

* Algorithm : Algorithm is a step-by-step procedure for solving a particular problem

* Types of data structure

i) Linear

- arrays
- stack
- queue
- linked list

ii) non-linear

- Ex ⇒
- Trees
 - graph

Q what are the operations performed on arrays
Ans Traversing, Searching, Merging, insertion, deletion, shorting.

* Data Structure : Logical organization of data in a particular manner.

* The logical organization of data to solve a particular problem in a particular manner is called data structure.

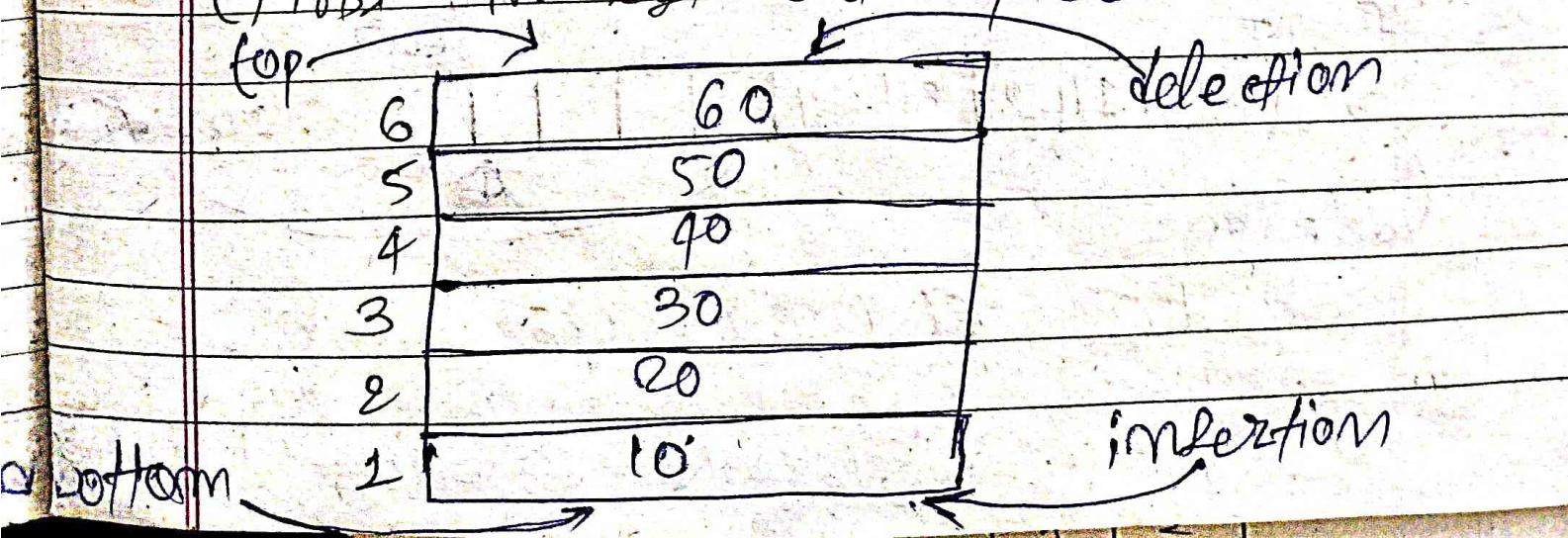
* What do you mean by time complexity?

Ans → Rate of growth of time taken by an algorithm to solve a particular problem with respect to input size.

* What do you mean by space complexity.

Ans → Space complexity refers to the amount of memory or storage space an algorithm uses to solve a problem with respect to input size.

* Stack : Stack is a linear data structure which operates in a LIFO (last in First Out) or FILO (first in last out) pattern.



Standard stack operations :

① push() - ② pop() : ③ isEmpty()

↓
Insert element

↓
Delete element

↓
Stack is empty

④ isFull() ⑤ peek() ⑥ count()

↓
Stack is full

↓
Access the item at the i position

↓
Get the no of item in the stack.

⑦ change() ⑧ display()

↓
change the item at the i position

↓
Display all items in the stack.

* What is Queue data structure?

Ans) Queue is a linear data structure which operates in a ~~fixed~~ (FIFO) first in first out or (LIFO) last in last out pattern.

front

rear



Standard Queue Operations

- ① enqueue() ② dequeue() ③ isEmpty()
- ④ isFull() ⑤ peek() ⑥ count() ⑦ change()
- ⑧ display()

Type of data structure

Data structures

Linear data structures

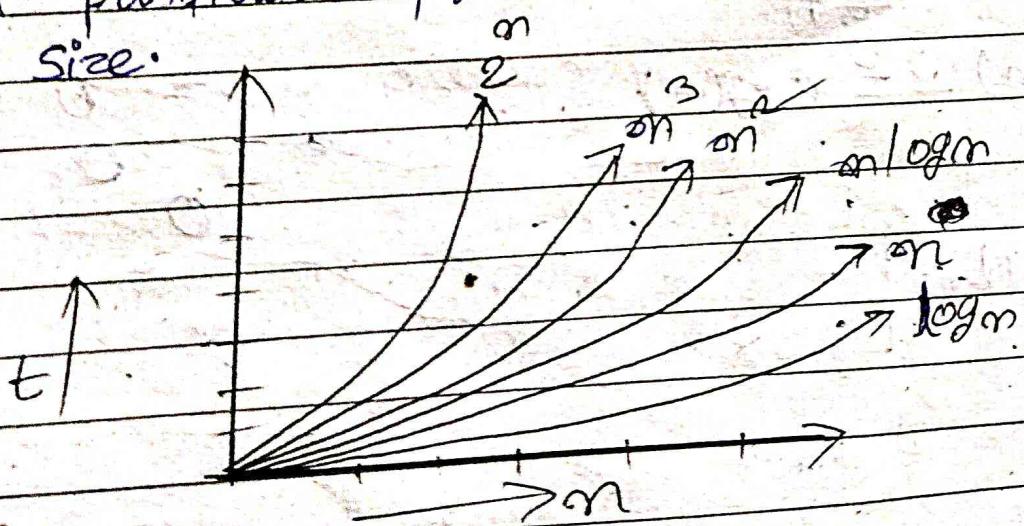
- Array
- Stack
- Queue
- Linked list

Non linear data structure

- Trees
- graph
- Tries

Complexity of algorithm

~~* Time complexity = Rate of growth of time taken by an algorithm to solve a particular problem with respect to input size.~~



Asymptotic notation ($O(1), O(n), \dots$)

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Big O Notation: The Big O notation is the mathematical way to express the upper bound or the longest amount of time an algorithm can possibly take to complete the program.

Constant time $\rightarrow O(1)$

Linear $\rightarrow O(n)$

Logarithmic $\rightarrow O(\log n)$

Quadratic $\rightarrow O(n^2)$ (worst case)

Cubic $\rightarrow O(n^3)$ (worst case)

y -axis $\rightarrow c \cdot g(n)$ upper bound

$f(n)$ tight bound

best case

lower bound

~~$c \cdot f(n) - g(n) >= 0$~~

$$f(n) = O(g(n)) \quad \text{on } x\text{-axis} \rightarrow f(n) = 5n^5 + 2n + 1$$

If

$$f(n) \leq c \cdot g(n) \quad \left\{ \begin{array}{l} \text{where } n \geq n_0 \\ c > 0 \\ n_0 \geq 1 \end{array} \right\}$$

Ex \Rightarrow

$O(1) \rightarrow O(n)$

$O(n)$

\rightarrow

$O(n^2)$

$O(\log n)$

\rightarrow

Solutions

$$f(m) = em^2 + 3m \Rightarrow O(m^2)$$

$$f(m) = 4m^4 + 3m^3 \Rightarrow O(m^4)$$

$$f(m) = N^2 + \log m \Rightarrow O(m^2)$$

$$f(m) = 12001 \Rightarrow O(1)$$

$$f(m) = 3m^3 + m^2 + 5 \Rightarrow O(m^3)$$

$$f(m) = \frac{m^3}{300} \Rightarrow O(m^3)$$

$$f(m) = 5m^2 + \log m \Rightarrow O(m^2)$$

$$f(m) = \frac{m}{q} \Rightarrow O(m)$$

$$f(m) = \frac{m+4}{q} \Rightarrow O(m)$$

Note $\Rightarrow O$ for $(\underbrace{\quad}_{\mathcal{E}} \underbrace{\quad}_{\mathcal{S}} \underbrace{\quad}_{\mathcal{Z}}) \Rightarrow O(m) \Rightarrow O(m+m)$

~~for $(\underbrace{\quad}_{\mathcal{E}} \underbrace{\quad}_{\mathcal{S}} \underbrace{\quad}_{\mathcal{Z}}) \Rightarrow O(m)$~~

for $(0 - n) \rightarrow O(n)$

$\sum_{\mathcal{E}} \text{ for } (0 - n) \rightarrow O(n)$

$\underbrace{\quad}_{\mathcal{Z}} \cdot O(n^2)$

for $(0 - n) \rightarrow O(m)$

$\sum_{\mathcal{E}} \underbrace{\quad}_{\mathcal{Z}} O(m^2) + O(m) \Rightarrow O(m+m)$

TC.

* What is the time complexity of the following code

```
a = 0;
for (i=0; i < n; i++) { } for (0 - n)
  { } for (j=n; j < i; j--) { } for (0 - n)
    { } 3  $\sum_{\mathcal{Z}} a = a + i + j;$  3  $\sum_{\mathcal{Z}} 3^3 \Rightarrow O(n^3)$ 
```

* What is the time complexity

int $a=0$; $b=0$;
 for ($i=0$; $i < n$; $i++$) { for ($0-n$) $O(n)$
 { $a=a+rand()$;
 } }
 for ($j=0$; $j < m$; $j++$) { for ($0-m$) $O(m)$
 { $b=b+rand()$;
 } }
 TC = $O(n+m)$

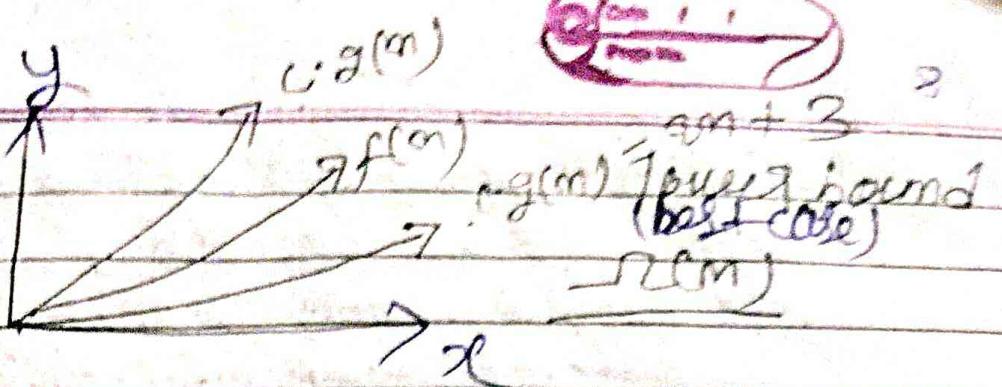
(3) int $a=0, b=0$;
 for ($i=0$; $i < n$; $i++$) { for ($0-n$) $O(n)$
 { for ($j=0$; $j < n$; $j++$) { for ($0-n$) $O(n)$
 { $a=a+j$;
 } } } }
 for ($k=0$; $k < n$; $k++$) { for ($0-n$) $O(n)$
 { $b=b+k$;
 } } }
 $O(n^3) + O(n^2) \Rightarrow O(n^3)$



Big Omega (Ω) = Omega notation
 Specifically describes best case scenario,
 it represents the lower bound time
 complexity of an algorithm.

$$f(n) = \Omega(g(n))$$

Iff
 $f(n) \geq c \cdot g(n)$ { where $n > n_0$, $c > 0$, $n_0 \geq 1$



Big theta (Θ) = Big theta notation
 Specifically describes average case scenario
 it represents the most ~~realistic~~ realistic
 time complexity of an algorithm.

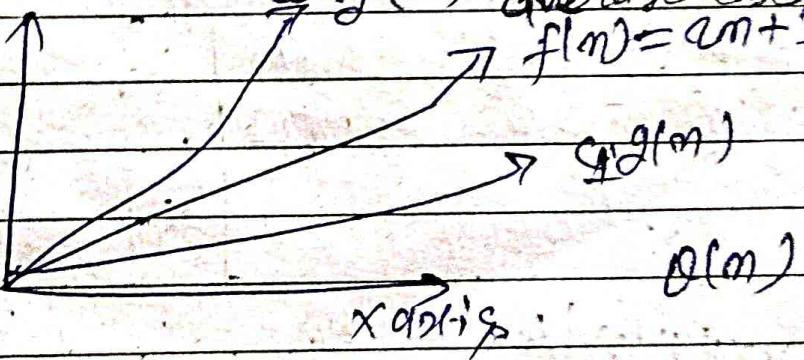
$$f(m) = \Theta(g(m))$$

iff

$$\Theta(g(m)) = C_1 \cdot g(m) \leq f(m) \leq C_2 \cdot g(m)$$

(where $n > n_0$, $C_1, C_2 > 0$, $n > n_0$, $n_0 \geq 1$)

y axis : $\frac{C_1 \cdot g(m)}{g(m)}$ average case $\frac{C_2 \cdot g(m)}{g(m)}$ tight bound



*** Space complexity** : Space complexity refers to the amount of memory an algorithm uses to solve ~~an~~ a problem with respect to input size.

Space complexity includes both Auxiliary space and space used by input.

Space $C = \text{input size} + \text{Auxiliary space}$

* **Auxiliary space:** Auxiliary space is the temporary space allocated by your algorithm to solve the problem, with respect to input size.
 $\text{Space } C = \text{input size} + \text{Auxiliary space}$

$A-1 \rightarrow \text{fun add}(m_1, m_2) \{$

$$\begin{cases} \text{sum} = m_1 + m_2 \\ \text{return sum} \end{cases}$$

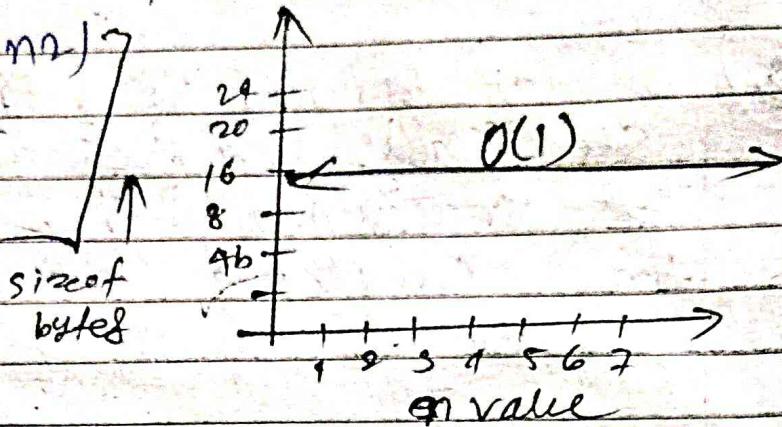
$m_1 \rightarrow 4 \text{ bytes}$

$m_2 \rightarrow 4 \text{ bytes}$

$\text{sum} = 4 \text{ bytes}$

$\text{Aux Sp} = 4 \text{ bytes}$

Total = 16 bytes



$A-2 \rightarrow \text{fun sumOfNumbers(arr}[i, N)] \{$

$$\begin{cases} \text{sum} = 0 \end{cases}$$

~~$\sum \text{sum} = \text{for } i=0 \text{ to } N$~~

$$\begin{cases} \text{sum} = \text{sum} + \text{arr}[i] \end{cases}$$

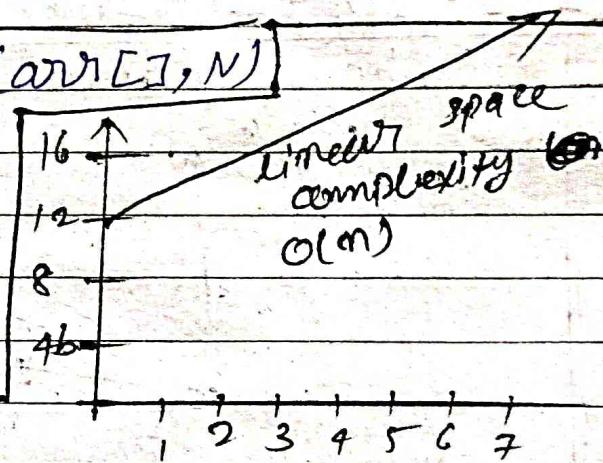
print(sum)

$\text{arr} \rightarrow N \text{ bytes} \quad \text{Aux Sp} = 4 \text{ bytes}$

$\text{sum} = 4 \text{ bytes} \quad \text{tot} = 4N + 12b$

$i = 4 \text{ bytes} \quad O(n)$

$O(n) + c$



$A-3 \rightarrow \text{int fact} = 1;$

$\text{for (int } i=1; i \leq n; i++) \{$

$\{ \text{fact} *= i; \}$

return fact;

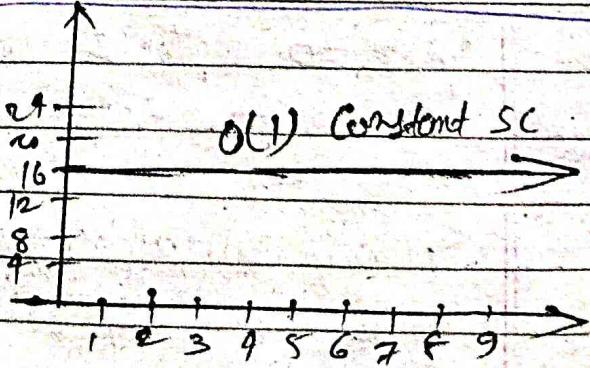
$\text{fact} = 4 \text{ bytes}$

$i = 4 \text{ bytes}$

$n = 4 \text{ bytes}$

$\text{fact} = 4 \text{ bytes}$

$O(1) \text{ constant sc}$



$n = 4 \rightarrow$ if ($n \leq 1$)

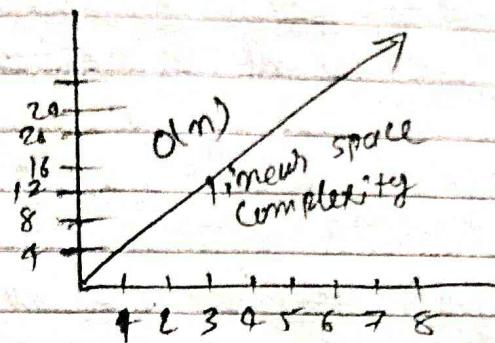
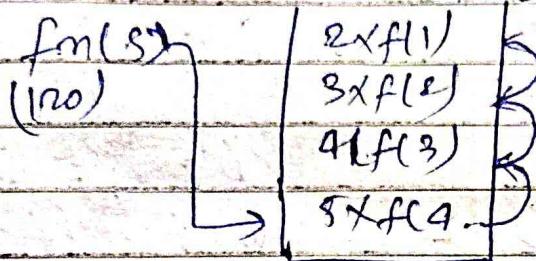
{ return 1;

else {
 ~~return~~

return ($n * factorial(2(n-1))$);

$3 \times f(4)$

// $n \rightarrow 4$ byte, $f(4) \rightarrow 4$ bytes



fact $\Rightarrow m \times 4$ bytes

$s = 4$ bytes + m bytes

Q What do you meant by time space Trade off
 Ans \Rightarrow Time space trade off is a way to solve a problem in less time by using more ~~or~~ storage space or by solving a problem in very little space by spending a long time.

Q difference b/w linear search and binary search.

~~i~~ Linear search

i Also called sequential search

binary search

Also called half-interval search & logarithmic search

ii less efficiency

High efficiency

iii less complex than binary

~~more~~ More complex than linear

iv time complexity ~~O(n)~~ $O(n)$

time complexity ~~O(n)~~ $O(\log n)$

v Best case to find the element in the first position

Best case to find the element in the middle position.

vi No required of sorted array.

Required of sorted array.



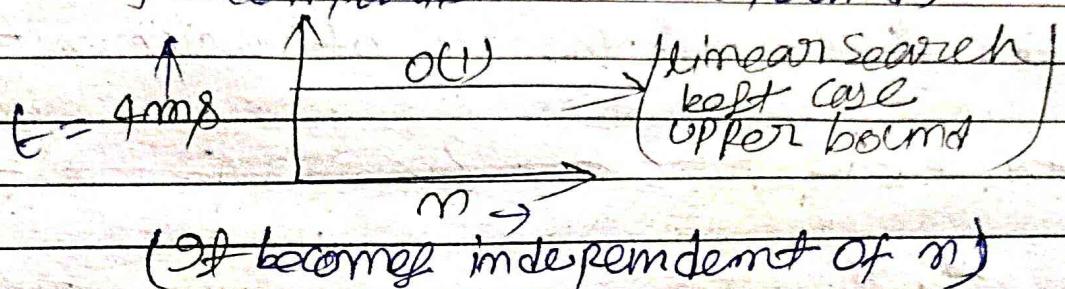
Topic Q1. Efficiency of binary search vs How binary search efficient to linear search.

Ans → linear search

$$x = 9$$

at index = 0 best case $O(1)$

No of comparison = 1 (found)



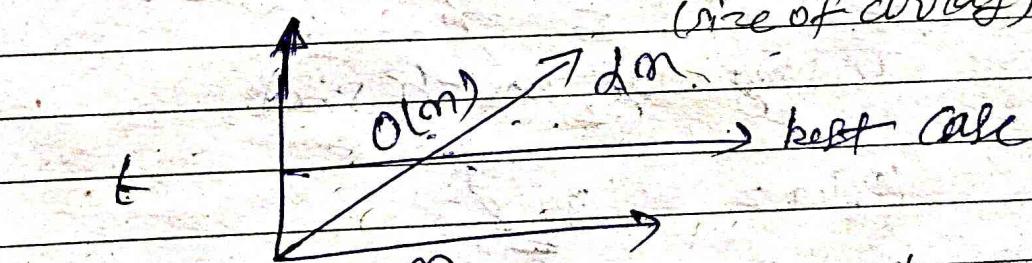
for worst case → i) last
ii) not found

linear search

[9 10 11 12]	Size = 4
0 1 2 3	

$x = 12$ No of Comparison = 4 → found $O(n)$

$x = 17$ No of Comparison = 4 → not found $O(n)$
(size of array)

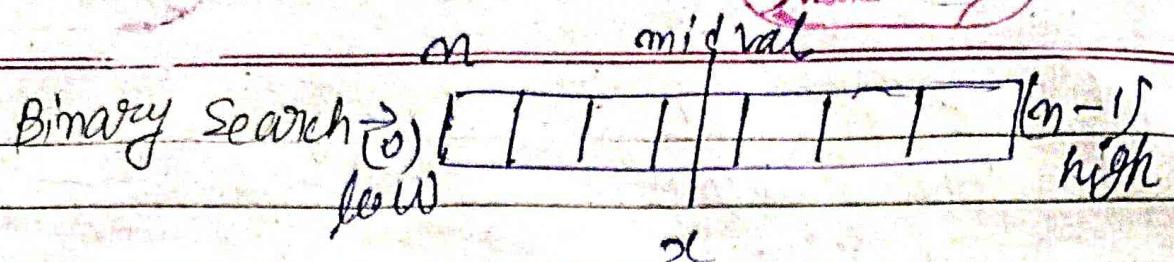


Analysis of linear search

$O(n)$ worst case of LS

$O(1)$ best case of LS

$n \rightarrow x$



$a[\text{mid}] == x$? found



Analysis Of binary Search

After first call the no of element = $\frac{n}{2} = \frac{n}{2}$

After second - = $\frac{n}{2} \times \frac{1}{2} = \frac{n}{2^2}$

1st third - = $\frac{n}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{n}{2^3}$

1st nth - = $\frac{n}{2^K}$

$$\frac{n}{2^K} = 1 \Rightarrow n = 2^K$$

Analysis of Worst Case

nth Call = $n = 2^K$

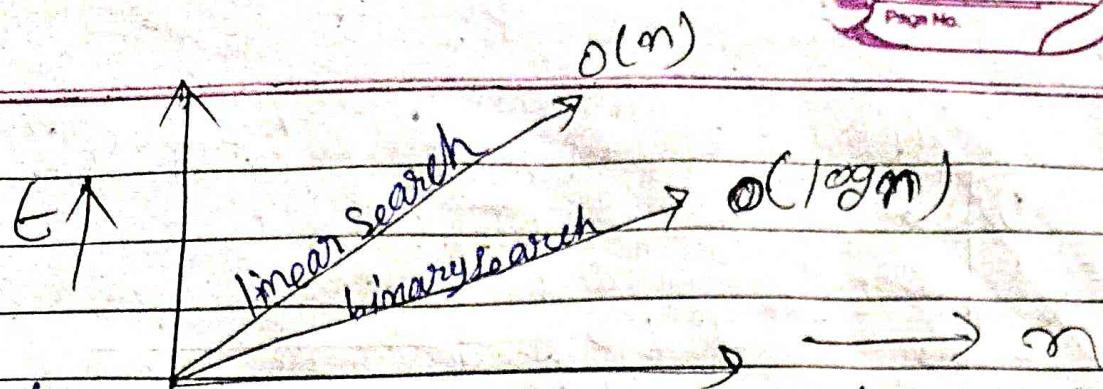
Taking \log_2 both sides

$$\log n = \log 2^K$$

$$\log n = K \cdot \log 2 \quad \left\{ \log_2 2 = 1 \right\}$$

$$\log n = K \times 1$$

$$K = \log(n) \Rightarrow O(\log n)$$



that's how binary search is efficient to linear search.

- * How a multidimensional array is stored in a computer system & representation of column and row major order

ans \Rightarrow

A multidimensional array is stored in a computer system in a linear fashion even if ~~it is~~ it is a multidimensional array because memory is a one dimensional thing.

There are two main ways to store a multidimensional array in memory

i) Row-major order ii) Column-major order

i) Row-major order: In row-major order, the elements of the array are stored in rows, from left to right and top to bottom. The first row is stored first, followed by the second row, and so on.

	0	1	2	3
0	6	9	8	3
1	0	7	9	4
2	8	3	7	0
3	5	1	6	1

left \rightarrow right / Top \rightarrow bottom

Row 1	Row 2	Row 3	Row 4
6	9	8	3
0	7	9	4
8	3	7	0
5	1	6	1

②

Column-major order: In column-major order, the elements of the array are stored in columns, from top to bottom and left to right. The first column is stored first, followed by the second column, and so on.

	0	1	2	3
0	6	9	8	3
1	0	7	9	4
2	8	3	7	0
3	5	1	6	1

6	0	8	5	1	9	7	3	1	8	9	7	6	3	4	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

col-1

col-2

col-3

col-4

→ Computer memory

↓ Column series

* Application of stack

(1) Conversion of infix to prefix and postfix expression using stack.

* Infix Expression: < operand > < operator > < operand >

(a) $a + b$ (b) h.

* Pre-fix expression: < operator > < operand > < operand >

$+ a b \rightarrow \text{prefix}$

* Post-fix expression: < operand > < operand > < operator >

$a b + \rightarrow \text{postfix}$

\wedge
 $*, /, \%$
 $+, -$

* Rules for convert infix to postfix.

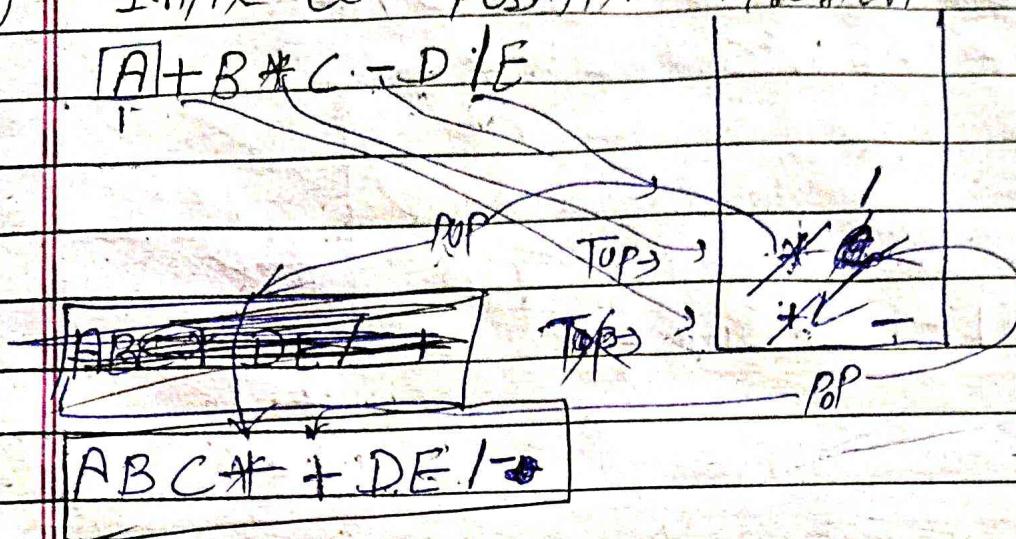
- ① Scan Expression from Left to Right.
- ② Print OPERANDS as they arrive.
- ③ If OPERATOR arrives & stack is empty, push this operator onto the stack.
- ④ If incoming OPERATOR has Higher precedence than the TOP of the stack, push it on stack.
- ⑤ If incoming Operator has lower precedence than TOP. ~~then stack top~~ of the stack then POP and print the TOP. Then test the incoming operator against the NEW TOP of stack.
- ⑥ If incoming operator has equal precedence with Top of stack, use Associativity rule.
- ⑦ For associativity of left to right - POP and print the Top of stack, then PUSH the incoming operator.
- ⑧ For associativity of right to left - PUSH incoming Operator on stack.
- ⑨ At the end of Expression, pop & print all operators from the stack.
- ⑩ If incoming SYMBOL is '(' Push it onto stack.
- ⑪ If incoming SYMBOL is ')' Pop the stack and print Operators till ')' is found & discard it.
- ⑫ If TOP of stack is ')' Push operator on stack.

① Application

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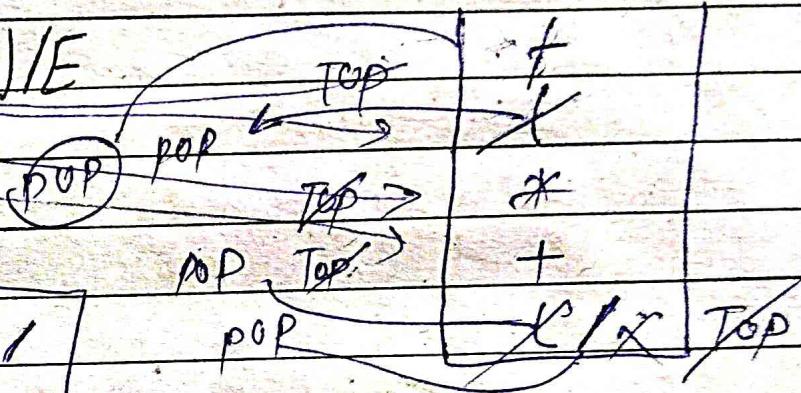
① Infix to Postfix expression

$A + B * C - D / E$



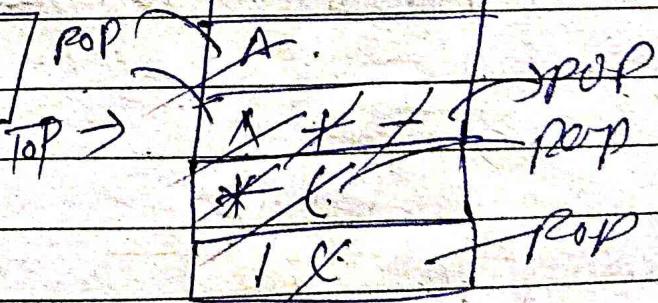
② $(A + B * (C - D)) / E$

$ABCD - * + / E$



* Infix: $A + (B * C - (D / E) \wedge F) * G \wedge H$
 ((a+b-c)*(d\wedge e\wedge f))/g

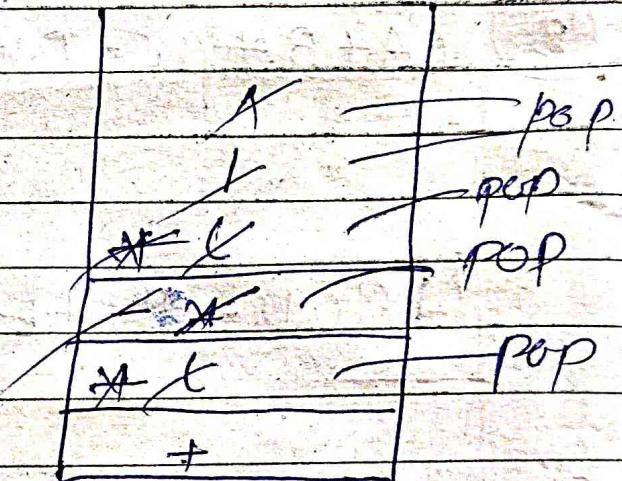
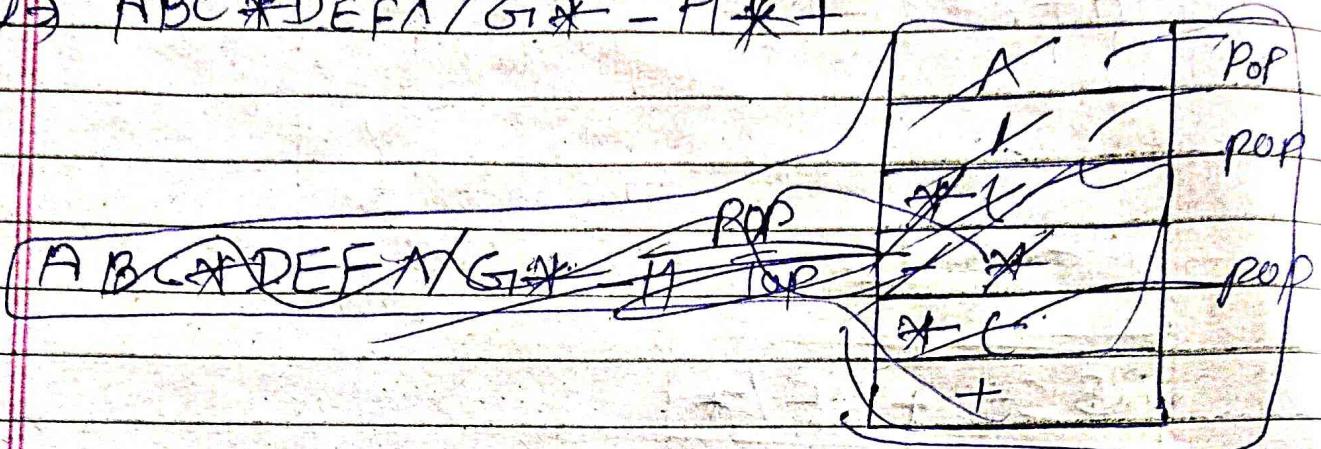
$ab + c - def \wedge g /$



(M)*

$$A + (B * C - (D \wedge E \wedge F) * G) * H$$

$$\text{only } ABC * DEF \wedge G * - H * +$$



$$ABC * DEF \wedge G * - H * +$$

*

$$A + (B * C - (D \wedge E \wedge F) * G) * H$$

$$ABC * DEF \wedge G * - H * +$$

2. Applic

* Convert infix to prefix expression.

Infix expression

Reverse & change $(\rightarrow) \rightarrow (\leftarrow)$

Infix to Postfix

Output

Reverse

Prefix Expression

1 Reverse infix expression & swap $(\rightarrow) \leftrightarrow (\leftarrow)$

2 Scan Expression from left to Right.

3 print Operands as they arrive.

4 If operands arrive & stack is empty, push to stack

5 Then the Top of the stack, push it on stack.

6 If incoming operator has higher precedence than the Top of the stack, push it on stack.

7 If incoming operator has equal precedence with Top of stack like incoming operator is \wedge pop & print Top of stack. Then test the incoming operator against the New Top of stack.

8 If incoming operator has equal precedence with Top of stack, push it on stack.

and same as ~~Infix to Postfix~~

at the end ~~Reverse output~~ ~~stack~~ string again.

② APP



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* Convert infix to prefix

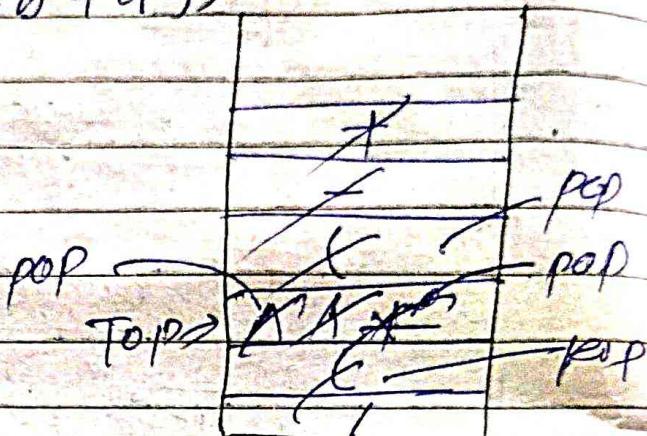
$$g((a+b-c)*d^e)f/g$$

Reverse & swap (\rightarrow) $\# \rightarrow$

$$g/\#(f^e\#d\#(c-b+a))$$

↓

C into in to postfix



~~gfed^acba+-*^~~ } output

Reverse

~~1 * - + abc^d^efg~~

③ APP

* Evaluation of postfix expression

6 0 6 1 5 2 * 5 - +

$$\text{pop} \rightarrow A = 6, B / A = 60 / 6 = 10$$

$$\text{pop} \rightarrow B = 60$$

$$\text{pop } A = 2, B * A \Rightarrow 5 * 2 = 10$$

$$\text{pop } B = 5$$

$$A = 57, B = 40$$

$$B - A = 10 - 5 = 5$$

$$A = 5$$

$$B = 10$$

$$B+A = 10+5 = 15$$

Angular = 15

→ stack में digit value वाले left से फिरवाये।
 नाव पर एक जो operator है उसके stack में उपरी एक element pop कर और A और B के बीच जो operator है उसके value find कर और उनके value of stack में सबसे ऊपरी जो value of stack
 का सामने कर दीजो तो, उसके digit value
 stack में किसी समीकरण के operator
 के बीच एक element pop कर
 value find कर समाप्त करें।

9pp

1

Balanced parenthesis checking

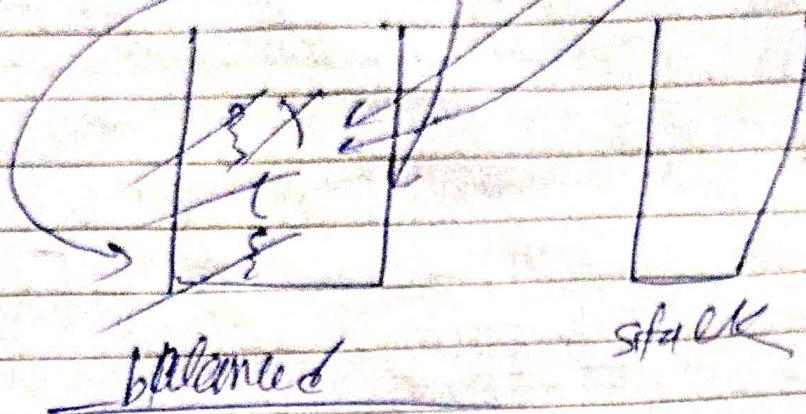
9

~~1~~

~~55.072~~

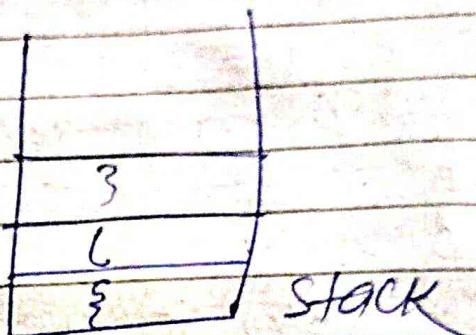
balanced

(ii) ~~$\{a + (b + c)\}$~~ $\{a + (b + c)\}$



(iii) ~~$\{a + (b + c\}$~~

$c \neq \}$



Not balanced

app

(5) factorial :

int factorial (n)

{ if ($n == 0$)

{ return 1;

else {

int main()

{

s = factorial (4);

cout << s;

}

return n * factorial (n-1);

$n = 4$

$f(4) = 4 * f(3)$

{

$f(3) = 3 * f(2)$

$f(1) = 1$

$f(2) = 2 * f(1)$

$f(2) = 2 * 1$

$f(1) = 1$

$f(1) = 1$

$f(0) = 1$

$f(0) = 1$

popped

$f(0) = 1$

main

stack

* Why we need to convert infix to postfix expression?

Ans →

Infix expression are readable and storable by humans. We can easily distinguish the order of operators and also ~~parenthesis~~ to solve we can use the parenthesis to solve that part first during mathematical expression. The computer cannot differentiate the operators and parenthesis easily. That's why we need to convert infix to postfix expression.

⊕ write Algorithm that how you insert & delete element in array.

Solⁿ:

To insert element in array.

1. Start
2. Enter your position of array to insert element.
3. input : pos.
4. pos = pos - 1.
5. for loop \rightarrow for($j = \text{size} - 1; j >= \text{pos}; j--$)
6. arr[j+1] = arr[j].
7. Enter element to be insert.
8. ~~input~~ the input element.
9. arr[pos] = ele;
10. print new array. 11. end

To delete element from array.

1. Start
2. Enter position of array to be delete.
3. input pos.
4. pos = pos - 1.
5. for loop \rightarrow for($i = \text{pos}; i < \text{size} - 1; i++$)
6. arr[i] = arr[i+1].
7. print new array.
8. end.

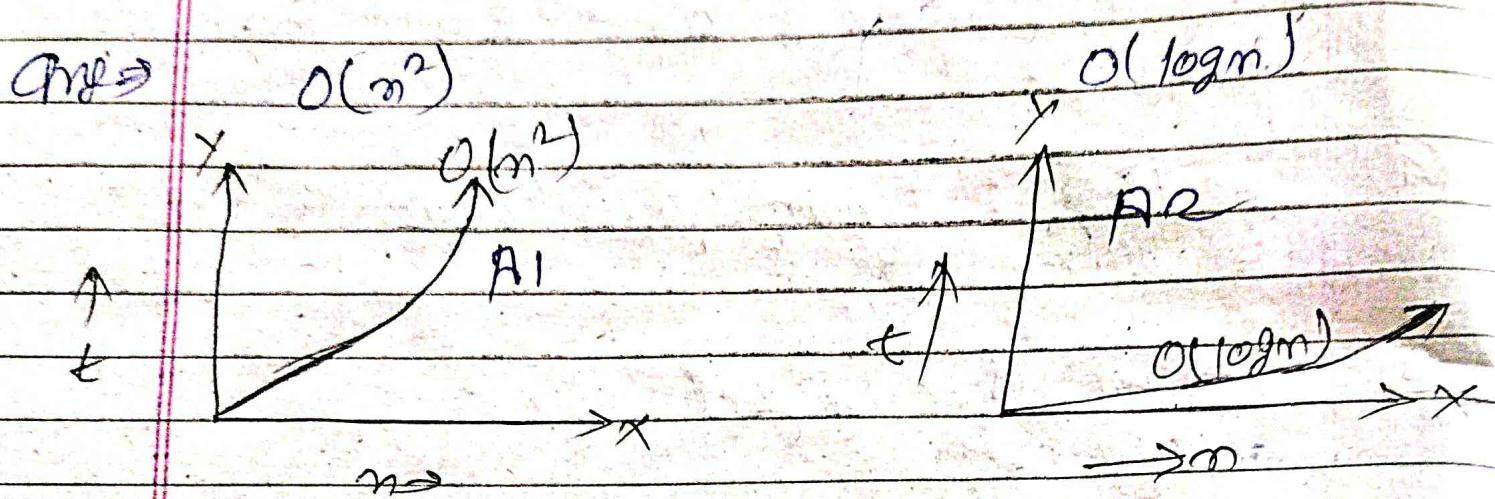
- Q Operation on stack, push, pop, overflow, underflow
- Q Enlist the various operations that can be performed on a data structure.

- ① Traversing : Visiting each element of a data structure one by one.
- ② Insertion : Adding a new element to a data structure.
- ③ Deletion : Removing an element from data structure.
- ④ Searching : Finding an element in a data structure.
- ⑤ Sorting : Arranging the elements of a data structure.
- ⑥ Merging : Combining two data structures into one.
- ⑦ Update : Changing the value of an element in a data structure.
- ⑧ Count : Counting the elements which are present in a data structure.
- ⑨ Reversal : Reversing the order of the elements in a data structure.
- ⑩ Minimum/Maximum : Finding the minimum or maximum element in a data structure.
- ⑪ Sum/Average : Finding the sum or average of the elements in a data structure.

Q For the same problem P , if you have two different algorithm A_1 and A_2 with time complexities as follows:

Algorithm $A_1 \Rightarrow O(n^2)$

Algorithm $A_2 \Rightarrow O(\log n)$ which algorithm you will choose?



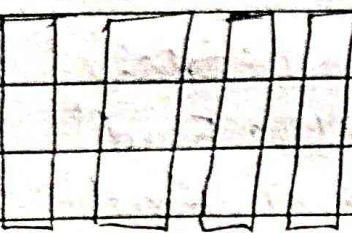
As we can see A_2 has lower bound and A_1 has tight bound that's why I choose A_2 Algorithm.

Q) What is difference between linear and a non linear data structure with example.

Linear

i) Examples: Array, stack queue, linkedlist.

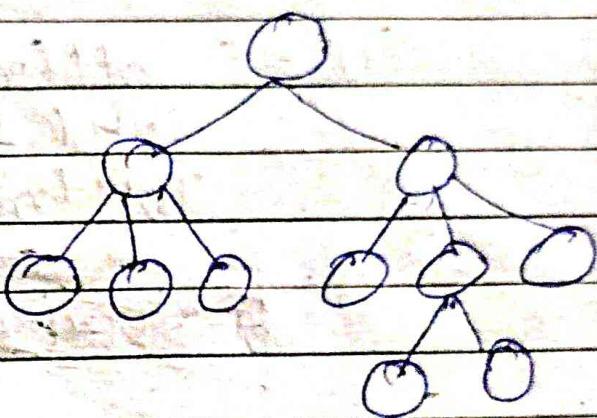
ii) Linear data structure



non-linear

Examples: Trees, graph sets, Tables

Non Linear Data Structure



iii) It can be traversed in single pass.

It cannot be traversed in a single pass.

iv) Not very memory efficient

More memory efficient

v) Used in simple application

Used in complex application

vi) data elements are arranged sequentially in a single line.

data elements are arranged hierarchically in multiple levels.

* Demonstrate the use of stack data structure is used to solve Tower of Hanoi problem.

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* Write a Recursive Algorithm to solve Tower of Hanoi problem.

sol →

TOH(N, Beg, Aux, End)

{ if (n == 1)

{ Beg → End;

return;

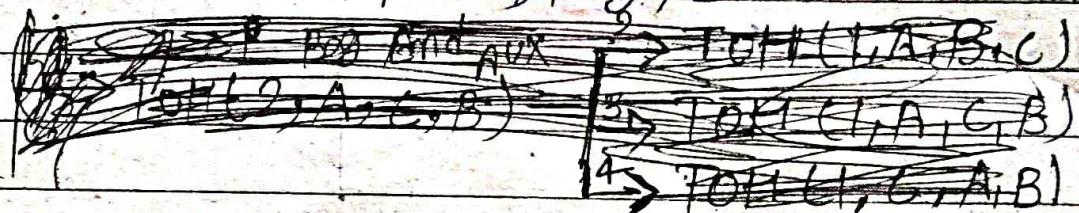
}

else { TOH(n-1, Beg, End, Aux)

TOH(1, Beg, Aux, End)

TOH(n-1, Aux, Beg, End)

}



Beg Aux End

~~TOH(3, A, B, C)~~ → ~~TOH(2, A, B, C)~~

~~disk 1 moved from A → C~~

~~disk 2 moved from A → B~~

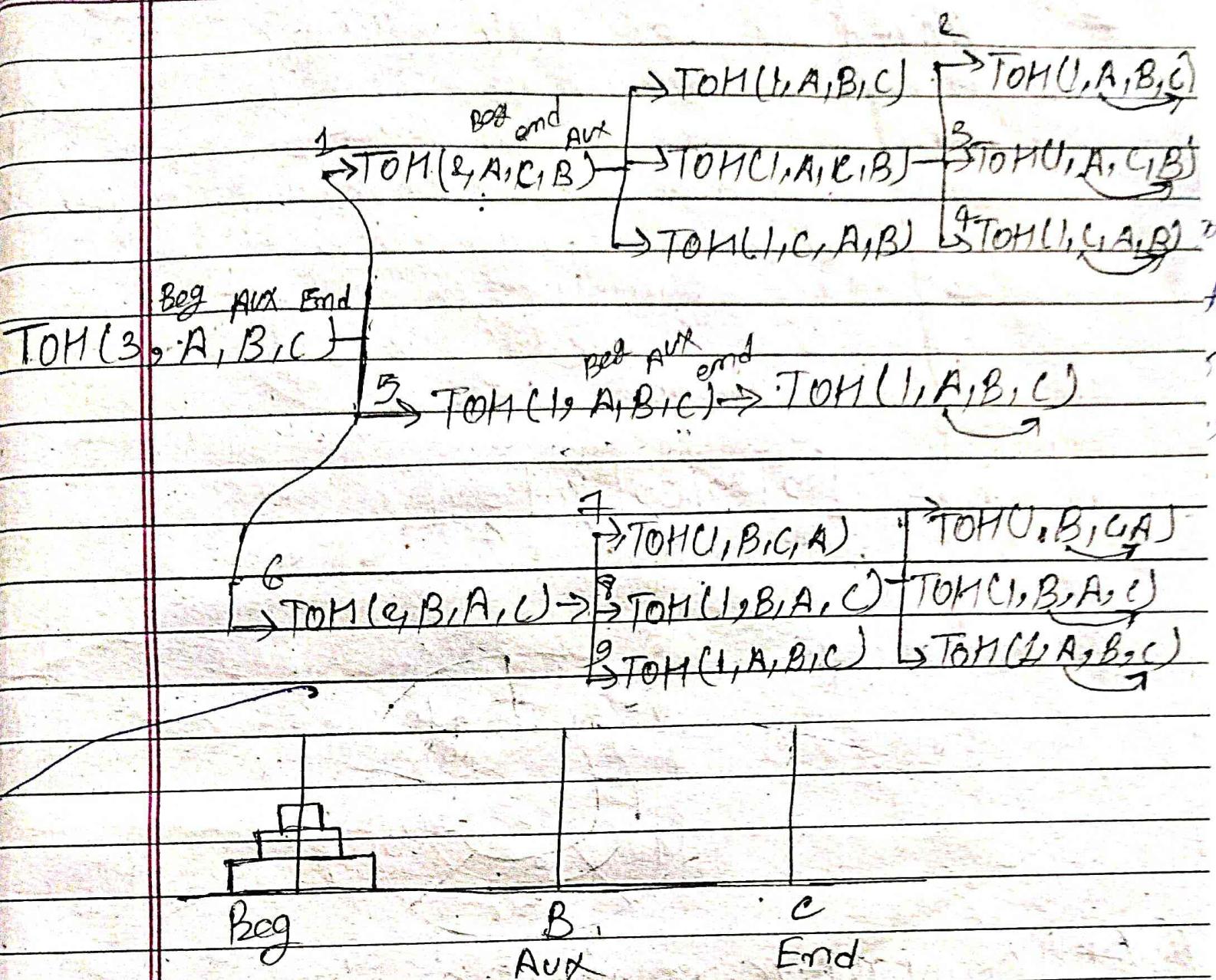
~~disk 1 moved from C → B~~

" 3 " " A → C

" 1 " " B → A

" 2 " " B → C

" 1 " " A → C



* What is TOH ~~stack~~.

Ans \Rightarrow Tower of Hanoi is a classic puzzle or mathematical problem that involves moving a stack of disks from one rod to another while adhering to certain rules.



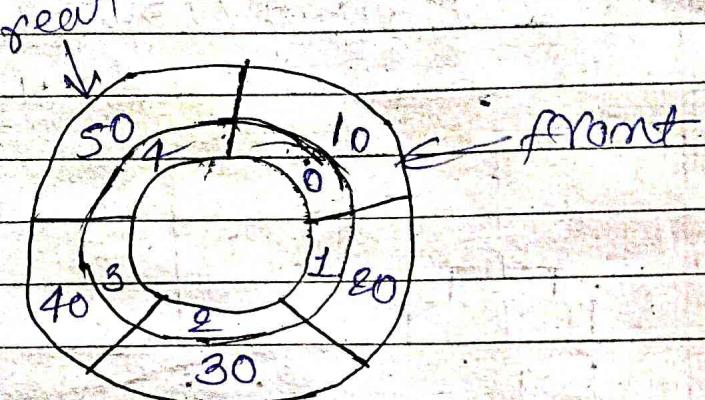
what is circular queue?

Ans A circular queue, also known as a circular buffer or ring buffer that serves as a linear collection of elements with a fixed size. Unlike a regular queue, it is a linear data structure in which the last position is connected back to the first position to make a circle.

Linear queue \Rightarrow

10	20	30	40	50
0	1	2	3	4

circular queue



Comparative analysis of linear queue and circular queue

Ans

$f \rightarrow 0 | 1 | 2 | 3 | 4 | 8$

initialization - `int arr[5];`

`int rear = -1;`

`int front = -1;`

`int size = 5 - 1;`

is empty()

{ if (`front == -1 && rear == -1`)

{ return true

}

else

{ return false

}

is full ()

```
{ if (rear == size )  
{ return true;  
}  
else {  
    return false;  
}
```

enqueue (value)

```
{ if (isfull ())  
{ return ; }  
else if (isempty ())  
{ rear = front = 0;  
arr [rear] = value;  
}  
else { rear++;  
arr [rear] = value;  
}
```

dequeue ()

```
{ int x=0;  
if (isempty ())  
{ return ; }  
else if ( front == rear )  
{ x = arr [front ]  
front = rear = -1;  
}  
else  
{ x = arr [front ]  
front++;  
}  
return ;  
}
```

Initialization -

01 11 89 . 3 | 4

f

89

```
int size=5; int arr[5];
N=size; int rear=-1;
int front=-1;
```

is empty()

{ if(front == -1 && rear == -1)

return true;

else

return false;

{

is full()

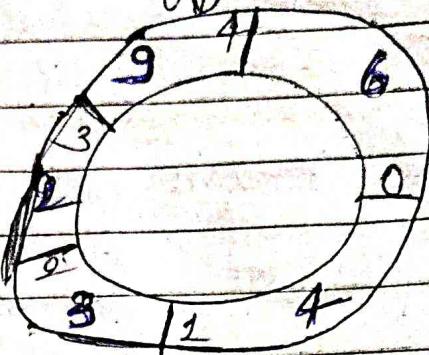
{ if((rear+1)%N==front)

return true;

else

return false;

{



enqueue(value)

{ if(!isfull())

return;

else if(!isempty())

{ rear = front = 0;

{ arr[rear]=value;

else { rear=(rear+1)%N;

{ arr[rear]=value;

{

dequeue()

{ front = (front+1)%N;

{ int x=0;

if(!isempty())

return;

else if(front == rear)

{ x=arr[front];

front=rear=-1;

{ else { x=arr[front];

front++;

return x;

{

What are sparse matrices? Discuss the type of sparse matrices. Mention the efficient method to store sparse matrices in a computer system.

⇒ A sparse matrix is a two-dimensional data object made of m rows and n columns, therefore having total $m \times n$ values. If most of the elements of the matrix have ~~non-zero~~ value, then it is called a sparse matrix.

Ex ⇒

$$\begin{bmatrix} 0 & 0 & 3 & 4 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 6 & 0 \end{bmatrix} \text{ mxm}$$

Types of Sparse matrices

1. Lower Triangular Sparse matrix
2. Upper triangular sparse matrix
3. Tri-diagonal matrix

1. Lower Triangular Matrix: In a lower triangular sparse matrix, all elements above the main diagonal have a zero value. This type of sparse matrix is also known as a lower triangular matrix. If you see its pictorial representation, then you find that all the elements having non-zero value appear below the diagonal.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 1 & 4 & 3 & 0 & 0 \\ 9 & 8 & 7 & 1 & 0 \\ 1 & 2 & 7 & 8 & 9 \end{bmatrix} \text{ 5x5}$$

② **Upper Triangular Matrix:** In the upper triangular sparse matrix, all elements below the main diagonal have a zero value. This type of sparse matrix is also known as an upper triangular matrix.

1	1	2	5	8
0	2	3	9	7
0	0	3	7	2
0	0	0	1	5
0	0	0	0	9

5x5

③ **Tri-diagonal matrix:** Tri-diagonal matrix is also another type of a sparse matrix, where elements with a non-zero value appear only on the diagonal or immediately below or above the diagonal.

1	1	0	0	0
5	2	8	0	0
0	8	3	2	0
0	0	4	1	5
0	0	0	7	9

5x5

The most efficient method to store sparse matrices in a computer system is to use a compressed storage format. Compressed storage formats store only the non-zero elements of the matrix, along with additional information to allow for efficient access to these elements.

In CSR format the matrix is stored as three arrays:

- **Values Array:** Store the non-zero values of the matrix row-wise.
- **Column Indices Array:** Store the column indices corresponding to each non-zero value in the same order as the values array.

Row Pointers Array: Store the index positions in the values cmd column indices arrays where each row starts.

Example:

	0	1	2	3	4
0	1	0	0	0	2
1	0	3	4	0	0
2	5	0	0	6	0
3	0	0	7	0	8

0,0	2,3
0,4	3,2
1,1	3,4
1,2	2,0
2,0	

- values array (A): [1, 2, 3, 4, 5, 6, 7, 8] [value]

- column indices array: [0, 4, 1, 2, 0, 3, 2, 4] [pos]

- Row pointers array: [0, 2, 4, 7, 8] [0 0 1 1 2 2 3 3]

Q2. Describe the implementation of priority queue.

Ans: There are two ways to implement priority queue.

i) one-way list implementation

ii) 2D-array implementation

A priority queue is a collection of elements such that each element has been assigned a priority and such that the order in which elements are deleted and processed comes from the following rules.

i) An element of higher priority is processed before any element of lower priority.

ii) Two elements with the same priority are processed according to the order in which they were added to the queue.

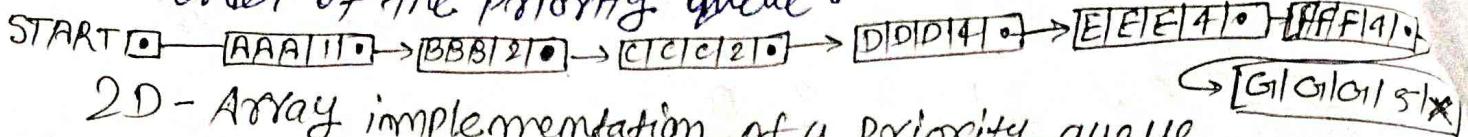
One-way List representation of a priority queue

a) Each node in the list will contain three items of information: an information field INFO, a priority number PRN and a link number LINK.

b) A node x precedes a node y in the list - ① when x has higher priority than y or ② when both have the same priority but x was added to the list before y.

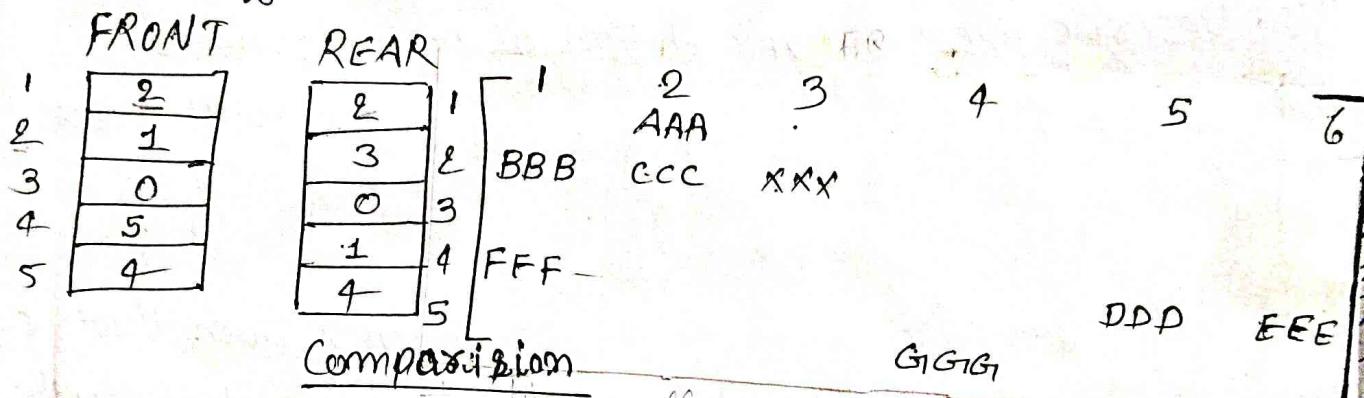
* space-efficient
* not time efficient

This means that the order in the one-way list ⁴⁵ corresponds to the order of the priority queue.



2D-Array implementation of a priority queue.

Another way to maintain a priority queue in memory is to use a separate queue for each level of priority. Each such queue will appear in its own circular array and must have its own pair of pointers, FRONT and REAR. In fact, if each queue is allocated the same amount of space a two dimensional array ~~QUEUE~~ can be used instead of the linear arrays.



One way list representation

- i) less time efficient
- ii) more space-efficient than array
- iii) overflow occurs only when the total number of elements exceeds the total capacity.

array representation of priority queue

- i) more time efficient
- ii) less space-efficient than ~~one~~ way
- iii) overflow occurs when the number of elements in any single priority level exceeds the capacity for that level.

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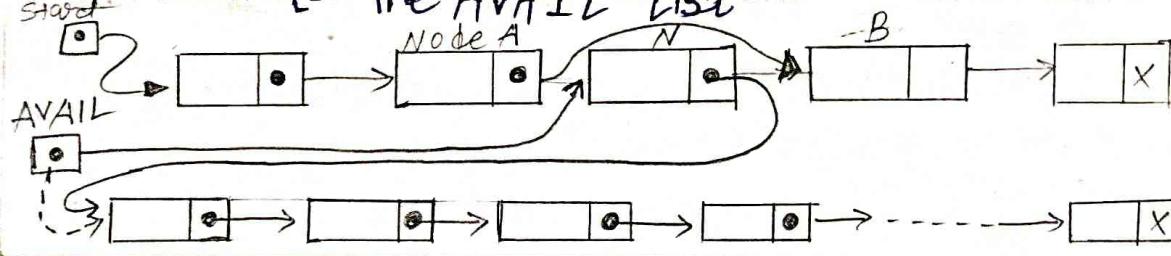
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Section : B

DSA Assignment - 2

(Q) what is AVAIL list? Describe the method of GARBAGE Collection which is performed by operating system for managing free-storage list.

Ans: AVAIL list is a free storage list maintained by the operating system to keep track of the available memory. It is a linked list of memory blocks that are not currently in use. When a process requests memory, the operating system allocates a block of memory from the AVAIL list. When a process terminates, the operating system returns the memory block it was using to the AVAIL list.



(Gc) Two steps process:

Step 1: Gc will sequentially visit all nodes in memory, and mark all nodes which are being used in program.

Step 2: It will collect all unmarked nodes and place them in free storage area.

- (2)
- (5)
- (7)
- (1)

Garbage collection: Garbage collection is an automatic dynamic memory management that identifies dead memory blocks and reallocates storage for reuse.

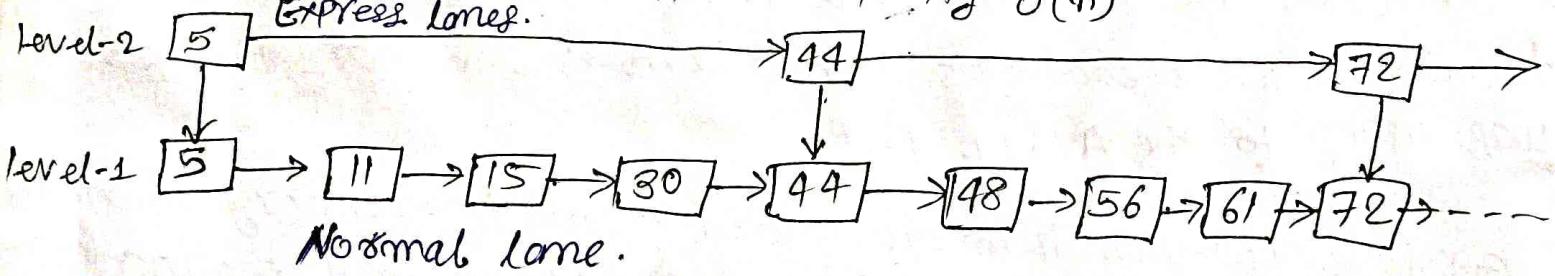
The operating system may periodically collect all the deleted space onto the free-storage list. This mechanism is called garbage collection.

Q2. Explain the implementation of skip list.

Ans: Skip list is a data structure that is used for storing a sorted list of items with the help of hierarchy of linked lists.

* What do you mean by skip list?

The skip list data structure skips over many of known as skip list. Time complexity $O(n)$



- It can consist of many levels
- All keys appear in level
- If key appears in level i then it also appears in all levels below i .

A SkipNode has $\frac{1}{2}$ \rightarrow val
 $\text{val} \rightarrow \text{next}$ \rightarrow next
 ↓ down \Rightarrow down

Algorithm : Searching in skip list

skipNode* search (int key)

curr = head

→ while curr != NULL

 → while curr → right != NULL & curr → right → val <= key

 → curr = curr → right

 if curr → val == key

 break;

 → curr = curr → down

return curr

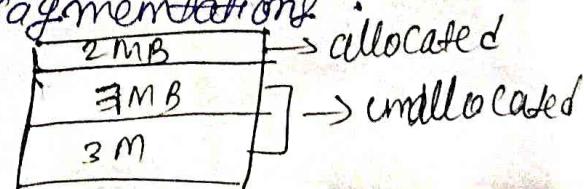
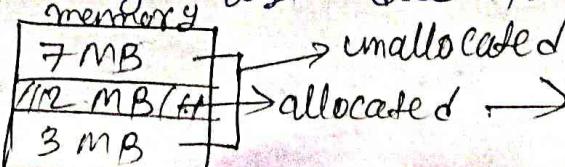
Advantage of Skip list

- The time complexity of search can become $O(\log n)$ in average case with $O(n)$ extra space.
- thus it improves the search.

Disadvantage of skip list

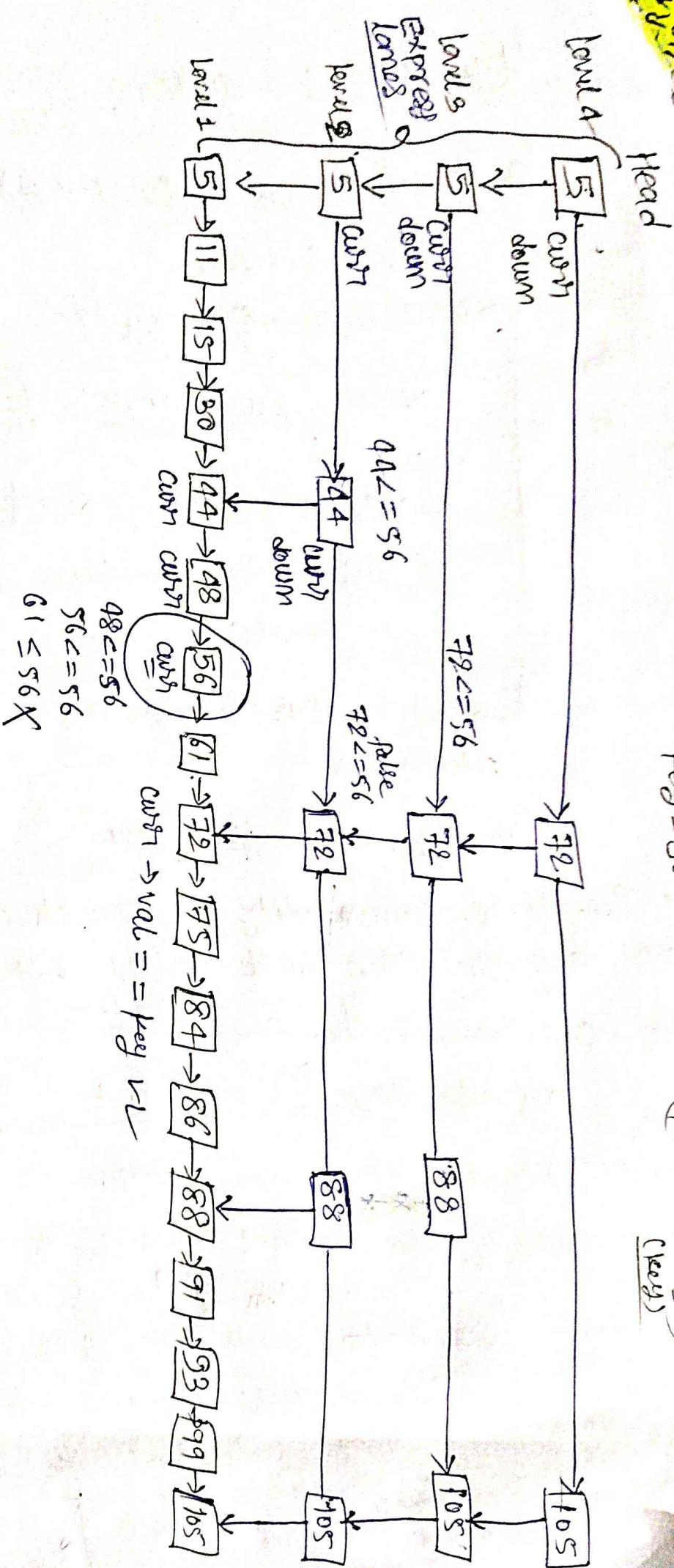
- It requires more memory.

→ Garbage compaction: Garbage compaction is the process of separation of allocated memory and ~~to~~ unallocated memory. This process is often used with garbage collection, it helps to recover memory lost due to fragmentation.



$\text{key} = 56$

To search 56
(key)



(PQ)

Illustrate the efficient way of storing sparse matrices in memory.

Ans -

0	0	0	0	5	0	0	8
1	0	0	4	0	0	6	0
2	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	8	0	0	0	0	0
5	0	0	0	0	0	0	2
6	0	0	10	0	7	0	0

7x8

Two efficient way of storing sparse matrices

i) Array representation

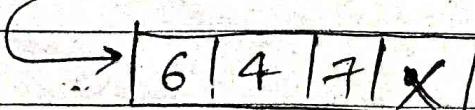
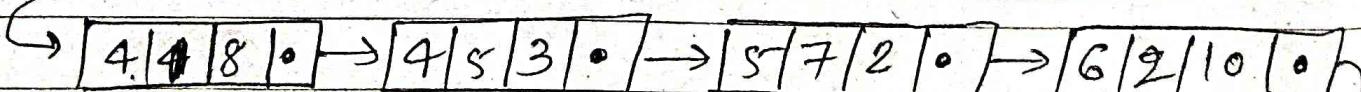
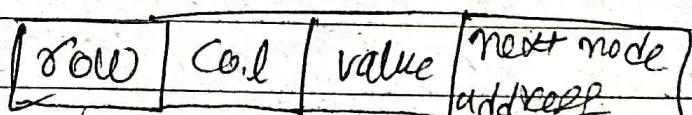
ii) Linked representation:

①

	0	1	2	3	4	5	6	7	8
row	0	1	1	2	4	4	5	6	6
col	4	2	6	0	1	5	7	2	4
value	5	4	6	1	8	3	2	10	7

②

Link list rep:



Assignment - 3

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①

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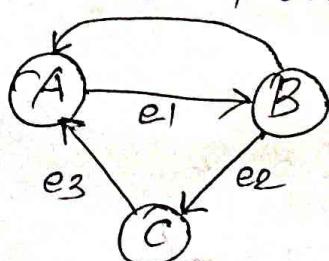
Raushan
Kumar)

②

Q.1 Q. what is a path matrix (or reachability matrix)?

Ans: The matrix containing information about the path exist or not between every pair of vertices or A path matrix is a matrix that is used to determine the shortest path between two nodes in graph.

Ex:



$$Q =$$

Adjacency matrix

	A	B	C
A	0	1	0
B	1	0	1
C	1	0	0

$$\Phi^2 = Q \times Q$$

	A	B	C
A	0	1	0
B	1	0	1
C	0	0	0

	A	B	C
A	0	1	0
B	1	0	1
C	1	0	0

	A	B	C
A	1	0	1
B	1	1	0
C	0	1	0

$$\Phi^3 = \Phi \times \Phi \times \Phi$$

	A	B	C
A	1	0	1
B	1	1	0
C	0	1	0

	A	B	C
A	0	1	0
B	1	0	1
C	1	0	0

	A	B	C
A	1	1	0
B	1	1	1
C	1	0	1

$$B = Q + \Phi + \Phi^3$$

	A	B	C
A	2	2	1
B	3	2	2
C	2	1	1

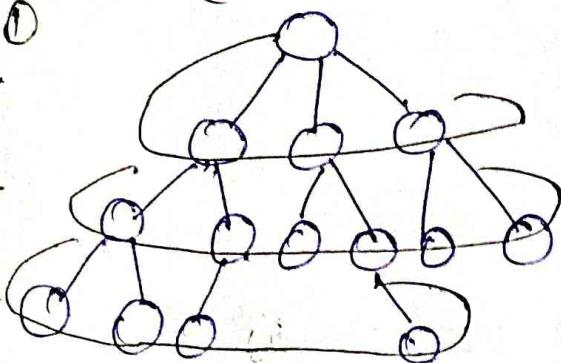
Now we can find path matrix using matrix B is:

	A	B	C
A	1	1	1
B	1	1	1
C	1	1	1

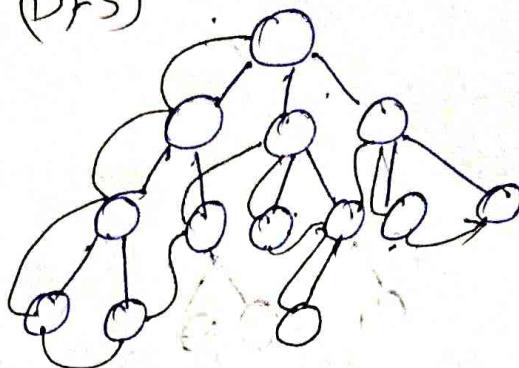
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Q1. Differentiate between the graph traversal techniques of BFS (Breath first search) & DFS (Depth first search). 35

(BFS)



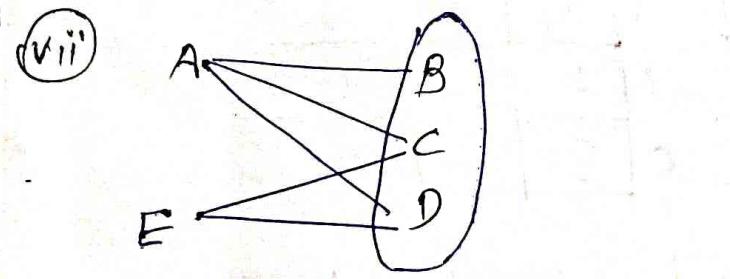
(DFS)



- i) Need queue data structure.
- ii) Need more space.
- iii) It focuses all the neighbours of a ~~node~~ at a time.
- iv) Time complexity is $O(V+E)$

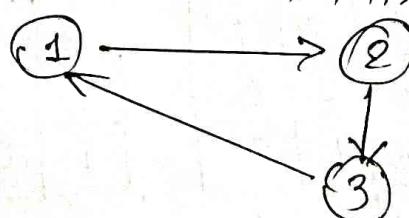
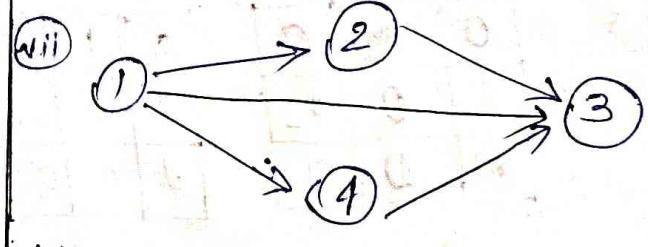
v) Applications: shortest path finding problem

vi) To check whether the graph is Bipartite or not.



- i) Need stack data structure.
- ii) Need less space than BFS.
- iii) It focuses on child of one node.
- iv) Time complexity is $O(V+E)$

v) DFS is used in topological sorting process, scheduling, cycle detection.



Q2. Compare the techniques and complexities of various sorting algorithms. → bubble sort, Shell sort, insertion sort, Selection sort, quick sort, merge sort, heap sort.

Comparison of Sorting \div Time complexity

Sorting Techniques	Best case	Average case	Worst case
1. Bubble sort	$O(n)$	$O(n^2)$	$O(n^2)$
2. Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
3. Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$
4. Quick sort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$
5. Merge sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
6. Shell sort	$O(n)$	$O(n(\log n)^f)$	$O(n(\log n)^2)$
7. Heap sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$

Sorting techniques	Space Complexity	Stability	Auxiliary Space	Locality of reference
1. Bubble sort	$O(1)$	Yes	Not required	Poor
2. Insertion sort	$O(1)$	Yes	Not required	Good
3. Selection sort	$O(1)$	No	Not required	Fair
4. Quick sort	$O(\log n)$	No	Not required	Good
5. Merge sort	$O(n)$	Yes	Required	Poor
6. Heap sort	$O(1)$	No	Not required	Fair
7. Shell sort	$O(1)$	No	Not required	Good.

⑩ Doubleded Ehme (Deauville)

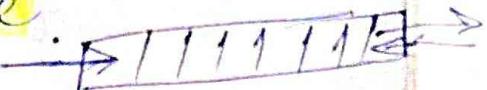
int a[5];



Input → Restricted sequence 

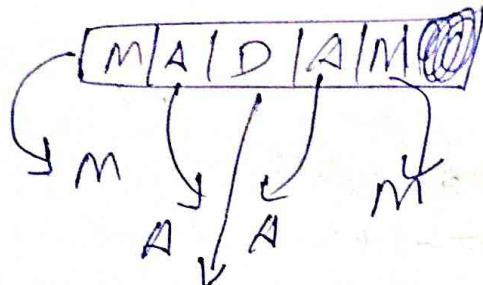
Output → Restricted Deletion

(Output only in ~~one~~ side
Delete both sides)



→ Example of input restricted beam
⑨ Palindromic locking

⑨ Palindrome checking

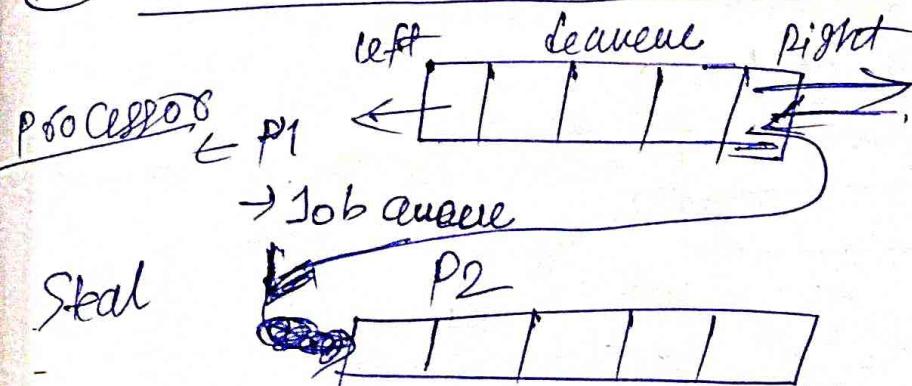


(b) undo and redo operations -



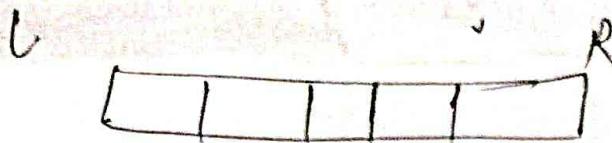
selection
very old
undo action

c) A - Steal job scheduling algorithm



Explain output restricted, draw the diagram input restricted.

Implementation of Dequeue



insertion at left where $N = \text{size of array}$
 $\text{left} = (\text{left} - 1 + N) \% N$

Dequeue

Deletion from left

$$\text{left} = (\text{left} + 1 + N) \% N$$

insertion at right

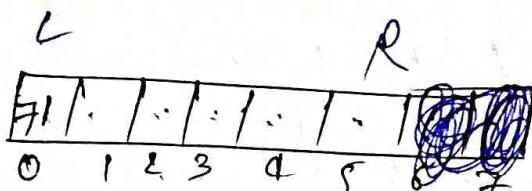
$$\text{right} = (\text{right} + 1) \% N$$

Deletion from right

$$\text{right} = (\text{right} - 1 + N) \% N$$

Implementing of dequeue using a circular array

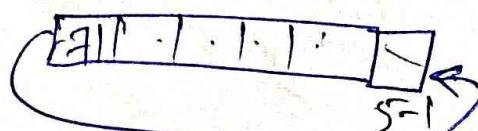
$$N=6$$



int $\text{left} = -1, \text{right} = -1$; [empty]

- ① insert -71 at left $\Rightarrow \text{left} = \text{right} = 0$
or -71 at right $\Rightarrow a[\text{left}] = -71$

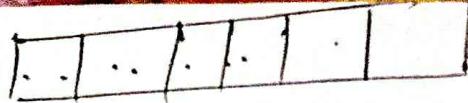
- ② ~~insert -80 at right~~
insert 69 at left.



$$\text{left} = 0$$

$$= (0 - 1 + 6) \% 6 = 5$$

$$= a[\text{left}] = 69$$



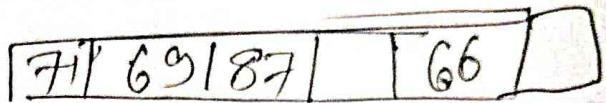
$$\text{right} = 0$$

$$(0+1) \times 6 =$$

$$\text{right} = 1$$

$$a[\text{right}] = 80$$

④ insert 66 at left



$$(2-1+6) \% 6 =$$

$$10 \% 6 = 4$$

$$a[\text{left}] = 66$$

⑤ insert 87 at right side

$$(1+1) \% 6 = 2 \% 6 = 2$$

$$\text{if}(2 == \text{left})$$

$$a[\text{right}] = 87$$

⑥ delete at right

$$(\text{old right} - 1 + N) \% N$$

$$(2-1+6) \% 6 = 7 \% 6 = 1$$

⑦ deletion at left

$$\text{old left} = 4$$

$$(\text{left} + 1 + N) \% N$$

$$(4+1+6) \% 6 = 11 \% 6 = 5$$

~~old left~~

Q why is circular queue better than linear queue?

Ans i) Efficient use of memory: In a circular queue, when the rear pointer reaches the end of the queue it wraps around to the beginning, which allows for efficient use of memory compared to a linear queue.

ii) Easier for insertion-deletion: In the circular queue, if the queue is not fully occupied then the elements can be inserted easily in the vacant locations. but in linear queue insertion not possible once the rear reaches the last index.

iii) Better performance: Circular Queue offers better performance in situations where data is frequently added and removed from the queue as compared to a linear queue.

iv) Reduced overflow risk: Circular queues are less likely to overflow than linear queues. This is because circular queues can use all of the available space even if the queue is not full.

Stack algorithm:

is-empty()

{ if($\text{top} == -1$)

{ return true;

} else

{ return false;

}

push(int value)

{ if(is-full())

{ cout << "overload"; }

} else { top++;

arr[top] = value;

}

deletion

				arr[5];
				top = -1
2	7			size = 4;
1	6			insertion

0 S

is-full()

{ if($\text{top} == \text{size}$)

{ return true;

}

} else

{ return false;

}

pop()

{ if(is-empty())

{ cout << "underflow"; }

}

else { int popv = arr[top];

arr[top] = 0;

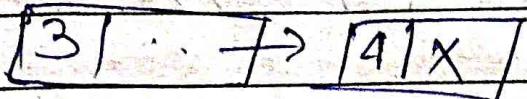
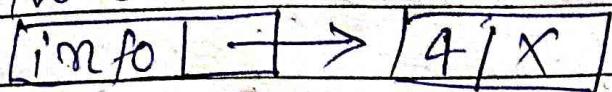
top--;

return popv;

}

To create a linear linked list

Node



#include <iostream>

#include <stdlib.h>

using namespace std;

struct Node

{ int data;

Node *next;

};

Node *head = NULL;

void insertinLL(int val)

{ Node *newnode = (Node*) malloc(sizeof(Node))

newnode->data = val;

newnode->next = NULL;

Node *last = head;

if (last == NULL)

{

~~newnode~~ head = newnode;

return;

}

else { while (last->next != NULL)

{ last = last->next;
}

last->next = newnode;
return;

}

}

struct node

{ int data;

Node *next;

};

Node *head = NULL;

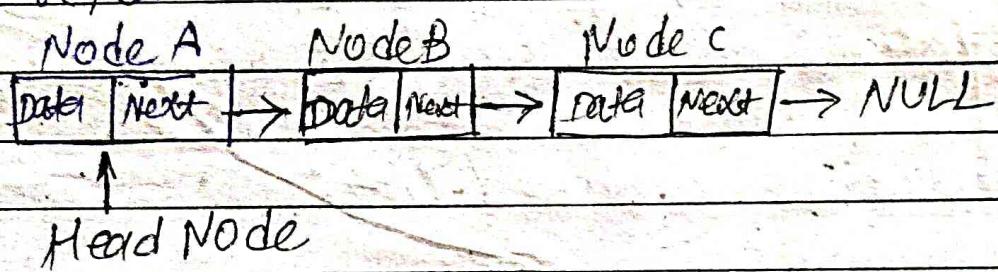
void insertionLL(int val)

{ Node *newnode = (Node*)malloc.

- * What is Linked List : A ^{ed}Link list is a linear data structure, in which the elements are not stored at contiguous memory locations. The elements in a linked list are linked using pointers (empty that point to the next element)

other words

A linked list consists of nodes where each node contains a data field and a reference (link) to the next node in the list.



Linked list consists of nodes where each node contains a data field and a reference (link) to the next node in the list.



Linked List vs Array

Advantages of linked list over arrays:

(i) Dynamic size

(ii) Ease of insertion / deletion

Disadvantages of Linked List over Array:

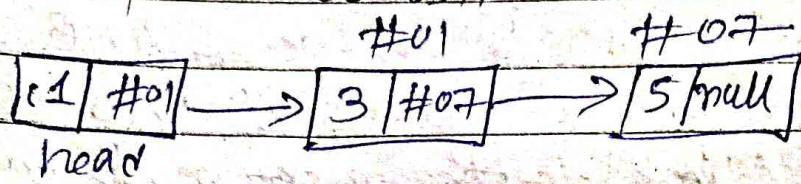
(i) Random access is not allowed. We have to access elements sequentially starting from the first node.

(2)

Extra memory space for a pointer is required with each element of the list.

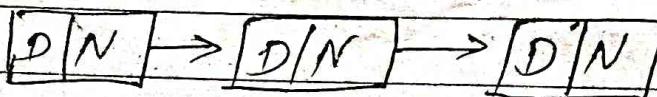
(3)

Not cache friendly. Since array elements are contiguous locations, there is locality of reference which is not there in case of linked lists.



Operations of linked list :-

1. Traversing a linked list:



2. Append a New node (to the end) of a list.

3. Prepend a new node (to start)

(4)

Inserting a new node to a specific position in the list.

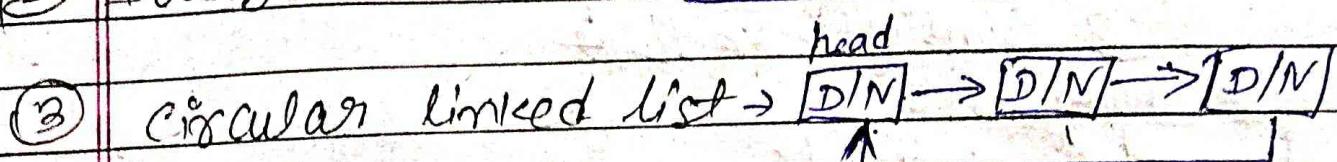
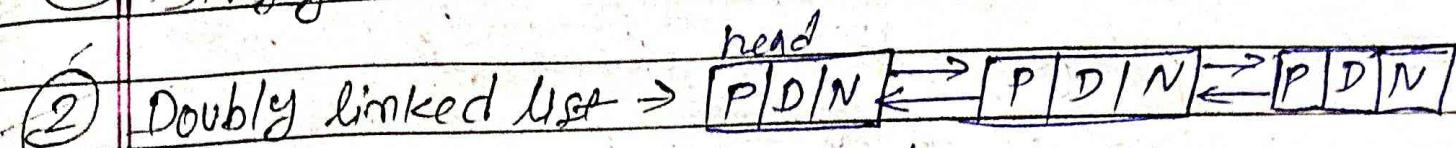
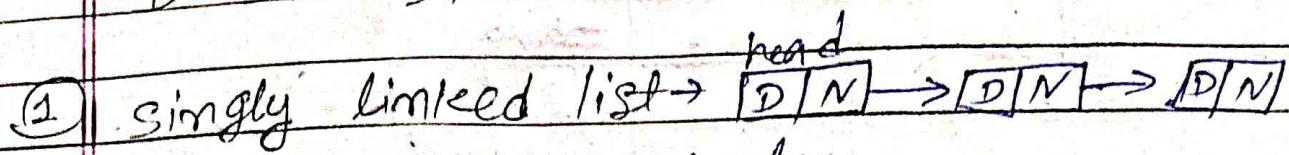
(5)

Deleting a node from the list.

(6)

Updating a node in the list.

Types of linked list

 $D = \text{Data} \Rightarrow P = \text{Previous } N = \text{Next}$ 

Some application of linked list

- Linked lists can be used to implement stack, queue
- Linked lists can also be used to implement graphs,
- Implementing Hash Tables
- undo functionality in photoshop or word

Creating a node

```
class Node {
public:
    int key;
    int data;
    Node* next;
    Node() {
        key = 0;
        data = 0;
        next = NULL;
    }
}
```

Node (int k, int d)

{ key = k;

data = d;

next = NULL;

}

};

int main()

{ Node n1(1, 10);

Node n2(2, 20);

}

Computer Memory

	#10			
m1	1 10		#70	
		2 20		m2

```

class SinglyLinkedList
{
public:
    Node * head;
    SinglyLinkedList(Node * h)
    void prepEndNode(Node * h)
    void appendNode(key)
    void insertNode(key)
    void deleteNode(key)
    void updateNode(key)
}

```

3) int main()

```

    Node n1(1, 10);
    Node n2(2, 20);
    Node n3(3, 30);

```

```

    SinglyLinkedList s(&n1);
    s.appendNode(&n2);
    s.prependNode(&n3);

```

3) Computer Memory

	#10			
m1	1 10		#70	
		2 20		m2

Single Linked List s.

31301#10

m1 #10 #70 → 2 | 20 | m2 →

```
#include <iostream>
#include <stdlib.h>
using namespace std;
struct Node
{
    int data;
    Node *next;
};

Node *head = NULL;

void insertInLL (int val)
{
    Node *newnode = (Node *) malloc(sizeof(Node));
    newnode->data = val;
    newnode->next = NULL;
    Node *last = head;
    if (last == NULL)
    {
        head = newnode;
        return;
    }
    else
    {
        while (last->next != NULL)
        {
            last = last->next;
        }
        last->next = newnode;
        return;
    }
}
```

```
void display()
{
    Node *ptr = head;
    if (ptr == NULL)
        cout << "No elements are there in
              Linked List";
    else
        cout << "Elements in linked list are: ";
        while (ptr != NULL)
            cout << ptr->data;
            ptr = ptr->next;
}
```

```
void deletedata (int todel)
{
    Node *temp = head;
    Node *prev;
    while (temp != NULL && temp->data != todel)
        {
            prev = temp;
            temp = temp->next;
        }
}
```

```
if (temp == NULL)
    cout << " " << todel << " not found ";
return;
```

else {

cout << "In deleting" << todel << endl from the linked
list m";

prev = temp => next;
free(temp);

}

int main()

{ cout << "Inserting 3 in the linked list m";
insertinLL(3);

cout << "Inserting 4 in the linked list m";
insertinLL(4);

cout << "Inserting 55 in the linked list m";
insertinLL(55);

cout << "Inserting 184 in the linked list m";
insertinLL(184);

cout << "Inserting -120 in the linked list m";
insertinLL(-120);

cout << "Currently the elements present in the
linked list are: m";

display();

cout << ".....m";

cout << "Delete 30";

deleteData(30);

cout << "Delete 55";

deleteData(55);

~~delete~~ cout << "Delete -20";

deleteData(-20);

~~del~~ cout << "Delete 7";

deleteData(7);

cout << "pp.....ooooo.....oooo...im'';

cout << "m Now, the elements present in the linked list are : im'';

display();

return 0;

}

Output

Inserting 3 in the linked list

" 4 " " 1 " " 3 " "

"

55

"

"

"

184

-

"

"

-20

-

"

"

Currently the elements present in the linked list are :

Elements in linked list are ;

3

4

55

184

-20

delete 30

30 not found

delete 55

deleting 55 from the linked list

deleting -20

deleting -20 from the linked list

delete 7

7 not found

Now, the elements present in the linked list are :

Elements in linked list are :

3

4

184

~~266 88 8~~

X

Q

Demonstrate with example an efficient method of storing a tridiagonal matrix in computer memory.

answ =

$$A = \begin{matrix} a_{11} \\ a_{21} \rightarrow a_{22} \\ a_{31} \rightarrow a_{32} \rightarrow a_{33} \\ \dots \dots \dots \dots \dots \dots \\ \dots \dots \dots \dots \dots \dots \\ a_{n1} \rightarrow a_{n2} \rightarrow a_{n3} \rightarrow \dots \rightarrow a_{nn} \end{matrix}$$

$$B[1] = a_{11}, B[2] = a_{21}, B[3] = a_{22}, B[4] = a_{31}, \dots$$

observe first B :-

$$1+2+3+4+\dots+m = \boxed{\frac{m(m-1)}{2}}$$

$$\boxed{B[l] = a_{l1}k}$$

l represent the no of ~~elements~~ elements in the list up to the $a_{l1}k$.

$$1+2+3+\dots+(j-1) = \boxed{\frac{j(j-1)}{2}}$$

element in the row j above $a_{l1}k$ and k are elements in row j .

$$\boxed{L = \frac{j(j-1)}{2} + k}$$



* Analyze and compare the mechanism to implement priority queue.

⇒ Compare

One way list implementation vs array implementation

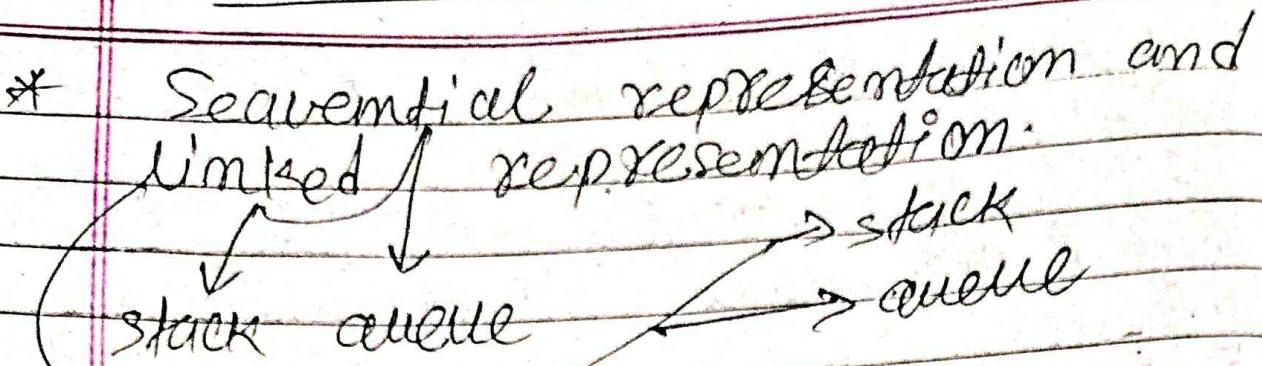
- | | |
|--|--|
| ① less time efficient | more time efficient |
| ② more space-efficient than array | less space-efficient than array. |
| ③ overflow occurs only when the total number of elements exceeds the total capacity. | overflow occurs when the total number of elements in any single priority level exceeds the capacity of that level. |

one way → START → A | 1 | 0 → B | 2 | 0 → C | 2 | 0

→ D | 4 | 0 → E | 4 | 0 → F | 4 | 0 → G | 5 | X

	1	2	3	4	5	6
array →	front	Rear	A	B	C	D
1	2	2				
2	1	2				
3	0	0		F		E
4	5	1			G	
5	4	4				

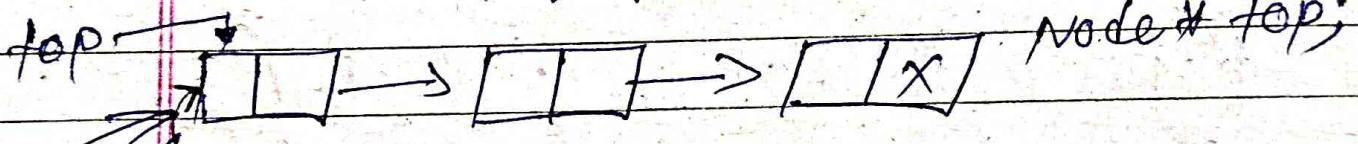
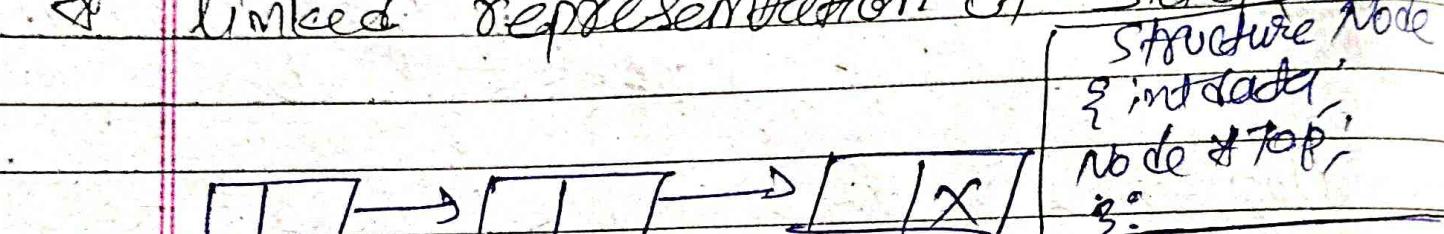
5x6



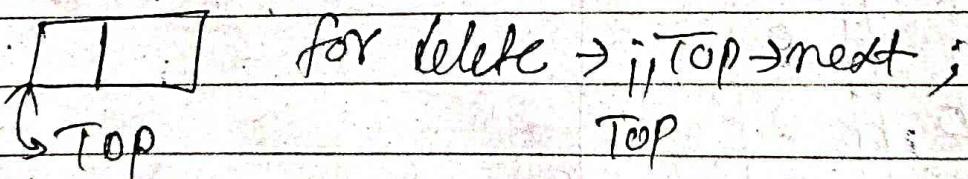
Implement stack and queue using:

- ① Sequential representation (array) → stack → queue
- ② Linked representation → stack → queue

* linked representation of stack

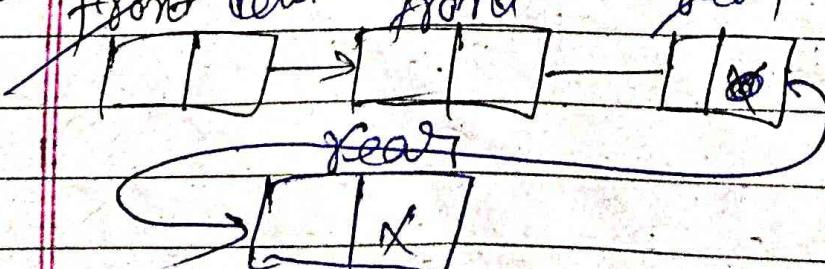


insertion &
deletion with
only (TOP)



* for queue

front delete front



for insertion

struct node

{ int data;

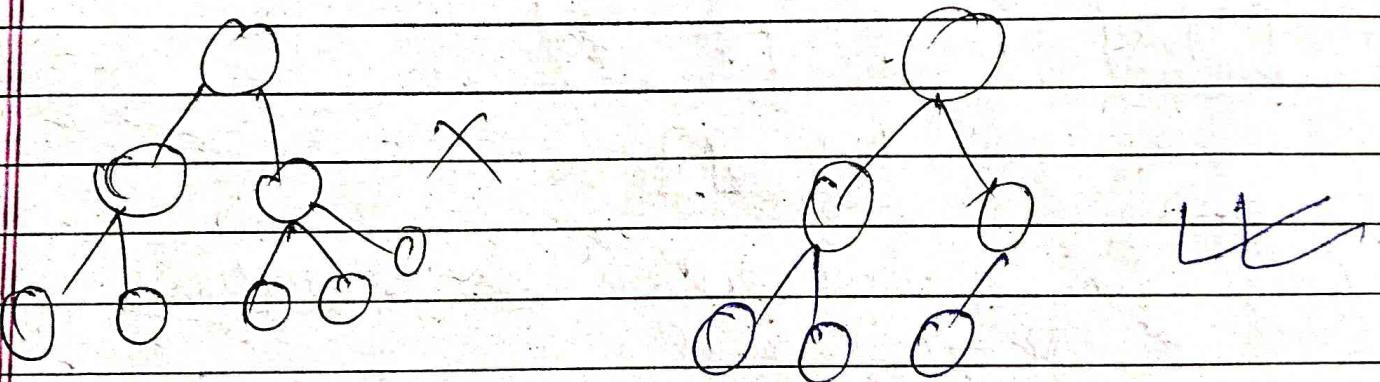
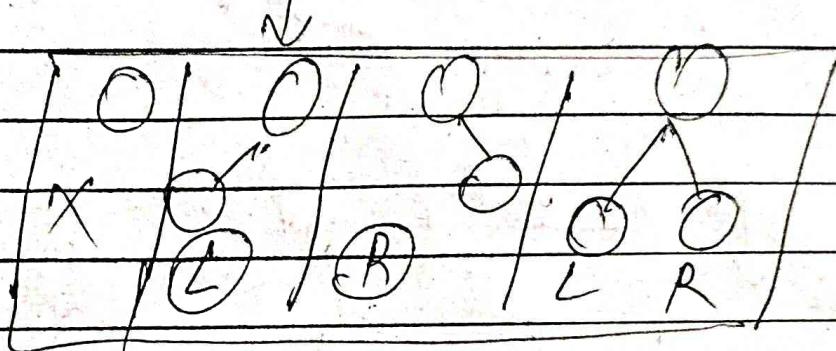
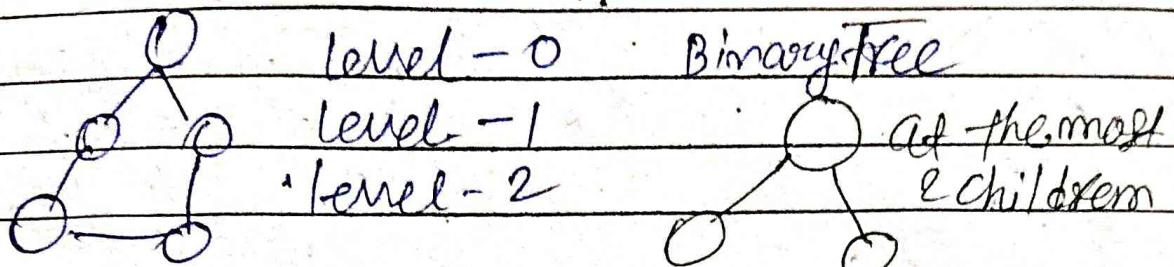
node * front;

node * rear;

};

Trees (non-linear data structure)

Hierarchical relationship



* Representation of Trees. (Array)

Array

Node $\rightarrow k^{th}$

left = $2k^{th}$

right = $(2k+1)^{th}$

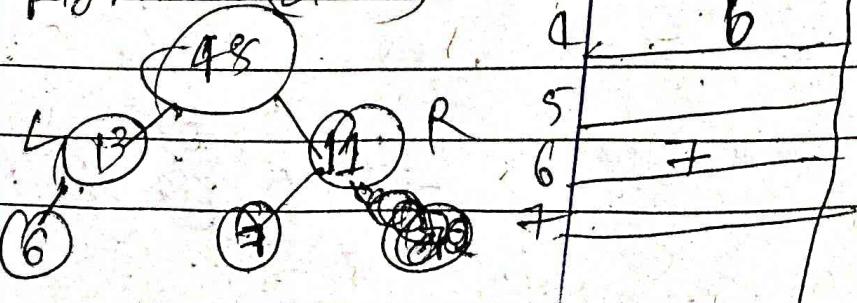
1 45 : $k=1$

2 13 : $2k=2 \times 1$

$2k+1=2 \times 1 + 1$

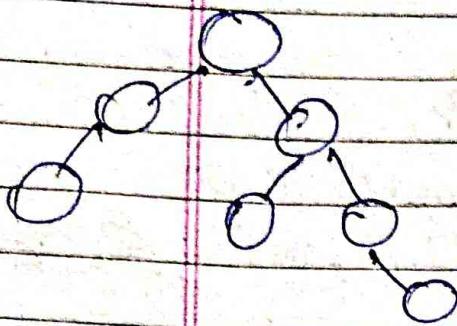
$2k=$

$2k+1$



* representation of tree using linked list

Root

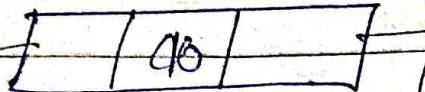


struct Node

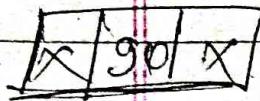
{ int data;
Node *left;
Node *right; }

Head

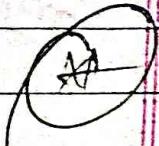
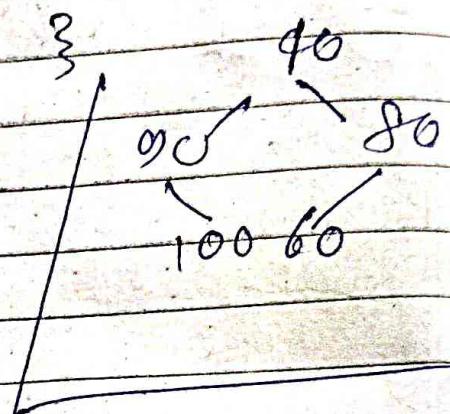
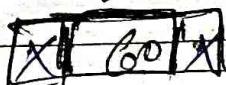
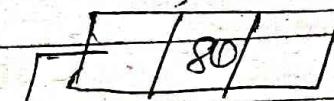
Root



left



right



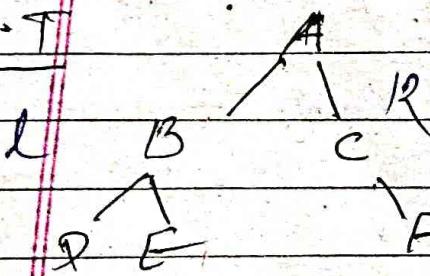
Tree traversal technique.

* pre-order

* in - order

* post - order

B.T



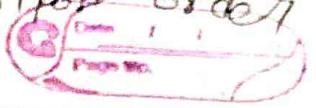
* pre - order \rightarrow Root \rightarrow left \rightarrow right

* In - \rightarrow left \rightarrow Root \rightarrow Right

Recursive in nature.

* Post - \rightarrow left \rightarrow right \rightarrow Root

2 Mots \rightarrow BTrees APPLY this pre-in-post order
and traverse



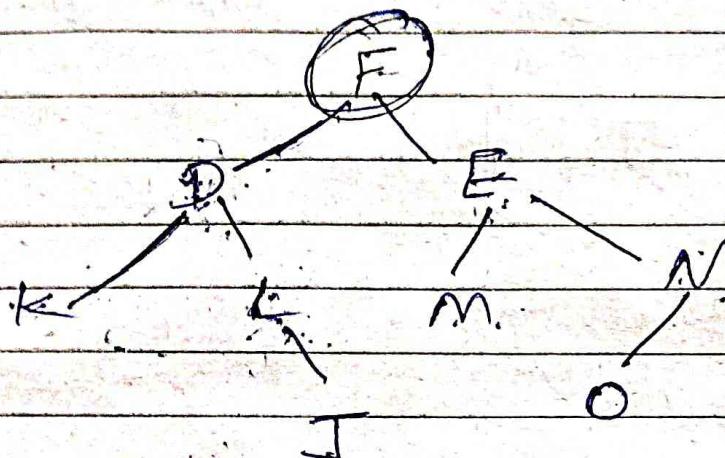
64

pre-order: A | BDE | CF \approx ABDECFC

In-order: B D E | A | C F \approx DBEACF

post-order: DEB | FC | A \approx DEBFCA

Q



pre: F | DKLJ | EMNO \approx FDKLJEMNO

In-order: KDLJ | F | MENO \approx KDLJFMEON

Post: Left | Right | Root

~~KILD~~

~~MONE~~

F

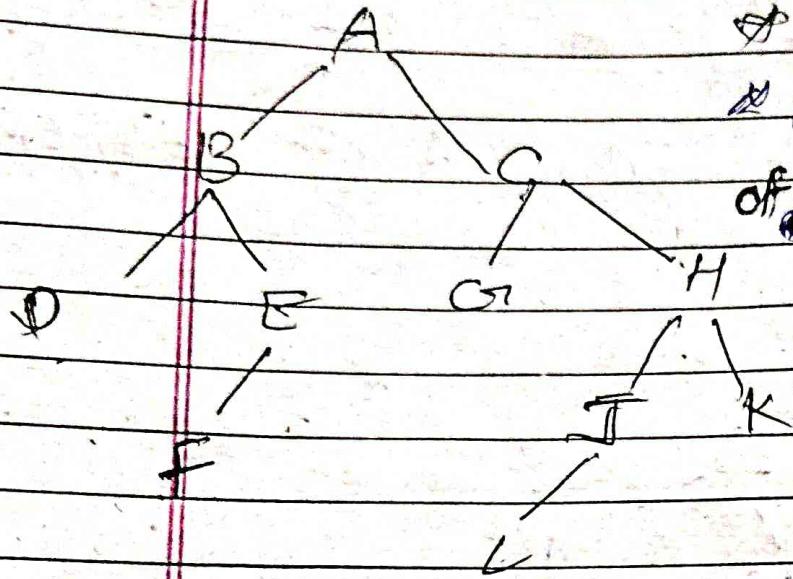
\approx KJLDMONEF

by default in order



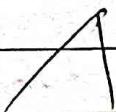
67

Threaded Binary Tree



* Avoid using NULL pointers
* Rather than using NULL
ptr. store the address
of the next node in the
favoural.

Threaded



one way

two way

threaded

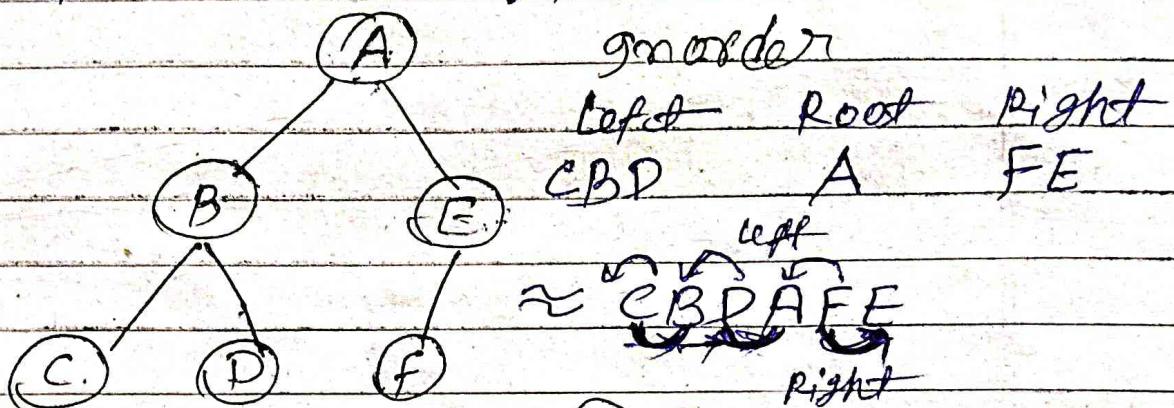
threaded

(only previous stored)
(left and right)

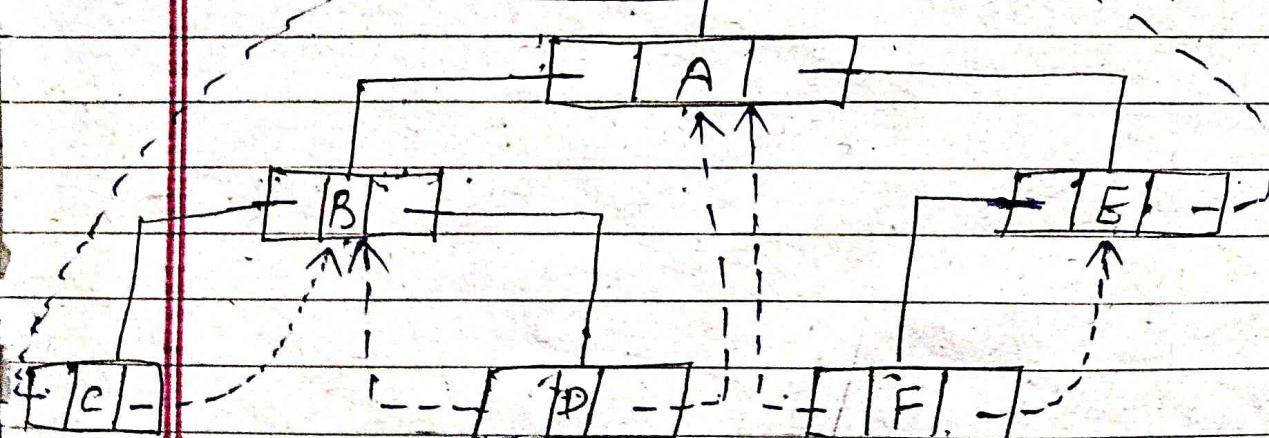
(previous & next both
address stored)

Ques: Apply two-way threading with header node in a tree.

- Ans:
 1) Left pointer of the first node and right pointer of last node will contain NULL value.
 → two way threading stores previous as well as next address. It avoids using Nullptr. Rather than using Null-pointer stores the address of the next node in the traversal.



HEADER

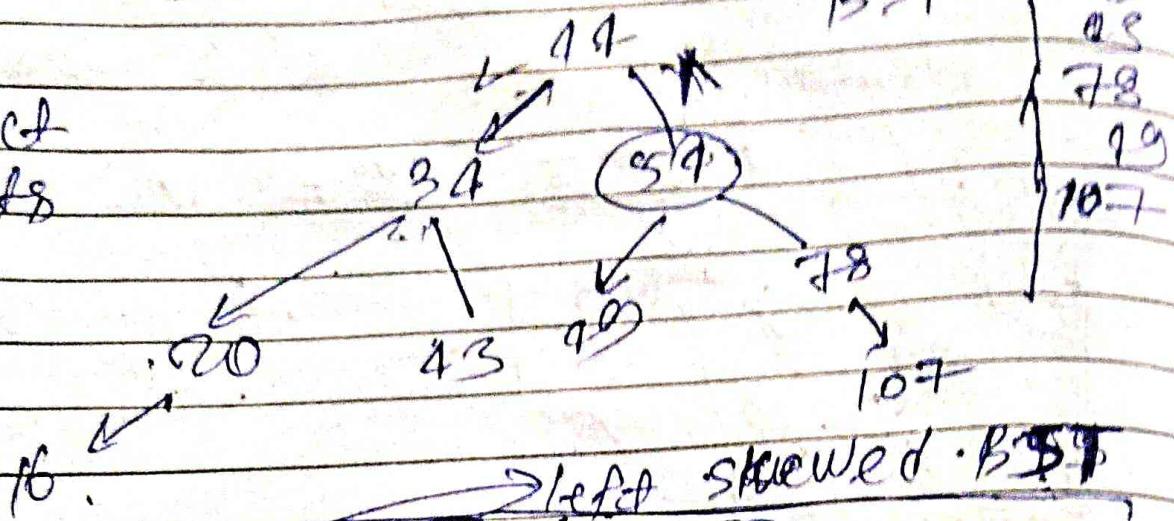


(Sentinel)

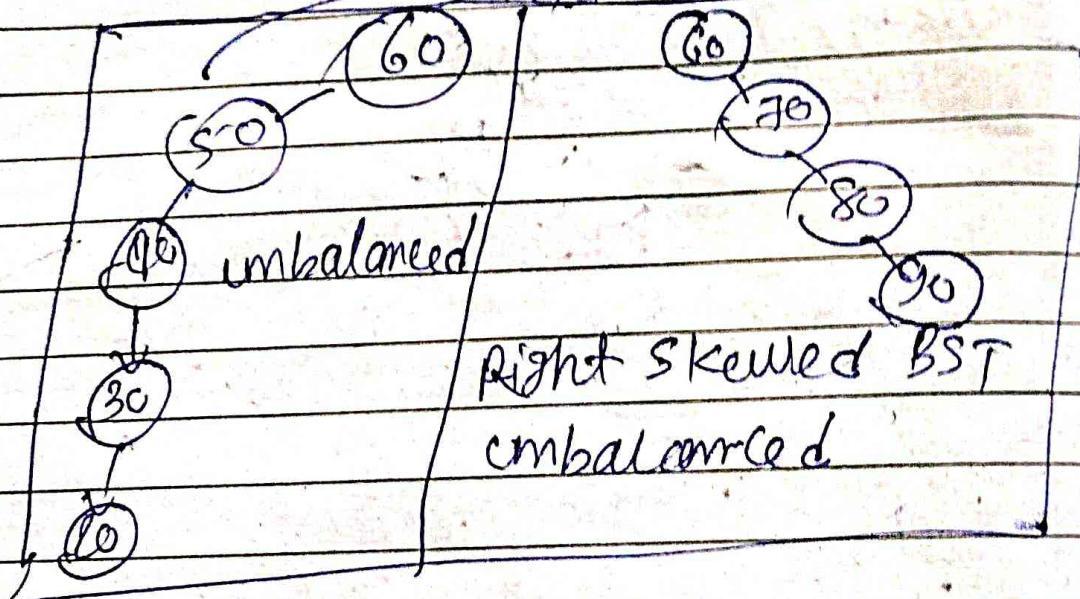
67

* Binary Search Tree (BST)

distinct elements



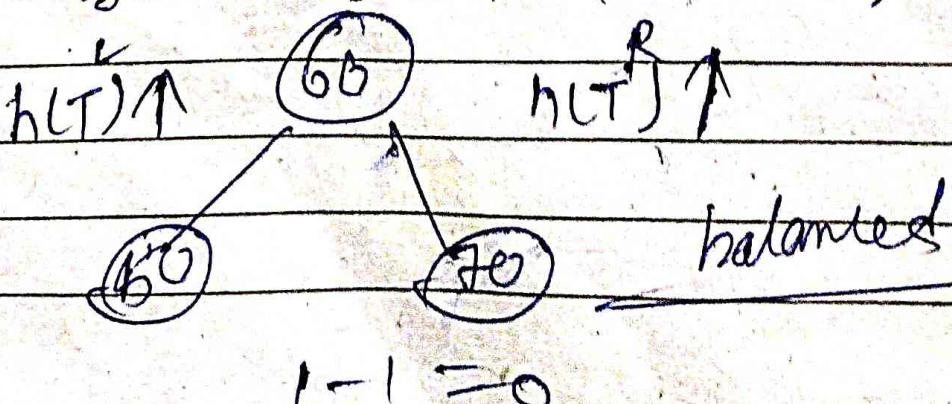
16. left skewed BST



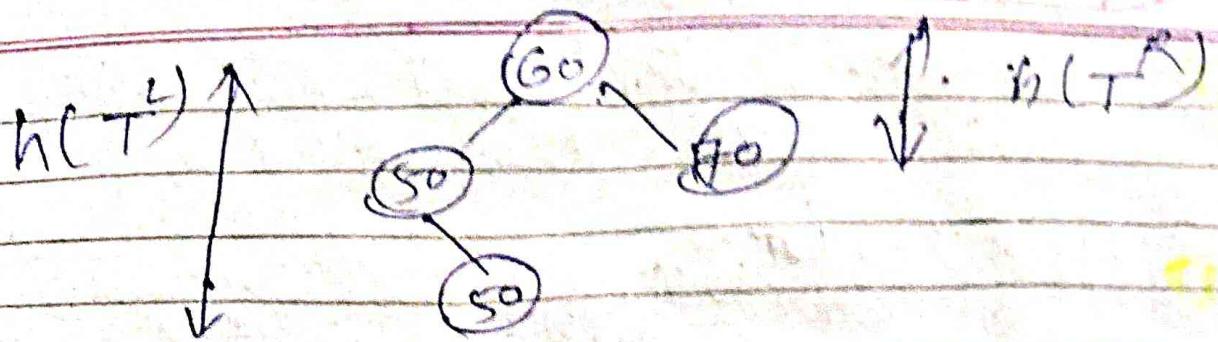
right skewed BST
unbalanced

* Balancing of BST.

AVL Tree : self-balancing BST.
Adelson Velski & Landef.



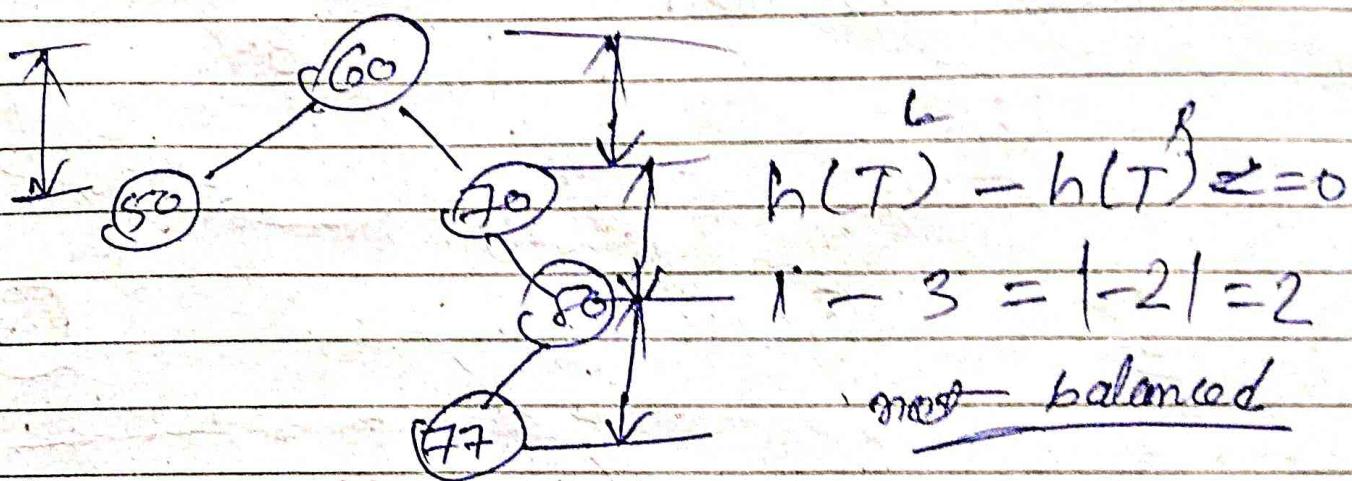
$$1 - 1 = 0$$



$$h(T^L) - h(T^R) = \\ 2 - 1 = 1$$

AVL Tree

$$[h(T^L) - h(T^R) \leq 1] \quad [\text{left subtree}]$$



* How can you say that a skewed tree does not give performance of AVL Tree

(P.M) Explain the way by which an imbalanced BST can be made
of an AVL Tree.

69

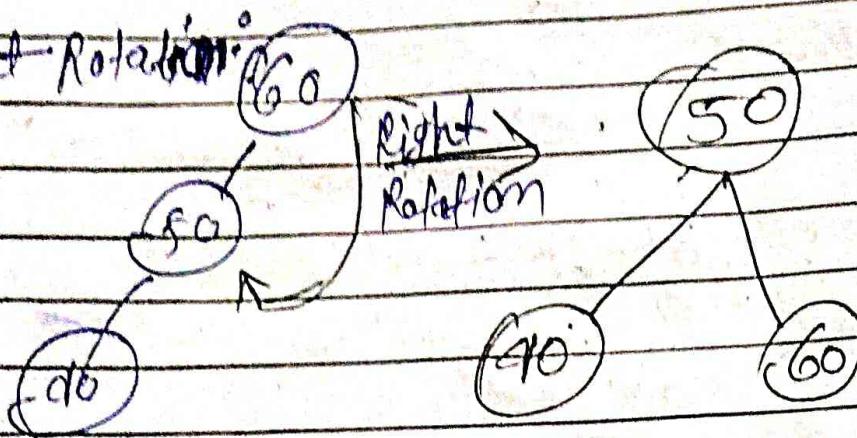


Rotations in BST to make it
an AVL Tree i.e. a balanced tree

Ans :-



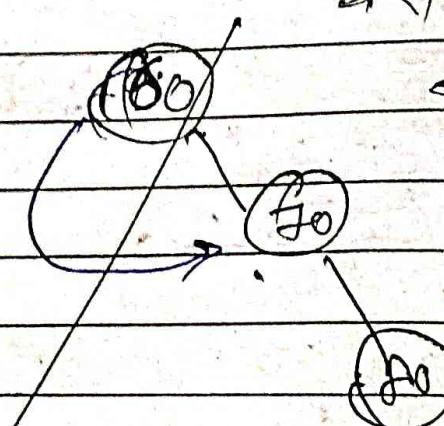
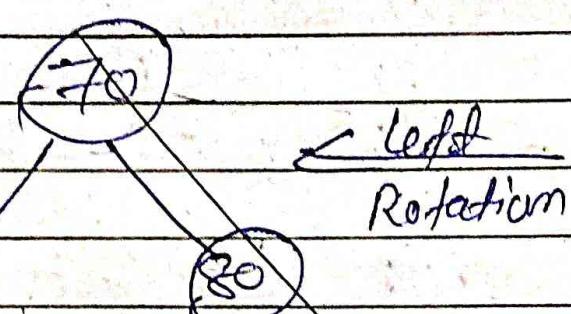
Right Rotation



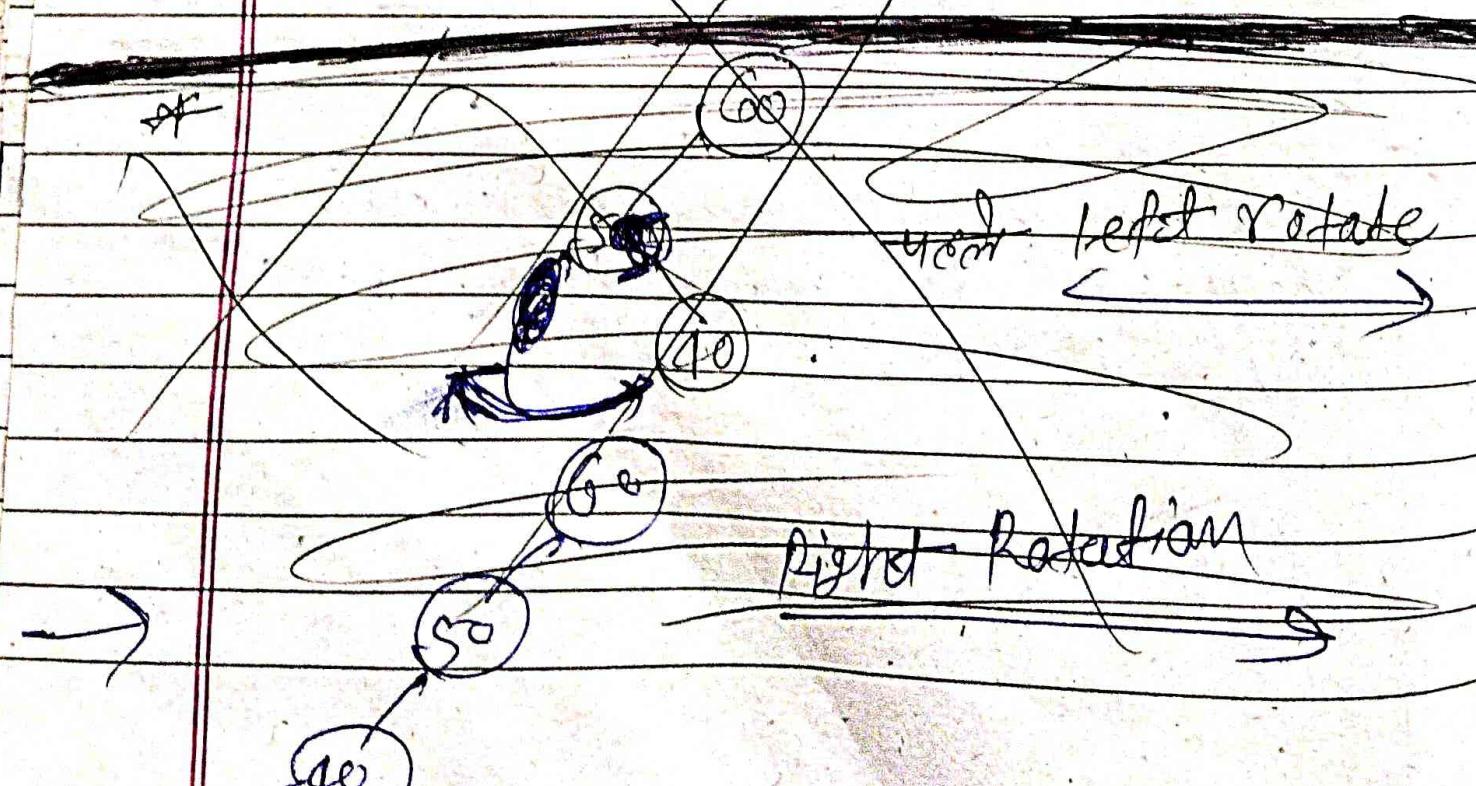
Left Rotation :



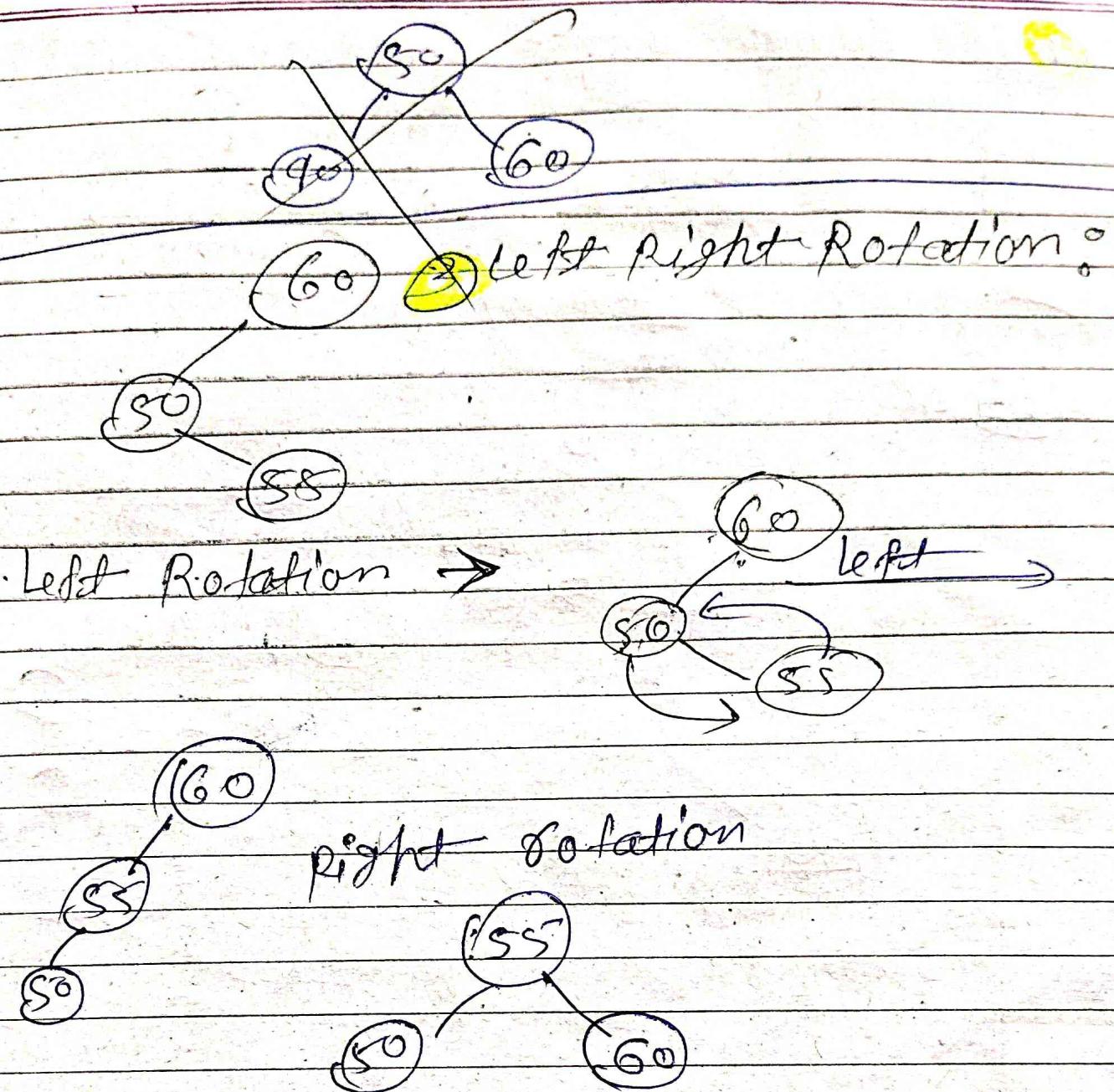
Right Left Rotation



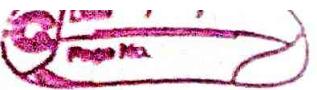
Right Left Rotate



Right - Right Rotation

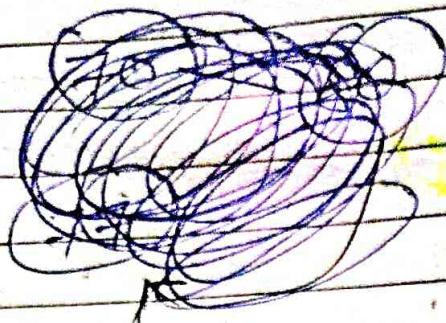
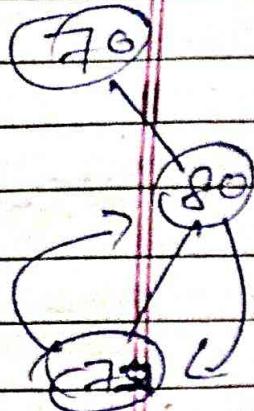


... PUNI SHAN KUMAR
OUT AND IN @ MURF
21-11-2021

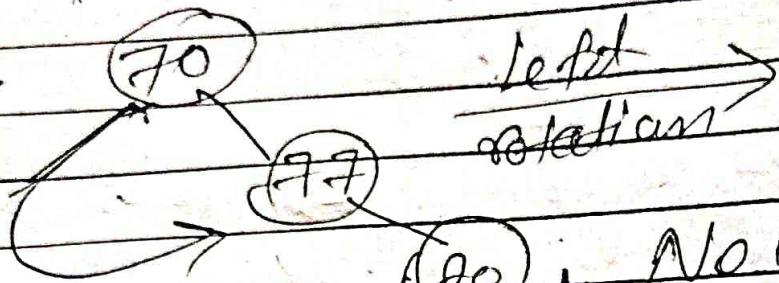


Right left rotation

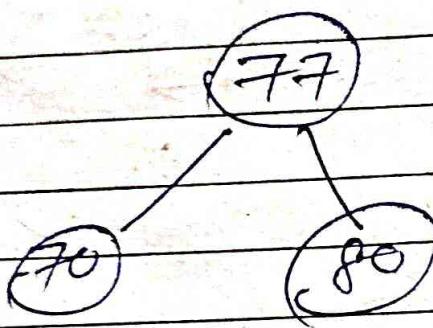
①



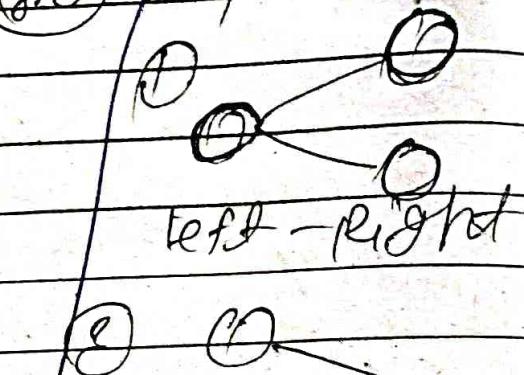
Right
rotation →



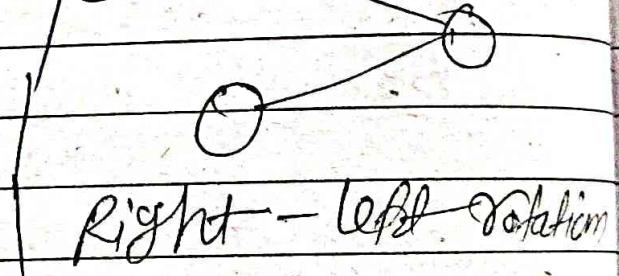
Left
rotation →



70, Note



left-right



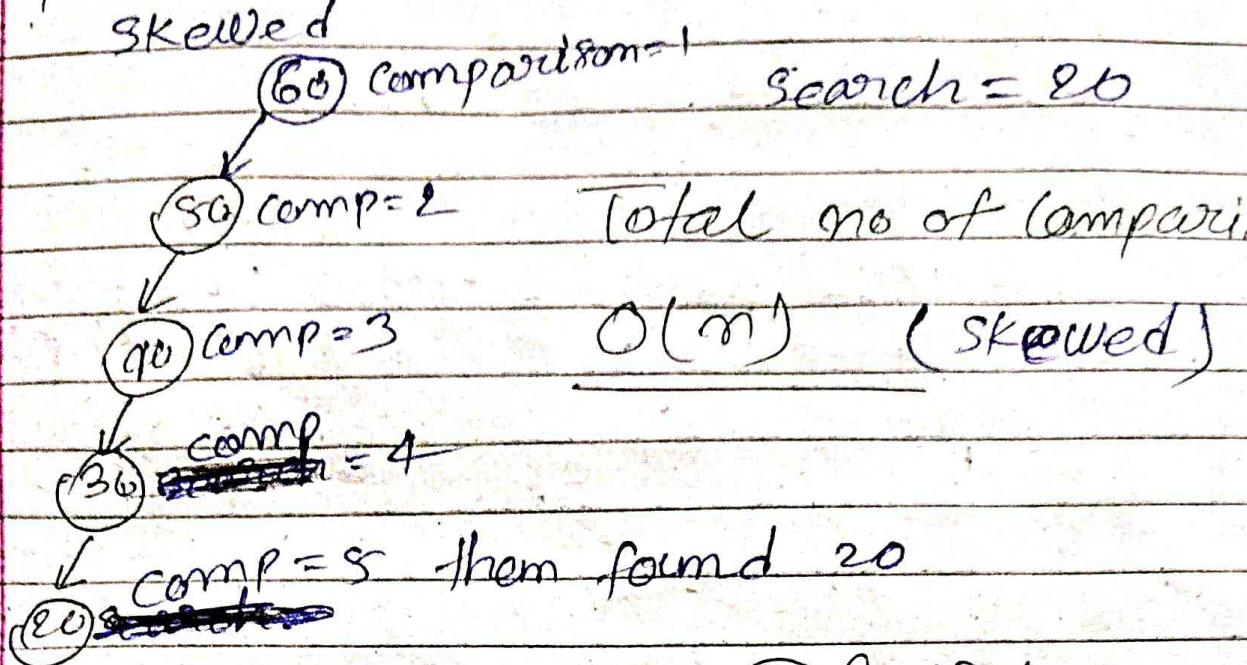
Right-left rotation

Q

* How can you say that a skewed does not give performance than AVL Tree.

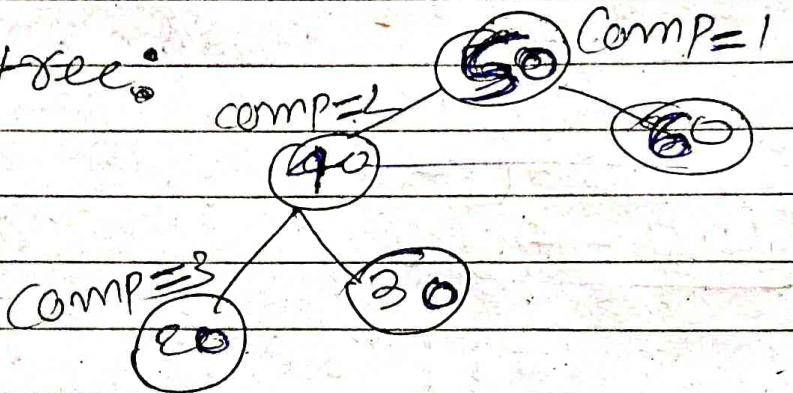
ans →

skewed



8

AVL tree:



No of Comparison = 3

$O(\log n)$ (AVL tree)

that's why we can say skewed does not give performance than AVL Tree.

Insertion] in BST
Deletion] ↑
balanced

AVL Tree = Balanced BST

#

m-way search tree : Every node can have at the most (m) child nodes.

5 way search tree
Ex :-

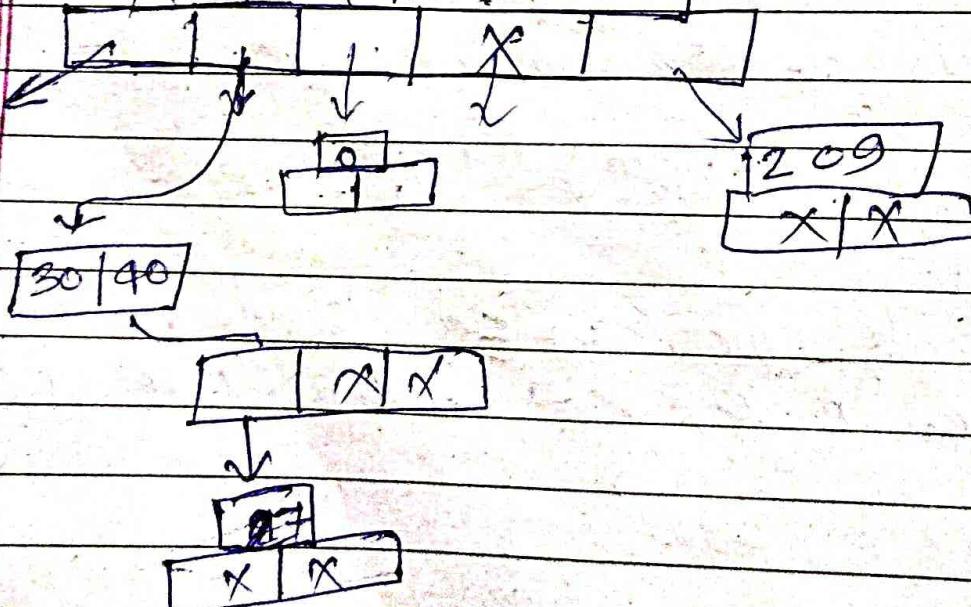
$m=5$, every node can have max 5 child nodes.

$$\text{key} = m-1$$

$$\text{key} = 5-1 = 4$$

$$\boxed{\text{key} = 4}$$

118 | 44 | 76 | 198 |



Mazik B-tree makes ~~an m-way search tree~~ balanced tree by maintaining a height close to $\log_m(m+1)$

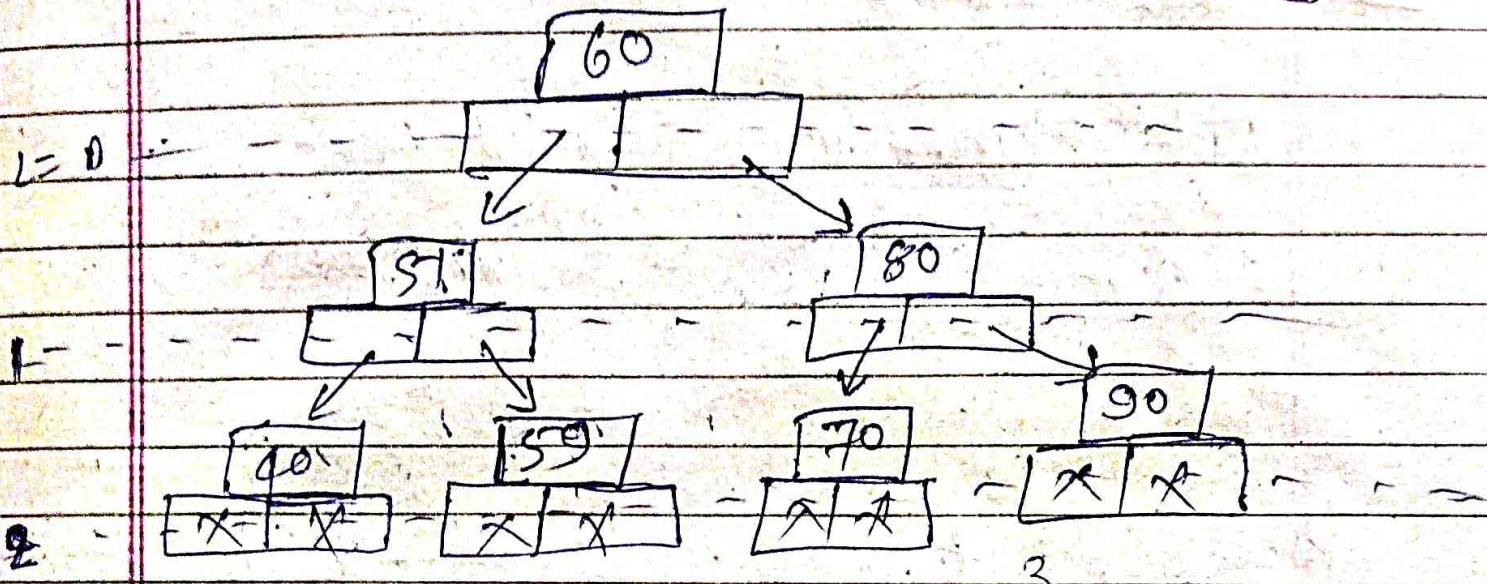
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(*) Analysis of m-way Search Tree.

m-way Search Tree $\rightarrow \text{key} = m-1$

$$\text{key} = 2-1$$

$$\boxed{\text{key} = 1}$$



$$\text{No. of nodes} \Rightarrow n = 7 = 2^3 - 1$$

$$\text{Height} = \text{No. of levels} + 1 \Rightarrow 2 + 1 = 3$$

$$n = 7 = 2^3 - 1$$

$$7 = 2^3 - 1$$

$$n = m^h - 1$$

$$\text{Height} = 2 + 1 = 3$$

$$n = m^h - 1$$

$$m^h = m + 1$$

Taking ~~logm both sides~~ \log_m both sides

$$\log_m(n^h) = \log_m(m + 1)$$

$$h \cdot \log_m n = \log_m(m + 1) \quad \left\{ \log_2^2 = 1 \right.$$

$$h = \log_m(m + 1)$$

$$h = \log_m(m + 1)$$

Application \rightarrow file indexing in disks.

Conclusion
m-way search tree may vary

$$h \Rightarrow \log_m(n+1) \leftrightarrow n \text{ (max)}$$

* B - Tree

Rules \Rightarrow m-way search tree

1. Root $\xrightarrow{\text{at least}} 2$ child nodes

~~$\xrightarrow{\text{at the most}}$~~ $\xrightarrow{\text{at the most}}$ m child nodes.

2. The, internal nodes $\xrightarrow{\text{at least}}$ $\lceil \frac{m}{2} \rceil$ child nodes
(except root)

Node

$\lfloor 2.5 \rfloor$

≈ 2

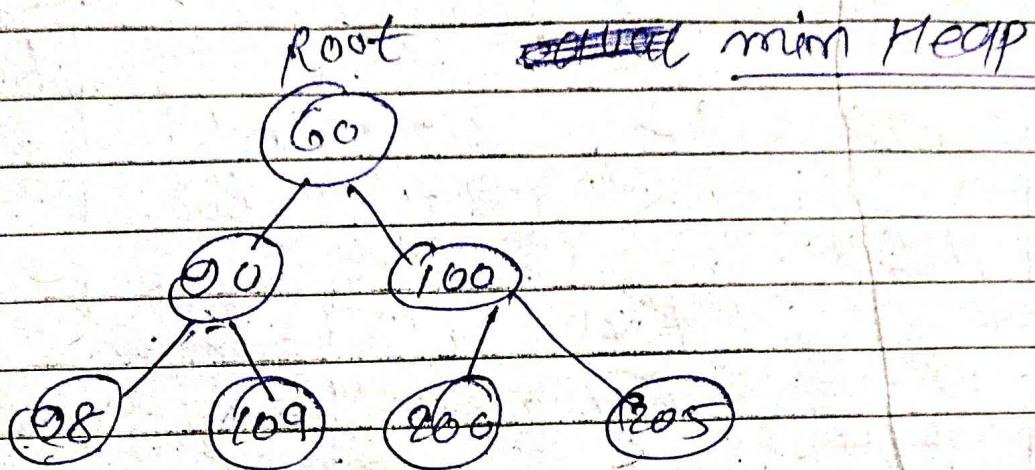
$$m = 5$$

$$\left\lceil \frac{5}{2} \right\rceil = \left\lceil 2.5 \right\rceil \cdot \text{Upper value} = 3$$

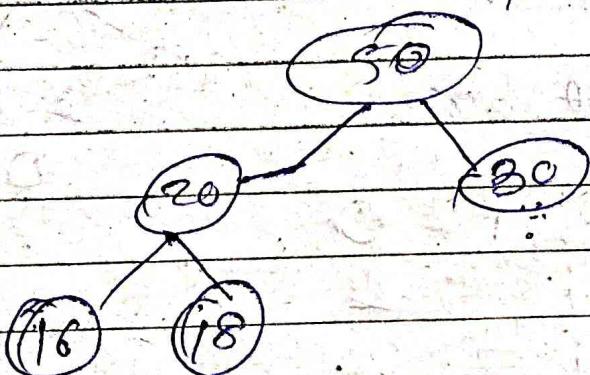
$$\textcircled{3} \quad \text{keys} = (m-1) \cdot 3 \text{ way.}$$

\textcircled{4} All leaf nodes should be at the same level.

* Heap Trees → min Heap
 GE 18 that binary tree → Max Heap

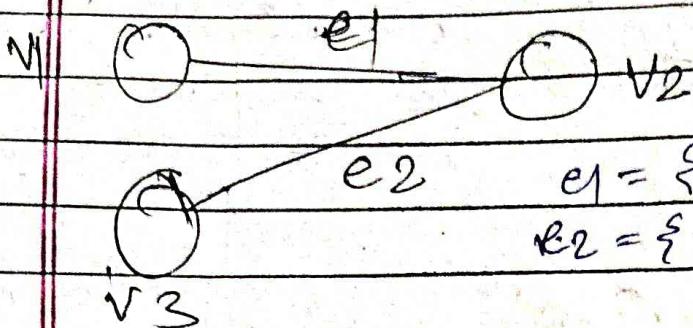


max Heap Root



Grasses

$$G = (V, E) \quad V = \{v_1, v_2, v_3\} \\ E = \{e_1, e_2\}$$



Term:

~~Decorated Graph~~

~~and the day before~~
directed 9/10.

~~directive group~~
~~undirected group~~

W.M. & C. 1894

$$[v_1, v_2, v_3] = v_{\text{source}}$$

$$[v_3, v_2, v_1] = x \text{ sink}$$

No of edges of A = 2

degree of A =

in degree out degree

$$\text{Indegree}(A) = \{0\}$$

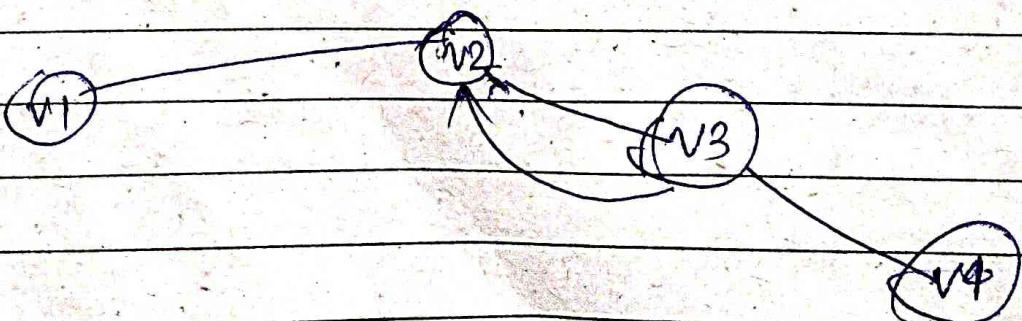
$$\text{Octdeg}(\Delta) = 2$$

$$\text{Indeg}(c) = 2 \text{, 3 sink}$$

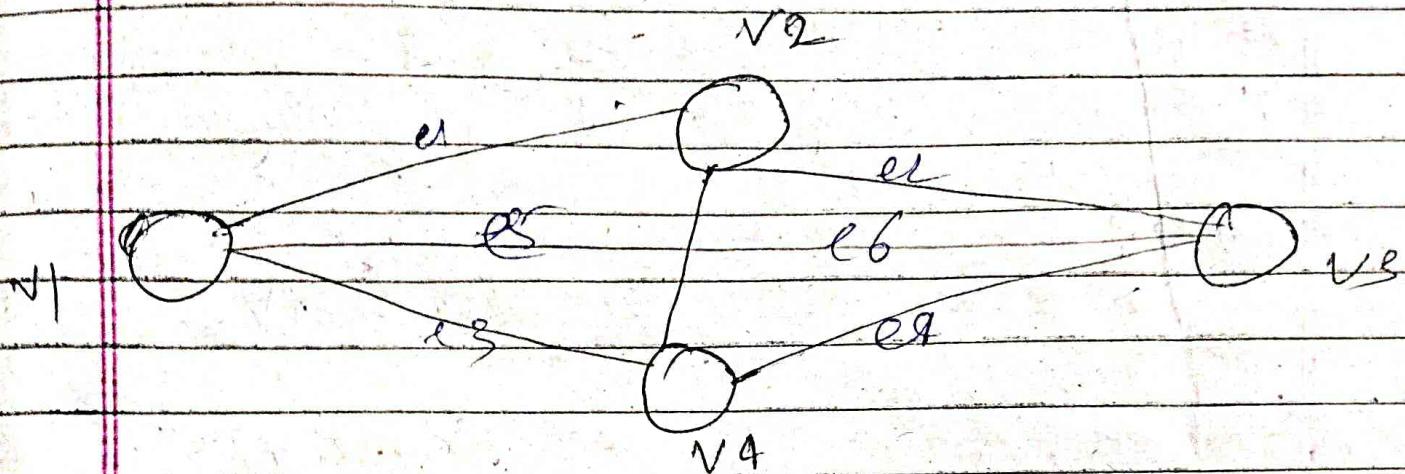
$$\text{outdeg}(c) = 0$$

$$\text{indeg}(b) = 0$$

Oct 29 (b) = 5



Cycle: It is simple closed path.



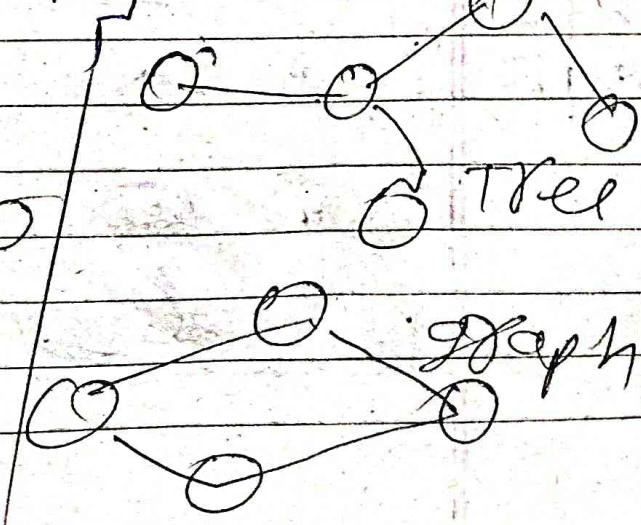
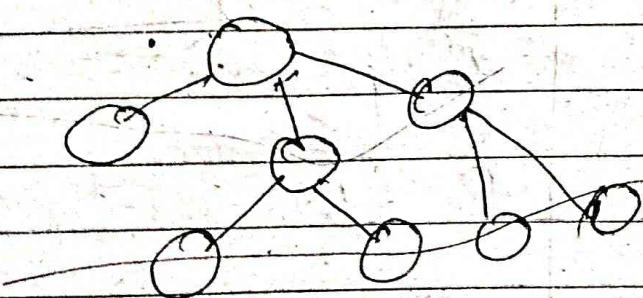
No of vertices = $n = 4$

No of edges in a complete graph: = e

$$n(n-1) = 4(4-1)$$

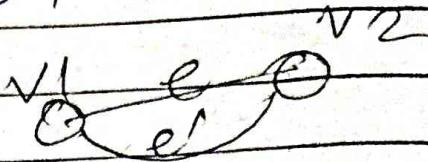
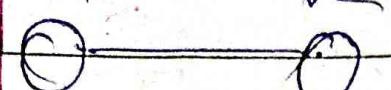
Q. Complete a graph of vertices 4
and compute the edges of 9
Complete graph.

Tree or Tree graph/a graph



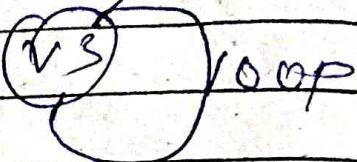
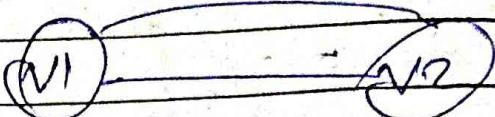
Q Multigraph : Allows (i) multiple edges

(ii) loop



v3

graph



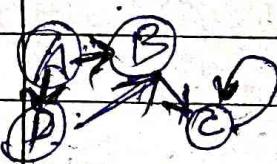
multiple

Q Representation of graph

Sequential rep (2D-Matrix)

Adjacency matrix

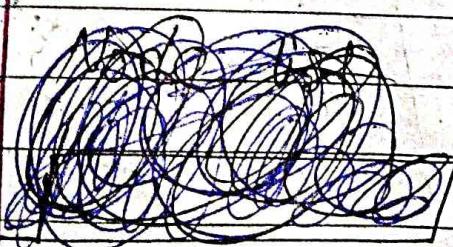
	A	B	C	D
A	0	1	0	1
B	0	0	1	0
C	0	0	1	0
D	0	1	0	0



linked rep
linked list (adjacency list)

node | Adjacency list

A	.B, D
B	.C
C	.C
D	B



1111 Node list

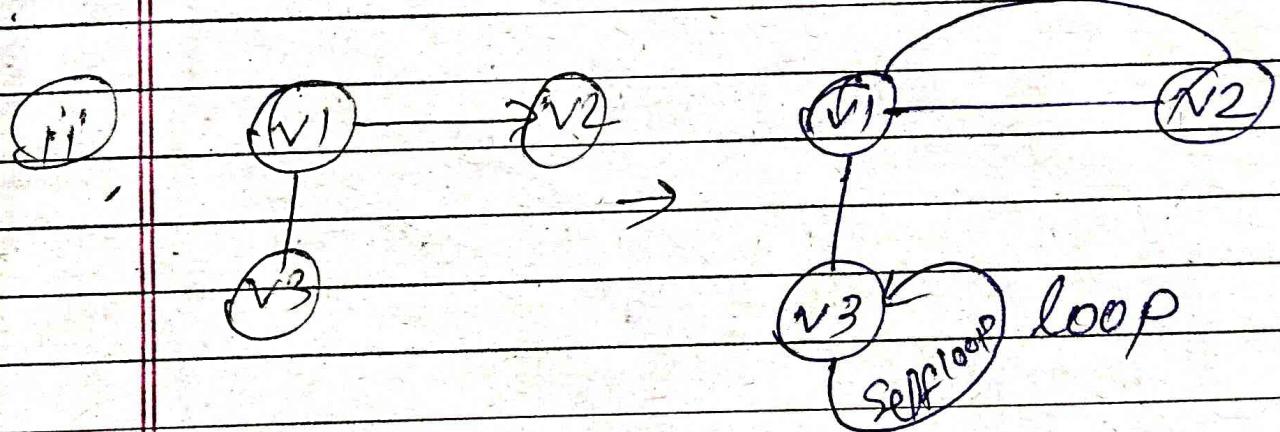
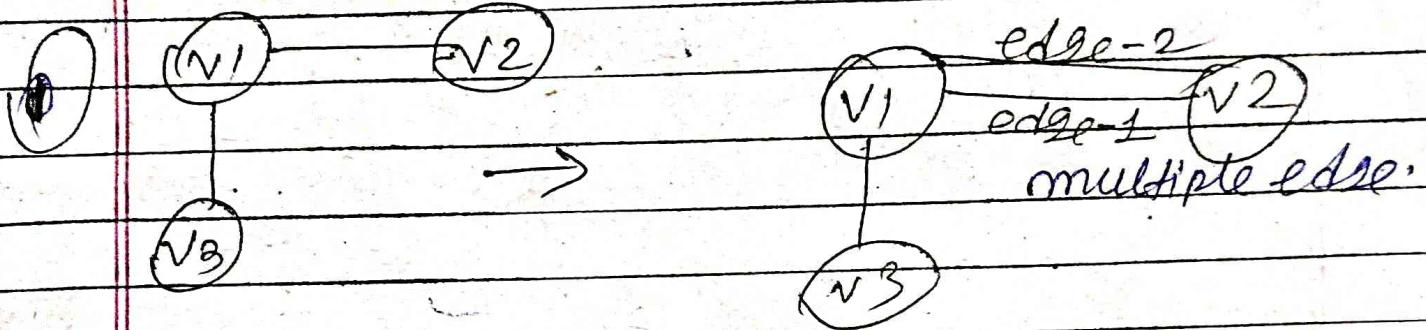
edge list

(PQ) Analyze the features that make a graph as a multigraph.

Ans ⇒ i) To make a graph as a multigraph

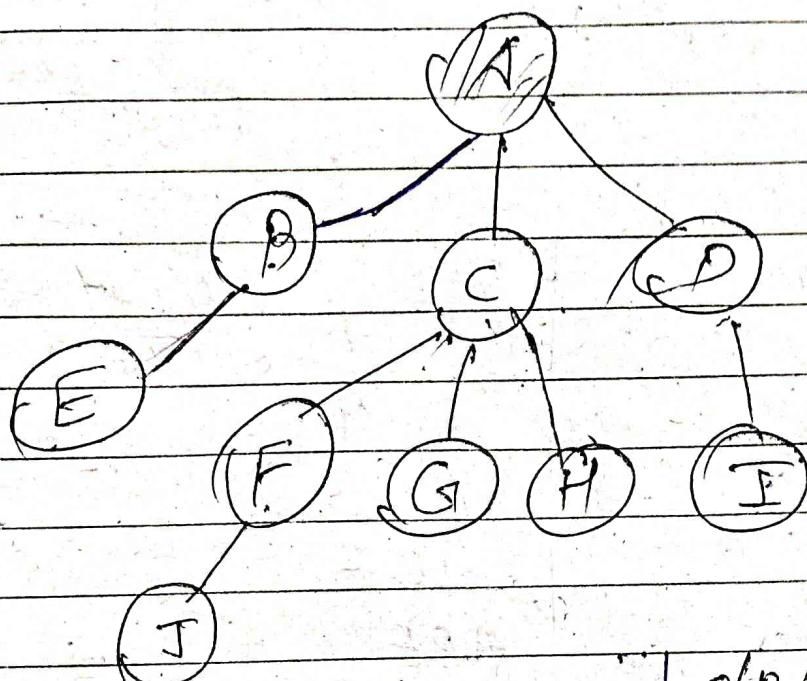
- i) graph should have multiple edges between two nodes.

- ii) graph can also have self loops.



* Apply BFS tech. [queue]

queue Q1 queue Q2



queue Q1

A
B C D
C D E
D E F G H
E F G H I

o/p:

Φ
A
AB
ABC
ABC D

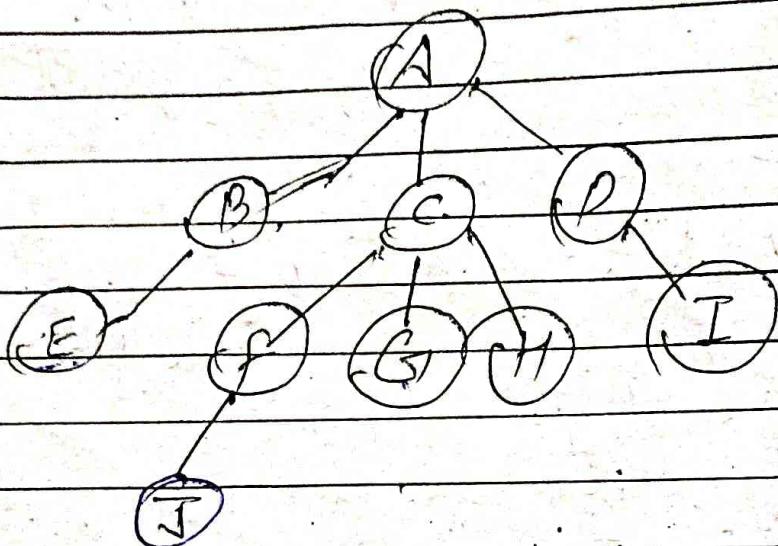
queue Q2

FGHI
 GHIJ
 HIJ
 IJ
 J

ABCDE
 ABCDEF
 ABCDEFGI
 ABCDEFGIH
 ABCDEFGHI
 ABCDEFGHIJ

* Apply DFS [stack]

only one stack required



Stack

D
 A
 B
 D C E
 D C
 D H G I F
 D H G J
 D H G
 D H
 D
 I

Op: Print Commi

A
 AB
 ABE

ABEC
 ABECF

ABECFJ
 ABECFJG

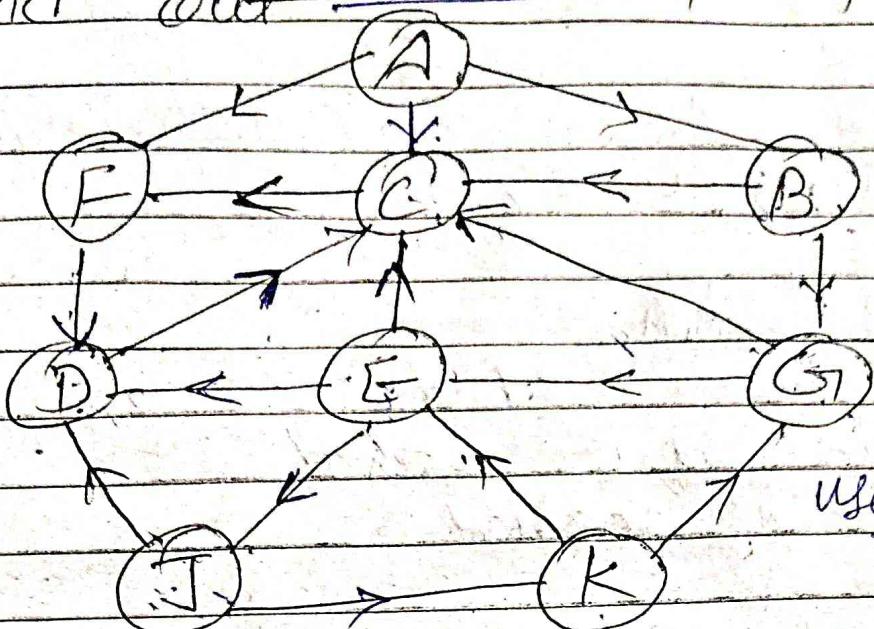
ABECFJGH
 ABECFJGHD
 ABECFJGHD

Explain Travel tech of
Graph
BST

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* find out minimum path from node A to J



use BFS

Q1

A
F C B
C B D
B D
D G
G
E
J
K

O/P.

Φ
A
AF
AFC
AFC.B
AFC.B.D
AFC.B.D.G
AFC.B.D.G.E
AFC.B.D.G.E.J

Q2

↓

BACK TRACK

A X B X G E J

A → B → G → E → J

Minimum path → A → B → G → E → J

* Hashing & saving the memory

* Hashing techniques [Hash Function]
 $\rightarrow H(k) : K - 1$

Hashing

- (1) Division method
- (2) Mid-square method
- (3) folding method

$$k = 3205$$

$$h = k \bmod m$$

$$h = 3205 \bmod$$

To represent 4 digit no. into two digit no. mod h with m
 $m \rightarrow$ prime no.

$m \rightarrow$ closest to maximum value
 and greater than record of the member

> m = 9699 (97)

> record

> no. of employee prime no. 81 AT 488

> 68

97) 3205 (04

X

(ii) Mid-Square method : $K = 3205 \rightarrow K^2$

$$K^2 = 10^4 / 272.888$$

(72)

(iii) Folding method : $① 3205 = 32 + 05 = 37$
 without
 converting

(iv) Reversing 3205
 $32 + 50 = 82$ with reversing

$$R \rightarrow ① 7148 = 71 + 48 = 119 = 19$$

$$② 7148 = 71 + 89 = 159 = 55$$

$$R \rightarrow 3232 = \cancel{3} \cancel{2} 32 + 23 = 55$$

* Collision in Hashing:

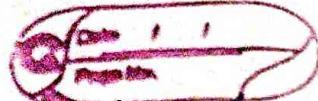
$$\begin{cases} H(K) = n \\ H(K') = n \end{cases} \quad H(K) \rightarrow \text{folding}$$

$$H(7148) \rightarrow 55$$

$$H(3232) \rightarrow 55$$

Coll. Resolution tech : (i) linear : probing
 (ii) quadratic Probing
 (iii) Double Hashing

the order n..



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practical formula

① Linear probing:

1

$h, h+1, h+2, h+3, \dots$

$h, (h+1) \% N, (h+2) \% N, \dots$

②

Quadratic probing:

$h, h+1^2, h+2^2, h+3^2, \dots$

$h, h+1, h+4, h+9, \dots$

double hashing

$H(k) = h$

$H'(k) = h'$

$h, (h+1)^2 \% N, (h+2)^2 \% N, \dots$

③

$h, h+h', h+2h', h+3h', \dots$

$h, (h+h') \% N, (h+2h') \% N, \dots$

④

Linear probing

$n=8$

	A	B	C	D	E	X	Y	Z
$h:$	4	8	2	11	4	11	5	1

Mem loc : $(9+1)\%11$

$(11+1)\%11$

1	L	3	4	5	6	7	8	9	10	11
X	C	Z	A	E	Y	-	B	-	D	-

Open addressing (closed Hashing)

[To resolve collision: involve probing]

Efficiency :

	A	B	C	D	E	X	Y	Z
HW	4	8	2	11	4	11	5	1
M loc.	1	2	3	4	5	6	7	8
	X	C	Z	A	E	Y	B	D

Step: 1

$$n = 8$$

$$m \text{ loc} = 11$$

$$\text{load factor} = \frac{n}{m} = 1$$

$$= \frac{8}{11} \Rightarrow H = 0.73$$

maximum = 11

Step : 2 Complete S(d) & U(d)

S(d) \rightarrow it is the average no of probes for successful search

U(d) \rightarrow it is the average no of probes for unsuccessful search.

$$S(d) = ?$$

Roughwork

$$U(d) = ?$$

$$S(d) = \frac{1}{2} \left[1 + \frac{1}{1 - \frac{1}{n}} \right] = 2.35 \quad | \quad S(d) = \frac{1}{2} \left[1 + \frac{1}{1 - \frac{1}{8}} \right]$$

$$U(d) = \frac{1}{2} \left[1 + \frac{1}{\left(1 - \frac{1}{8} \right)^2} \right] = 7.36$$

$$S = \frac{2.35}{8}$$

$$S = 0.29$$

$$U = \frac{7.36}{8}$$

$$U = 0.669$$

$$S = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{2}{8} + \frac{2}{8} + \frac{2}{8} + \frac{3}{8}$$

$$S = \frac{13}{8} \quad S = 1.625 \quad \approx 1.63$$

$K \rightarrow 1$ ~~the average value is 1.63~~ $1.63 < 2.35$

$$S < S(d)$$

$$U = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11$$

$$7 + 6 + 5 + 4 + 3 + 2 + 1 + 9 + 1 + 1 + 8$$

$$11 \text{ (cm)}$$

$$U = \frac{40}{11} \quad U = 3.63$$

$$UV \leftarrow \cdot V(d) \quad 3.63 < 7.36$$

* open chaining (hashing)

Records	A	B	C	D	E	X	Y	Z
4	8	2	11	4	11	5	1	

$$m = 8, m = 11$$

dim K	8
1	3
2	0
3	0
4	1, 5, 2
5	7
6	0
7	0
8	2
9	0
10	0
m =	11 4 6

Date _____
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i	Info P	Link
1	A	0
2	B	0
3	C	0
4	D	0
5	E	1
6	X	4
7	Y	0
8	Z	0
9		0
10		0
11		

faster method than arrays
but not space efficient because it uses
memory to save the pointers.

efficiency

$$S(d) = 1 + \frac{1}{2} d \quad U(d) = \frac{-d}{e} + 1, d = \frac{n}{\text{error}}$$

$$S_d = ? \quad U_d = ? \quad \rightarrow \text{Euler's no. } 2.718$$

$$\boxed{\begin{aligned} d &= \frac{8}{11} \\ d &= 0.73 \end{aligned}}$$

$$S(d) = 1 + \frac{1}{2} \times 0.73, U(d) = \frac{-d}{e} + 1$$

$$S(d) = 1.36$$

$$S(d) = 1.37$$

$$U(d) = 0.08 + 0.73 = 1.21$$

$$S = A + B + C + D + E + F + G + H + I + J + K + L$$

$$m = 8$$

$$S = \frac{10}{8}$$

$$S = 1.25$$

$s(d)$

S

$$1.37 > 1.25$$

efficient

$$U = 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11$$

$$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$$

$$m = 11$$

$$U = \frac{11}{11} \quad U = 1$$

$U(d)$

$$1.21 > 1$$

efficient

U

Analysis of load factor:

open Addressing

closed hashing

[Hash Table]

$$[m \geq n]$$

$$11 = m$$

$$d = \frac{n}{m} \leq 1$$

chaining

open - hashing

[m < n]

$$m$$

$$(m < n)$$

$$d = \frac{n}{m} \rightarrow 1$$

Rehashing:

(4) Quick Sort Algorithm

Step 1 (25), 57, 48, 37, 12, 92, 86, 33

Set first element as pivot

Step 2 $dm \rightarrow$

$dm = 0$

$UP = 7$

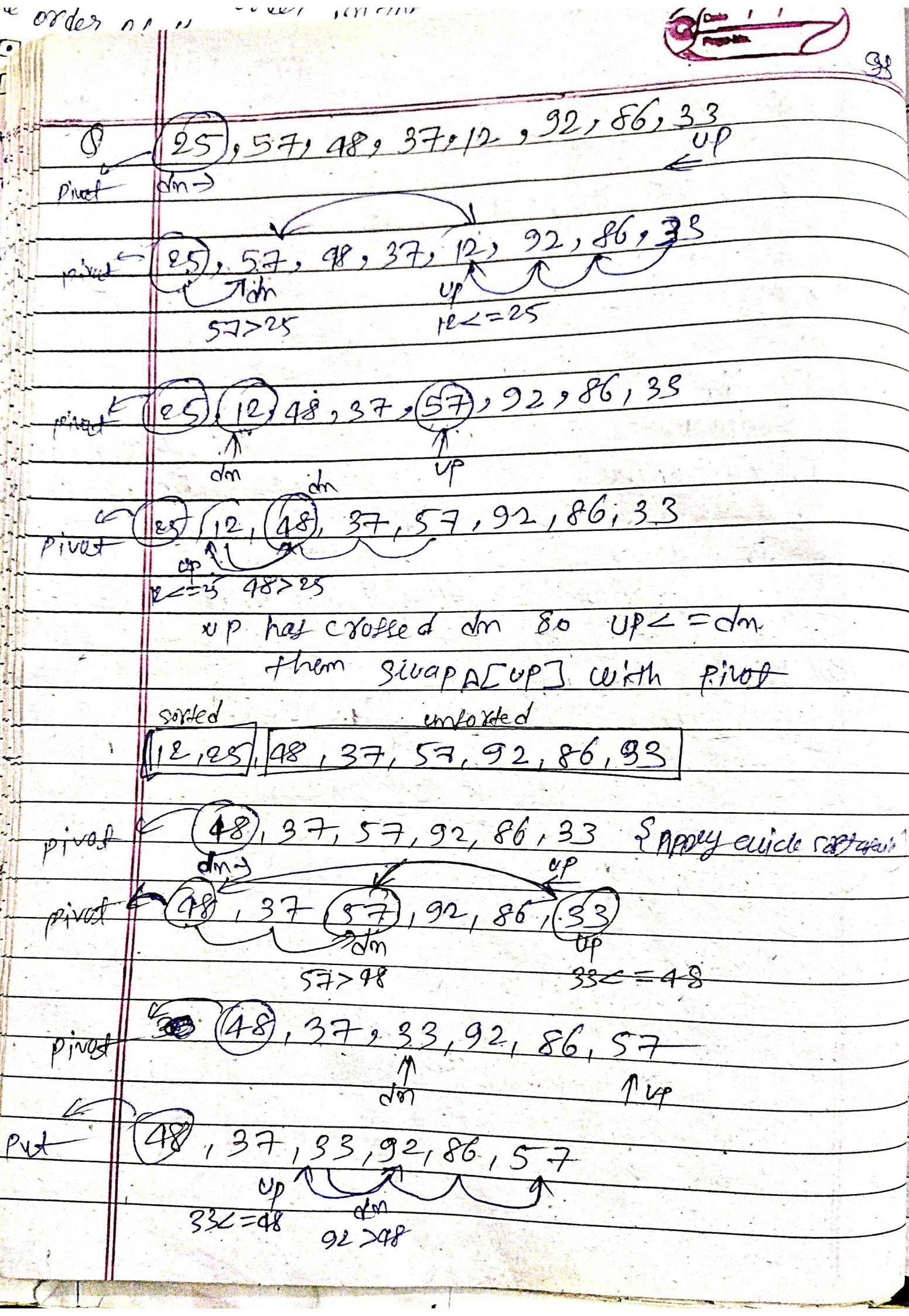
UP will decrease

$A[dm] > \text{pivot}$

till $A[UP] \leq \text{pivot}$

Step 3 if $UP > dm$ swap $A[dm]$ with $A[UP]$

but if $UP \leq dm$ swap $A[UP]$ with pivot



up has crossed dn

$up \leftarrow dm$ so swap it with pivot

A[up] swap with pivot

[33, 37, 48, 92, 86, 57]

sorted

unsorted

pivot ↗
33, 37
↓
dm ↑
up

pivot ↗
33, 37
↑
dm
up ↗
up ↘
do not have crossed dm
 $up \leftarrow dm$

then A[up] swap with pivot

[33, 37] 48 | 92 86 57

sorted

unsorted

pivot ↗
92, 86, 57
↓
dm ↘
up ↘
pivot ↗
92, 86, 57 + ↗
dm ↘
up ↗
92 ↗ 92

[57, 86] | 92
↓
sorted
unsorted

UP crossed dm
swap A[up] with pivot.
 $up \leftarrow dm$

pivot ↗
57, 86
dm ↗
up ↗

pivot ↗
57, 86
dm ↗
up ↗
up crossed dm
 $up \leftarrow dm$
A[up] swap with pivot

[57, 86]
sorted

finally we got

[12, 25, 33, 37, 48, 53, 86, 92]
sorted list

Time Complexity of quicksort (normal case)

$$= O(n \log n)$$

worst case = $O(n^2)$

(5) Merge Sort :

1. 2. 3. 4. 5. 6. 7. 8. 9. 10
310, 285, 179, 652, 351, 423, 861, 254, 450, 520

$$\text{complete mid} = \left\lceil \frac{\text{low} + \text{high}}{2} \right\rceil$$

$$\text{low} = 1$$

$$\text{high} = 10$$

$$\text{mid} = \left\lceil \frac{1+10}{2} \right\rceil \Rightarrow \text{mid} = 5$$

low high
[1, 10]

19.5. 6, 10

310, 285, 179, 652, 351

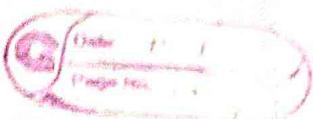
423, 861, 254, 480, 520

~~Do same~~

do same process

fill individual

elements are not obtained.



~~310, 285, 179, 652, 351~~

low = 1

high = 5

$$\text{mid} = \lceil \frac{1+5}{2} \rceil \quad \lceil \text{mid} \rceil = 3$$

1	2	3	4	5
310	285	179	652	351

[1, 10]

[1, 5]

[6, 10]

[1, 3] [9, 5]

~~310, 285, 179~~ $l=1, h=3$

$$m = \lceil \frac{1+3}{2} \rceil \quad \lceil m \rceil = 2$$

1	2	3
310	285	179

[1, 3]

[1, 2] [3, 3]

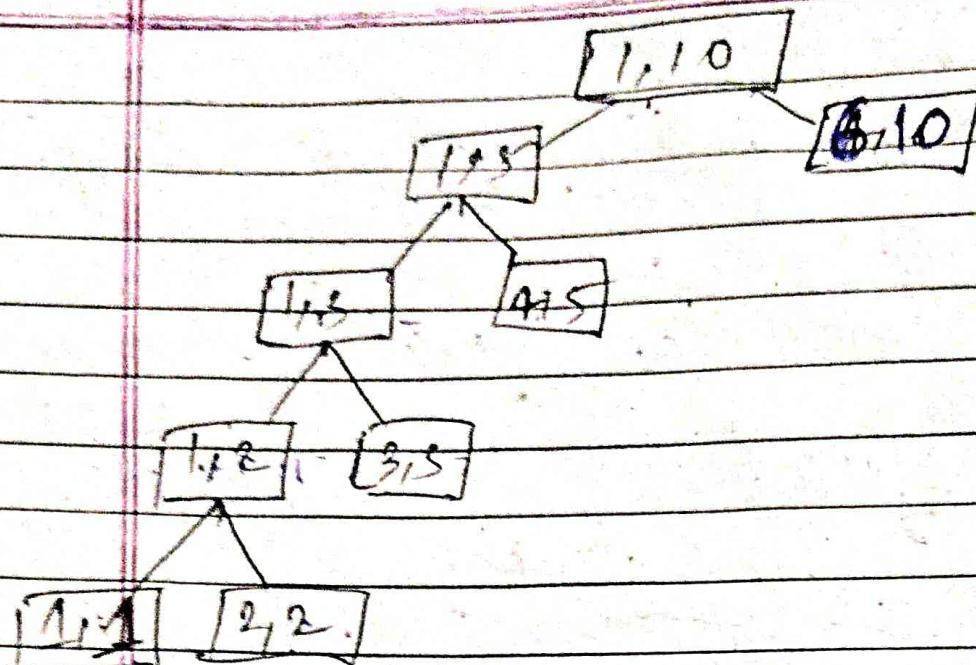
~~310, 285~~ $l=1, h=2$ $m = \frac{3}{2} = \lceil m \rceil = 2$

1	2	3
310	285	

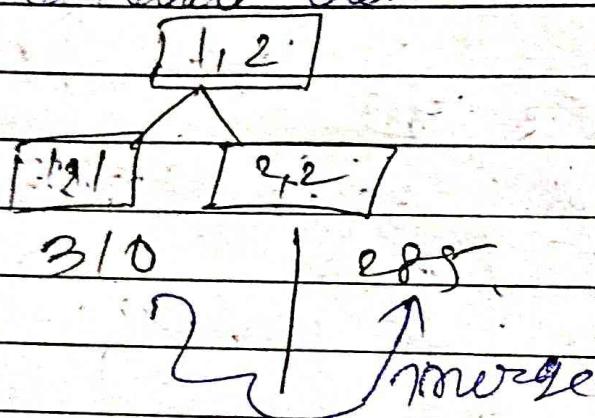
[1, 2]

[1, 1]

[2, 2]



merge
individual element



~~310, 285~~ 285, 310

1, 2 | 3, 3
285, 310 | 179

merge

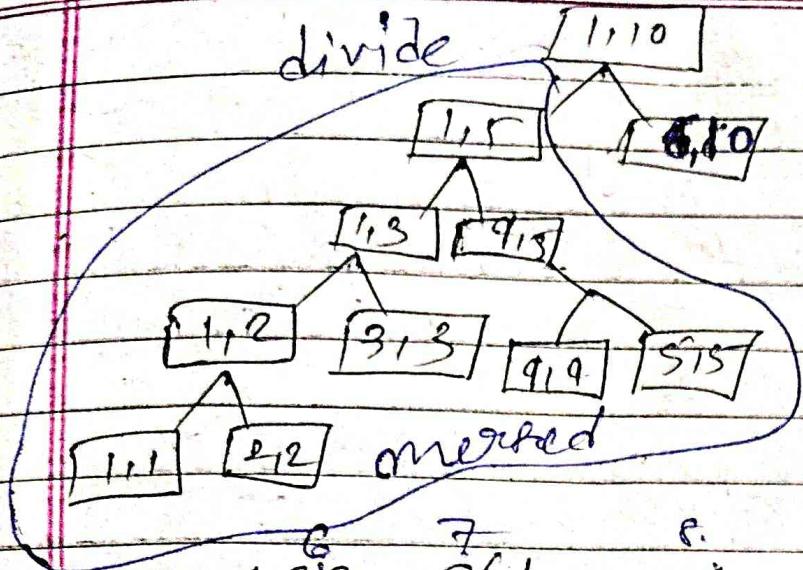
179, 285, 310

~~179, 285, 310~~, split 415
652, 351
 $i=4$ $m = \frac{9}{2}$ $m = 4$
 $n=5$

merge

179, 285, 310, 351, 652

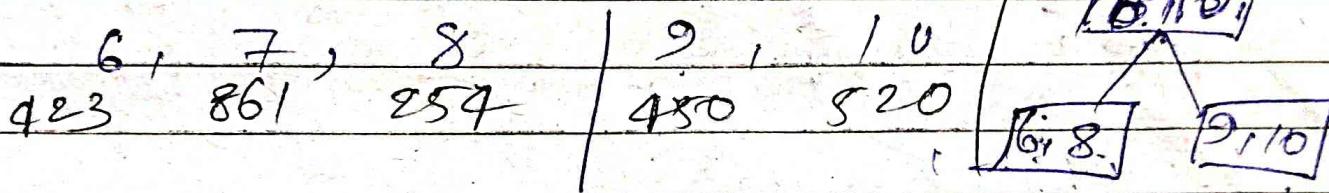
merge 1915
1915
518



6, 7, 8, 9, 10
423, 861, 254, 450, 520

$$l=6 \\ h=10$$

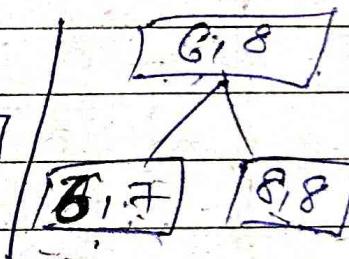
$$m = \left\lceil \frac{6+10}{2} \right\rceil \quad m = 8$$



6, 7, 8
423 861 254

$$l=6 \\ h=8 \\ \cancel{m=6}$$

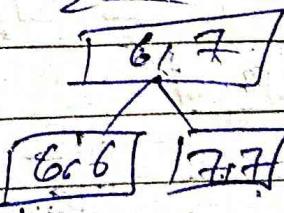
$$m = \left\lceil \frac{6+8}{2} \right\rceil \quad m = 7$$



6
423

7
861

$$m = \left\lceil \frac{6+7}{2} \right\rceil \quad m = 6$$

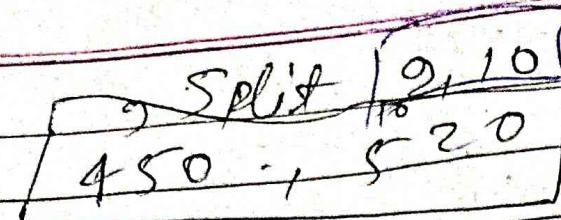


6
423 | 861

merged | 8
423, 861 | 254

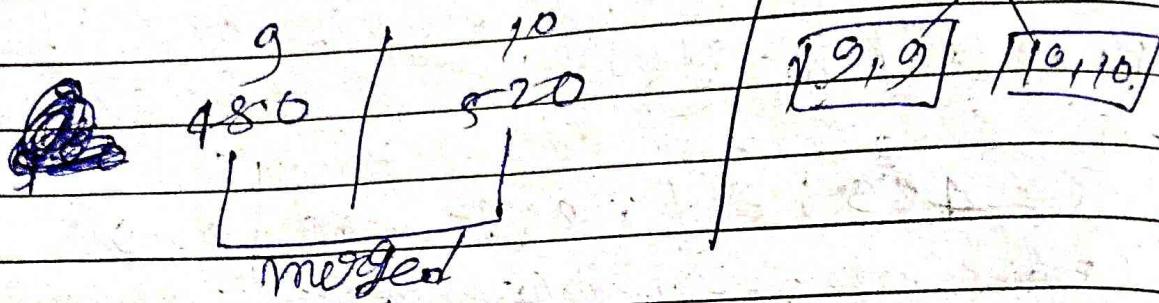
merged

6, 7, 8
254, 423, 861



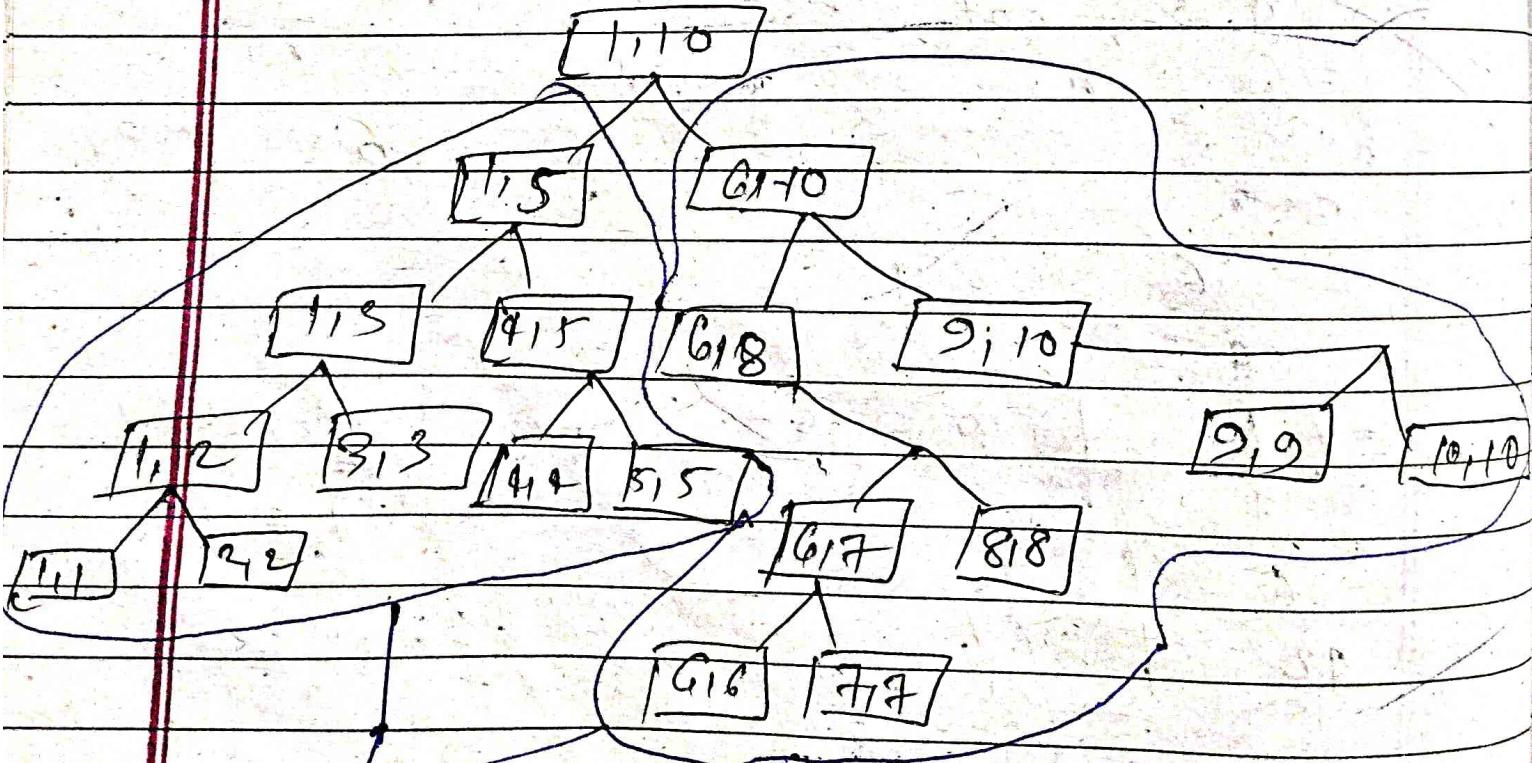
$$l=9 \quad m = \frac{10+9}{2} \quad m = \frac{19}{2} \quad (m=9)$$

$$h=10$$

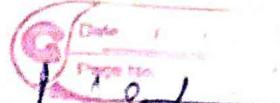


6, 7, 8
254, 423, 861

merged {254, 423, 950, 520, 861}



finally merge both



finally merging the two list

[179, 285, 310, 351, 652] [859, 423, 450, 520, 861]

for merging process an extra array is required

[179, 285, 310, 351, 652] [859, 423, 450, 520, 861]

Compare i & j and increment them

if $i < j$ insert i in newarray | if $j < i$ insert j in newarray
 & increment i again | and increment j again
 compare

[179, 284, 285, 310, 351, 423, 450, 520, 652, 861]

final merged & sorted
 Array list

Time Complexity of Merge Sort

= $O(n \log n)$ in normal,

Average, Worst case

but not space efficient

because it takes one extra array to merge both list.

⑥ Heapsort

40, 80, 35, 90, 45, 50, 70.

phase ① Create max Heap

(a) 40

(b) 80

compare left & right child

which ever is greater

compare it with parent

→ swap (if required) to get max heap }

Note: ~~insert~~
~~left & right~~
~~right & left~~
 delete right →

(c) 40

(d) 80

(e)

90

90 35

(f)

80 40
35

(g)

80
90 35

(h)

80
40 35

(i)

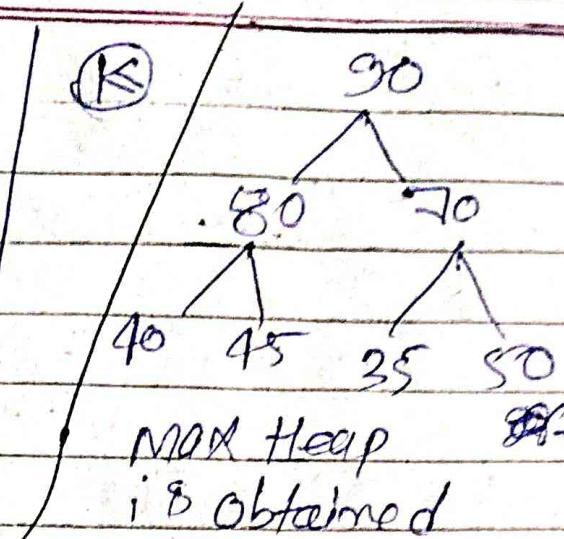
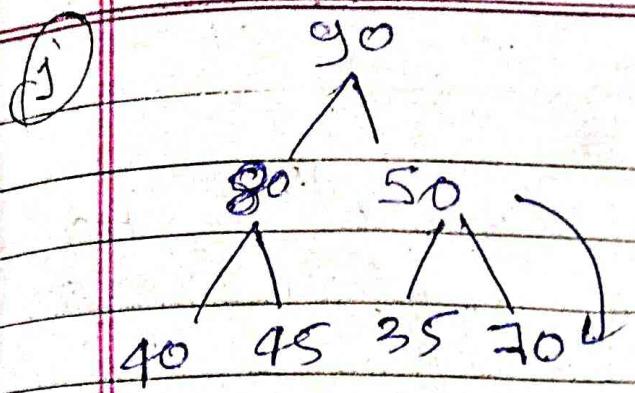
90
80 35

(j)

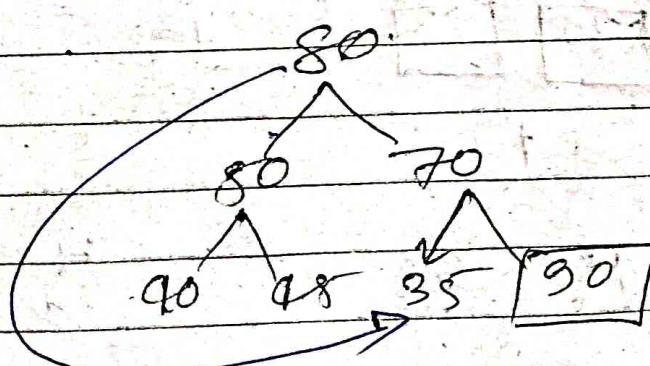
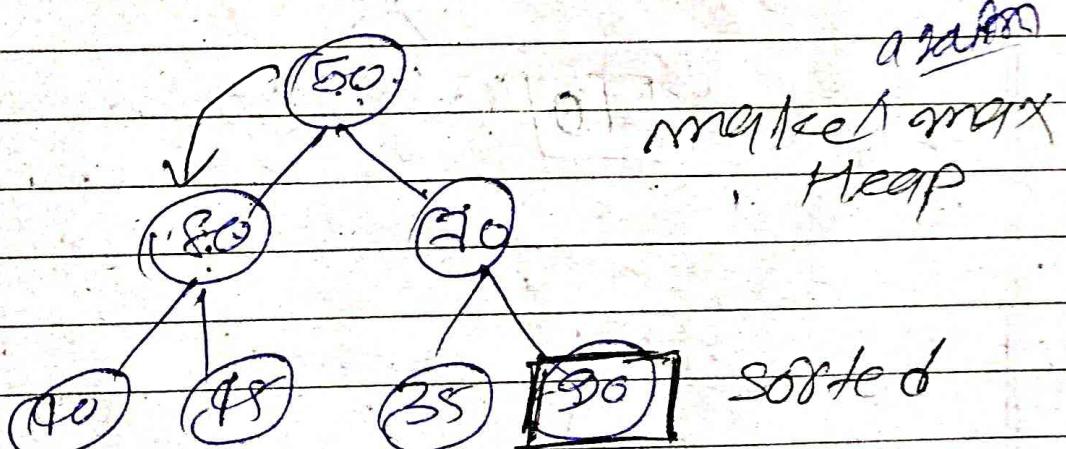
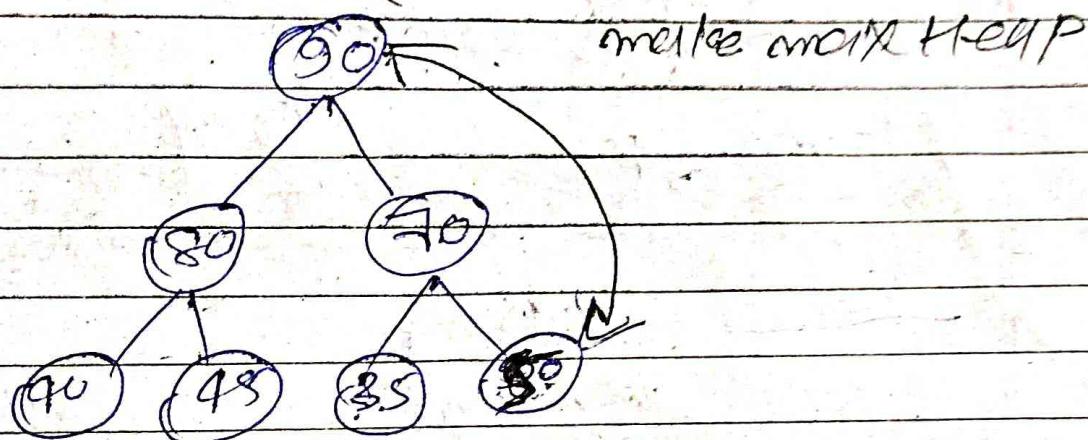
90
80 35
45 50

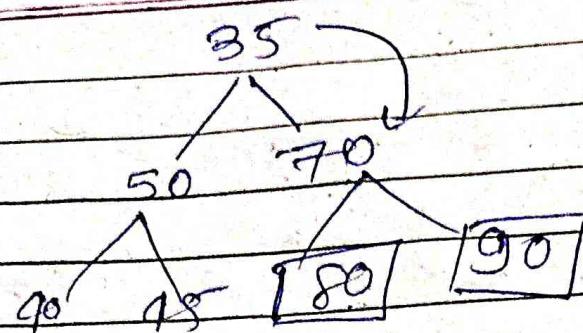
(k)

90
80 45
90 45
35

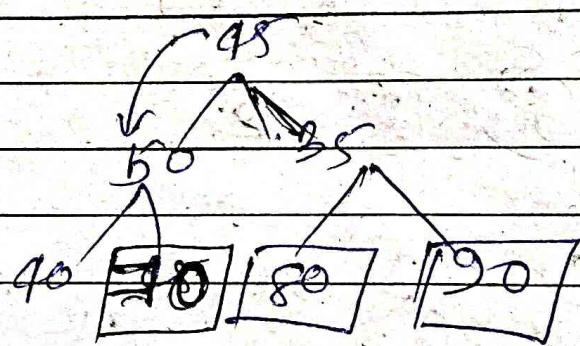
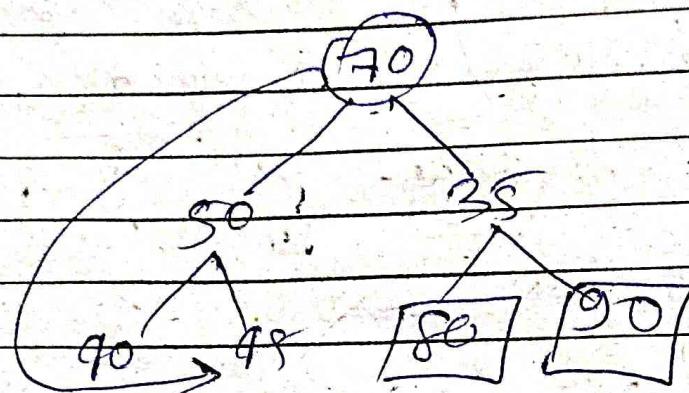


Phase 2) Repeat Deleting Root & swap with last element

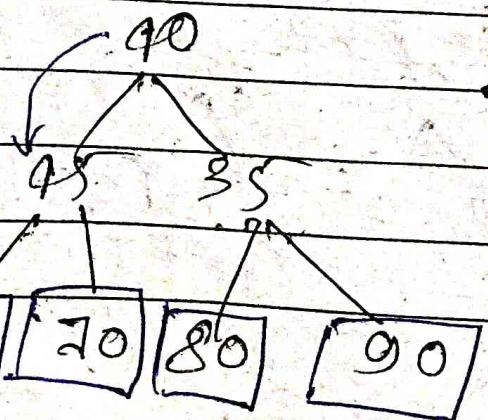
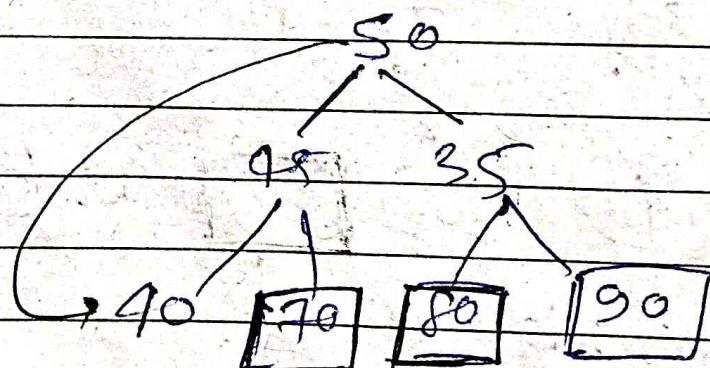




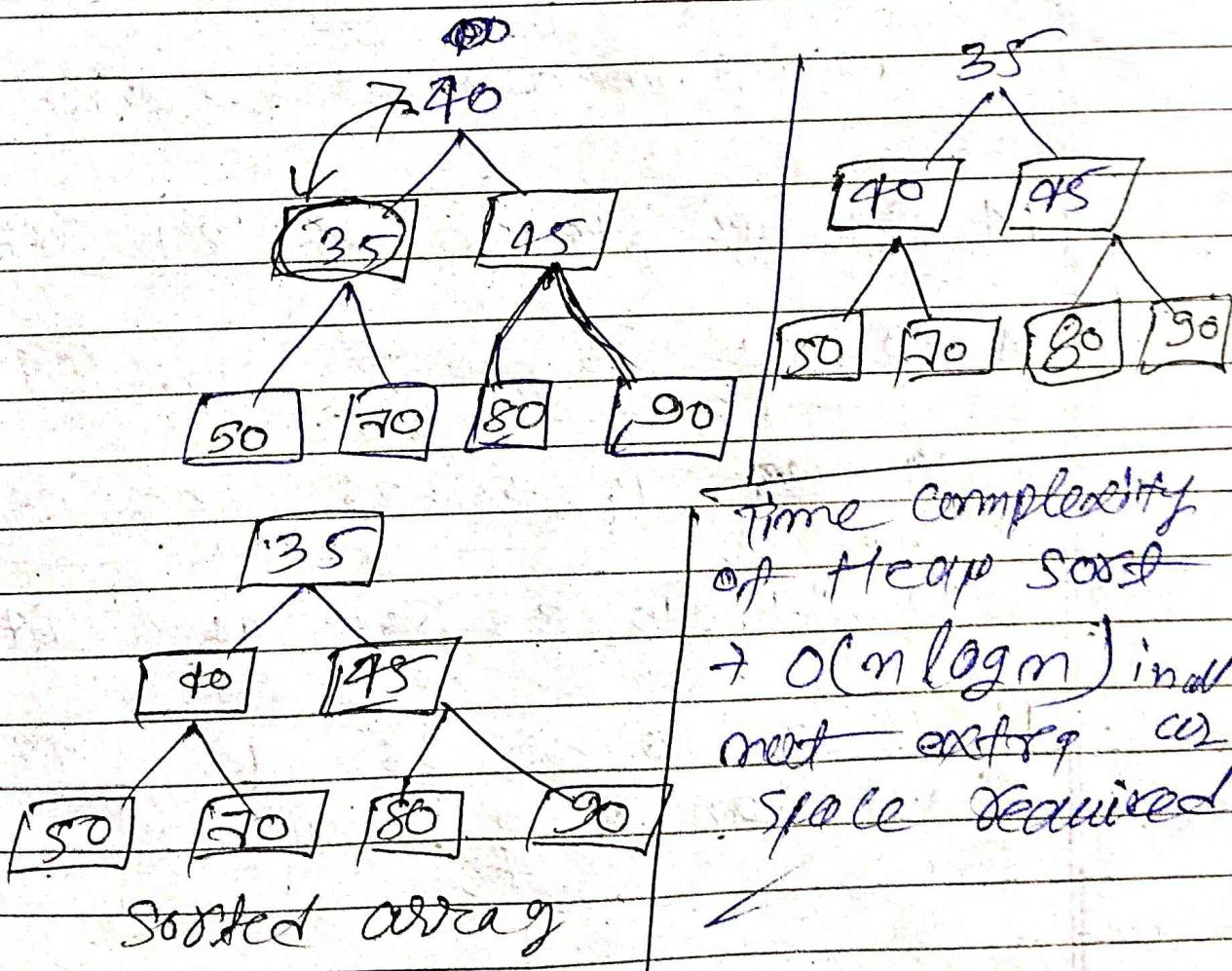
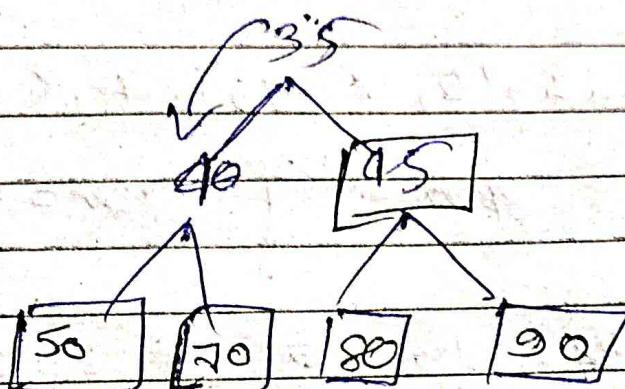
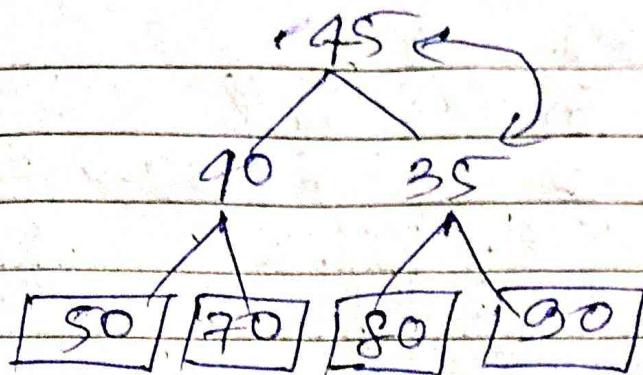
make
max heap
again



make max
heap
again



make max
heap
again



(7)

Shell sort :

① By Donald Shell

choice of increment $a^{\frac{d}{2}}$

$$\left[\frac{N}{2} \right], \left[\frac{N}{4} \right], \left[\frac{N}{8} \right], \dots, \frac{1}{1}$$

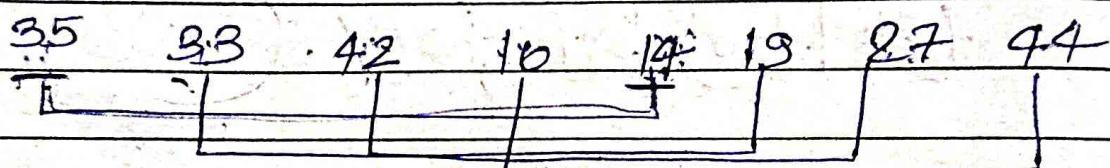
No of elements.

①

35, 33, 42, 10, 14, 19, 27, 44

$$N = 8$$

$$\text{Span} = \left\lceil \frac{8}{2} \right\rceil \quad \text{Span} = 4$$



Swapping the elements if required.

14 33 42 10 35 19 27 44

Pass 1

14 19 42 10 35 33 27 44

14 19 27 10 35 33 42 44
no swap

14 19 27 10 35 33 42 44

14 19 27 10 35 33 42 44

$$\text{Next Span} = \frac{8}{4} = 2$$

14	19	27	10	35	33	42	44
14	19	27	10	35	33	42	44
14	10	27	19	35	33	42	44
14	10	27	19	35	33	44	44
14	19	27	10	35	33	42	44

Performing insertion sort on

→ 14 27 35 42

→ 19 10 33 44

→ [14] 27 35 42

sorted → [14] 27 35 42

[14] 27 35 42

sorted → [14] 27 35 42

[14] 27 35 42

sorted → [14] 27 35 42

[14] 27 35 42

→ [19] 10 33 44

[19] 10 33 44

→ [19] 10 33 44

[19] 10 33 44

→ [19] 10 33 44

[19] 10 33 44

→ [19] 10 33 44

80 Sorted Result of pass 2

14 10 27 19 35 33 42 44

Pass-3: Span = $\frac{8}{8} = 1$

14 10 27 19 35 33 42 44
1 1 1 1 1 1 1 1

Performing insertion sort on the elements

14 10 27 19 35 33 42 44

$\rightarrow 1$

14 10 27 19 35 33 42 44

~~→ 1~~

10 14 27 19 35 33 42 44

$\rightarrow 1$

10 14 27 19 35 33 42 44

$\rightarrow 1$

10 14 19 27 35 33 42 44

$\rightarrow 1$

10 14 19 27 35 33 42 44

10 14 19 27 35 33 42 44

10 14 19 27 35 33 42 44

10 14 19 27 35 33 42 44

Time Complexity of shell sort is

$$O(m(\log n)^e)$$

Practical Q : Shell sort

54 26 93 17 77 31 44 55 20

$$N = 9$$

$$\text{Span} = \frac{9}{2} = 4$$

54 26 93 17 77 31 44 55 20

No swap

swaps

54 26 93 17 20 31 44 55 77

↑ A

No swap

NOSWAP

54 26 44 17 20 31 93 55 77

↑

No SWAP

54 26 44 17 20 31 93 55 77

↑

No SWAP

$$\text{Next Span} = \frac{9}{2} = 4$$

54 26 44 17 20 31 93 55 77

Insertion on

54 20 93 77

26 17 31 55



77

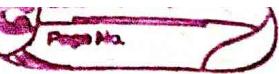
54 26 44 20 93 77

sorted

44 26 20 93 77

sorted

44 26 20 93 77



20 44 54 93 77

Sorted

20 44 54 93 77

20 44 - 54 93 77
Sorted

20 44 54 93 77

20 44 54 - 77 93
Sorted

26 17 31 55

Sorted

26 17 31 55

17 26 31 55

→

17 26 31 55

17 26 31 55

Sorted

20 17 44 26 54 31 77 55 93

$$\text{MCRT Span} = \frac{9}{8} = 1$$

20 17 44 26 54 31 77 55 93

20 17 44 26 54 31 77 55 93
 5 →

20 17 44 26 54 31 77 55 93
 17 20 → 44 26 54 31 77 55 93

17 20 44 26 54 31 77 55 93
 →

17 20 44 26 54 31 77 55 93

17 20 26 44 54 31 77 55 93
 sorted →

17 20 26 44 84 31 77 55 93
 →

17 20 26 40 84 31 77 55 93

17 20 26 31 40 54 77 55 93
 →

17 20 26 31 40 54 77 55 93
 →

17 20 26 31 40 84 77 55 93

17 20 26 31 40 54 55 77 93
 →

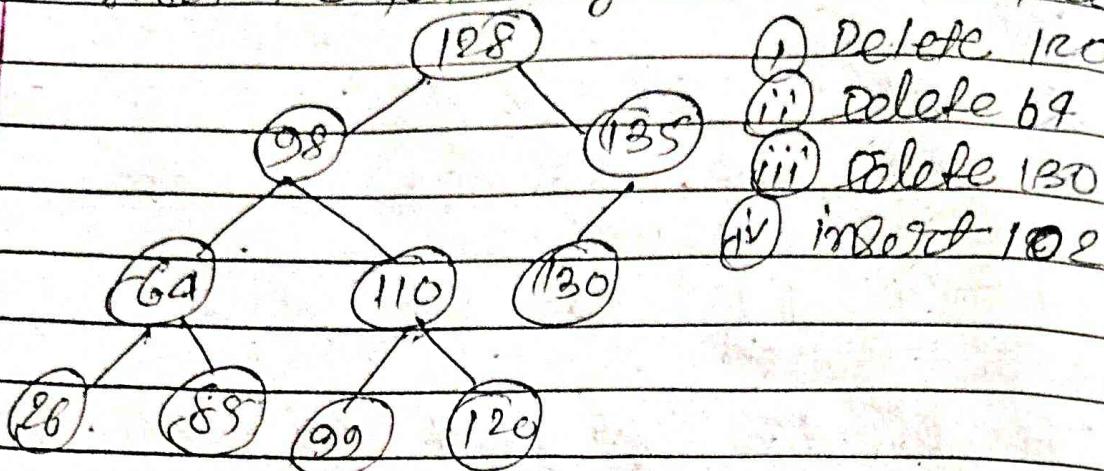
17 20 26 31 40 54 55 77 93
 →

sorted

Time Complexity $O(n \log n)$

(Q3)

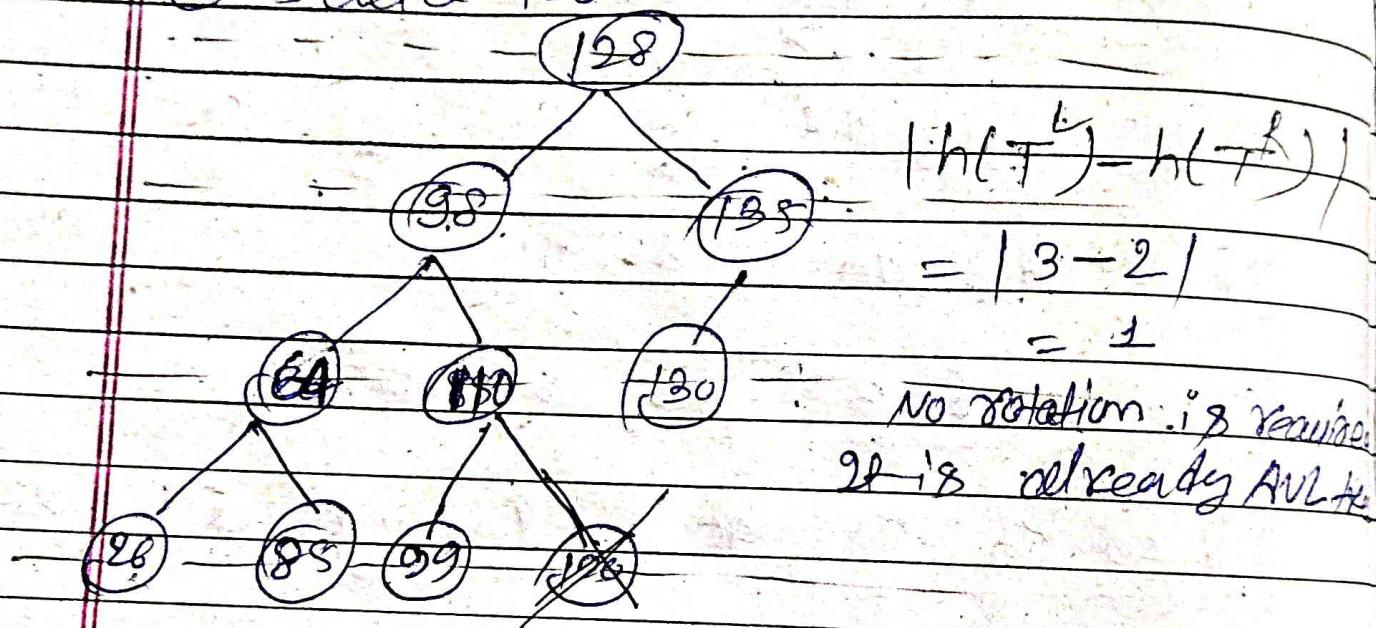
Consider the following AVL Search tree



- i) Delete 120
- ii) Delete 64
- iii) Delete 130
- iv) insert 102

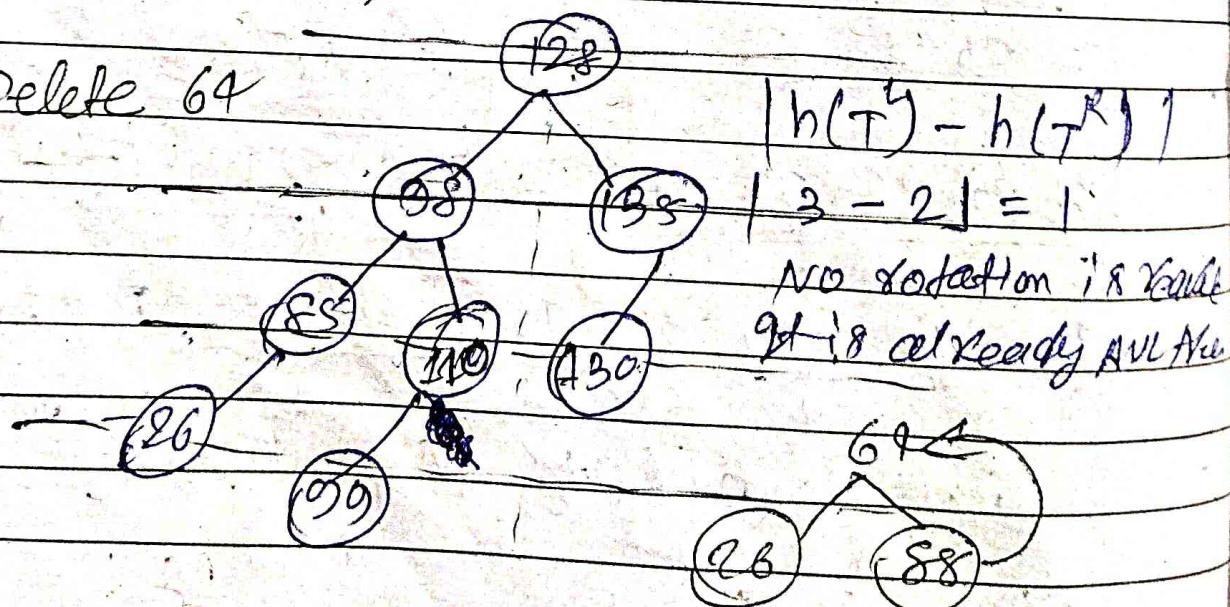
Ans =

① Delete 120



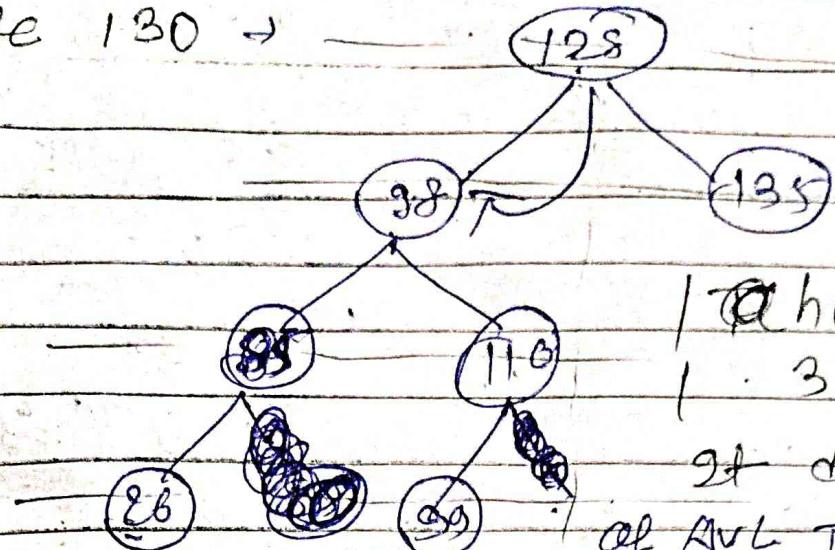
②

Delete 64



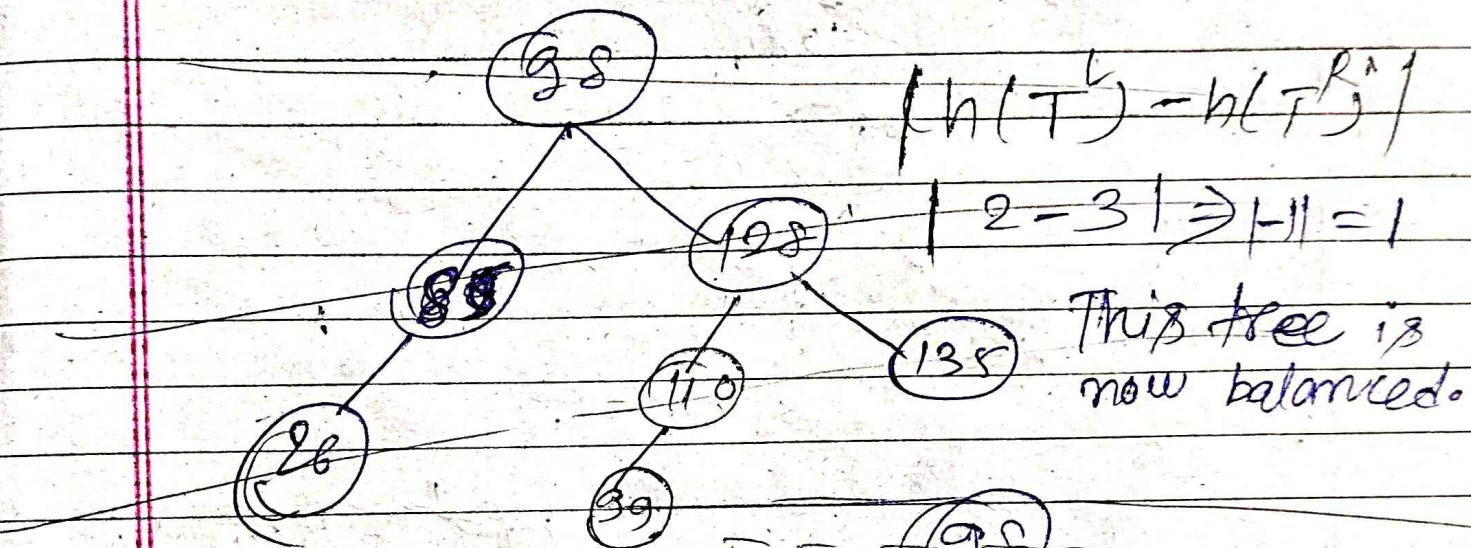
③

Delete 130 →



$$\begin{aligned} & |h(T^L) - h(T^R)| \\ & |3 - 1| = 2 \end{aligned}$$

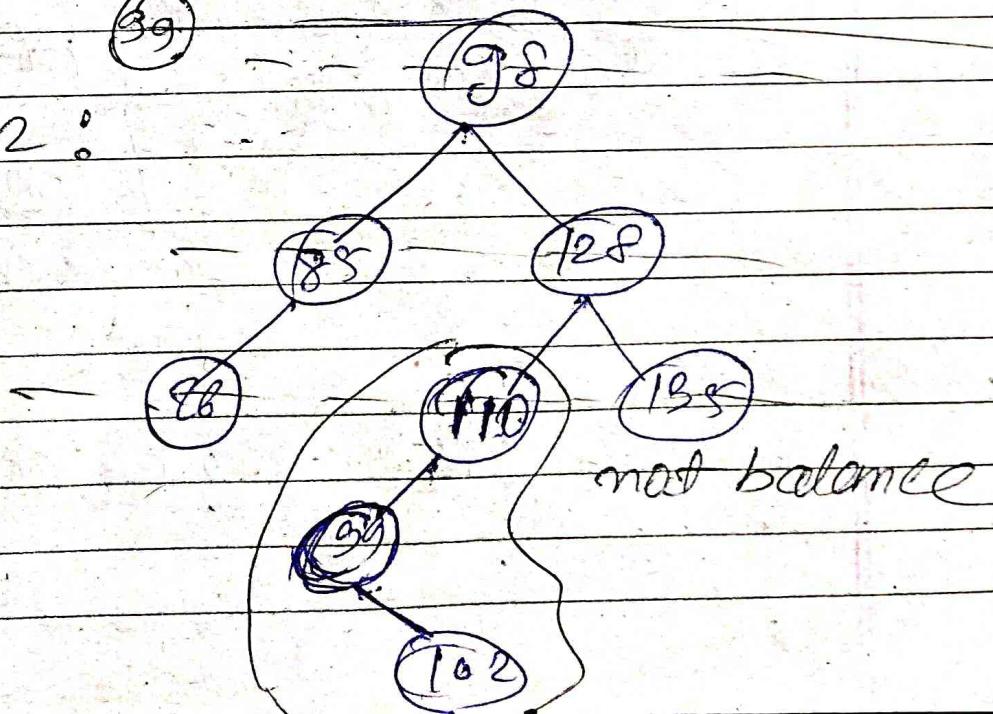
It does not remain
of AVL tree defined
to perform right
rotation to balance
the tree.



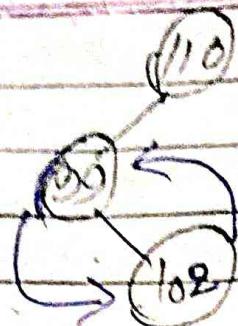
$$\begin{aligned} & |h(T^L) - h(T^R)| \\ & |2 - 3| \geq |1| = 1 \end{aligned}$$

This tree is
now balanced.

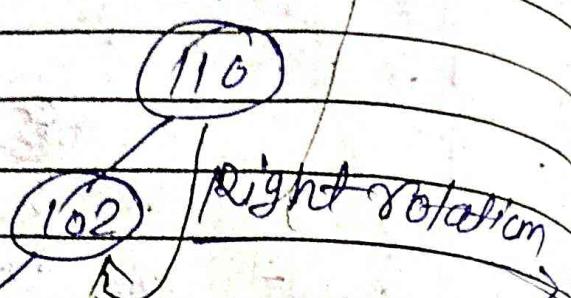
~~for~~ insert 102 :



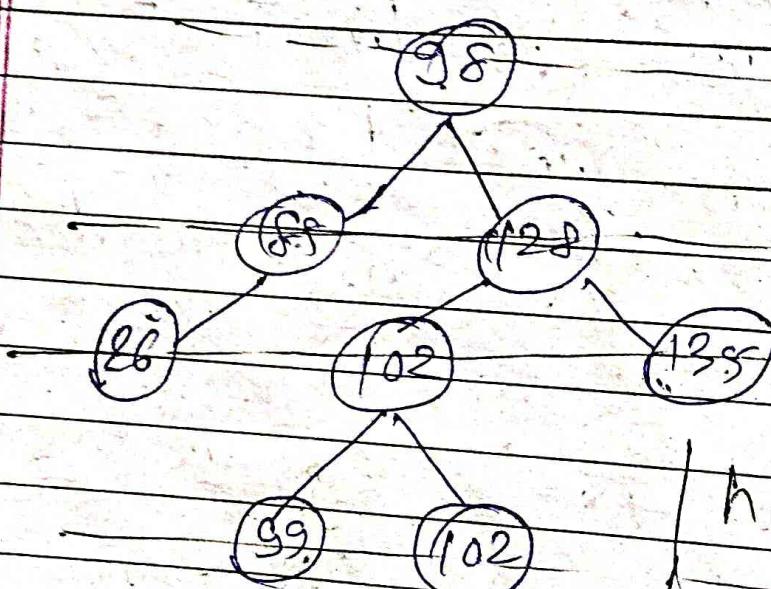
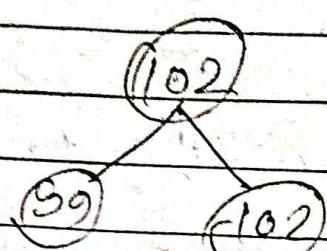
not balanced



Left rotation



Right rotation



$$h(T^L) - h(T^R)$$

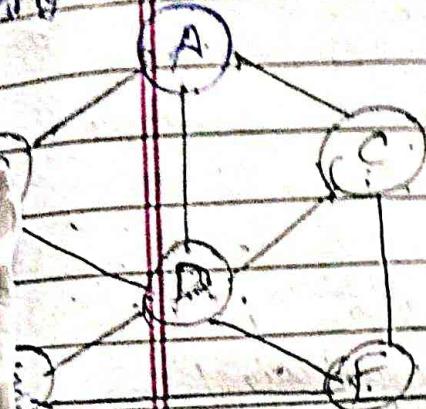
$$|2 - 3| = |-1| = 1$$

This is a balanced tree on AVL tree

Practice more from maam's pdf

X Traverse the following graph using BFS starting from node B.

ans =



	A	B	C	D	E	F
A	0	1	1	1	0	0
B	1	0	0	1	1	0
C	1	0	0	1	0	1
D	1	1	1	0	1	1
E	0	1	0	1	0	1
F	0	0	1	1	1	0

Q1

B

EDA

DAF

ADF

AFG

FC

C

(PQ) Illustrate the use of adjacency matrix and list to represent a graph in memory

Sequential representation

(i) Adjacency Matrix

	1	2	3	4	5	6
1	0	1	0	0	0	0
2	0	0	0	1	0	0
3	1	1	0	0	1	0
4	0	0	0	0	0	1
5	0	0	0	1	0	1
6	0	0	0	0	0	0

The adjacency matrix uses more space but allows for constant time to check whether edge exists b/w two vertices.

output

Ø

BE

BED

BEDA

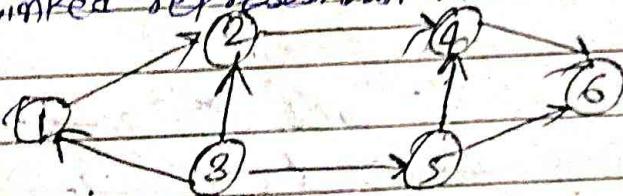
~~BEDAF~~

BEDAAFF

BEDAAFC

(ii) ~~BEDAFC~~

linked representation



(ii) Adjacency List

Node	Adjacency List	Notes
1	2	The Adj. list is more
2	4	space efficient
3	1, 2, 5	but requires
4	6	more time for
5	4, 6	check whether edge
6	•	exists b/w vertices

(PYS)

specify the situation(s) when operating system performing the garbage collection mention the steps involved in performing collection.

Ans \Rightarrow operating system perform garbage collection in the following situation.

- when a process terminates : when a process terminates, the operating system ~~reclaims/collect~~ all the memory that was allocated to that process.
- when a program requests more memory : if a program requests more memory than the operating system may perform garbage collection.

steps involve in performing collection

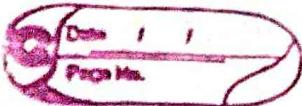
Step 1 \Rightarrow GC will sequentially visit all nodes in memory and mark all nodes which are being used in program.

Step 2 \Rightarrow It will collect all unmarked nodes and place them in free storage area.

(PYS)

what is adjacency list?

Ans \Rightarrow An adjacency list is a data structure used to represent a graph where each node in the graph stores a list of its neighboring vertices.

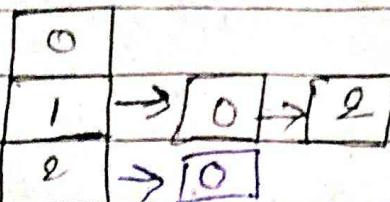


(Q) Ex:

① → ② Array

③

→

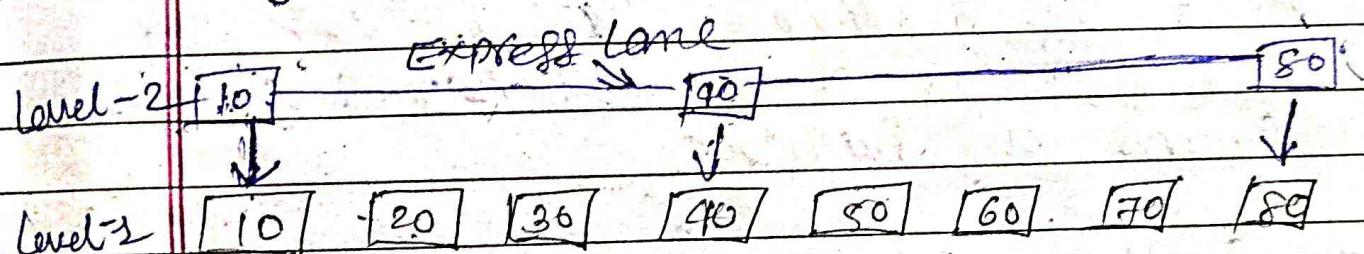


↓

(P) what do you mean by skip list?

(Q)

A skip list is a data structure that allows for efficient search, insertion and deletion of elements in a sorted list. It skips over many of items in the full list in one step, that's why it is called skip list.



(P) why balancing is important in Binary search tree?

- Faster search time

- More efficient for insertion/deletion.

- Reduced memory usage.

- Time complexity $O(n \log_2 n)$

(Q)

How rebalancing is done?

(Q) Step 1: Create a new hash table with twice the no of buckets of the old table.

Step 2: Iterate over the elements in the old table and add them to the new table using their new hash values.

Step 3: Discard the old table.

NAME: RAUSHAN
m-
rehashing is the process of increasing the size of a hashmap and redistributing the elements to new buckets.

* what is rehashing & when is rehashing
Ans: Rehashing is a technique used in hash tables to improve performance as the number of elements in the table increases.

* when rehashing is done:

Rehashing is done when the load factor exceeds a certain threshold.

$$\text{load factor} = \frac{\text{no. of elements in the table}}{\text{no. of buckets}}$$

* Need of hashing: hashing is needed to index and retrieve information from a database.

(Q) Records: P Q R A B C V W X
H(R): 5 4 5 6 8 11 11 1 4

Suppose the records are entered into the table T in the above order.

(a) Examine the efficiency of the given hash function with linear probing as the collision resolution technique.

(b) formulate the memory organization if the records are stored using chaining.

B Q

W P R

A B X C

mloc: 1 2 3 4 5 6 7 8 9 10 11

Step: 1 $n = 9$, $m = 11$

load factor $d = \frac{n}{m}$ $d = \frac{9}{11} [d = 0.81]$

Step: 2 $S(d) = ?$ $U(d) = ?$

$$S(d) = \frac{1}{2} \left[1 + \frac{1}{(1-d)} \right], U(d) = \frac{1}{2} \left[1 + \frac{1}{(1-d)^2} \right]$$

$$S(d) = \frac{1}{2} \left[1 + \frac{1}{1-0.81} \right], U(d) = \frac{1}{2} \left[1 + \frac{1}{(1-0.81)^2} \right]$$

$$[S(d) = 3.13] \quad [U(d) = 14.35]$$

$$P Q R A B C V W X$$

$$S = 1 + 1 + 2 + 2 + 1 + 1 + 2 + 2 + 6$$

$$m(9) \qquad \qquad \qquad S \qquad \qquad S(d)$$

$$S = \frac{18}{9} \quad [S = 2] \quad 2 < 3.13$$

efficient

$$V = 1 2 3 4 5 6 7 8 9 10 11$$

$$3 + 2 + 1 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 4$$

$m(11)$

$$V = \frac{38}{11}$$

$$[V = 3.45]$$

V

$U(d)$

$$3.45 < 14.35 \quad \text{efficient}$$

(b)

link

1	8
2	0.
3	0.
4	8.9
5	7.3
6	4
7	0
8	5
9	0
10	0
11	7

Info link

1	P	0
2	Q	0
3	R	1
4	A	0
5	B	0
6	C	0
7	V	6
8	W	0
9	X	2
10	"	"
11	"	"

Efficiency

efficiency

 $m=9, m=11$

load factor $d = \frac{m}{m} = \frac{9}{11} = \frac{9}{11} \quad [d = 0.81]$

$$S(d) = 1 + \frac{1}{2}d \quad [U(d) = \bar{e} + d]$$

$$S(d) = 1 + \frac{1}{2} \times 0.81, \quad U(d) = 0.4459 + 0.81 \\ [U(d) = 1.259]$$

$$S(d) = 1 + 0.40$$

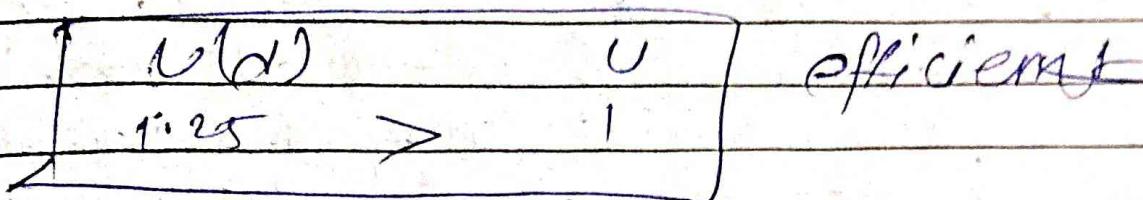
$$[S(d) = 1.40]$$

$$S = P + Q + R + A + B + C + V + W + X \\ e + 9 + 1 + 1 + 1 + 8 + 1 + 1 + 1 + 1 \\ n(9)$$

$$S = \frac{12}{9} \quad [S = 1.33] \quad [S(d) = S \\ 1.40 > 1.33 \\ \text{efficient}]$$

$$U = \frac{1+1+1+1+1+1+1+1+1+1}{m(11)}$$

$$U = \frac{11}{11} \quad [U=1]$$



(Q1) List the areas of applications of data structure.

- Ans \Rightarrow
- Databases : data storage and retrieval.
 - operating system
 - compiler design
 - Artificial intelligence
 - Game development
 - Machine learning
 - Blockchain

(Q2) with the help of code snippet explain complexity analysis.

Ans \Rightarrow

```

int main()
{
    int i, n=8;
    for (i=1; i<=n; i++)
        cout << "Hello" << endl;
    return 0;
}
  
```

\therefore The time complexity of above code is $O(n)$
 because hello is printed n times on the screen.
 Auxiliary space: $O(1)$

You can give one more example ~~acum~~ your own
 in 4 marks.

(Q4)

which data structure is ideal to perform recursion operation and why?

Ans) Stack data structure is ideal for perform recursion operation because it allow for easily ~~remove~~ main-frame functions calls and returns. Each function call is pushed onto stack and when a function completes it is popped off the stack.

(Q5)

why it is said that searching a node in a binary search tree is efficient than that of a simple binary tree?

Ans) Searching a node in a binary search tree (BST) is more efficient than a simple binary tree because a BST has a specific property: for each node, all in its left subtree have values less than the node, and all nodes in its right subtree have values greater than the node. This property allows for a more efficient search.

(Q6)

Discuss how circular queue overcomes limitations over linear queue with example:

Ans) A circular queue overcomes limitations over linear queue by addressing the problem of wasted space at the front of the queue. In a linear queue, when elements are dequeued the front space becomes unusable.

~~Front~~ ~~Rear~~

POD example:

Consider both linear and circular queue of size 6 elements

0	1	2	3	4	5
29	21	72	15	54	24

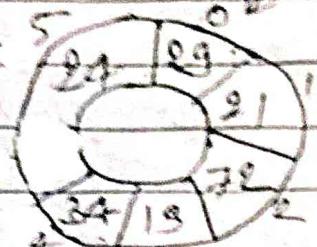
↑
Front

↑
Rear

linear queue

Rear

Front



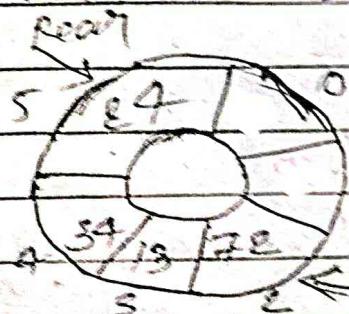
circular queue

- When the dequeue operation is performed on the both the queues: consider the first 2 elements that are deleted from both the queues.

0	1	2	3	4	5
		72	15	54	24

↑
front

↑
Rear



0

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

18

19

20

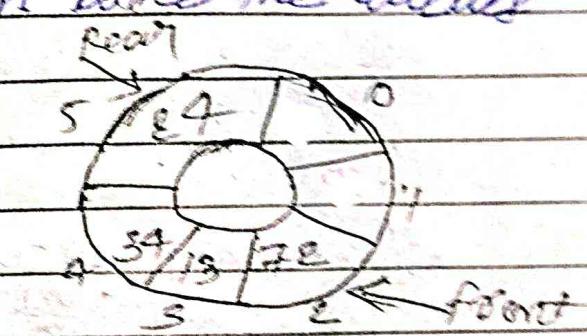
21

22

23

24

25



Rear

0

1

2

3

4

5

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- Now enqueue operation is performed: consider an element with a value of 100, the insertion of element 100 in linear queue is not possible but in circular queue rear will be gone at the 0-th index and value will be inserted.

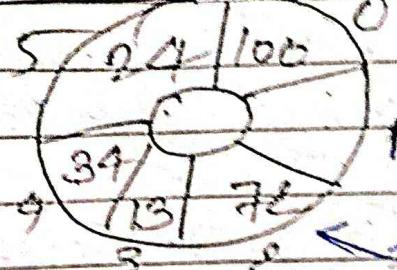
0	1	2	3	4	5
		72	15	54	24

↑

Front

↑

X



0

1

2

3

4

5

6

7

8

9

10

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12

13

14

15

16

17

18

19

20

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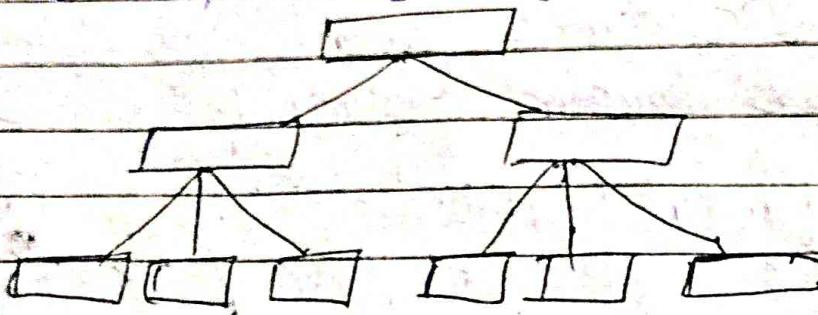
(Q3)

write a brief note on B-Trees.

B-tree also called m-way tree. It is a self-balanced tree in which every node contains multiple keys and has ~~more than~~ at least two children and at the most m children.

B-Tree of order m has the following properties

- All the leaf nodes must be at same level.



- All nodes except root node must have at least $\lceil \frac{m}{2} \rceil - 1$ keys and at the most $(m-1)$ keys.

- All the key value in a node must be in ~~order~~ ascending order.

- B-Tree have at least 2 child nodes and at the most m child nodes.

- Insertion of a node in B-Tree happens only at leaf Node.

- Time Complexity of B-Tree in
[Searching] → O(log n)
• Inserting
• Deleting

(Ans)

What is hashing, need of hashing?
 Discuss the various hash functions.

Ans → Hashing is the process of generating a fixed-size output from a variable size input using mathematical formulas known as hash functions.

Need of hashing:

- Data retrieval for index
 - retrieve information from a database
- hash functions

There are many hash functions

(1) Division Method

(2) Mid square Method

(3) folding Method

(i) Division method: In division method hash function divides the value k by m and then uses the remainder obtained as

hash value

$$\begin{array}{r} K = 9699 \\ \times 97 \\ \hline 96990 \\ - 9699 \\ \hline 0 \end{array}$$

97 → Not prime
 98 → Not prime
 97 → prime

$$K = 1276 \quad m = 11$$

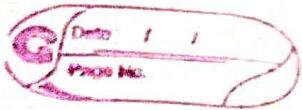
$$h(1276) = 1276 \bmod 11$$

$$1276 \bmod 11 = 0$$

(ii) mid square method: square the value of the key $\rightarrow k^2$ and extract the middle

2 digits as the hash value.

$$\text{Ex} \rightarrow K = 60, k^2 = 3600 \Rightarrow 60$$



(iii)

folding method: It involve two methods

(i)

without reverse

(ii)

with reverse

Ex →

~~Ex~~

$$32/05 \quad 32 + 05 = 37$$

(ii)

Ex → with reverse: $32/05 \rightarrow 32/50 \rightarrow$
 $32 + 50 = 82$

Ex →

~~Ex~~

7148

$$71 + 48 = 119 = 19$$

Ex → (i)

$$71 + 84 = 155 = 55$$

addressing

* *

open addressing of a technique to resolve
 collision in hashing.

Ex →

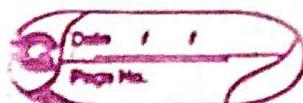
In linear probing, the table is searched sequentially that starts from the original location of the hash. If in case the location that we get is already occupied, then we check for the next location.

Ex →

~~Ex~~

Index

values



$$H(k) = S \% N = 0$$

Linear probing: $h, (h+1)\%N, (h+2)\%N, \dots$

Ex →	A	B	C	D	E	X	Y	Z
	4	8	2	11	4	11	5	1
Mloc →	X	C	Z	A	E	Y	B	D
	1	8	3	4	5	6	7	10

for E $\Rightarrow (h+1)\%N \Rightarrow (4+1)\%11 = 5\%11 = 5$

for X $\Rightarrow (h+1)\%N \Rightarrow (11+1)\%11 = 12\%11 = 1$

for Y $\Rightarrow (h+1)\%N \Rightarrow (5+1)\%11 = 6\%11 = 6$

for Z $\Rightarrow (h+1)\%N \Rightarrow (1+1)\%11 = 2\%11 = 2$

for A $\Rightarrow (h+2)\%N \Rightarrow (1+2)\%11 = 3\%11 = 3$

(Q2) Describe quadratic probing as a technique to resolve collision in hashing

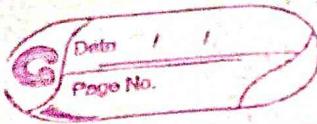
Ans → Quadratic probing is a collision resolution technique in hash tables, when a collision occurs, quadratic probing searches for the next slot by using a quadratic function.

$$H(k) = h, (h+1^2)\%N, (h+2^2)\%N, (h+3^2)\%N, \dots$$

Ex →	A	B	C	D	E	X	Y	Z
	4	8	2	11	4	11	5	1

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Mloc	1	2	C	3	4	A	E	Y	B	Z	D
										10	11

$$\text{for } E \rightarrow (h+1^2) \% N = (4+1^2) \% 11 \\ = 5 \% 11 = 5$$

$$\text{for } X \rightarrow (h+1^2) \% N = (11+1) \% 11 = 12 \% 11 = 1$$

$$\text{for } Y \rightarrow (h+1^2) \% N = (5+1) \% 11 = 6 \% 11 = 6$$

$$\text{for } Z \rightarrow (h+1^2) \% N = (1+1) \% 11 = 2 \% 11 = 2 \quad X$$

$$\text{again } (h+2^2) \% N = (7+4) \% 11 = 5 \% 11 = 5 \quad X$$

$$(h+3^2) \% N = (1+9) \% 11 = 10 \% 11 = 10$$

* double hashing: In double hashing two hash functions are used the formula of double hashing is

$$h(k) = h, (h+h') \% N, (h+2h') \% N, \dots$$