

\* Stokes theorem: It states that line integration of a vector field function around a closed surface is equal to surface integration of the curl of vector field function taken over an open surface bounded by closed path.

$$\oint_C \vec{E} \cdot d\vec{l} = \iint_S (\nabla \times \vec{E}) \cdot d\vec{S}$$

\* Gauss divergence theorem: It states that the surface integration of a vector field function over an closed surface is equal to volume integration of the divergence of the function over the volume of the closed surface.

$$\iint_S \vec{E} \cdot d\vec{S} = \iiint_V (\nabla \cdot \vec{E}) dV$$

## \* Maxwell equations of integral form

i) Gauss law for electrostatic

$$\oint_S \vec{E} \cdot d\vec{s} = \iiint_V \frac{\rho}{\epsilon_0} dv$$

ii) Gauss law of magnetic field

$$\oint_S \vec{B} \cdot d\vec{s} = 0$$

iii) Faraday's law of electromagnetic induction

$$\oint_C \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \left( \iint_S \vec{B} \cdot d\vec{s} \right)$$

iv) Ampere's law for varying currents

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \iint_S \vec{j}_c \cdot d\vec{s}$$

$\vec{j}_c \rightarrow$  conduction current density

\* Maxwell's equations of differential form

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$\rightarrow$  Gauss law e.s.

$$\nabla \cdot \vec{B} = 0$$

$\rightarrow$  Gauss Mag

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$\rightarrow$  Faraday's law of emf

$$\nabla \times \vec{B} = \mu_0 (\vec{J}_c + \vec{J}_D)$$

Ampere's law

\* Maxwell equations in free space diff form

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

i)

Derive Gauss law of electrostatic

$$\oint \vec{E} \cdot d\vec{s} = \iiint_{V} \frac{\rho}{\epsilon_0} \cdot dV$$

Applying Gauss divergence theorem ~~to~~

$$\oint \vec{E} \cdot d\vec{s} = \iiint (\nabla \cdot \vec{E}) \cdot dV$$

$$\iiint \nabla \cdot \vec{E} dV = \iiint \frac{\rho}{\epsilon_0} dV$$

$$\iiint \nabla \cdot \vec{E} dV - \iiint \frac{\rho}{\epsilon_0} dV = 0$$

$$\iiint_{V} (\nabla \cdot \vec{E} - \frac{\rho}{\epsilon_0}) dV = 0$$

This integration is zero independent of the volume of the surface

$$\nabla \cdot \vec{E} - \frac{\rho}{\epsilon_0} = 0$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

PROOFED

 Gauß law of Magnetic field.

$$\oint \vec{B} \cdot d\vec{s} = 0$$

 Applying gauß divergence theorem

$$\iint_S \vec{B} \cdot d\vec{s} = \iiint_V (\nabla \cdot \vec{B}) dv$$

$$\iiint_V (\nabla \cdot \vec{B}) dv = 0 \quad [dv \neq 0]$$

 This integration is zero independent of the volume of the surface

$$\boxed{\nabla \cdot \vec{B} = 0} \quad \text{prooved}$$

 Faraday's law of electro magnetic induction

$$\oint_C \vec{E} \cdot d\vec{l} = - \frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{s}$$

Applying Stokes's theorem

$$\oint_C \vec{E} \cdot d\vec{l} = \iint_S (\nabla \times \vec{E}) \cdot d\vec{s}$$

$$\iint_S (\nabla \times \vec{E}) \cdot d\vec{s} = - \frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{s}$$

$$\iint_S (\vec{J} \times \vec{E}) \cdot d\vec{s} = - \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\iint_S (\vec{J} \times \vec{E} + \frac{\partial \vec{B}}{\partial t}) \cdot d\vec{s} = 0 \quad \text{if } S \neq 0$$

This integration is zero independent of the surface.

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\boxed{\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}}$$

**IV** Ampere's Circuital law of varying current  
(Steady Current)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \iint_S \vec{J}_c \cdot d\vec{s} \quad (\text{R.H.S.})$$

Applying Stokes theorem

$$\oint_C \vec{B} \cdot d\vec{l} = \iint_S (\vec{J} \times \vec{B}) \cdot d\vec{s} \quad (\because H.S. = R.H.S.)$$

$$\iint_S (\nabla \times \vec{B}) \cdot d\vec{s} = \mu_0 \iint_S \vec{J} \cdot d\vec{s}$$

$$\iint_S (\nabla \times \vec{B} - \mu_0 \vec{J}) \cdot d\vec{s} = 0$$

This integration is zero independent of the surface

$$\vec{\nabla} \times \vec{B} - \mu_0 \vec{j} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$$

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}}$$

\* Ampere circuital law for time varying current (FIR 9 FIR current density)

Taking divergence both sides

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \mu_0 (\vec{\nabla} \cdot \vec{j}_c)$$

Divergence of curl = 0

$$0 = \mu_0 (\vec{\nabla} \cdot \vec{j}_c)$$

$$\therefore \mu_0 \neq 0$$

$$\vec{\nabla} \cdot \vec{j}_c = 0$$

that means some dependent term is missing on RHS

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}_c + \vec{x} \quad \text{--- (1)}$$

again taking divergence both sides

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \mu_0 (\vec{\nabla} \cdot \vec{j}_c) + (\vec{\nabla} \cdot \vec{x})$$

$$0 = \mu_0 (\vec{\nabla} \cdot \vec{j}_c) + \vec{\nabla} \cdot \vec{x}$$

$$\mu_0 \neq 0$$

divide  $\mu_0$  both sides

$$0 = (\nabla \cdot \vec{J}_c) + \frac{1}{\mu_0} (\nabla^2 \cdot \vec{x})$$

$$\therefore \nabla \cdot \vec{J}_c + \frac{\partial \vec{x}}{\partial t} = 0$$

$$\boxed{\nabla \cdot \vec{J}_c = - \frac{\partial \vec{x}}{\partial t}}$$

equation of  
continuity  
continuity

$$0 = - \frac{\partial \vec{x}}{\partial t} + \frac{1}{\mu_0} (\nabla^2 \cdot \vec{x})$$

$$\frac{1}{\mu_0} (\nabla^2 \cdot \vec{x}) = \frac{\partial \vec{x}}{\partial t}$$

$$\nabla^2 \cdot \vec{x} = \mu_0 \frac{\partial \vec{x}}{\partial t}$$

$$\therefore \text{Gauss law} \quad \nabla \cdot \vec{E} = \frac{S}{\epsilon_0}$$

$$S = \epsilon_0 (\nabla \cdot \vec{E})$$

$$\nabla \cdot \vec{x} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

integrate w.r.t. space

$$\vec{x} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{--- (2)}$$

(2) put in (1)

$$\nabla \times \vec{B} = \mu_0 \vec{j}_c + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \left( \vec{j}_c + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) : \left\{ \left[ j_D = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right] \right\}$$

$$\boxed{\nabla \times \vec{B} = \mu_0 (\vec{j}_c + \vec{j}_D)} \quad \text{proven}$$

## \* Significance of Maxwell's equations

$$\nabla \cdot \vec{E} = \frac{q}{\epsilon_0}$$

(1) single charge can be exist.

(ii) It is time independent.

(iii)  $\nabla \cdot \vec{E}$  is scalar quantity.

$$\nabla \cdot \vec{B} = 0$$

(i) It tells that isolated poles does not exist.

(ii) It is a time independent.

(iii) divergence of  $\vec{B}$  is always zero.

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

(i) It relates electric field vector and magnetic flux density vector.

(ii) It is time dependent.

(iii) It describes Faraday's law of electromagnetic induction along with Lenz's law.

$$\therefore B = \mu_0 H, H = \frac{B}{\mu_0}$$

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4)  $\vec{\nabla} \times \vec{B} = \mu_0 (\vec{j} + \frac{\partial \vec{D}}{\partial t})$  /  $\vec{\nabla} \times \vec{H} = \vec{j} + \vec{A}$

i) It is time dependent.

ii) It relates magnetic field vector with displacement vector

iii)  $\vec{\nabla} \times \vec{B}$  is also called modified Ampere's law

**\* Derive Maxwell's electromagnetic wave equation for free space and show that speed of em waves in free space is  $3 \times 10^8$  m/s. Also comment on the nature of em waves from Faraday's law**

$$\text{Sol} \rightarrow \nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \text{--- (1)}$$

Taking curl both sides

$$\nabla \times (\nabla \times \vec{E}) = - \frac{\partial}{\partial t} (\nabla \times \vec{B}) \quad \text{--- (2)}$$

from vector property

$$\nabla \times (\nabla \times \vec{E}) = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$$

$$\nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = - \frac{\partial}{\partial t} (\nabla \times \vec{B})$$

$$[\nabla \cdot \vec{E} = 0] \text{ gaus law } E \text{ for free space}$$

Multiply by (-)  $0 - \nabla^2 \vec{E} = - \frac{\partial}{\partial t} (\nabla \times \vec{B}) \quad \left\{ \because \vec{B} = \mu_0 \vec{H} \right.$

$$\nabla^2 \vec{E} = \frac{\partial}{\partial t} (\nabla \times \mu_0 \vec{H}) \quad \text{for free space}$$

$$\nabla^2 \vec{E} = \mu_0 \frac{\partial}{\partial t} (\nabla \times \vec{H})$$

$$\left\{ \begin{array}{l} \nabla \times \vec{B} = \mu_0 \frac{\partial \vec{H}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \frac{\partial \vec{D}}{\partial t} \end{array} \right.$$

$$\nabla^2 \vec{E} = \mu_0 \frac{\partial}{\partial t} \left( \frac{\partial \vec{D}}{\partial t} \right)$$

$$\nabla^2 \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

~~Maxwell's eqns for free space~~

$$\therefore \vec{D} = \epsilon_0 \vec{E}$$

~~Maxwell's eqns~~

$$\vec{\nabla} \cdot \vec{E} = \mu_0 \frac{\partial}{\partial t} \left( \frac{\partial (\epsilon_0 E^2)}{\partial t} \right)$$

$$\vec{\nabla} \cdot \vec{E} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left( \frac{\partial E}{\partial t} \right) \quad (3)$$

$$\boxed{\vec{\nabla} \cdot \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \rightarrow \text{wave eqn for free space}$$

$$\delta \boxed{\vec{\nabla} \cdot \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}} \quad \text{where } \vec{E} = \vec{E}_0 \sin(\omega t - \vec{k} \cdot \vec{r}) \\ \vec{B} = \vec{B}_0 \sin(\omega t - \vec{k} \cdot \vec{r})$$

Speed of em waves in free space is  $3 \times 10^8 \text{ m/s}$

for general wave eqn

$$\boxed{\vec{\nabla} \cdot \vec{\psi} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}} \quad (4)$$

by comparing eqn (3) and (4)

$$\vec{\nabla} \cdot \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \quad \boxed{\frac{1}{v^2} = \mu_0 \epsilon_0}$$

$$\vec{\nabla} \cdot \vec{\psi} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad \boxed{\frac{1}{v^2} = \frac{1}{\mu_0 \epsilon_0}}$$

$$v = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c}$$

$$\boxed{v = c}$$

$$\boxed{v = 3 \times 10^8 \text{ m/s}} \quad \text{proven}$$

phy

## Nature of em waves

The electromagnetic wave has a 'transverse' nature. Electric and magnetic field vector in electromagnetic wave are perpendicular to each other and also perpendicular to the wave's propagation direction. This type of electromagnetic wave is referred as transverse nature.

### \* Transverse nature of EM Waves

So

from Maxwell's 2nd eqn in free space:

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \text{and} \quad \vec{\nabla} \cdot \vec{B} = 0$$

$\therefore$  We know that em wave eqn

$$\vec{\nabla}^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \& \quad \vec{\nabla}^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

so, The general solution of this equation is

$$\vec{E}(\vec{r}, t) = E_0 e^{i(k \vec{r} - \omega t)}$$

$$\text{and } \vec{B}(\vec{r}, t) = B_0 e^{i(k \vec{r} - \omega t)}$$

where  $E_0$  &  $B_0$  are complex amplitude  
 $k$  is a wave propagation vector

$$\therefore \vec{\nabla} \cdot \vec{E} = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) (\vec{E}_0 \cdot e^{i(kx+ky+kz-wt)}) \quad (A)$$

where  $\vec{E}_0 = \vec{E}_{0x} + \vec{E}_{0y} + \vec{E}_{0z} \quad (B)$

$$\vec{k} = i k_x + j k_y + k^z$$

$$\vec{e} = i \vec{x} + j \vec{y} + k \vec{z}$$

$$\vec{k} \cdot \vec{e} = (i k_x + j k_y + k^z)(i x + j y + k z)$$

$$\vec{k} \cdot \vec{e} = k_x x + k_y y + k_z z \quad (C)$$

Put eqn B and C in eq A

$$\vec{\nabla} \cdot \vec{E} = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) \cdot (i E_{0x} e^{i(kx+ky+kz-wt)}) \quad (Kx + Ky + Kz - wt)$$

$$\vec{\nabla} \cdot \vec{E} = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) \cdot (i E_{0x} e^{i(kx+ky+kz-wt)}) + i E_{0y} e^{i(kx+ky+kz-wt)} + i E_{0z} e^{i(kx+ky+kz-wt)}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial}{\partial x} (E_{0x} e^{i(kx+ky+kz-wt)}) + \frac{\partial}{\partial y} (E_{0y} e^{i(kx+ky+kz-wt)}) + \frac{\partial}{\partial z} (E_{0z} e^{i(kx+ky+kz-wt)})$$

$$\vec{\nabla} \cdot \vec{E} = E_{0x} e^{i(kx+ky+kz-wt)} + i k_x E_{0y} e^{i(kx+ky+kz-wt)} + i k_y E_{0z} e^{i(kx+ky+kz-wt)}$$

$$= E_{0z} e^{i(kx+ky+kz-wt)} + i k_z E_{0x} e^{i(kx+ky+kz-wt)} + i k_x E_{0y} e^{i(kx+ky+kz-wt)}$$

$$\vec{\nabla} \cdot \vec{E} = (i k_x E_{0x} + i k_y E_{0y} + i k_z E_{0z}) e^{i(kx+ky+kz-wt)}$$

$$\vec{\nabla} \cdot \vec{E} = i (k_x E_{0x} + k_y E_{0y} + k_z E_{0z}) e^{i(kx+ky+kz-wt)} \quad (1)$$

Now we calculate  $\vec{k} \cdot \vec{E} \rightarrow$  (put eqn d in place of  $\vec{E}$ )

$$\vec{k} \cdot \vec{E} = (i k_x + j k_y + k^z) \cdot (i E_{0x} e^{i(kx+ky+kz-wt)})$$

$$\vec{k} \cdot \vec{E} = (i k_x + j k_y + k^z k_z) \cdot (E_0 e^{i(k_z z - \omega t)})$$

Put eqn (B) & (C) in place of  $E_0$  or  $(k^z \vec{x})$

$$\vec{k} \cdot \vec{E} = (i k_x + j k_y + k^z k_z) \cdot (i E_{0x} + j E_{0y} + k^z E_{0z}) \cdot e^{i(k_z z - \omega t)}$$

$$\vec{k} \cdot \vec{E} = (k_x E_{0x} + k_y E_{0y} + k_z E_{0z}) e^{i(k_z z + k_y y + k_x x - \omega t)}$$

(2)

Put eq (2) in (1)

$$\vec{\nabla} \cdot \vec{E} = i (\vec{k} \cdot \vec{E})$$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \left\{ \text{from maxwell eqn}$$

$\vec{\nabla} \cdot \vec{E} = 0 \quad \left\{ \text{in free space} \right. \right)$

$$0 = i (\vec{k} \cdot \vec{E})$$

$$\vec{k} \cdot \vec{E} = 0 \quad \& \quad \vec{k} \cdot \vec{B} = 0$$

$$\vec{k} \perp \vec{E} \quad \& \quad \vec{k} \perp \vec{B}$$

So hence proved electric and magnetic fields are perpendicular to the direction of propagation vector ( $\vec{k}$ ).

This is Electromagnetic waves are transverse in nature.

## and its Significance

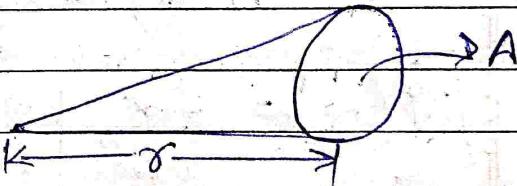
\* Poynting vector  $\vec{P}$   $\equiv$  Energy or power transformed per unit area is called poynting vector, poynting vector is denoted by  $\vec{P}$  and its unit is  $\text{watt/m}^2$ .

$\vec{P} = \frac{\text{Energy or power transformed}}{\text{area}} \text{ and significance}$

unit:  $P = \frac{\text{watt}}{\text{m}^2}$

\* Solid angle  $\Omega$   $\equiv$  Solid angle is the ratio of area of the sphere and square of the distance  $r$ . It is called solid angle, it is denoted by  $\Omega$  and its unit is steradian.

$$\Omega = \frac{A}{r^2}$$



Dimension less

$\Omega = \text{unit steradian}$

other words  $\Omega$   $\equiv$  solid angle subtended by the surface area of a sphere subtended by the line connecting the radius of that sphere is called solid angle.

$$\Omega = \frac{A}{r^2}$$

# Relationship between Electric field & potential:

Once  $\Rightarrow$  Consider a point charge

$+q$ , the electric potential  $+q$  at a point  $P$  distance  $x$  from  $+q$  and given by

$$\therefore V = \frac{1}{4\pi\epsilon_0} \frac{q}{x} \quad (A)$$

and

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} \vec{x} \quad (B)$$

We know that

$$\nabla(\frac{1}{x}) = -\frac{\vec{x}}{x^3}$$

$$\nabla(\frac{1}{x}) = -\frac{\vec{x}}{x^3} \quad \left\{ \begin{array}{l} x^1 = x \\ x^2 \end{array} \right. \quad (C)$$

$$\vec{E} = -\frac{1}{4\pi\epsilon_0} \frac{q}{x^2} \vec{x}$$

Multiply by  $q$  both sides

$$\vec{V} = -\frac{1}{4\pi\epsilon_0} \frac{q}{x} = -\frac{\vec{x}}{x^2} \cdot \frac{q}{4\pi\epsilon_0}$$

Mul by  $-$  both sides

$$-\nabla \cdot \vec{V} = \vec{x} \cdot \frac{1}{4\pi\epsilon_0} \frac{q}{x^2}$$

from eq<sup>n</sup> (A) & (B)

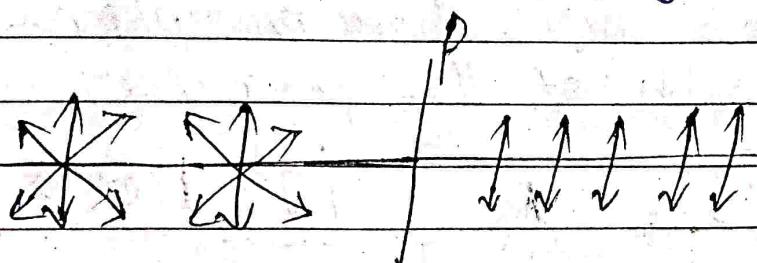
$$-\nabla \cdot \vec{V} = E$$

Thus the electric field ( $E$ )

can be expressed as -ve of  
proven Gradient of electric  
potential ( $V$ ).

$$\vec{E} = -\nabla \cdot \vec{V}$$

- \* What is polarization  $\rightarrow$  short note  
 Ans) polarization is the property of electromagnetic radiations in which the direction and magnitude of the vibrating electric fields are related in a specified way.



When electromagnetic radiation of sunlike is to be come to earth or in a space then electric fields are oscillation in all direction perpendicularly the propagation of the em wave and when this wave pass through polariser then ~~oscillation~~ oscillation of em wave in only one direction this process called polarization.

- \* Brewster's law: The refractive index of a medium is equal to the tangent sine of the polar angle.

$$n = \tan i_p \quad \text{--- (i)}$$

$$\text{From Snell law } n = \frac{\sin i_p}{\sin r} \quad \text{--- (ii)}$$

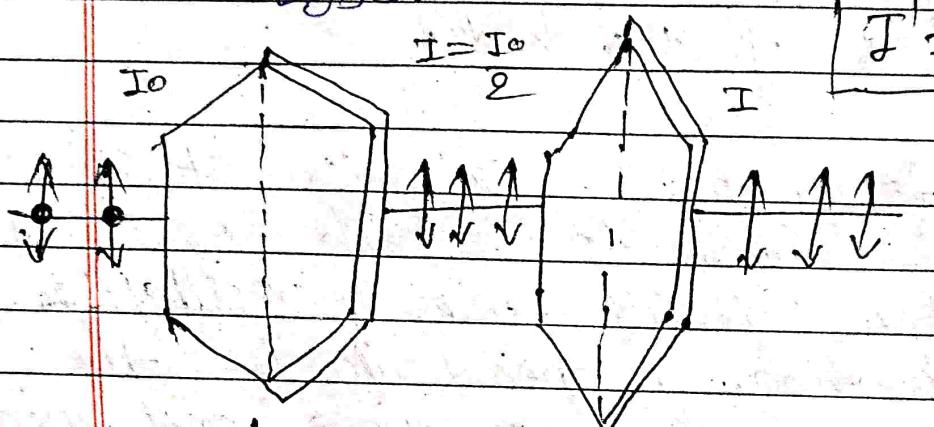
$$\text{From eq (i) \& (ii)} \quad \cancel{\sin r = \sin(90 - i_p)}$$

$$\tan i_p = \frac{\sin i_p}{\sin r} \Rightarrow \boxed{i_p + r = 90}$$

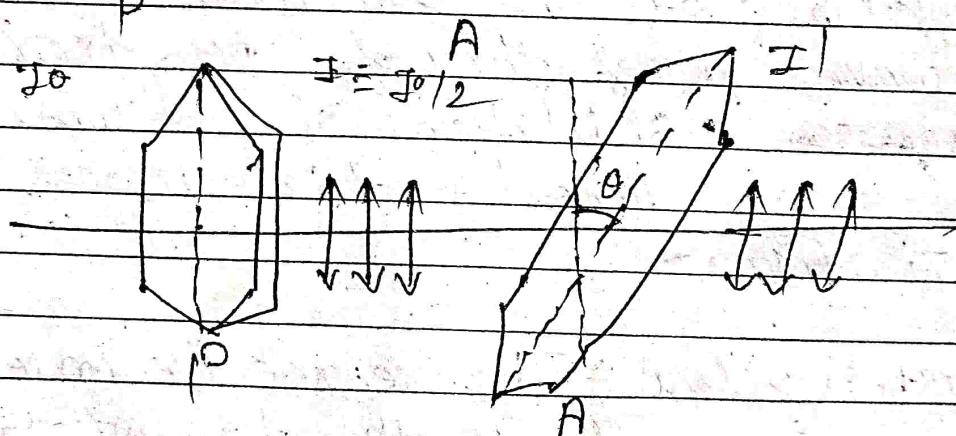
$$\frac{\sin i_p}{\cos i_p} = \frac{\sin i_p}{\sin r}$$

$$\sin r = \cos i_p$$

\* Malus law  $\therefore$  It states that when a beam of plane polarized light from a polarizer is incident on analyser, the intensity of light emerging from analyser varies as the square of the cosine of angle between plane of transmission of the polarizer and analyser:

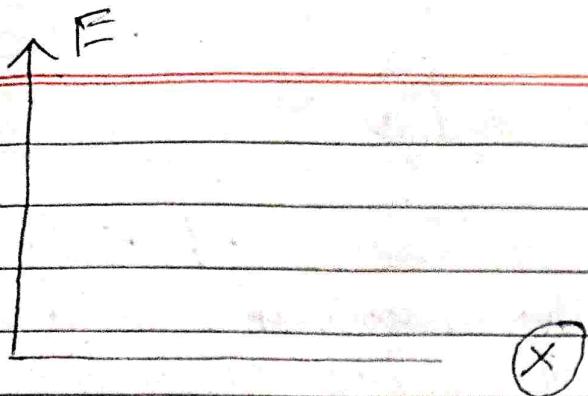


$$I' = I \cos^2 \theta$$



(P.W.Q)

What is the concept of displacement current?



\* Type ① and Type ② Superconductor and difference

\* अनुकृति, अन्तर्गत, अंतर्गत प्रभाव

\* Concept of energy gap insulator, conductor and Superconductor

\* Semiconductor

\* Nonresonance  $H(T)$ ;  $I_C(T)$

\* Nonresonance based on de Broglie hypothesis of Nonresonance.

(iv) Define Poynting vector and its significance. Energy as power transformed per unit area is called Poynting vector, Poynting vector is denoted by  $\vec{P}$  and its unit is  $\text{Watt}/\text{m}^2$ .  $\vec{P} = \text{Energy transformed} \rightarrow \text{unit} = \frac{\text{Watt}}{\text{m}^2}$  area (direction of  $\vec{P}$  is always normal to the area.)

- i) it represents the flow of energy in a given direction.
- ii) it indicates the rate of transfer of energy per unit area.
- iii) it measures power per unit area transformed by energy wave.



## Definition of Gradient

→ Gradient is a vector that points in the direction of the maximum rate of increase of the field and its magnitude represents the rate of change of the field in that direction.

$$\nabla f = \hat{i} \frac{\partial f}{\partial x} + \hat{j} \frac{\partial f}{\partial y} + \hat{k} \frac{\partial f}{\partial z}$$



Definition of Divergence = Divergence measures that how much the field spreads out from a point

(div  $\vec{f}$ )

$$\nabla \cdot \vec{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$



Definition of ~~Divergence~~ = curl

→ curl measures that the amount of local rotation of the field at a point

(curl  $\vec{f}$ )

$$\nabla \times \vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

- Note →  $\nabla \cdot \vec{A} = 0 \rightarrow$  solenoidal  $E = -\nabla V \rightarrow$  irrotational
- $\nabla \times \vec{A} = 0 \rightarrow$  irrotational
- $\nabla \times \vec{A} \neq 0 \rightarrow$  rotational
- $|\nabla \times \vec{A}| = 0 \rightarrow$  conservative field
- $|\nabla \times \vec{A}| \neq 0 \rightarrow$  non-conservative field

Divergence  $\Rightarrow$  Divergence of vector field at a point is defined as flux per unit volume provided the volume is taken to be as small as possible if  $\vec{A}$  is a vector field then

$$\text{Divergence of } \vec{A} = \nabla \cdot \vec{A} = \lim_{V \rightarrow 0} \frac{\text{flux}}{\text{volume}}$$

$$\therefore \nabla \cdot \vec{A} = \lim_{V \rightarrow 0} \frac{\iint \vec{A} \cdot d\vec{S}}{V}$$

Significance  $\Rightarrow$

① It gives in the spreading tendency of field.

If  $\nabla \cdot \vec{A}$  is +ve so it is  $\rightarrow$  source

If  $\nabla \cdot \vec{A}$  is -ve so it is  $\rightarrow$  sink field

If  $\nabla \cdot \vec{A}$  is zero so it is  $\rightarrow$  dipole field and the field is solenoidal.

$$\nabla \cdot \vec{A} = \frac{\partial(A_x)}{\partial x} + \frac{\partial(A_y)}{\partial y} + \frac{\partial(A_z)}{\partial z}$$

Curl  $\Rightarrow$  Curl of vector field  $\vec{A}$  at a point is defined as maximum circulation per unit area, provided the area is taken to be as small as possible.

Its direction is normal to the surface.

$$\text{curl } \vec{A} = \nabla \times \vec{A} = \lim_{S \rightarrow 0} \frac{\text{max circulation}}{\text{Area}} = \lim_{S \rightarrow 0} \frac{\oint \vec{A} \cdot d\vec{l}}{S}$$

$$\text{magnitude of curl of } \vec{A} = |\nabla \times \vec{A}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

If  $|\nabla \times \vec{A}| \neq 0$  Rotational field

$= 0$  irrotational field

& conservative field

It measures how much a field rotates/circulates

Significance  $\Rightarrow$  It gives the rotational tendency of the field

**AP** Gradient  $\hat{\nabla}\phi$  of scalar field  $\phi$  is defined as maximum rate of change of scalar field with space coordinates and its direction is always normal to the surface.

$$\hat{\nabla}\phi = \left( \frac{d\phi}{dx} \right)_{\max} \hat{n} = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$$

significance  $\rightarrow$  gradient gives the direction of maximum rate of change of scalar field.

eg.  $E = -\hat{\nabla}V$

Divergence  $\dagger$ : Divergence of vector field can be defined as

$$\text{divergence} = \lim_{V \rightarrow 0} \frac{\text{flux}}{\text{volume}}$$

flux per unit volume where volume  $V \rightarrow 0$

divergence of a vector field is scalar for a vector ~~field~~  $\vec{A}$  divergence can be defined as

$$AB \Rightarrow \rightarrow$$

$$\nabla \cdot \vec{A} = \frac{\partial(A_x)}{\partial x} + \frac{\partial(A_y)}{\partial y} + \frac{\partial(A_z)}{\partial z}$$

for solenoidal field divergence of vector  $= 0$

for example a vector  $\vec{A}$  to be solenoidal

$$\nabla \cdot \vec{A} = 0$$

Significance of divergence

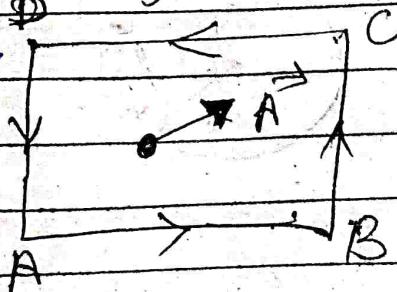
- (i) the indication of the spreading of the vector from a particular point
- (ii) it measures density of charge

Define curl  $\dagger$ : ~~closed~~ line integral of vector field over the close loop per unit area is known as curl.

$$\text{curl}(\vec{A}) = \lim_{\text{area} \rightarrow 0} \frac{1}{\text{area}} \oint \vec{A} \cdot d\vec{l}$$

significance

- (i) it measure how much a vector field circulate or rotates
- (ii) it measure how a fluid may rotate



**P** ✓ Gradient : gradient measure of the rate of change or the slope of a scalar or vector field. The gradient is often denoted by the symbol  $\nabla$  (del)

$$\nabla f = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$$

Significance  $\rightarrow$  it measures rate of change of one variable with respect to another

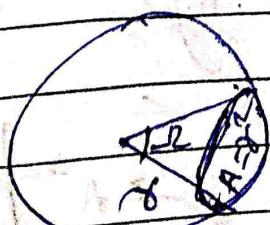
It measures magnetic field at a point.

**Q** Solid angle : Solid angle is the angle subtended by any part of a spherical surface of unit radius at its center. It is 3D angle. Its symbol is  $\Omega$ . Unit is steradian.

$$\text{Generally } \Omega = \frac{A}{r^2}$$

$$\text{for a sphere } \Omega = \frac{4\pi r^2}{r^2}$$

$$\Omega = 4\pi \text{ steradian}$$



A vector field is given as  $\vec{A} = xy\hat{i} + yz\hat{k}$ . Find  $\nabla \times \vec{A}$  and tell whether the field is conservative or not.

$$\begin{array}{|ccc|} \hline & \hat{i} & \hat{j} & \hat{k} \\ \hline \hat{i} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \hat{j} & \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ \hat{k} & 0 & \frac{\partial}{\partial y} & 0 \\ \hline \end{array}$$

$$\Rightarrow \hat{i} \left( \frac{\partial(yz)}{\partial y} - 0 \right) - \hat{j} \left( \frac{\partial(yz)}{\partial x} - 2(xy) \right) + \hat{k} \left( 0 - 2(xy) \right)$$

$$\Rightarrow \hat{i}z - \hat{j}(0 - 0) + \hat{k}(-2x)$$

$$\Rightarrow \hat{i}z + \hat{k}(-x) \Rightarrow \hat{z}\hat{i} - x\hat{k}$$

non-conservative field

Given  $E = 3x^2\hat{i} - 6xy\hat{j} + 12zy^2\hat{k}$  find divergence of the given vector field and comment on its nature whether field is converging or diverging.

$$\nabla \cdot f = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} =$$

$$\nabla \cdot f = \frac{\partial(3x^2)}{\partial x} + \frac{\partial(-6xy)}{\partial y} + \frac{\partial(12zy^2)}{\partial z}$$

$$\nabla \cdot f = 6x - 6x + 24zy^2$$

$$\nabla \cdot f = 24zy^2$$

because  $x$  and  $z$  positive  
so vector field is diverging.

Diverging : if  $x$  is positive and  $z$  is positive then divergence is positive so vector field is diverging.

Converging : if  $x$  is negative and  $z$  is positive then divergence is negative so vector field is converging.

Ques

Given  $\vec{F} = x^2y\vec{i} + yx\vec{j} + z^2y\vec{k}$ . find the divergence and curl of this vector field.

i) divergence  $\nabla \cdot \vec{F} =$

$$\Rightarrow \frac{\partial(x^2y)}{\partial x} + \frac{\partial(yx)}{\partial y} + \frac{\partial(z^2y)}{\partial z} =$$

$$\Rightarrow 2xy + 2xy + 2yz$$

curl of  $\vec{F} =$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & yx & z^2y \end{vmatrix}$$

$$\Rightarrow i \left( \frac{\partial(z^2y)}{\partial y} - \frac{\partial(y^2x)}{\partial z} \right) - j \left( \frac{\partial(z^2y)}{\partial x} - \frac{\partial(x^2y)}{\partial z} \right) + k$$

$$\left( \frac{\partial(y^2x)}{\partial x} - \frac{\partial(x^2y)}{\partial y} \right)$$

$$\Rightarrow i((z^2) - 0) - j(0 - 0) + k((y^2) - (x^2))$$

$$\Rightarrow z\vec{i} + k(y^2 - x^2)\vec{k}$$

$$\Rightarrow \boxed{2\vec{i} + y^2\vec{k} - x^2\vec{k}}$$

(PQ) find divergence and curl of position vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\text{divergence } \nabla \cdot \vec{f} = \frac{\partial(x)}{\partial x} + \frac{\partial(y)}{\partial y} + \frac{\partial(z)}{\partial z}$$

$$\nabla \cdot \vec{f} = 1 + 1 + 1 \quad \boxed{\nabla \cdot \vec{f} = 3}$$

$$\text{curl } \vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$\hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0-0)$$

$$\boxed{\text{curl } \vec{f} = 0}$$

(PQ) The electric potential given by  $V(x, y, z) = x^2 + 3y^2 + 9z^2$   
calculate the associated electric field at point (1, 1, 0)

Sol  $\rightarrow$  we know that  $E = -\nabla V$

$$E = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (x^2 + 3y^2 + 9z^2)$$

$$E = -\left(\frac{\partial^2(4x^2)}{\partial x^2}\hat{i} + \frac{\partial^2(12y^2)}{\partial y^2}\hat{j} + \frac{\partial^2(18z^2)}{\partial z^2}\hat{k}\right)$$

$$E = -\left(8x\hat{i} + 16y\hat{j} + 18z\hat{k}\right) \text{ at point (1, 1, 0)}$$

$$E = -\left(8(1)\hat{i} + 16(1)\hat{j} + 18(0)\hat{k}\right)$$

$$\boxed{E = -8\hat{i} + 16\hat{j}}$$

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electric

The potential in a certain region of space is given by  $V(x, y, z) = 20x^2 + 10y + 5z^3$ , find the electric field intensity vector? Check the field thus obtained is (i) uniform (ii) solenoidal.

$$\text{only } V = 20x^2 + 10y + 5z^3$$

$$\vec{E} = -\nabla V$$

$$\vec{E} = -(\overset{\circ}{i}(40x) + \overset{\circ}{j}(10) + \overset{\circ}{k}(15z^2))$$

$$\boxed{\vec{E} = -40x\overset{\circ}{i} + 10\overset{\circ}{j} + 15z^2\overset{\circ}{k}}$$

$$\begin{aligned} \textcircled{i} \quad \vec{E} \cdot \vec{A} &= \frac{\partial(20x^2)}{\partial x} + \frac{\partial(10y)}{\partial y} + \frac{\partial(5z^3)}{\partial z} \\ &= 40x + 10 + 15z^2 \end{aligned}$$

\* Calculate the electric field at point (1, 1, 1) in an electric field the electric potential is given by  $V(x, y, z) = (4x^2 + 3y^2 + 9z^2)^{-1/2}$

$$\vec{E} = -\nabla V \quad V = \frac{1}{\sqrt{4x^2 + 3y^2 + 9z^2}}$$

$$\vec{E} = -\left(\frac{\overset{\circ}{i}}{\overset{\circ}{x}}\right) + \left(\frac{\overset{\circ}{j}}{\overset{\circ}{y}}\right) + \left(\frac{\overset{\circ}{k}}{\overset{\circ}{z}}\right) \quad V = \frac{1}{\sqrt{4x^2}} + \frac{1}{\sqrt{3y^2}} + \frac{1}{\sqrt{9z^2}}$$

$$\vec{E} = -\left(\frac{\overset{\circ}{i}}{2x} + \frac{\overset{\circ}{j}}{\sqrt{3}y} + \frac{\overset{\circ}{k}}{3z}\right)$$

$$\frac{\partial}{\partial x} \left( \frac{1}{2x} \right) \rightarrow \vec{E} = -\left(\frac{\overset{\circ}{i}}{2x^2} + \overset{\circ}{j}\left(\frac{1}{3y^2}\right) + \overset{\circ}{k}\left(-\frac{1}{3z^2}\right)\right)$$

$$\boxed{\vec{E} = \frac{1}{2x^2}\overset{\circ}{i} + \frac{1}{3y^2}\overset{\circ}{j} - \frac{1}{3z^2}\overset{\circ}{k}}$$

(Q) find the constants  $a, b$  and  $c$  such that the vector field.

$$\text{Ans} \rightarrow f = (x+2y+az)i + (bx-3y-2)j + (4x+cy+2z)k$$

$$(\vec{i} \times \vec{f}) = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x+2y+az) & (bx-3y-2) & (4x+cy+2z) \end{vmatrix}$$

$$i \left( \frac{\partial(4x+cy+2z)}{\partial y} - \frac{\partial(bx-3y-2)}{\partial z} \right) - j \left( \frac{\partial(4x+cy+2z)}{\partial x} - \frac{\partial(x+2y+az)}{\partial z} \right) + k \left( \frac{\partial(bx-3y-2)}{\partial x} - \frac{\partial(x+2y+az)}{\partial y} \right) = 0$$

$$\Rightarrow i(c-0) - j(4-a) + k(b-2) = 0$$

$$ci - j(4-a) + k(b-2) = bi + aj + ok$$

$$ci = 0i, -j(4-a) = 0j, k(b-2) = 0k$$

$$[c=0], [-4+a=0], [b-2=0]$$

$$[a=4], [b=2], [c=0]$$

\* Show that gravitation free field is irrotational & conservative.

(PQ) Newton's law of universal gravitation is represented  $\vec{F} = \frac{GMm}{r^2} \hat{r}$

where  $f$  is the magnitude of the gravitational force exerted by one object on another.  $M$  and  $m$  are the masses of the objects, and  $r$  is a distance. find the curl of gravitational field.

$$\vec{f} = -\frac{GMm}{r^2} \hat{r}, \quad \hat{r} = \frac{\vec{r}}{r}$$

if a vector field is to be irrotational & conservative then  $\nabla \times \vec{F} = 0$

$$\vec{f}_G = \frac{GMm}{r^3} \hat{r}, \quad r = \sqrt{x^2 + y^2 + z^2}$$

Let's find  $|\nabla \times \vec{f}_G|$

$$\vec{f} = \frac{GMm}{(x^2 + y^2 + z^2)^{3/2}}$$

$$\vec{f}_x = \frac{GMm \cdot x}{(x^2 + y^2 + z^2)^{3/2}}$$

$$f_x \hat{x} = \frac{GMm \cdot x}{(x^2 + y^2 + z^2)^{3/2}}$$

$$\vec{f}_z = \frac{GMm \cdot z}{(x^2 + y^2 + z^2)^{3/2}} \left[ \frac{\partial}{\partial x} \left( \frac{z}{(x^2 + y^2 + z^2)^{3/2}} \right) - \frac{\partial}{\partial z} \left( \frac{x}{(x^2 + y^2 + z^2)^{3/2}} \right) \right] + \vec{k}$$

$$= GMm \left[ \frac{\partial}{\partial y} \left( \frac{z}{(x^2 + y^2 + z^2)^{3/2}} \right) - \frac{\partial}{\partial z} \left( \frac{y}{(x^2 + y^2 + z^2)^{3/2}} \right) \right] - \vec{k}$$

$$= GMm \left[ \frac{2}{x^2 + y^2 + z^2} \left( x^2 + y^2 + z^2 \right)^{-3/2} - \frac{2}{x^2 + y^2 + z^2} \left( x^2 + y^2 + z^2 \right)^{-3/2} \right]$$

$$= \frac{2}{x^2 + y^2 + z^2} \left[ \frac{y^2}{x^2 + y^2 + z^2} - \frac{x^2}{x^2 + y^2 + z^2} \right]$$

$$\Rightarrow GMm \left[ \frac{2}{x^2 + y^2 + z^2} \left( x^2 + y^2 + z^2 \right)^{-3/2} - \frac{2}{x^2 + y^2 + z^2} \left( x^2 + y^2 + z^2 \right)^{-3/2} \right] + \vec{k}$$

$$= \frac{2}{x^2 + y^2 + z^2} \left[ \frac{y^2}{x^2 + y^2 + z^2} - \frac{x^2}{x^2 + y^2 + z^2} \right]$$

$$\Rightarrow GMm \left[ \frac{2}{x^2 + y^2 + z^2} \left( x^2 + y^2 + z^2 \right)^{-3/2} - \frac{2}{x^2 + y^2 + z^2} \left( x^2 + y^2 + z^2 \right)^{-3/2} \right] + \vec{k}$$

$$= \frac{2}{x^2 + y^2 + z^2} \left[ \frac{y^2}{x^2 + y^2 + z^2} - \frac{x^2}{x^2 + y^2 + z^2} \right]$$

$$\Rightarrow \frac{GMm}{(x^2 + y^2 + z^2)^{5/2}} \left( -\frac{3}{2} (x^2 + y^2 + z^2) \cdot 2y - \frac{3}{2} (x^2 + y^2 + z^2) \cdot 2z \right)$$

$$= \frac{GMm}{(x^2 + y^2 + z^2)^{5/2}} \left[ -\frac{3}{2} (x^2 + y^2 + z^2)^{-3/2} \cdot 2x - \frac{3}{2} (x^2 + y^2 + z^2)^{-3/2} \cdot 2y \right] + \vec{k}$$

$$\Rightarrow GMm \left[ -\frac{3}{2} \left( \frac{\partial}{\partial x} \left( \frac{y^2}{x^2 + y^2 + z^2} \right) - \frac{\partial}{\partial y} \left( \frac{x^2}{x^2 + y^2 + z^2} \right) \right) \right]$$

$$\Rightarrow \frac{GMm}{(x^2 + y^2 + z^2)^{5/2}} \left( -\frac{3}{2} \left( \frac{2y^2}{x^2 + y^2 + z^2} - \frac{2x^2}{x^2 + y^2 + z^2} \right) \right)$$

$$\boxed{|\nabla \times \vec{f}_G| = 0}$$

$\vec{f}_G$  is irrotational & conservative.

(Q1)  $\vec{A} = x^2y\hat{i} + (x-y)\hat{k}$ . find  $\nabla \times \vec{A}$  and  $\nabla \cdot \vec{A}$

$$\nabla \cdot \vec{A} = \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) (x^2y\hat{i} + (x-y)\hat{k})$$

$$\nabla \cdot \vec{A} = 2(x^2y) + 0$$

$$\nabla \cdot \vec{A} = 2xy + 0 \quad \boxed{\nabla \cdot \vec{A} = 2xy}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & 0 & x-y \end{vmatrix}$$

$$i \left( \frac{\partial(x-y)}{\partial y} - 0 \right) + k \left( 0 - \frac{\partial(x^2y)}{\partial y} \right)$$

$$i(-1) + k(0 - 2x^2) \Rightarrow \boxed{-i - 2x^2k}$$

Gradient of scalar field  $\phi(x, y, z) = \vec{\nabla}\phi$

(Q2) Given  $\phi(x, y, z) = 3x^2 - y^3 + yz$  find gradient

of  $\phi(x, y, z)$

$$\text{Given } \phi = 3x^2 - y^3 + yz$$

$$\vec{\nabla}\phi = i \frac{\partial \phi}{\partial x} + j \frac{\partial \phi}{\partial y} + k \frac{\partial \phi}{\partial z}$$

$$\vec{\nabla}\phi = i \frac{\partial(3x^2 - y^3 + yz)}{\partial x} + j \frac{\partial(3x^2 - y^3 + yz)}{\partial y} + k \frac{\partial(3x^2 - y^3 + yz)}{\partial z}$$

$$\vec{\nabla}\phi = (6xi + j(-3y^2) + kz) \text{ at point } (1, 2, 3)$$

$$\vec{\nabla}\phi = 6i - 3j + kz$$

$$\boxed{\vec{\nabla}\phi = 6i - 3j + kz}$$

Teacher's Signature \_\_\_\_\_

Given electric scalar function  $V(x, y, z)$ ,

$v = -x^2 + xy \frac{e}{z} + z^3$  find electric field intensity  
at pt  $(1, -1, 0)$

So  $\therefore \boxed{\vec{E} = -\nabla v}$

$$\vec{E} = -\left[ i \frac{\partial v}{\partial x} + j \frac{\partial v}{\partial y} + k \frac{\partial v}{\partial z} \right]$$

$$\vec{E} = -\left[ i \frac{\partial}{\partial x} (-x^2 + xy \frac{e}{z} + z^3) - j \frac{\partial}{\partial y} (-x^2 + xy \frac{e}{z} + z^3) - k \frac{\partial}{\partial z} (-x^2 + xy \frac{e}{z} + z^3) \right]$$

$$\vec{E} = -i(-2x + y \frac{e}{z}) - j(xe) - k(xy + 3z^2)$$

$$\vec{E}_{(1, -1, 0)} = -i(-2 + 0) - j \cdot 0 - k(1 + 0)$$

$$\boxed{\vec{E} = 2i - 0j - k} \text{ V/m}$$

$$\text{magnitude of } \vec{E} = \sqrt{4+0+1} = \sqrt{5} \text{ V/m}$$

\* De broglie Nomenclature -

$$\lambda = \frac{h}{P}$$

i) Case I In terms of kinetic energy

$$\lambda = \frac{h}{\sqrt{2mK \cdot E}}$$

Q1 An  $\bar{e}$ , an  $\alpha$  particle & a proton have some energy. Arrange them in order of decreasing  $\lambda$ .

De-Broglie 1.

$$\lambda = \frac{h}{\sqrt{2mK}} \quad \lambda \propto \frac{1}{\sqrt{m}}$$

$$\lambda_e > \lambda_p > \lambda_\alpha$$

mass  $\rightarrow$

ii) Case 2 : In terms of potential

$$S.C \rightarrow \lambda = \frac{\sqrt{150}}{\sqrt{V}} \quad \text{in } \textcircled{A}$$

$$\lambda = \frac{h}{\sqrt{2emqV}}$$

$$\lambda = \frac{h}{\sqrt{2emqV}}$$

$$h = 6.626 \times 10^{-31} \text{ Js}$$

$$\text{mass of } \bar{e} = 9.1 \times 10^{-31} \text{ kg}$$

Q2 find de broglie  $\lambda$  associated with an  $\bar{e}$  accelerated at 600V

$$\text{Ans} \rightarrow \lambda = \frac{\sqrt{150}}{\sqrt{V}} \quad \lambda = \frac{\sqrt{150}}{\sqrt{600}}$$

$$\lambda = 0.5 \text{ A}$$

-31  
-13  
-804

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Ques

Calculate the de-Broglie wavelength of a virus particle accelerated by a potential difference of 30,000V.

Ans

$$\lambda = h$$

Jemtoev

$$\lambda = \frac{h}{p}$$

$$h = 6.62 \times 10^{-34} \text{ Js}$$

$$p = \frac{6.62 \times 10^{-34}}{\sqrt{18 \times 1.6 \times 3 \times 10^{-96}}} \text{ kg m/s}$$

$$p = \frac{6.62 \times 10^{-34}}{9.29 \times 10^{-23}} \text{ kg m/s}$$

$$p = \frac{6.62 \times 10^{-34}}{\sqrt{54 \times 1.6 \times 10^{-96}}} \text{ kg m/s}$$

$$p = 6.62 \times 10^{-11} \text{ kg m/s}$$

$$p = \frac{6.62 \times 10^{-34}}{\sqrt{86.4 \times 10^{-96}}} \text{ kg m/s}$$

$$p = 0.71 \times 10^{-11} \text{ kg m/s}$$

$$p = 0.071 \times 10^{-11} \text{ kg m/s}$$

$$p = 0.071 \text{ A.U.}$$

Through short cuts  $\lambda = \sqrt{\frac{180}{V}}$

$$\lambda = \sqrt{\frac{180}{30000}}$$

$$\lambda = \sqrt{\frac{180}{3000}}$$

$$\lambda = \sqrt{0.005}$$

$$\lambda = 0.07 \text{ A.U.}$$

Ques

An electron and a proton are moving with some speed of 300m/s compare the magnitude of de-Broglie wavelength for these particles.

Given mass of an electron  $9.1 \times 10^{-31} \text{ kg}$ , mass of proton  $= 1.67 \times 10^{-27} \text{ kg}$  and plank's constant  $h = 6.62 \times 10^{-34} \text{ Js}$

$$\text{Ans} \quad \lambda_e = \frac{h}{mv} \Rightarrow \lambda_e = \frac{h}{J \cdot m \cdot J \cdot m \cdot m^2 v^2} \quad \lambda_e = \frac{h}{mv}$$

-34

$$de = 6.62 \times 10^{-34}$$

$$\cancel{9.1 \times 10^{-31} \times 300 \text{ m/s}}$$

$$de = 6.62 \times 10^{-34} \frac{9.1}{10} \times 10^{-31} \times 10^{-2}$$

$$de = 0.2424 \times 10^{-5}$$

$$\boxed{de = 0.2424 \times 10^{-5}}$$

$$\cancel{de = 2.424 \text{ nm}} \quad \cancel{= 0.00002424 \text{ nm}}$$

$$dp = \frac{6.62 \times 10^{-34}}{1.67 \times 10^{-27} \times 300} \quad dp = 6.62 \times 10^{-34} \times 10^{-27} \times 10^{-2}$$

$$dp = 1.3213 \times 10^{-9} \quad \boxed{dp = 1.32 \text{ nm}}$$

So wavelength of electron is greater than wavelength of proton.

\* Calculate the potential difference through which an electron should be accelerated to achieve a speed of 1600 m/s and also find the wavelength of the matter wave associated with the moving electron.

$$V = \sqrt{180} \quad \boxed{V = 136}$$

$$V = \frac{h}{mv} \quad d = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 16000}$$

$$d = \frac{6.62 \times 10^{-34} \times 10^{-31}}{9.1 \times 10^{-31} \times 16000} \quad \boxed{d = 0.0484 \times 10^{-6}}$$

$$\cancel{d = 4.84 \times 10^{-7}} \quad \cancel{= 4.84 \text{ nm}}$$

$$V = 180 \quad \cancel{V = 70.42 \text{ V}} \quad d = 45.4 \text{ nm}$$

$$6.073 \quad V = 22.28 \text{ V}$$

(Q) (Pys) An electron is accelerated with low acceleration voltage  $V$ , show that the expression for wavelength of the matter waves associated with the electron is given by  $\frac{12.27}{\sqrt{V}} \text{ Å}$

Ans

$$\lambda = \frac{h}{\sqrt{2mV}} \quad d = 6.62 \times 10^{-31} \times \frac{1.6 \times 10^{-19}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$\lambda = \frac{6.62 \times 10^{-31}}{\sqrt{18 \times 10^{-50} \times V}} \quad d = \frac{6.62 \times 10^{-31}}{\sqrt{10^{-25} \times V}}$$

$$\lambda = \frac{6.62 \times 10^{-31} \times 10^{-28}}{\sqrt{5 \times 10^{-39} \times V}}$$

$$\lambda = \frac{1.227 \times 10^{-9}}{\sqrt{V}}, \quad d = \frac{1.227 \times 10^{-10}}{10}$$

$$\lambda = \frac{12.27 \times 10^{-9} \times 10^1}{\sqrt{V}}, \quad d = \frac{12.27 \times 10^{-10}}{\sqrt{V}}$$

$$\boxed{\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}}$$

\* Calculate the de-Broglie wavelength of an e. whose kinetic energy is 80 eV, given  $h = 6.62 \times 10^{-34} \text{ Joule-sec}$   $m_e = 9.1 \times 10^{-31} \text{ kg}$ ,  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joule}$

Ans

$$\lambda = \frac{h}{\sqrt{2mV}} \quad d = \frac{6.62 \times 10^{-31}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 50 \times 1.6 \times 10^{-19}}}$$

$$\lambda = \frac{6.62 \times 10^{-31}}{\sqrt{1456 \times 10^{-50}}} \quad d = \frac{6.62 \times 10^{-31}}{38 \times 10^{-25}}$$

~~$$\lambda = 0.173 \times 10^{-30} \times 10^{-25}$$~~

$$\lambda = 0.17 \times 10^{-30} \quad \boxed{d = 0.17 \text{ mm}}$$

(Q4) Calculate the velocity and de Broglie wavelength of a proton emerges  $10^5$  eV. Given that mass of proton =  $1.66 \times 10^{-34}$  g; Planck's constant =  $6.62 \times 10^{-34}$  Js, and charge on electrons  $4.8 \times 10^{-10}$  coulombs.

$$d = \frac{h}{mv} \quad \text{seamok}$$

$$d = \frac{h}{\sqrt{5.312 \times 10^{24}}} \quad \text{2.30}$$

$$d = 2.08782 \times 10^{-10}$$

$$d = 2.08 \text{ nm}$$

$$d = \frac{h}{\sqrt{2m \cdot 1.6 \times 10^{-19}}}$$

$$d = \frac{h}{mv}$$

$$v = \frac{h}{md} \quad 16$$

$$v = \frac{6.62 \times 10^{-34}}{1.66 \times 10^{-34} \times 2.08 \times 10^{-10}} \rightarrow v = 6.62 \times 10^{10} \text{ m/s} \quad 4.64$$

$$v = 1.42 \times 10^{10} \text{ m/s}$$

# Quantum mechanics

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\* what is de-Broglie hypothesis or Matter waves

Ans According to de-Broglie hypothesis every moving particle in a medium is associated by a wave. This associated wave is called matter waves.

$$\lambda = \frac{h}{P}$$

Expression for de-Broglie wavelength from Einstein's relation  $E = mc^2$  (1)  
for a light particle  $E = hv$  (2)

Comparing (1) & (2)  $\therefore v = \frac{c}{\lambda}$

$$mc^2 = hv$$

$$mc^2 = \frac{h\lambda}{d}$$

$$mc\lambda = h \quad \lambda = \frac{h}{mc} \quad \therefore P = m \cdot V \quad P = m \cdot c$$

$$\lambda = \frac{h}{P}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{h}{\sqrt{2mc^2V}}$$

$$h = 6.6 \times 10^{-34} \text{ Js}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$e = 1.6 \times 10^{-19} \quad \textcircled{D} = m_0$$

\* calculate the de-Broglie wavelength associated with ~~infinite electrons~~, which are accelerated by a voltage of 50 kV.

$$\text{Ans} \quad \lambda = \frac{h}{P}$$

$$V = 50 \times 10^3 \text{ V}$$

$$\lambda = \frac{6.66 \times 10^{-34}}{\sqrt{2mc^2V}}$$

$$\lambda = \frac{6.66 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 50 \times 10^3}}$$

$$\lambda = 0.55 \times 10^{-11} \quad \textcircled{D} = 0.0055 \text{ nm}$$

(PYQ)

Define phase velocity and group velocity, prove that particle velocity is equal to group velocity for non-relativistic speeds.

→ Phase velocity  $\equiv$  That velocity in which a plane monochromatic wave moves through a medium is called phase velocity or it is denoted by  $v_p$ .

$$\text{Phase velocity } v_p = \lambda d \quad \text{according to Einstein } E = mc^2 \quad \text{--- (1)}$$

$$\text{Energy of photon particle } E = h\nu \quad \text{--- (2)}$$

equate eq (1) & (2)

$$mc^2 = h\nu$$

$$\nu = \frac{mc^2}{h}$$

Now  $\because$  phase velocity  $v_p = \lambda d$

$$v_p = \frac{mc^2}{h} \times d \quad \left\{ \because d = \frac{h}{mv} \right.$$

$$v_p = \cancel{mc^2} \times \frac{h}{\cancel{m}}$$

$$\therefore v_p = \frac{c^2}{v} \quad \text{where } \frac{c^2}{v} > c$$

Phase velocity can be greater than ' $c$ '.

according to Einstein  $E = \gamma m c^2$  (8)

square on

$$E^2 = h\nu$$

$$mc^2 = h\nu$$

$$\nu = \frac{mc^2}{h}$$

put in

$\therefore$  Phase velocity  $[v_p = \nu]$  from (8)

$$v_p = \frac{mc^2}{\kappa} \times \frac{1}{m\nu} \quad [v_p = \frac{c}{\sqrt{\kappa}}]$$

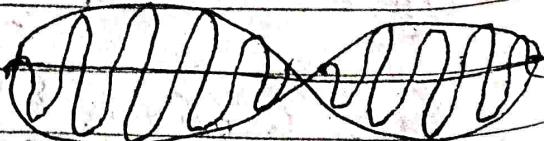
\* group velocity  $\therefore$  That velocity in which group of waves (a wave packet) moves in a medium is called group velocity. It is represented by ( $v_g$ ).

Derivation

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{\omega_2 - \omega_1}{k_2 - k_1}$$

$$g_1 = a \sin(k_1 x - \omega_1 t)$$

$$g_2 = a \sin(k_2 x - \omega_2 t)$$



5/2 → The square of the absolute value of wave function gives probability density to locate the particle at point xyz and time t.

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(Q) What is wave function? Write about significance of wave function. It is mathematical way of representation of given wave & actually represents the state of particle once complete known.

- A wave function is a function which describes the behaviour of a matter wave. It is a function with respect to position & time. It is denoted by  $\psi(\vec{r}, t) = \psi = |\psi|^2$

Significance of wave function

- ① It describes the state of quantum system.
- ② It helps to find probability of a particle at a given point xyz and time t.
- ③ It is used to calculate the energy levels of atoms.

Condition for well behaved wave function

- ① The wave function must be finite.
- ② Wave function must be single valued.
- ③ Wave function must be continuous.
- ④ Wave function must be square integrable.

What is normalisation of wave function?

Total probability to locate the particle in this universe should come to be 1.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P dV = 1, P \rightarrow \text{probability}$$

Normalisation of the particle

$$P \propto |\psi|^2 \rightarrow P = N |\psi|^2$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} N |\psi|^2 dV \quad \text{if } N = 1$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\psi|^2 dV = 1, \psi \text{ is normalise}$$

# (A) Particle in one D - box

Consider a particle is moving in one D - within a box from

$$x=0 \text{ to } x=L$$

$$\begin{cases} V=0 & \text{for } 0 < x < L \\ V=\infty & \text{for } 0 > x > L \end{cases}$$

$$n=0$$

$$x=L$$

From Schrödinger equation,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad \left( \frac{K^2}{\hbar^2} = \frac{8mE}{\hbar^2} \right) \quad (1)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad \left[ \frac{\partial^2 \psi}{\partial x^2} + K^2 \psi = 0 \right] \quad (A)$$

Sol<sup>M</sup> of eq (A)

$$\psi = A \cos kx + B \sin kx \quad (B)$$

$$\text{at } x=0, \psi=0$$

$$\psi = A \cdot$$

$$\text{again } \psi = A \cos kx + B \sin kx$$

$$\text{put } A=0$$

$$\psi = B \sin kx \quad (C)$$

$$\text{at } x=L, \psi=0$$

$$0 = B \sin kL$$

$$\sin kL = 0$$

$$\sin kL = \sin(n\pi) \quad (1, 3, 5, \dots)$$

$$kL = n\pi$$

$$k = \frac{n\pi}{L}$$

put in C

$$\psi = B \sin \left( \frac{n\pi}{L} x \right)$$

(D)

(E)

from eq ② and m  $k = \frac{e^2 m E}{h^2}$   $\therefore h = \frac{\hbar}{\sqrt{k}}$

$$E = \frac{k^2 h^2}{8m}$$

put eq ③ in ②

$$E = \frac{m^2 \pi^2 \cdot h^2}{L^2} \quad E = \frac{n^2 \pi^2 \hbar^2}{L^2 \cdot 8m} \quad E = \frac{n^2 \hbar^2}{8m L^2}$$

$$E = \frac{n^2 \hbar^2}{8m L^2}$$

$$\psi = B \sin\left(\frac{n\pi x}{L}\right) \quad \text{Energy value}$$

$$\int_0^L B^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

→ Normalisation constant

Normalisation Condition

$$\int_0^L B^2 \sin^2\left(\frac{n\pi x}{L}\right) \cdot dx = B^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) \cdot dx = 1$$

$$B^2 \int_0^L 1 - \cos\left(\frac{2n\pi x}{L}\right) \cdot dx = 1$$

$$\frac{B^2}{2} \int_0^L 1 - \cos\left(\frac{2n\pi x}{L}\right) \cdot dx = 1$$

$$\Rightarrow \frac{B^2}{2} \left[ x - \sin\left(\frac{2n\pi x}{L}\right) \right]_0^L = 1 \Rightarrow \frac{B^2}{2} \left[ x - \sin\left(\frac{2n\pi x}{L}\right) \cdot \frac{L}{2n\pi} \right]_0^L = 1$$

$$\Rightarrow \frac{B^2}{2} \left[ L - \sin\left(\frac{2n\pi L}{L}\right) \cdot \frac{L}{2n\pi} \right] = 1 \Rightarrow \frac{B^2}{2} [L - 0] = 1$$

$$\Rightarrow B^2 \frac{L}{2} = 1 \quad ; \quad B^2 L = 2$$

$$B = \sqrt{\frac{2}{L}} \quad \text{④ put in eq ④}$$

$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad \text{⑤}$$

$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$n=1$  ground state

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$$\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

for maximum value

$$\sin\left(\frac{\pi x}{L}\right) = \sin\left(\frac{\pi}{2}\right)$$

$$\frac{\pi x}{L} = \frac{\pi}{2}$$

$$x = \frac{L}{2}$$

$$m=3$$

$$m=2$$

$$m=1$$

$$x=0$$

max

G.

for first excited state

$$m=2$$

$$\psi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$$

for max value  $\Rightarrow \sin\left(\frac{2\pi x}{L}\right) = \sin\left(\frac{\pi}{2}\right)$

$$\frac{2\pi x}{L} = \frac{\pi}{2} \quad \boxed{x = \frac{L}{4}}$$

$$\text{again } \psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

for second excited state

$$m=3 \quad \psi = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right)$$

for max value

$$\sin\left(\frac{3\pi x}{L}\right) = \sin\left(\frac{\pi}{2}\right)$$

$$\frac{3\pi x}{L} = \frac{\pi}{2} \quad \boxed{x = \frac{L}{6}}$$

what is

~~Discuss~~ dispersion relation and discuss various cases of dispersion giving its mathematical expression.

pyro

Schrodinger wave equation for time independent

sol: A general wave expression in differential form

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{r^2} \frac{\partial^2 \psi}{\partial t^2} \quad (1)$$

Given 3D

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{r^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\psi \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) = \frac{1}{r^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\nabla^2 \psi = \frac{1}{r^2} \frac{\partial^2 \psi}{\partial t^2} \quad (2)$$

$$\psi(x, y, z) = \psi_0 e^{-iwt} \quad (3) \text{ this is the sol of eq (2)}$$

diff eq (3) with respect to t

$$\frac{\partial \psi}{\partial t} = \psi_0 (-i\omega) e^{-iwt}$$

again diff wrt to t

$$\frac{\partial^2 \psi}{\partial t^2} = \psi_0 (-i\omega)(-i\omega) e^{-iwt}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \psi_0 (-1)\omega^2 e^{-iwt} \quad \left\{ \begin{matrix} i^2 = -1 \\ \text{from eq (3)} \end{matrix} \right.$$

$$\frac{\partial^2 \psi}{\partial t^2} = \psi_0 e^{-i\omega t} - \omega^2$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \quad \left\{ \begin{matrix} \therefore \omega = 2\pi V \\ \omega = \frac{2\pi V}{h} \end{matrix} \right. \quad \left. \begin{matrix} \therefore V = \frac{\hbar^2}{8m} \\ I.D. = \frac{V}{h} \end{matrix} \right\}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\psi \left( \frac{2\pi V}{h} \right)^2$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\psi \frac{4\pi^2 V}{h^2} \quad \left\{ \begin{matrix} h = \hbar \\ \rho \end{matrix} \right\} \quad \left. \begin{matrix} \frac{\partial^2 \psi}{\partial t^2} = -\psi \frac{4\pi^2 V}{h^2} \\ \frac{\partial^2 \psi}{\partial t^2} = -\psi \frac{222}{h^2} \end{matrix} \right\}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{44\pi^2 \hbar^2 p^2}{h^2} \quad \left\{ \hbar = \frac{\hbar}{2\pi} \Rightarrow \frac{\partial \pi}{\partial t} = \frac{1}{\hbar} \right.$$

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = -\frac{4v^2 p^2}{\hbar^2}}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{4v^2 m^2 v^2}{\hbar^2} \quad \left\{ p = mv \right\}$$

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = -\frac{v^2 m^2 v^2 \psi}{\hbar^2}} \quad (4)$$

Put eq (4) in eq (2)

~~$$\nabla^2 \psi = \frac{1}{\hbar^2} \left[ -v^2 m^2 v^2 \psi \right]$$~~

$$\boxed{\nabla^2 \psi = -\frac{(m^2 v^2 \psi)}{\hbar^2}} \quad \left\{ E = K.E + V \text{ potential Energy} \right.$$

$$E = \frac{1}{2}mv^2 + Vp$$

$$\nabla^2 \psi = -\frac{e^2 m (E - V)}{\hbar^2} \psi \quad (6)$$

$$E = \frac{mv^2}{2} + Vp$$

$$E - V = \frac{mv^2}{2}$$

$$E(E - V) = \frac{mv^2}{2}$$

$$2m(E - V) = mv^2$$

Put in (6) (5)

$$\boxed{\nabla^2 \psi + \frac{e^2 m (E - V)}{\hbar^2} \psi = 0}$$

Time independent

$$\boxed{\nabla^2 \psi + \frac{e^2 m (E - V)}{\hbar^2} \psi = 0}$$

what is dispersion relation and discuss various cases of dispersion give its mathematical expression.

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8 Relation between phase velocity and group velocity or prove that particle velocity  $v_p$  equal to group velocity.

Relationship b/w  $v_g$  &  $v_p$  is known as dispersion relation.

We know that for group velocity

$$v_g = \frac{\Delta \omega}{\Delta k} = \frac{d\omega}{dk} \quad \text{--- (1)}$$

from phase velocity

$$v_p = \frac{\omega}{k} \rightarrow \omega = v_p k \quad \text{--- (2)}$$

diff w.r.t.  $k$

$$\frac{d\omega}{dk} = v_p + k \frac{dv_p}{dk} \quad \text{--- (3)}$$

from eq (1)

$$v_g = v_p + k \frac{dv_p}{dk} \quad \left\{ \begin{array}{l} k = \frac{2\pi}{\lambda}, d = \frac{2\pi}{R} \\ \end{array} \right.$$

$$\lambda = \frac{2\pi}{k}$$

Diff w.r.t.  $\lambda$

$$\frac{dv_p}{d\lambda} = -\frac{2\pi}{k^2} \quad \text{--- (4)}$$

4 multiply by  $d\lambda$   
and divide in last term

$$v_g = v_p + k \frac{dv_p}{dk} \times \frac{d\lambda}{d\lambda} \rightarrow \text{from eq (8)}$$

$$v_g = v_p + k \frac{dv_p}{dk} \cdot \left( -\frac{2\pi}{k^2} \right) \Rightarrow v_g = v_p + \frac{dv_p}{dk} \left( -\frac{2\pi}{k} \right)$$

$$v_g = v_p + \frac{dv_p}{dk} (-1)$$

? from (6)

$$v_g = v_p - \frac{dv_p}{dk} \cdot 1$$

$\frac{dv_p}{dk} = 0$  when

$$v_g = v_p - (0) \cdot 1$$

$$v_g = v_p - 1 \frac{dv_p}{dk}$$

medium is independent  
of frequency

Hence  $\rightarrow v_p \rightarrow$  phase velo

$v_g \rightarrow$  group velo

(a) (i)

VP increases with increasing  $d$  or VP decreases with decreasing  $d$ , then  $\frac{dVP}{dV} \text{ is } +ve \rightarrow \text{Normal dispersion}$   
 $V_g < VP$

(ii) VP does not depend on  $d$ , then  $\frac{dVP}{dV} = 0$

$VP = V_g \rightarrow \text{non dispersive medium.}$

(iii) VP increases with decreasing  $d$  or vice versa  
then  $\frac{dVP}{dV} \text{ is } -ve \Rightarrow V_g > VP \rightarrow \text{Anomalous dispersion.}$

## Eigen function & Eigen value

If  $\hat{x}$  is any operator acting on some function  $f$  and in answer we get function  $f$  itself multiplied by constant ( $\lambda$ ) then equation is known as eigen equation,

$\hat{x}$  is known as eigen operator,  $f$  is known as eigen function and constant  $\lambda$  is known as eigen value.

Ex)  $\frac{d(e^x)}{dx} = 2 \cdot e^x$  is an eigen eq.  
 $\frac{d}{dx}$  is eigen operator for fun<sup>n</sup>  $e^x$  and  $2$  is eigen value.

$\frac{d(\sin x)}{dx} = \cos x$ , here  $\frac{d}{dx}$  is not eigen fun<sup>n</sup>; we are not getting same fun<sup>n</sup> in answer.

~~Q1~~ what do you understand by wave packet

~~Ans~~ A wave packet refers to the case where two or more waves exist simultaneously.

**Q2**

difference between phase velocity & group velocity

i) That velocity in which plane monochromatic wave moves through a medium.

ii) It is represented by  $v_p$

iii) Its formula is

$$v_p = \frac{\omega}{k}$$

That velocity in which group of waves moves in a medium.

It is represented by  $v_g$

Its formula is

$$v_g = \frac{\Delta\omega}{\Delta k}$$

iv) phase velocity is the characteristic of individual wave

group velocity is the characteristic of group of waves.

~~Q3~~ ~~Q4~~

formation of wave packet discuss.

\* ~~electron wave function :-~~

Numerical on One 1D - box

(a)  $E = \frac{n^2 h^2}{8mL^2}$  energy

(b) wave function  $\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$   $n=1, 2, 3, \dots$

(c) Normalization  $\int_{-L}^{+L} \psi^* \psi \cdot dx = 1$

(d) Probability =  $\int_{-L}^{+L} \psi^* \psi \cdot dx$   $\{ \rightarrow \text{d}t/3\}$

(px) find the energy of an electron in a box of width 1 nm in the ground state.

ans  $E = \frac{n^2 h^2}{8mL^2}$  where  $h = 6.62 \times 10^{-31}$   
~~mass  $\rightarrow 9.1 \times 10^{-31}$~~   
 $L^2 = \text{width, length}$

$E = n^2 (6.62 \times 10^{-31})^2$   $L = 1 \text{ nm}$   
 $8 \times 9.1 \times 10^{-31} \times (10^{-9} \text{ m})^2$   $L = 1 \times 10^{-10} \text{ m}$

$E = n^2 (6.62 \times 10^{-31})^2$   $-68$   
 $8 \times 9.1 \times 10^{-31} \times (10^{-9})^2$   $E = \frac{n^2 43.92 \times 10^{-68}}{72 \times 8 \times 10^{-51}}$

$E = n \times 0.60 \times 10^{-17}$

$E = 1 \times 0.60 \times 10^{-17}$

for electron no 6

$E = 0.60 \times 10^{-17} \times 1.6 \times 10^{-19}$   $E = 37.5 \text{ eV}$

pxo

Find the de Broglie wavelength corresponding to the ground state energy for the particle trapped in a 1-dimensional box of length 5 nm.   
 particle  $\rightarrow$  an electron

$$\text{Given } n = 1$$

and  $\Rightarrow$

$$l = 5 \text{ nm}$$

$$l = 5 \times 10^{-9} \text{ m}$$

$$E = \frac{n^2 h^2}{8m l^2}$$

$$E = n^2 \times 0.60 \times 10^{-17} \rightarrow 1$$

~~$E = 1 \times 0.60$~~

$$E = \frac{n^2 \times (6.62 \times 10^{-34})^2}{l^2 (8 \times 9.1 \times 10^{-31})} \rightarrow E = n^2 \left( \frac{43.82 \times 10^{-37}}{72.8} \right)$$

$$E = \frac{n^2 \times 43.82 \times 10^{-37}}{72.8 \times l^2}$$

$$E = \frac{n^2 \times 0.60 \times 10^{-17}}{l^2}$$

$$E = \frac{n^2 \times 0.60 \times 10^{-17}}{25 \times 10^{-18}}$$

$$E = 0.60 \times 10 \times 10^{-18}$$

$$E = 0.024 \times 10^{-19} \quad [ ]$$

$$d = 6.62 \times 10^{-34} \times 3 \times 10^8$$

$$d = \frac{0.024 \times 10^{-19}}{0.024} \times 10^{-26} \times 10^{19}$$

$$d = 827.5 \times 10^{-7} \text{ m}$$

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E}$$

**Q1.** An electron is confined to a one-dimensional potential box of length 0.1 nm. Calculate the difference in energies corresponding to the ground state and first excited state in eV, given me:

$$l = 0.1 \text{ nm} \quad [l = 0.1 \times 10^{-9} \text{ m}]$$

find energy at ground & first excited state

$$\text{mass} = 9.1 \times 10^{-31} \text{ kg} \quad E = \frac{n^2 h^2}{8ml^2}$$

$$E = \frac{n^2 \times 0.60 \times 10^{-37}}{l^2} \text{ J}$$

at ground state  $n=1$

$$E = \frac{1 \times 0.60 \times 10^{-37}}{0.1 \times 10^{-9}} \Rightarrow E = 6 \times 10^{-19} \text{ J}$$

$$E = \frac{6 \times 10^{-19}}{1.6 \times 10^{-19}} \quad | \quad E_g = 3.75 \times 10^{-9} \text{ eV}$$

$$E_{es} = \frac{n^2}{l^2} \times 0.60 \times 10^{-37} \quad E_{eg} = \frac{(2)^2}{0.1 \times 10^{-9}} \times 0.60 \times 10^{-37}$$

$$E_{eg} = \frac{4 \times 0.60 \times 10^{-28}}{1.6 \times 10^{-19}} \Rightarrow E_{eg} = 24 \times 10^{-9} \text{ J}$$

$$E_{es} = 15 \times 10^{-9} \text{ J}$$

$$E_{es} - E_{eg} = (15 \times 10^{-9}) - (3.75 \times 10^{-9})$$

$$E_{eg} = 11.25 \times 10^{-9} \text{ J}$$

(PYO) An electron is confined to a one dimensional potential box of length  $2\text{Å}$ . Calculate the energies corresponding to the second and fourth quantum state in eV.

$$m_e \Rightarrow l = 2\text{Å} \quad [l = 2 \times 10^{-10} \text{ m}]$$

Calculate energies at second and third excited states.

$$E_{\text{exc.state}} = \frac{n^2 h^2}{l^2 8m}$$

$$E_{\text{SS}} = \frac{n^2}{l^2} \times 0.60 \times 10^{-37} \text{ J} \Rightarrow \frac{(1.3)^2}{(2 \times 10^{-10})^2} \times 0.60 \times 10^{-37}$$

$$E_{\text{S.S}} = \frac{9}{4 \times 10^{-20}} \times 0.60 \times 10^{-37} \text{ J} \Rightarrow E_{\text{S.S}} = \frac{50.4}{4} \times 10^{-17} \text{ J}$$

$$\boxed{\frac{E_{\text{S.S}} = 1.35 \times 10^{-17}}{1.6 \times 10^{-19}}} \quad \boxed{E_{\text{SS}} = 0.84 \times 10^{-2} \text{ eV}}$$

$$E_{\text{4th state}} = \frac{n^2}{l^2} \times 0.60 \times 10^{-37} \text{ J}$$

$$= \frac{(5)^2}{(2 \times 10^{-10})^2} \times 0.60 \times 10^{-37} = \frac{25 \times 0.60}{4 \times 10^{-20}} \times 10^{-37}$$

$$E_{\text{4ths}} = \frac{15}{4} \times 10^{-37} \times 10^{-20} \Rightarrow 3.75 \times 10^{-17} \text{ J}$$

$$E_{\text{4th.S}} = \frac{3.75 \times 10^{-17}}{1.6 \times 10^{-19}} \quad \boxed{E_{\text{4th.S}} = 2.34 \times 10^2 \text{ eV}}$$

Energy corresponding to second quantum state =  $\boxed{0.84 \times 10^{-2} \text{ eV}}$

Energy corresponding to fourth quantum state =  $\boxed{2.34 \times 10^2 \text{ eV}}$

(Q) Find the probability of finding a particle trapped in a 1-dimensional box of length  $L$  between  $0.45L$  &  $0.55L$ . Assume the particle to be in ground state and first excited state

$$\Rightarrow \text{probability} = \int_{m_1}^{m_2} \psi^* \psi \cdot dx \quad \& m_1 = 0.45L \\ m_2 = 0.55L \\ n = 1$$

$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \Rightarrow \psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$P = \int_{0.45L}^{0.55L} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \times \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) dx$$

$$P = \int_{0.45L}^{0.55L} \sqrt{\frac{2}{L}} \sqrt{\frac{2}{L}} \cdot \sin^2\left(\frac{n\pi x}{L}\right) dx \quad \begin{aligned} \cos 2x &= 1 - 2\sin^2 x \\ \sin^2 x &= 1 - \cos 2x \end{aligned}$$

$$P = \frac{2}{L} \int_{0.45L}^{0.55L} \frac{1}{2} (1 - \cos(2n\pi x)) \cdot dx$$

$$P = \frac{2}{L} \times \frac{1}{2} \left[ x - \frac{\sin(2n\pi x)}{2n\pi} \right]_{0.45L}^{0.55L}$$

$$P = \frac{2}{L} \left[ x - \frac{\sin(2n\pi x)}{2n\pi} \right]_{0.45L}^{0.55L}$$

Put  $n=1$

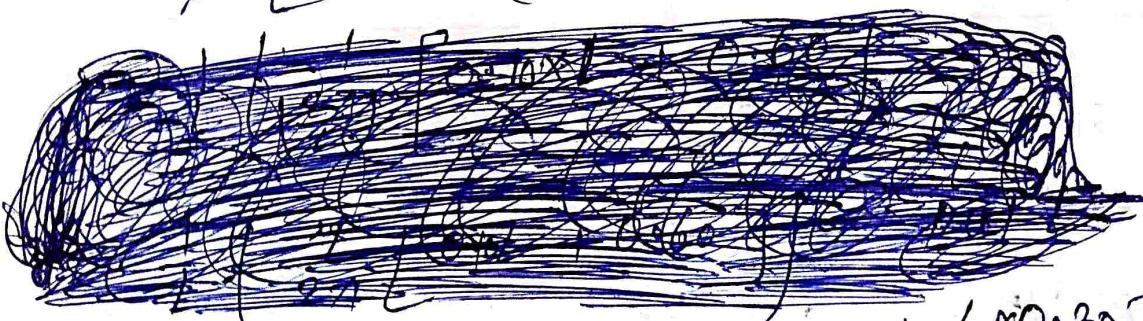
0.55L

$$P = \frac{1}{L} \left[ \text{circled } [x - \sin\left(\frac{2n\pi}{L}\right)] \right]_{0.45L}^{0.55L}$$

$$P = \frac{1}{L} \left[ \text{circled } [0.55L - \sin\left(\frac{2(0.55\pi)}{L}\right)] \right]_{0.45L}^{0.55L} \\ - \left[ 0.45L - \left(\frac{L}{2\pi}\right) \sin\left(\frac{2(0.45L)\pi}{L}\right) \right]_{0.45L}^{0.55L}$$

$$P = \frac{1}{L} \left[ \text{circled } [0.55L - \left(\frac{L}{2\pi}\right) \sin(1.01\pi)] - [0.45L - \left(\frac{L}{2\pi}\right) \sin(0.9\pi)] \right]$$

$$P = \frac{1}{L} \left[ \text{circled } [0.55L + \frac{0.30}{\frac{L}{2\pi}}] - [0.45L + \frac{0.30}{\frac{L}{2\pi}}] \right]$$



$$P = \frac{1}{L} \left[ 0.55L + \frac{L}{2\pi} \times 0.30 - 0.45L + \frac{L}{2\pi} \times 0.30 \right]$$

$$P = \frac{1}{L} \left[ 0.10L + \frac{L}{2\pi} \times 0.60 \right]$$

$$P = \frac{1}{L} \left[ 0.10 + \frac{0.60}{2\pi} \right]$$

$$P = \left[ 0.10 + \frac{0.60}{2 \times 3.14} \right]$$

$$P = [0.10 + 0.095],$$

$$\boxed{P = 0.1954}$$

$$\text{im } \therefore P = 0.1954 \times 100$$

$$\boxed{P = 19.5\%}$$

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$$P = \frac{1}{L} \left[ x - L \sin \left( \frac{2\pi n x}{L} \right) \right] \quad \begin{matrix} 0.55L \\ 0.45L \end{matrix}$$

for excited state put  $n=2$

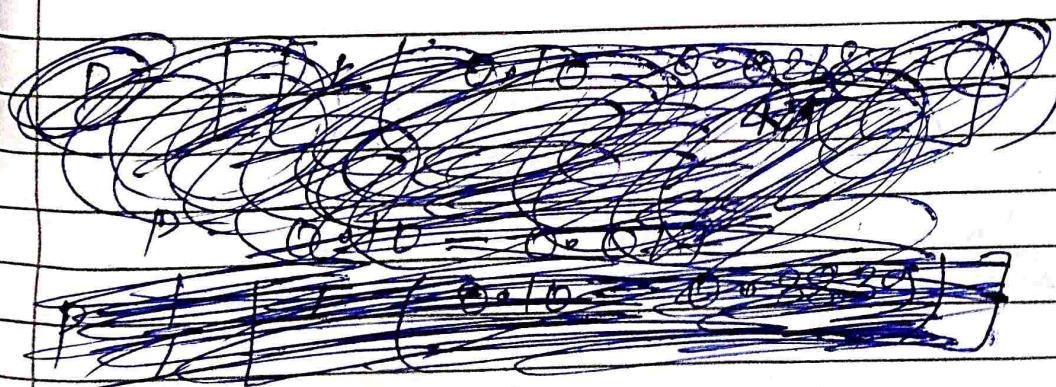
$$P = \frac{1}{L} \left[ x - L \sin \left( \frac{4\pi x}{L} \right) \right] \quad \begin{matrix} 0.55L \\ 0.45L \end{matrix}$$

$$P = \frac{1}{L} \left[ 0.55L - L \sin \left( \frac{4\pi (0.55L)}{L} \right) - (0.45L - L \sin \left( \frac{4\pi (0.45L)}{L} \right)) \right]$$

$$= \frac{1}{L} \left[ 0.55L - L \sin (6.911) - 0.45L + L \sin (5.65) \right]$$

$$P = \frac{1}{L} \left[ 0.55L - L \times \cancel{0.20327} - 0.45L + \cancel{0.177300} \right] \quad (-0.59)$$

$$P = \frac{1}{L} \left[ 0.10L - L \times \cancel{0.21877} \right] \quad P = \frac{1}{L} [0.10L - 1.773L]$$



$$P = \frac{1}{L} \times 0 [0.10 - \cancel{0.21877} - 0.10] = 0.0062 \times 100$$

$$\text{int. } P = 0.0062 \times 100$$

**P = 0.62 %**  
Teacher's Signature: \_\_\_\_\_

in 1 m in excited state.

PXO

Consider a 1D box of length  $1/4$  m in which a particle is trapped. find the particle wave function and the corresponding energy for its 2<sup>nd</sup> excited state.

Sol<sup>m</sup>:  $L = 1/4$   $\boxed{\psi = ?}$   $\boxed{E = ?}$  for excited state

wave function  $\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$

$$\psi = \sqrt{\frac{2}{1/4}} \sin\left(\frac{n\pi x}{1/4}\right)$$

$$\psi = \sqrt{\frac{8}{1}} \sin\left(\frac{4n\pi x}{1}\right)$$

$$\boxed{\psi = 2\sqrt{\frac{8}{1}} \sin\left(\frac{4n\pi x}{1}\right)}$$

$$E = \frac{n^2 h^2}{8mL^2} \quad E = \frac{n^2}{1^2} (0.60 \times 10^{-37} \text{ J}) \quad \text{excited state } n=3$$

$$E = \frac{(2)^2}{(1)^2} \times 0.60 \times 10^{-37} \text{ J}$$

$$E = \frac{64}{1^2} \times 0.60 \times 10^{-37} \text{ J}$$

$$E = \frac{38.4 \times 10^{-37}}{1^2} \text{ J}$$

in terms of eV

$$E = \frac{24}{1^2} \times 10^{-18} \text{ eV}$$

**Ques** The wave function of a certain particle is  $\psi = A \cos x$  for  $-\pi/2 < x < \pi/2$ . Find the value of  $A$ . Also find the probability that a particle be found between  $x=0$  and  $x=\pi/4$ .

(Ans) Sol : Probability

$$P = \int_0^{\pi/4} \psi \cdot \psi^* \cdot dx \Rightarrow \int_0^{\pi/4} \psi \cdot \psi^* \cdot dx \text{ where } \psi = \sqrt{\frac{2}{L}} \sin\left(\frac{m\pi x}{L}\right)$$

$$P = \int_0^{\pi/4} \sqrt{\frac{2}{L}} \sin\left(\frac{m\pi x}{L}\right) \cdot \sqrt{\frac{2}{L}} \sin^2\left(\frac{m\pi x}{L}\right) \cdot dx$$

$$P = \frac{1}{L} \int_0^{\pi/4} 1 - \cos\left(\frac{m\pi x}{L}\right) \cdot dx$$

~~$$P = \frac{L}{2} \left[ x - \frac{1}{m\pi} \sin\left(\frac{m\pi x}{L}\right) \right]_0^{\pi/4}$$~~

~~$$P = \frac{1}{L} \left[ \frac{\pi}{4} - \frac{L}{m\pi} \sin\left(-\frac{m\pi \cdot 0}{L}\right) \right] = 0.7$$~~

$$P = \frac{1}{L} \left[ \frac{\pi}{4} - \frac{L}{m\pi} \sin\left(\frac{m\pi^2}{4L}\right) \right]$$

**PxQ** Give a detailed description of spontaneous and stimulated emission and hence derive the relation for the Einstein coefficients for the same.

Sohin  $\rightarrow$  Page [1, 2 & 3]

**PxQ** obtain the relation between transition probabilities of spontaneous and stimulated emission.

Mention characteristic properties of a laser beam

- i) monochromatic (single frequency, single wavelength)
- ii) intense beam focused on a very small area
- iii) coherent (highly coherent)

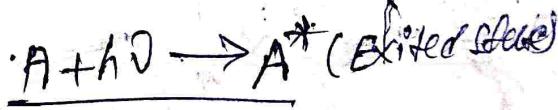
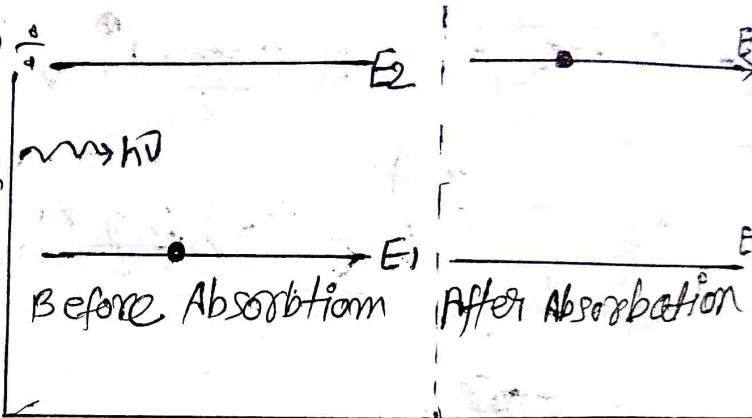
Page  $\rightarrow$  0.01, 1 & 2

### \* Stimulated Absorption

Consider that initially an atom is in lower energy state having energy  $E_1$ , if a photon of energy  $h\nu$  is incident on an atom in lower state, then atom absorbs this energy & jumps to upper state whose energy  $E_2$ .

$$h\nu = E_2 - E_1$$

This process is known as stimulated absorption.



**PxQ** Describe how laser radiation is different from ordinary light? and laser light is better than ordinary light. Why?

Ans: Ordinary light is divergent and incoherent whereas laser radiation is highly intense, highly coherent, highly directional this is the how laser radiation is different from ordinary light?

Laser light is better than ordinary light because of laser light is highly intense, highly directional and highly coherent that's why better than ordinary light.

On which factor stimulated absorption depends

- i) The no of atoms in the ground state.
- ii) The energy of the incoming photon.
- iii) The intensity of the incoming photon.
- iv) nature of atom
- v) energy difference

# Laser

Page No. ①

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What is laser and its full form.

Ans ⇒ Laser is a device that emits a monochromatic, highly intense, unidirectional beam of light.

The full form of laser is

L → Light

A → Amplification by

S → Stimulated

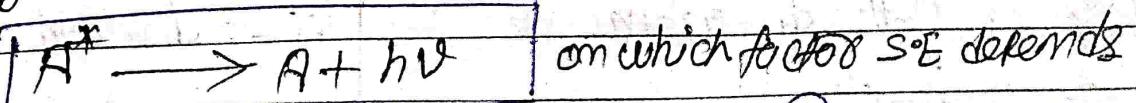
E → Emission of

R → Radiations

Spontaneous Emission

Consider that initially an atom is in higher energy state  $E_2$ , atom is excited before emission. After emission state lives for  $10^{-8}$  sec, so ~~the~~ atom will make a transition to come into lower energy state  $E_1$ . During this transition atom will release a photon of energy  $h\nu$  where  $h\nu = E_2 - E_1$ , this process is called spontaneous emission.

Thus we can say that the process of emission of a photon by an atom without any external energy is called Spontaneous emission.

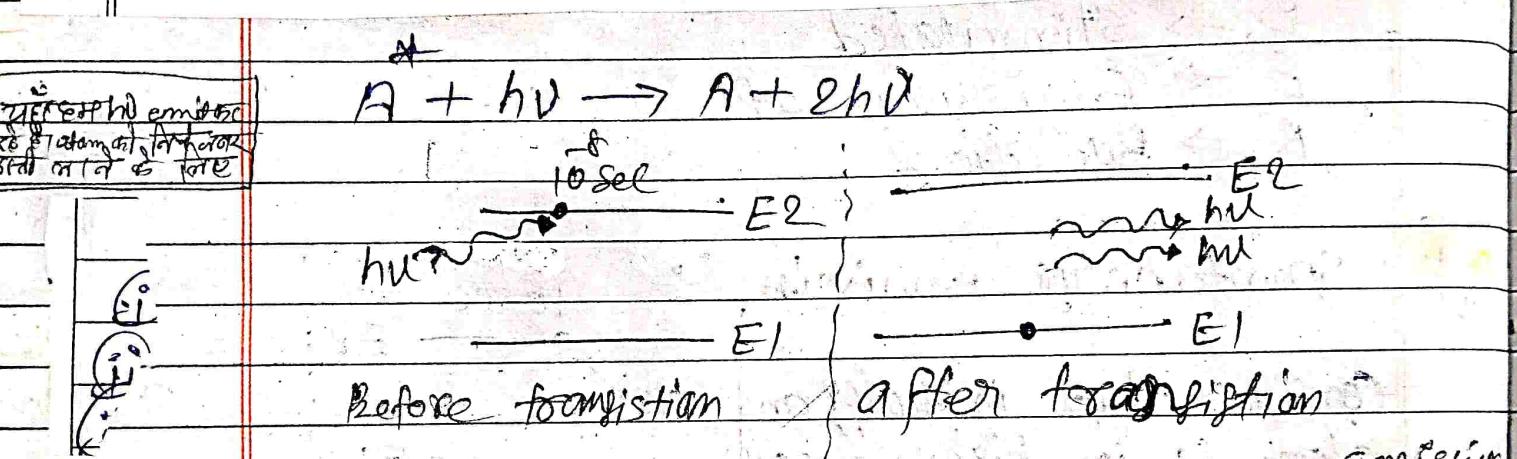


- ① Energy difference (ii) Nature of atom
- ii) Excited state lifetime (iii) Temperature
- ① no. of atom in excited state.

\* stimulated emission : At high energy state when energy  $h\nu$  is given to the atom, it falls back to the ground energy state by emitting two photons. It is called and these two photons have same frequency, same direction same phase is called stimulate emission.

on which factor stimulated emission depends.

- (i) no of atoms present in the excited state.
- (ii) nature of atom.
- (iii) energy of given photon.



RX 10

Differentiate spontaneous emission and stimulated emission

(i) emitted photons move in the same direction.

Spontaneous emission emitted photons move in all directions.

(ii) the rate of transition is

No need of external energy the rate of transition is

$$\text{Rate} = B_2 N_2 E(v)$$

$$\text{SPE} = A_2 N_2$$

it emits two photons.

it emits only one photon.

(iii) light emitted through this process is coherent.

light emitted through this process is incoherent.

(iv) High directional

less directional.

(v) High intense

less intense.

(vi) Coherent radiation

incoherent.

\* Some basic background for Einstein's coefficients:

Three process of (E.C) (Three Einstein's coefficients)

- (i) Stimulated ~~emission~~ Absorption
- (ii) Spontaneous emission
- (iii) Stimulated emission

Absorption

i)  $B_{12} \rightarrow$  Einstein coefficient for stimulated ~~emission~~

ii)  $B_{21} \rightarrow$  " " " " emission

iii)  $A_{21} \rightarrow$  " " " " Spontaneous emission

Note → Rate of Stimulated Absorption  $\rightarrow (R_{SA})$

Rate of Stimulated emission  $\rightarrow (R_{SE})$

Rate of spontaneous emission  $\rightarrow (R_{SPE})$

PYQ) find the units of Einstein coefficients.

[Jule  $\text{sec}^{-2} \text{m}^3$ ]

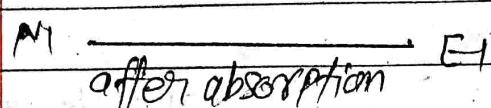
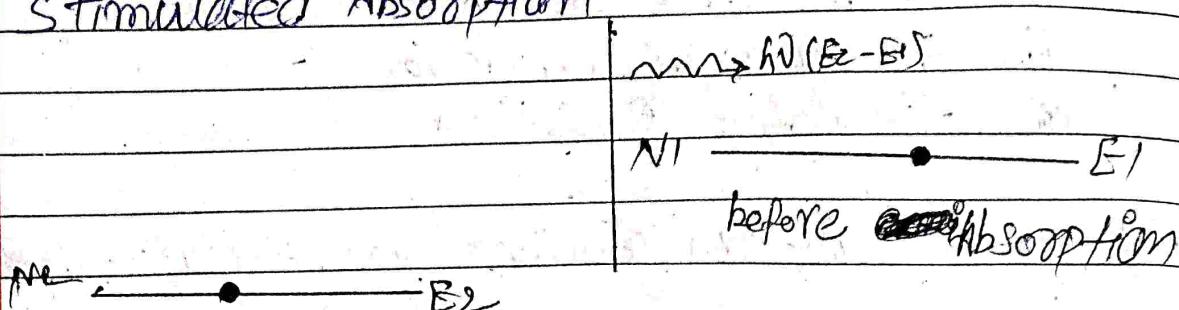
Q) Explain principle of working of laser.

Ans) The principle of working of laser is based on the process of stimulated emission of radiation.

**p.v**

Einstein's Coefficients & 3rd right condition  
necessary for losing action to take place.

### (1) Stimulated Absorption



**p.1** i) Rate of Stimulated absorption is directly proportional to ( $N_1$ ) How much atoms present in ground state.  
 $R_{STA} \propto N_1$

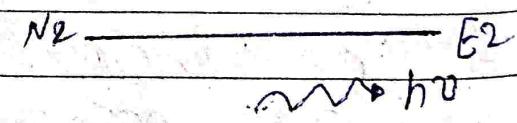
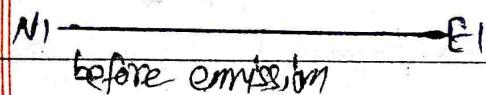
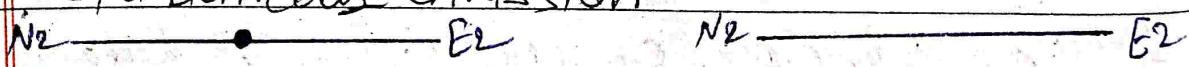
ii) Rate of Stimulate absorption is directly proportional to incident radiation of energy density.

$$R_{STA} \propto N_1 E(v)$$

where  $B_{12}$  is proportionality coefficient

$$R_{STA} = B_{12} N_1 E(v) \quad (A)$$

### (2) Spontaneous emission



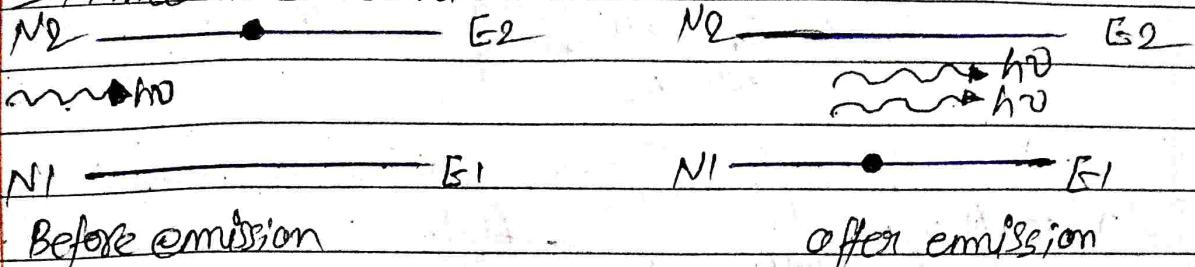
i) Rate of Spontaneous emission is directly proportional to ( $N_2$ ) how much atoms present in excited state

$$R_{SPE} \propto N_2$$

where  $A_{21}$  is proportionality constant

$$R_{SPE} = A_{21} N_2 \quad (B)$$

## (3) Stimulated emission



D 3

(i) Rate of stimulated emission is directly proportional to  $(N_2)$  how much atoms present in excited state.

$$R_{STE} \propto N_2$$

(ii) Rate of stimulated emission is directly proportional to  $E(v)$  incident radiation of energy density.

$$R_{STE} \propto E(v)$$

$\left\{ \begin{array}{l} B_21 \rightarrow \text{is proportionality} \\ \text{constant} \end{array} \right.$

$$R_{STE} \propto N_2 E(v)$$

$$R_{STE} = B_21 N_2 E(v) \quad (c)$$

$$R_{STA} = B_12 N_1 E(v) \quad (A)$$

$$R_{SPE} = A_21 N_2 \quad (B)$$

In thermodynamic equilibrium

$$R_{STA} = R_{SPE} + R_{STE} \quad \text{by using eq A, B, C}$$

$$B_12 N_1 E(v) = A_21 N_2 + B_21 N_2 E(v) \quad \text{According to Maxwell Boltzmann law}$$

$$B_12 N_1 E(v) - B_21 N_2 E(v) = A_21 N_2 \quad (N_1 = (E_2 - E_1) / k_B T)$$

$$E(v) (B_12 N_1 - B_21 N_2) = A_21 N_2 \quad \left\{ \begin{array}{l} N_2 \\ e \\ \therefore E_2 - E_1 = h\nu \end{array} \right.$$

$$E(v) = \frac{A_21 N_2}{B_12 N_1 - B_21 N_2} \quad (E)$$

$$E(v) = \frac{A_21 N_2}{B_21 N_2 / B_12 N_1 - 1} \quad \text{using eq D \& E put eq E in}$$

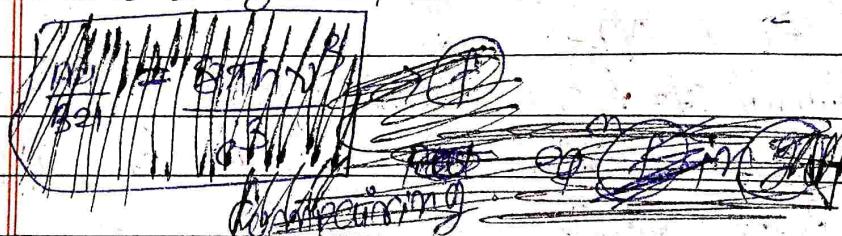
$$E(v) = A_21 \quad (D)$$

$$B_21 \left( \frac{B_12 N_1}{B_21 N_2} - 1 \right)$$

$$E(v) = \frac{A_{21}}{B_{21} [B_{12} N_1 - 1]} \quad \therefore \frac{N_1}{N_2} = \frac{h^v / k_B T}{e}$$

$$E(v) = \frac{A_{21} \times 1}{B_{21} \left[ \frac{B_{12}}{B_{21}} e^{\frac{h^v}{k_B T}} - 1 \right]} \quad \text{Eq. 9}$$

according to Planck's radiation law,



External pumping is required to supply energy to the system

$$E(v) = \frac{8\pi h v^3}{c^3} \left[ e^{\frac{h^v}{k_B T}} - 1 \right] \quad \text{Eq. h}$$

and achieve the necessary condition for lasing.

by Comparing eq. (9) & h

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3} \quad \& \quad \frac{B_{12}}{B_{21}} = 1$$

$$B_{12} = B_{21}$$

PQ

Discuss on what factors the rate of stimulated absorption per unit volume ( $R_{SA}$ ) and rate of stimulated emission per unit volume ( $R_{STE}$ ) depend. Write their mathematical expression. Hence find condition when rate of stimulated emission per volume will be more than rate of stimulated absorption per unit volume.

Calling (P.1, P.2, P.3) here  $R_{SA} \leftrightarrow R_{STE}$

$R_{STE} \rightarrow R_{STA}$

$$\text{For } R_{STE} = B_{21} N_2 E(v)$$

$$\begin{array}{l} R_{STE} \\ "R_{STA} \end{array} \rightarrow \gg 1$$

$$R_{STA} = B_{12} N_1 E(v) \quad \text{Eq. B}$$

Q. Compare the properties of He-Ne and ruby laser.

ans →

Property	Ruby laser	He-Ne laser
1. Active medium	Ruby-crystal	He- Ne-gass 10:1
2. Pumping mechanism	optical pumping	Electric discharge
3. Output power	Up to 100 watts	Up to 5 milliwatt
4. Beam quality	poor	good
5. Applications	laser surgery, Spectroscopy, Rangefinding, Holography, Research and development	display technology, Barcode scanning, interferometry.

Q. Compare the properties of He-Ne and CO<sub>2</sub> laser -

Property	CO <sub>2</sub> laser	He-Ne laser
Active medium	N <sub>2</sub> & He	He- Ne-gass 10:1
Pumping mechanism	Electric discharge	Electric discharge
Output power	Up to 100 kW	Up to 50 milliwatt
Beam quality	good Up to 100 kW	Excellent <del>Up to 50 milliwatt</del>
Applications	Cutting, welding, drilling, marking.	display technology, Barcode scanning, interferometer

⑧ Application of optical fiber

ans → Telecommunication, medical, Defense, industry  
High speed Net service.

Q metastable state  
Ans An excited state of an atom that remains stable for a relatively long time before emitting a photon & returning to its ground state.

Q what do you mean by total internal reflection  
give two example of TIR in daily life.

Ans The angle at which the incident ray travelling from denser to rare medium is equal to or greater than critical angle then ray will graze & reflected back into the same medium. This phenomenon is called TIR.

Q write three transition in He-Ne laser

- i)  $E_6 - E_5$  - infrared -  $3.39 \mu\text{m}$
- ii)  $E_6 - E_3$  - visible (Red) -  $\lambda = 6328 \text{\AA}$
- iii)  $E_4 - E_3$  - infrared -  $1.15 \mu\text{m}$

→ ① Fiber optic communication  
② Diamond sparkling.

Q Metastable State → A metastable state refers to an excited state of an atom that has a relatively long lifetime compared to other excited states.

dividing eq (A)  $\Rightarrow$  (B)

$$\frac{B_{21} N_2 E(v)}{B_{12} N_1 E(v)} \gg 1^{\circ} \quad \text{Gingstein Coefficient}$$

$$B_{12} = B_{21}$$

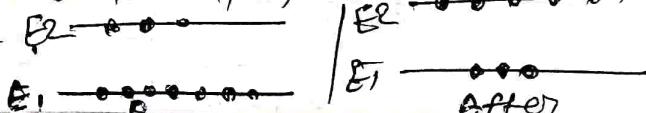
$$\frac{B_{21} N_2}{B_{12} N_1}$$

$$\frac{N_2}{N_1} \gg 1 \quad N_2 \gg N_1 \quad \text{Population}$$

~~Population inversion~~

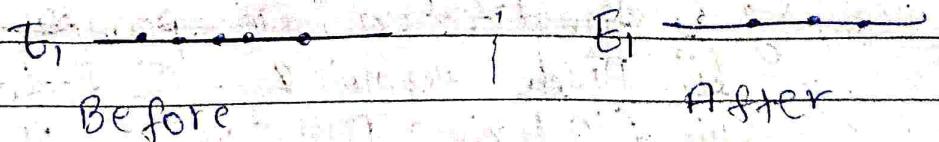
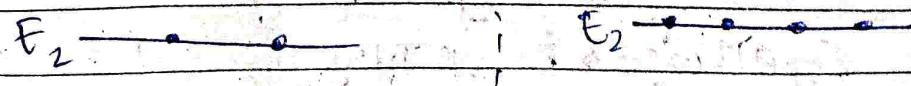
Q) What is population inversion and what is need to achieve population inversion?

Ans) population inversion: population inversion in a laser refers to the condition where no of atoms in excited state is greater than ground state, which is necessary for laser amplification and emission of coherent light



To achieve population inversion, it is necessary to excite the atoms or molecules into the higher energy states by supplying energy from an external source such as optical pumping. this is need.

The situation in which the no. of atoms in the higher energy state exceeds that in the lower energy state ( $N_2 > N_1$ ) is known as "population inversion".

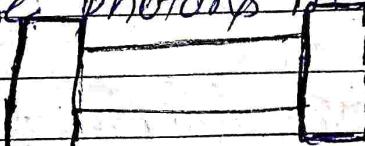


Ques

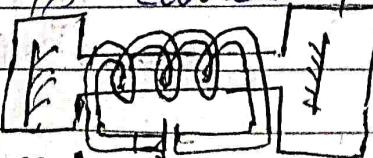
How does a laser work? Explain active medium, optical pumping and optical resonators.

Ans → lasers produce a narrow beam of light in which all of the light wave have similar wavelength.

Laser / active medium : Active medium is a material in which laser action takes place. The active medium may be solid, liquid or gases. It contains atoms or molecules that can be excited into a higher energy state so when these excited atoms or molecules return to their lower energy state they release photons in the form of laser light



Optical pumping : Optical pumping is the process used to excite the atoms or molecules in the active medium. This is done by using light source which can be form of lamp. The light source provides energy to the atoms or molecules exciting them into a higher energy state. This process is called population inversion.

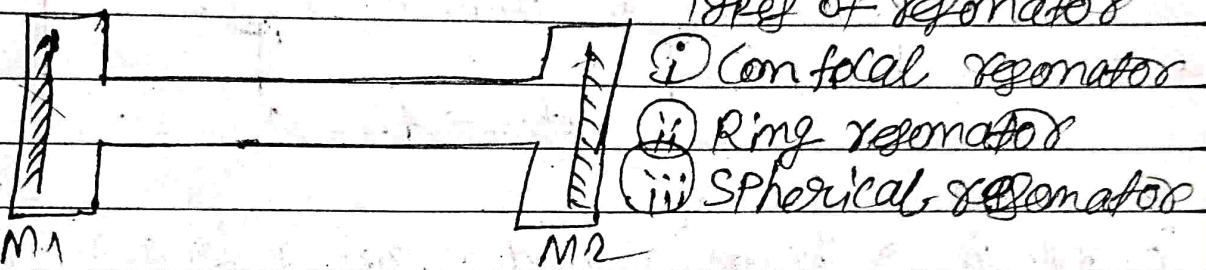


Optical resonators : Types of O.P

- i) Discharge lamp pumping
- ii) Stimulated emission pumping
- iii) Diode pumping
- iv) Electrical flashlamp pumping

Optical resonator: An optical resonator is a device used to reflect the light to convert light into a laser radiation, resonator consists two mirrors placed at end of active medium. One mirror is totally reflective another is partially reflective, laser light passes through partially reflective mirror.

### Types of resonator



i) Confocal resonator

ii) Ring resonator

iii) Spherical resonator

(PQ) In LASER, in place of 'A', it should be 'O' why?

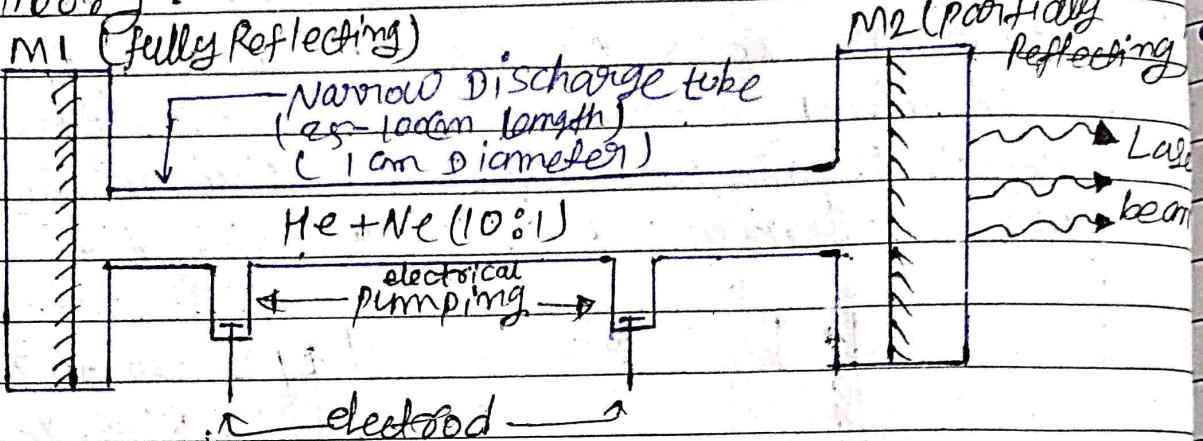
ans  $\Rightarrow$  'A' refers to Amplifiers it is always either no feed back or negative feed back this is employed on the other hand stimulated emission is basically oscillation process so 'O' refers to oscillation. Hence in place of (A) there must be 'O' in the word LASER.

(PQ) What is the role of helium in He-Ne laser

ans  $\Rightarrow$  The role of the Helium(gas) in He-Ne laser is to increase the efficiency of the lasing process.

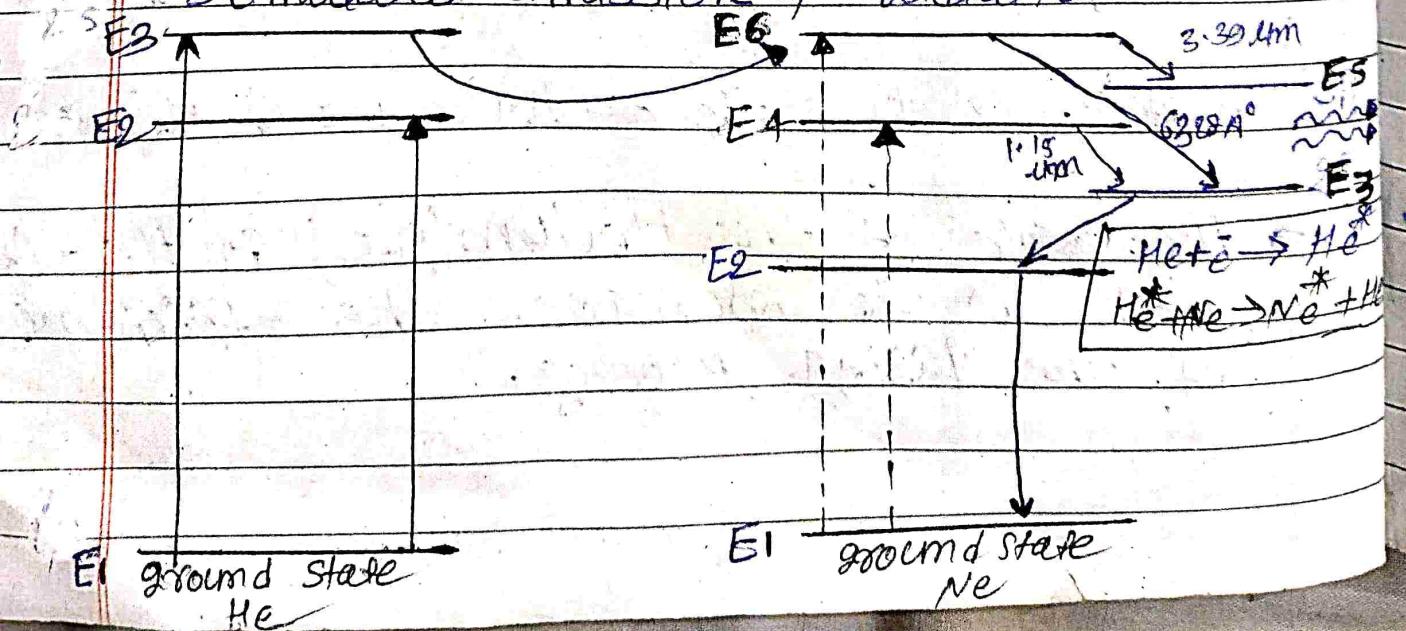
PYQ

(He-Ne laser) or (4 level laser) or (gasous laser)  
 it's construction, working, diagram, principle, theory.

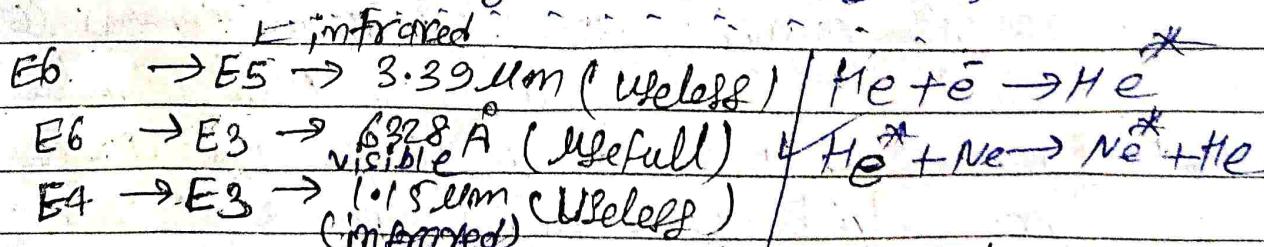


Construction  $\Rightarrow$  Shown in above diagram a long narrow discharge tube length 25-100 cm & diameter 1 cm. We filled Helium (He) & Neon (Ne) gasses of amount (10:1) at 1 mm of Hg and 0.1 mm of Hg pressure. There are two mirror ( $M_1$  &  $M_2$ ) at the corner of the tube makes resonator cavity,  $M_1$  mirror is fully reflective and  $M_2$  mirror is partially reflective. There are two electrical pumping attached with tube having two different electrode.

principle  $\therefore$  He-Ne laser works on the principle of stimulated emission of radiation.



Working :- As we apply dc voltage in tube than there electron will be produce in tube, Now there are two atoms He & Ne in tube So Helium atom will be excited by electron because mass of Helium is 4 and mass of neon is 20 So firstly Helium will be excited and again neon. So Helium will be gone from ground state to excited state, so excited Helium atom will ~~transfor~~ emerge to the neon (Ne) present in ground state So neon will be gone to excited state from ground state after this neon atom will show transition and come to lower level than when a electron is coming from E<sub>6</sub> level to E<sub>5</sub> level than it emits a photon of 3.39 nm when electron will jump from E<sub>6</sub> to E<sub>3</sub> level than again it will be emitted a photon of 6328 Å and this photon will bring the new atoms of the second excited of neon atom from upper level to the ground state, then another photon will be emitted So this photon has 6328 Å and when a electron jumped from E<sub>6</sub> level to E<sub>3</sub> level that ~~the~~ those electron was emitted a photon of 6328 Å So both photon will be same wavelength and these two photon will be recombined these two photon will be reflected between two mirror M<sub>1</sub> & M<sub>2</sub> again and again than we will get laser light of red colour



Application :- ① 3D images of objects | to read bar codes.

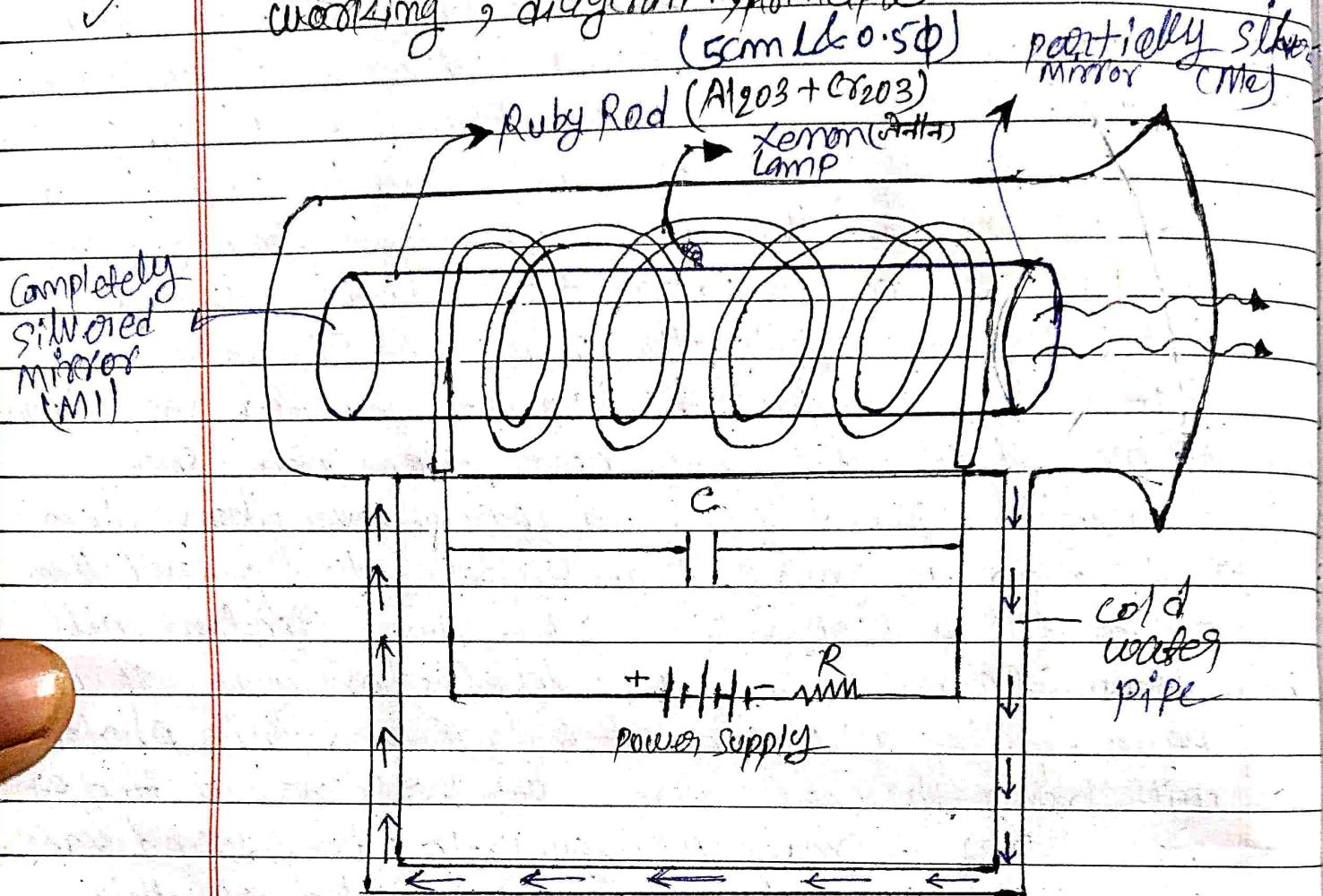
- (i) in laboratory
- (ii) in library to scan bar code.

## (Solid laser)

Pyro

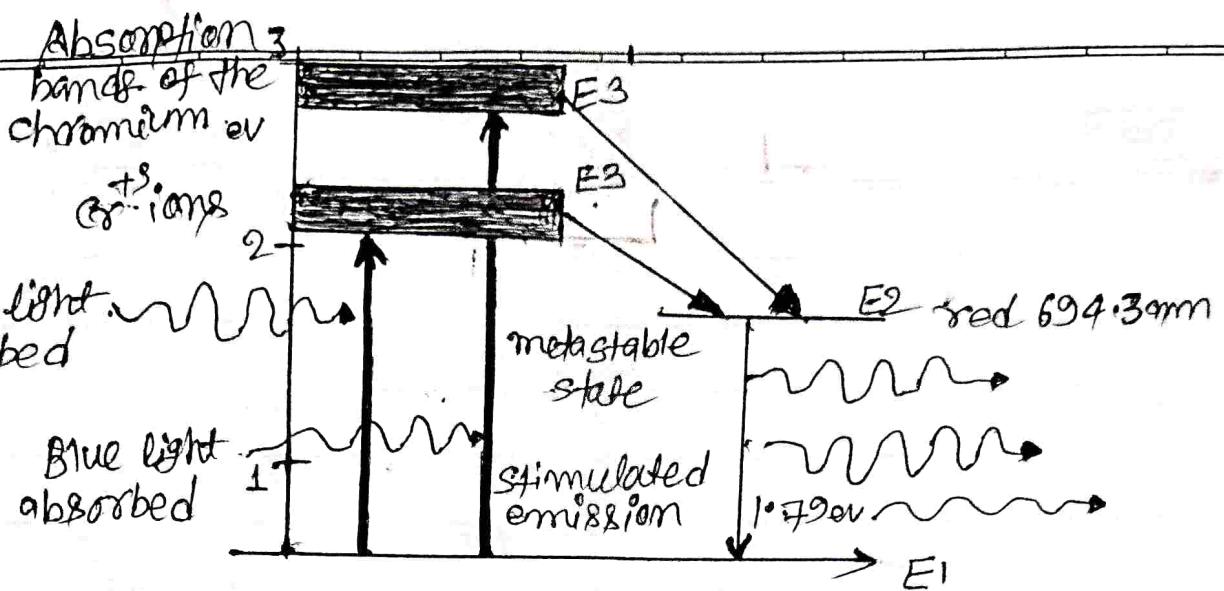
(Ruby laser) or three level laser Construction, working, diagram, principle

(5cm L &amp; 0.5φ)

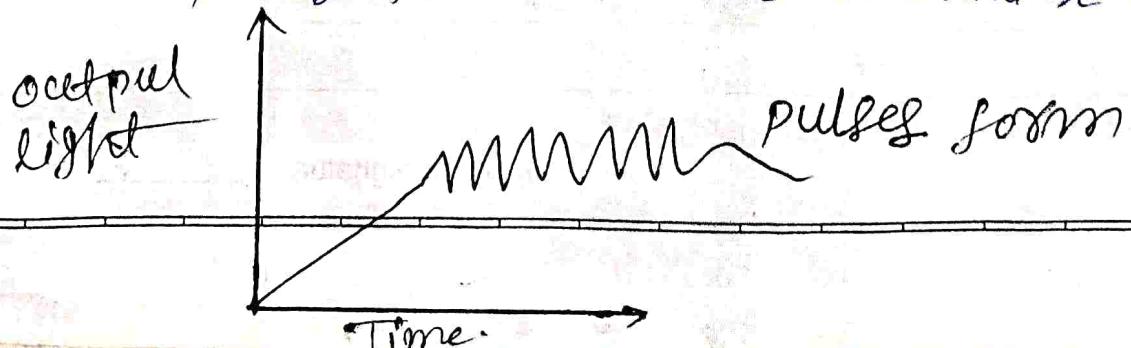
partially silvered mirror (M<sub>2</sub>)

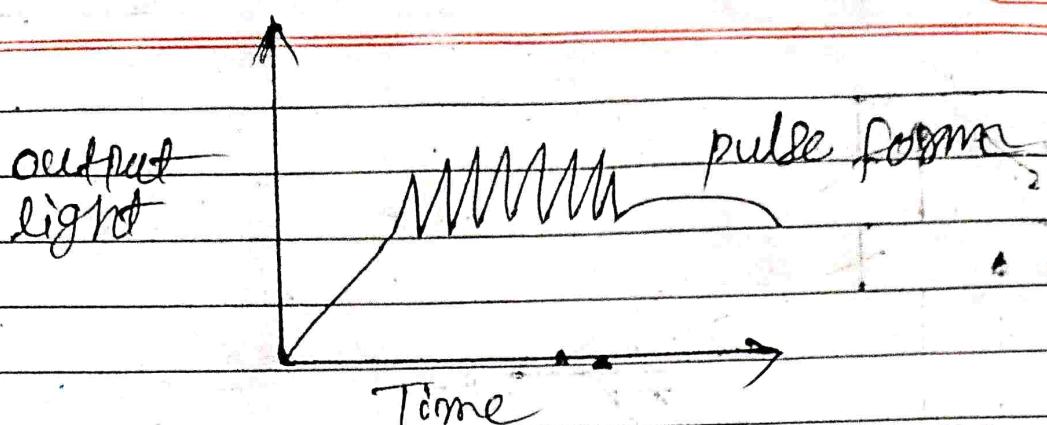
Construction: Shown above diagram a ruby rod of length 5cm and diameter 0.5 cm, ruby rod is made of two substance ( $\text{Al}_2\text{O}_3$  &  $\text{Cr}_2\text{O}_3$ ). There are two mirrors at the corner of the rod, first one is completely silvered mirror ( $M_1$ ) second one is partially silvered mirror ( $M_2$ ) both mirror  $M_1$  &  $M_2$  will make resonator cavity and there is a optical pumping attached with xenon lamp which is connected by a power supply & capacitor we kept ruby rod and optical lamp in a chamber in which cold water pipe is present to cool ruby laser.

Principle : Ruby laser works on the principle of stimulated emission of radiation.



When power supply is turned on capacitor starts getting discharge due to which xenon lamp emits thousand of joules of energy in milliseconds. This energy is absorbed by  $\text{Cr}^{+3}$  ions in ruby laser and they go from ground state to excited state. The ions which absorb 0.85 nm component from light go to green band and ions which absorb 0.42 nm component go to blue band. The lifetime of excited state is 10<sup>-8</sup> s therefore these ions jump ~~from~~ to metastable. The ions of both ~~level~~ blue & green level go to metastable level therefore population inversion is achieved when these ions travel from metastable state to ground state than it shows transition from metastable state to ground state and emits two photons which are coherent and same wavelength which is called spontaneous emission & these two photons travel in resonator cavity again & again than we get laser light of red colour. Found laser light is not ~~on~~ in the form of continuous light rather in the form of pulses from this is the disadvantage of Ruby laser.





Applications  $\rightarrow$  Cutting, drilling, welding, etc.

(P.W)

Explain why He-Ne laser is superior to a ruby laser?

Ans  $\rightarrow$  Ruby laser requires high power pumping source whereas Helium-Neon laser requires low power pumping source. Efficiency of He-Ne laser is more than ruby laser so He-Ne laser is superior to a ruby laser.

(P.W)

Explain spiking in Ruby laser.

Ans  $\rightarrow$

~~The Ruby laser is made up of spikes of high intensity emission. This phenomena is called spiking.~~

The term 'spiking' in ruby laser refers to a phenomenon that can occur when the laser is operated normally. The output power is stable and constant.

## Basics for numericles

$$\textcircled{1} \quad h\nu = E_2 - E_1 \quad \textcircled{2} \quad \text{population inversion} \quad n = n_0 e^{\frac{-E}{kT}}$$

$$\textcircled{3} \quad \text{population inversion ratio} \Rightarrow \frac{n_2}{n_1} = e^{-\frac{h\nu}{kT}}$$

$$\textcircled{4} \quad \text{Intensity} \Rightarrow I = \frac{P}{A} w, \quad \textcircled{5} \quad P = \frac{I \cdot \delta}{A}, \quad \textcircled{6} \quad \bar{E} = h\nu \quad \text{energy}$$

\textcircled{7} Total energy  $\Rightarrow E = \text{no of atom} \times \text{energy of one photon}$

$$E = nh\nu \quad \nu = \frac{c}{\lambda} \quad \alpha_{eq} = \pi \delta^2$$

$$\textcircled{8} \quad \text{no of ions} \Rightarrow n = \frac{Ed}{hc} \quad k = 1.038 \times 10^{-23} \frac{m^2 \cdot kg}{s^2 \cdot K}$$

$$\Rightarrow k = 8.6 \times 10^{-5} \frac{eV}{K}$$

$$\textcircled{9} \quad \text{rate of SPE} = A \nu n_2$$

$$\textcircled{10} \quad \text{rate of STE} = B \nu n_2 E(n)$$

$$\textcircled{11} \quad \text{To find } P \text{ (power)} \Rightarrow P = R = N \times E$$

where  $R \rightarrow$  Rate of emission of energy

$N \rightarrow$  No of total laser ing particle

$\hookrightarrow$  Energy

(PQ)

Compute the ratio of population of the two states in a He-Ne laser that produces light of wavelength  $6.328 \times 10^{-5}$  cm at  $27^\circ\text{C}$ .

$$\text{any} \Rightarrow \text{ratio of population} = \frac{-hc}{kT}$$

$$\frac{N_2}{N_1} = \frac{-h^0/kT}{e} \Rightarrow \frac{N_2}{N_1} = e$$

$$\text{given} \Rightarrow d = 6.328 \times 10^{-5} \text{ cm}$$

$$d = \frac{6.328 \times 10^{-5}}{10^{-2}} \text{ m}$$

$$d = 6.328 \times 10^{-7} \text{ m}$$

$$h = 6.62 \times 10^{-34}, c = 3 \times 10^8$$

$$T = 27^\circ\text{C}, T = 27 + 273 \text{ K}$$

$$T = 300 \text{ K}$$

$$K = 1.38 \times 10^{-23}$$

$$\Rightarrow hc = 6.62 \times 3 \times 10^{-26}$$

$$hc = 19.86 \times 10^{-26}$$

$$\Rightarrow \frac{-hc}{d} = \frac{-19.86 \times 10^{-26}}{6.328 \times 10^{-7}} = 3.0138 \times 10^{-19}$$

$$K_T = 1.38 \times 10^{-23} \times 300$$

$$K_T = 1.38 \times 3 \times 10^{-21}$$

$$K_T = 4.14 \times 10^{-21}$$

$$= (3.0138 \times 10^{-19}) / (4.14 \times 10^{-21})$$

$$\frac{N_2}{N_1} = e$$

$$= (0.7579 \times 10^2) \Rightarrow \frac{N_2}{N_1} = e$$

$$\frac{N_2}{N_1} = e$$

-75.79

pxq

In a given laser, total number of lasing particle is  $2.8 \times 10^{19}$ . If laser emits a wavelength of  $6328 \text{ Å}$ , then calculate the energy of one photon being emitted by the laser. If the laser beam is focused on an area equal to the square of its wavelength for 1s, find intensity of the ht focused beam, Assume the efficiency of laser to be 100%.

ans  $\Rightarrow$  Given  $\therefore n = 2.8 \times 10^{19}$  find  $E = ?$

$$\lambda = 6328 \text{ Å} \quad I = ?$$

$$\text{Area} = \lambda^2$$

$$E = h\nu \quad E = \frac{hc}{\lambda} \quad E = \frac{19.86 \times 10^{-26}}{6328 \times 10^{-16} \text{ m}}$$

$$E = 0.003138 \times 10^{-16} \text{ J}$$

$$E = 3.13 \times 10^{-19} \text{ J} \quad \text{or} \quad I = \frac{P}{A}$$

$$A = \lambda^2 \quad A = 6.328 \times 6328 \times 10^{-20} \text{ m}^2$$

$$A = 40043584 \times 10^{-20} \text{ m}^2$$

$$A = 4 \times 10^{-13} \text{ m}^2$$

$$P = R = N \times E$$

$$P = 2.8 \times 10^{19} \times 3.13 \times 10^{-19}$$

$$P = 8.764 \text{ W}$$

$$I = \frac{P}{A} \quad I = \frac{8.764}{4 \times 10^{-13}}$$

$$I = 2.19 \times 10^{13} \text{ W/m}^2$$

(PQ) find the ratio of rate of spontaneous emission to rate of stimulated emission at 300K corresponding to emission of green light photon with  $\lambda = 550 \text{ nm}$ .

given  $T = 300 \text{ K}$

$$\text{ans} \Rightarrow \lambda = 550, d = 550 \times 10^{-9} \text{ m}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3} \quad \text{where } v = c$$

$$\text{solution} = \frac{8\pi h c^3}{d^3} \Rightarrow \frac{8\pi h c^3}{c^3} \times \frac{1}{d^3}$$

$$= \frac{8\pi h}{d^3} \Rightarrow \frac{8 \times 3.14 \times 6.62 \times 10^{-31}}{(550 \times 10^{-9})^3}$$

$$= \frac{166.37 \times 10^{34}}{166375 \times 10^{24}}$$

$$= 1 \times 10^{13} \text{ answer}$$

(PQ) find the intensity of a laser beam of 10mW power having laser beam diameter of 1cm.

$$\text{ans} \Rightarrow I = \frac{P}{A} \quad P = 10 \text{ mW} \quad P = 10^{-2} \text{ W}$$

$$A = \pi r^2 \quad r = 0.005 \text{ m}$$

$$d = 1 \text{ cm}$$

$$d = 1 \times 10^{-2}$$

$$r = \frac{10^{-2}}{2} \quad r = \frac{1}{200}, r = 0.005 \text{ m}$$

$$I = \frac{10^{-2}}{7.85 \times 10^{-5}}$$

$$I = 127.38 \frac{\text{W}}{\text{m}^2}$$

$$A = \pi r^2$$

$$A = 3.14 \times (0.005)^2$$

$$A = 7.85 \times 10^{-5} \text{ m}^2$$

(PQ)

Calculate the ratio of stimulated absorption to stimulated emission for a material at 800K if the population density of an excited state is 1000 times that of a ground state.

Given

$$N_1 = 1000 N_2$$

$$\frac{N_1}{N_2} = \frac{1}{1000} \quad (\text{given})$$

$$\frac{\text{Rate of Rste}}{\text{Rate of Rste}} = \frac{N_1 B_1 E(h)}{N_2 B_2 E(h)}$$

$$\frac{N_1}{N_2}$$

$$\frac{1}{1000}$$

(PQ) A laser beam having intensity  $2 \text{ mW/m}^2$  passed through a circular cross-sectional area of  $1 \text{ mm}^2$ . Find energy of the photons emitted by laser in 1s.

$$\text{Given } I = 2 \times 10^3 \text{ W/m}^2 \quad a = 1.5 \times 10^{-6} \quad b = 1 \text{ m}^2$$

$$\therefore I = \frac{E}{A \cdot t}$$

$$2 \times 10^3 = \frac{E}{1.5 \times 10^{-6} \times 1}$$

$$E = 2 \times 10^9$$

$$1.5 \times 10^{-6} \times 1 \quad | E = 2 \times 10^9 \rightarrow |$$

$$I = \frac{P}{A}$$

~~work~~  $P = \text{the rate of doing work}$   
 $\text{Rate mean per unit time}$

$$\text{so } \frac{1}{t} = 1$$

Sol:  $T = 300K$ ,  $d = 550 \times 10^9 m$

$$R_{SP} = \frac{hc/KT}{R_{ST}} = \left( e^{-\frac{hc/KT}{R_{ST}}} - 1 \right)$$

$$\Rightarrow \frac{(hc/KTd)}{\left( e^{-\frac{hc/KTd}{R_{ST}}} - 1 \right)}$$

$$\therefore [hc = 10.086 \times 10^{-26}]$$

$$KTd = 1.038 \times 10^{-23} \times 300 \times 550 \times 10^9$$

$$KTd = 1.038 \times 3 \times 55 \times 10^{-23} \times 10^9 \times 10^3$$

$$[KTd = 227.7 \times 10^{-29}]$$

$$\frac{R_{SP.}}{R_{ST}} = \frac{(10.086 \times 10^{-26})}{(227.7 \times 10^{-29})}$$

$$= (e^{-\frac{87.22}{10}})$$

# Optics Fiber

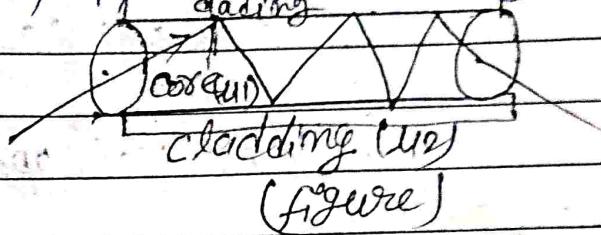
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Date / 20

(Q) Define acceptance angle and numerical aperture with its significance hence find expression for numerical aperture and acceptance angle. physical significance and Merit of acceptance angle & numerical aperture.

Acceptance angle : Acceptance angle is the maximum angle of incident light at which incident light propagates inside the core of optical fibre by total internal reflection.

$$\text{io} = \sin^{-1} [n_1^2 - n_2^2]$$



Merit : i) Increased light collection : A large acceptance angle means optical fibers can collect light from a wider range of angles.

ii) Improved flexibility : By increasing the acceptance angle, optical fibers become more flexible and easier to use.

Physical significance of acceptance angle : To transmit as much of light from the source as possible.

Numerical aperture : It is measurement of sin of acceptance angle and this value is always based on refractive index values of core clad denoted by NA.

$$NA = \sin \text{io}$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

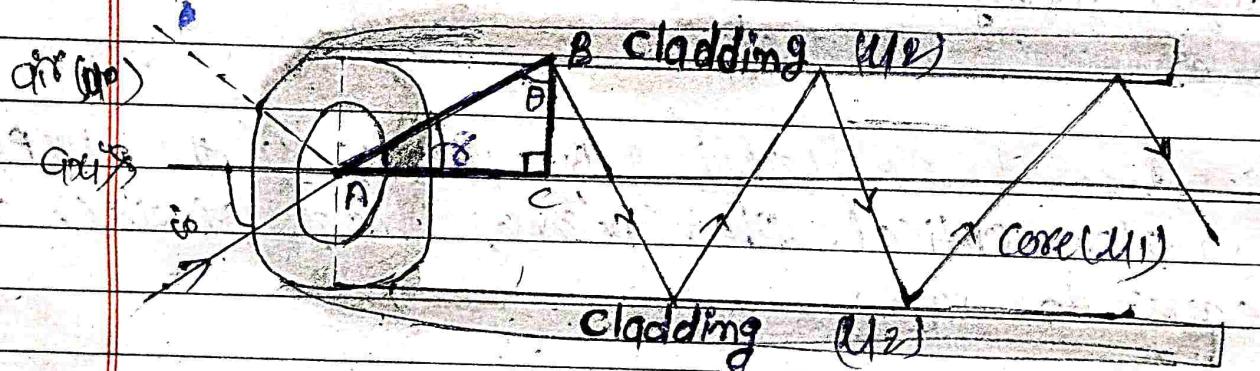
Q. Merit of numerical aperture :-

- (i) Improved resolution
- (ii) Increased brightness

\* physical significance of numerical aperture:-

- (i) Numerical aperture is a measure of the acceptance angle of the fiber.
- (ii) Numerical aperture is used in microscopy.

Mathematical expression of acceptance angle is numerical aperture.



from Snell's law at incident side,

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_0} \Rightarrow \sin i = \left(\frac{n_1}{n_0}\right) \sin r \quad \text{--- A}$$

from  $\triangle ACB$

$$\angle r = (90 - \theta)$$

$$\sin \theta = \sin (90 - \alpha)$$

$$\sin r = \cos \theta \quad \text{--- B} \quad \text{put in A}$$

$$\sin i = \left(\frac{n_1}{n_0}\right) \cos \theta \quad \text{--- C}$$

for maximum acceptance angle  
 $\theta = 90^\circ$

express numerical aperture in terms of  $\Delta$ , that is fractional change in a refractive index of fiber.

$$NA \xrightarrow{\Delta}$$

$$\therefore NA = \sqrt{u_1^2 - u_2^2}$$

$$NA = \sqrt{\frac{(u_1 + u_2)(u_1 - u_2)}{2u_1}}$$

$$NA = \sqrt{\frac{\left(\frac{u_1 + u_2}{2}\right)\left(\frac{u_1 - u_2}{2}\right)2u_1}{u_1}}$$

$$\therefore \boxed{\Delta = \frac{u_1 - u_2}{u_1}}$$

$$NA = \sqrt{u_1 \Delta 2u_1} \quad \left\{ \frac{u_1 + u_2}{2} = \text{Average of } u_1 \right.$$

$$NA = \sqrt{2 \Delta u_1^2}$$

$$\boxed{NA = u_1 \sqrt{2 \Delta}}$$



Teacher's Signature

SMF

i) Its core diameter is very small.

ii) It has only one path.

iii) It has only one direction.

iv) It carry one wavelength.

v) Its V number is less than 2.405

MMF

i) Its core diameter is very large.

ii) It has more than one path.

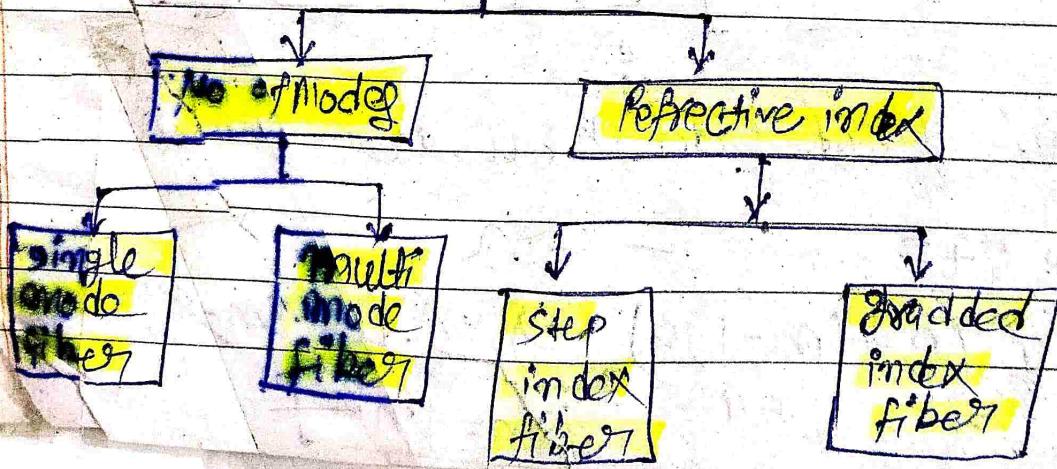
iii) It has more than one direction.

iv) It carry more than one d.

v) Its V no is more than 2.405

Q) Classify the various types of optical fiber with diagram.

### Optical fiber



$$\sin i = \left(\frac{n_1}{n_0}\right) \cos \theta_c \Rightarrow \sin i \left(\frac{n_1}{n_0}\right) = \sqrt{1 - \sin^2 \theta_c}$$

$\therefore$  for critical angle  $\sin \theta_c = \frac{n_2}{n_1} \therefore$

$$\frac{n_1}{n_0} \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$

$$\sin i = \left(\frac{n_1}{n_0}\right) \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$\sin i = \frac{n_1}{n_0} \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin i = \frac{4\pi}{\lambda} \times \sqrt{n_1^2 - n_2^2} \Rightarrow \sin i = \frac{1}{\lambda} \sqrt{n_1^2 - n_2^2}$$

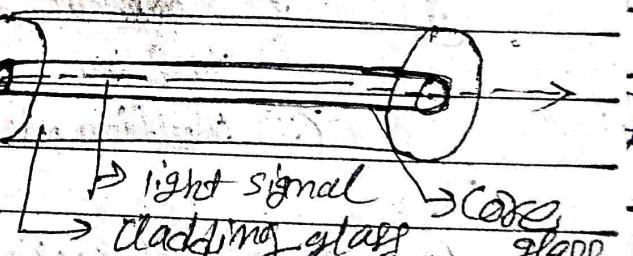
$$\boxed{\sin i = \sqrt{n_1^2 - n_2^2}} \rightarrow NA$$

$$\boxed{i = \sin^{-1} \sqrt{n_1^2 - n_2^2}} \quad i_c = i_o = i_{\max}$$

$\therefore \{ n_0 \approx 1 \}$   
 into air

Q) What do you understand about single mode fiber, and Multimode fiber:

i) Single mode fiber (SMF):



Single mode fiber has a very small core diameter  $\phi$  and it has only one path and only one direction through light propagates, it can carry only one wavelength of light. They do not show dispersion effect. If  $V$  number is less than 2.405.

(ii)

Multimode fiber

Multimode fiber has very large diameter  $\phi$  and it have many path and directions through which lights propagate, can carry more than one wavelength of light.

cladding (C12)

core (C11)

cladding (C12)

axis

C11 &gt; C12

Q

difference between step index fiber and graded index fiber? use suitable diagram.

Step index fiberGraded index

(i)

It's refractive index of core is greater than cladding by some step so it is called of step index.

core will change gradually as we go away from axis so it is called graded index.

(ii)

It has two types

(i) Single mode fiber

(ii) Multimode fiber

It has one type

(i) Multimode fiber

(iii)

It's refractive index of core is uniform.

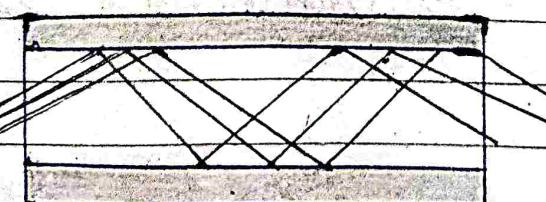
It's refractive index of core is non-uniform.

(iv)

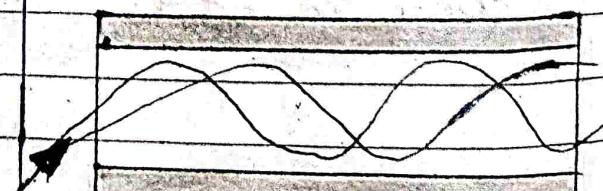
It's path is zig-zag

It's path is Helical

(v)



Step index



Graded index fiber

\*  $V$  - number (Normalised frequency) or cutoff parameter

$$V = \frac{2\pi c}{\lambda} \sqrt{n_1^2 - n_2^2}$$

$$\text{or } V = \frac{DT}{\lambda} \sqrt{n_1^2 - n_2^2}$$

$$N \cdot A = M \sqrt{2\Delta}$$

$$\text{where } \Delta = \frac{\rho n_1 - n_2}{n_1}$$

$$V = \frac{2\pi c}{\lambda} \sqrt{2\Delta}$$

$\lambda$  - radius of core

$\lambda$  - wavelength of incident light

$n_1$  - refractive index of core

$n_2$  - ref. index of cladding

if  $V \leq 2.405$  (only one, single mode)

if  $V > 2.405$  (multimodes are possible)

$N \approx \frac{V}{2}$  (for step index multi-mode fiber)

$N \approx \frac{V^2}{4}$  (for Graded index Multi-mode)

\* No. of Modes in multi-mode fiber:

$$N = \frac{V^2}{2}$$

} for step index multimode

$$N = \frac{V^2}{4}$$

} for graded index MMf

$$\approx 70.89 \Rightarrow 70$$

$$\approx 70.09 \Rightarrow 70$$

Note  $\rightarrow$  light gathering ability =  $NA$

(PQ)

Enlist various types of losses in optical fiber

- i
- ii
- iii
- iv

scattering of light

Absorption of light by materials  
waveguide & Micro bend losses  
Splicing losses

(PQ)

Enlist various types of losses in optical fiber

- i
- ii
- iii
- iv

Scattering loss

Absorption loss

Bend loss

~~Dispersion loss~~ polarization loss

(i)

Scattering loss :-

This loss occurs when

light is scattered by

impurities of the fiber

, causing a loss of signal strength.

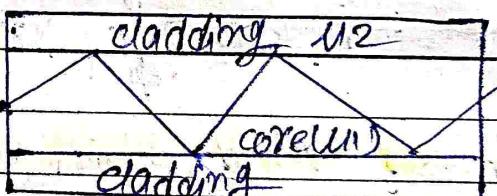


fig → 17.6 (Define)

(ii)

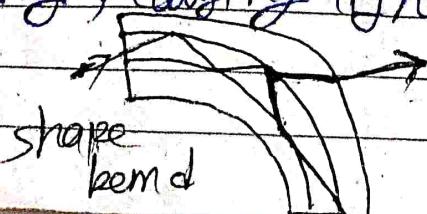
Absorption loss :- Calling fig 17.6;

Absorption loss occurs when light is absorbed by impurities in the fiber, causing a loss of signal strength, intensity of signal.

(iii)

Bend loss :- Calling fig 17.6;

bend loss occurs when the fiber is bent too sharply, causing light to leak out of the fiber.



(iv) Polarization loss or polarization loss occurs when the polarization of the light wave changes as it travels through the fiber, causing a loss of signal strength.

Calling fig. 17.6; later here.

(v) Give four advantages of optical fibers over copper wires:

i) Greater bandwidth

ii) Immunity to interference

iii) Security

iv) Durability

Q Explain pulse dispersion & its three types.

$\rightarrow$  Pulse dispersion is the spreading of a pulse of light as it travels through an optical fiber. This is caused by the different wavelengths of light traveling at different speeds within the fiber.

Goton Types

i) Modal dispersion ii) Intermodal dispersion  
iii) Waveguide dispersion

i) Intermodal dispersion: It is caused by light rays travelling in different modes of a multimode fiber have different propagation delays

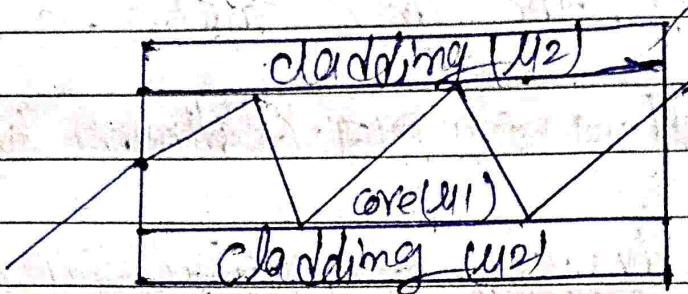
ii) Intermodal dispersion: It is caused by the refractive index of the fiber material varies with wavelength

iii) Waveguide dispersion: It is caused by the waveguide structure of the fiber. It is a small effect in single-mode fibers, but it can be significant in multimode fibers.

Q

Discuss the propagation mechanism of light waves in optical fiber.

Sol:



Mechanism: The propagation of light waves in optical fibers is based on the principle of total internal reflection.

Optical fiber consists of a core, which is made of transparent material such as silicon surrounded by a cladding layer which refractive index is less than core refractive index.

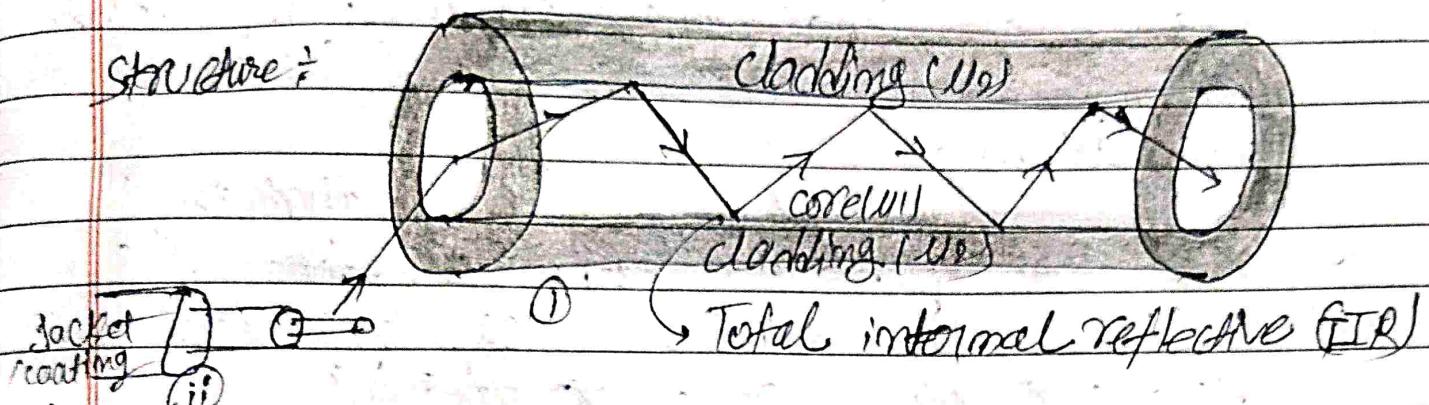
When a light wave enters the core of the fiber, it is refracted towards normal to the surface, if the angle of incidence is greater than the critical angle then wave undergoes total internal reflection.

As the light wave propagates down the fiber, it experiences some loss due to absorption and scattering.

However, the propagation of light waves in an optical fiber is a complex process that works on total internal reflection. This process allows for efficient and high-speed transmission of light signals over long distance.

**PQ** write the principle of optical fiber. Explain its working and hence ~~work~~ structure of optical fiber! Explain structure of an optical fiber.

structure :-



Optical fiber is a thin, flexible and transparent ~~sheet of~~ glass used to transmitting light signals over long distance, it consists of several layers of material, it works on the principle of TIR.

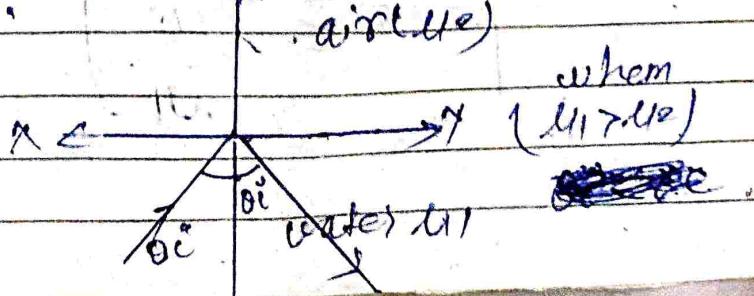
i) core :- This is the central part of the fiber where light travels. it made of glass and it has refractive index greater than cladding ( $n_1 > n_2$ )

ii) cladding :- This is a layer over the core and it is made of glass and it has refractive index less than the core ( $n_2 < n_1$ )  
or jacket → protective sheath

iii) Coating :- This layer protects the fiber and provides mechanical strength.

\* principle The principle of optical fiber is Total internal reflection.

\* working :-



working:

When a light wave enters the core of the fiber, it is refracted towards normal the surface, if the angle of incident is greater than the critical angle, then wave undergoes total internal reflection.

As the light wave propagates through the fiber it experiences some loss due to absorption and scattering

all over the propagation of light wave in an optical fiber is a complex process that works on total internal reflection. This process allows for efficient and high-speed transmission of light signals over long distance.

Note → cut off wavelength of air is  $\lambda_c$   
 $N = \sqrt{v}$  value is 1.205 &  $v = N^2$   
 Given,  $\lambda_c$  given  $v = \text{no of modes}$   
 for which we find,

8.

Relative refractive index :

$$\frac{n_2}{n_1}$$

## Basic concepts of Numerical fiber

① acceptance angle  $i^{\circ} = \sin^{-1} \sqrt{u_1^2 - u_2^2}$

② Numerical aperture  $NA = \sqrt{u_1^2 - u_2^2}$

③ critical angle  $\theta_c = \sin^{-1} \left( \frac{u_2}{u_1} \right)$

④ V number (Normalise frequency) cut off parameter

i)  $V = 2\pi \sqrt{u_1^2 - u_2^2}$  for V number find

ii)  $V = D\pi \sqrt{u_1^2 - u_2^2}$  for diameter find

iii)  $V = \frac{2\pi \Delta u_1 \sqrt{2\Delta}}{1}$  where  $\Delta = u_1 - u_2$

if  $V \leq 2.405$  (only one single mode)

$V > 2.405$  (multi-mode are possible)

$u_1 \rightarrow$  core ( $u_1 > u_2$ )  
 $u_2 \rightarrow$  clad

5

$\Delta = \frac{(n_1 - n_2)}{n_1}$  fractional change  
in refractive index

6 To calculate number of modes  $= N = \frac{V^2}{\Delta}$  {for step index mmf}

$N = \frac{V^2}{\Delta}$  {for graded index MMF}

7 Cladding is doped to give the index difference of

$$\Delta = \frac{n_1 - n_2}{n_1}$$

(P x Q)

If the wavelength of carrier waves for step index fiber is 800 nm and Vnumber is 4.5, then calculate the diameter of the fiber, if the refractive index of core and clad is 1.50 and 1.49 respectively.

$$\text{Sol:} \quad \text{given: } d = 800 \text{ nm}$$

$$d = 800 \times 10^{-9} \text{ m}$$

$$V = 4.5 \quad n_1 = 1.50$$

$$n_2 = 1.49$$

$$V = \frac{d}{\lambda} \sqrt{n_1^2 - n_2^2}$$

$$4.5 = \frac{D \times \pi}{800 \times 10^{-9}} \sqrt{(1.50)^2 - (1.49)^2}$$

$$4.5 = D \times 3.14 \sqrt{2.25 - 2.201}$$

$$8 \times 10^{-7}$$

$$8 \times 10^{-7} \times 4.5 = D \sqrt{0.0299}$$

$$8 \times 10^{-7} \times 4.5 = D \times 0.1729$$

$$3.14 =$$

$$D = \frac{8 \times 10^{-7} \times 4.5}{3.14 \times 0.1729}$$

$$D = \frac{8 \times 4.5 \times 10^{-7}}{0.5429} \Rightarrow D = 36 \times 10^{-7}$$

$$D = 66.31 \times 10^{-7} \text{ m}$$

(pxp) A step index fiber has normalized frequency  $v = 26.6$  at 1800nm wavelength. If the core radius is 25  $\mu\text{m}$ , calculate the numerical aperture and hence find the value of acceptance angle.

Ans → Given  $v = 26.6$

$$\lambda = 1800\text{nm}$$

$$NA = \sqrt{\mu_1^2 - \mu_2^2}$$

$$[d = 1800 \times 10^{-9} \text{m}]$$

$$r = 25 \mu\text{m}$$

$$[r = 25 \times 10^{-6} \text{m}]$$

$\therefore$  we know that  $v = 2\pi r \sqrt{\mu_1^2 - \mu_2^2}$

$$26.6 = \frac{2 \times \pi \times 25 \times 10^{-6}}{1800 \times 10^{-9}} \sqrt{\mu_1^2 - \mu_2^2}$$

$$\frac{1800 \times 26.6 \times 10^{-9}}{2 \times \pi \times 25 \times 10^{-6}} = \sqrt{\mu_1^2 - \mu_2^2}$$

$$\sqrt{\mu_1^2 - \mu_2^2} = \frac{345.8 \times 10^{-6}}{157.07 \times 10^{-6}} \Rightarrow 345.8 \times 10 \times 10^{-6}$$

$$\boxed{\text{NA} = 2.020 \times 10^{-1}} \quad [\text{NA} = 0.22]$$

$$i_c = \sin^{-1}(\sqrt{\mu_1^2 - \mu_2^2})$$

$$i_c = \sin^{-1}(0.22)$$

$$i_c = \sin^{-1}(0.22)$$

$$\boxed{i_c = 12.70^\circ}$$

(PQ)

An optical fiber has numerical aperture of 0.2 and cladding refractive index of 1.59 determine (i) acceptance angle for the fiber in liquid with refractive index 1.42 and (ii) critical angle between core - clad interface.

Ans ⇒

$$NA = 0.2, \mu_2 = 1.59$$

$$\text{Acceptance angle } i^\circ = \sin^{-1}(\mu_1^2 - \mu_2^2)$$

$$\sin i^\circ = \sqrt{\mu_1^2 - \mu_2^2}$$

$$\sin i^\circ = NA = \sqrt{\mu_1^2 - \mu_2^2}$$

$$0.2 = \sqrt{\mu_1^2 - \mu_2^2}$$

$$(0.2)^2 = \mu_1^2 - \mu_2^2$$

$$\mu_1^2 = (0.2)^2 + \mu_2^2$$

$$\mu_1^2 = 0.04 + (1.59)^2$$

$$\mu_1 = \sqrt{0.04 + 2.5281}$$

$$\mu_1 = \sqrt{2.5681} \quad \mu_1 = 1.60$$

$$\mu_1 = 1.60$$

i) acceptance angle

$$i^\circ = \sin^{-1} \frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_0}$$

$$i^\circ = \sin^{-1} \left( \frac{\sqrt{(1.60)^2 - (1.59)^2}}{1.42} \right)$$

$$i^\circ = \sin^{-1} \left( \frac{\sqrt{2.56 - 2.5281}}{1.42} \right)$$

$$i^\circ = \sin^{-1} \left( \frac{\sqrt{0.0319}}{1.42} \right) \quad i^\circ = \sin^{-1} \left( \frac{0.1786}{1.42} \right)$$

$$i^\circ = \sin^{-1}(0.1257)$$

$$i^\circ = 7.22^\circ$$

(ii) critical angle  $\theta_c = \sin^{-1} \left( \frac{\mu_2}{\mu_1} \right)$

$$\theta_c = \sin^{-1} \left( \frac{1.59}{1.60} \right)$$

$$\theta_c = \sin^{-1}(0.99)$$

$$\boxed{\theta_c = 81.89^\circ}$$

(iv) Calculate the refractive index of the core and cladding material of a fiber from the following data : NA = 0.22 ; relative refractive index is 0.012 where NA is numerical aperture

~~$m_{\text{Sol}} =$~~  given  $\therefore NA = 0.22$ ,  ~~$\mu_2 = 1.012$~~

~~$\therefore \mu_1^2 - \mu_2^2 = \frac{NA^2}{1 - NA^2}$~~

~~$\mu_1^2 - \mu_2^2 = 0.012^2 / (1 - 0.012^2)$~~

~~$\therefore NA = \sqrt{\mu_1^2 - \mu_2^2} \Rightarrow 0.22 = \sqrt{\mu_1^2 - \mu_2^2}$~~

~~$(0.22)^2 = \mu_1^2 - \mu_2^2$~~

~~$(0.22)^2 = (1.012 \cdot \mu_2)^2 - \mu_2^2$~~

~~$0.0484 = 1.024144 \mu_2^2 - \mu_2^2$~~

~~$0.0484 = \mu_2^2 (1.024 - 1)$~~

~~$\mu_2^2 = \frac{0.0484}{0.024}$~~

$$\mu_2^2 = 2.016$$

$$\boxed{\mu_2 = 1.41}$$

$$\mu_1 = 1.012 \times 1.41$$

$$\boxed{\mu_1 = 1.43}$$

(PQD)

An optical fiber has a N.A. of 0.20 and a cladding refractive index of 1.59. Determine the acceptance angle for the fiber in water which has a refractive index of 1.33.

$$\text{given} \rightarrow n_2 = 1.59$$

$$n_0 = 1.33$$

$$\text{NA} = 0.20$$

$$i^{\circ} = \sin^{-1}(n_1^2 - n_2^2)$$

$$\sin i^{\circ} = \sqrt{n_1^2 - n_2^2} = \text{NA}$$

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

$$(0.20)^2 = n_1^2 - (1.59)^2$$

$$n_1^2 = (0.20)^2 + (1.59)^2$$

$$n_1^2 = 0.04 + 2.5281$$

$$n_1^2 = 2.5681$$

$$n_1 = \sqrt{2.5681}$$

$$\boxed{n_1 = 1.60}$$

$$i^{\circ} = \sin^{-1} \left( \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \right)$$

$$i^{\circ} = \sin^{-1} \left( \frac{\sqrt{(1.60)^2 - (1.59)^2}}{1.33} \right)$$

$$i^{\circ} = \sin^{-1} \left( \frac{\sqrt{2.56 - 2.5281}}{1.33} \right)$$

$$i^{\circ} = \sin^{-1} \left( \frac{\sqrt{0.0319}}{1.33} \right) \Rightarrow i^{\circ} = \sin^{-1} (0.0239)$$

$$i^{\circ} = \sin^{-1} \left( \frac{0.17860}{1.33} \right) \quad i^{\circ} = \sin^{-1} (0.1342)$$

$$\boxed{i^{\circ} = 7.71^{\circ}}$$

(PQ)

A step index fiber having length 2km is found to have  $n_1 = 1.55$  and  $n_2 = 1.50$ . Find acceptance angle and numerical aperture of the fiber. If the radius of the core is 15  $\mu\text{m}$  and the wavelength of the carrier is 850 nm, then check whether the fiber is SMF or MMF.

$$l = 2\text{ km}$$

$$\text{Given } n_1 = 1.55$$

$$n_2 = 1.50$$

$$d = 850 \text{ nm} \quad [d = 850 \times 10^{-9} \text{ m}]$$

$$r = 15 \mu\text{m} \quad [r = 15 \times 10^{-6} \text{ m}]$$

find  $i^\circ = ?$  NA = ? check SMF ?  
MMF ?

$$\text{we know that } NA = \sqrt{n_1^2 - n_2^2}$$

$$NA = \sqrt{(1.55)^2 - (1.50)^2}$$

$$NA = \sqrt{2.4025 - 2.25} \Rightarrow NA = \sqrt{0.1525}$$

$$NA = 0.39$$

$$i^\circ = \sin^{-1} \sqrt{n_1^2 - n_2^2}$$

$$i^\circ = \sin^{-1}(0.39) \quad i^\circ = 22.98^\circ$$

$$\boxed{i^\circ = 23^\circ}$$

$$V = \frac{2\pi l}{1} \sqrt{n_1^2 - n_2^2} \quad \therefore V = \frac{2 \times \pi \times 15 \times 10^{-6}}{8.5 \times 10^{-8}} \times 0.39$$

$$V = \frac{36.75 \times 10^{-6}}{8.5} \quad V = 0.4384 \times 10^{-2}$$

$$\boxed{V = 43.24 \quad \therefore V > 80 \text{ ps}}$$

So it is MMF?

(pno) A manufacturer wishes to make a silica core step index fiber with  $N = 75$  and numerical aperture  $NA = 0.30$  to be used at 820nm. If  $n_1 = 1.458$ , what should the core size and cladding index be? Also find the value of critical angle and acceptance angle of the given fiber.

$$\text{Given: } N = 75 \quad [d = 820\text{nm}] \\ NA = 0.30$$

$$\therefore n_1 = 1.458$$

$$n_2 = ? \quad \theta_c = ? \quad i_o = ?$$

$$\therefore NA = 0.30 \quad NA = \sqrt{n_1^2 - n_2^2}$$

$$(0.30)^2 = n_1^2 - n_2^2$$

$$(n_2)^2 = (n_1^2) - (0.30)^2$$

$$(n_2)^2 = (1.458)^2 - (0.30)^2$$

$$(n_2)^2 = \sqrt{(1.458)^2 - (0.30)^2}$$

$$n_2 = \sqrt{2.0125764 - 0.09}$$

$$n_2 = \sqrt{2.035764}$$

$$\boxed{n_2 = 1.426}$$

$$\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \quad \theta_c = \sin^{-1} \left( \frac{1.426}{1.458} \right)$$

$$\theta_c = \sin^{-1} (0.978052)$$

$$\theta_c = 77.97^\circ \quad \boxed{\theta_c = 78^\circ}$$

$$i_o = \sin^{-1} \sqrt{n_1^2 - n_2^2}$$

$$i_o = \sin^{-1} (\sqrt{0.092288}) \quad i_o = \sin^{-1} (0.303789)$$

$$\boxed{i_o = 17.68^\circ}$$

(Q) find acceptance angle, numerical aperture, critical angle and v-number of the optical fiber from the data given below:

Refractive index of core = 1.48, fractional change in refractive index = 0.005, core radius  $r_1 = 50\text{ mm}$ , wavelength of radiation  $\lambda = 850\text{ nm}$ . Check whether the fiber is single mode or multimode.

$$i_o = ?, \text{NA} = ?, \theta_c = ? \quad v = ?$$

any  $\Rightarrow$  check (SMF) or (MMF)

given data

$$\therefore [n_1 = 1.48], [\Delta = 0.005]$$

$$r_1 = 50\text{ mm} \quad r_1 = 50 \times 10^{-3}\text{ m}$$

$$d = 850\text{ nm}$$

$$[d = 850 \times 10^{-9}\text{ m}]$$

$$\because \Delta = n_1 - n_2 \Rightarrow 0.005 = 1.48 - n_2$$

$$\frac{n_1}{n_1} \quad \frac{1.48}{1.48}$$

$$0.0074 = 1.48 - n_2$$

$$n_2 = 1.48 - 0.0074$$

$$[n_2 = 1.47]$$

$$i_o = \sin^{-1}(n_1^2 - n_2^2) \quad i_o = \sin^{-1}(\sqrt{2.1904 - 2.1609})$$

$$i_o = \sin^{-1}(\sqrt{0.0295}) \Rightarrow i_o = \sin^{-1}(0.1717)$$

$$i_o = 9.88^\circ \approx [i_o = 9^\circ]$$

$$[\text{NA} = 0.1717] \quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \quad \theta_c = \sin^{-1}\left(\frac{1.47}{1.48}\right)$$

$$\theta_c = \sin^{-1}(0.993243)$$

$$[\theta_c = 83.33^\circ] \quad v = 2\pi d \sqrt{(n_1)^2 - (n_2)^2}$$

$$v = \frac{2\pi d \times 50 \times 10^{-3}}{850 \times 10^{-9}} \times 0.1717 = 53.94 \times 16 \times 10 \times 10^{-6} \quad -6 \quad 9 \quad -1$$

(o)

$$\Rightarrow 0.6345 \times 10^2$$

$$V = 63.45$$

$\therefore V > 2.405$  So it is (MMF.)

- (PQ) A glass clad fiber is made with core glass of refractive index 1.5 and cladding is supposed to give the index difference of 0.05. Determine the refractive index of the cladding, numerical aperture of the fiber, critical angle of reflection and critical acceptance angle.

Sol:

$$\text{Given: } n_1 = 1.5 \quad \Delta = 0.05$$

$$n_2 = ? \quad NA = ?, \quad \theta_c = ?, \quad i_o = ?$$

$$\Delta = \frac{n_1 - n_2}{n_1} \Rightarrow 0.05 = \frac{1.5 - n_2}{1.5}$$

$$\Rightarrow 0.075 = 1.5 - n_2$$

$$n_2 = 1.5 - 0.075$$

$$n_2 = 1.4 \quad NA = \sqrt{(n_1)^2 - (n_2)^2}$$

$$NA = \sqrt{(1.5)^2 - (1.4)^2} \quad NA = \sqrt{2.25 - 1.96}$$

$$NA = \sqrt{0.29} \quad [NA = 0.53]$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \Rightarrow \theta_c = \sin^{-1}\left(\frac{1.4}{1.5}\right)$$

$$\theta_c = \sin^{-1}(0.93333) \quad [\theta_c = 68.96^\circ]$$

$$i_o = \sin^{-1}(n_1) \approx n_2 \quad [i_o = \sin^{-1}(0.53)]$$

$$[i_o = 32^\circ]$$

An optical fiber of graded index type is made up of a core, where light travels, made of glass of refractive index  $n_1 = 1.5$  surrounded by another layer of glass of lower refractive index  $n_2$ .

Find

- i)  $n_2$  of the cladding so that the critical angle at the core-cladding interface is  $80^\circ$ .
- ii) Numerical Aperture of the fiber.
- iii) V-parameter for core radius 50  $\mu\text{m}$  and operating wavelength of 0.850  $\mu\text{m}$ .
- iv) Number of modes guided in the core.

Given  $n_1 = 1.5$   $\theta_c = 80^\circ$   $r = 50 \mu\text{m}$

$r = 5 \times 10^{-5} \text{ m}$   $\lambda = 0.850 \mu\text{m}$

$d = 0.850 \times 10^{-6} \text{ m}$

i)  $n_2 = ?$   $\theta_c = 80^\circ \therefore \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$

$80^\circ = \sin^{-1} \left( \frac{n_2}{n_1} \right)$

$\sin 80^\circ = \frac{n_2}{n_1}$   $n_2 = 0.98 \times 1.5$

$n_2 = 1.47$

ii)  $NA = (n_1)^2 - (n_2)^2$   $NA = \sqrt{(1.5)^2 - (1.47)^2}$

$NA = \sqrt{2.25 - 2.0609}$   $NA = \sqrt{0.0891}$

$NA = 0.29$

(iii)  $V = \frac{2\pi r}{d} \left( \text{core} - \text{cladding} \right)^2$

$$V = \frac{2 \times \pi \times 5 \times 10^{-5}}{0.850 \times 10^{-6}} \times 0.29$$

$$V = \frac{9.11 \times 10 \times 10^{-5}}{0.850}$$

$$V = 10.71 \times 10^1 \Rightarrow V = 107.17 \text{ / (MMF)}$$

(iv) Number of modes  $\rightarrow \because$  Graded index

$$N = \frac{\pi^2}{4} \quad N = (107.17)^2$$

$$N = \frac{11485.40}{4}$$

$$N = 2871.35$$

no. of modes

- (px) A step index fiber with large core diameter compared with the wavelength of the transmitted index light has an acceptance angle in air of  $30^\circ$  and fractional refractive index difference of  $0.03$ . Determine (i) numerical aperture of the fiber (ii) the critical angle at the core cladding interface (iii) necessary core radius for fiber to be multimode if the wavelength of transmitted light is  $950 \text{ nm}$ .

Sol: Given:  $i_o = 30^\circ \quad \Delta = 0.03$

we know that

$$\hat{c}_0 = \sin^{-1} (\sin^2 u_{11} - \sin^2 u_{12}) \Rightarrow 30^\circ = \sin^{-1} NA$$

$$\sin 30^\circ = \sqrt{\sin^2 u_{11} - \sin^2 u_{12}}$$

$$(1) \quad \underline{u_{12}^2 = u_{11}^2 - 0.25} \Rightarrow 0.25 = u_{11}^2 - u_{12}^2$$

$$\boxed{u_{12}^2 = u_{11}^2 - 0.25} \quad (1)$$

$$\Delta = \frac{u_{11} - u_{12}}{u_{11}} \Rightarrow 0.03 = \frac{u_{11} - u_{12}}{u_{11}}$$

$$0.03 u_{11} = u_{11} - u_{12} \Rightarrow -u_{12} = 0.03 u_{11} - u_{11}$$

$$+u_{12} = +0.97 u_{11} \quad | \quad \boxed{u_{12} = 0.97 u_{11}} \quad (2)$$

put in (1)

$$(0.97 u_{11})^2 = u_{11}^2 - 0.25$$

$$0.9409 u_{11}^2 = u_{11}^2 - 0.25$$

$$0.9409 u_{11}^2 - u_{11}^2 = -0.25$$

$$-0.0591 u_{11}^2 = -0.25$$

$$u_{11}^2 = \frac{0.25}{0.0591} \Rightarrow u_{11}^2 = 4.23$$

$$\boxed{u_{11} = \sqrt{4.23}} \\ \boxed{u_{11} = 2.05}$$

$$u_{12} = 0.97 \times 2.05$$

$$\boxed{u_{12} = 1.99}$$

$$(1) \quad NA = \sqrt{(u_{11})^2 - (u_{12})^2}$$

$$NA = \sqrt{4.23 - 1.99}$$

$$NA = \sqrt{0.8424}$$

$$\boxed{NA = 0.91}$$

(ii)

$$\theta_c = ? \quad \theta = \sin^{-1} \left( \frac{M_r}{M_u} \right)$$

$$\theta_c = \sin^{-1} \left( \frac{1.99}{2.05} \right)$$

$$\theta_c = \sin^{-1} (0.970731)$$

$$\theta_c = 76.10^\circ$$

(iii)

$$V = 2\pi r NA$$

$$2.405 > \frac{2\pi r \times 0.49}{d}$$

$$2.405 > \frac{2\pi \times 8 \times 0.49}{950 \times 10}$$

$$2\pi \times 8 \times 0.49 < 2.405 \times 950 \times 10$$

~~$$8 < \frac{2.405 \times 950 \times 10}{2\pi \times 0.49}$$~~

$$8 <$$

(Q1)

$$\mu_1 = 1.55, \mu_2 = 1.51, d = 50 \text{ nm}$$

$$r = 0.8 \text{ nm}$$

(a) NA (b)  $i_o$  (c) V-number

$$\text{Sol} \rightarrow NA = \sqrt{\mu_1^2 - \mu_2^2} \quad NA = \sqrt{2.25 - 0.2801}$$

$$NA = \sqrt{0.0301}$$

$$NA = 0.173$$

$$i = \sin^{-1}(\sqrt{\mu_1^2 - \mu_2^2}) \Rightarrow i_o = \sin^{-1}(0.17349)$$

$$i_o = 9.99^\circ \approx 10^\circ$$

$$V = \frac{2\pi r}{d} \sqrt{\mu_1^2 - \mu_2^2}$$

$$d = 50 \times 10^{-6} \text{ m}$$

~~$$d = 50 \times 10^{-6} \text{ m}$$~~

$$d = 25 \times 10^{-6} \text{ m}$$

$$d = 0.8 \times 10^{-6} \text{ m}$$

$$V = 2 \times \pi \times 25 \times 10^{-6} \times 0.173$$

$$0.8 \times 10^{-6}$$

$$V = 33.96$$

(Q2)

An optical fiber has NA of 0.15 and cladding refractive index is ~~is~~ equal to 1.50. Find NA of the fiber in a liquid of refractive index 1.30.

(Q3)

$$NA = 0.15, \mu_e = 1.50$$

$$\mu_o = 1.30 \quad NA = ?$$

$$NA = \sqrt{U_1^2 - U_2^2}$$

$$(0.15) = U_1 - U_2$$

$$U_1^2 = (0.15)^2 + (U_2)^2$$

$$U_1^2 = (0.15)^2 + (4.50)^2$$

$$U_1^2 = 0.0225 + 8.025$$

$$U_1^2 = 8.2725$$

$$U_1 = \sqrt{8.2725}$$

$$U_1 = 1.507$$

$$NA = \sqrt{(1.507)^2 - 4.50^2}$$

1.30

$$NA = \sqrt{8.2725 - 2.25}$$

$$NA = 1.30$$

$$NA = \frac{\sqrt{0.02249595}}{1.30}$$

$$NA = \frac{0.149986}{1.30}$$

$$NA = 0.1153$$

(Q8) Calculate the refractive indices of the core & cladding material of a fiber if numerical aperture is 0.22 and fractional refractive index changes is 0.012.

ans →

$$NA = 0.22$$

$$\Delta = 0.012$$

$$NA = \sqrt{U_1^2 - U_2^2} \quad (0.22) = U_1 - U_2$$

$$\Delta = U_1 - U_2$$

$$U_1 = \frac{U_1 - U_2}{\Delta}$$

$$U_1^2 = 0.0484 + U_2^2 \quad \text{--- (A)}$$

$$0.012 = \frac{U_1 - U_2}{U_1}$$

$$0.012 U_1 = U_1 - U_2$$

$$0.012 U_1 - U_1 = -U_2$$

$$U_1 (-0.988) = -U_2$$

$$U_1 (0.988) = U_2 \quad \text{--- (B)}$$

$$U_2 = U_1 (0.988) \quad \text{--- (B) put in A}$$

$$U_1^2 = 0.0484 + (U_1 (0.988))^2$$

$$U_1^2 = 0.0484 + U_1^2 (0.976144)$$

$$U_1^2 - U_1^2 (0.976144) = 0.0484$$

$$U_1^2 (0.023856) = 0.0484$$

$$U_1^2 = \frac{0.0484}{0.023856} \quad U_1^2 = 20.288$$

$$U_1 = \sqrt{20.288}$$

$$U_1 = 4.2 \quad \text{put (ii) in (A)}$$

$$U_2 = 4.2 \times (0.988)$$

$$U_2 = 4.0$$

(PQ) A step index fiber has numerical aperture of 0.26 & core refractive index 1.5 and diameter 100μm. Calculate refractive index of plating angle of acceptance and critical angle at core clad interface, fractional change in a refractive index and cut off wavelength that can be passed through optical fibre with minimum 1088.

$$\text{Ans} \Rightarrow NA = 0.26, n_1 = 1.5, d = 100\mu\text{m}$$

$$\checkmark n_2 = ?, \checkmark i_c = ?, \checkmark \theta_c = ?, \checkmark \lambda = ?$$

$$d = ? \quad 1088 = ?$$

$$0.26 = \sqrt{n_1^2 - n_2^2}$$

$$(0.26)^2 = (1.5)^2 - n_2^2$$

$$n_2^2 = (1.5)^2 - (0.26)^2$$

$$n_2^2 = 2.25 - 0.0676$$

$$n_2^2 = 2.1824$$

$$n_2 = \sqrt{2.1824}$$

$$n_2 = 1.47$$

$$i_o = \sin^{-1} NA$$

$$i_o = \sin^{-1} \sqrt{(1.5)^2 - (1.47)^2}$$

$$i_o = \sin^{-1} \sqrt{2.25 - 2.01609}$$

$$i_o = \sin^{-1} (0.0891)$$

$$i_o = \sin^{-1} (0.298496)$$

$$i_o = 17.36^\circ$$

$$\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$

$$\theta_c = \sin^{-1} \left( \frac{1.47}{1.5} \right) \quad \theta_c = \sin^{-1} (0.98)$$

$$\boxed{\theta_c = 78.52^\circ} \quad \Delta = 1.5 - 1.47$$

$$\Delta = \frac{n_1 - n_2}{n_1} \quad \Delta = \frac{1.5 - 1.47}{1.5}$$

$$\boxed{\Delta = 0.02}$$

Minimum loss  $\Rightarrow$  Single mode

$$\Rightarrow V \leq 2.405$$

$$\frac{2\pi r(4.52\Delta)}{\lambda} \leq 2.405$$

given

$$r = 50\text{ um}$$

$$= 50 \times 10^{-6}\text{ m}$$

$$\text{or } \frac{2\pi r(\Delta - 1)}{\lambda} \leq 2.405$$

$$\frac{2 \times 3.14 \times 50 \times 10^{-6} \times 0.26}{2.405} \leq \lambda$$

$$\lambda \geq$$

## Total internal reflection

→ when an incident light travels from denser to rare medium and if incident angle is greater than critical angle then light reflect back in denser medium this phenomena is known as Total internal reflection.

(Q3) The light gathering ability of an optical fiber is 0.479. The fractional refractive index b/w core & cladding is 0.0005. Calculate the refractive index of the cladding.

Sol.  $\rightarrow$  Note light gathering ability = NA

$$\therefore NA = \mu_1 \sqrt{2\Delta}$$

$$0.479 = \mu_1 \sqrt{2 \times 0.0005}$$

$$\mu_1 = \frac{0.479}{0.031602}$$

$$\boxed{\mu_1 = 15.15} \quad NA = \sqrt{\mu_1^2 - \mu_2^2}$$

$$NA^2 = \mu_1^2 - \mu_2^2$$

$$(0.479)^2 = (15.15)^2 - \mu_2^2$$

$$0.229441 = 229.5825 - \mu_2^2$$

$$\mu_2^2 = 229.29 \quad \mu_2 = \sqrt{229.29}$$

$$\boxed{\mu_2 = 15.14}$$

- i) Doping = Adding impurities to the semiconductor material to increase its conductivity.
- ii) Temperature increasing = Increasing the temperature of a semiconductor can increase its conductivity by increasing the number of free electrons and holes.

# Semiconductors

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What are

(PQ) Intrinsic semiconductor? Name the ways by which we can enhance the conductivity of intrinsic semiconductor. Discuss in detail the extrinsic semiconductors.

Intrinsic semiconductor: An intrinsic semiconductor is a pure semiconductor material that has no any impurities or doping two important example

i) Pure Silicon & pure Germanium, Intrinsic semiconductor has equal numbers of electrons and holes thus has no net electrical charge.

The electrical conductivity of intrinsic semiconductor is generally low at room temperature.

P 2.3

→ At higher temperature we can enhance the conductivity of intrinsic semiconductor.

Extrinsic: If in intrinsic semiconductor we add calculated amount of impurity in pure semiconductor two important example i) Boron and phosphorus is called extrinsic semiconductor and the process of adding impurity in pure semiconductor is called doping. Extrinsic semiconductor are two types i) n-type and ii) p type semiconductor.

(PQ) What are Semiconductors? write two way to increase the conductivity of intrinsic semiconductors.

Ans → Semiconductors are materials that have electrical conductivity between a conductor and an insulator Ex: silicon, germanium.

Ques

What is difference between intrinsic and extrinsic semiconductors? Explain two types of extrinsic semiconductors.

Intrinsic

i) It is a pure semiconductor material.

Extrinsic  
It is impure semiconductor materials.

ii) It has no any impurity and doping.

It has impurity and doping.

iii) It has equal numbers of electron and holes.

It has not equal numbers of electrons and holes.

iv) It has low electrical conductivity at room temperature.

It has higher electrical conductivity at room temperature.

v) Its example is pure silicon, pure Germanium.

Its example is boron, phosphorous.

vi) It has no any type of semiconductor.

It has two types of semiconductor n-type P-type semiconductor.

Extrinsic semiconductor

- i) n-type semiconductor
- ii) p-type semiconductor

\* N-type semiconductor : N-type Semiconductor is a type of Semiconductor doped with a pentavalent impurity, the pentavalent impurity are added in the N-type Semiconductor so it increases the numbers of electrons for conduction.

Ex → phosphorous, arsenic, antimony.

\* P-type semiconductor : P-type Semiconductor is a type of Semiconductor doped with a trivalent impurity in a small amount and result as a large number of holes are created in it. example → aluminium, boron, gallium.

DQ) Differentiate n-type and p-type semiconductor?

n-type

p-type

i) It is doped with a pentavalent impurity

It is doped with a trivalent impurity.

ii) In n-type electrons act as a majority charge carriers

In P-type holes act as a majority charge carrier

iii) In n-type Semiconductor Conductivity is mainly due to electrons.

In P-type Semiconductor conductivity is mainly due to holes.

iv) It is donor type

It is acceptor type

v) Impurity atoms is pentavalent.

Impurity atoms is trivalent

Ex → phosphorous, arsenic

Ex → aluminium  
boron, gallium.

(PQ)

Define Fermi level and give its physical significance, and how it varies with temperature in intrinsic semiconductors, Explain with diagram.

ans →

Highest energy level occupied by electron in band at 0 K is called Fermi level.

→ Significance of Fermi level

- (i) Determines the electrical conductivity of a material.
- (ii) Affects the number of electrons available for conduction.
- (iii) Determines whether a material is a conductor, semiconductor, insulator in Fermi level

→ varies with term in intrinsic semiconductor calling - (5.7)

(PYQ)

Discuss the effect of temperature on conductors and semiconductors in terms of Conductivity.

ans → Conductivity of Semiconductors increases with increasing in temperature. Reason → with increase in temperature the number of electrons from the valence bond can jump to the conduction band in semiconductors.

→ Conductivity of conductors decreases with increasing in temperature.

(PQ) Derive an expression for fermi energy in intrinsic Semiconductor. what is the effect of temperature on fermi level in an intrinsic semiconductor.

$$\Rightarrow \text{no of } e^- \text{ in conduction band} = (E_C - E_F)/kT$$

$$n = N_C e^{-\frac{(E_C - E_F)}{kT}}$$

conduction band  $E_C$

$E_F$

$$\text{no of } e^- \text{ in valence band} = (E_F - E_V)/kT$$

valence band  $E_V$

$$P = N_V e^{-\frac{(E_C - E_F)}{kT}} \quad \text{for intrinsic Semiconductors}$$

$$N_C e^{-\frac{(E_C - E_F)}{kT}} = N_V e^{-\frac{(E_F - E_V)}{kT}} \quad n = P$$

$$\frac{N_C}{N_V} = \frac{e^{-\frac{(E_F - E_V)}{kT}}}{e^{-\frac{(E_C - E_F)}{kT}}} \Rightarrow \frac{N_C}{N_V} = e^{\frac{(-E_F + E_V + E_C - E_F)}{kT}}$$

Taking log both sides

$$\log \left( \frac{N_C}{N_V} \right) = \log \left( e^{\frac{(-E_F + E_V + E_C)}{kT}} \right)$$

$$\log \frac{N_C}{N_V} = \frac{-E_F + E_V + E_C}{kT}$$

$$-2E_F = -(E_V + E_C) + kT \log \frac{N_C}{N_V}$$

$$2E_F = E_V + E_C - kT \log \frac{N_C}{N_V}$$

$$E_F = \frac{E_C + E_V}{2} - \frac{kT \log \frac{N_C}{N_V}}{2}$$

$$E_F = \frac{E_C + E_V}{2} + \frac{kT \log \frac{N_V}{N_C}}{2}$$

Define  $\rightarrow$  507.

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→ Effect of temperature

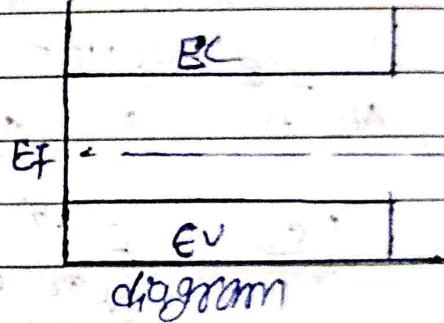
Case-1 at  $T = 0\text{K}$

$$E_f = \frac{E_C + E_V}{2} - \frac{kT \ln N_C}{N_V} \text{ fermi}$$

$E_f = \frac{E_C + E_V}{2}$  at mean $\pm 1$  Energy level  
middle of energy gap.

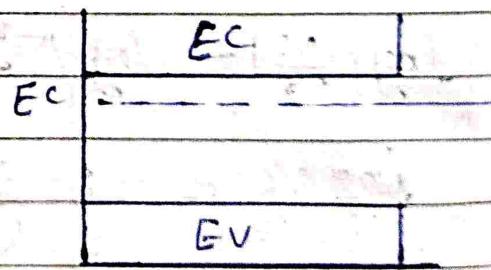
Case-2 → at  $T = 300\text{K}$

Fermi energy level below middle of  
energy gap



Case-3 → at  $T = -300\text{K}$

Fermi energy level above middle of energy  
gap



Q1 How the resistivity of semiconductor  
& conductor vary with temperature

Semi  $\rightarrow T \uparrow | C \uparrow | R \downarrow$

Condu  $\rightarrow T \uparrow | C \downarrow | R \uparrow$

(Q) Show the fermi level in case of intrinsic semiconductors lies in the middle of conduction and valence band. Also explain its variation with temperature.

Ans no. of  $e^-$  in conduction band

$$n = N_c e^{-\frac{(E_C - E_F)}{kT}}$$

no. of  $e^-$  in valence band

$$p = N_V e^{-\frac{(E_F - E_V)}{kT}} \quad \text{for intrinsic semiconductors}$$

$$n = p$$

$$N_c e^{-\frac{(E_C - E_F)}{kT}} = N_V e^{-\frac{(E_F - E_V)}{kT}}$$

$$\frac{N_c}{N_V} = \frac{e^{-\frac{(E_C - E_F)}{kT}}}{e^{-\frac{(E_F - E_V)}{kT}}} \quad \text{at equilibrium } N_c = N_V$$

$$\frac{N_c}{N_V} = e^{\frac{(-2E_F + E_V + E_C)/RT}{kT}} = e^{\frac{-2EF + EV + EC}{kT}} = 1$$

$$\frac{(-2E_F + E_V + E_C)/kT}{e} = e^0$$

$$\frac{-2E_F + E_V + E_C}{kT} = 0 \Rightarrow -2E_F + E_V + E_C = 0$$

$$2E_F = E_C + E_V$$

$$E_F = \frac{E_C + E_V}{2}$$

$E_C, C_B$

proved that fermilevel lies in the middle of conduction and valence band.

--- middle

$E_B, V_B$

→ variation with temperature → calling (5.7)

(Q1)

position of fermi level in intrinsic semiconductor

Ans  $\Rightarrow$  The position of fermi level in intrinsic semiconductors depends on factors such that mass of electrons and holes, temperature.

Case - 1  $\Rightarrow m_n = m_p$  (holes)

$$(e) \text{ then } N_C = N_V$$

$$\boxed{\text{Ef} = \frac{E_C + E_V}{2} - \frac{kT}{N_C} \log \frac{N_C}{N_V}}$$

$$\text{Ef} = \frac{E_C + E_V}{2} - \frac{kT}{N_C} \log \frac{N_C}{N_V}$$

$$\text{Ef} = \frac{E_C + E_V}{2} - \frac{kT}{N_C} \log 1 \quad (\because \log 1 = 0)$$

$$\text{Ef} = \frac{E_C + E_V}{2} - 0$$

$$\boxed{\text{Ef} = \frac{E_C + E_V}{2}}$$

Fermi energy lies in the middle conduction band and valence band.

~~scribbled~~

(PQ)

for a particular mass condition the position of the fermi energy level for intrinsic semiconductor is below the center of the intermediate energy gap justify.

Ans  $\Rightarrow$  for a particular mass fermi energy level is below center of the intermediate energy gap. it implies that the density of states in the valence band is higher than that in the conduction band, this situation can occur in certain materials where the effective mass of electrons is higher than that of holes.

(PQ)

Discuss the location of fermi levels under suitable limiting conditions with necessary theory.

under suitable limiting conditions, the location of the fermi level can be determined by the following

- ① At absolute zero temperature  $\therefore$  fermi level lies in the center of the band gap b/w Valence and Conduction band.
- ② At higher temperature  $\therefore$  The fermi level shifts towards the conduction band due to the thermal excitation of electrons.
- ③ In a heavily doped semiconductor  $\Rightarrow$  the fermi level lies close to the donor or acceptor energy level.

(PQ)

Intrinsic semiconductor behave as insulator at OK, comment and add your suggestion

$$\text{only } \Rightarrow E_F = \frac{E_C + E_V}{2} - \frac{kT}{N} \log \frac{N}{M}$$

at  $\boxed{\quad}$   $T = 0K$

$$E_F = \frac{E_C + E_V}{2}$$

It is middle of energy gap.

$E > E_F$  no  $\bar{e}$  found }

$E < E_F$  :  $\bar{e}$  found }

if energy state is less than Fermi energy level  $\bar{e}$  (electron) found if energy state is greater than Fermi energy level so no  $\bar{e}$  found

so at OK any intrinsic semiconductor behave as insulator.

(ii) if we increase in the temperature then it behaves like conductor.

(PQ)

What is the physical meaning of Fermi level?

The physical meaning of Fermi level is that it represents the boundary between the occupied and unoccupied energy states of the electron in a material.

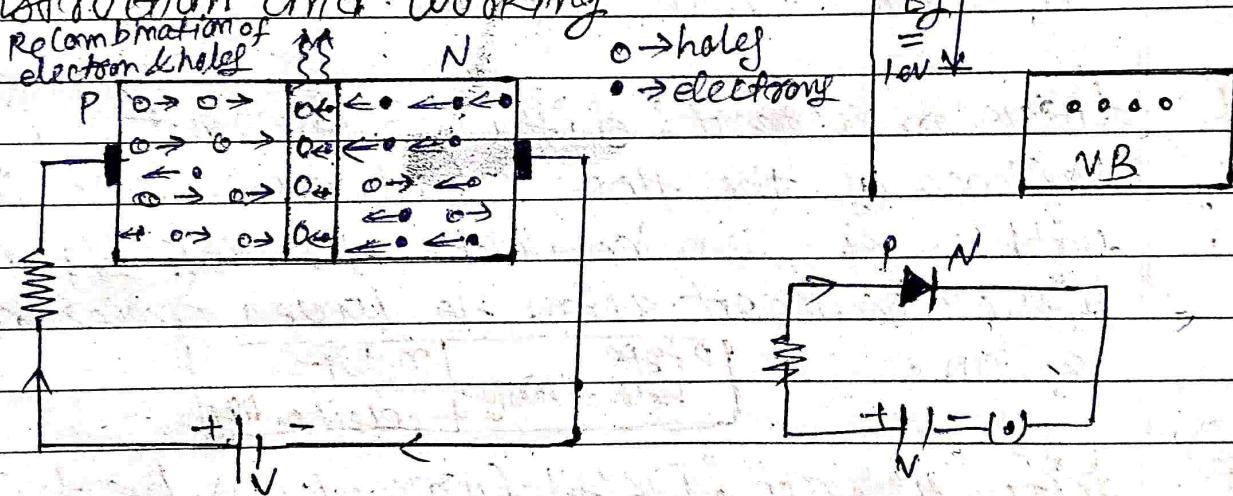
**Ques** Discuss the working of a LED with proper energy band diagram.

**Ans** LED is a semiconductor light source that emits light when current flows through it.

~~principle: Energy is required to move an electron from valence band to conduction band to generate electron and hole pair, energy is emitted when an electron and hole recombine in semiconductors. Energy is emitted in the form of heat but some special semiconductors emit energy in the form of light. This is the basic principle of LED.~~

C.B.

### Construction and Working



LED consists of PN junction diode. A resistance  $R$  is connected in series to control the current through diode. In PN junction diode there is excess of holes in P-region and excess of electrons in N-region. When voltage is applied across the junction diode, the diffusion of holes from P to N region and electrons from N to P region

take place while crossing the junction es and holes recombine with each other and energy released. The energy released due to recombination process is in the form of photon of energy  $h\nu = E_g$  where  $E_g$  is bandgap.

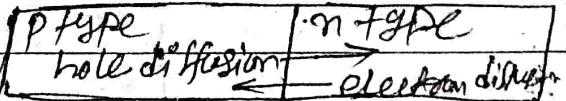
$$\therefore h\nu = E_g \quad [E_g = \frac{hc}{\lambda}] \quad \nu = \frac{c}{\lambda}$$

$$\lambda = \frac{hc}{E_g}$$

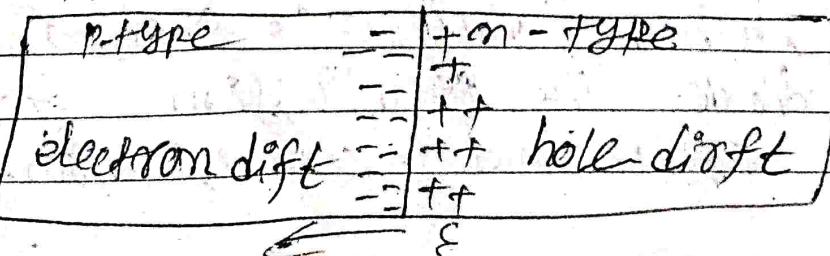
Some application of it

- ① Aviation lightning
- ② Automobile headlamps
- ③ Camera flashes
- ④ Mobile phone backlight

 diffusion current : diffusion current can be defined as the flow of charge carriers within a semiconductor travel from a higher concentration to lower concentration region.



 drift current : Drift Current can be defined as the charge carriers move in a Semiconductor because of the electric field. There are two kinds of charge carriers in a semiconductor like hole and electron.



### Diffusion current

i) The diffusion current can be occurred because of the diffusion in charge carriers.

ii) It obeys Fick's law.

iii) It is independent of permittivity.

iv) Some amount of external energy is enough for the process of diffusion current.

### Drift current

The drift current can be occurred because of movement of charge carriers.

It obeys ohm's law.

It depends on the permittivity.

It requires electrical energy for the process of drift current.



Q Where does fermi level lie in intrinsic semiconductor and in n-type & p-type semiconductors.

Ans  $\Rightarrow$  In intrinsic semiconductor, the fermi level lies in the middle of the band gap. This is because of no of electrons in the conduction band is equal to no of holes in the valence band.

The fermi level in n-type semiconductor lies just below the conduction band.

The fermi level in a p-type semiconductor lies close to the top of the valence band.

Q) Stating the principle of Solar Cell, briefly explain its working.

A Solar Cell or Photovoltaic cell is a device that converts light energy from sun light

principle → Solar cell work on the principle of photovoltaic effect.

working → When photons depletion layer

of energy greater than  $E_g$  fall at the junction

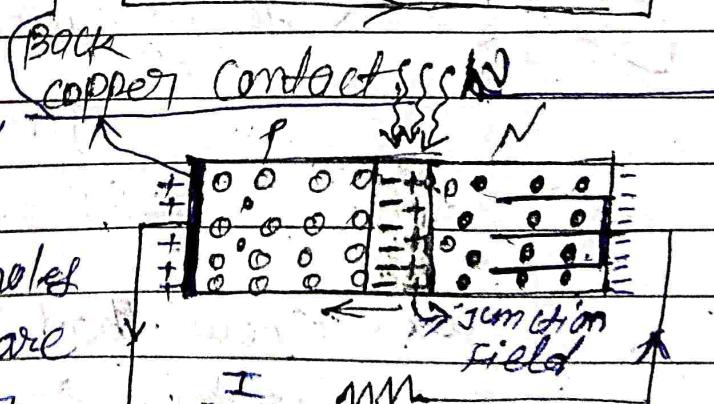
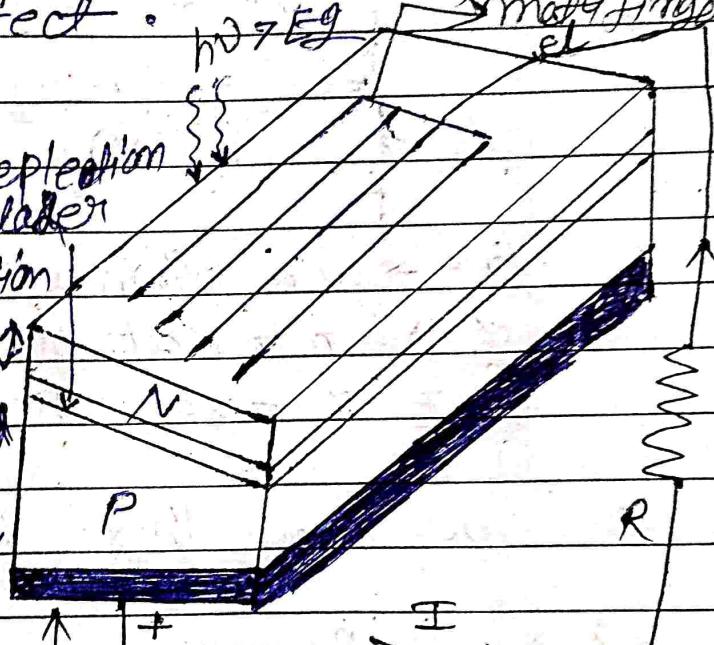
e-hole pairs are generated in the depletion layer. The e-holes produced

and moves in opposite direction due to junction field.

The electrons move towards n-side of P-N junction are collected by the metal fingers.

Holes moves towards p-side are collected by back copper contact.

thus the p-side becomes positive and n-side becomes negative when external load is connected then current flows through the load R.



Expt

M

~~Different~~ Derive the differential equation for harmonic oscillator. Also show that total energy of the harmonic oscillator is constant at any instant of time.

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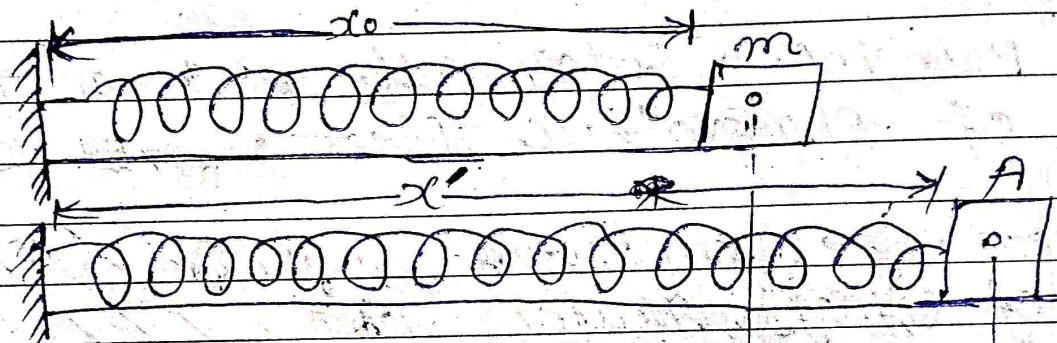
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Roshan paper last year

PyQ

Establish the equation of simple harmonic oscillating  $x(t) = A \sin(\omega t + \phi)$  with symbols having their usual meaning. Find kinetic energy and potential energy associated with SHM. Hence show that total energy of a simple harmonic oscillator is constant at any instant of time.

Ans →

 $x_0 \rightarrow$  equilibrium length $x' \rightarrow$  length at time t $\Delta x = x' - x_0$  change in length (extension)

we know that in simple harmonic oscillation

Restoring force  $\propto -x$  - displacement $f \propto -x \quad \because k - \text{force constant}$ 

$$f = -kx \quad (1)$$

we know that from Newton's second law of motion  $F = ma$  where  $m = \text{mass}$ 

$$F = -kx$$

 $a = \text{acceleration}$ 

$$ma = -kx \quad \left\{ \therefore a = \frac{d^2x}{dt^2} \right.$$

$$\frac{d^2x}{dt^2} = -\frac{kx}{m}$$

$$\frac{d^2x}{dt^2} + \frac{kx}{m} = 0$$

equation of SHO.

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad \left\{ \begin{array}{l} \omega^2 = \frac{k}{m} \\ \Rightarrow \omega = \sqrt{\frac{k}{m}} \end{array} \right. \text{ angular velocity}$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad (2)$$

Sol of SHM eq

Multiplying by  $2 \frac{dx}{dt}$  in eq (2) both sides

$$2 \frac{dx}{dt} \frac{d^2x}{dt^2} + \omega^2 2 \frac{dx}{dt} x = 0$$

$$\frac{d}{dt} \left( \frac{dx}{dt} \right)^2 + \omega^2 \frac{d(x^2)}{dt} = 0 \quad (3)$$

$$\therefore \frac{d}{dt} \left( \frac{dx}{dt} \right)^2 = \frac{d}{dt} \left( \frac{dx}{dt} \right)^2$$

$$\text{and } \frac{d(x^2)}{dt} = \frac{d(x^2)}{dt}$$

Integrating both sides in (3)

$$\int \frac{d}{dt} \left( \frac{dx}{dt} \right)^2 + \omega^2 \int \frac{d(x^2)}{dt} = C$$

$$\left( \frac{dx}{dt} \right)^2 + \omega^2 x^2 = C \quad (4)$$

$$\text{At } x=A; v=0; \frac{dx}{dt}=0$$

$$(0)^2 + \omega^2 A^2 = C \quad (5) \text{ put 4 in } (3)(4)$$

$$\left( \frac{dx}{dt} \right)^2 + \omega^2 x^2 = \omega^2 A^2 = \left( \frac{dx}{dt} \right)^2 = \omega^2 A^2 - \omega^2 x^2$$

$$\left( \frac{dx}{dt} \right)^2 + \omega^2 x^2 = \omega^2 (A^2 - x^2)$$

$$\left(\frac{dx}{dt}\right)^2 = w^2 (A^2 - x^2)$$

$$\frac{dx}{dt} = \pm w \sqrt{A^2 - x^2}$$

$$\frac{dx}{dt} = w \sqrt{A^2 - x^2}$$

$$\frac{dx}{\sqrt{A^2 - x^2}} = w dt$$

Integrating both sides

$$\int \frac{dx}{\sqrt{A^2 - x^2}} = w \int dt + \phi$$

$$\sin^{-1} \frac{x}{A} = wt + \phi$$

$$\frac{x}{A} = \sin(wt + \phi)$$

$$x = A \sin(wt + \phi)$$

Sol. of diff eq  
of sinm.

Using the equation  $x = A \sin(\omega t + \phi)$  for natural vibrations of a stretched spring in horizontal plane. Show that the total energy of a vibrator is a constant quantity.

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The equation  $x(t) = A \sin(\omega t + \phi)$  represents a simple harmonic motion, where  $x(t)$  is the displacement of an object from its equilibrium position at time  $t$ ,  $A$  is the amplitude of the motion,  $\omega$  is the angular frequency of the motion, and  $\phi$  is the phase angle.

→ we know that Kinetic energy is

$$K.E = \frac{1}{2} m v^2 - \textcircled{A}$$

value of  $v$  in SHM we know that

$$v = A \omega \cos \omega t$$

$$v = A \omega \sqrt{1 - \sin^2 \omega t}$$

$$\text{put } \sin \omega t = \frac{y}{A}$$

$$v = A \omega \sqrt{1 - \frac{y^2}{A^2}} \quad v = A \omega \sqrt{A^2 - y^2}$$

$$| v = \omega \sqrt{A^2 - y^2} | - \textcircled{1} \text{ put in A}$$

$$K.E = \frac{1}{2} \times m \times \left[ \omega \sqrt{A^2 - y^2} \right]^2$$

$$K.E = \frac{1}{2} m \cdot \omega^2 (A^2 - y^2)$$

$$K.E = \frac{1}{2} m \omega^2 A^2 - \frac{1}{2} m \omega^2 y^2$$

∴ we know that Potential energy  $\Rightarrow P.E = \frac{1}{2} K y^2$

$$P.E = \frac{1}{2} K y^2 - \textcircled{A} \quad \{ \text{in SHM}$$

$$\rightarrow K = m \omega^2$$

$$P.E = \frac{1}{2} \times m \omega^2 y^2 - \textcircled{C}$$

$$\therefore J = A (\text{max})$$

$$P.E = \frac{1}{2} m \omega^2 A^2$$

PQ → Prove by mathematical analysis that the mechanical energy of free oscillations of a simple harmonic is conserved.

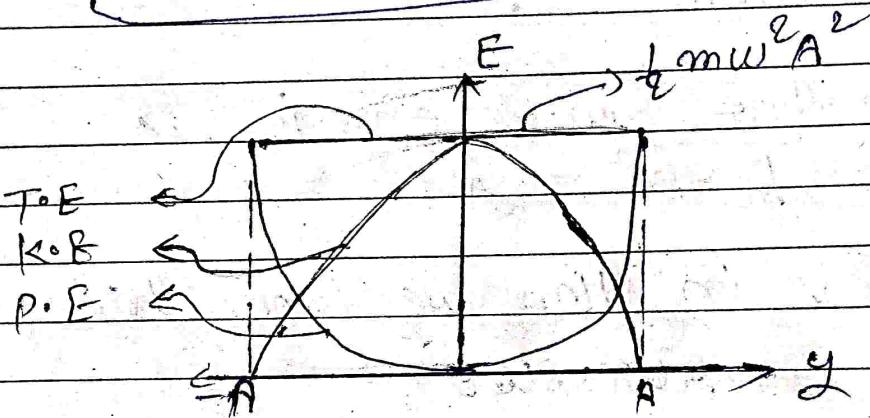
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$$\text{Total energy} = K \cdot E + P \cdot E$$

$$= \frac{1}{2} m \omega^2 A^2 - \cancel{\frac{1}{2} m \omega^2 x^2} + \cancel{\frac{1}{2} m \omega^2 y^2}$$

$$\boxed{T \cdot E = \frac{1}{2} m \omega^2 A^2}$$



Here  $T \cdot E$  (total energy) is constant at any instant of time.

PQ Define Simple Harmonic motion? Give examples.

Ans → Simple harmonic motion is a type of periodic motion or oscillation motion where the restoring force is directly proportional to the displacement acts in opposite direction.

$$F \propto -x$$

$$\boxed{F = -kx} \quad k \rightarrow \text{force constant}$$

$$x \rightarrow \text{displacement}$$

Ex → Motion of a mass attached to a spring

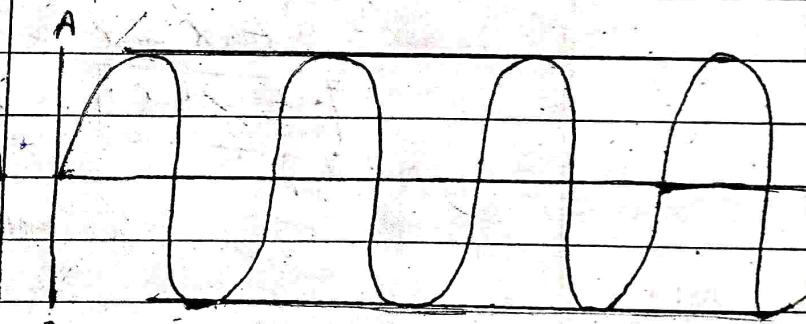
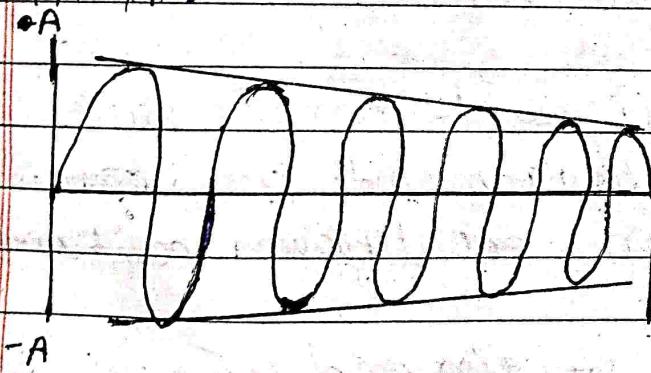
**PYP** Differentiate damped and undamped oscillations.  
 damped undamped

i) The oscillations whose amplitude decreases with time called damped oscillation.

The oscillations whose amplitude remains constant with time called undamped oscillation.

ii) There is external force. There is no external force.

iii) An example of damped oscillation is pendulum in Air. An example of undamped oscillation is pendulum in vacuum.



**SVO** What are oscillations?

What do you understand by free oscillations and forced oscillations?

Oscillation is a periodic movement around an equilibrium point. There are two types of oscillation i) free oscillation ii) forced oscillations.

Free oscillations = The oscillation of a body when it oscillates with its own frequency are called ~~are~~ free oscillation.

Ex  $\Rightarrow$  a simple pendulum.

(ii)

forced oscillation : when a body is subjected to periodic force, it oscillates with the frequency of the periodic forces are called forced oscillation.

Ex  $\Rightarrow$  vibration of a guitar string.

(pxo)

Define Hooke's law

any  $\Rightarrow$  The force needed to extend or compress a spring is directly proportional to the distance it is stretched or compressed from its equilibrium position.

$$F \propto -x$$

$f = -kx$  where  $k$  force constant  
 $x$  displacement from its own equilibrium position.

Ans T  
pxo

Define stress and strain and give their types.

any  $\Rightarrow$  stress  $\hat{=}$  stress refers to the internal force per unit area that arises in a material.

Types of stress  $\hat{=}$  i) Tensile stress  
 ii) Compressive stress  
 iii) Shear stress

strain  $\hat{=}$  strain is a measure of the change in shape that occurs in a material when it is subjected to stress.

Types of stress  $\hat{=}$  i) Tensile strain  
 ii) Compressive strain  
 iii) Shear strain

(PQ) Discuss briefly, the motion of a lightly damped oscillator

$\Rightarrow$  The motion of a lightly damped oscillator is affected by its initial conditions and the characteristics of the damping force. For example, if the oscillator is initially displaced from its equilibrium position and released, it will oscillate with [redacted] decreasing amplitude until it comes to rest.

✓ eq<sup>n</sup> of motion of damped oscillator

(PQ) Derive [redacted] differential equation of damped harmonic oscillations and discuss special cases of oscillatory motion. Discuss the case of critically and specially cases of oscillatory motion.

$\Rightarrow$  Restoring force  $f_R \propto -x$

$$f_R = -c x \quad (A) \quad \begin{matrix} c \rightarrow \text{force} \\ \text{constant} \end{matrix}$$

frictional force  $f_D \propto -\dot{x}$

$$f_D = -y \cdot \frac{dx}{dt} \quad \left. \begin{matrix} y \text{ is damping} \\ \text{coefficient} \end{matrix} \right\} \quad (B)$$

$$F = f_R + f_D \quad (C)$$

$$F = -cx - y \frac{dx}{dt} \quad \left. \begin{matrix} \text{According to Newton's} \\ \text{and law} \end{matrix} \right\} \quad (D)$$

$$f = ma \rightarrow f = m \frac{d^2x}{dt^2} \quad (D)$$

equating eq<sup>n</sup> (C) & (D)

$$\frac{m d^2x}{dt^2} = -cx - y \frac{dx}{dt}$$

$$\frac{m d^2x}{dt^2} + cx + y \frac{dx}{dt} = 0 \quad \left. \begin{matrix} \text{divide by } m \text{ both} \\ \text{sides} \end{matrix} \right\}$$

$$\frac{c}{m} \frac{dx^2}{dt^2} + \frac{y}{m} \frac{dx}{dt} + \frac{c}{m} x = 0$$

put  $\frac{y}{m} = 2K$  (K is damping const.)  
 $\frac{c}{m} = w_0^2$

$$\boxed{\frac{d^2x}{dt^2} + 2K \frac{dx}{dt} + w_0^2 x = 0} \quad \text{diff eq}$$

Critically damped  $\therefore$  critically damping is the point at which the system's motion returns to equilibrium as quickly as possible.

Special damped  $\therefore$  in this type of motion the amplitude of the oscillation decreases with respect to time and oscillation frequency reduces until the system reaches its equilibrium position.

Lightly damped  $\therefore$  lightly damped motion refers to a system that has a small amount of resistance to motion.

(px) Q Lightly damped oscillations are simple harmonic or not.

Ans ~~Yes~~  $\rightarrow$  lightly damped oscillations are not simple harmonic but they are same in some cases.

## (PYP) Application of superconductivity

- Ans → i) Magnetic levitation (Maglev)  
 ii) Power transmission  
 iii) Super computer  
 iv) MRI / NMR devices  
 v) motors / generators.

(PYP) Differentiate hard and soft magnetic materials.

Soft magnetic material

hard magnetic material

Ans → It can be easily magnetized or demagnetized.

It cannot be easily magnetized or demagnetized.

i) Its coercivity and Retentivity are small.

Its coercivity and retentivity are large.

iii) It has high permeability.

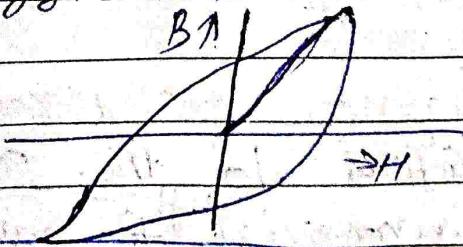
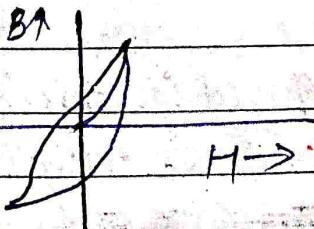
It has low ~~permeability~~.

iv) eddy current loss is ~~high~~ low.

eddy current loss is high.

v) hysteresis loss is low.

hysteresis loss is high.



(PYP) What is hysteresis? Based on magnetic material

Ans → When an external magnetic field is applied to a magnetic material, the material's magnetic domain align with the direction of the applied field. This is known as hysteresis.

# Nanomaterial

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PYO

write short note on Dielectric materials.

Ans ⇒

Dielectric materials are often used as insulators because they can withstand high electric fields without conducting electricity, they can store electric charge. The capacitance of a capacitor depends on the dielectric constant of the material between plates.

Ex = capacitor → plastic, glass

\* \*

write short note on ferroelectric material

Ans ⇒

Ferroelectric materials have a unique combination of electrical, mechanical, and optical properties, which make them useful in a wide range of applications. They are used in electronic devices such as capacitors, transducers.

PYO

write short notes on i) Magnetic Anisotropy  
ii) Magnetostriiction.

i)

Magnetic Anisotropy : Magnetic anisotropy refers to the property of a magnetic material ~~that~~ to have a ~~preferred~~  $\theta$  particular direction of magnetization. This means that the material is easier to magnetize in one particular direction. Magnetic anisotropy is used in various applications such as magnetic data storage devices, magnetic sensors.

(ii) **Magnetostriction:** When a magnetic field is given to a magnetic material, it experiences <sup>change</sup> in all its shape or size or length or dimension with property of some magnetic materials is known as **Magnetostriction**. Application of magnetostriction is magnetic sensors, transformer.

**Q10)** What are nanomaterials? How can we classify them? Application of nanomaterials.

**Ans**) Nanomaterial is a material made up of between 1 and 100 nanometers. It has at ~~at least~~ one dimension in the nanoscale. Nanomaterial used in electronics, medicine.

Classification of Nanomaterials.

- (a) carbon based nanomaterials
- (b) metal based nanomaterials
- (c) Dendrimers
- (d) Nanocomposites

**Q10)** Write some important applications and risks of nano materials.

Applications

- i) **Electronics:** Nano materials are used in electronic devices to improve their performance  
ex  $\Rightarrow$  transistor.
- ii) **Biomedical:** Nano materials are used in bio-medical - ex  $\Rightarrow$  drug delivery, imaging.

③ Energy : Nano materials are used in emerging applications such as solar cells, fuel cells and batteries.

④ Water treatment : Nano materials are used in water treatment to remove pollutants ~~and~~ ex → titanium dioxide.

### Risks:

① ~~Environmental~~ Environmental Impact

→ Nano materials can also have an environmental impact such as polluting water and soil.

② Toxicity : Nano materials can be toxic to living organisms, including humans.

(px) Explain factors responsible for change in properties when we change from bulk to nanomaterial or nanoscale.

Ans → When we move from bulk materials to nanomaterials, there are several factors that can contribute to changes in their properties.

i) Size : ~~The~~ size of the nano material decreases, the surface area to volume ratio increases which can significantly affect the properties of the material.

- (ii) Surface area: The high surface area of nanomaterials can lead to changes in their physical and chemical properties.
- (iii) Quantum effects: At the nanoscale quantum effects can become dominant and can lead to unique electronic and optical properties.  
Ex  $\Rightarrow$  band gap of a material can change as the size of the material decreases.
- (iv) Crystal structure: As the size of a material decreases, its crystal structure can also change ex  $\Rightarrow$  change in crystal structure can result in changes in the material's hardness, strength and conductivity.

Q Why these properties are different from bulk materials?

Ans  $\Rightarrow$

Nano materials have different properties from bulk materials due to the increased surface area-to-volume ratio, quantum effects, and size dependent properties that arise at the nanoscale.



ferrites : Ferrites are a type of ceramic material that have magnetic properties. They are composed of iron oxide ( $\text{Fe}_2\text{O}_3$ ) combined with other metal oxides, such as nickel oxide ( $\text{NiO}$ ); zinc oxide ( $\text{ZnO}$ ). The specific combination of metal oxides determines the properties of the ferrite.

Ferrites are known for their high electrical resistances and low eddy current losses which makes them useful in various applications in electronics & telecommunication.

## SHM

## Base of Harmonic

$$\omega = 2\pi$$

$T \rightarrow$  Time period

$t \rightarrow$  given time

$$\phi = \omega t$$

$$K = \frac{\omega^2}{T}$$

Eq of SHM

$$x = A \sin(\omega t + \phi) \quad [\text{mean position}]$$

$$x = A \cos(\omega t + \phi) \quad [\text{extreme position}]$$

$$v = \frac{dx}{dt}, \quad a = \frac{d^2x}{dt^2}$$

at mean position [displacement  $x = 0$ ]

at extreme position

[displacement  $x = \pm A$ ]

$$\text{frequency} \quad \omega = 2\pi f$$

$$f = \frac{1}{T}$$

angular frequency

$$\text{Time Period} = T = \frac{1}{f}$$

$$\text{unit} = \text{Hz} = \text{sec}^{-1}$$

Amplitude = Mean position  $\leftrightarrow$  extreme position  $\leftrightarrow$  distance  $\leftrightarrow (A)$

time  $\rightarrow$  small ( $t$ )

time period ( $T$ )

Note  $\rightarrow$  mean position  $\Leftrightarrow \phi = 0$  &  $\dot{x} = \ddot{x} = 0$   
extreme position  $\Leftrightarrow \phi = \pm \frac{\pi}{2}$

$$K \cdot E = \frac{1}{2} mv^2$$

$$B \cdot P \cdot E = \frac{1}{2} K x^2$$

(Q5)

An electromagnetic beam of 100 photons having wavelength of  $1\text{ Å}$ , passes through  $1\text{ mm}^2$  area of cross section. Find the intensity of em waves that crosses the area in 5 sec.

ans  $\rightarrow$  given  $\rightarrow n$  of photons = 100

$$\underline{1\text{ mm} \times 1\text{ mm}} \quad d = 1\text{ Å} \quad \text{area} = 1\text{ mm}^2 = (10^{-3})^2 = 10^{-6}\text{ m}^2$$

$$t = 5 \quad I = ?$$

$$\therefore \text{formulae of Intensity} \Rightarrow I = E \frac{W}{m^2}$$

$$E = h\nu$$

$$E = \frac{hc}{d}$$

$$E = \frac{hc}{d}$$

$$E = \frac{m \cdot hc}{d} \quad E = \frac{100 \times 6.63 \times 10^{-34}}{10^{-10}} \times 3 \times 10^8$$

$$E = 1989 \times 10^{-16} \text{ J}$$

$$I = \frac{1989 \times 10^{-16}}{10^6 \times 5}$$

$$I = 397.8 \times 10^{-10} \text{ W/m}^2$$

(Q6)

A car weighing  $200\text{ kg}$  is moving with speed  $100\text{ km/h}$  find de-Broglie wavelength associated with moving car.

$$m = 200\text{ kg}$$

$$v = 100\text{ km/h}$$

$$\lambda = \frac{h}{mv}$$

$$v = ?$$

$$v = \frac{100 \times 1000}{60 \times 60} \text{ m/s}$$

$$v = 30$$

$$V = \frac{500}{18}$$

$$V = \frac{280}{9}$$

$$V = 87.77 \text{ m/s}$$

$$d = \frac{6.63 \times 10^{-39}}{2000 \times 27.27}$$

$$d = \frac{6.63 \times 10^{-39} \times 10}{27.27 \times 2}$$

$$d = \frac{6.63 \times 10^{-39} \times 10}{54.54 \times 36}$$

~~$d = 0.1215 \times 10^{-39} \text{ m}$~~

$$d = 0.1215 \times 10^{-39} \text{ m}$$

$$d = 1.215 \times 10^{-40} \text{ m}$$

$$\begin{aligned} v &= 100 \text{ km/h} \\ v &= 100 \times \frac{1000}{3600} \text{ m/s} \\ &= \frac{100}{36} \text{ m/s} \\ &= \frac{5}{18} \text{ m/s} \end{aligned}$$

(Q8) de-broglie wavelength associated with a moving neutron and an electron is the same.  $d = 1 \text{ Å}$ . Compare their energy.

$$\text{Ans} \Rightarrow \because d = \frac{h}{\text{v}_{\text{RME}}} \quad \text{v}_{\text{RME}} = \frac{h}{d}$$

$$\left[ E_n = \frac{h^2}{8\pi m_n d^2} \right] \quad \left[ E_e = \frac{h^2}{8\pi m_e d^2} \right]$$

$$E_n = \frac{h^2}{8\pi m_n d^2}, \quad E_e = \frac{h^2}{8\pi m_e d^2}$$

$$\frac{E_n}{E_e} = \frac{\frac{h^2}{8\pi m_n d^2} \times \text{v}_{\text{RME}}}{\frac{h^2}{8\pi m_e d^2} \times \text{v}_{\text{RME}}} = \frac{E_n}{E_e} = \frac{m_e}{m_n}$$

$$\text{em} = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{mm} = 1.66 \times 10^{-27} \text{ kg}$$

-27

$$\frac{E_n}{E_e} = \frac{9.1 \times 10^{-31}}{1.66 \times 10^{-27}} = \frac{1.66 \times 10^{-27}}{9.1 \times 10^{-31}}$$

$$\frac{E_n}{E_e} = \frac{1.66 \times 10^{-27}}{9.1 \times 10^{-31}}$$

$$\frac{E_n}{E_e} = 0.1824 \times 10^{14} \times 10^{-27} \times 10^{37}$$

$$\frac{E_n}{E_e} = 0.1824 \times 10^{14} \times 10^{-27} \times 10^{37}$$

$$\frac{E_n}{E_e} = 1.82 \times 10^{-5}$$

Fe

$$\boxed{\frac{E_n}{E_e} = 1.82 \times 10^{-5} \times E_e}$$

(Q15) A swing in park executes SHM whose eq<sup>n</sup> of motion is expressed as  $s(t) = 7 \sin(0.8t + 5\pi)$ . find amplitude, angular frequency ( $\omega$ ), natural frequency ( $f$ ) initial phase of this SHM. At 180° find K.E & P.E of these oscillation at  $t = 38$

ans  $\Rightarrow s(t) = 7 \sin(0.8t + 5\pi) \quad \left\{ \begin{array}{l} \omega = 2\pi f \\ T \end{array} \right.$

$$x = A \sin(\omega t + \phi)$$

$$\boxed{A=7}, \boxed{\omega=0.8}$$

$$\therefore \boxed{\omega = 2\pi f} \quad \omega = 2\pi f$$

$$0.8 = 2\pi f$$

$$f = \frac{0.8}{2\pi T}$$

$$f = \dots$$

$$f = 1.28 \text{ s}^{-1}$$

$$f = 1.28 \text{ s}^{-1}$$

initial phase ( $\phi = 5\pi$ )

$$K.E = ? \quad P.E = ? \quad [K.E = \frac{1}{2}mv^2]$$

$$P.E = \frac{1}{2} K a^2$$

$$P.E = \frac{1}{2} K [7 \sin(0.8t + 5\pi)]$$

$$P.E = \frac{1}{2} K \times 49 \sin^2(0.8t + 5\pi)$$

$$P.E = 24.5 K \sin^2(0.8 \times 3 + 5\pi) \text{ J}$$

$$P.E = 24.5 K \sin^2(2.4 + 5\pi) \text{ J}$$

$$K.E = \frac{1}{2} mv^2 \quad K.E = \frac{1}{2} m \left[ \frac{d}{dt} [7 \sin(0.8t + 5\pi)] \right]^2$$

$$K.E = \frac{1}{2} m [7 \cdot 0.8 \cos(0.8t + 5\pi) \cdot 0.8]^2$$

$$K.E = \frac{1}{8} m [56 \cos(0.8t + 5\pi)]^2$$

$$K.E = \frac{1}{8} m [31.36 \cos^2(0.8t + 5\pi)] \text{ J}$$

$$K.E = 15.68m \cos^2(0.8t + 5\pi) \text{ J}$$

- (Q6) Give SHM with time period of 2 second  
 - find the frequency & wavelength of SHM, also find angular frequency & velocity. ~~Amplitude~~ - 5 m/s.

Soln) given  $T = 2 \text{ sec}$ ,  $v = 5 \text{ m/s}$

$$\text{frequency} = \frac{1}{T} = \frac{1}{2} = 0.5 \text{ Hz/s}^{-1}$$

angular frequency

$$\omega = 2\pi f$$

$$\omega = 2\pi \times 0.5$$

$$\omega = \frac{\pi}{2} \times 1$$

$$\boxed{\omega = \pi \text{ rad.}}$$

wavelength = ~~velocity~~  $\frac{\text{velocity}}{\text{frequency}}$

$$\text{wavelength} = \frac{5}{0.5} = 10 \text{ m}$$

$$\therefore d = 10 \text{ m}$$

$$\text{if } \Rightarrow d = \frac{c}{f}$$

where  $c \rightarrow \text{speed of light}$

then

~~$d = 3 \times 10^8 \text{ m}$~~

~~$d = 3 \times 10^8 \text{ m}$~~

(17) A Spring has spring constant  $K = 500 \text{ N/m}$   
 find extension developed in the spring  
 if restoring force of  $10 \text{ N}$  is acting on it.

$$\text{ans} \Rightarrow K = 500 \text{ N/m} \quad f = 10 \text{ N}$$

$$\therefore f = -Kx \quad (\text{magnitude})$$

$$10 = -500 \times x$$

$$x = \frac{-10}{500} \quad x = \frac{1}{50}$$

$$[x = 0.02 \text{ m}] \quad (\text{magnitude})$$

(18) How much extension is spring will be produced if a man of  $10 \text{ kg}$  is attached to one end of a spring? Given  $K = 500 \text{ N/m}$

$$g = 9.8 \text{ m/s}^2$$

$$\text{ans} \Rightarrow \text{Given } m = 10 \text{ kg} \\ K = 500$$

$$f = Kx$$

$$mg = Kx$$

$$x = \frac{mg}{K} \quad x = \frac{10 \times 9.8}{500}$$

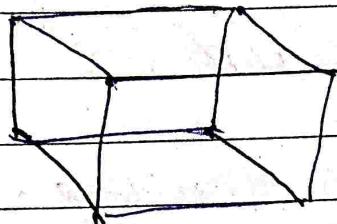
$$x = \frac{98}{500}$$

$$[x = 0.196 \text{ m}]$$

(28)

Given cube with side 1 cm is divided into 1000 small cubes. Find surface area to volume ratio in both cases.

$m$   
Sol:



$$a = 1 \text{ cm}$$

$$a = 10^{-2} \text{ m}$$

$$V = a^3 = V = (10^{-2})^3$$

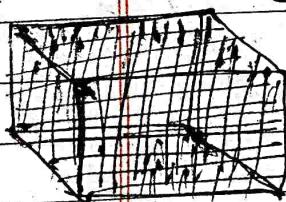
$$V = 10^{-6} \text{ m}^3$$

$$S.A = 6 \times a^2 = 6 \times 10^{-2} \times 10^{-2}$$

$$S.A = 6 \times 10^{-4} \text{ m}^2$$

$$\text{Surface area to volume ratio} = \frac{6 \times 10^{-4}}{10^{-6}} = 6 \times 10^2$$

$$\frac{S.A}{V} = 600 \text{ m}$$



initial volume = final volume

$$a^3 = 1000 a_1^3$$

$$a_1^3 = \frac{a^3}{1000} \quad a_1 = \left(\frac{a}{10}\right)^3$$

$$a_1 = \sqrt[3]{\left(\frac{a}{10}\right)^3} \quad a_1 = \frac{10^{-2}}{10^{-6}}$$

$$a_1 = 10^{-3} \text{ m} \quad V = 1000 \times 10^{-3} \times 10^{-3} \times 10^{-3} = 10^{-9} \text{ m}^3$$

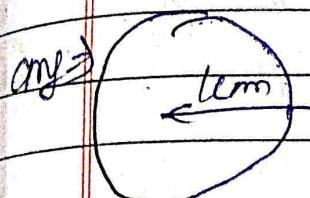
$$S.A = 1000 \times 6 \times 10^{-3} \times 10^{-3} = 1000 \times 6 \times 10^{-6}$$

$$S.P = 6 \times 10^{-3} \text{ m}^2$$

$$\frac{S.A}{V} = \frac{6 \times 10^{-3}}{10^{-6}} = 6 \times 10^3 = 6000 \text{ m}$$

$$\frac{S.A}{V} = 6000$$

(94) Given sphere with radius 1 cm is divided into 1000 small spheres. find Surface area to volume ratio in both cases. Does this new ratio same/mod/less than the original ratio?



$$r = 1 \text{ cm. } V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi r^3 \quad S.A. = \frac{4 \pi r^2}{V} = \frac{4 \pi r^2}{\frac{4}{3} \pi r^3} = \frac{3}{r}$$

$$\frac{S.A.}{V} = \frac{3 \times 4 \pi r^2}{4 \times \frac{4}{3} \pi r^3} = \frac{3}{r}$$

$$\frac{S.A.}{V} = \frac{3}{10^{-2}} \quad \frac{S.A.}{V} = 300$$

initial volume = final volume

~~$\frac{4}{3} \pi r^3 = 1000 \times \frac{4}{3} \pi r_1^3$~~

$$\frac{r^3}{1000} = r_1^3 \quad r_1^3 = \left(\frac{r}{10}\right)^3$$

$$r_1 = \sqrt[3]{\frac{r}{10}} \quad r_1 = \frac{r}{10}$$

$$r_1 = \frac{10^{-2}}{10} \quad r_1 = 10^{-3} \text{ m}$$

$$S.A. = 4 \pi r^2 \quad S.A. = 4 \pi \times 10^{-6} \quad S.A. = 4 \pi \times 10^{-6}$$

$$V = \frac{4}{3} \pi \times 10^{-9}$$

$$\frac{S.A.}{V} = \frac{4 \pi \times 10^{-6}}{4 \pi \times 10^{-9}} \quad \frac{S.A.}{V} = 3 \times 10^3$$

$$\frac{S.A.}{V} = 3000$$

new ratio is more than original ratio.

Q

Given a simple harmonic motion defined by  $x(t) = 12 \sin(0.7\pi t + 5\pi/2)$  find the amplitude, natural frequency, time period, wavelength of the given SHM.

$$\text{given: } x(t) = 12 \sin(0.7\pi t + 5\pi/2)$$

ans  $\Rightarrow$ 

Comparing  $x(t) = A \sin(\omega t + \phi_0)$   
with

we find  $\Rightarrow$  Amplitude  $A = 12$

$$\therefore \omega = 0.7\pi$$

$$\therefore \omega = 2\pi f$$

$$0.7\pi = 2\pi f$$

$$f = \frac{0.7}{2} \quad | f = 0.35 \text{ Hz} / \text{s}$$

$$\therefore f = \frac{1}{T} \quad | T = \frac{1}{f}$$

$$| T = 2.85 \text{ s}$$

$$\therefore kx = 5\pi x$$

$$k = 5\pi$$

$$\therefore k = \frac{2\pi}{T} \quad | d = \frac{2\pi}{K}$$

$$d = \frac{2\pi}{5\pi}$$

$$| d = 0.4 \text{ m}$$

Q A particle of mass  $9.1 \times 10^{-31}$  kg has a kinetic energy of 80 eV. Find the associated de-Broglie wavelength, group velocity, phase velocity.

Given:

$$\text{m} = 9.1 \times 10^{-31} \text{ kg}$$

$$E = 80 \times 1.6 \times 10^{-19} \text{ J}$$

ans  $\Rightarrow$

Find  $d$ ,  $v_g$ ,  $v_p = ?$

$$d = \frac{h}{\text{momentum}} = \frac{h}{\cancel{10^{-34}}} = 6.62 \times 10^{-34}$$

$$6.62 \times 10^{-34} \text{ m} \times 9.1 \times 10^{-31} \text{ kg} \times 80 \times 1.6 \times 10^{-19}$$

$$d = \lambda$$

$$\text{now } v_g = \frac{dE}{dp}$$

$$v_g = \frac{P}{m}$$

$$v_g = \frac{h}{dm}$$

$$v_g = \frac{6.62 \times 10^{-34}}{2 \times 9.1 \times 10^{-31}}$$

$$v_g = \cancel{6.62} \times 10^{-31} \text{ m/s}$$

$$v_p = \frac{E}{P}$$

$$\frac{dE}{dp} = \frac{1}{2m} \times dp$$

$$\frac{dE}{dp} = \frac{dp}{2m}$$

$$\boxed{\frac{dE}{dp} = \frac{P}{m}}$$

$$v_p = \frac{80 \times 1.6 \times 10^{-19}}{2 \times 9.1 \times 10^{-31}}$$

$$v_p = \frac{80 \times 1.6 \times 10^{-19}}{2 \times 9.1 \times 10^{-31}} \times 80 \times 1.6 \times 10^{-19}$$

$$\boxed{v_p = 6.62 \times 10^{-31} \text{ m/s}}$$

**What is superconductivity?**

Ans  $\Rightarrow$  A superconductivity is the ability of certain materials to conduct electric current with practically zero resistance.

Ex  $\Rightarrow$  Al, Hg.

**What are superconductors?**

Ans  $\Rightarrow$  A superconductor is defined as a substance that offers no resistance to the electric current when it becomes colder ~~than~~ <sup>below</sup> a critical temperature.

Ex  $\Rightarrow$  mercury, barium, calcium

**What is conductor?**

Ans  $\Rightarrow$  A conductor is a material that allows electricity to flow through it is called conductors. Ex  $\Rightarrow$  Fe, Al, Cu

**What is insulator?**

Ans  $\Rightarrow$  A insulator is a material that do not allows electricity to flow through it is called insulator.

Ex  $\Rightarrow$  Plastic, glass, wood

**Critical temperature**  $\div$  **Critical temperature** is that temperature at which the resistance of a conductor becomes zero is called critical temperature. Ex  $\Rightarrow$  Hg is  $T_c$  is  $4.2\text{K}$

## Application of Superconductor

- (1) Magnetic Resonance imaging (MRI)
- (2) Power transmission.
- (3) Energy storage.
- (4) Quantum Computing.
- (5) Magnetic levitation.
- (6) fusion energy.

\* What is superconductivity?

Ans → A superconductivity is the ability of certain materials to conduct electric current with practically zero resistance.

Ex → Al, Hg.

\* What are superconductors?

Ans → A superconductor is defined as a substance that offers no resistance to the electric current when it becomes colder ~~than~~<sup>below</sup> a critical temperature.

Ex → mercury, barium, calcium

\* What is conductor?

Ans → A conductor is a material that allows electricity to flow through it; it is called conductor. Ex → Fe, Al, Cu

\* What is insulator?

Ans → A insulator is a material that does not allow electricity to flow through it; it is called insulator.

Ex → plastic, glass, wood

\* Critical temperature is that temperature at which the resistance of a conductor becomes zero; it is called critical temperature. Ex → Hg  $T_c = 4.2K$

\* What is Semiconductor?

Ans) A semiconductor is a substance which lies between the conductor and insulator.  
Ex) silicon, germanium.

\* What is an ideal conductor?

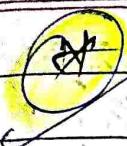
Ans) An ideal conductor is a hypothetical material whose resistance is ideally zero. In ideal conductor a current will continuously flow through it even when the power source is removed.

\* What is a perfect conductor?

Ans) A perfect conductor is an idealized material that has zero electrical resistance and can conduct electricity infinitely well without any loss of energy.

\* What are the properties of superconductivity?

- i) zero resistance
- ii) diamagnetic material
- iii) persistent current



Names few properties which do not change when a conductor becomes Superconductor ?

Electrical charge

- i) Electrical charge : ~~Superconductivity~~ is a property that arises due to the interaction between electrons in a material, however the charge of the material remains the same before and after the transition to superconductivity
- ii) Atomic structure : The atomic structure of material remains unchanged when it transitions to a superconductor
- iii) Mass : The mass of the material also remains the same before and after transition to a superconductor
- iv) Volume : The volume of material does not change when it becomes a superconductor
- v) Elasticity : Elasticity property of a material does not change when a conductor becomes a superconductor

\* Name some property which ~~changes~~ changes when a conductor becomes superconductor?

- i) Zero resistance : The electrical resistance of a superconductor goes to zero, which means that an electrical current can flow through it without any loss of energy. ~~it is~~  
but zero resistance can not possible in conductors
- ii) Critical temperature : At critical temperature, ~~superconductors~~ the ~~temperature~~ of the <sup>super</sup>conductor is zero. but at critical temperature, ~~the resistance~~ ~~not~~ the ~~resistance~~ of conductor does not zero. necessary to
- iii) Meissner effect : A normal conductor cannot repel magnetism but when a conductor becomes a superconductor than they repel the magnetic field and behave like diamagnetic.
- iv) Persistence current : When we keep rings of superconductors in applied magnetic field and we connect ring with power supply than there is current induced in ring and ~~a~~ steady current flows through superconductor for years even if power supply is off, but this things does not work in normal conductor.

AM



what do you mean by Meissner effect? prove superconductors are perfect diamagnetic materials? It also called flux expulsion  
 Application of meissner effect (Same) for d

Q) When a superconducting material is placed in a magnetic field and cooled below the critical temperature, it behaves than superconducting material repels magnetic field lines and behaves like diamagnetic material is known as meissner effect

from magnetic property

$$B = \mu_0(I + H) \quad \text{--- (1)}$$

$\rightarrow$  intensity of ~~magnetic field~~

$\oplus$  magnetisation

$H \rightarrow$  applied magnetic field

when material becomes a superconductor

then  $B = 0$

$$0 = \mu_0(I + H)$$

$$\therefore I = -H \quad \text{--- (2)}$$

Now,

$$\therefore \chi_m = \frac{I}{H}$$

$$\chi_m = -H \quad [ \chi_m = -1 ]$$

$$[ \chi_m = -1 ] \quad \text{--- (3)}$$

magnetic susceptibility  $\chi_m = -1$

so relative permeability  $[ \mu_r = 1 + \chi_m ]$

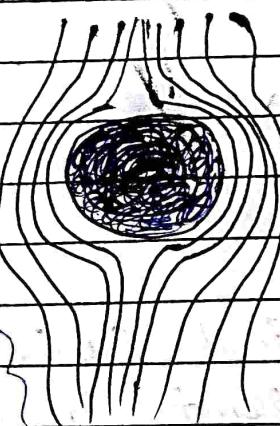
$$\mu_r = 1 - 1$$

$$[ \mu_r = 0 ] \quad \text{--- (4)}$$

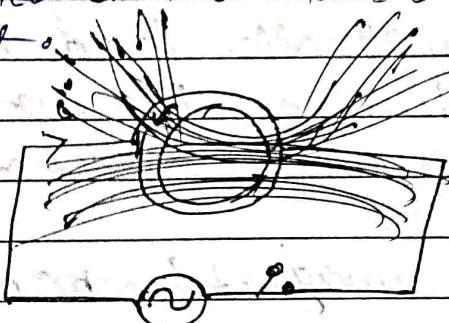
so from eqn. (3) and (4) it indicates that

Superconductors are perfect diamagnetic Application of meissner effect material.

Magnetic levitation, magnetic shielding, superconducting magnets.

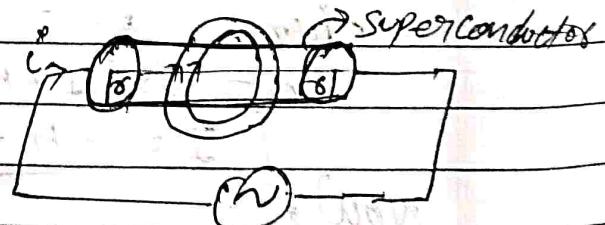


**\* Persistent current** : When a superconducting ring is placed in a magnetic field, a current is induced in the ring. When ring connected through a power supply and if we remove that power supply, we find the magnitude of current is constant without powersupply. Such steady current flowing in superconductor is called persistent current.



**\* Critical current** : The maximum applied current in circuit which can be flow through a superconductor without destroying its superconductivity. is called critical current ( $I_c$ )

Conductor radius =  
from amperes law



$$\int \vec{B} \cdot d\vec{l} = \mu_0 i$$

$$\int H \cdot d\vec{l} = i \quad \left. \begin{array}{l} \{ \\ \end{array} \right\} \vec{B} = \mu_0 i$$

$$H(2\pi r) = C$$

$$H = H_c, i = I_c$$

$$I_c = 2\pi r H_c$$

critical current refers to maximum current that a superconductor can carry without experiencing resistance. is called critical current.

$$I_c = 2\pi r H_c$$

\* What are applications of the superconductors?  
use in

- ans  $\Rightarrow$
- i Super Computer.
  - ii Use in magnetic levitation. (train)
  - iii Superconducting magnets
  - iv Power transmission
  - v Medical diagnosis

✓ \* What is critical magnetic field ( $H_c$ )

ans  $\Rightarrow$  The strong magnetic field at which the superconducting material loses its superconductivity is called critical magnetic field

\* What is the effect of Magnetic field on super-conductor

ans  $\Rightarrow$  When a strong magnetic field is applied to a superconductor superconductor loses its superconductivity.

(\*) flux penetration/define London penetration depth

ans  $\Rightarrow$  When the temperature of very thin film of superconductor decreased below critical temperature then field lines are not expelled out of material, that is there line continue to pass through the material this phenomena is called flux penetration.

\* For Sample & for a Material become a superconductor there are two necessary conditions to be obeyed in respect to each other

- any  $\Rightarrow$  i) Resistance of material should become zero at a given value of temperature.
- ii) Material should be diamagnetic.

\* Define critical value of magnetic field

any  $\Rightarrow$  The critical value of magnetic field refers to the specific value of the magnetic field at which a material undergoes a phase transition from one magnetic state to another.

$$H_{c(T)} = H_{c(0)} \left[ 1 - \frac{T}{T_c} \right]$$

\* Define critical value of current

any  $\Rightarrow$  The critical value of current is the maximum current that a device can carry before it undergoes a significant change in its behavior, critical value of current varies widely depending on the type of device.



Explain superconductors are non ohmic devices.

Ans  $\Rightarrow$  we know that from maxwell eq<sup>n</sup>

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \text{--- (1)}$$

from ohm's law in vector form

$$\vec{J} = \sigma \vec{E} \quad \text{--- (2)}$$

Let's consider superconductors obey ohm's law

but for superconductor

$$\text{Resistivity } [\rho = 0] \quad \therefore [\sigma = \infty]$$

$\sigma$  put in (2)

$$\vec{J} = \frac{\vec{E}}{\rho} \Rightarrow [\vec{E} = \rho \vec{J}]$$

But  $\rho = 0$

$$[\vec{E} = 0] \text{ put in (1)}$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \Rightarrow - \frac{\partial \vec{B}}{\partial t} = 0$$

Integrate both sides  
w.r.t. time

$$[\vec{B} = \vec{C}] \text{ constant}$$

$\therefore$  when material becomes superconductor then magnetic flux density within the superconductor should be remain constant

So when  $\rho$  becomes zero inside material or superconductor then  $B$  also become zero.

Hence our assumption is wrong.  
Hence Superconductor does not obey Ohm's law. So they are non ohmic devices.

\* Define Cooper pair : When a material goes below critical temperature then electrons which are present in material electrons they make pair with together and that pair of electrons in superconductor are called cooper pair.

\* Isotope effects : The critical temperature of superconductor varies with isotope mass on the other words Mass of isotope is inversely related to critical temperature.

$$\frac{T_c \propto}{m^2}$$

(\*) significance of London equation.

- ① They provide a simple description of the Meissner effect.
- ii) They explain why superconductors have zero resistance to electric current.
- iii) They have been used in MRI machines, superconducting magnets.

Ques Deduce London equations and define London penetration depth

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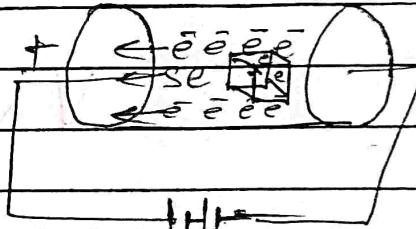
London equation and its significance

- In 1935, London brothers, F London & H London modified Maxwell's equation to explain the phenomena of superconductivity & gave London equation. According to London theory every superconductor contains two types of electron  
 (i) Normal electron (ii) Super electron

When the temperature of superconductor is increased, no of normal electrons ( $e^-$ ) increased and when temperature of superconductor decreased, no of superelectrons increased. at 0-Kelvin all the  $e^-$  in superconductors are super

0-Kelvin

Let Super electron = nse



$$f = -e\vec{E}$$

according to Newton second law

$$\vec{F} = m\vec{a} \quad (\vec{a} = \frac{d\vec{s}}{dt})$$

$$\vec{E} = \frac{\vec{F}}{m} = \frac{\vec{F}}{e}$$

$$1 \quad -e\vec{E} = m\left(\frac{d\vec{s}}{dt}\right)$$

$$\int \vec{F}^2 = -e\vec{E}$$

$$\frac{d\vec{s}}{dt} = -e\vec{E} \quad 2$$

$$\therefore \vec{I} = \frac{e}{R}$$

$$\therefore \vec{I} = -neAV \Rightarrow \vec{I} = nev \Rightarrow \vec{I} = nev$$

$$\vec{I} = -nevs \quad 3$$

diff eq(3) wrt t

$$\frac{d\vec{I}}{dt} = -nse \frac{d\vec{s}}{dt} \quad 4$$

Put eq<sup>n</sup> 2 in 4

$$\frac{d\vec{I}}{dt} = -nse \left(-\frac{e\vec{E}}{m}\right)$$

$$\frac{d\vec{J}_S}{dt} = \frac{e^2 n_S \vec{E}}{m} \rightarrow \text{London 1st equation}$$

(5)

from eq<sup>n</sup> (5) we find  $\vec{E}$

$$\vec{E} = \frac{m}{n_S e^2} \frac{d\vec{J}_S}{dt} \quad (6)$$

from Maxwell law eq

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (7)$$

put eq (6) in (7)

$$\nabla \times \left( \frac{m}{n_S e^2} \frac{d\vec{J}_S}{dt} \right) = - \frac{\partial \vec{B}}{\partial t}$$

$$\frac{m}{n_S e^2} \left( \nabla \times \frac{d\vec{J}_S}{dt} \right) = - \frac{\partial \vec{B}}{\partial t}$$

Integrating both sides

$$\frac{m}{n_S e^2} \int \left( \nabla \times \frac{d\vec{J}_S}{dt} \right) dt = - \int \frac{\partial \vec{B}}{\partial t} \cdot dt$$

$$\frac{m}{n_S e^2} (\vec{J} \times \vec{J}_S) = - \left[ \vec{B} \right]_{B_0}^{B}$$

$$\frac{m}{n_S e^2} (\vec{J} \times \vec{J}_S) = - (\vec{B} - \vec{B}_0) \quad \begin{cases} \text{According to} \\ \text{Misson effect} \end{cases}$$

$$\frac{m}{n_S e^2} (\vec{J} \times \vec{J}_S) = - \vec{B} \quad \begin{cases} \vec{B}_0 = 0 \end{cases}$$

$$\nabla \times \vec{J}_S = - \frac{\vec{B} n_S e}{m} \quad \text{London second equation}$$

from maxwell eq<sup>n</sup>  $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

Taking curl both side

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_0 (\vec{\nabla} \times \vec{J})$$

by a vector property  $(\vec{A} \times (\vec{B} \times \vec{C})) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \vec{B}(\vec{\nabla} \cdot \vec{\nabla}) = \mu_0 (-\frac{\vec{B} m s e^2}{m})$$

$$\therefore \vec{\nabla} \cdot \vec{B} = 0 \text{ so}$$

$$-\vec{B} + \vec{\nabla}^2 \vec{B} = +\frac{\mu_0 m s e^2}{m} \vec{B}$$

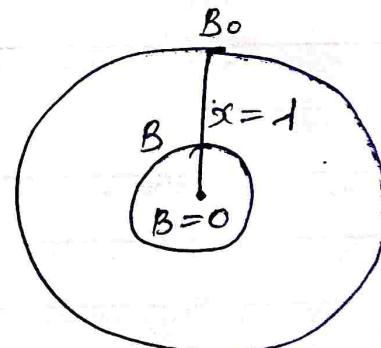
Comparing with  $\vec{\nabla}^2 \vec{B} = \frac{1}{r^2} \vec{B}$

$$\frac{1}{r^2} = \frac{\mu_0 m s e^2}{m}$$

where  $r$  is penetration depth

$$\vec{\nabla}^2 \vec{B} = \frac{1}{r^2} \vec{B}$$

$$\Rightarrow \text{so } r = \frac{1}{B(x)} = \frac{1}{B_0 e^{-x/r}}$$



$$\begin{aligned} x &= 1 \\ B(x) &= B_0 e^{-x/r} \\ B(x) &= B_0 e^{-1} \\ B(x) &= \frac{B_0}{e} \end{aligned}$$

so,  $B$  is decreasing by  $\frac{1}{e}$

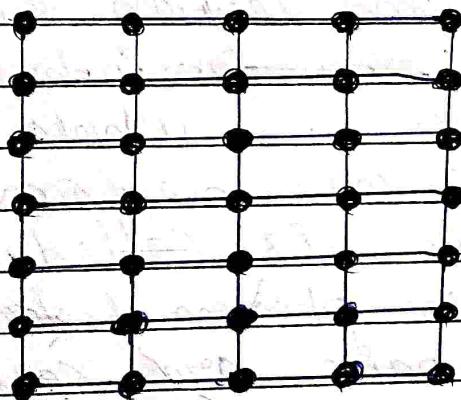
so, the depth  $x$  at which  $B$  becomes zero is known as penetration depth ..

\* BCS theory : The quantum theory of superconductivity was given combinedly by three scientists Bardeen, Cooper and Schrieffer in 1957 this theory is called BCS theory.

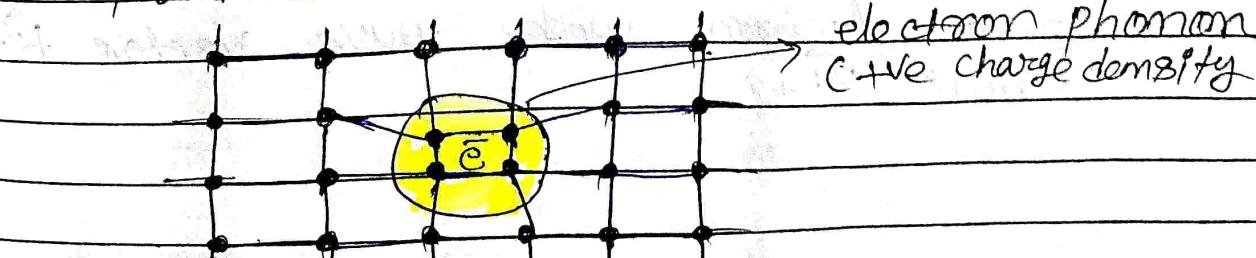
BCS theory explain superconductivity phenomena by 2 term

- ① electron ( $e^-$ ) phonon electron ( $e^-$ ) interaction
- ② Cooper pair

Let Consider a normal lattice

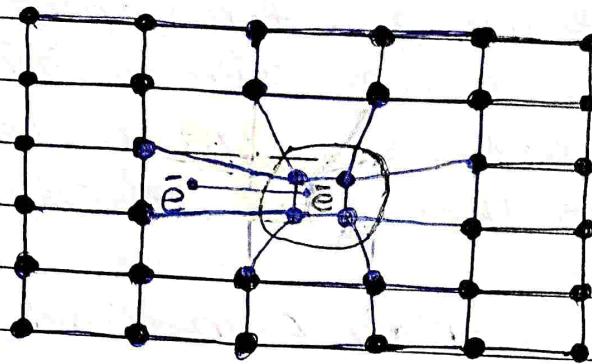


Now, if any electron will entered in lattice, then so the positive ions around the the electron will be attracted to the electron and after attraction positive ions will be displaced from own position and the bond of ions will be distorted so distortion will be produce in lattice.

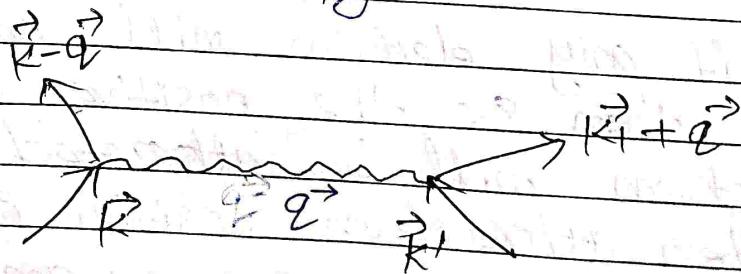


electron phonon  
c+v charge density

In the lattice around the electron high +ve charge density will be produced called electron phonon.

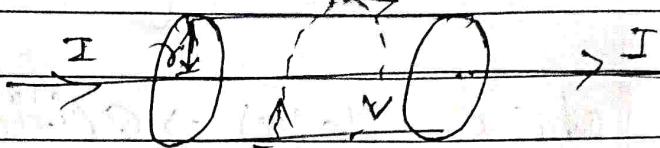


When another electron enters in the lattice so it will be attracted by the charge density and form a weak link between electron and phonon and this attraction between phonon and electron called electron-phonon interaction. Now 1st electron moves in whole lattice along with second electron in terms of pair this is called cooper pair so this electron-phonon-electron interaction and cooper pairs are responsible for superconductivity.



Let the 1st electron have wave vector  $\vec{k}$  if it emits a phonon  $\vec{q}$  which is absorbed by second electron with wave vector  $\vec{k}'$  and emits  $\vec{k} + \vec{q}$ .

## \* Silsbee's rule :

$r \rightarrow$  radius of wire  $\rightarrow$  

$I \rightarrow$  Current of wire

$B \Rightarrow$  mag flux density

$\therefore$  By applying Ampere circuital law

$$\oint B \cdot dl = \mu_0 I$$

$$\oint B dl \cos 0^\circ = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I$$

$$\frac{B(2\pi r)}{\mu_0} = I \Rightarrow \boxed{H(2\pi r) = I} \quad \text{--- (2)}$$

Let  $H_c \rightarrow$  critical field

$H \leq H_c$  for material to become a superconductor

$$2\pi r H \leq 2\pi r H_c$$

$$I \leq 2\pi r H_c$$

$I_c =$  max allowed current

$I_c = 2\pi r H_c \rightarrow$  absence of external magnetic field

in the presence of external magnetic field.  
 $I_c$  decreases

$$\text{So } I_c = 2\pi r (H_c - 2H)$$

This is called Silsbee's rule.

\* Numerical on critical temperature and critical magnetic field.

$$H_c(T) = H_c(0) \left[ 1 - \frac{T}{T_c^2} \right]$$

where  $H_c(T) \rightarrow$  critical magnetic field at any  $(T)$  temperature

$H_c(0) \rightarrow$  critical magnetic field at  $0K$  temperature  
 or maximum magnetic field.  
 $T \rightarrow$  Temperature  
 $T_c \rightarrow$  critical temperature or transition temperature

\* Ques The critical magnetic field for a superconductor at absolute zero is  $9 \times 10^4$  Am $^{-1}$  and at 6K is  $5 \times 10^4$  Am $^{-1}$  find the critical temperature and energy required to break cooper pair at absolute zero.

Ans  $\rightarrow$  Given  $H_c(0) = 9 \times 10^4$  Am $^{-1}$   $\frac{4}{5} = 1 - \frac{36}{T_c^2}$   
 $H_c(6K) = 5 \times 10^4$  Am $^{-1}$   $\frac{9}{5} = 1 - \frac{36}{T_c^2}$

Find  $T_c = ?$   $\frac{36}{T_c^2} = \frac{9-5}{9}$

$$H_c(T) = H_c(0) \left[ 1 - \frac{T}{T_c^2} \right] \quad \frac{36}{T_c^2} = \frac{9-5}{9}$$

$$5 \times 10^4 = 9 \times 10^4 \left[ 1 - \frac{36}{T_c^2} \right] \quad \frac{36}{T_c^2} = \frac{4}{9}$$

~~$5 \times 10^4 = 9 \times 10^4 \left[ 1 - \frac{36}{T_c^2} \right]$~~

$$4 T_c^2 = 36 \times 9$$

$$T_c^2 = \frac{36 \times 9}{4}$$

$$T_c^2 = 81$$

$$T_c = 9K$$

$$\boxed{T_c = 9K}$$

~~energy required to break cooper pair at absolute zero~~

Note  $\rightarrow$  To find the energy required to break a cooper pair at absolute zero, we can use BCS theory

$$\Delta = 1.76 \times K \times T_C \quad E = 3.54 \times 1.38 \times 10^{-23} \times T_C^{-2}$$

$$K = \text{Boltzmann Constant} = (1.38 \times 10^{-23} \text{ J/K})$$

$$\Delta = 1.76 \times 1.38 \times 10^{-23} \times 9$$

$$E = 2\Delta \quad \Delta = 21.85 \times 10^{-23}$$

energy required to break a cooper pair at absolute zero form =  $2\Delta$

$$E = 2 \times 21.85 \times 10^{-23} \quad E = \frac{hc}{d} \quad \begin{array}{l} \text{current} \\ \text{density} \end{array}$$

$$E = 43.71 \times 10^{-23} \text{ J}$$

$$d = \frac{hc}{E} \quad H_C = \frac{I_C}{2\pi r} \quad E_C = H_C \cdot \pi r^2$$

you can get wavelength also

**Q8** Given critical magnetic field of material at 0K is  $15 \times 10^3 \text{ A/m}$  calculate critical magnetic field at ~~70 m~~ 5K for the same material. Given  $T_C = 7 \text{ K}$ .

$$H_C(0) = 15 \times 10^3 \text{ A/m}$$

$$H_C(5K) = ?$$

$$T_C = 7 \text{ K}$$

$$H_C(T) = H_C(0) \left[ 1 - \frac{T^2}{T_C^2} \right]$$

$$H_C(5K) = 15 \times 10^3 \left[ 1 - \frac{25}{49} \right]$$

$$H_C(5K) = 15 \times 10^3 \left[ 1 - 0.51 \right] \quad \text{Alm}$$

$$H_C(5K) = 15 \times 10^3 [0.49] \Rightarrow H_C(5K) = 7.35 \times 10^3 \text{ Alm}$$

Pyc

Load in superconducting state has critical temperature of  $7\text{ K}$  at zero magnetic field and critical field of  $8.5\text{ T}$  at  $0\text{ K}$ . Determine the critical field at  $4\text{ K}$ .

$$\text{Ans} \Rightarrow T_c = 7\text{ K}, H_c(0) = 8.5\text{ T (Tesla)} \\ H_c(4\text{ K}) = ?, T = 4\text{ K}$$

Note  $\rightarrow$  To convert tesla to Ampere Permetre  
A/m  
a formula here

$$H = \frac{\chi T}{4\pi \times 10^7} \text{ A/m}$$

$$8.5\text{ T} = 6.76 \times 10^{-8} \text{ A/m}$$

$$H_c(T) = H_c(0) \left[ 1 - \frac{T}{T_c} \right]^2$$

$$H_c(4\text{ K}) = 6.76 \times 10^{-8} \left[ 1 - \frac{4}{7} \right]$$

$$= 6.76 \times 10^{-8} [1 - 0.32]$$

$$H_c(4\text{ K}) = 6.76 \times 10^{-8} [0.68]$$

$$H_c(4\text{ K}) = 4.60 \times 10^{-8}$$

$$H_c(4\text{ K}) = 4.60 \times 10^{-8} \text{ A/m}$$

Pyc  
for a given Superconductor sample value of critical magnetic field corresponding to  $14\text{ K}$  and  $13\text{ K}$  respectively are  $2.8 \times 10^{-8} \text{ A/m}$  and  $5.6 \times 10^{-8} \text{ A/m}$  find critical temperature and critical magnetic field value at  $0\text{ K}$ .

~~Ans~~

$$H_c(14) = 2.8 \times 10^8 \text{ A/m}$$

$$H_c(13) = 5.6 \times 10^8 \text{ A/m}$$

$$T_c = ? , H_c(0K) = ?$$

~~for~~ for at tem 14K

$$H_c(14K) = H_c(0) \left[ 1 - \frac{T^2}{T_c^2} \right]$$

$$2.8 \times 10^8 = H_c(0) \left[ 1 - \frac{14^2}{T_c^2} \right]$$

$$5.6 \times 10^8 = H_c(0) \left[ \frac{T_c^2 - 169}{T_c^2} \right] \quad A$$

for at tem 13K

$$H_c(13K) = H_c(0) \left[ 1 - \frac{T^2}{T_c^2} \right]$$

$$5.6 \times 10^8 = H_c(0) \left[ \frac{T_c^2 - 169}{T_c^2} \right] \quad B$$

divide eq<sup>n</sup> B ÷ A

~~$$5.6 \times 10^8 = H_c(0) \left[ \frac{T_c^2 - 169}{T_c^2} \right]$$~~

~~$$H_c(0) \left[ \frac{T_c^2 - 169}{T_c^2} \right]$$~~

~~$$2 = \frac{T_c^2 - 169}{T_c^2} \times \frac{T_c^2}{T_c^2 - 169}$$~~

$$2 = \frac{T_c^2 - 169}{T_c^2 - 196} \Rightarrow 2T_c^2 - 392 = T_c^2 - 169$$

$$2T_c^2 - T_c^2 = -169 + 392$$

$$T_c^2 = 223 \quad T_c = 14.93 \text{ K}$$

$$H_c(14K) = H_c(0K) \left[ \frac{1 - 196}{(14.93)^2} \right]$$

$$2.8 \times 10^5 = H_c(0K) \left[ \frac{223 - 196}{223} \right]$$

$$2.8 \times 10^5 = H_c(0) \left[ \frac{27}{223} \right]$$

$$27 H_c(0) = 223 \times 2.8 \times 10^5$$

$$H_c(0K) = \frac{223 \times 2.8 \times 10^5}{27}$$

$$H_c(0K) = 8.25 \times 2.8 \times 10^5$$

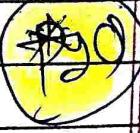
$$\boxed{H_c(0K) = 23.1 \times 10^5 \text{ A/m}}$$

→ Shortest

$$2.8 \times 10^5 = H_c(0) (0.1210)$$

$$H_c(0) = \frac{2.8 \times 10^5}{0.1210}$$

$$\boxed{H_c(0) = 23.1 \times 10^5 \text{ A/m}}$$

 For a given superconducting sample, the values of critical magnetic field corresponding to 0K and 5K respectively are  $10 \times 10^6 \text{ A/m}$  and  $5 \times 10^6 \text{ A/m}$ . Find wavelength of a photon required to break Cooper pairs in the superconductor given Boltzmann's constant  $K = 1.38 \times 10^{-23} \text{ J/K}$ .

Ans

$$H_c(0) = 10 \times 10^6 \text{ A/m}$$

$$H_c(5K) = 5 \times 10^6 \text{ A/m}$$

$$A = ? \quad K = 1.38 \times 10^{-23} \text{ J/K}$$

$$E = 3.54 \times 1.38 \times 10^{-23} \times T_c$$

$$E = \frac{hc}{\lambda} \quad \lambda = \frac{hc}{E}$$

$$H_c(T) = H_c(0) \left[ 1 - \frac{T}{T_c} \right]$$

$$5 \times 10^6 = 1.0 \times 10^6 \left[ 1 - \frac{25}{T_c^2} \right]$$

$$\frac{5 \times 10^6}{10^6 \times 10^6} = \left[ 1 - \frac{25}{T_c^2} \right] \Rightarrow \frac{1}{2} = 1 - \frac{25}{T_c^2}$$

~~$$\frac{25}{T_c^2} = 1 - 0.5$$~~

$$\frac{25}{T_c^2} = \frac{0.5}{1}, T_c^2(0.5) = 25$$

$$T_c^2 = \frac{25}{0.5}, T_c^2 = 50$$

$$T_c = 7.07 \text{ K}$$

$$E = 3.54 \times k \times T_c$$

$$E = 3.54 \times 1.38 \times 10 \times 7.07 / E = \frac{hc}{\lambda}$$

$$E = 34.53 \times 10^{-23} \text{ J}$$

$$A = \frac{hc}{E}$$

$$A = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{34.53 \times 10^{-23}}$$

$$A = 19.86 \times 10^{-34} \times 10^8$$

$$34.53$$

~~$$A = 0.57 \times 10^{-34} \text{ J}$$~~

~~$$A = 0.575152041 \times 10^{-34} \text{ J}$$~~

~~$$A = 0.575152041 \times 10^{-34} \text{ J}$$~~

(PQ) Calculate the Wavelength of the photon which will be required to break cooper pair in superconducting Zn with  $T_c = 0.56 \text{ K}$

$$E = 3.54 \times 1.38 \times 10 \times 0.56 \text{ K}$$

$$E = 2.73 \times 10^{-23} \text{ J}$$

$$E = 2.73 \times 10^{-23} \text{ J}$$

$$A = 7.27 \times 10^{-1} \text{ m}$$

~~$$A = 7.27 \times 10^{-1} \text{ m}$$~~

P 20

## Differentiate type-1 and type-2 superconductors

### Type - 1

(i) It is also known as Soft Superconductors

(ii) There is only one ( $H_c$ ) critical magnetic field

(iii) For this critical magnetic field is very low.

(iv) It shows perfect Meissner effect.

(v) It is complete diamagnetic material.

(vi) Ex  $\rightarrow$  Pb

### Type - 2

(i) It is also known as hard superconductors

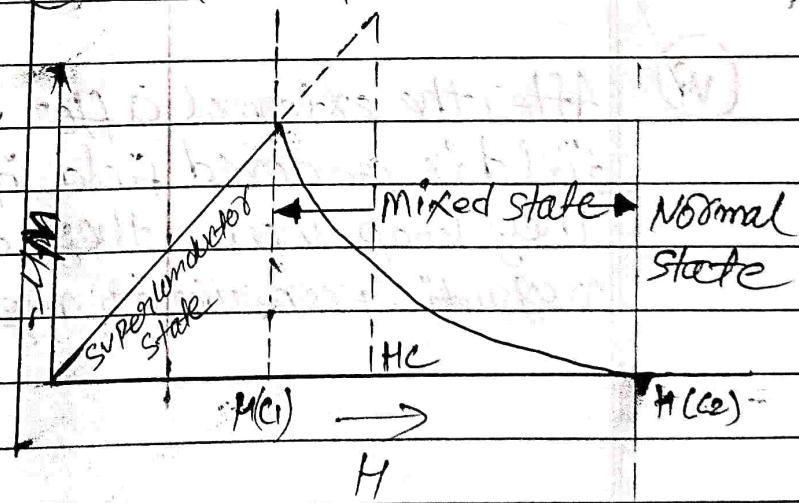
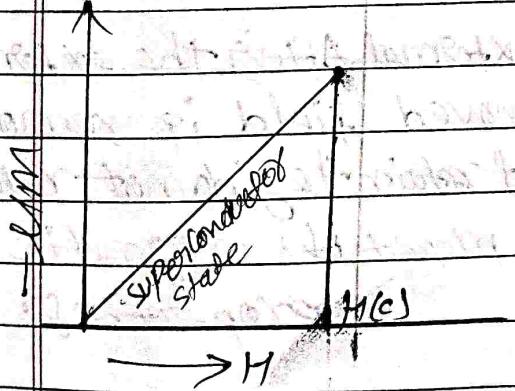
(ii) There are two critical magnetic fields ( $H_{c1}$ ) & ( $H_{c2}$ )

(iii) In this critical magnetic field is very high.

(iv) It does not show perfect Meissner effect.

(v) It does not complete diamagnetic material.

(vi) Ex  $\rightarrow$  Ba



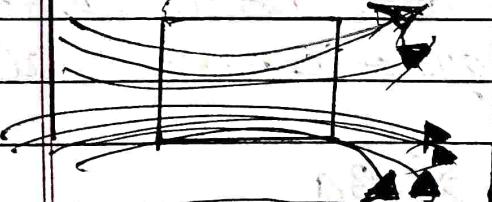
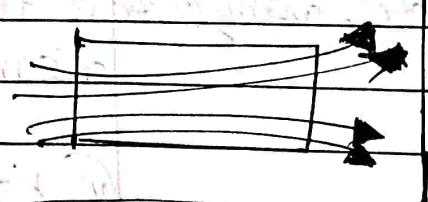
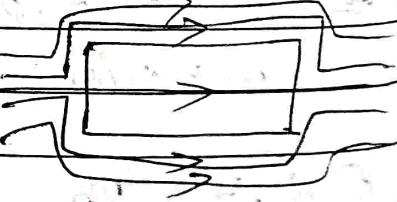
**PYO**

Compare any four properties of diamagnetic, paramagnetic and ferromagnetic substances.

Ferromagnetic

Paramagnetic

Diamagnetic

i	They are solid	They can be solid, liquid or gas	They can be solid, liquid or gas.
ii	They are strongly attracted by a magnet.	They weakly attract by a magnet	They weakly repelled by a magnet.
iii	They tend to move from low to high field region.	They tend to move from low to high field region.	They tend to move from high to low field region.
iv	Ex $\Rightarrow$ Iron	Ex $\Rightarrow$ Lithium	Ex $\Rightarrow$ Copper
v			
vi	After the external field is removed they keep their magnetic properties	After the external field is removed they do not retain its magnetic properties	After the external field is removed they do not retain its magnetic properties

8) dielectric materials : A dielectric material is a poor conductor of electricity but an efficient supporter of electrostatic fields.

I can store electrical charges, have a high resistance and negative temperature coefficient of resistance.

9) B-H Curve  $\rightarrow$  free hand notebook

10) Magnetostriction : Coupling between the magnetization and the lattice does result in deformation, an effect called magnetostriction.

11) Magnetic anisotropy  $\rightarrow$  Magnetic anisotropy refers to the property of a magnetic material that makes it exhibit different magnetic properties along different directions.

( ~~for all~~  $\rightarrow$  exhibit )