Ans 1)

$$A = \begin{array}{cc} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} & \begin{array}{l} \Rightarrow R_1 \\ \Rightarrow R_2 \\ \Rightarrow R_3 \\ \Rightarrow R_4 \end{array} \end{array}$$

step ① $\Rightarrow R_2 - 2R_1 ,\ R_3 - 3R_1 ,\ R_4 - 6R_1$

we get, Also $R_2 \leftrightarrow R_3$ [swap]

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

step ② $\Rightarrow$ $\cancel{R_4 - R_2}$ $R_4 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 3 & 2 \end{bmatrix}$$

step ③ $\Rightarrow R_4 + R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

As we can see here there is 4 non-zero rows so rank of the Matrix $A = 4$

A2) The dimension of the domain W is $n = 2 \times 2$

Now, Nullity $[T]$ = dim (Ker (T))

A symetrix matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ will map to zero polynomial by satisfying the following conditions -

$a - b = 0$
$b - c = 0$
$c - a = 0$

This implies $a = b = c$

So the matrix = $\begin{bmatrix} a & a \\ a & d \end{bmatrix}$

The dim of this space is 1 so nullity of T is 1.

Now Rank (T) + Nulity (T) = dim (W)

Rank (T) = 3

Rank is 3.

A3) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$a=2$, $b=-1$ $c=-1$ $d=2$

so

$$A^{-1} = \frac{1}{4-1} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A^{-1} = 1/3 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Now for eigen value and eigen vectors of A^{-1}

$$\det(A^{-1} - \lambda I) = 0$$

$$\det\left(\frac{1}{3}\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}\right) = 0$$

$$1/3\left((2-\lambda)^2 - 1\right) = 0$$

$$\underset{a^2}{\underline{(2-\lambda)^2}} - \underset{b^2}{1} = 0$$

$a^2 - b^2 = (a-b)(a+b)$

so $(2-\lambda-1)(2-\lambda+1) = 0 \Rightarrow (3-\lambda)(1-\lambda) = 0$

eigen values are $\lambda_1 = 3$ $\lambda_2 = 1$

For $\lambda_1 = 3$

$$\cancel{A^{-1} - 3I = \frac{1}{3}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}}$$

$$(A^{-1} - 3I)v = 0$$

$$A + 4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

Now $\det(A + 4I - \lambda I) = 0$

~~det (A+4~~ $\det\left(\begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right) = 0$

$$(6-\lambda)^2 - 1 = 0$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$(6-\lambda-1)(6-\lambda+1) = 0 \Rightarrow (5-\lambda)(7-\lambda) = 0$$

$\lambda_1 = 5, \quad \lambda_2 = 7$

A4) $3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

Given,

$x(0) = 0,\ y(0) = 0,\ z(0) = 0$

After 3 Iterations

$x \approx 3.013$

$y \approx -2.944$

$z \approx 7.007$

Iteration 1 :

$x(1) = \dfrac{7.85 + 0.1(0) + 0.2(0)}{3} \approx 2.616$

$y(1) = \dfrac{-19.3 - 0.1(2.616) + 0.3(0)}{7} \approx -2.825$

$z(1) = \dfrac{71.4 - 0.3(2.616) + 0.2(-2.825)}{10} \approx 7.306$

Iteration 2:

$x(2) = [7.85 + 0.1(-2.825) - 0.2(7.306)]/3 \approx 2.974$

$y(2) = [-19.3 - 0.1(2.974) + 0.3(7.306)]/7 \approx -2.883$

$z(2) = [71.4 - 0.3(2.974) + 0.2(-2.883)]/10 \approx 7.001$

Iteration 3 :

$x(3) = [7.85 + 0.1(-2.883) - 0.2(7.001)]/3 \approx 3.013$

$y(3) = [-19.3 - 0.1(3.013) + 0.3(7.001)]/7 \approx -2.944$

$\cancel{z(3) = [-19.3 - 0.1(3.013) + 0.3(7.001}$

$z(3) = [71.4 - 0.3(3.013) + 0.2(-2.944)]/10 \approx 7.007$

A5) Given
$$x + 3y + 2z = 0$$
$$2x - y + 3z = 0$$
$$3x - 5y + 4z = 0$$
$$x + 17y + 4z = 0$$

Augomented Matrix :

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 - 2R_1$, $R_3 - 3R_1$, $R_4 - R_1$ [Row reduing]

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right] \xrightarrow[R_4 + 2R_2]{R_3 - 2R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

the system has a row of zeroes without a leading 1 in the last row. ∴ system is consistent

From 2nd row :

$$-7y - z = 0 \Rightarrow z = -7y$$

from 1st row :

$$x + 3y + 2z = 0$$
$$x + 3y - 14y = 0 \Rightarrow x = 11y$$

$x = 11y$, $y = y$, $z = -7y$

∴ by changing value of y we can get infinite solutions

A6) If $T(u+v) = T(u) + T(v)$ for all u, v in P_2 and $T(ku) = kT(u)$ for all u in P_2 and all scalar k

then we can determine whether Linear Transformation

So,

LHS

$$T([a+bx+cx^2] + [d+ex+fx^2]) = T([a+d] + [b+e]x + [c+f]x^2)$$

$$= (a+d+1) + (b+e+1)x + (c+f+1)x^2$$

RHS

$$T(a+bx+cx^2) + T(d+ex+fx^2) = (a+1) + (b+1)x + (c+1)x^2$$

$$+ (d+1) + (e+1)(x) + (f+1)x^2$$

$$= (a+d+1) + (b+e+1)x + (c+f+1)x^2$$ $\therefore$ It satisfies 1^{st} Condition

LHS

$$T(k(a+bx+cx^2)) = T(ka + kbx + ckx^2)$$

$$= (ka + kb + 1) + (kc + k)x + (kc + k)x^2$$

$$= k(a+1) + k(b+1)x + k(c+1)x^2$$

linearly

basis = { (1,

$+ 3(18))/7 = 5$

$$kT(a+bx+cx^2) = k\left((a+1) + (b+1)x + (c+1)x^2\right)$$

$$= k(a+1) + k(b+1)x + k(c+1)x^2$$

$\therefore$ It satisfies 2^{nd} Condition

$\therefore$ It is Indeed Linear Transformation

A7) so $\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$

Row reducing

$R_2 - 2R_1 \quad R_3 - 3R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix}$$

$R_3 \quad -\frac{9}{5} R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

From Here we can see that 3^{rd} row is a linear combination of first two rows

$\therefore$ the set S in linearly dependent

Since it is linearly dependent it cannot be a basis for $V_3(R)$ Now,

the dim of the subspace spanned by S is 2

and To find basis for this subspace we can take remaining linearly independent vectors which are (1,2,3) and (3,1,0)

basis = { (1,2,3) , (3,1,0) }

A8) Given $3x - 6y + 2z = 23$

$4x + y - z = -15$

$x - 3y + 7z = 16$

initial $x(0) = 1$, $y(0) = 1$, $z(0) = 1$

$$x^{(k+1)} = [23 + 6y^{(k)} - 2z^{(k)}] / 3$$

$$y^{(k+1)} = [-15 - 4x^{(k)} + z^{(k)}] / -1$$

$$z^{(k+1)} = [16 - x^{(k)} + 3y^{(k)}] / 7$$

Iteration 1:

$x^{(1)} = 23 + 6(1) - 2(1)/3 = (23+6-2)/3 = 9$

$y(1) = -15 - 4(1) + 1/-1 = (-15 - 4 + 1)/-1 = 18$

$z(1) = (16 - 1 + 3(1))/7 = 3$

Iteration 2:

$x^{(2)} = 23 + 6(18) - 2(3)/3 = 23$

$y(2) = \frac{-15 - 4(9) + 3}{-1} = 24$

$z(2) = 16 - 9 + 3(18))/7 = 5$

Iteration 3:

$$x^{(3)} = \frac{23 + 6(24) - 2(5)}{3} = 38$$

$$y(3) = \frac{-15 - 4(23) + 5}{-1} = 48$$

$$z(3) = 16 - 23 + 3(24)/7 = 6$$

After 3 iterations approx value of $x \approx 38$, $y \approx 48$, $z \approx 6$

A9) Application of matrix operations in image processing is in color manipulation such as adjusting brightness, contrast and saturation.

For example, let's consider adjusting brightness of an img. Given an image represented as a matrix of pixel values, we can increase or decrease the brightness of the image by adding or subtracting a constant value from each pixel's intensity.

New pixel value = Original pixel value + Brightness Offset

By applying this matrix operation to each pixel in the image we can effectively adjust its brightness level. Similar can be done for contrast, saturation, etc.