}
 1. \quad |\psi\rangle &= CZ\left(\alpha|0\rangle + \beta|1\rangle\right)|1\rangle \\
 &= CZ\left(\alpha|01\rangle + \beta|11\rangle\right) \\
 &= CZ\alpha|01\rangle + CZ\beta|11\rangle
 \end{aligned}$$

$$= \alpha|01\rangle - \beta|11\rangle$$

$$= \left(\alpha|0\rangle - \beta|1\rangle \right) |1\rangle$$

Therefore a relative phase of -1 was added to the control qubit

2. Phase kickback does not occur since the state $|0\rangle$ is not an eigenvector of the Hadamard Gate