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# EXERCISE #3 - Distributions

1. **What is a distribution?**

Ans: A probability distribution is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range. This range will be bounded between the minimum and maximum possible values, but precisely where the possible value is likely to be plotted on the probability distribution depends on a number of factors. These factors include the distribution's mean (average), standard deviation, skewness, and kurtosis.

2. **What is the difference between a discrete and a continuous distribution?**

Ans: A discrete distribution describes the probability of occurrence of each value of a discrete random variable. A discrete random variable is a random variable that has countable values, such as a list of non-negative integers.

A continuous distribution describes the probabilities of the possible values of a continuous random variable. A continuous random variable is a random variable with a set of possible values (known as the range) that is infinite and uncountable.

3. **Fill in the graph below to show the probability mass function of the outcomes of a fair die. What is the name of this type of distribution where the probability of each event is the same?**

Ans:
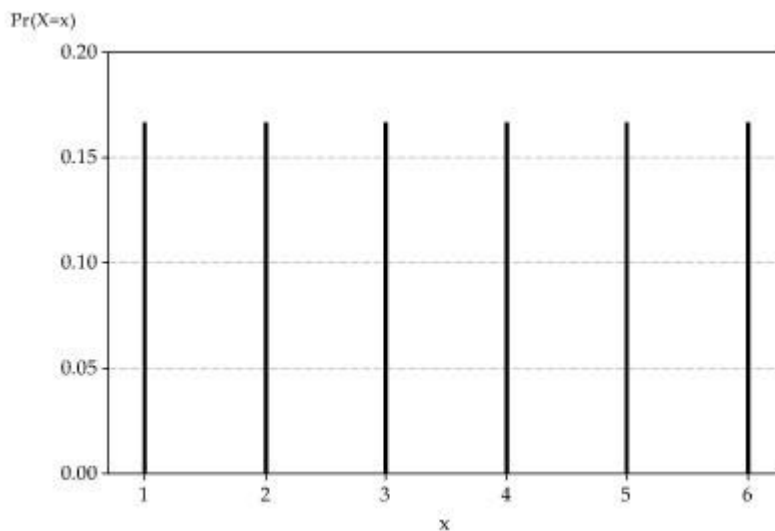


4. **In the above graph, does changing the number of trials change the appearance of the graph? Why or why not?**

Ans: No the graph will not change appearance because probability of each number stays same i.e
1/6

BINOMIAL DISTRIBUTIONS

5. **If you flip a fair coin five times, what is the probability that heads come up twice?**

Ans:

$$P(x:n,p) = \left(\frac{n!}{x!(n-x)!}\right)(p)^x(1-p)^{(n-x)} \quad x=\underline{\quad 2 \quad} \quad n=\underline{\quad 5 \quad} \quad p=\underline{10/32}$$

6. **What is the difference between a Binomial Distribution and a Poisson Distribution?**

Ans: Binomial distribution describes the distribution of binary data from a finite sample. Thus, it gives the probability of getting r events out of n trials. Poisson distribution describes the distribution of binary data from an infinite sample. Thus, it gives the probability of getting r events in a population.

7. **On average, five hurricanes form in the Atlantic Ocean each year. Given this, what is the probability that 7 hurricanes form next year?**

Ans:

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad e = 2.71828 \quad x=\underline{\quad 7 \quad} \quad \lambda=\underline{\quad 5 \quad}$$

$P(x) = 5^7.e^{-5} / 7! = 0.1044$

NORMAL DISTRIBUTIONS

### 8. In a Normal Distribution, what is meant by the "68-95-99.7 Rule"?

Ans: A normal distribution, sometimes called the bell curve, is a distribution that occurs naturally in many situations. For example, the bell curve is seen in tests like the SAT and GRE. The bulk of students will score the average (C), while smaller numbers of students will score a B or D. An even smaller percentage of students score an F or an A. This creates a distribution that resembles a bell (hence the nickname). The bell curve is symmetrical. Half of the data will fall to the left of the mean; half will fall to the right.

The empirical rule, also referred to as the three-sigma rule or 68-95-99.7 rule, is a statistical rule which states that for a normal distribution, almost all observed data will fall within three standard deviations (denoted by σ) of the mean or average (denoted by μ).

In particular, the empirical rule predicts that 68% of observations falls within the first standard deviation (μ ± σ), 95% within the first two standard deviations (μ ± 2σ), and 99.7% within the first three standard deviations (μ ± 3σ).

### 9. The graph below shows a Standard Normal Distribution curve. Shade in the area that represents 95.45% of values (that is, the area inside of 2 standard deviations).
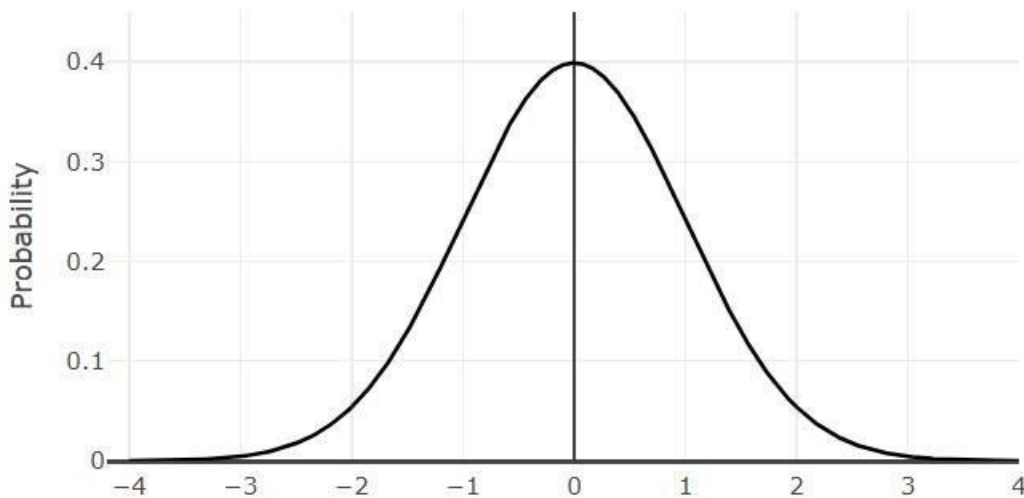


Ans:

### 10. The IQ Test is designed to have a mean score of 100 with a standard deviation of 15 points. A score above 140 is considered to be genius level. What is the calculated z-score for an IQ of 140?

Ans: Standard Deviation = 15 points

Mean = 100 points

Z score for $140 = (X - Mean) / S.D = (140 - 100) / 15 = 2.67$

11. **Into what percentile does this put people who have an IQ of 140?**

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|------|------|------|------|------|------|------|------|------|------|
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |

Ans: Since our Z score is 2.67 then our percentile is z [2.6] [0.07] = 0.9962

Therefore, people with 140 IQ will be put in the 99.62 percentile.